Stock Crash and $R^2$ around a Catastrophic Event: Evidence from the Great East Japan Earthquake

**Abstract:** We investigate the effects of opacity on stock price synchronicity, and frequency and severity of a stock crash. We develop a simple analytical model for valuing stocks in the presence of management earnings forecasts and market-wide information. Stock price synchronicity is predicted to increase with opacity. Our model also reveals that stock crashes are more frequent and more severe for opaque firms. These predictions are confirmed empirically. Further, our model predicts that after a catastrophic event, stock price becomes synchronous with the market and the frequency and severity of a stock crash increase because of severely limited investor attention. We use the Great East Japan Earthquake as a representative catastrophe to examine these implications, and provide support for the model. Finally, we find that the stock price of a firm disclosing opaque information tends to covariate with the market, and hence suffers more serious damage from the earthquake, which is consistent with the model.

**Keywords:** Financial reporting opaqueness, Stock price synchronicity, Stock crash, the Great East Japan Earthquake, Limited attention

**JEL classification:** G12, G14, M41
1 Introduction

We examine the effects of opacity to investors, in the sense that they cannot understand what a firm intrinsic value really is, on stock price synchronicity and crash risk. Since Roll (1988) found that stock prices mostly do not co-move with the whole market and argued that informed traders acting on private information play an important role, there has been considerable research concerning the determination of stock price synchronicity. Morck, Yeung and Yu (2000) show that stock returns are more synchronous in poor economies than in rich economies. They state that the presence of public investor property rights causes this difference.

Jin and Myers (2006) argue that it is not enough that the reason for high $R^2$s can be attributed to just poor protection of investors. They investigate the relation between $R^2$ and opaqueness (lack of transparency) for 40 stock markets from 1990 to 2001 and find theoretical and empirical evidence that more opaque markets have higher $R^2$s, leading to their being more likely to crash. Based on a model by Jin and Myers (2006), Hutton, Marcus and Tehranian (2009) test the firm-level relation between $R^2$ and opaqueness using the U.S. firms from 1991 to 2005. Using abnormal accruals (i.e., earnings management) as a proxy for opacity, they find that more opaque firms have higher $R^2$s and tend to experience stock crashes. This result suggests that $R^2$s and crash risks are driven by accounting opaqueness.

Researchers in accounting and finance have used various kinds of measures to indicate accounting opaqueness. For example, Bhattacharya, Daouk and Welker (2003) use earnings aggressiveness (i.e., the opposite concept of accounting conservatism), loss avoidance, and earnings smoothness as proxies for differences in accounting opacity and explain the association between accounting opacity and cost of capital and trading volume world-wide. Francis, LaFond, Olsson and Schipper (2004) examine the link between the cost of equity capital and accounting opacity by using the following earnings attributes: accruals quality proposed by Dechow and Dichev (2002), persistent, predictability, smoothness, value relevance, timeliness, and conservatism. Thus, empirical research so far has basically used reported earnings to quantify accounting opacity 1.

Although we also use reported earnings-based opacity measures and examine the relation between opacity, $R^2$, and crash risk, those measures have potential weaknesses. First, as Hutton et al. (2009) point out, the relation between discretionary accruals and both $R^2$ and crash risk essentially disappears in the post-Sarbanes-Oxley Act (SOX) years. They interpret this result as

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indicating that earnings management has substantially subsided due to greater monitoring and scrutiny of accounting practices after SOX, and then, earnings-based opacity measures could no longer be used as a proxy for opaqueness after SOX. Because many developed countries including Japan have recently crafted regulations similar to SOX, we need to employ an alternative measure of opaqueness to replace it.

Japanese listed firms have a distinctive financial reporting system in that they report actual earnings for each year. In addition, almost all of them release point-estimate of management earnings forecasts for each following year. Utilizing this unique setting, we use the precision of management earnings forecasts as the opacity measure. It is reasonable that regardless of whether the causes of less precise earnings forecasts are intentional biases or major changes in the business environment, firms reporting less precise earnings forecasts are regarded as being more opaque by investors, who form their expectations on the earnings forecasts, especially in uncertain situations.

We first develop a simple approach to valuing stocks in the presence of management earnings forecasts and market-wide information under “usual” conditions. Our model predicts that (1) stock crashes are likely to be observed in firms whose management reports opaque forecasts and (2) stock price synchronicity increases with management-forecast-based opacity. These predictions are confirmed empirically.

We next develop a model of describing investor attention and allocation under “unusual” conditions in which investor attention is severely limited due to an unexpected catastrophic shock, and provide predictions about the impact of the limited attention on stock return co-movement and individual stock crashes. This model implies that after a catastrophe, (3) stock prices become more synchronous and (4) crash risks become higher in the market. In addition, our model predicts that (5) opaque firms are more prone to stock price collapse around an unexpected catastrophe. We use the Great East Japan Earthquake (GEJE) as a representative catastrophe to examine these implications from (3) to (5), and provide evidence that is consistent with our model.

This study contributes to the extant literature in several ways. First, while Jin and Myers (2006) developed an analytical model that explains that opaqueness increases stock price synchronicity by shifting firm-specific risk to managers, they show the relation between firm-level opaqueness and stock crashes anecdotally. They nearly describe that a greater frequency of large, negative, firm-specific returns occurs in countries where firms are more opaque to investors. We developed a simple model that explains the relation between firm-level opaqueness and its stock
crash risk, and then, show empirical evidence that the more opaque the firms, the more prone they are to crash.

Second, recent years have seen an increase in studies investigating the determinants of crash risk (e.g., Chen et al., 2001; Hutton et al., 2009; Kim et al., 2011a,b; An and Zhang, 2013). This research provides surprising results that larger firms are more crash-prone than smaller firms. This seems counter intuitive and casts doubt on the ability of the measures used in prior literature to accurate stock crash risk. We regard the frequency with which weekly returns exceed a certain threshold and severity of stock crashes (i.e., minimum weekly returns) during the year as a measure of crash risk. We show that our crash risk measures are intuitively plausible. For example, larger and more profitable firms are not likely to be experience stock crash. In contrast to previous studies, we use such intuitively plausible measures and provide additional empirical evidence to the growing literature explaining the relation between opacity and crash risks.

Third, Peng and Xiong (2006) developed a model showing the learning process of a representative investor who has limited attention. They view this representative investor as one of many retail investors in the stock market and show the effect of limited attention on return co-movement by some investors in the market. Unlike Peng and Xiong (2006), we theoretically and empirically show that when all investors regardless whether they have philological bias or not, face limited attention due to exogenous shock, stock prices become synchronous with the market and the frequency and severity of individual stock crashes become higher. We believe that this paper has implications for situations in which any exogenous shock, such as an earthquake, a tsunami, or exogenous financial crisis occurs.

Finally, our study contributes to growing literature on the linkage between unexpected shocks and financial reporting. For example, Francis et al. (2013) and Watts and Zuo (2012) show that firms with more conservative financial reporting experience less negative “long-run” stock returns during financial crises. However, there is little evidence documenting what factors affect “short-run” stock returns when unexpected shocks actually happen. We develop the theory that stock price synchronicity is a key factor explaining cross-sectional difference in stock returns in the event of shocks, and then that synchronicity is positively associated with opacity. As a result, we predict that opaque firms tend to have large negative returns after the shock has occurred. Ample support for this prediction is found in the data relating to a representative unexpected shock, the GEJE. This study demonstrates the importance of transparency when unexpected shocks have occurred.
The paper proceeds as follows. The next section develops theoretical predictions. In Section 3, we present the research design and data description. Section 4 reports the empirical results using data from Japan. Finally, Section 5 presents the summary and conclusion.

2 Model and Hypotheses

Our model is partly based on Peng and Xiong (2006). Their model focused on investors’ limited attention and overconfidence, and concluded that these two factors increase the return comovement (i.e., high synchronicity). We focus on opacity, investors’ limited attention, crash risk, and synchronicity. First, we clarify what effects the opacity of financial information has on the crash risk and the synchronicity by using the Bayes rule. Second, we clarify what effects investors’ limited attention has on the synchronicity by revised Peng and Xiong (2006) model. Third, we clarify that the stock price of less opaque firms is less synchronous and less likely to crash even where investors’ attention is limited.

2.1 The effects of opacity on crash risk and synchronicity

We assume that all investors are risk-averse. Let \( \theta_i \) be the value of firm \( i \). Investors do not know \( \theta_i \), but know its probability distribution \( \tilde{\theta}_i \). We assume that \( \tilde{\theta}_i \) has a normal distribution with a mean of \( \mu_{0,i} \) and a variance of \( \sigma_{0,i}^2 \).

\[
\tilde{\theta}_i \sim N(\mu_{0,i}, \sigma_{0,i}^2) \quad (i = 1, 2, \ldots, n)
\]

The probability density function of \( \tilde{\theta}_i \) is

\[
p(\theta_i) = \frac{1}{\sqrt{2\pi\sigma_{0,i}}} \exp\left\{ -\frac{(\theta_i - \mu_{0,i})^2}{2\sigma_{0,i}^2} \right\}
\]

The investor updates her belief about \( \tilde{\theta}_i \) based on firm specific-information \( \tilde{y}_i \) and market information \( \tilde{m} \).

The manager knows \( \theta_i \) and adds his intentional bias \( \tilde{\varepsilon}_i \) to it to make noisy specific-information \( \tilde{y}_i \). We assume that \( \tilde{\varepsilon}_i \) has a normal distribution with a mean of 0 and a variance of \( \sigma_i^2 \), and that the investor knows the distribution.

\[
\tilde{y}_i = \theta_i + \tilde{\varepsilon}_i \quad , \quad \tilde{\varepsilon}_i \sim N(0, \sigma_i^2) \quad (i = 1, 2, \ldots, n)
\]

Hence \( \tilde{y}_i \) has a a normal distribution with a mean of \( \theta_i \) and a variance of \( \sigma_i^2 \). We further assume that \( \tilde{\varepsilon}_i \) and \( \tilde{\varepsilon}_j \) are independent across different firms (\( i \neq j \)).

\[
\tilde{y}_i \sim N(\theta_i, \sigma_i^2) \quad , \quad \tilde{\varepsilon}_i \perp \tilde{\varepsilon}_j \quad (i \neq j)
\]
The conditional probability density function of $\tilde{y}_i$ given $\theta_i$ is

$$p(y_i \mid \theta_i) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp \left\{ -\frac{(y_i - \theta_i)^2}{2\sigma_i^2} \right\}$$

Assuming that the market-wide information $\tilde{m}$ consists of the sum of the firms’ value in the market and the error term $\tilde{\varepsilon}_m$. We assume that $\tilde{\varepsilon}_m$ has a normal distribution with a mean of 0 and a variance of $\sigma_m^2$, and that the investor knows the distribution.

$$\tilde{m} = \sum_{i=1}^{n} \theta_i + \tilde{\varepsilon}_m = \theta_i + \sum_{j \neq i} \theta_j + \tilde{\varepsilon}_m, \quad \tilde{\varepsilon}_m \sim N(0, \sigma_m^2)$$

Hence $\tilde{m}$ has a normal distribution with a mean of $\sum_{i=1}^{n} \theta_i$ and a variance of $\sigma_m^2$. Assuming that $\tilde{\varepsilon}_m$ and $\tilde{\varepsilon}_i$ are independent ($i = 1, 2, \ldots, n$), $\tilde{m}$ and $\tilde{y}_i$ are independent.

$$\tilde{m} \sim N(\theta_i + \theta_{-i}, \sigma_m^2) \quad \text{where} \quad \theta_{-i} \equiv \sum_{j \neq i} \theta_j$$

$\tilde{m} \perp \tilde{\varepsilon}_i, \tilde{m} \perp \tilde{y}_i$ $(i = 1, 2, \ldots, n)$

The conditional probability density function of $\tilde{m}$ given $\theta_i$ is

$$p(m \mid \theta_i) = \frac{1}{\sqrt{2\pi}\sigma_m} \exp \left\{ -\frac{(m - (\theta_i + \theta_{-i}))^2}{2\sigma_m^2} \right\}$$

Since $\tilde{y}_i$ and $\tilde{m}$ are independent. The joint conditional probability density function of $\tilde{y}_i$ and $\tilde{m}$ is

$$p(y_i, m \mid \theta_i) = p(y_i \mid \theta_i)p(m \mid \theta_i)$$

$$= \left( \frac{1}{\sqrt{2\pi}\sigma_i} \right)^2 \frac{1}{\sqrt{2\pi}\sigma_m} \exp \left\{ -\frac{1}{2} \left[ \frac{(y_i - \theta_i)^2}{\sigma_i^2} + \frac{(m - (\theta_i + \theta_{-i}))^2}{\sigma_m^2} \right] \right\}$$

Applying the Bayes rule, the conditional probability density function of $\tilde{\theta}_i$ given the firm-specific information $y_i$ and the market-wide information $m$ is

$$p(\theta_i \mid y_i, m) = \frac{p(\theta_i)p(y_i, m \mid \theta_i)}{\int_{-\infty}^{\infty} p(\theta_i)p(y_i, m \mid \theta_i) \, d\theta_i}$$

$$\propto \exp \left\{ -\frac{1}{2} \left( \frac{y_i - \theta_i}{\sigma_i^2} + \frac{m - (\theta_i + \theta_{-i})}{\sigma_m^2} \right)^2 \right\}$$

where $A = \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}}$

$$B = \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}} \left( \frac{y_i}{\sigma_i^2} + \frac{m - \theta_{-i}}{\sigma_m^2} + \frac{\mu_{0,i}}{\sigma_{0,i}} \right).$$
Appendix 1 provides the proof. Hence,

\[ \tilde{\theta}_i \mid y_i, m \sim N \left( w_{y,i}y_i + w_{m,i}(m - \theta_{-i}) + w_{0,i}\mu_{0,i}, \frac{1}{p_{y,i} + p_m + p_{0,i}} \right) \]  \hspace{1cm} (1)

where

\[ w_{y,i} = \frac{p_{y,i}}{p_{y,i} + p_m + p_{0,i}}, \hspace{0.5cm} w_{m,i} = \frac{p_m}{p_{y,i} + p_m + p_{0,i}}, \hspace{0.5cm} w_{0,i} = \frac{p_{0,i}}{p_{y,i} + p_m + p_{0,i}} \]

\[ p_{y,i} = \frac{1}{\sigma_i^2}, \hspace{0.5cm} p_m = \frac{1}{\sigma_m^2}, \hspace{0.5cm} p_{0,i} = \frac{1}{\sigma_{0,i}^2} \]

where \( p_{y,i}, p_m \) and \( p_{0,i} \) are respectively the precision of the firm specific information, the market-wide information and the investor’s prior belief, and \( p_{y,i} + p_m + p_{0,i} \) is the precision of investors’ posterior belief about the firm value given the firm specific information \( y_i \) and the market-wide information \( m \).

From Equation (1), the lower the precision of the firm specific information is, the higher the conditional variance of investors’ posterior belief \( \text{Var} \left( \tilde{\theta}_i \mid y_i, m \right) = \frac{1}{p_{y,i} + p_m + p_{0,i}} \) is. That means higher crash risk. Therefore we have the first hypothesis.

**Hypothesis 1.** The higher the opacity of information that a firm discloses, the higher the crash risk is.

Focusing on \( \text{Var} \left( \tilde{\theta}_i \mid y_i, m \right) = \frac{1}{p_{y,i} + p_m + p_{0,i}} \), the lower precision of firm-specific information \( y_i \) means that it has a weaker impact on, and the market-wide information \( m \) has a stronger impact on the conditional variance \( \text{Var} \left( \tilde{\theta}_i \mid y_i, m \right) \). If the firm-specific information is completely unreliable \( (p_{yi} \rightarrow 0) \), the conditional variance is explained only by the precision of market-wide information and the investor’s prior belief, and the conditional expectation \( \text{E} \left( \tilde{\theta}_i \mid y_i, m \right) \) converges to a value that the firm-specific information does not have any impacts on.

\[ \text{E} \left( \tilde{\theta}_i \mid y_i, m \right) = \frac{p_m}{p_m + p_{0,i}}(m - \theta_{-i}) + \frac{p_{0,i}}{p_m + p_{0,i}}\mu_{0,i} , \hspace{0.5cm} \text{Var}(\tilde{\theta}_i \mid y_i, m) = \frac{1}{p_m + p_{0,i}} \]

Taken together, concerning a firm which discloses opaque information, the investor’s expectation about the firm value becomes highly covariated with the market-wide information. Hence the stock price has higher synchronicity. Therefore we have the second hypothesis.

**Hypothesis 2.** The more opaque information a firm discloses, the more synchronous the stock price is.

### 2.2 The effects of catastrophe on crash risk and synchronicity

In this section, we investigate an investor’s decision making under the situation after a catastrophic event. The time-line is as follows. First firm values \( \theta_i \) \( (i = 1, 2, \ldots, n) \) that an investor...
does not know are realized. Second the investor forms a portfolio based on the prior belief about the firm value \( \tilde{\theta}_i \), the firm-specific information \( \tilde{y}_i \) \( (i = 1, 2, \ldots, n) \), and the market-wide information \( \tilde{m} \). Third the firm-specific information \( y_i \) \( (i = 1, 2, \ldots, n) \) and the market-wide information \( m \) are realized. Finally the investor liquidates her portfolio.

Since from Equation (1), the posterior conditional expectation given \( y_i \) and \( m \) is

\[
E\left( \tilde{\theta}_i \mid y_i, m \right) = w_{0,i} \mu_{0,i} + w_{y,i} y_i + w_{m,i} (m - \theta_{-i}),
\]

the prior conditional expectation can be written as

\[
E\left( \tilde{\theta}_i \mid \tilde{y}_i, \tilde{m} \right) = w_{0,i} \mu_{0,i} + w_{y,i} \tilde{y}_i + w_{m,i} (\tilde{m} - \theta_{-i}) = \alpha_i + c_i \tilde{y}_i + k_i \tilde{m},
\]

where \( k_i \) is the impact of the market-wide information \( \tilde{m} \) on, and \( c_i \) is the impact of the firm-specific information \( \tilde{y}_i \) on the prior conditional expectation of the investor.

Let \( \{w_1, w_2, \ldots, w_n\} \) be the investor’s portfolio \( (\sum_{i=1}^{n} w_i = 1) \). The variance \( V \) of her portfolio is

\[
V = \text{Var} \left( \sum_{i=1}^{n} w_i E(\tilde{\theta}_i \mid \tilde{y}_i, \tilde{m}) \right) = \text{Var} \left( \sum_{i=1}^{n} w_i (\alpha_i + c_i \tilde{y}_i) \right) = \text{Var} \left( \tilde{m} \sum_{i=1}^{n} w_i k_i + \sum_{i=1}^{n} w_i c_i \tilde{y}_i \right)
\]

\[
= \left( \sum_{i=1}^{n} w_i k_i \right)^2 \text{Var}(\tilde{m}) + \sum_{i=1}^{n} w_i^2 c_i^2 \text{Var}(\tilde{y}_i) = \left( \sum_{i=1}^{n} w_i k_i \right)^2 \sigma_m^2 + \sum_{i=1}^{n} w_i^2 c_i^2 \sigma_i^2
\]

where \( \sigma_m^2 = \text{Var}(\tilde{m}) \) and \( \sigma_i^2 = \text{Var}(\tilde{y}_i) \).

Since the investor is risk-averse, her problem is to minimize the variance of her portfolio’s value given a certain level of the expected value. When she knows \( \sigma_m^2 \) and \( \sigma_i^2 (i = 1, 2, \ldots, n) \) and has the ability enough to process all the available information immediately, her problem can be written as

\[
\min_{w_i} V = \left( \sum_{i=1}^{n} w_i k_i \right)^2 \sigma_m^2 + \sum_{i=1}^{n} w_i^2 c_i^2 \sigma_i^2
\]

\[
(\text{s.t.}) \quad \sum_{i=1}^{n} w_i E \left( \tilde{\theta}_i \mid \tilde{y}_i, \tilde{m} \right) = \text{const.}
\]

However, consider a situation after a catastrophic shock (e.g., a big earthquake, a tsunami, etc). She must make her decision quickly to avoid a big loss. Her attention is severely limited. In this situation, she cannot get the exact information of \( \sigma_m^2 \) and \( \sigma_i^2 (i = 1, 2, \ldots, n) \) that she
would have in an ordinary situation, and has to estimate the distributions of \( \tilde{m} \) and \( \tilde{y}_i \) and get the subjective variances \( \sigma_{s,m}^2 \) and \( \sigma_{s,i}^2 \), respectively.

Assuming that an investor believes that the more attention she pays to the market or the specific firms, the more exact information she can get about \( \tilde{m} \) and \( \tilde{y}_i \) (i.e., the less \( \sigma_{s,m}^2 \) and \( \sigma_{s,i}^2 \) are).

We use an information theory to formalize \( \sigma_{s,m}^2 \) and \( \sigma_{s,i}^2 \). Concerning an event \( E \) that occurs with probability \( p \), the information value \( I(E) \) that \( E \) occurred is defines as\(^2\)

\[
I(E) = -a \log_2 p \quad (a > 0).
\]

The message of Equation (2) is that the more probability \( p \) that an event \( E \) occurs, the less information value \( I(E) \) that it occurred.

Let \( E \) be an event that an investor estimation about \( \sigma_{s,i}^2 \) is right \( (\sigma_{s,i}^2 = \sigma_i^2) \), \( p \) the probability that \( \sigma_{s,i}^2 = \sigma_i^2 \), and \( I(E) \) the information value that \( \sigma_{s,i}^2 = \sigma_i^2 \). Let \( \lambda_i \) be the amount of the investor’s attention to firm \( i \) divided by the total amount of her attention. The more attention she pays to firm \( i \), the higher \( p \) is. Hence, we use \( \lambda_i \) as a proxy for \( p \). On the other hand, the small \( \sigma_{s,i}^2 \) means that the investor believes the uncertainty of \( \tilde{y}_i \) is small. In that case, even if the event \( E \) occurs, the investor does not regard the information that \( E \) occurred as valuable. That is, the information value \( I(E) \) is small for the investor. Hence we use \( \sigma_{s,i}^2 \) as a proxy of \( I(E) \). Further we use a natural logarithm for simplicity of calculation instead of a logarithm whose base is two.

\[
\sigma_{s,i}^2 = -a_i \ln \lambda_i \quad (a_i > 0, \ i = 1, 2, \ldots, n)
\]

Similarly we specify the subjective variance \( \sigma_{s,m}^2 \) as

\[
\sigma_{s,m}^2 = -a_m \ln \lambda_m \quad (a_m > 0)
\]

where \( \lambda_m \) is the amount of the attention to the market divided by the total amount of her attention. She estimates \( a_i \) and \( a_m \) based on the precision of the firm specific information and the market-wide information, respectively, that she got before.

Figure 1 illustrates the curves of subjective variance \( \sigma_{s,i}^2 \) depending upon \( \lambda_i \) when \( a_i = 1 \) and \( 2 \). Given a certain level of her attention \( \lambda_i \), the smaller \( a_i \) is, the smaller the subjective variance \( \sigma_{s,i}^2 \) is. This means that a firm with smaller (larger) \( a_i \) discloses more transparent (opaque) information\(^3\). The curve of \( \sigma_{s,m}^2 \) can be illustrated as well.

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\(^2\) The first seminal paper is Shannon (1948).

\(^3\) Similarly, as to the market-wide information, the smaller \( a_m \) is, the more precise the market-wide information is.
Since $\sigma_{s,m}^2$ and $\sigma_{s,i}^2$ are decreasing in $\lambda_m$ and $\lambda_i$ respectively, there exist certain levels of her attention $\Lambda_{s,m}^*$ and $\Lambda_{s,i}^*$ where $\sigma_{s,m}(\Lambda_{s,m}^*) = \sigma_{s,m}^2$ and $\sigma_{s,i}(\Lambda_{s,i}^*) = \sigma_{s,i}^2$ respectively. Since she does not know $\sigma_{s,m}^2$ and $\sigma_{s,i}^2$, she does not know $\Lambda_{s,m}^*$ and $\Lambda_{s,i}^*$.

In such situation, she makes decision by two steps. The first step is to decide how she distributes her attention to the market and the specific firms (to decide $[\lambda_m, \lambda_1, \cdots, \lambda_n]$). The second step is to decide how she distributes her wealth to the specific firms (to decide $[w_1, \cdots, w_n]$). Hence her problem can be written as

$$\min_{w_i} \left( \min_{\lambda_m, \lambda_i} V_s \right)$$

where $V_s = \left( \sum_{i=1}^{n} w_i k_i \right)^2 \sigma_{s,m}^2 + \sum_{i=1}^{n} w_i c_i^2 \sigma_{s,i}^2$

(s.t.) $\sum_{i=1}^{n} w_i E \left( E(\tilde{\theta}_i | \tilde{y}_i, \tilde{m}) \right) = \text{const}$, $0 < \lambda_i < 1$, $0 < \lambda_m < 1$, $\lambda_m + \sum_{i=1}^{n} \lambda_i = 1$.

$V_s$ can be regarded as a subjective variance of her portfolio.

The first step of her decision is to solve

$$\min_{\lambda_m, \lambda_i} V_s = \left( \sum_{i=1}^{n} w_i k_i \right)^2 \sigma_{s,m}^2 + \sum_{i=1}^{n} w_i c_i^2 \sigma_{s,i}^2$$

$$= -\left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m \ln \lambda_m - \sum_{i=1}^{n} w_i c_i^2 a_i \ln \lambda_i$$

(s.t.) $0 < \lambda_m < 1$, $0 < \lambda_i < 1$, $\lambda_m + \sum_{i=1}^{n} \lambda_i = 1$.

We use the Lagrangian multiplier method.

$$L = -\left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m \ln \lambda_m - \sum_{i=1}^{n} w_i c_i^2 a_i \ln \lambda_i - \nu \left( 1 - \lambda_m - \sum_{i=1}^{n} \lambda_i \right)$$

$$\frac{\partial L}{\partial \lambda_m} = -\frac{\left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m \lambda_m}{\lambda_m} + \nu = 0$$

$$\frac{\partial L}{\partial \lambda_i} = -\frac{w_i^2 c_i^2 a_i}{\lambda_i} + \nu = 0 \quad (i = 1, 2, \cdots, n)$$

$$\frac{\partial L}{\partial \nu} = 1 - \lambda_m - \sum_{i=1}^{n} \lambda_i = 0$$

From the three equations above,

$$\frac{\left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m}{\lambda_m} = \frac{w_1^2 c_1^2 a_1}{\lambda_1^*} = \cdots = \frac{w_n^2 c_n^2 a_n}{\lambda_n^*} \equiv \kappa^* \quad (\kappa^* \neq 0)$$

$$\lambda_m^* + \sum_{i=1}^{n} \lambda_i^* = 1$$
The solutions are

\[
\lambda^*_m = \left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m, \quad \lambda^*_1 = \frac{w_1^2 c_1^2 a_1}{\kappa^*}, \quad \cdots, \quad \lambda^*_n = \frac{w_n^2 c_n^2 a_n}{\kappa^*}
\]

where \( \kappa^* = \left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m + \sum_{i=1}^{n} w_i^2 c_i a_i \).

Since \( 0 < \lambda^*_m < 1 \) and \( 0 < \lambda^*_i < 1 \), \( \lambda^*_m \) and \( \lambda^*_i \) are the interior solutions and are the functions of \( w_i \) (\( i = 1, \cdots, n \)).

\[
\lambda^*_m = \lambda^*_m(w_1, \cdots, w_n), \quad \lambda^*_1 = \lambda^*_1(w_1, \cdots, w_n), \quad \cdots, \quad \lambda^*_n = \lambda^*_n(w_1, \cdots, w_n)
\]

From Equation (3), we can derive

\[
\frac{\left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m}{\lambda^*_m} = \frac{\sum_{i=1}^{n} w_i^2 c_i^2 a_i}{\sum_{i=1}^{n} \lambda^*_i}.
\]

Appendix 2 provides the proof.

For sufficient large \( n \),

\[
\left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m > \sum_{i=1}^{n} w_i^2 c_i^2 a_i
\]

Appendix 3 provides the proof.\(^4\)

From Equation (5) and (6), we have

\[
\lambda^*_m > \sum_{i=1}^{n} \lambda^*_i
\]

Equation (7) means that the investor pays more attention to the market-wide information than to all the firm-specific information disclosed by the firms in the market. That leads to higher synchronicity in their stock prices. Therefore we have the third hypothesis.

**Hypothesis 3.** After a catastrophe (e.g., an big earthquake, a tsunami) where investors’ attention is severely limited, the stock prices become synchronous.

Since soon after a catastrophe bad news is pervasive over the market , we also hypothesize that the crash risk becomes higher where investors’ attention is severely limited.

\(^4\)We assume that the more firms are there in the market, the smaller portion of her wealth is distributed to firm \( i \) (\( i = 1, 2, \ldots, n \)). That is

\[
\forall i \in \{1, 2, \ldots, n\}, \quad \lim_{n \to \infty} w_i = 0.
\]
Hypothesis 4. After a catastrophe (e.g., an big earthquake, a tsunami) where investor attention is severely limited, the crash risk becomes higher in the market.

The more firms are there in the market, the more likely it is that Equation (6) holds. Appendix 5 provides the proof. Therefore we have a corollary.

Corollary 1. The more firms there are in the market, the more attention an investor pays to the market-wide information than the firm specific information.

2.3 The effects of transparency on crash risk after a catastrophic shock

The second step of an investor’s decision is to solve

$$
\min_{w_i} V_s = - \left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m \ln \lambda_m^*(w_1, \ldots, w_n) - \sum_{i=1}^{n} w_i^2 c_i^2 a_i \ln \lambda_i^*(w_1, \ldots, w_n)
$$

(s.t.) \( \sum_{i=1}^{n} w_i E(\tilde{\theta}_i | y, m) = \text{const} \), \( \sum_{i=1}^{n} w_i = 1 \).

Since \( \lambda_m^* \) and \( \lambda_i^* (i = 1, 2, \ldots, n) \) functions of \( w_i (i = 1, 2, \ldots, n) \), the solution of her problem can be written as

\[
[w_1^*, \ldots, w_n^*, \lambda_m^*(w_1^*, \ldots, w_n^*), \lambda_1^*(w_1^*, \ldots, w_n^*), \ldots, \lambda_n^*(w_1^*, \ldots, w_n^*)] .
\]

Now we compare synchronicity between a transparent firm and an opaque firm. Let firm 1 be the transparent firm \( (a_1 = 1) \) and firm 2 be the opaque firm \( (a_2 = 2) \).

\[
\sigma_{s,i}^2 = -a_i \ln \lambda_i \quad (i = 1, 2 ; a_1 = 1, a_2 = 2)
\]

For simplicity, we assume that the investor’s unknown variances are the same between the two firms \( (\sigma_1^2 = \sigma_2^2 = \sigma^2) \). Let \( \Lambda_i \) be the amount of her attention to firm \( i \) where the subjective variance is equal to the unknown variance \( (\sigma_{s,i}^2(\Lambda_i) = \sigma^2(i = 1, 2)) \). Since she does not know \( \sigma^2 \), she does not know \( \Lambda_i (i = 1, 2) \). Figure 2 shows the curves of the two firms and contrasts the magnitude relations of the widths between \( [0, \Lambda_1] \) and \( [0, \Lambda_2] \), and between \( [\Lambda_1, 1] \) and \( [\Lambda_2, 1] \). In Figure 2, the curve of firm 2 is located above that of firm 1.

Since the range of \( [\Lambda_1, 1] \) is wider than that of \( [\Lambda_2, 1] \), it is more likely that \( \Lambda_1 < \lambda_1^*(w_1^*, \ldots, w_n^*) < 1 \) than that \( \Lambda_2 < \lambda_2^*(w_1^*, \ldots, w_n^*) < 1 \). On the other hand, since the range of \( [0, \Lambda_2] \) is wider than that of \( [0, \Lambda_1] \), it is more likely that \( 0 < \lambda_2^*(w_1^*, \ldots, w_n^*) < \Lambda_2 \) than that \( 0 < \lambda_1^*(w_1^*, \ldots, w_n^*) < \Lambda_1 \).

In the case that \( \Lambda_i < \lambda_i^*(w_1^*, \ldots, w_n^*) < 1 \), the investor pays more attention than is required to equalize \( \sigma_{s,i}^2(\Lambda_i) \) with \( \sigma_i^2 \). It can be considered that all the specific information about firm
is reflected in her decision since she pays more attention to firm $i$ than $Λ_i$. Even though the amount of attention paid to the firm is small, as far as she pays more attention than $Λ_i$, her belief about the firm value does not tend to covariate with the market. Hence, the stock price of the firm is less synchronous and less likely to crash even after a catastrophic shock.

In the case that $0 < \lambda^*_i(w^*_1, \ldots, w^*_n) < Λ_i$, the investor pays less attention than is required to equalize $σ^2_x(Λ_i)$ with $σ^2_i$. She could get more firm specific information by paying more attention to firm $i$. However she pays more attention to the market since the cost of paying attention to firm $i$, $a_i$, is high. Hence her belief about the firm value tends to covariate with the market, and the stock price of the firm is more synchronous and more likely to crash after a catastrophic shock. Therefore we have the fifth hypothesis.

**Hypothesis 5.** The stock price of a firm that discloses opaque information is more likely to covariate with the market, and hence is more likely to crash after a catastrophe.

### 3 Data, Variable Measurement, and Research Design

#### 3.1 Measuring financial reporting opacity using management earnings forecasts

Firms listed on the Tokyo Stock Exchange (TSE) have a unique practice of annual earnings announcements. The TSE requires listed firms to report not only the actual earnings of each year and the current year but also the management earnings forecasts for the following year at the earnings announcements so as to provide information that is useful in decision making. Furthermore, these forecasts are released in the form of point estimates, because firms are recommended not to issue range or qualitative estimates (e.g., TSE, 2006). Actually, almost all firms report point estimates of the management earnings forecasts for the following year in accordance with the TSE requirement. This fact implies that disclosure of management earnings forecasts are effectively mandated in Japan.

In Japan, market expectations substantially depend on earnings guidance by management who are the most familiar with future prospects of firms. This is because a competing source of earnings expectations, analyst forecasts, are not sufficiently provided for a wide range of firms—about two thirds of firms have zero coverage, and management earnings forecasts dominate, on average, analysts forecast in accuracy for firms with coverage. In this situation, firms that are provided forecasts with lower accuracy are more opaque to investors in the sense that they cannot see what a firm’s intrinsic value really is. Since most firms provide management earnings forecasts in Japan, management forecasts are generally accurate work well as a proxy for explaining the
cross-sectional difference of opacity.

We measure management forecast accuracy (ACCURACY) for each firm-year observation based on the absolute value of the difference between the initial management earnings forecast of period $t$ made at the earnings announcement of period $t - 1$ and the actual current earnings, deflated by the market value of equity as of the end of fiscal year $t - 1$:

$$ACCURACY_{j,t} = \left| \frac{f e_{j,t-1} - e_{j,t}}{MVE_{j,t-1}} \right|,$$

where $ACCURACY_{j,t}$ is management forecast accuracy for firm $j$ of period $t$, $f e_{t-1}$ is the management forecast of period $t$ earnings made at the earnings announcement of period $t - 1$, $e_t$ is the actual earnings in period $t$, and $MVE_{t-1}$ is the market value of equity at the end of fiscal year $t - 1$.

We define our opacity measure using management forecasts ($MF_{OPACITY}$) as the sum of the management forecast accuracy over the past three-year:

$$MF_{OPACITY}_{j,t} = \sum_{k=1}^{3} ACCURACY_{j,t-k}$$

The simple idea behind this measure is that firms that consistently provide lower accuracy forecasts (i.e., higher value of $MF_{OPACITY}$) are regarded as more opaque to investors.

In our framework, considering that fundamental firm value, $\theta$, is an increasing function of forthcoming earnings and that $y$ is management earnings forecasts, absolute value of signed error, $\varepsilon_y$, reflects future ACCURACY. $MF_{OPACITY}$ is essentially identical concept to $\sigma_y^2$ to the extent that higher ACCURACY in the past presage higher ACCURACY in the future. In Japan, the bias in management earnings forecast made at the earnings announcement in the previous year has a strong autocorrelation structure (e.g., Kato et al., 2009; Shimizu, 2007). Therefore, it seems reasonable that the past ACCURACY is a signal for the future ACCURACY and $MF_{OPACITY}$ is a proxy for $\sigma_y^2$.

### 3.2 Measurement of stock price synchronicity

A stock volatility comprises the following two components: (1) those tied to market and/or industry-wide information and (2) those tied to firm-specific information. Using weekly data, we first estimate the $R^2$ for each observations for firm-year $t$ from the expanded index model in Equation (9), which allows us to decompose total return variations into these two components.

$$r_{j,w} = \alpha_j + \beta_{1,j}r_{m,w-1} + \beta_{2,j}r_{i,w-1} + \beta_{3,j}r_{m,w} + \beta_{4,j}r_{i,w} + \beta_{5,j}r_{m,w+1} + \beta_{6,j}r_{i,w+1} + \varepsilon_{j,w},$$

(9)
where \( r_{j,w} \) is the stock return in week \( w \) of firm \( j \) in industry \( i \), \( r_{m,w} \) is the Tokyo Stock Price Index (TOPIX) return in week \( w \), and \( r_{i,w} \) is the value-weighted return of industry \( i \), based on two-digit NIKKEI Industrial Code, in week \( w \). We correct for nonsynchronous trading by including one lead and lag terms for the market and industry indexes, following Dimson (1979).

Given the bounded nature of \( R^2 \), we use a logistic transformation of \( R^2 \), which can range from negative to positive infinity (e.g., Morck et al., 2000; Gul et al., 2010). We define stock price synchronicity as follow:

\[
\Psi_{j,t} = \log \left( \frac{R^2_{j,t}}{1 - R^2_{j,t}} \right),
\]

where \( \Psi_{j,t} \) is our measure of synchronicity for firm in year \( t \). Higher value of \( \Psi \) means more synchronous.

### 3.3 Crash measure

Negative coefficient of skewness (NCSKEW), down-to-up volatility (DUVOL), and Hutton et al. (2009) measure that based on the number of the firm-specific returns exceeding certain standard deviations below its mean value have become popular and widely used measures of crash risk (e.g., Chen et al., 2001; Kim et al., 2011a,b; An and Zhang, 2013). This research provides a surprising result that large firms are associated with a higher likelihood of a crash, suggesting large firms appear to be more crash-prone than small firms. This seems counter to intuitive. However, previous paper does not seem to tell well convincing.

One possible reason is due to artifact of crash definition. With regard to Hutton et al. (2009) measure, since size and standard deviation of stock are highly negatively correlated, the threshold return that qualifies as a crash is smaller in absolute value for larger firms. Thus, for large firms, Hutton et al. (2009) measure are likely to easily qualify as a crash even though the negative returns are not so high in severity. This fact raises doubts regarding the ability of these measures used in prior research to capture crash phenomenon.

Therefore, we focus on the different aspects of stock crash and use the following new measures. The first is frequency of stock crash and the second is the severity. We define the frequency of crash \((CRASH_{x\%j,t})\) as indicator variable equal to one if firm \( j \) within year \( t \) experiences one or more weekly return fall \( x\% \) and equal to zero otherwise. For robustness, we report the results using the threshold \( x \), 15%, 20%, 25%, and 30%. Another aspect of stock crash, severity \((SEVERITY_{j,t})\), is defined as minimum weekly return for firm \( j \) in year \( t \).
3.4 Timeline and our research design

Figure 3 presents a timeline of when major variables is measured. Hypothesis 3 and Hypothesis 4 predict that stock prices become synchronous and stock crashes become higher, respectively, after the GEJE that occurred on March 11. To test these hypotheses, we define year \( t \) as the period from week after Mach 11 in year \( t - 1 \) to week before March 10 in year \( t \) (52 or 53 weeks each year) throughout all analyses.

To ensure that the accounting-based variables are known to investors before synchronicity and crash in year \( t \) are measured, we match the accounting data for December fiscal year-end in our year \( t - 2 \) to November fiscal year-end in our year \( t - 1 \) with the explained variables in year \( t \).

3.5 Sample selection

We obtained management forecasts data reported at the time of earnings announcement from the Nikkei NEEDS-BULK management forecasts database, and other accounting data from the Nikkei NEEDS-FinancialQUEST. Market data, such as market prices, number of shares outstanding, and stock returns, is from the Financial Data Solutions NPM daily returns database\(^5\).

In Japan, the Financial Services Agency requires listed firms to report the statement of cash flows from 2000. To calculate over the past three-year accruals directly from the statement of cash flows, our sample period begin with 2004. Our sample period ends in 2011, after which accounting and stock market-based data are not available at the time of our data collection.

This study includes firm-year observations from 2004 to 2011 for the firms listing on Japanese stock markets for synchronicity and crash analysis. We require a minimum of 24 week data for measuring stock price synchronicity and the frequency and severity of crash to avoid the problems from small sample issues. In addition, we exclude financial firms and firm-year with insufficient data to calculate both MF\_OPACITY and ACC\_OPACITY as well as several control variables. The accumulative sample is 18,395 firm-years from 2004 to 2011 with the number of observations from 1,702 in 2004 to 2,520 in 2011.

In regard to an event study for the GEJE, the sample observations meet the following sample selection criteria: (1) non-financial firms, (2) firms with stock returns around the GEJE, and (3) firms with sufficient data to both MF\_OPACITY and ACC\_OPACITY as well as several control variables. These criteria yield a final sample of 2,559.

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\(^5\)This returns database basically corresponds to the CRSP in the U.S.
4 Empirical Analysis

4.1 Financial reporting opacity and our crash measures

We begin by showing the relation between stock crashes and our management-forecast-based opacity measure. In Panel A of Table 2, we highlight the distribution of stock crashes based on four-thresholds for each MF_OPACITY deciles. To construct this table, break points for the sort are computed on the basis of the annual rank of MF_OPACITY. Panel A of Table 2 shows that for any threshold, the percentage of crash firms monotonically increases across MF_OPACITY deciles and chi-squared tests confirm that stock crashes are significantly concentrated in the high MF_OPACITY portfolios. For example, the percentage of our sample that experienced at least more than one −20% crash for the lowest MF_OPACITY decile is 10.5%, while that for the highest decile is 30.1%. The difference between those two groups of 19.7% is statistically significant at a 1% level. In Hutton et al. (2009), 17.1% of their sample experience at least one crash based on their criteria. This percentage is the nearest when we use −20% as a crash threshold (i.e., 15.0% of our sample experience at least one crash based on our criteria). Hence, compared to Hutton et al. (2009), the −15% (−25% and −30%) crash threshold in our analysis might be less (more) strict.

Panel B of Table 2 provides summary statistics of minimum weekly returns during the year (i.e., SEVERITY) for each MF_OPACITY decile. Mean and median value of SEVERITY decrease monotonically between the lowest and highest deciles. In addition, we can easily reject the null hypothesis that the mean and median differences between SEVERITY for the lowest and highest decile is equal to zero at 1% level (t-stat. = −23.3, z-stat. = −27.6). These results suggest that more opaque firms, on average, tend to experience at least one large negative return. In sum, results in Table 2 indicates that stock crashes are more frequent and more severe for opaque firms, which is consistent with Hypothesis 1.

Next, to test Hypothesis 4, we investigate whether the frequency and severity of individual stock crashes become higher due to the GEJE. Figure 4 highlights the frequency of stock crashes based on −20% threshold across the year. If Hypothesis 4 is correct, then we can observe an abnormally high frequency of stock crashes in the GEJE year (i.e., 2011). However, Figure 4 shows that abnormally high frequency of stock crashes is observed in 2008 rather than in 2011, although 2011 seems high. Recall that 2008 was the height of the Global Financial Crisis. This crisis affected not only the U.S. stock market but also the Japanese market. The Nikkei Stock Average was 12,214 Japanese Yen on the day before the bankruptcy of Lehman.
Brothers (September 12, 2008), the price on October 28, 2011 reached levels not seen since before October 1982. Considering the great effect of the crisis on Japanese stock market, abnormally high frequency of stock crash in 2008 is not surprising.

During a crisis period, investors must do the same after a catastrophic event such as the GEJE. As we describe in Section 2, such a situation leads to severely limited attention by investors. Therefore, Hypothesis 3 and 4 apply to not only 2011 but also 2008. Stated differently, if these two hypotheses are correct, stock prices should have become more synchronous during 2008 due to severely limited investor attention, and since 2008, stock crashes should be more frequent and more severe during 2008, because the news spread in the Japanese market about the Global Financial Crisis. As the result, the fact that an abnormally high frequency of stock crashes is observed in 2008 is consistent with our model. In addition to 2008, it makes sense that we observe a high frequency of stock crashes in 2007. This is because the crisis began in December 2007 as defined by the National Bureau of Economic Research. To mitigate omitted variable bias arising from not including important regressors, we include a 2007 and 2008 dummy as control variables when we conduct regression analyses to test Hypothesis 1 to 4.

Figure 5 gives year-by-year mean data for SEVERITY. The mean of SEVERITY for three-years, 2007, 2008, and 2011 are well below the mean across our sample period (−13.3%). The mean of SEVERITY for those year is −13.9%, −23.1%, and −14.1%, respectively. We believe that the results of 2007 and 2008 are due to the effect of the Global Financial Crisis on the Japanese stock market. Given that there is a linkage between investors’ limited attention, stock price synchronicity, and stock crashes, it represents the most convincing evidence. In fact, we show that stock price synchronicity increases for 2007 and 2008 in the next section. Given the wide-spread bad news across the markets, greater stock price synchronicity should lead to more stock crashes as described in our model. Therefore, strong evidence shows that more frequent and severe of stock crashes are likely to be observed in 2007 and 2008, for supporting our model.

Besides 2007 and 2008, we observed abnormally low SEVERITY in 2011, which is consistent with Hypothesis 4. This evidence suggests that after the GEJE, firms tended to experience at least one severe large negative returns. Overall, the results of Figure 4 and 5 support Hypothesis 4 (i.e., the frequency and severity of individual stock crashes become higher after the GEJE).

The key question is whether the effect of opacity and the GEJE on stock crashes remains even after controlling for the known determinants of the frequency and severity of stock crashes. This issue is investigated next in a regression framework. With regard to the analysis for the frequency of stock crashes, we assume that the marginal probability of the frequency follows a
logistic distribution and is given by
\[
\text{Prob}(\text{CRASH}x\%_{j,t} = 1) = \frac{1}{1 + \exp(-x_j^T\beta)},
\] (10)
where \(\text{CRASH}x\%_{j,t}\) is an indicator variable equal to one if firm \(j\) within year \(t\) experiences one or more weekly return fall \(x\%\) \((x \in \{-15\%, -20\%, -25\%, -30\%\})\) and equal to zero otherwise, \(x_j\) is firm \(j\)’s \(K \times 1\) vector of explanatory variables including intercept, and \(\beta\) is \(K \times 1\) unknown coefficient vector. A higher level of \(x_j^T\beta\) implies a higher probability of stock crash during the year \(t\). Our main focus among explanatory variables is \(MF\_\text{OPACITY}_{j,t}\) and \(D\_\text{EARTHQUAKE}_{j,t}\) for testing Hypothesis 1 and 4, respectively. The former is the sum of the management forecast accuracy over the past three years, and the latter is an indicator variable equal to one year after earthquake (i.e., 2011). Following Hutton et al. (2009), we include a discretionary-accruals-based opacity measure (\(ACC\_\text{OPACITY}\)), firm size (\(\log(MVE)\)), book-to-market ratio (\(\log(BM)\)), return on net operation asset (\(RNOA\)), and leverage (\(LEV\)) as control variables. In addition, we include a 2007 and 2008 dummy as control variables to avoid the omitted variables bias. Detail definition for these control variables are provided in Table 1.

Regarding the severity of stock crashes, we estimate the following model:
\[
SEVERITY_{j,t} = \beta_0 + \beta_1 D\_\text{EARTHQUAKE}_{j,t} + \beta_2 MF\_\text{OPACITY}_{j,t} + \sum_k \gamma_k \text{Control variable}_k + \varepsilon_{j,t},
\] (11)
where \(SEVERITY_{j,t}\) is a minimum weekly return for firm \(j\) in year \(t\). In this model, we include the same control variables as in Equation (10).

Table 3 reports these two regression results for alternative specifications and dependent variables. Regression 1 shows the results when we estimate Equation (10) with \(CRASH - 15\%\) as a dependent variable. The coefficient on \(MF\_\text{OPACITY}\) is positive \((= 1.564)\) and statistically significant \((p < 0.01)\). This result suggests that stock crashes are more frequent for firms with consistently providing lower accuracy management forecasts, which is consistently Hypothesis 1. In addition, we find that there is also a significantly positive coefficient on \(ACC\_\text{OPACITY}\). Overall, we find that both mandatory-management forecast-based and discretionary-accruals-based opacity measures can explain the cross-sectional variation in stock crash frequency. As a robustness check, we estimate Equation (10) using annual decile rank, scaled to range between 0 and 1, for \(MF\_\text{OPACITY}\) and \(ACC\_\text{OPACITY}\), to mitigate outlier effect and/or consider non-linearity relation as shown in Regression 2. The pattern of coefficients estimated in terms of the sign and significance is similar to Regression 1. Hence, our results are a robust measurement of those two variables.
A dependent variable of Regression 3, 4, and 5 is $CRASH - 20\%$, $CRASH - 25\%$, and $CRASH - 30\%$, respectively. The coefficient of MF\_OPACITY is positive and significant at 1\% level in all three specifications. These results indicate that Hypothesis 1 is supported when we use any four values as stock crash thresholds.

In regard to Hypothesis 4, the coefficient on D\_EARTHQUAKE is positive and significant at 1\% level in Regression 1 to 5. Considering the magnitude of the coefficient, the economic impact of the GEJE on the frequency of stock crashes appears to be high. For example, the coefficient of D\_EARTHQUAKE in Regression 3 is 1.061 and it implies that after the GEJE, the $-20\%$ crash frequency increase by x\% even after controlling for other well-known determinants of stock crashes. Hence, we conclude that Hypothesis 4 is supported.

To test the impact of opacity and the GEJE on the stock crash severity, we estimate Equation (11). Regression 6 of Table 3 shows the OLS estimator with $t$-statistics based on the two-way cluster robust standard errors in parentheses following Petersen (2009). The coefficient of MF\_OPACITY is negative and significantly different from zero. To confirm the economics impact of it, we estimate Equation 11 using annual decile rank, scaled to range between 0 and 1, for MF\_OPACITY as well as ACC\_OPACITY. By this coding scheme, the coefficient on MF\_OPACITY can be interpreted as the difference in minimum returns during the year between the portfolio for the most opaque firms and that for the least opaque firms. Regression 7 shows that the coefficient on MF\_OPACITY is $-0.033$ and statistically significant at 1\% level. This result means that the difference in minimum return between the lowest and highest MF\_OPACITY deciles is 3.3\%. We believe that these differences are also economically significant.

Turning to the coefficient on D\_EARTHQUAKE, we find that there is a significantly negative coefficient on it (coeff. = $-0.025$, $p < 0.01$) in Regression 6. It implies that minimum returns decreased 2.5\% in the GEJE year, even after accounting for other known determinants of stock crash severity. The conclusion drawn from Regression 7 remains basically unchanged in term of sign and magnitude by other coding scheme for MF\_OPACITY and ACC\_OPACITY.

The results of Table 3 strongly support Hypothesis 1 and 4. Overall, these results suggest that stock crashes are more frequent and more severe for opaque firms after the GEJE.

\subsection{Financial reporting opacity and stock price synchronicity}

In this section, we test Hypothesis 2 and 3 related to stock price synchronicity. Panel A of Table 4 reports the results of the univariate relationship between $R^2$ and MF\_OPACITY. To construct this table, we sort the sample firms into deciles based on the annual rank of MF\_OPACITY, and
then, compute the average $R^2$ for each portfolio. Panel A of Table 4 shows that $R^2$ monotonically decreases across MF.OPACITY deciles, which is inconsistent with Hypothesis 1. Given Roll (1988)'s finding that firm size is an important factor of why $R^2$s differ, this result is not surprising. In line with our intuition, MF.OPACITY is strongly correlated with firm size ($\log(MVE)$) ($\rho = -0.33, p < 0.01$) for our sample, suggesting that opaque firms are concentrated in small firms. Therefore, the negative correlation between MF.OPACITY and $R^2$ may be spurious due to the firm size effect on $R^2$.

To control firm size, we form 25 portfolios by independently sorting firms into five quintiles based on their annual firm size ranking and five quintiles based on their annual MF.OPACITY ranking. Panel B of Table 4 reports mean values of $R^2$ for each portfolios. Looking down each column, the result indicates that the larger firm tend to have higher $R^2$, which confirm Roll (1988)'s finding in our sample. Conversely, looking across each row, we observe higher opacity is associated with higher $R^2$ except for the lowest quintile and Q2 quintile. For Q2 quintile, we also observe that the highest $R^2$ is in the highest quintile. In sum, our management forecast based opacity measure appears to independently affect $R^2$ even after controlling for firm size.

Figure 6 shows mean values $R^2$ from Equation (9) for each year over our sample period. Noteworthy is that the high $R^2$s are observed in 2007, 2008, and 2011. This evidence is consistent with our model. In Section 2, we argued that stock prices become more synchronous when investor attention is severely limited. As described in the previous section, investor attention is severely limited in the three-years corresponding to the Global Financial Crisis and the GEJE. Hence, the evidence on Figure 6 supports our model is consistent with Hypothesis 3.

Next, we conduct regression analysis to control for potential effect of the various firm characteristics including firm size on the $R^2$. We estimate that the following model examines the effect of opacity and the GEJE on stock price synchronicity:

$$\Psi_{j,t} = \beta_0 + \beta_1 D_{\text{EARTHQUAKE}}_{j,t} + \beta_2 MF_{\text{OPACITY}}_{j,t} + \sum_k \gamma_k \text{Control variable}_k + \varepsilon_{j,t}, \quad (12)$$

where $D_{\text{EARTHQUAKE}}_{j,t}$ is an indicator variable equal to one year after earthquake (i.e., 2011) and equal to zero otherwise. According to the previous studies (e.g., Hutton et al., 2009), we control for discretionary-accruals-based opacity measure ($ACC_{\text{OPACITY}}$), firm size ($\log(MVE)$), book-to-market ratio ($\log(BM)$), the variance of weekly returns of industry index ($\log(VARIND)$), skewness ($SKEW$) and kurtosis ($KURT$) of the firm-specific weekly returns, return on net operation asset ($RNOA$), and leverage ($LEV$). Detailed definition for these control variables are provided in Table 1. In addition, we include a 2007 and 2008 dummy as control
variables to control the effect of the U.S. sub-prime mortgage financial crisis on the Japanese market. Hypothesis 2 predicts the coefficient of MF\textsubscript{OPACITY} should be significantly positive. With regard to Hypothesis 3, as long as the catastrophic shock increases the stock price synchronicity, the coefficient on D\textsubscript{EARTHQUAKE} should be significantly positive.

Table 5 reports the estimation results of Eq (12). Regression 3 shows that the coefficient on MF\textsubscript{OPACITY} is positive ($= 0.221$) and statistically significant ($p < 0.01$) while coefficient on ACC\textsubscript{OPACITY} is negative ($= -0.049$) and insignificantly different from zero at a conventional level. The former result indicates that firms with opaque financial reports in terms of management forecasts have stock returns that are more synchronous. On the other hand, the latter result suggests that opacity based on earnings management has no explanatory power for stock price synchronicity, which is inconsistent with Hutton \textit{et al}. (2009).

As a robustness check, we transform two opacity measures to the annual decile rank, scaled to range between 0 and 1, to mitigate the effect of outliers. Regression 4 of Table 5 presents estimates using ranked measures. We find that the pattern of the estimated coefficients on ranked measures in terms of the sign and significance is similar to Regression 3 in Table 5. Hence, our results are robust to an alternative treatment of the opacity measures. Overall, these results strongly support our Hypothesis 2.

Regarding Hypothesis 3, the coefficient on D\textsubscript{EARTHQUAKE} is positive and statistically significant in both Regression 3 and 4 as expected. The implied value impact of $R^2$ using the parameter from Regression 3, holding all other variables at their mean, after the GEJE (38.0\%) increases by around 8.6\% as compared to $R^2$ before the GEJE (29.5\%). This impact is an economically as well as statistically significant amount. Hence, the results strongly support our Hypothesis 3.

\subsection*{4.3 Financial reporting opacity and crash severity around the GEJE}

In this section, we examine the association between management-forecast-based opacity and the behavior stock price around the GEJE to test Hypothesis 5. Table 6 is reports the mean of stock returns for each portfolios classified by MVE and MF\textsubscript{OPACITY}. The positive relation between firm size and stock returns around the GEJE is intuitively appealing: i.e., large ships are less affected by a big wave compared to small ships. Therefore, it is important to control firm size when we examine the stock returns around the GEJE.

To construct Table 6, we independently sort sample firms (i.e., 2,559 firm) into quintiles based on rank of each variable, and then compute the average stock returns for each portfolio.
Panel A of Table 6 shows the mean of raw return on the GEJE day (day 0) for each portfolios. We can point out that for all MVE quintiles, stock returns are likely to almost monotonically decrease across MF_Opacity quintiles and the difference in stock returns between the lowest and highest MF_Opacity deciles is statistically significant at 1% level.

These results suggest that MF_Opacity quintiles have predictive power for cross-sectional stock returns on the day the quake struck. This is a surprising result for given the time of 2:46 p.m occurred the GEJE. Since the Japanese stock market close 3:00 p.m., the collapse of the price occurred between 2:46 p.m. and 3:00 p.m. as shown in Figure 7. This figure plots the price movement of the Nikkei 225 in the afternoon session, called the “Goba”, between 12:30 p.m. and 3:00 p.m. on March 11, 2011. MF_Opacity were able to predict the severity level of “flash” crash for just 14 minutes. Turning to MVE, looking down each column, we see that a largely monotonic and statistically significant negative relation between MVE and stock return, which is inconsistent with our intuition. In contrast, in Panel B and C of Table 6 indicating cumulative returns from days 0 to +1 and from days 0 to +2, respectively, we observe a largely monotonic and statistically positive relation between MVE and stock return. This result is in line with our inference.

With regard to the relation between MF_Opacity and stock returns around the GEJE, looking across each row, we observe very large negative returns for extremely opaque firms for all MVE quintiles in Panel B of Table 6. For example, the average CR[0; +1] of the lowest MF_Opacity quintile is −9.5% while that of the highest MF_Opacity quintile is −17.4%. The difference of −7.9% is statistically significant at the 1% level. Portfolios with higher MF_Opacity have a tendency for lower returns as shown in Panel B, which is similar to Panel C of Table 6. In sum, Table 6 suggests that the predictive power of MF_Opacity for stock returns around the GEJE is not due to the correlation between firm size and MF_Opacity. Rather, given the fact that the average stock returns do not relate to MVE in three MF_Opacity portfolios (i.e., Q2 MF_Opacity quintiles in Panel B and Q1 and Q2 MF_Opacity quintiles in Panel C), MF_Opacity itself can explain cross-sectional variation in stock returns around the GEJE.

Finally, the following model examines whether our inference from the analyses in Table 7 remains unchanged, even after controlling for the potential effect of various firm characteristics on stock returns;

\[
Ret_j = \beta_0 + \beta_1 MF\_Opacity_{j,t} + \sum_k \gamma_k Control\ variable_k + \varepsilon_{j,t},
\]  

(13)
where \( Ret_j \in \{R[0]_j, CR[0 : +1]_j, CR[0 : +2]_j\} \), \( R[0]_j \) represents the raw return for stock \( j \) on the day occurred the GEJE (day 0), and \( CR[0 : +k]_j \) indicates cumulative return stock \( j \) from day 0 to \(+k\) \((k \in \{+1, +2\})\) relative to day 0.

Table 7 reports the OLS coefficient estimates and White (1980) heteroscedasticity consistent \( t \)-statistics in parentheses. Regression 1 shows the result without considering the two opacity measures based on management forecast precision and abnormal accruals. The coefficient of determination is 4.2% and slightly low as in event studies in accounting and finance. However, with two opacity measures, the coefficient of determination increase by 1.7% and 3.2% in Regression 2 and 3, respectively. This implies that opacity measures are important factors in explaining the stock returns on the day of the GEJE. The results of the coefficient on ACC\_OPACITY in Regression 2 and 3 are noteworthy. While the coefficient is not negative but positive and statistically significant at marginal level in Regression 2, it is insignificant in Regression 3. Therefore, increasing \( R \)-squared from Regression 1 to 2 and 3 is mainly due to MF\_OPACITY.

In both Regression 2 and 3, we observed that coefficients on MF\_OPACITY are negative and significant at 1% level. For example, the coefficient on rank(MF\_OPACITY) is \(-0.011\) in Regression 3. This result means that the difference in stock returns on the GEJE day between the lowest and highest MF\_OPACITY is 1.1%, even after controlling for other determinants. The difference between these two groups grows larger as the event period lengthens. The difference is 8.7% (13.2%) when we set event period from day 0 to day +1 (+2) as shown in Regression 6 (9). These results suggest that MF\_OPACITY is economically important as well as statistically significant. Moreover, we can point out that the coefficient of determination with MF\_OPACITY has increased 1.5 times than the coefficient of determination without MF\_OPACITY for both event periods. This result implies that MF\_OPACITY has stronger explanatory power for stock returns around the GEJE than any other variables.

Overall, we interpret the evidence in Table 7 as suggesting that the stock price of firms disclosing opaque management forecasts tend to co-move with the market around the GEJE and, hence catastrophe-based collapse occurred. This is because investors fell into a crisis of limited attention due to the earthquake and were likely to pay little attention to specific firms and more attention to the market as a whole, and as a result, the stock price of such firms were more covariated with the market than transparent firms. Management-forecast-based opacity measures were able to predict stock returns when the unexpected catastrophic shock occurred. We conclude that the results in this section strongly support Hypothesis 5.
5 Conclusions

This study provides theory and empirical evidence to demonstrate that opacity of firms’ financial reports plays a role in influencing the distribution of stock returns. Developing a simple bayesian model in the presence of management earnings forecasts and market-wide information, we predict that at normal time, (1) firms with opaque financial reports have stock returns that are more synchronous with the market and (2) stock crashes are more frequent and more severe for opaque firms. Using the precision of effectively mandated management forecasts as a measure of opacity, we obtain empirical results consistent with these predictions.

In addition, we extend Peng and Xiong (2006) model to examine how investors’ limited attention arising from catastrophic event (i.e., an big earthquake and a tsunami) affects stock price synchronicity and stock crash in the market. Our model reveals that (3) investors are likely to pay more attention to the market-wide information than to firm-specific information, thus leading to higher synchronicity and (4) higher crash risk, and (5) the stock price of firms disclosing opaque financial reports tend to more co-move with the market around a catastrophic event and, hence catastrophe-based collapse occurred. Using the GEJE as a representative catastrophic event, ample support for these predictions is found in the data.

Our results have important implication for capital allocation. Since prior work shows that less stock price synchronicity (i.e., lower $R^2$) exhibit a better allocation of capital (e.g., Wurgler, 2000), our results suggest that the allocation efficiency improve through disclosing greater transparent of financial information and deteriorate around a catastrophic event. Besides, our results have implication for managers who wish to avoid a stock crash. Our results suggest that improving financial reporting reduce any collapse in a firm’s stock price.

References


This figure illustrates that given a certain level of an investor’s attention, the smaller $a_i$, the smaller the subjective variance is.
Figure 2: Magnitude relations of the widths of the ranges

This figure illustrates that the range $[\Lambda_1, 1]$ is wider than $[\Lambda_2, 1]$ and that the range $[0, \Lambda_2]$ is wider than $[0, \Lambda_1]$. 

\[ \sigma^2_{S,i} = -a_i \cdot \ln \lambda_i \]
Figure 3: Timeline for measurement of main variables.
Figure 4: Yearly cross-sectional distribution stock crash based on CRASH—20%

This figure plots the cross-sectional means of SEVERITY over our sample period from 2004 to 2011. SEVERITY is defined as a minimum weekly return during the year. We define year $t$ as the period from week after March 11 in year $t - 1$ to week before March 10 in year $t$. 
Figure 5: Yearly cross-sectional means of SEVERITY

This figure plots the cross-sectional means of SEVERITY over our sample period from 2004 to 2011. SEVERITY is defined as a minimum weekly return during the year. We define year $t$ as the period from week after Mach 11 in year $t-1$ to week before March 10 in year $t$.

Figure 6: Yearly cross-sectional means of $R^2$

This figure plots the cross-sectional means of $R^2$ estimated by Equation (9) over our sample period from 2004 to 2011. We define year $t$ as the period from week after Mach 11 in year $t-1$ to week before March 10 in year $t$. 
The GEJE occurred at 2:46 p.m., March 11, 2011.

Figure 7: Nikkei 225 during the Great East Japan Earthquake

This figure shows the price movement of the Nikkei 225 in the afternoon session, called the “Goba”, between 12:30 p.m. and 3:00 p.m. on March 11, 2011. The GEJE occurred at 2:46:18 p.m. The price of Nikkei 225 was 10,360.21 Japanese Yen before the GEJE, and collapsing by 1.02% between 2:46 p.m. and 3:00 p.m.
### Dependent variables

**Synchronicity Measure**  \( \Psi_{j,t} \)

\( \Psi_{j,t} \) is our measure of annual synchronicity based on \( R^2 \) from Equation (9) for firm \( i \) in year \( t \). Given the bounded nature of \( R^2 \), we use a logistic transformation of \( R^2 \), which can range from negative to positive infinity.

**Stock Crash Measure**  \( CRASH_{x\%j,t} \)

\( CRASH_{x\%j,t} \) is an indicator variable equal to one if firm \( j \) within year \( t \) experiences one or more weekly return fall \( x\% \) (\( x \in \{-15\%, -20\%, -25\%, -30\%\} \)) and equal to zero otherwise.

**SEVERITY_{j,t}**

\( SEVERITY_{j,t} \) is a minimum weekly return for firm \( j \) in year \( t \).

**Event Period Returns**  \( Ret_j \)

\( Ret_j \in \{R[0]_j, CR[0 : +1]_j, CR[0 : +2]_j\} \), \( R[0]_j \) represents the raw return for stock \( j \) on the day occurred the GEJE (day 0), and \( CR[0 : +k]_j \) indicates cumulative return stock \( j \) from day 0 to \( +k \) (\( k \in \{+1, +2\} \)) relative to day 0.

### Independent Variables

**Opacity Measure**  \( MF\_OPACITY_{j,t} \)

\( MF\_OPACITY_{j,t} \) is our opacity measure using management forecasts as the sum of the management forecast accuracy over the past three-year.

**ACC\_OPACITY_{j,t}**

\( ACC\_OPACITY_{j,t} \) is another opacity measure based on discretionary accruals as the sum of absolute abnormal accruals over the past three-year. Normal accruals are estimated in cross-section for each two-digit Nikkei Industrial Code and year combination using Modified Jones model.

**The GEJE Dummy**  \( D\_EARTHQUAKE_{j,t} \)

\( D\_EARTHQUAKE_{j,t} \) is an indicator variable equal to one year after earthquake (i.e., year 2011) and equal to zero otherwise.

**Control Variables**

\( \log(MVE)_{j,t-1} \)

\( \log(MVE)_{j,t-1} \) is the natural log of the market value of equity at the beginning of fiscal year \( t \).

\( \log(BM)_{j,t-1} \)

\( \log(BM)_{j,t-1} \) is the natural log of book-to-market ratio measured at the beginning of fiscal year \( t \).

\( RNOA_{j,t-1} \)

\( RNOA_{j,t-1} \) is return on net operation assets for year \( t - 1 \).

\( LEV_{j,t-1} \)

\( LEV_{j,t-1} \) is the book value of all liabilities divided by total asset, measured at the beginning of the fiscal year \( t - 1 \).

\( \log(VARIND)_{j,t} \)

\( \log(VARIND)_{j,t} \) is the natural log of the variance of the weekly industrial returns based on two-digit Nikkei Industrial Code during the firm’s fiscal year \( t \).

**SKEW_{j,t}**

\( SKEW_{j,t} \) is the skewness of the weekly returns for firm \( j \) over the fiscal year \( t \).

**KURT_{j,t}**

\( KURT_{j,t} \) is the kurtosis of the weekly returns for firm \( j \) over the fiscal year \( t \).

---

Table 1: Variable Definitions
### Panel A: Distribution of stock crash for each MF.OPACITY deciles

<table>
<thead>
<tr>
<th>Decile</th>
<th>CRASH - 15%</th>
<th>CRASH - 20%</th>
<th>CRASH - 25%</th>
<th>CRASH - 30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1 (Less opacity)</td>
<td>20.5%</td>
<td>10.5%</td>
<td>4.9%</td>
<td>2.1%</td>
</tr>
<tr>
<td>D2</td>
<td>20.3%</td>
<td>10.2%</td>
<td>4.7%</td>
<td>1.9%</td>
</tr>
<tr>
<td>D3</td>
<td>23.6%</td>
<td>11.4%</td>
<td>5.2%</td>
<td>2.4%</td>
</tr>
<tr>
<td>D4</td>
<td>24.4%</td>
<td>11.8%</td>
<td>5.0%</td>
<td>2.2%</td>
</tr>
<tr>
<td>D5</td>
<td>25.3%</td>
<td>12.5%</td>
<td>6.1%</td>
<td>2.9%</td>
</tr>
<tr>
<td>D6</td>
<td>27.3%</td>
<td>12.7%</td>
<td>6.0%</td>
<td>2.8%</td>
</tr>
<tr>
<td>D7</td>
<td>30.6%</td>
<td>14.6%</td>
<td>6.4%</td>
<td>2.8%</td>
</tr>
<tr>
<td>D8</td>
<td>34.3%</td>
<td>15.8%</td>
<td>8.1%</td>
<td>3.7%</td>
</tr>
<tr>
<td>D9</td>
<td>39.1%</td>
<td>20.1%</td>
<td>11.2%</td>
<td>5.7%</td>
</tr>
<tr>
<td>D10 (More opacity)</td>
<td>52.7%</td>
<td>30.1%</td>
<td>16.2%</td>
<td>9.5%</td>
</tr>
<tr>
<td>All Firms</td>
<td>29.8%</td>
<td>15.0%</td>
<td>7.4%</td>
<td>3.6%</td>
</tr>
<tr>
<td>Chi-squared statistics</td>
<td>797.3***</td>
<td>484.6***</td>
<td>326.2***</td>
<td>261.6***</td>
</tr>
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</table>

### Panel B: Mean and median value of SEVERITY for portfolios formed by MF.OPACITY

<table>
<thead>
<tr>
<th>Decile</th>
<th>Mean</th>
<th>Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1 (Less opacity)</td>
<td>-11.1%</td>
<td>-9.0%</td>
</tr>
<tr>
<td>D2</td>
<td>-11.3%</td>
<td>-9.3%</td>
</tr>
<tr>
<td>D3</td>
<td>-11.9%</td>
<td>-10.0%</td>
</tr>
<tr>
<td>D4</td>
<td>-12.0%</td>
<td>-10.2%</td>
</tr>
<tr>
<td>D5</td>
<td>-12.7%</td>
<td>-10.9%</td>
</tr>
<tr>
<td>D6</td>
<td>-12.9%</td>
<td>-11.1%</td>
</tr>
<tr>
<td>D7</td>
<td>-13.5%</td>
<td>-11.7%</td>
</tr>
<tr>
<td>D8</td>
<td>-14.0%</td>
<td>-12.2%</td>
</tr>
<tr>
<td>D9</td>
<td>-15.4%</td>
<td>-13.0%</td>
</tr>
<tr>
<td>D10 (More opacity)</td>
<td>-18.1%</td>
<td>-15.5%</td>
</tr>
<tr>
<td>All Firms</td>
<td>-13.3%</td>
<td>-11.3%</td>
</tr>
<tr>
<td>D10-D1</td>
<td>-7.6%***</td>
<td>-6.5%***</td>
</tr>
</tbody>
</table>

Table 2: Association between MF.OPACITY and stock crash
<table>
<thead>
<tr>
<th>Dependent Var.</th>
<th>Regression 1</th>
<th>Regression 2</th>
<th>Regression 3</th>
<th>Regression 4</th>
<th>Regression 5</th>
<th>Regression 6</th>
<th>Regression 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.752</td>
<td>1.012</td>
<td>0.095</td>
<td>-0.561</td>
<td>0.484</td>
<td>-0.189***</td>
<td>-0.188***</td>
</tr>
<tr>
<td></td>
<td>(0.91)</td>
<td>(0.62)</td>
<td>(0.030)</td>
<td>(-0.19)</td>
<td>(0.18)</td>
<td>(-4.57)</td>
<td>(-4.56)</td>
</tr>
<tr>
<td>MF.OPACITY</td>
<td>1.564***</td>
<td>1.295***</td>
<td>0.921***</td>
<td>1.010***</td>
<td>-0.055***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(5.33)</td>
<td>(4.36)</td>
<td>(3.67)</td>
<td>(4.38)</td>
<td>(-7.83)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACC.OPACITY</td>
<td>3.668***</td>
<td>3.846***</td>
<td>3.752***</td>
<td>3.820***</td>
<td>-0.160***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(8.25)</td>
<td>(7.08)</td>
<td>(7.48)</td>
<td>(7.67)</td>
<td>(-7.07)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rank(MF.OPACITY)</td>
<td>1.159***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.033***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(8.54)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-8.26)</td>
<td></td>
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<tr>
<td>Rank(ACC.OPACITY)</td>
<td>0.935***</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.034***</td>
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<tr>
<td></td>
<td>(7.46)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-6.04)</td>
<td></td>
</tr>
<tr>
<td>D.EARTHQUAKE</td>
<td>1.131***</td>
<td>1.302***</td>
<td>1.061***</td>
<td>0.947***</td>
<td>0.608***</td>
<td>-0.025***</td>
<td>-0.032***</td>
</tr>
<tr>
<td></td>
<td>(8.07)</td>
<td>(9.23)</td>
<td>(9.51)</td>
<td>(8.82)</td>
<td>(4.56)</td>
<td>(-6.36)</td>
<td>(-8.44)</td>
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<tr>
<td>Year Dummy 2007</td>
<td>1.114***</td>
<td>1.070***</td>
<td>1.048***</td>
<td>0.924***</td>
<td>0.988***</td>
<td>-0.032***</td>
<td>-0.030***</td>
</tr>
<tr>
<td></td>
<td>(8.71)</td>
<td>(9.50)</td>
<td>(9.35)</td>
<td>(8.69)</td>
<td>(4.85)</td>
<td>(-6.92)</td>
<td>(-8.10)</td>
</tr>
<tr>
<td>Year Dummy 2008</td>
<td>3.707***</td>
<td>3.721***</td>
<td>3.744***</td>
<td>3.451***</td>
<td>3.156***</td>
<td>-0.126***</td>
<td>-0.125***</td>
</tr>
<tr>
<td>log(MVE)</td>
<td>-0.188**</td>
<td>-0.175**</td>
<td>-0.169</td>
<td>-0.183</td>
<td>-0.260***</td>
<td>0.005***</td>
<td>0.005***</td>
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<tr>
<td></td>
<td>(5.37)</td>
<td>(5.50)</td>
<td>(5.28)</td>
<td>(5.32)</td>
<td>(-6.76)</td>
<td>(6.85)</td>
<td>(6.86)</td>
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<tr>
<td>log(BM)</td>
<td>-0.296</td>
<td>-0.351*</td>
<td>-0.321**</td>
<td>-0.284**</td>
<td>-0.186</td>
<td>0.007</td>
<td>0.010*</td>
</tr>
<tr>
<td></td>
<td>(7.10)</td>
<td>(7.54)</td>
<td>(7.60)</td>
<td>(7.65)</td>
<td>(7.32)</td>
<td>(7.34)</td>
<td>(7.35)</td>
</tr>
<tr>
<td>RNOA</td>
<td>-1.116***</td>
<td>-1.287***</td>
<td>-1.326***</td>
<td>-1.859***</td>
<td>-1.567***</td>
<td>0.054***</td>
<td>0.066***</td>
</tr>
<tr>
<td>LEV</td>
<td>0.425*</td>
<td>0.345</td>
<td>0.348</td>
<td>0.590*</td>
<td>0.516</td>
<td>-0.018**</td>
<td>-0.016*</td>
</tr>
<tr>
<td></td>
<td>(4.72)</td>
<td>(4.57)</td>
<td>(4.56)</td>
<td>(4.59)</td>
<td>(4.55)</td>
<td>(4.57)</td>
<td>(4.56)</td>
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<tr>
<td>Observations</td>
<td>18,395</td>
<td>18,395</td>
<td>18,395</td>
<td>18,395</td>
<td>18,395</td>
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<td>18,395</td>
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<tr>
<td>Pseudo R²</td>
<td>0.2626</td>
<td>0.2579</td>
<td>0.3085</td>
<td>0.2835</td>
<td>0.2572</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Adjusted R²</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td>0.3632</td>
<td>0.3416</td>
</tr>
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</table>

Table 3: Regression results of stock crash on MF.OPACITY and the GEJE
### Panel A: Mean $R^2$ for portfolios formed by MF.OPACITY

<table>
<thead>
<tr>
<th>MF.OPACITY</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1 (Less opacity)</td>
<td>39.5%</td>
</tr>
<tr>
<td>D2</td>
<td>38.1%</td>
</tr>
<tr>
<td>D3</td>
<td>36.2%</td>
</tr>
<tr>
<td>D4</td>
<td>35.6%</td>
</tr>
<tr>
<td>D5</td>
<td>35.0%</td>
</tr>
<tr>
<td>D6</td>
<td>34.2%</td>
</tr>
<tr>
<td>D7</td>
<td>32.9%</td>
</tr>
<tr>
<td>D8</td>
<td>31.7%</td>
</tr>
<tr>
<td>D9</td>
<td>30.7%</td>
</tr>
<tr>
<td>D10 (More opacity)</td>
<td>27.8%</td>
</tr>
<tr>
<td>All Firms</td>
<td>34.2%</td>
</tr>
</tbody>
</table>

### Panel B: Mean $R^2$ for portfolios formed by MF.OPACITY and firm size

<table>
<thead>
<tr>
<th>MF.OPACITY</th>
<th>Q1 (Small)</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5 (More opacity)</th>
<th>All Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1 (Small)</td>
<td>21.8%</td>
<td>21.3%</td>
<td>20.3%</td>
<td>22.0%</td>
<td>21.4%</td>
<td>21.4%</td>
</tr>
<tr>
<td>Q2</td>
<td>26.7%</td>
<td>25.9%</td>
<td>26.6%</td>
<td>26.4%</td>
<td>28.8%</td>
<td>26.9%</td>
</tr>
<tr>
<td>Q3</td>
<td>30.6%</td>
<td>31.6%</td>
<td>32.6%</td>
<td>33.1%</td>
<td>35.2%</td>
<td>32.6%</td>
</tr>
<tr>
<td>Q4</td>
<td>35.7%</td>
<td>37.8%</td>
<td>37.7%</td>
<td>38.8%</td>
<td>41.3%</td>
<td>38.3%</td>
</tr>
<tr>
<td>Q5 (Large)</td>
<td>50.0%</td>
<td>49.3%</td>
<td>50.2%</td>
<td>52.8%</td>
<td>56.1%</td>
<td>51.7%</td>
</tr>
</tbody>
</table>

All Firms    | 33.0%      | 33.2% | 33.5% | 34.6% | 36.6% |

Table 4: Association between $R^2$ and MF.OPACITY
<table>
<thead>
<tr>
<th>Dependent Var.</th>
<th>Regression 1</th>
<th>Regression 2</th>
<th>Regression 3</th>
<th>Regression 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Constant</strong></td>
<td>$-7.609^{***}$</td>
<td>$-7.922^{***}$</td>
<td>$-7.929^{***}$</td>
<td>$-8.002^{***}$</td>
</tr>
<tr>
<td></td>
<td>(−20.5)</td>
<td>(−16.8)</td>
<td>(−16.6)</td>
<td>(−14.5)</td>
</tr>
<tr>
<td><strong>MF_OPACITY</strong></td>
<td>0.355***</td>
<td>0.221***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.88)</td>
<td>(3.75)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ACC_OPACITY</strong></td>
<td>−0.049</td>
<td>−0.048</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(−0.96)</td>
<td>(−0.85)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>rank(MF_OPACITY)</strong></td>
<td></td>
<td></td>
<td>0.153***</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(2.60)</td>
<td></td>
</tr>
<tr>
<td><strong>rank(ACC_OPACITY)</strong></td>
<td></td>
<td></td>
<td>−0.041</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(−0.79)</td>
<td></td>
</tr>
<tr>
<td><strong>D_EARTHQUAKE</strong></td>
<td></td>
<td></td>
<td>0.388***</td>
<td>0.411***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(6.89)</td>
<td>(7.33)</td>
</tr>
<tr>
<td><strong>YearDummy2007</strong></td>
<td></td>
<td></td>
<td>−0.096**</td>
<td>−0.102**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(6.57)</td>
<td>(7.03)</td>
</tr>
<tr>
<td><strong>YearDummy2008</strong></td>
<td>0.111*</td>
<td>0.107</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>log(MVE)</strong></td>
<td>0.357***</td>
<td>0.370***</td>
<td>0.364***</td>
<td>0.365***</td>
</tr>
<tr>
<td></td>
<td>(6.89)</td>
<td>(7.33)</td>
<td>(6.57)</td>
<td>(7.03)</td>
</tr>
<tr>
<td><strong>log(BM)</strong></td>
<td>0.186***</td>
<td>0.198***</td>
<td>0.176***</td>
<td>0.165***</td>
</tr>
<tr>
<td><strong>RNOA</strong></td>
<td>−0.241*</td>
<td>−0.110</td>
<td>−0.126</td>
<td>−0.142</td>
</tr>
<tr>
<td><strong>LEV</strong></td>
<td>0.505***</td>
<td>0.444***</td>
<td>0.489***</td>
<td>0.470***</td>
</tr>
<tr>
<td><strong>log(VARIND)</strong></td>
<td>0.270***</td>
<td>0.270***</td>
<td>0.254***</td>
<td>0.249***</td>
</tr>
<tr>
<td><strong>SKEW</strong></td>
<td>−0.123***</td>
<td>−0.126***</td>
<td>−0.099**</td>
<td>−0.098**</td>
</tr>
<tr>
<td><strong>KURT</strong></td>
<td>0.004</td>
<td>0.003</td>
<td>−0.005</td>
<td>−0.004</td>
</tr>
<tr>
<td><strong>Observations</strong></td>
<td>18,395</td>
<td>18,395</td>
<td>18,395</td>
<td>18,395</td>
</tr>
<tr>
<td><strong>Adjusted $R^2$</strong></td>
<td>0.3994</td>
<td>0.4033</td>
<td>0.4220</td>
<td>0.4223</td>
</tr>
</tbody>
</table>

Table 5: Regression results of stock price synchronicity on MF\_OPACITY and the GEJE
Panel A: Mean $R[0]$ for portfolios formed by MF\_OPACITY and firm size

<table>
<thead>
<tr>
<th>Portfolio Type</th>
<th>Q1 (Less opacity)</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5 (More opacity)</th>
<th>Q5–Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1 (Small)</td>
<td>−0.6%</td>
<td>−0.5%</td>
<td>−1.2%</td>
<td>−1.5%</td>
<td>−1.5%</td>
<td>−0.9%***</td>
</tr>
<tr>
<td>Q2</td>
<td>−0.7%</td>
<td>−0.8%</td>
<td>−0.7%</td>
<td>−1.1%</td>
<td>−1.6%</td>
<td>−0.9%***</td>
</tr>
<tr>
<td>Q3</td>
<td>−0.9%</td>
<td>−1.0%</td>
<td>−1.1%</td>
<td>−1.4%</td>
<td>−1.8%</td>
<td>−0.9%***</td>
</tr>
<tr>
<td>Q4</td>
<td>−1.3%</td>
<td>−1.1%</td>
<td>−1.4%</td>
<td>−1.8%</td>
<td>−2.1%</td>
<td>−0.8%***</td>
</tr>
<tr>
<td>Q5 (Large)</td>
<td>−1.2%</td>
<td>−1.4%</td>
<td>−1.7%</td>
<td>−1.8%</td>
<td>−2.0%</td>
<td>−0.9%***</td>
</tr>
<tr>
<td>Q5–Q1</td>
<td>−0.6%**</td>
<td>−0.9%***</td>
<td>−0.5%**</td>
<td>−0.2%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel B: Mean CR[0 : +1] for portfolios formed by MF\_OPACITY and firm size

<table>
<thead>
<tr>
<th>Portfolio Type</th>
<th>Q1 (Less opacity)</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5 (More opacity)</th>
<th>Q5–Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1 (Small)</td>
<td>−9.5%</td>
<td>−8.9%</td>
<td>−11.7%</td>
<td>−13.6%</td>
<td>−17.4%</td>
<td>−7.9%***</td>
</tr>
<tr>
<td>Q2</td>
<td>−7.9%</td>
<td>−9.5%</td>
<td>−8.8%</td>
<td>−12.7%</td>
<td>−15.7%</td>
<td>−7.9%***</td>
</tr>
<tr>
<td>Q3</td>
<td>−9.0%</td>
<td>−10.2%</td>
<td>−10.6%</td>
<td>−12.2%</td>
<td>−15.8%</td>
<td>−6.8%***</td>
</tr>
<tr>
<td>Q4</td>
<td>−9.9%</td>
<td>−9.2%</td>
<td>−9.9%</td>
<td>−13.7%</td>
<td>−15.8%</td>
<td>−5.8%***</td>
</tr>
<tr>
<td>Q5 (Large)</td>
<td>−7.5%</td>
<td>−7.3%</td>
<td>−9.2%</td>
<td>−9.7%</td>
<td>−12.0%</td>
<td>−4.6%***</td>
</tr>
<tr>
<td>Q5–Q1</td>
<td>2.0%**</td>
<td>1.7%</td>
<td>2.5%**</td>
<td>3.9%***</td>
<td>5.4%***</td>
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</tr>
</tbody>
</table>

Panel C: Mean CR[0 : +2] for portfolios formed by MF\_OPACITY and firm size

<table>
<thead>
<tr>
<th>Portfolio Type</th>
<th>Q1 (Less opacity)</th>
<th>Q2</th>
<th>Q3</th>
<th>Q4</th>
<th>Q5 (More opacity)</th>
<th>Q5–Q1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1 (Small)</td>
<td>−20.0%</td>
<td>−19.5%</td>
<td>−25.8%</td>
<td>−28.4%</td>
<td>−33.9%</td>
<td>−13.9%***</td>
</tr>
<tr>
<td>Q2</td>
<td>−18.8%</td>
<td>−20.8%</td>
<td>−21.3%</td>
<td>−27.0%</td>
<td>−33.3%</td>
<td>−14.5%***</td>
</tr>
<tr>
<td>Q3</td>
<td>−19.7%</td>
<td>−21.6%</td>
<td>−23.8%</td>
<td>−25.1%</td>
<td>−32.6%</td>
<td>−13.6%***</td>
</tr>
<tr>
<td>Q4</td>
<td>−20.5%</td>
<td>−20.7%</td>
<td>−22.0%</td>
<td>−27.4%</td>
<td>−30.4%</td>
<td>−9.9%***</td>
</tr>
<tr>
<td>Q5 (Large)</td>
<td>−18.2%</td>
<td>−17.7%</td>
<td>−20.0%</td>
<td>−19.5%</td>
<td>−23.8%</td>
<td>−5.7%***</td>
</tr>
<tr>
<td>Q5–Q1</td>
<td>1.8%</td>
<td>1.8%</td>
<td>5.8%***</td>
<td>8.8%***</td>
<td>10.1%***</td>
<td></td>
</tr>
</tbody>
</table>

Table 6: Association between stock returns around the GEJE and MF\_OPACITY
<table>
<thead>
<tr>
<th>Dependent Var.</th>
<th>Regression 1</th>
<th>Regression 2</th>
<th>Regression 3</th>
<th>Regression 4</th>
<th>Regression 5</th>
<th>Regression 6</th>
<th>Regression 7</th>
<th>Regression 8</th>
<th>Regression 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.009*</td>
<td>0.019***</td>
<td>0.027***</td>
<td>-0.380***</td>
<td>-0.284***</td>
<td>-0.242***</td>
<td>-0.631***</td>
<td>-0.473***</td>
<td>-0.389***</td>
</tr>
<tr>
<td></td>
<td>(1.74)</td>
<td>(3.36)</td>
<td>(4.81)</td>
<td>(-14.7)</td>
<td>(-10.2)</td>
<td>(-8.36)</td>
<td>(-16.9)</td>
<td>(-11.9)</td>
<td>(-9.42)</td>
</tr>
<tr>
<td>$MF_OPACITY$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.009***</td>
<td>-0.082***</td>
<td></td>
<td></td>
<td>-0.126***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-5.39)</td>
<td>(-9.47)</td>
<td></td>
<td></td>
<td>(-11.0)</td>
</tr>
<tr>
<td>$ACC_OPACITY$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.006*</td>
<td>0.037</td>
<td></td>
<td></td>
<td>0.037</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(1.84)</td>
<td>(1.54)</td>
<td></td>
<td></td>
<td>(1.14)</td>
</tr>
<tr>
<td>$rank(MF_OPACITY)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.011***</td>
<td>-0.087***</td>
<td>-0.132***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(-9.86)</td>
<td>(-13.3)</td>
<td>(-14.2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$rank(ACC_OPACITY)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.001</td>
<td>0.005</td>
<td></td>
<td></td>
<td>-0.010</td>
</tr>
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<td></td>
<td></td>
<td>(0.73)</td>
<td>(0.84)</td>
<td></td>
<td></td>
<td>(-1.12)</td>
</tr>
<tr>
<td>log($MVE$)</td>
<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.002***</td>
<td>0.010***</td>
<td>0.006***</td>
<td>0.005***</td>
<td>0.016***</td>
<td>0.010***</td>
<td>0.008***</td>
</tr>
<tr>
<td>log($BM$)</td>
<td>0.004***</td>
<td>0.003***</td>
<td>0.004***</td>
<td>0.046***</td>
<td>0.044***</td>
<td>0.049***</td>
<td>0.058***</td>
<td>0.054***</td>
<td>0.060***</td>
</tr>
<tr>
<td>$RNOA$</td>
<td>0.012***</td>
<td>0.007**</td>
<td>0.007**</td>
<td>0.099***</td>
<td>0.053***</td>
<td>0.057***</td>
<td>0.165***</td>
<td>0.093***</td>
<td>0.103***</td>
</tr>
<tr>
<td>$LEV$</td>
<td>0.000</td>
<td>0.003**</td>
<td>0.005**</td>
<td>0.043***</td>
<td>0.071***</td>
<td>0.075***</td>
<td>0.018</td>
<td>0.061***</td>
<td>0.066***</td>
</tr>
<tr>
<td>Observations</td>
<td>2,559</td>
<td>2,559</td>
<td>2,559</td>
<td>2,559</td>
<td>2,559</td>
<td>2,559</td>
<td>2,559</td>
<td>2,559</td>
<td>2,559</td>
</tr>
<tr>
<td>Adjusted $R^2$</td>
<td>0.0421</td>
<td>0.0592</td>
<td>0.0740</td>
<td>0.1080</td>
<td>0.1546</td>
<td>0.1648</td>
<td>0.1054</td>
<td>0.1604</td>
<td>0.1731</td>
</tr>
</tbody>
</table>

Table 7: Regression results of stock returns around the GEJE on MF\_OPACITY
Appendix 1

\[
p(\theta_i | y_i, m) = \frac{p(\theta_i)p(y_i, m \mid \theta_i)}{\int_{-\infty}^{\infty} p(\theta_i)p(y, m \mid \theta_i)d\theta_i} \\
\propto p(\theta_i)p(y_i, m \mid \theta_i) \\
= \left( \frac{1}{\sqrt{2\pi}} \right)^3 \frac{1}{\sigma_i \sigma_m \sigma_{0,i}} \exp \left\{ -\frac{1}{2} \left[ \frac{(y_i - \theta_i)^2}{\sigma_i^2} + \frac{(m - (\theta_i + \theta_{-i}))^2}{\sigma_m^2} + \frac{(\theta - \mu_{0,i})^2}{\sigma_{0,i}^2} \right] \right\} \\
\]

\[
\frac{(y_i - \theta_i)^2}{\sigma_i^2} + \frac{(m - (\theta_i + \theta_{-i}))^2}{\sigma_m^2} + \frac{(\theta - \mu_{0,i})^2}{\sigma_{0,i}^2} \\
= \frac{1}{\sigma_i^2} (\theta_i^2 - 2y_i \theta_i + y_i^2) + \frac{1}{\sigma_m^2} (\theta_i^2 - 2(m - \theta_{-i}) \theta_i + m^2) + \frac{1}{\sigma_{0,i}^2} (\theta_i^2 - 2\mu_{0,i} \theta_i + \mu_{0,i}^2) \\
= \left( \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}^2} \right) \left\{ \theta_i^2 - 2 \left( \frac{1}{\sigma_i} + \frac{1}{\sigma_m} + \frac{1}{\sigma_{0,i}} \right) \left( \frac{y_i}{\sigma_i} + \frac{m - \theta_{-i}}{\sigma_m} + \frac{\mu_{0,i}}{\sigma_{0,i}} \right) \theta_i \right\} \\
+ \frac{y_i}{\sigma_i^2} \frac{m}{\sigma_m} + \frac{\mu_{0,i}}{\sigma_{0,i}} + \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}^2} \left( \frac{y_i}{\sigma_i} + \frac{m - \theta_{-i}}{\sigma_m} + \frac{\mu_{0,i}}{\sigma_{0,i}} \right)^2 \\
= \left( \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}^2} \right) \left\{ \theta_i^2 - \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}^2} \left( \frac{y_i}{\sigma_i} + \frac{m - \theta_{-i}}{\sigma_m} + \frac{\mu_{0,i}}{\sigma_{0,i}} \right)^2 \right\} \\
+ \frac{y_i}{\sigma_i^2} \frac{m}{\sigma_m} + \frac{\mu_{0,i}}{\sigma_{0,i}} \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}^2} \left( \frac{y_i}{\sigma_i} + \frac{m - \theta_{-i}}{\sigma_m} + \frac{\mu_{0,i}}{\sigma_{0,i}} \right)^2 \right\} \\
\]

Hence,

\[
p(\theta_i \mid y_i, m) \propto \left( \frac{1}{\sqrt{2\pi}} \right)^3 \frac{1}{\sigma y \sigma_m \sigma_{0,i}} \exp \left\{ -\frac{1}{2} \left[ A (\theta - B)^2 + C \right] \right\} \\
\propto \exp \left\{ -\frac{1}{2} \left[ A (\theta - B)^2 + C \right] \right\} = \exp \left\{ -\frac{1}{2} \left( \frac{1}{A} \right) (\theta - B)^2 \right\} \\
A = \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}^2} \\
B = \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}^2} \left( \frac{y_i}{\sigma_i^2} + \frac{m - \theta_{-i}}{\sigma_m^2} + \frac{\mu_{0,i}}{\sigma_{0,i}^2} \right) \\
C = \frac{y_i}{\sigma_i^2} \frac{m}{\sigma_m} + \frac{\mu_{0,i}}{\sigma_{0,i}} \frac{1}{\sigma_i^2} + \frac{1}{\sigma_m^2} + \frac{1}{\sigma_{0,i}^2} \left( \frac{y_i}{\sigma_i^2} + \frac{m - \theta_{-i}}{\sigma_m^2} + \frac{\mu_{0,i}}{\sigma_{0,i}^2} \right)^2 \\
\]

Therefore,

\[
p(\theta_i \mid y_i, m) \propto \exp \left\{ -\frac{1}{2} \left[ A (\theta - B)^2 + C \right] \right\} \\
= \exp \left\{ -\frac{1}{2} C \right\} \exp \left\{ -\frac{1}{2} \left( \frac{1}{A} \right) (\theta - B)^2 \right\} \\
\propto \exp \left\{ -\frac{1}{2} \left( \frac{1}{A} \right) (\theta - B)^2 \right\} \quad \blacksquare \\
\]

40
Appendix 2

From Equation (3),
\[ w_1^2 c_1^2 a_1 = \kappa^* \lambda_1^* , \quad \cdots , \quad w_n^2 c_n^2 a_n = \kappa^* \lambda_n^* . \]

\[ \frac{\sum w_i^2 c_i^2 a_i}{\sum \lambda_i^*} = \frac{\kappa^* \sum \lambda_i^*}{\sum \lambda_i^*} = \kappa^* \]

(A1)

From Equation (3) and (A1),
\[ \frac{(\sum w_i k_i)^2 a_m}{\lambda_m^*} = \frac{\sum w_i^2 c_i^2 a_i}{\sum \lambda_i^*} . \]

(5)

Appendix 3

We prove
\[ \forall i \in \{i = 1, 2, \ldots, n\}, \quad \left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m > \sum_{i=1}^{n} w_i^2 c_i^2 a_i . \]

(6)

Let \( k_{\text{min}} \) be the minimum value of \( k_i \).
\[ \left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m \geq \left( \sum_{i=1}^{n} w_i k_{\text{min}} \right)^2 a_m = \left( k_{\text{min}} \sum_{i=1}^{n} w_i \right)^2 a_m = k_{\text{min}}^2 a_m > 0 \]

This means that the minimum value of \( (\sum w_i k_i)^2 a_m \) is a positive value \( k_{\text{min}}^2 a_m \).

On the other hand, let \( c_{\text{max}} \) be the maximum value of \( c_i^2 a_i \).
\[ \sum_{i=1}^{n} w_i^2 c_i^2 a_i \leq \sum_{i=1}^{n} w_i^2 c_{\text{max}} = c_{\text{max}} \sum_{i=1}^{n} w_i^2 \]

Since \( \lim_{n \to \infty} \sum_{i=1}^{n} w_i^2 = 0 \) (Appendix 4 provides the proof) and \( \sum_{i=1}^{n} w_i^2 c_i^2 a_i > 0 \),
\[ \lim_{n \to \infty} \sum_{i=1}^{n} w_i^2 c_i^2 a_i = 0 . \]

Taken together, for sufficiently large \( n \),
\[ \left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m \geq k_{\text{min}}^2 a_m > c_{\text{max}} \sum_{i=1}^{n} w_i^2 \geq \sum_{i=1}^{n} w_i^2 c_i^2 a_i . \]

Appendix 4

We can write
\[ \forall i \in n, \quad w_i = \frac{1}{n} + \varepsilon_i \quad \text{where} \quad \sum_{i=1}^{n} \varepsilon_i = 0 . \]
Since \( \forall i \in \{1, 2, \ldots, n\} \), \( \lim_{n \to \infty} w_i = 0 \),

\[
\lim_{n \to \infty} \left( \frac{1}{n} + \varepsilon_i \right) = 0.
\]

Hence,

\[
\forall i \in \{1, 2, \ldots, n\}, \lim_{n \to \infty} \varepsilon_i = 0
\]

Using this,

\[
\lim_{n \to \infty} n \sum_{i=1}^{n} w_i^2 = \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{1}{n} + \varepsilon_i \right)^2
\]

\[
= \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{1}{n^2} + \frac{2\varepsilon_i}{n} + \varepsilon_i^2 \right)
\]

\[
= \lim_{n \to \infty} \left( \frac{1}{n} + \frac{2}{n} \sum_{i=1}^{n} \varepsilon_i + \varepsilon_i^2 \right)
\]

\[
= \lim_{n \to \infty} \frac{1}{n} + 2 \lim_{n \to \infty} \frac{1}{n} \sum_{i=1}^{n} \varepsilon_i + \lim_{n \to \infty} \varepsilon_i^2
\]

\[
= \lim_{n \to \infty} \frac{1}{n} + 2 \left( \lim_{n \to \infty} \frac{1}{n} \right) \sum_{i=1}^{n} \lim_{n \to \infty} \varepsilon_i + \left( \lim_{n \to \infty} \varepsilon_i \right) \left( \lim_{n \to \infty} \varepsilon_i \right)
\]

\[
= 0
\]

**Appendix 5**

\[
\left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m > \sum_{i=1}^{n} w_i^2 c_i^2 a_i \quad (i = 1, \ldots, n) \quad (6)
\]

We prove that the larger \( n \) is, the more likely Equation (6) holds. Let \( c_{\text{max}} \) be the maximum value of \( c_i^2 a_i \). The LHS of Equation (6) is

\[
\left( \sum_{i=1}^{n} w_i k_i \right)^2 a_m \geq \left( \sum_{i=1}^{n} w_i k_{\text{min}} \right)^2 a_m = \left( k_{\text{min}} \sum_{i=1}^{n} w_i \right)^2 a_m = k_{\text{min}}^2 a_m > 0.
\]

(A2)

On the other hand,

\[
\sum_{i=1}^{n} w_i^2 c_i^2 a_i \leq c_{\text{max}} \sum_{i=1}^{n} w_i^2
\]

Since from Appendix 4, \( \lim_{n \to \infty} \sum_{i=1}^{n} w_i^2 = 0 \),

\[
\lim_{n \to \infty} \sum_{i=1}^{n} w_i^2 c_i^2 a_i \leq 0
\]

However, since \( w_i^2 c_i^2 a_i > 0 \),

\[
\lim_{n \to \infty} \sum_{i=1}^{n} w_i^2 c_i^2 a_i \geq 0
\]
Taken together,

$$\lim_{n \to \infty} \sum_{i=1}^{n} w_i^2 c_i^2 a_j = 0$$  \hspace{1cm} (A3)

From (A2) and (A3), the larger $n$ is, the more likely Equation (6) holds.