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Abstract

This study examines how for-profit firms finance capital from investors through environmental, social, and governance (ESG) effort. It examines a situation with two types of investors: socially responsible and for-profit investors. In this scenario, firms outnumber all investors in the market and they must attract socially responsible investors to successfully obtain the capital they require. Regardless of whether socially responsible investors' search has significant effects, this study demonstrates that when a firm is willing to take ESG actions, regulators aiming to promote ESG may encourage investors to prioritize ESG performance in their investment choices; meanwhile, strengthening shareholders' rights or promoting corporate governance reform may not necessarily be ideal for them.

JEL Classification: D83, G23, G32, M14.

Keywords: ESG, matching intensity, search, social impact, socially responsible investor.

1. Introduction

The PwC report forecasts that in 2026, environmental, social, and governance (ESG)-oriented assets under management in the United States, Europe, and the Asia-Pacific region will respectively reach \$10.5 trillion, \$19.6 trillion, and \$3.3 trillion, representing an increase of more than double that in 2022, a 53% rise, and a triple rise, respectively.[1],[2] While many managers consider ESG actions to be less efficient than alternative investment strategies or cast doubt on current ESG evaluation methodologies, the recent trend in ESG investments implies that firms that do not act on ESG issues might risk losing their investors (see Eccles and Klimenko (2019)). This study develops a simple theoretical model for studying how recent trends in investors' prioritization of ESG affect firms' ESG actions. In particular, if a fraction of socially responsible investors increase in the market, or in other words, if an amount of the socially responsible capital increases, do firms invest more in ESG related activities? If such investors gain more power to influence the firm management, do firms increase their ESG actions? If the matching technology between firms and socially responsible investors is improved, do firms boost their ESG actions? In these scenarios, do socially responsible investors increase their costly engagement in the firms they invest in? Furthermore, if firms take ESG actions, will doing so reduce their profit which eventually results in less ESG actions in the future? All of these questions are examined in two different models: one in which only the firms make costly effort in search for the socially responsible investors and the other in which both the firms and the socially responsible investors make costly effort to match with each other.

This paper attempts to answer these questions by setting up a search model in which for-profit firms subject to financial constraints require capital for a project and search for investors to raise the required capital. Two types of investors are considered: for-profit investors who are purely interested in pecuniary payoffs and socially responsible investors who are interested in pecuniary payoffs and ESG performance. Owing to the limited pool of potential investors, firms do not want to miss out on the opportunity to

[1] See Michelson (2020).

[2] According to OECD (2021), "The growth of sustainable finance, including the increasing array of financial products, has attracted the attention of investors,... In particular, ESG investing has become a leading form of sustainable finance and has shifted from early stages of development towards mainstream finance in a number of OECD jurisdictions. This generally refers to the process of considering environmental, social and governance (ESG) factors when making investment decisions."

be matched with any investor in the market. For-profit investors and for-profit firms can match without incurring any costs because their interests are aligned; however, socially responsible investors wish to avoid investing in firms that do not actively engage in ESG. Hence, firms wish to attract capital from socially responsible investors by increasing the matching probability with them. One way to attract such investors is to take ESG-related action, that is, to increase the ESG effort and thereby improve social performance. Hence in this paper, the firm's ESG actions are synonymous to the firm's search cost to match up with the socially responsible investor. When determining the level of the ESG actions, firms need to consider two key elements. Firstly, they consider whether such actions would increase the chance of being matched with a socially responsible investor. Secondly, they evaluate whether such actions could marginally increase the firm's total returns and at what level.[3]

In sum, when a firm takes ESG actions in the current study, it affects not only the pecuniary returns and the social performance or reputation of the firm but also the matching probability. As such, the firm's expected payoff is modeled as a combination of the case in which the firm is matched with either of the two types of investors and the case in which no matching occurs. After a firm is matched with an investor, they negotiate the distribution of their expected payoffs through generalized Nash bargaining. Up to the end of Section 4, socially responsible investors exert a costly engagement effort after the bargaining but their search cost to match up with the for-profit firms is not significant or zero. This model setup is hereinafter referred to as the basic model. After Section 5, this paper explores an extension in which both the socially responsible investors and the for-profit firms make significant costly effort in the search process for each other. However, in this extension, for simplicity, the socially responsible investors do not make a costly ESG engagement in the firm they invest in.

With the basic model, the following results are obtained under one common condition: the for-profit firm is willing to take a positive level of ESG actions. The first result is that an increase in the number of socially responsible investors increases the firm's ESG

[3] Total returns are defined as the sum of pecuniary return and social impact created by the firm, both of which are affected by the firm's ESG actions. Socially responsible investors can also increase a part of social impact, which is a function of their engagement efforts. This part of social impact can be included in the definition of the firm's total returns. However, because this part of social impact is independent of the firm's ESG actions, its inclusion or exclusion in the firm's total returns does not affect the results.

actions, if an increase in the actions decreases the pecuniary returns of the project. This result holds even if an increase in the firm's ESG actions reduces the total returns of the project. However, the socially responsible investor's engagement effort is independent of the size of ESG capital.

Secondly, if the socially responsible investor gains more influence over firm management, it does not necessarily increase the firm's ESG actions. However, it always increases his engagement effort in firm management.

Thirdly, an improvement in the matching technology between firms and socially responsible investors induce firms to take more ESG actions. However, investor's engagement effort is independent of the improvement in the matching technology.

Intuitively, an increase in the number of ESG concerned investors increases the probability of a match between a firm and a socially responsible investor. Thus, when they are matched, even if an increase in a firm's ESG actions decreases the total returns of its project, an increase in the number of socially responsible investors raises the firm's marginal expected payoff from its ESG actions, if the ESG actions elasticity of the matching intensity (EAEMI) is sufficiently large.[4] Indeed, if the firm's ESG actions are within a positive range, the EAEMI is sufficiently large. On the other hand, the increase in the number of the socially responsible investor reduces the matching probability between the firm and the for-profit investors. This results in the increase in the firm's marginal expected payoff from its ESG actions if the marginal pecuniary returns are negative. In sum, whether the firm is matched with the for-profit or the socially responsible investor, the larger the number of socially responsible investors, the more the firm takes ESG action. By contrast, the socially responsible investor chooses his engagement effort after being matched with a firm where he is no longer concerned about the probability of being matched with a firm. Therefore, the engagement effort of the socially responsible investor is independent of the size of socially responsible capital.

The intuition for the results on the increased influence of the socially responsible investor over firm management is as follows. To start with, the increased influence on the management implies an increase in his bargaining power over the firm. An increase in the investor's bargaining power always increases his engagement effort because it increases his

[4]EAEMI is defined as the ratio of the percentage change in matching intensity to the percentage change in the firm's ESG actions. EAEMI is only applicable to the socially responsible investor because the matching intensity of for-profit investors does not depend on the firm's ESG actions.

marginal expected payoff with respect to his effort. However, the effect of an increase in the investor's bargaining power on a firm's ESG actions is uncertain. The reason is that although the firm's marginal expected payoff with respect to its ESG actions increases in response to the increase in the investor's engagement effort, the firm will receive less from its total returns as the investor's bargaining power increases.

The intuition for the result of the improvement in the matching technology on ESG action depends on the fact that the firm's marginal expected payoff with respect to its ESG actions increases in the improvement in the matching technology if the firm is willing to make a positive level of ESG actions. By contrast, the socially responsible investor's engagement effort is independent of the improvement in the matching technology because his engagement decision is made after the matching and is therefore not dependent on the matching function.

Below are the results from the extended model. Similar to the basic model, all results hold when the firm is willing to take a positive level of ESG actions. Firstly, an increase in socially responsible capital increases both the firm's ESG actions and the socially responsible investor's search effort if an increase in the firm's ESG actions raises the total returns of the project whilst reducing its pecuniary returns.

Secondly, an increase in the influence of the socially responsible investor over firm management reduces the firm's ESG actions but raises the socially responsible investor's search effort if an increase in the firm's ESG actions reduces the total returns of the project.

Thirdly, an improvement in the matching technology increases both the firm's ESG actions and the socially responsible investor's search effort if an increase in the firm's ESG actions raises the total returns of the project.

Several remarks about the results of the extended model are in order. Firstly, when the firm is matched with a socially responsible investor, the firm's marginal expected payoff from its ESG actions depends upon the marginal total returns of the project with respect to the firm's ESG actions. Hence, an additional condition is required to obtain the result of the extended model. Secondly, as for the case of socially responsible investor gaining more bargaining power over firm management, the result of the extended model is not ambiguous as in the basic model.

The findings regarding the increase in socially responsible capital and the increased influence of socially responsible investors have important policy implications. If the search effort of socially responsible investors does not have significant effects, a regulator that prefers to increase ESG actions from firms may adopt policies that encourage investors to be more conscious of firms' ESG performance, particularly if firms' pecuniary returns decrease with their ESG actions. Such behavior is a current trend in practice; an example is a regulator supporting the activities of nongovernmental organizations (NGOs) and nonprofit organizations (NPOs) that are in line with ESG and Sustainable Development Goals (SDGs). Meanwhile, a policy that grants socially responsible investors more power to the firm management may not be useful for regulators aiming to promote ESG investment. For example, corporate governance reforms or strengthening shareholder rights may not enhance firms' ESG actions.

By contrast, when the search effort of socially responsible investors has significant effects, a regulator aiming to increase firms' ESG actions may need to consider marginal total returns (an addition of the marginal pecuniary returns and the marginal social impact) with respect to firms' ESG actions. More specifically, even if the marginal pecuniary returns are negative, the regulator will benefit from increasing the number of investors interested in firms' ESG performance. This can be done by supporting the activities of NGOs and NPOs that are geared toward ESG and SDGs, because as long as the marginal social impact is sufficiently large the marginal total returns become positive even if the marginal pecuniary returns are negative. However, if the marginal social impact is not large enough to make marginal total returns positive, the regulator cannot benefit from making or supporting policies that give more power to socially responsible investors to influence firms. Such policies are for example, corporate governance reforms or strengthening shareholder rights.

Finally, an improvement in the matching technology may imply an augmentation of ESG information. Thus, the findings indicate that the augmentation of ESG information increases firms' ESG actions if the marginal total returns with respect to firms' ESG actions are positive, regardless of whether the search effort of socially responsible investors has significant effects.

The model in this paper is related to existing theoretical studies on sustainable investing and the implications for socially responsible investors. Deriving an ESG factor in an equilibrium asset pricing model when investors are interested in ESG, Pástor, Stambaugh, and Taylor (2021) indicate that sustainable investing exerts positive social impact by making firms greener and shifting real investment towards green firms. Adachi-Sato and Osano (2023) examine the engagement roles of sustainable and passive fund managers.

Several studies have examined how the presence of socially responsible investors affects equilibrium allocation when firms are subject to financing constraints and need to raise funds from socially responsible investors and traditional for-profit investors. Chowdhry, Davies, and Waters (2019), show that to alleviate project owners' incentives to overemphasize profits, socially responsible investors must hold financial claims if the project owners cannot commit to social aims when raising capital from for-profit investors alone. Studying the situation in which socially responsible and commercial investors compete to finance for-profit entrepreneurs, Green and Roth (2021), characterize the strategies for socially responsible investors that result in high social welfare and financial returns. Oehmke and Opp (2020), examine the optimal investment choices for socially responsible investors that bargain with entrepreneurs with interest in ESG. They show that total surplus is generally the highest in an economy with a balance between socially responsible and for-profit financial capital. This result implies that socially responsible and for-profit financial investors complement each other. Landier and Lovo (2022) discuss how investors' preferences, production technologies, and capital market frictions affect the size, emission threshold, and investment policies of socially responsible funds. Illustrating how socially responsible funds can exploit capital market friction in a model wherein firms search for capital, they indicate that a firm's incentive to reduce negative externalities increases with the size of the socially responsible capital.[5]

By contrast, this study focuses on firms' ESG actions which not only increase social impact but also serve as a way to attract socially responsible investors. Specifically, this paper illustrates that if socially responsible investors are not exhibiting search cost to match with the firms, the growth of socially responsible capital may increase firms' ESG actions if EAEMI is sufficiently large. Although their actions reduce their pecuniary returns, if socially responsible investors are engaged in a costly search for the match with the firms, the growth of socially responsible capital may increase firms' ESG actions.

[5] Heinkel, Kraus, and Zechner (2001); Edmans, Levit, and Schneemeier (2022); and Broccardo, Hart, and Zingales (2022) examine the problem of divestment. When a social planner sets a minimum susceptibility standard that all investments and production must satisfy, Inderst and Opp (2022) ask whether such labelling is socially optimal.

This occurs if the marginal social impact of firms with respect to their ESG actions is larger than the absolute value of the marginal pecuniary returns of firms with respect to their ESG actions and also when the marginal pecuniary returns of the firm are negative. Moreover, this study is the first to examine the effect of the bargaining power of proactive socially responsible investors on firm management.

The remainder of this paper is organized as follows. Section 2 describes the basic model. Section 3 characterizes the equilibrium of the basic model and Section 4 discusses the comparative static results. Section 5 extends the basic model by considering the search effort of socially responsible investors. Section 6 concludes the paper. The proofs for all the propositions, corollaries, and lemmas are provided in the Appendix.

2. The Basic Model

The economy consists of three groups of agents, namely, for-profit entrepreneurs, socially responsible investors, and for-profit investors. Time t is 0 to 5. All players are risk neutral. The presence of a safe asset normalized to deliver a zero net return is assumed.

There exists a measure S of socially responsible investors whose investment strategy considers not only financial performance but also social impact (performance). A measure P represents the for-profit investors who are purely interested in financial performance. It is also assumed that $S + P = N$, where N is a fixed measure of investors. Each investor is given one unit of capital but lacks the skill to run a firm.

There are a measure F of firms. Each firm requires one unit of capital to start a project but is not endowed with capital.[6] If a firm cannot raise capital, its payoff is 0. However, if a firm receives the requisite capital, it produces observable pecuniary returns $\pi \geq 0$ and observable social impact $w \geq 0$ at the final stage.

This section presents the basic model that characterizes the optimal ESG action decision of the firm and the optimal engagement effort decision of the socially responsible investor. In Section 5, the model is extended in which the socially responsible investor chooses the search effort instead of the engagement effort.

At the beginning of $t = 0$, a firm chooses an ESG action level $e \geq 0$ to maximize its expected payoff minus the ESG action cost. At $t = 1$, the firm searches for investors.

[6] The firm has only a limited amount of fund that enables it to pay for the cost of ESG actions.

After the firm finds an investor, at $t = 2$, the two parties negotiate the distribution of project returns: if the firm is matched with a socially responsible investor, they bargain over the sum of pecuniary returns, π, and social impact, w, enjoyed by the investor; if it is matched with a for-profit investor, they bargain over only the pecuniary returns. The justification for this bargaining procedure is discussed later in this section. Then, at $t = 3$, the firm begins the project with the capital provided by the investor. At $t = 4$, if the firm is matched with a socially responsible investor, the investor chooses an engagement effort level, $a \geq 0$, to the firm to maximize his payoff less the engagement effort cost. The socially responsible investor cannot commit to a level a at any stage prior to $t = 4$. Finally, at $t = 5$, the project's pecuniary returns and social impact are realized. In the subsequent sections, the participation constraints for socially responsible and for-profit investors are assume to be satisfied. Hence, this paper assumes that their expected payoff at $t = 2$ exceeds the unit capital cost.

Pecuniary returns, π, are a function of the observable ESG actions of a firm, e, at $t = 0$. Social impact, w, is a function of the observable ESG actions of a firm, e, and the ESG engagement effort of a socially responsible investor, a. Specifically, I assume that $\pi(0) \geq 0$ and $\pi''(e) < 0$ and that $w(e, a) = w_F(e) + w_S(a)$, where $w_F(0) \geq 0$, $w'_F(e) > 0$, $w''_F(e) < 0$, $w_S(0) \geq 0$, $w'_S(a) > 0$, and $w''_S(a) < 0$. As the firm's ESG actions e may lead to either $\pi'(e) \geq 0$ or $\pi'(e) < 0$, I do not predetermine the positive or negative sign of $\pi'(e)$. However, for simplicity, I assume that $\pi(e) \geq 0$ for any $e \geq 0$.

If a firm takes ESG actions e, it bears a cost $c_F(e)$, where $c_F(0) = 0$, $c'_F(e) > 0$, and $c''_F(e) > 0$. If a socially responsible investor exerts an effort a, it incurs a cost $c_S(a)$, where $c_S(0) = 0$, $c'_S(a) > 0$, and $c''_S(a) > 0$.[7]

This paper assumes that firms must first search for investors with whom to raise their required capital, and assume that $S + P = N < F$. Then, a firm that takes an ESG action e is randomly matched with a socially responsible investor with a probability $\frac{S}{F}\kappa\lambda_S(e)$, where $\kappa > 0$, $\lambda_S(0) > 0$, $\lambda'_S(e) > 0$, $\lambda''_S(e) < 0$, and $\lambda_S(\infty) < 1$. The firm is also randomly matched with a for-profit investor with a probability $\frac{P}{F}\lambda_P$, where $\lambda_P \in (0, 1)$. $\kappa\lambda_S(e)$ and λ_P denote the matching intensity. The matching intensity between a firm and a socially

[7] In fact, the engagement effort of the socially responsible investor may involve a firm's effort or actions in the period prior to $t = 5$. Consequently, the firm may bear additional cost $c_A(a)$. Nevertheless, this consideration does not affect our main results if it does not affect the matching probability discussed below.

responsible investor, $\kappa\lambda_S(e)$, increases with the firm's ESG actions, e, as the greater the ESG actions, the higher the probability of being matched with a socially responsible investor.

After a firm is matched with a socially responsible investor, the two parties negotiate the distribution of the sum of the pecuniary returns and social impact of the project through generalized Nash bargaining. Then, the pecuniary amount paid to the firm as compensation for the social impact directly comes out of bargaining between the firm and the socially responsible investor. Note that this additional pecuniary amount—(share of the firm)×(social impact)—in turn decreases the pecuniary returns of the socially responsible investor.[8] If a firm is matched with a for-profit investor, the two parties only consider the pecuniary returns in the bargaining. A firm has bargaining power $1 - \beta_S \in (0, 1)$ when it bargains with a socially responsible investor and $1 - \beta_P \in (0, 1)$, when it bargains with the for-profit investor. A socially responsible investor has bargaining power β_S, and for-profit investor has β_P. If bargaining fails, the outside option values of the firm and the investor are equal to zero because neither side has an exit option once the firm is matched with the investor.

If a firm with ESG actions e bargains with a socially responsible investor at $t = 0$, the application of the generalized Nash bargaining dictates that for a given engagement effort a, the firm's predicted payoff is $(1 - \beta_S)[\pi(e) + w_F(e) + w_S(a)]$ at $t = 2$, whereas the socially responsible investor's predicted payoff is $\beta_S[\pi(e) + w_F(e) + w_S(a)]$ at $t = 2$. By contrast, if the firm bargains with a for-profit investor, the application of the generalized Nash bargaining dictates that the firm's predicted payoff is $(1 - \beta_P)\pi(e)$ at $t = 2$, whereas the for-profit investor's predicted payoff is $\beta_P\pi(e)$ at $t = 2$.

3. Equilibrium

In the subsequent analysis, I work backward to derive the optimal ESG action decision of the firm and the optimal engagement effort decision of the socially responsible investor. Firstly, the maximization problem of the socially responsible investor with respect to the engagement effort at $t = 4$ is explored. Then, the maximization problem of the firm with respect to the ESG action level at $t = 0$ is examined.

[8] We assume that $\pi(e)$ is sufficiently large so that the pecuniary payoff of the socially responsible investor is positive in this bargaining.

I begin by discussing the maximization problem of the socially responsible investor at $t = 4$. As shown in the the preceding section, his maximization problem after bargaining is represented by

$$\max_{a \geq 0} \beta_S[\pi(e) + w_F(e) + w_S(a)] - c_S(a). \tag{1}$$

Hence, the first-order condition for problem (1) is given by

$$\beta_S w_S^{'}(a) - c_S'(a) = 0. \tag{2}$$

From the assumptions of $w_S(a)$ and $c_S(a)$, the solution to (1) is positive and satisfies the second-order condition.[9]

Next, I investigate the maximization problem of the firm at $t = 0$. As argued in the preceding section, the maximization problem of the firm is expressed as

$$\max_{e \geq 0} \frac{S}{F}\kappa\lambda_S(e)(1-\beta_S)\left[\pi(e) + w_F(e) + w_S(a)\right] + \frac{N-S}{F}\lambda_P(1-\beta_P)\pi(e) - c_F(e), \tag{3}$$

where the first term is the firm's expected payoff when it bargains with a socially responsible investor, the second term is the firm's expected payoff when it bargains with a for-profit investor, and the third term is the ESG action cost. As has been discussed in Section 2, note that the bargaining outcome includes the reward to social impact when the firm is matched with the socially responsible investor. $\frac{S}{F}\kappa\lambda_S(e)$ and $\frac{P}{F}\lambda_P$ are the probabilities that the firm is matched with a socially responsible investor and a for-profit investor, respectively. Thus, $1 - \frac{S}{F}\kappa\lambda_S(e) - \frac{P}{F}\lambda_P$ is the probability of the case in which the firm is not matched with any investor; hence, the payoff is 0.

The first-order condition for (3) is expressed as

$$\begin{aligned} \frac{S}{F}\kappa(1-\beta_S)\left\{\lambda_S'(e)\left[\pi(e) + w_F(e) + w_S(a)\right] + \lambda_S(e)\left[\pi'(e) + w_F'(e)\right]\right\} \\ + \frac{N-S}{F}\lambda_P(1-\beta_P)\pi'(e) - c_F'(e) = 0. \end{aligned} \tag{4}$$

The left-hand side of (4) is the firm's expected marginal payoff with respect to its ESG actions e. I assume that the solution to (3) satisfies the second-order condition. In fact,

[9] More precisely, we need the conditions $\lim_{a\to 0} \beta_S w_S'(a) > \lim_{a\to 0} c_S'(a)$ and $\lim_{a\to\infty} \beta_S w_S'(a) < \lim_{a\to\infty} c_S'(a)$ to ensure that the solution to (1) is positive.

if $\pi'(e) + w'_F(e) < 0$ and if $\lambda_S(e)$ were to be independent of e (and is thus treated as if it were a constant), the value of (3) is decreasing in e, because the assumption of $w'_F(e) > 0$ implies $\pi'(e) < 0$ if $\pi'(e) + w'_F(e) < 0$. As the smallest e can be 0, the optimal solution becomes the corner solution. In fact, the matching intensity λ depends on e and $\lambda'_S(e) > 0$. Thus, I can prove that the solution to (3) becomes positive under a certain condition.

To this end, define $\zeta(e) = \frac{e}{\kappa\lambda_S(e)}\frac{d[\kappa\lambda_S(e)]}{de}$ and $\eta(a,e) = \frac{e}{\pi(e)+w_F(e)+w_S(a)}\frac{\partial[\pi(e)+w_F(e)+w_S(a)]}{\partial e}$. Here, $\zeta(e)$ is the ESG action elasticity of matching intensity (EAEMI), which indicates how much the matching intensity increases given an increase in one unit of ESG actions.[10] On the other hand, $\eta(a,e)$ is the ESG action elasticity of the firm's total returns, where $\pi(e) + w_F(e)$ is referred to as the firm's total returns. It measures the extent to which the total returns are affected by a unit increase in the firm's ESG actions.[11] Under some reasonable assumptions, $\zeta(e)$ is always positive, whereas $\eta(a,e)$ can be negative if an increase in e reduces the total returns of the project, implying the marginal total returns of the project with respect to e are negative (i.e., $\pi'(e) + w'_F(e) < 0$).

The following lemma is a necessary condition for the ESG actions, e, to be positive:

Lemma 1: *If the solution to (3) is positive,* $\zeta(e) > -\eta(a,e)$ *holds.*

Lemma 1 shows that if the firm's optimal ESG actions are positive (i.e., $e^* > 0$), $\zeta(e)$ must be sufficiently more elastic than $-\eta(a,e)$.

Intuitively, if $\zeta(e) \leq -\eta(a,e)$, the firm's expected marginal payoff—the left-hand side of (4)—becomes negative, which implies that the firm cannot choose a positive level of ESG actions.

This study can now assume that the firm's optimal ESG actions are positive. Therefore, the equilibrium values of the ESG actions of the firm, e^*, and the engagement effort of the socially responsible investor, a^*, are simultaneously determined by (2) and (4).

4. Comparative Statics

This paper discusses the effects of the key parameters of the model on the ESG actions of the firm, e^*, and the engagement effort of the socially responsible investor, a^*, in equilibrium. The key parameters are the measure of socially responsible investors, S; the

[10] Recall that only the matching intensity for the socially responsible investor is affected by the firm's ESG effort. Thus, ζ only applies to the socially responsible investor.

[11] The investor's engagement effort, a, is fixed in $\eta(a,e)$.

bargaining power of socially responsible investors relative to that of firms, β_S; and the shift of the matching intensity function between firms and socially responsible investors, κ.

I first explore the effect of an increase in the measure of socially responsible investors, S, on e^* and a^*, respectively. Parameter S can be viewed as the size of socially responsible capital. Thus, the recent trend characterized by investors' growing concern about firms' ESG performance can be captured as an increase in S.

By conducting comparative statics using (2) and (4) with $\zeta(e)$ and $\eta(a, e)$, the following proposition is obtained:

Proposition 1:

Suppose that the firm's optimal ESG actions are in the positive range, $e^* > 0$. Then:

(i) An increase in S increases e^* if $\pi'(e) < 0$;

(ii) a^* is independent of S.

Indeed, even if the firm's marginal total returns with respect to e are negative, that is, $\pi'(e) + w_F'(e) < 0$, which results in $\eta(a, e) \leq 0$, the result of Proposition 1(i) indicates that the following corollary holds.

Corollary to Proposition 1: *Even if* $\pi'(e) + w_F'(e) < 0$, *an increase in S increases e^* if the firm is willing to make a positive level of ESG actions.*[12]

Proposition 1 shows that as the measure of socially responsible investors/capital increases (i.e., a larger S), the firm's equilibrium ESG actions, e^*, increase if the firm is willing to take a positive level of ESG actions and if the firm's marginal pecuniary returns with respect to e are negative (i.e., $\pi'(e) < 0$). The corollary to Proposition 1 also indicates that even if the marginal total returns with respect to e are negative, an increase in S increases e^* if the firm is willing to take a positive level of ESG actions, that is, $\zeta(e)$ is sufficiently large.[13] Meanwhile, Proposition 1 shows that an increase in S has no effect on the socially responsible investor's engagement effort, a^*.

The rationale for Proposition 1 is as follows. An increase in S leads to a higher probability of a firm being matched with a socially responsible investor. Hence, if the firm's marginal total returns with respect to e are nonnegative (i.e., $\pi'(e) + w_F'(e) \geq 0$) which

[12]Note that $\pi'(e) + w_F'(e) < 0$ automatically implies $\pi'(e) < 0$ under the assumption of $w_F'(e) > 0$.

[13]Note that $\eta(a, e) < 0$ if $\pi'(e) + w_F'(e) < 0$. Then, Lemma 1 implies that $\zeta(e)$ needs to be large enough to ensure $e^* > 0$.

always leads to $\eta(a, e) \geq 0$ and thereby yields $\zeta(e) > -\eta(a, e)$, an increase in S increases the firm's expected marginal payoff attained by being matched with a socially responsible investor. Furthermore, if the marginal pecuniary returns with respect to e are negative (i.e., $\pi'(e) < 0$), an increase in S also raises the firm's expected marginal payoff attained by being matched with a for-profit investor. Accordingly, an increase in S increases e^*. Even if the firm's marginal total returns are negative (i.e., $\pi'(e) + w_F'(e) < 0$) and, therefore, $\eta(a, e) \leq 0$ and $\pi'(e) < 0$, the increasing effect of a rise in S on the firm's marginal payoff continues to hold if $\zeta(e)$ is sufficiently large so as to satisfy $\zeta(e) > -\eta(a, e)$. Indeed, Lemma 1 indicates that the condition of $\zeta(e) > -\eta(a, e)$ must be automatically satisfied if $e^* > 0$. Consequently, the increase in S increases e^* within the range of $e^* > 0$.

For the intuition behind the effect of S on a^*, note that the socially responsible investor determines a after being matched with the firm. Thus, the socially responsible investor's decision does not depend on S. Hence, S does not affect a^*.

The theoretical implications of Proposition 1 and its corollary are as follows. As mentioned previously, an increase in S can be viewed as an increase in ESG-focused capital. Thus, Proposition 1 suggests that when the firm is willing to take some positive actions on ESG, its actions grow when the number of ESG concerned investors grow. However, Proposition 1 also implies that a growing number of such investors do not affect their engagement effort toward firm management.

In particular, the corollary to Proposition 1 suggests that even if the marginal total returns of the project with respect to e are negative (i.e., an increase in the firm's ESG actions decreases the pecuniary returns of the project more than it increases the project's social impact), if the number of ESG investors increases, the firm's ESG actions increase as long as $\zeta(e)$ is sufficiently larger than $-\eta(a, e)$.

Next, I examine the effect of the bargaining power of socially responsible investors over firms, β_S, on e^* and a^*. β_S can be interpreted as the extent of the influence of socially responsible investors on firm management. Hence, an increase in β_S may imply extensive development in a social trend that gives socially responsible investors greater influence over firms because socially responsible investments are beginning to be publicly considered as significantly valuable for regulators seeking sustainable development.

The following proposition is with regards to an increase in β_S.

Proposition 2:

Suppose that the firm's optimal ESG actions are in the positive range, $e^* > 0$. Then:
(i) The effect of β_S on e^* is ambiguous;
(ii) An increase in β_S increases a^*.

Even if $\pi'(e) + w'_F(e) < 0$ to satisfy $\eta(a, e) < 0$, the following corollary holds.

Corollary to Proposition 2: *Even if $\pi'(e) + w'_F(e) < 0$, the effect of marginal increase in β_S on e^* is ambiguous if the firm's ESG actions are in the positive range.*

Proposition 2 indicates that the increased bargaining power of the socially responsible investor (i.e., larger β_S) does not necessarily increase the ESG actions of the firm, e^*, if the firm is willing to take ESG actions. This result holds even if the firm's marginal total returns are negative. Meanwhile, Proposition 2 states that a larger β_S always increases the engagement effort of the socially responsible investor, a^*.

The rationale behind Proposition 2 is as follows: I start with the effect of a^*. An increase in β_S implies greater bargaining power for the socially responsible investor, thus increasing the expected marginal payoff for the socially responsible investor with respect to a. Hence, an increase in β_S increases a^*.

To examine the rationale behind the effect of β_S on e^*, firstly, an increase in a^* in response to an increase in β_S also raises the firm's marginal expected payoff with respect to e, and therefore, e^*. However, if the firm takes a positive level of ESG actions to satisfy $\zeta(e) > -\eta(a, e)$ (see Lemma 1), the first two terms on the left-hand side of (4) are positive. Thus, an increase in β_S has an effect to reduce the firm's expected marginal payoff when it is matched with a socially responsible investor because he will take a larger share from the total returns of the project. Due to these two competing effects, the total effect of β_S on e^* is ambiguous. Even if a firm's marginal total returns are negative (i.e., $\pi'(e) + w'_F(e) < 0$), the effect of β_S on e^* remains ambiguous because an increase in a^* in response to an increase in β_S raises the firm's expected marginal payoff.

The theoretical implications of Proposition 2 and its corollary are as follows. As discussed earlier, an increase in β_S can be considered as a social trend that gives socially responsible investors greater influence on firm management. Hence, Proposition 2 suggests that the extensive development in this social trend does not necessarily increase the firm's ESG actions even if the firm is willing to take a positive level of ESG actions. However, Proposition 2 indicates that such extensive development always increases the engagement effort level of the existing socially responsible investors.

In addition, the corollary to Proposition 2 suggests that even if the marginal total returns of the project with respect to e are negative, giving socially responsible investors more power to engage in firm management does not have a clear effect on the firm's ESG actions.[14]

By comparing the results of Propositions 1 and 2, the study demonstrates that if the firm's ESG actions are in the positive range but reduces its pecuniary returns, the firm's ESG actions increase with the number of investors interested in firm's ESG performance. Meanwhile, giving socially responsible investors more power to influence firm management does not necessarily increase the firm's ESG actions. Consequently, if the firm is willing to take a positive level of ESG actions that may reduce its pecuniary returns, the regulator aiming to increase the firm's ESG actions should adopt policies that encourage investors to be conscious of firm's ESG performance. An example of such policies is to support the activities of NGOs and NPOs that are geared toward ESG and SDGs. By contrast, this regulator may not necessarily benefit from making/supporting policies that give socially responsible investors greater influence on firms. Examples of such policies include corporate governance reform and strengthening shareholder rights.

Finally, this paper explores the effect of the change in the matching intensity between firms and socially responsible investors on e^* and a^*. The change in the matching intensity (i.e., an increase in κ) can indicate a variation in the efficiency of the matching and search technologies of firms and socially responsible investors.

The following proposition with regard to the upper shift of κ is derived.

Proposition 3:

Suppose that the firm's optimal ESG actions are in the positive range, $e^* > 0$. Then:

(i) An increase in κ increases e^*;

(ii) a^* is independent of κ.

Proposition 3 demonstrates that the upper shift of the matching intensity (an increase in κ) increases the ESG actions of the firm, e^*. Proposition 3 also states that the upper shift of the matching intensity has no effect on the socially responsible investor's engagement effort, a^*.

[14] Recall that total returns are defined as $\pi(e) + w_F(e)$. As $w'_F(e) > 0$ always hold, when the marginal total returns being negative implies that $\pi'(e) < 0$ and $|\pi'(e)| > |w'_F(e)|$.

Intuitively, a larger κ increases the firm's expected marginal payoff when $\zeta(e) > -\eta(a, e)$. Hence, Lemma 1 ensures that a larger κ increases e^* if $e^* > 0$. As the decision of the socially responsible investor is made after being matched with the firm, the investor's decision does not depend on the matching intensity. Hence, a^* is independent of κ.

The theoretical implication of Proposition 3 is as follows. An increase in κ may imply an augmentation of ESG information. Thus, Proposition 3 suggests that an augmentation of ESG information increases the firm's ESG actions if the firm is willing to take a positive level of ESG actions. Proposition 3 also indicates that an augmentation of ESG information does not affect the engagement effort level of the socially responsible investor.

5. Search Efforts of Socially Responsible Investors

In this section, the basic model is extended by considering a two-sided matching model in which socially responsible investors can choose an effort level to search for ESG-related firms at $t = 0$. However, for simplicity, socially responsible investors do not make an ESG engagement effort at $t = 4$.

It is assumed that each firm attracts socially responsible investors to raise capital by expending an ESG action e at $t = 0$, while each socially responsible investor searches for ESG-related firms by expending a search effort ℓ at $t = 0$.

As in the basic model, if a firm takes ESG actions e, it bears a cost $c_F(e)$, where $c_F(0) = 0$, $c'_F(e) > 0$, and $c''_F(e) > 0$. If a socially responsible investor exerts a search effort ℓ, it incurs a nonpecuniary cost $c_M(\ell)$, where $c_M(0) = 0$, $c'_M(\ell) > 0$, and $c''_M(\ell) > 0$.

At $t = 1$, a firm that has taken an ESG action e is randomly matched with a socially responsible investor that has made an effort $\ell \geq 0$, with a probability $\frac{S}{F}\kappa\lambda_S(e, \ell) \equiv \frac{S}{F}\kappa[\lambda_{S1}(e) + \lambda_{S2}(\ell)]$, where $\lambda_{Sj}(0) > 0$, $\lambda'_{Sj}(i) > 0$, $\lambda''_{Sj}(i) < 0$, and $\lambda_{Sj}(\infty) < 1$ for $i = e, \ell$ and $j = 1, 2$. As in the basic model, the firm is still randomly matched with a for-profit investor with a probability $\frac{P}{F}\lambda_P$, where $\lambda_P \in (0, 1)$. The matching intensity between a firm and a socially responsible investor, $\kappa\lambda_S(e, \ell)$, increases with both the firm's ESG action e and the socially responsible investor's effort ℓ.

As the socially responsible investor does not make an engagement effort in this section, it is assumed that the social impact of the project depends on only a firm's ESG action e, that is, $w_F(e)$. I also assume that $w_F(0) \geq 0$, $w'_F(e) > 0$, and $w''_F(e) < 0$. Given that a socially responsible investor chooses the search effort at $t = 0$, his maximization problem at $t = 0$ is now represented by

$$\max_{\ell \geq 0} \frac{S}{F}\kappa[\lambda_{S1}(e) + \lambda_{S2}(\ell)]\beta_S\left[\pi(e) + w_F(e)\right] - c_M(\ell), \tag{5}$$

where the first term is his expected payoff enjoyed when he is matched with a firm, and the second term is his search cost. Note that $\frac{S}{F}\kappa[\lambda_{S1}(e) + \lambda_{S2}(\ell)]$ is the probability of the socially responsible investor being matched with a firm. In addition, the socially responsible investor needs to pay $(1-\beta_S)w_F(e)$ to the firm as a result of bargaining when he is matched with the firm.

The first-order condition for problem (5) is provided by

$$\frac{S}{F}\kappa\lambda'_{S2}(\ell)\beta_S\left[\pi(e) + w_F(e)\right] - c'_M(\ell) = 0. \tag{6}$$

It follows from the assumptions of $\lambda_{S2}(\ell)$ and $c_M(\ell)$ that the solution to (6) is positive and satisfies the second-order condition.[15]

Next, the maximization problem of the firm with respect to the ESG actions at time 0 is given as follows. As in the basic model, the firm's maximization problem is given by

$$\max_{e \geq 0} \frac{S}{F}\kappa[\lambda_{S1}(e) + \lambda_{S2}(\ell)](1-\beta_S)\left[\pi(e) + w_F(e)\right] + \frac{N-S}{F}\lambda_P(1-\beta_P)\pi(e) - c_F(e), \tag{7}$$

where the first term is its expected payoff when it bargains with a socially responsible investor, the second term is its expected payoff when it bargains with a for-profit investor, and the third term is the ESG action cost. Note that $1 - \frac{S}{F}\kappa[\lambda_{S1}(e) + \lambda_{S2}(\ell)] - \frac{N-S}{F}\lambda_P$ is the probability of the case in which the firm is not matched with any investors, and hence the payoff is 0 in this case.

The first-order condition for (7) is expressed as

$$\frac{S}{F}\kappa(1-\beta_S)\left\{\lambda'_{S1}(e)\left[\pi(e) + w_F(e)\right] + [\lambda_{S1}(e) + \lambda_{S2}(\ell)]\left[\pi'(e) + w'_F(e)\right]\right\}$$

[15] More precisely, the conditions $\lim_{a\to 0} \frac{S}{F}\kappa\beta_S\left[\pi(e) + w_F(e)\right]\lambda'_{S2}(\ell) > \lim_{a\to 0} c'_M(\ell)$ and $\lim_{a\to\infty} \frac{S}{F}\kappa\beta_S\left[\pi(e) + w_F(e)\right]\lambda'_{S2}(\ell) < \lim_{a\to\infty} c'_M(\ell)$ must be considered for any $e \geq 0$ to ensure that (6) has a positive solution.

$$+\frac{N-S}{F}\lambda_P(1-\beta_P)\pi'(e)-c_F'(e)=0. \tag{8}$$

I assume that the solution to (8) satisfies the second-order condition, which is represented by $\Gamma^\circ < 0$, where Γ° is defined by (10) below.

To show that the solution to (8) is positive, the following condition is necessary. Define $\zeta^\circ(e,\ell)=\frac{e}{\kappa[\lambda_{S1}(e)+\lambda_{S2}(\ell)]}\frac{\partial\{\kappa[\lambda_{S1}(e)+\lambda_{S2}(\ell)]\}}{\partial e}$ and $\eta^\circ(e)=\frac{e}{\pi(e)+w_F(e)}\frac{\partial[\pi(e)+w_F(e)]}{\partial e}$. Here, $\zeta^\circ(e,\ell)$ is the EAEMI in the extended model. The firm's elasticity of the total returns with respect to e is expressed as $\eta^\circ(e)$ in this extended model. Again, under our assumptions, $\zeta^\circ(e,\ell)$ is always positive, whereas $\eta^\circ(e)$ is negative if $\pi'(e)+w_F'(e)<0$.

Then, the following lemma similar to Lemma 1 is obtained.

Lemma 2: *If the solution to (8) is positive, then $\zeta^\circ(e,\ell) > -\eta^\circ(e)$.*

The intuition is similar to that of Lemma 1.

As in the main model, the firm's optimal ESG actions are assumed to be positive. Then, the equilibrium values of the ESG actions of the firm, e^*, and the search effort of the socially responsible investor, ℓ^*, are simultaneously determined by (6) and (8).

In order to discuss the comparative static results of S, β_S, and κ on e^* and ℓ^*, I impose the following assumption, which is a sufficient condition for l and e to decrease as the effort increases.

Assumption 1:

$$\Psi^\circ \equiv \Gamma^\circ\left\{\frac{S}{F}\kappa\beta_S\lambda_{S2}''(\ell)\left[\pi(e)+w_F(e)\right]-c_M''(e)\right\}$$

$$-\beta_S(1-\beta_S)\left\{\frac{S}{F}\kappa\lambda_{S2}'(\ell)\left[\pi'(e)+w_F'(e)\right]\right\}^2>0. \tag{9}$$

Here,

$$\begin{aligned}\Gamma^\circ \equiv\ & \frac{S}{F}\kappa(1-\beta_S)\left\{\lambda_{S1}''(e)\left[\pi(e)+w_F(e)\right]+2\lambda_{S1}'(e)\left[\pi'(e)+w_F'(e)\right]\right.\\ & \left.+[\lambda_{S1}(e)+\lambda_{S2}(\ell)]\left[\pi''(e)+w_F''(e)\right]\right\}+\frac{N-S}{F}\lambda_P(1-\beta_P)\pi''(e)-c_F''(e)\\ & <0, \end{aligned} \tag{10}$$

where the inequality is derived because the solution to (7) is assumed to satisfy the second-

order condition.

This assumption is a sufficient condition that ensures that an increase in the ESG action cost of the firm decreases the ESG actions, formally represented by $\frac{de^*}{dc'_F} < 0$, and an increase in search effort of the socially responsible investor decreases search effort, formally represented by $\frac{d\ell^*}{dc'_E} < 0$. For this proof, see the Appendix.

By executing comparative statics using (6) and (8) under Assumption 1, the following proposition regarding the effect of an increase in S is obtained:

Proposition 4:
Suppose that the firm's optimal ESG actions are in the positive range, $e^* > 0$.[16] *Then, an increase in S increases e^* and ℓ^* if $\pi'(e) \leq 0$ and $\pi'(e) + w'_F(e) \geq 0$.*

Proposition 4 demonstrates that as the fraction of socially responsible capital increases (i.e., a larger S), both the firm's ESG actions, e^*, and the socially responsible investor's search effort, ℓ^*, increase if the firm is willing to take a positive level of ESG actions and if the firm's marginal total returns with respect to e are nonnegative (i.e., $\pi'(e) + w'_F(e) \geq 0$) whilst the firm's marginal pecuniary returns with respect to e are nonpositive ($\pi'(e) \leq 0$). Thus, if the firm is willing to take a positive level of ESG actions, the sufficient condition for Proposition 4 is that the firm's marginal social impact with respect to e must be sufficiently larger than the absolute value of the firm's marginal pecuniary returns with respect to e when the latter is negative. In contrast to the basic model, this proposition suggests that an increase in S does not necessarily increase e^* if $\pi'(e) + w'_F(e) < 0$. In addition, note that a larger S can affect ℓ^* because the socially responsible investor chooses ℓ before being matched with the firm.

The intuition for Proposition 4 is as follows. An increase in S implies a higher probability of a firm being matched with a socially responsible investor. Hence, if the firm's marginal total returns with respect to e are nonnegative (i.e., $\pi'(e) + w'_F(e) \geq 0$) such that they always satisfy $\zeta^\circ(e) > -\eta^\circ(e)$, an increase in S raises the firm's expected marginal payoff with respect to e and ℓ through matching. In addition, given $\lambda'_{S2}(\ell) > 0$, the increase in ℓ (e) as a result of the direct effect of an increase in S further raises the firm's expected marginal payoff with respect to e and ℓ through matching if $\pi'(e) + w'_F(e) \geq 0$. Thus, these two effects together induce the firm to increase e^* and the socially responsible

[16] Although $\pi'(e) + w'_F(e) \geq 0$ implies $\zeta^\circ(e, \ell) > -\eta^\circ(e)$, the assumption of $e^* > 0$ is necessary to ensure that (7) has an interior solution.

investor to raise ℓ^*. In fact, an increase in S conversely causes a lower probability of a firm being matched with a for-profit investor. However, if the firm's marginal pecuniary returns with respect to e are nonpositive ($\pi'(e) \leq 0$), this effect also induces the firm to increase e^* because the firm's marginal expected payoff with respect to e attained by being matched with a for-profit investor is then increasing in S.

In contrast, if the firm's marginal total returns with respect to e are negative (i.e., $\pi'(e) + w'_F(e) < 0$), the increase in ℓ (e) conversely reduces the firm's (socially responsible investor's) marginal expected payoff with respect to e (ℓ) attained by their matching. Hence, an increase in S does not necessarily increase e^* or ℓ^* if $\pi'(e) + w'_F(e) < 0$.

Next, this study discusses the effect of β_S on e^* and ℓ^*. The following proposition represents the increase in β_S.

Proposition 5:
Suppose that the firm's optimal ESG actions are in the positive range, $e^* > 0$. Then, an increase in β_S decreases e^* but increases ℓ^* if $\pi'(e) + w'_F(e) \leq 0$.

In contrast to the basic model, Proposition 5 indicates that the increased bargaining power of the socially responsible investor (i.e., larger β_S) decreases the ESG actions of the firm, e^*, but increases the search effort of the socially responsible investor, ℓ^*, if the firm is willing to take a positive level of ESG actions and if the firm's marginal total returns with respect to e are nonpositive (i.e., $\pi'(e) + w'_F(e) \leq 0$). The latter condition implies that the firm's marginal social impact with respect to e is smaller than the firm's marginal pecuniary returns with respect to e. However, these results do not generally hold if the firm's marginal total returns with respect to e are positive.

Intuitively, a larger β_S directly increases the expected marginal payoff for the socially responsible investor with respect to ℓ. On the other hand, if the firm takes a positive level of ESG actions to satisfy $\zeta^\circ(e) > -\eta^\circ(e)$ (see Lemma 2), the first two terms on the left-hand side of (8) are positive. Thus, a larger β_S directly reduces the firm's expected marginal payoff with respect to e when the firm is matched with a socially responsible investor because the larger share out of the total returns of the project will be taken by the investor. These two direct effects caused by an increase in β_S induce the socially responsible investor to increase ℓ and the firm to decrease e. In addition, given $\lambda'_{S2}(\ell) > 0$, if $\pi'(e) + w'_F(e) \leq 0$, such an increase in ℓ further reduces the firm's expected marginal payoff with respect to e, whilst a decrease in e further raises socially responsible investor's expected marginal payoff with respect to ℓ. Hence, combining these effects together, a larger β_S decreases e^* but increases ℓ^* if $\pi'(e) + w'_F(e) \leq 0$.

Finally, this paper investigates the effect of an increase in κ.

Proposition 6:

Suppose that the firm's optimal ESG actions are in the positive range, $e^* > 0$.[17] *An increase in κ increases e^* and ℓ^* if $\pi'(e) + w'_F(e) \geq 0$.*

Proposition 6 demonstrates that the upper shift of the matching intensity ($\kappa > 0$) increases the ESG actions of the firm, e^*, and the search effort of the socially responsible investor, ℓ^*, if the firm is willing to take a positive level of ESG actions and if the firm's marginal total returns with respect to e are nonnegative ($\pi'(e) + w'_F(e) \geq 0$). This is in contrast to Proposition 3, which only required one condition; $e \gtreqless 0$.

Intuitively, the upper shift of the matching intensity directly increases the firm's expected marginal payoff with respect to e if $\pi'(e) + w'_F(e) \geq 0$, and also increases the socially responsible investor's expected marginal payoff with respect to ℓ. Then, applying arguments similar to those of Proposition 4, I can give the intuition behind the result of Proposition 6 if $\pi'(e) + w'_F(e) \geq 0$.

The theoretical implications for the case in which the socially responsible investor's search cost is significant, as represented by Propositions 4–6 are summarized as follows. Suppose that firms are willing to take some ESG actions. Then, Proposition 4 suggests that a growing number of socially responsible investors increases both the ESG actions of firms and the search effort of socially responsible investors, if the firms' marginal total returns with respect to e are positive while the firms' marginal pecuniary returns are negative. Proposition 5 implies that the extensive development in a social trend that gives socially responsible investors greater influence on firm management decreases the ESG actions of firms but increases the search effort of socially responsible investors, if the firms' total returns with respect to e are negative. Proposition 6 states that an improvement in the search technology between firms and socially responsible investors increases the ESG actions of firms and the search effort of socially responsible investors if the firm's marginal total returns with respect to e are positive.

[17] The same remark as footnote 16 is applied to this proposition.

In particular, a comparison of the results of Propositions 4 and 5 yield interesting results. The more investors are interested in firms' ESG performance, the more the firm invests in the ESG actions, if $\pi'(e) \leq 0$ and $\pi'(e) + w'_F(e) \geq 0$. Meanwhile, the more the socially responsible investors are influential on firm management, the less the firm invests on ESG actions if $\pi'(e) + w'_F(e) \geq 0$. Consequently, if socially responsible investors' search for ESG-committed firms has significant effects and if firms are willing to take a positive level of ESG actions, the regulator aiming to increase the ESG actions of firms should adopt policies according to the relation between the firms' marginal social impact and their marginal pecuniary returns with respect to e.

For example, if the firm's marginal social impact with respect to e is sufficiently larger than the absolute value of the firm's marginal pecuniary returns with respect to e when the latter is negative, the regulator should increase the number of investors interested in the firm's ESG performance by supporting the activities of NGOs and NPOs that are geared toward ESG and SDGs. By contrast, under similar situations, the regulator cannot benefit from making/supporting policies that give socially responsible investors greater influence on firms. Examples of such policies include corporate governance reform and strengthening shareholder rights.

5. Conclusion

Using a search model, this study theoretically examines how for-profit firms finance capital from socially responsible investors by taking a positive level of ESG actions. ESG actions affect a firm's pecuniary returns and social performance. If the firm does not take any ESG actions, the chance of not being matched with any investor increases and may result in zero payoff. Therefore, the firm may be better off taking a positive level of ESG actions even though doing so could decrease its pecuniary payoff (while keeping it positive).

The comparative statics derive the following results:
All the results hold with a common condition that the firm is willing to take a positive level of ESG actions.

(1) Basic model: Socially responsible investors do not make any effort in the search process to match with the firms.

(i) As the number of investors prioritizing in ESG increases, the firm acts more on ESG, if more ESG actions decrease the pecuniary returns of the project.
(ii) Even if socially responsible investors are given more power to influence firm management, the firm does not necessarily act more in ESG.
(iii) As the matching technology improves, the firm takes more ESG actions.
(2) Extended model: Both socially responsible investors and firms make costly effort in matching with each other.
(i) As a growing number of investors prioritizing in ESG increases, the firm takes more ESG actions, if more ESG actions raise the total returns of the project while reducing the pecuniary returns of the project.
(ii) As the socially responsible investors gain more power to influence the firm management, the firm takes less ESG actions, if more ESG actions reduce the total returns of the project.
(iii) As the matching technology improves, the firm takes more ESG actions if more ESG actions raise the total returns of the project.

These results yield novel testable implications regarding the effects of various ESG trends and issues on the firm's ESG actions.

Appendix

Proof of Lemma 1: Rearranging (4) with $\zeta(e)$ and $\eta(a,e)$, I obtain

$$\frac{S}{F}(1-\beta_S)\left[\pi(e)+w_S(a)+w_F(e)\right]\frac{\kappa\lambda_S(e)}{e}\left[\zeta(e)+\eta(a,e)\right]$$

$$+\frac{N-S}{F}\lambda_P(1-\beta_P)\pi'(e)-c_F'(e)=0. \tag{A1}$$

If $\zeta(e)\leq-\eta(a,e)$, then it follows from $\zeta(e)>0$ that $\eta(a,e)<0$. The definition of $\eta(a,e)$ implies that $\frac{\partial[\pi(e)+w_F(e)+w_S(a)]}{\partial e}=\pi'(e)+w_F'(e)<0$. Given $w_F'(e)>0$, I must have $\pi'(e)$ <0. As $\zeta(e)\leq-\eta(a,e)$ and $\pi'(e)<0$, it follows from $c_F'(e)>0$ that the left-hand side of (A1) is negative. Consequently, if $e^*>0$ is a solution to (4) or (A1), the necessary condition is $\zeta(e)>-\eta(a,e)$. ‖

Proof of Propositions 1–3: Define

$$\begin{aligned}\Gamma\equiv{}&\frac{S\kappa(1-\beta_S)}{F}\left\{\lambda_S''(e)\left[\pi(e)+w_F(e)+w_S(a)\right]+2\lambda_S'(e)\left[\pi'(e)+w_F'(e)\right]\right.\\&\left.+\lambda_S(e)\left[\pi''(e)+w_F''(e)\right]\right\}+\frac{N-S}{F}\lambda_P(1-\beta_P)\pi''(e)-c_F''(e)\\&<0,\end{aligned} \tag{A2}$$

where the final inequality is derived because the solution to (3) is assumed to satisfy the second-order condition. Then, totally differentiating (2) and (4) with respect to e, a, S, β_S, and κ yields

$$\begin{bmatrix}\Gamma & \frac{S\kappa(1-\beta_S)}{F}\lambda_S'(e)w_S'(a)\\ 0 & \beta_S w_S''(a)-c_S''(a)\end{bmatrix}\begin{bmatrix}de\\ da\end{bmatrix}$$
$$=\begin{bmatrix}-\frac{\kappa(1-\beta_S)}{F}\left\{\lambda_S'(e)\left[\pi(e)+w_F(e)+w_S(a)\right]+\lambda_S(e)\left[\pi'(e)+w_F'(e)\right]\right\}+\frac{(1-\beta_P)}{F}\lambda_P\pi''(e)\\ 0\end{bmatrix}dS$$
$$+\begin{bmatrix}\frac{S\kappa}{F}\left\{\lambda_S'(e)\left[\pi(e)+w_F(e)+w_S(a)\right]+\lambda_S(e)\left[\pi'(e)+w_F'(e)\right]\right\}\\ -w_S'(a)\end{bmatrix}d\beta_S$$

$$+\begin{bmatrix} -\frac{S}{F}(1-\beta_S)\left\{\lambda'_S(e)\left[\pi(e)+w_F(e)+w_S(a)\right]+\lambda_S(e)\left[\pi'(e)+w'_F(e)\right]\right\} \\ 0 \end{bmatrix} d\kappa. \tag{A3}$$

By solving (A3) and rearranging it using (A2) and the definitions of $\zeta(e)$ and $\eta(a,e)$, I obtain

$$\begin{aligned}\frac{de}{dS} &= \frac{1}{\Psi}\left\{-\frac{(1-\beta_S)}{F}\left[\pi(e)+w_F(a)+w_S(e)\right]\frac{\kappa\lambda_S(e)}{e}\left[\zeta(e)+\eta(a,e)\right]\right. \\ &\quad \left.+\frac{1-\beta_P}{F}\lambda_P\pi'(e)\right\}\left[\beta_S w''_S(a)-c''_S(a)\right] \\ &\quad > 0, \qquad \text{if } \zeta(e)>-\eta(a,e) \text{ and } \pi'(e)<0,\end{aligned} \tag{A4}$$

$$\frac{da}{dS}=0, \tag{A5}$$

$$\begin{aligned}\frac{de}{d\beta_S} &= \frac{S}{F\Psi}\left[\pi(e)+w_F(e)+w_S(a)\right]\frac{\kappa\lambda_S(e)}{e}\left[\zeta(e)+\eta(a,e)\right]\left[\beta_S w''_S(a)-c''_S(a)\right] \\ &\quad +\frac{S\kappa}{F\Psi}(1-\beta_S)\lambda'_S(e)\left[w'_S(a)\right]^2,\end{aligned} \tag{A6}$$

$$\frac{da}{d\beta_S}=-\frac{\Gamma}{\Psi}w'_S(a)>0, \tag{A7}$$

$$\begin{aligned}\frac{de}{d\kappa} &= -\frac{S(1-\beta_S)}{F\Psi}\left[\pi(e)+w_F(e)+w_S(a)\right]\frac{\lambda_S(e)}{e}\left[\zeta(e)+\eta(a,e)\right]\left[\beta_S w''_S(a)-c''_S(a)\right] \\ &\quad > 0, \qquad \text{if } \zeta(e)>-\eta(a,e),\end{aligned} \tag{A8}$$

$$\frac{da}{d\kappa}=0, \tag{A9}$$

where $\Psi = \Gamma\left[\beta_S w''_S(a) - c''_S(a)\right] > 0$. Note that $\lambda'_S(e) > 0$, $w''_S(a) < 0$, and $c''_S(a) > 0$. It follows from (A4)–(A9) that Propositions 1–3 are verified. The corollaries to Propositions 1 and 2 are also evident. ‖

Proof of Lemma 2: Rearranging (8) with $\zeta^\circ(e,\ell)$ and $\eta^\circ(e)$, I obtain

$$\frac{S}{F}\kappa(1-\beta_S)\frac{\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi(e)+w_F(e)\right]}{e}\left[\zeta^\circ(e,\ell)+\eta^\circ(a)\right]$$

$$+\frac{N-S}{F}\lambda_P(1-\beta_P)\pi'(e)-c_F'(e)=0. \tag{A10}$$

Then, repeating the argument of Lemma 1, I can verify that if $e^*>0$ is a solution to (8) or (A10), the necessary condition is $\zeta^\circ(e,\ell)>-\eta^\circ(a)$. ‖

Proof of Propositions 4–6 and the implication of Assumption 1: Given (10), totally differentiating (6) and (8) with respect to S, β_S, and κ yields

$$\begin{bmatrix} \Gamma^\circ & \frac{S\kappa(1-\beta_S)}{F}\lambda_{S2}'(\ell)\left[\pi'(e)+w_F'(e)\right] \\ \frac{S\kappa\beta_S}{F}\lambda_{S2}'(\ell)\left[\pi'(e)+w_F'(e)\right] & \frac{S\kappa\beta_S}{F}\lambda_{S2}''(\ell)\left[\pi(e)+w_F(e)\right]-c_M''(\ell)\end{bmatrix}\begin{bmatrix} de \\ d\ell\end{bmatrix}$$
$$=\begin{bmatrix} -\frac{\kappa(1-\beta_S)}{F}\left\{\lambda_{S1}'(e)\left[\pi(e)+w_F(e)\right]+\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi'(e)+w_F'(e)\right]\right\}+\frac{\lambda_P(1-\beta_P)\pi'(e)}{F} \\ -\frac{\kappa\beta_S}{F}\lambda_{S2}'(\ell)\left[\pi(e)+w_F(e)\right]\end{bmatrix}dS$$
$$+\begin{bmatrix} \frac{S\kappa}{F}\left\{\lambda_{S1}'(e)\left[\pi(e)+w_F(e)\right]+\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi'(e)+w_F'(e)\right]\right\} \\ -\frac{S\kappa}{F}\lambda_{S2}'(\ell)\left[\pi(e)+w_F(e)\right]\end{bmatrix}d\beta_S$$

$$+\begin{bmatrix} -\frac{S(1-\beta_S)}{F}\left\{\lambda_{S1}'(e)\left[\pi(e)+w_F(e)\right]+\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi'(e)+w_F'(e)\right]\right\} \\ -\frac{S\beta_S}{F}\lambda_{S2}'(\ell)\left[\pi(e)+w_F(e)\right]\end{bmatrix}d\kappa. \tag{A11}$$

To evaluate the comparative static results, let us note that $\pi'(e)+w_F'(e)\geq 0$ implies $\zeta^\circ(e,\ell)+\eta^\circ(e)>0$ because of $\eta^\circ(e)\geq 0$. Then, solving (A11) and rearranging it with Assumption 1 and the definitions of $\zeta^\circ(e,\ell)$ and $\eta^\circ(e)$, I obtain

$$\begin{aligned}\frac{de}{dS}=&-\frac{1}{\Psi^\circ}\left\{\frac{\kappa(1-\beta_S)}{F}\frac{\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi(e)+w_F(e)\right]}{e}\left[\zeta^\circ(e,\ell)+\eta^\circ(e)\right]\right.\\ &\left.-\frac{\lambda_P(1-\beta_P)}{F}\pi'(e)\right\}\left\{\frac{S\kappa\beta_S}{F}\lambda_{S2}''(\ell)\left[\pi(e)+w_F(e)\right]-c_M''(\ell)\right\}\\ &+\frac{S}{\Psi^\circ}\left[\frac{\kappa\lambda_{S2}'(\ell)}{F}\right]^2\beta_S(1-\beta_S)\left[\pi'(e)+w_F'(e)\right]\left[\pi(e)+w_F(e)\right]\\ &>0,\qquad \text{if } \pi'(e)+w_F'(e)\geq 0 \text{ and } \pi'(e)\leq 0,\end{aligned} \tag{A12}$$

$$
\begin{aligned}
\frac{d\ell}{dS} &= -\frac{\Gamma^\circ}{\Psi^\circ}\frac{\kappa\beta_S}{F}\lambda'_{S2}(\ell)\left[\pi(e)+w_F(e)\right] \\
&+\frac{1}{\Psi^\circ}\left\{\frac{(1-\beta_S)\kappa}{F}\frac{\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi(e)+w_F(e)\right]}{e}\left[\zeta^\circ(e,\ell)+\eta^\circ(e)\right]\right. \\
&\left.-\frac{\lambda_P(1-\beta_P)}{F}\pi'(e)\right\}\frac{S\kappa\beta_S}{F}\lambda'_{S2}(\ell)\left[\pi'(e)+w'_F(e)\right] \\
&> 0, \qquad \text{if } \pi'(e)+w'_F(e)\geq 0 \text{ and } \pi'(e)\leq 0,
\end{aligned} \tag{A13}
$$

$$
\begin{aligned}
\frac{de}{d\beta_S} &= \frac{S\kappa}{\Psi^\circ F}\frac{\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi(e)+w_F(e)\right]}{e}\left[\zeta^\circ(e,\ell)+\eta^\circ(e)\right] \\
&\times\left\{\frac{S\kappa\beta_S}{F}\lambda''_{S2}(\ell)\left[\pi(e)+w_F(e)\right]-c''_M(\ell)\right\} \\
&+\frac{1}{\Psi^\circ}\left[\frac{S\kappa\lambda'_{S2}(\ell)}{F}\right]^2(1-\beta_S)\left[\pi'(e)+w'_F(e)\right]\left[\pi(e)+w_F(e)\right] \\
&< 0, \qquad \text{if } \zeta^\circ(e,\ell)>-\eta^\circ(e) \text{ and } \pi'(e)+w'_F(e)\leq 0,
\end{aligned} \tag{A14}
$$

$$
\begin{aligned}
\frac{d\ell}{d\beta_S} &= -\frac{\Gamma^\circ}{\Psi^\circ}\frac{S\kappa}{F}\lambda'_{S2}(\ell)\left[\pi(e)+w_F(e)\right] \\
&-\frac{1}{\Psi^\circ}\left(\frac{S\kappa}{F}\right)^2\lambda'_{S2}(\ell)\beta_S\left[\pi'(e)+w'_F(e)\right] \\
&\times\frac{\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi(e)+w_F(e)\right]}{e}\left[\zeta^\circ(e,\ell)+\eta^\circ(e)\right] \\
&> 0, \qquad \text{if } \zeta^\circ(e,\ell)>-\eta^\circ(e) \text{ and } \pi'(e)+w'_F(e)\leq 0,
\end{aligned} \tag{A15}
$$

$$
\begin{aligned}
\frac{de}{d\kappa} &= -\frac{1}{\Psi^\circ}\frac{S(1-\beta_S)}{F}\frac{\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi(e)+w_F(e)\right]}{e}\left[\zeta^\circ(e,\ell)+\eta^\circ(e)\right] \\
&\times\left\{\frac{S\kappa\beta_S}{F}\lambda''_{S2}(\ell)\left[\pi(e)+w_F(e)\right]-c''_M(\ell)\right\} \\
&+\frac{1}{\Psi^\circ}\left[\frac{S\lambda'_{S2}(\ell)}{F}\right]^2\kappa\beta_S(1-\beta_S)\left[\pi'(e)+w'_F(e)\right]\left[\pi(e)+w_F(e)\right] \\
&> 0, \qquad \text{if } \pi'(e)+w'_F(e)\geq 0,
\end{aligned} \tag{A16}
$$

$$
\begin{aligned}
\frac{d\ell}{d\kappa} &= -\frac{\Gamma^\circ}{\Psi^\circ}\frac{S}{F}\beta_S\lambda'_{S2}(\ell)\left[\pi(e)+w_F(e)\right] \\
&\quad + \frac{1}{\Psi^\circ}\left(\frac{S}{F}\right)^2\lambda'_{S2}(\ell)\kappa\beta_S(1-\beta_S)\left[\pi'(e)+w'_F(e)\right] \\
&\quad \times \frac{\left[\lambda_{S1}(e)+\lambda_{S2}(\ell)\right]\left[\pi(e)+w_F(e)\right]}{e}\left[\zeta^\circ(e,\ell)+\eta^\circ(e)\right] \\
&\quad > 0, \qquad \text{if } \pi'(e)+w'_F(e) \geq 0.
\end{aligned} \tag{A17}
$$

Note that $\lambda'_{S2}(\ell) > 0$, $\lambda''_{S2}(\ell) < 0$, and $c''_M(\ell) > 0$. It follows from (A12)–(A17), Propositions 4–6 are verified.

Finally, to derive the implication of Assumption 1, let us consider the shifts of the cost functions from $c_F(e)$ to $c_F(e) + \varepsilon_1 e$ and from $c_M(\ell)$ to $c_M(\ell) + \varepsilon_2\ell$. Then, totally differentiating (6) and (8) with respect to e, ℓ, ε_1, and ε_2, I obtain

$$
\frac{de}{d\varepsilon_1} = \frac{1}{\Psi^\circ}\left\{\frac{S\kappa\beta_S}{F}\lambda''_{S2}(\ell)\left[\pi(e)+w_F(e)\right] - c''_M(\ell)\right\} \text{ and } \frac{de}{d\varepsilon_2} = \frac{\Gamma^\circ}{\Psi^\circ}.
$$

Hence, to obtain the well-behaved result given in the text, I need to assume $\Psi^\circ > 0$ because of $\lambda''_{S2}(\ell) < 0$, $c''_M(\ell) > 0$, and $\Gamma^\circ < 0$. ‖

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