Gains from Policy Cooperation in Capital Controls and Financial Market Incompleteness

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\textbf{ABSTRACT}

We examine how the degree of financial market incompleteness affects welfare gains from policy cooperation in capital controls. When financial markets are incomplete, international risk sharing is disturbed. However, the optimal global policy significantly reverses the welfare deterioration due to inefficient risk-sharing. We find that when financial markets are more incomplete, the welfare gap between the optimal global policy and the Nash equilibrium increases, and the welfare gains from policy cooperation in capital controls then become larger.

\textbf{KEYWORDS}

financial markets; incomplete markets; policy cooperation; capital controls; optimal policy; welfare; Ramsey policy; open-loop Nash game

\textbf{JEL CLASSIFICATION}

D52; E61; F32; F38; F42; G15

1. Introduction

Many studies discuss the gains from monetary policy coordination. Among them, several employ two-country models with complete financial markets, which implies that international risk sharing is perfect (e.g., Clarida et al. 2001, 2002; Benigno and Benigno 2003, 2006; Corsetti and Pesenti 2001, 2005; Fujiwara and Teranishi 2017). On the other hand, some studies employ two-country models with incomplete financial markets, and recent studies extend this literature in several directions. Corsetti et al. (2010) show that incomplete markets break an open-economy version of divine coincidence and cause policy trade-offs. Benigno (2009) shows that gains from the optimal monetary policy increase with cross-country asymmetries in initial net international positions. Rabitsch (2012) studies welfare gains from monetary policy cooperation under three types of international financial market structures: complete markets, financial autarky, and incomplete markets.

Although many studies examine gains from monetary policy coordination from many different perspectives, as mentioned above, few authors analyze the gains from policy coordination in capital controls.\textsuperscript{1} Noteworthy exceptions are De Paoli and Lipinska (2013) and Heathcote and Perri (2016). De Paoli and Lipinska (2013) show that capital controls can be beggar-thy-neighbor policies, and policy coordination in capital controls can yield gains. This is because individual countries have incentives to manage their terms of trade to stabilize their own output fluctuations, but the uncoordinated use of capital controls disturbs international consumption risk sharing...
and deteriorates global welfare. Using a two-country model augmented with capital accumulation, Heathcote and Perri (2016) show that for certain parameterizations, capital controls can lead to better international risk sharing. Through this improved risk sharing, symmetric capital controls can be welfare improving for both countries compared to free international capital mobility, which implies that capital controls can be Pareto-improving.

In this study, we also examine the welfare gains from policy coordination in capital controls. However, we focus on how the degree of financial market incompleteness affects welfare gains from policy cooperation in capital controls. Incomplete markets disturb international risk-sharing, while the optimal global policy mitigates risk-sharing inefficiency. We find that when financial markets are more incomplete, global welfare deteriorates under the Nash equilibrium; however, the optimal global policy mitigates the welfare loss due to financial market inefficiencies. When financial markets are more incomplete, we find that the welfare gap between the optimal global policy and the Nash equilibrium increases, and welfare gains from policy cooperation in capital controls then become larger. To the best of our knowledge, no previous work addresses this relationship between the gains from policy coordination in capital controls and the degree of financial market incompleteness in the related literature.

2. Model setup

We consider a familiar two-country model populated with a continuum of agents of unit mass, in which the population in segment \([0, n]\) belongs to country \(H\) and that in segment \((n, 1]\) belongs to country \(F\).

The household in country \(H\) maximizes the following expected life utility:

\[
E_0 \sum_{t=0}^{\infty} \beta^t \{u(C_t) - v(L_t)\},
\]

where \(E_0\) denotes the mathematical expectations operator conditional on the information available at time 0, and \(\beta \in (0, 1)\) is the discount factor. The functions \(u\) and \(v\) are increasing in the composite consumption index \(C_t\) and the labor supply \(L_t\), respectively.

The budget constraint for a household in country \(H\) is

\[
C_t + (1 + \tau_{t-1}) \frac{q_{t}}{q_{t-1}} D_{t-1} + \frac{\delta}{2} D_t^2 = D_t + p_t^H Y_t + T_t,
\]

where \(D_t\) denotes the foreign debt position, \(R_t^*\) denotes the gross interest rate on foreign debt, \(q_t\) denotes the real exchange rate, \(\tau_t\) denotes the tax rate of the foreign debt, and \(T_t\) denotes a lump-sum transfer from the government. Following many previous two-country studies (Benigno 2009; De Paoli and Lipinska 2013; Kim and Kim 2018), we introduce quadratic adjustment costs, \(\frac{\delta}{2} D_t^2\), to characterize incomplete financial markets.

Following Viani (2010), we define the “risk-sharing gap,” which is a key variable in our study, as

\[
gap_t = \log \left( \frac{SDF_t}{SDF_t^*} \right).
\]
Herein, $SDF_t$ denotes the “stochastic discount factor” of country $H$:

$$SDF_t = \beta \frac{u_C(C_{t+1})}{u_C(C_t)} \frac{q_{t+1}}{q_t}. \quad (4)$$

$SDF^*_t$ denotes the “stochastic discount factor” of country $F$:

$$SDF^*_t = \beta^* \frac{u_C(C^*_{t+1})}{u_C(C^*_t)}. \quad (5)$$

Under complete markets, the risk-sharing gap is zero, since perfect cross-border risk-sharing makes the stochastic discount factors of the two countries equal. However, under incomplete markets, the gap deviates from zero. Since the gap indicates a deviation from the allocation under complete markets, we can measure the lack of risk sharing (i.e., risk-sharing inefficiency) with the risk-sharing gap in Eq.(3).

Following Schmitt-Grohé and Uribe (2006), we measure the conditional welfare of country $H$ by computing the expected welfare conditional on the initial non-stochastic steady state:

$$W_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, L_t)$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t U((1 + \eta)C, L), \quad (6)$$

where

$$U(C_t, L_t) = u(C_t) - v(L_t), \quad (7)$$

and $C$ and $L$ denote their non-stochastic steady state levels. With $\eta$, we evaluate and compare conditional welfare levels under different parameters or policies.

Similarly, we compute conditional global welfare by defining

$$W^W_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U^W(C_t, C^*_t, L_t, L^*_t)$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t U^W((1 + \eta)C, L), \quad (8)$$

where $U^W(C_t, C^*_t, L_t, L^*_t) = nU(C_t, L_t) + (1 - n)U(C^*_t, L^*_t)$. The transition from the first line to the second follows from the fact that $C = C^*$ and $L = L^*$ in their non-stochastic steady states by structure.

Since our model is a standard two-country model with incomplete asset markets, which is similar to those in previous studies (e.g., Benigno 2009; Corsetti et al. 2010; Rabitsch 2012; De Paoli and Lipinska 2013; Kim and Kim 2018), we add a detailed description of our model in Appendix A1 instead of developing the full model in the main text. We fix the parameter values as in Table A1, which are common in the literature. However, we vary the parameter value of $\delta$ to examine how the asset market incompleteness affects the dynamics of the two economies in Section 4. We also choose a different value for the elasticity of substitution between domestic and imported goods $\theta$ to show the robustness of our analysis results in Appendix A5. We perform numerical
experiments using a second-order approximation of the model with the perturbation method following Schmitt-Grohé and Uribe (2004).³

3. Optimal global policy and Nash equilibrium

We characterize the optimal global policy by analyzing welfare-maximizing Ramsey policies with commitment. Under the optimal global policy, a global policy maker optimizes capital controls to maximize global welfare, considering the private sector’s response to the implemented policies. Setting up a Lagrangian problem in which the global policy maker maximizes global welfare subject to the first-order conditions of the private agents and the market-clearing conditions of the two economies, we obtain the first-order conditions for the global policy maker. The first-order conditions for the global policy maker, the first-order conditions of the private agents, and the market-clearing conditions of the two economies characterize the cooperation equilibrium. Appendix A3 explains the cooperation equilibrium in detail.

Under a Nash equilibrium, the Home policy maker optimizes capital controls in country H to maximize country H’s welfare level, while the Foreign policy maker optimizes capital controls in country F to maximize country F’s welfare level. Thus, the strategic interaction between the two countries’ policy makers leads to different outcomes from those under the welfare-maximizing cooperative policy. Specifically, we consider an open-loop Nash equilibrium in which the actions of each country’s policy maker are best response to the other policy maker’s best response.

Setting up two Lagrangian problems in which the Home and Foreign policy makers maximize their own country’s welfare subject to the first-order conditions of the private agents and the market-clearing conditions of the two economies, we obtain the first-order conditions for each country’s policy maker. The first-order conditions for each country’s policy maker, the first-order conditions of the private agents, and the market-clearing conditions of the two economies characterize an open-loop Nash equilibrium. Appendix A4 explains the Nash equilibrium in detail.

We obtain the relevant first-order conditions that characterize the optimal global policy and Nash equilibrium problems using Bodenstein et al. (2018)’s Matlab procedures.⁴

4. Results

Figure 1 compares the impulse responses of the main variables to a negative home productivity shock. Figure 1(a) is the case when $\theta = 0.5$, which implies that the home and foreign goods are complements. Figure 1(b) is the case when $\theta = 3$, which implies that the home and foreign goods are imperfect substitutes. In both cases, the solid and dotted curves indicate the case with high ($\delta = 0.1$) and low frictions ($\delta = 0.001$), respectively. As Corsetti et al. (2010) and De Paoli and Lipinska (2013) argue, the elasticity of substitution between the home and foreign goods, $\theta$, is a critical parameter to dynamics in the two-country models. In Figures 1(a) and (b), the position of the solid and dotted curves critically depends on the parameter $\theta$. For example, in the case of “Home output,” the solid curve is lower than the dotted curve in (a), but the solid curve is higher than the dotted curve in (b). However, in the “Risk-sharing gap” case, although the position of the two curves is still reversed between (a) and (b), we should note that the solid curve deviates from zero further than the dotted curve.
Figure 1. Responses to a negative (one standard deviation) home productivity shock.

(a) $\theta = 0.5$

(b) $\theta = 3$

does in either case. Compared to the low-friction case ($\delta = 0.001$), the international financial markets are more incomplete in the high-friction case ($\delta = 0.1$). Therefore, international risk sharing is more disturbed in the high-friction case, and the “Risk-sharing gap” deviates further from zero compared to the low-friction case. Related with this, in the “Foreign debt holdings” case, the solid curve remains closer to zero than the dotted curve does in either case. This is again because in the high-friction case, the international financial markets are more incomplete and international risk sharing is more disrupted compared to the low-friction case. The disruption of international risk sharing restricts changes in foreign debt holdings.

In Figure 1, we find that in case (a) or (b), a higher degree of friction causes the impulse response of the “Risk-sharing gap” to deviate further. Figure 2 (a) plots the standard deviation of the risk-sharing gap defined by Equation (3) under different degrees of friction. The solid curve in Figure 2 (a) depicts the case without capital controls. Consistent with Figure 1, the solid curve in Figure 2 (a) indicates that the standard deviation of the risk-sharing gap increases as the degree of friction increases. Figure 2 (b) depicts the associated conditional global welfare level, which we measure as a percentage of steady state consumption. The solid curve in Figure 2 (b) depicts the case without capital controls, which indicates that the conditional global welfare decreases as the degree of friction increases because the two countries suffer from more insufficient risk-sharing as the degree of friction increases.

In Figure 2 (a), the dashed curve plots the standard deviations of the risk-sharing gap under the Nash equilibrium, in which the standard deviation of the risk-sharing gap increases when the degree of friction increases, as well as in the case without capital controls. However, comparing the dashed and solid curves, we see that the standard deviation of the risk-sharing gap under the Nash equilibrium is higher than that in the case without capital controls. In Figure 2 (b), the dashed curve plots the global welfare level under the Nash equilibrium. Consistent with the dashed curve in Figure 2 (a), the dashed curve in Figure 2 (b) indicates that global welfare deteriorates as the degree of friction increases due to the resulting insufficient risk-sharing. Comparing the dashed and solid curves in Figure 2 (b), we see that the global welfare level under the Nash equilibrium is lower than that in the case without capital controls, which is consistent with the comparison between the solid and dashed curves in Figure 2 (a).

The dashed dotted curve in Figure 2 (a) plots the standard deviations of the risk-sharing gap under the optimal global policy. Compared with the Nash equilibrium case
and the no capital controls case, the optimal global policy keeps the standard deviation of the risk-sharing gap close to zero. In Figure 2 (b), we see that compared to the other two cases, the optimal global policy prevents global welfare from deteriorating, even when the degree of friction is high, which is consistent with the dashed dotted curve in Figure 2 (a).

Figure 3 (a) plots the difference in the standard deviations of the risk-sharing gap between the Nash equilibrium and the optimal global policy (i.e., the difference between the dashed curve and the dashed dotted curve in Figure 2 (a)). Figure 3 (a) shows that as the degree of friction increases, the difference in the standard deviations of the risk-sharing gap between the Nash equilibrium and the optimal global policy increases. Figure 3 (b) plots the difference in the (conditional) global welfare levels between the optimal global policy and the Nash equilibrium, in which the difference in global welfare levels increases when financial markets are less efficient, which is consistent with the results in Figure 3 (a). This implies that the gain from international policy cooperation in capital controls is larger when financial markets are less efficient.

In Figure 3, we set the elasticity of substitution between the home and foreign goods, $\theta$, to 3. We next consider cases with different values of $\theta$ based on Rabitsch (2012)'s finding that welfare gains from monetary coordination critically depend on the elasticity of substitution between the home and foreign goods. Similar to Figure 3 (a), Figure 4 (a) plots the differences in the standard deviations of the risk-sharing gap between the Nash equilibrium and the optimal global policy, but for different values of $\delta$ and $\theta$. As Figure 4 (a) shows, as $\theta$ decreases to 1, the difference in the standard deviation of the risk-sharing gap decreases, but it starts to increase for lower values of $\theta(<1)$ (for any value of $\delta$).

Similar to Figure 3 (b), Figure 4 (b) plots the difference in the global welfare levels between the optimal global policy and the Nash equilibrium, but for different values of $\delta$ and $\theta$. As Figure 4 (b) shows, as $\theta$ decreases to 1, the difference in the global welfare levels decreases, but it starts to increase for lower values of $\theta(<1)$ (for any value of $\delta$).

In Figure 3 (b), we already showed that the difference in global welfare levels between the optimal global policy and the Nash equilibrium increases as the degree of friction increases. However, Figure 4 (b) shows that for lower values of $\theta(<1)$, the welfare
gains from policy cooperation in capital controls, which we measure as the difference in global welfare levels between the optimal global policy and the Nash equilibrium, can be much larger than that in the benchmark case with $\theta = 3$.

Figure 3. (a) Difference in the standard deviations of the risk-sharing gap between the Nash equilibrium and optimal global policy for different values of $\delta$. (b) Difference in conditional global welfare levels between the optimal global policy and the Nash equilibrium for different values of $\delta$.

Figure 4. (a) Difference in the standard deviations of the risk-sharing gap between the Nash equilibrium and optimal global policy for different values of $\delta$ and $\theta$. (b) Difference in conditional global welfare levels between the optimal global policy and the Nash equilibrium for different values of $\delta$ and $\theta$. 

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5. Conclusions

We investigated how the degree of financial market incompleteness affects the welfare gains from policy cooperation in capital controls. Under higher degrees of incompleteness, policy cooperation in capital controls becomes more welfare improving compared to the Nash equilibrium case. The intuition is straightforward. When financial markets are less efficient, international risk sharing is more disturbed. However, since policy cooperation significantly reverses the welfare deterioration due to inefficient risk-sharing, policy cooperation in capital controls becomes more critical when financial markets are more incomplete.

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Appendix

A1. Detailed model setting and equilibrium conditions

We assume that the household’s utility functions $u(C_t)$ and $v(L_t)$ are iso-elastic functions:

\[
    u(C_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma},
\]

and

\[
    v(L_t) = \frac{L_t^{1+\chi} - 1}{1+\chi}.
\]

We define the home composite consumption index $C_t$ as

\[
    C_t = \left[ \nu \frac{s}{\tau} (C_{H,t})^{\frac{\theta-1}{\tau}} + (1 - \nu) \frac{s}{\tau} (C_{F,t})^{\frac{\theta-1}{\tau}} \right]^{\frac{\tau}{\theta-1}},
\]

where $1 - \nu = (1 - n)\lambda$. The parameter $\lambda$ indicates the degree of openness. The corresponding home consumption-based price index is

\[
    P_t = \left[ \nu (P_{H,t})^{1-\theta} + (1 - \nu) (P_{F,t})^{1-\theta} \right]^{\frac{1}{1-\theta}}.
\]

Similarly, the foreign composite consumption index $C_t^*$ is

\[
    C_t^* = \left[ \nu s \frac{\theta-1}{\tau} (C_{H,t}^*)^{\frac{\theta-1}{\tau}} + (1 - \nu s) \frac{\theta-1}{\tau} (C_{F,t}^*)^{\frac{\theta-1}{\tau}} \right]^{\frac{\tau}{\theta-1}},
\]
where $\nu^* = n\lambda$. The corresponding foreign consumption-based price index is

$$
P^*_t = \left[ \nu^*(P^*_{H,t})^{1-\theta} + (1 - \nu^*)(P^*_{F,t})^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{A6}
$$

All goods are traded and the law of one price holds:

$$
P_{H,t} = S_t P^*_{H,t}, \tag{A7}
$$

and

$$
P_{F,t} = S_t P^*_{F,t}, \tag{A8}
$$

where $S_t$ denotes the nominal exchange rate. However, purchasing power parity does not hold due to the home bias specification given by (A4) and (A6). We define the real exchange rate as

$$
q_t = \frac{S_t P^*_t}{P^*_t}, \tag{A9}
$$

which can deviate from one. It follows from Eqs. (A4), (A6), and (A9) that the international relative prices satisfy

$$
p^H_t^{\theta-1} = \nu + (1 - \nu) \left( \frac{p^F_t}{p^H_t} \right)^{1-\theta}, \tag{A10}
$$

and

$$
\left( \frac{p^H_t}{q_t} \right)^{\theta-1} = \nu^* + (1 - \nu^*) \left( \frac{p^F_t}{p^H_t} \right)^{1-\theta}, \tag{A11}
$$

where $p^H_t \equiv \frac{P^H_t}{P^*},$ and $p^F_t \equiv \frac{P^F_t}{P^*}$. We define the terms of trade as

$$
ToT_t = \frac{p^F_t}{p^H_t}. \tag{A12}
$$

The tax revenue from capital controls in Eq. (2) are transferred to the household

$$
T_t = \tau_{t-1} R^*_t \frac{q_t}{q_{t-1}} B_{t-1}, \tag{A13}
$$

such that capital controls have no effect on the economy’s resource constraint.

The production technology is given by

$$
Y_t = A^\frac{1}{\lambda} \tilde{L}_t, \tag{A14}
$$

and

$$
Y^*_t = A^\frac{1}{\lambda^*} \tilde{L}^*_t. \tag{A15}
$$
Maximizing the lifetime utility (1) with respect to $D_t$ (and $C_t$) subject to the budget constraint (2) (and considering (A1), (A2), and (A14)) yield the Euler equation in country $H$:

$$1 - \delta D_t = \beta (1 + \tau_t) R_t^\sigma \frac{q_{t+1}}{q_t} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma}.$$  \hfill (A16)

Maximizing the lifetime utility (1) with respect to $Y_t$ (and $C_t$) subject to the budget constraint (2) (and considering (A1), (A2), and (A14)), we also obtain

$$p_t^H C_t^{-\sigma} = A_t^{-x} Y_t^x.$$  \hfill (A17)

The household in country $F$ maximizes the following expected life utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \{u(C_t^*) - v(L_t^*)\}.$$  \hfill (A18)

The budget constraint for a household in country $F$ is

$$C_t^* + (1 + \tau_{t-1}^*) R_{t-1}^* D_{t-1}^* = D_t^* + p_{t}^F Y_t^* + T_t^*.$$  \hfill (A19)

The transfer policy in country $F$, which is similar to (A13), is

$$T_t^* = \tau_{t-1}^* R_{t-1}^* B_{t-1}^*.$$  \hfill (A20)

Maximizing the lifetime utility (A18) with respect to $D_t^*$ (and $C_t^*$) subject to the budget constraint (A19) (and considering the equivalent functional forms to (A1), (A2), and (A15)) yield the Euler equation in country $F$:

$$1 = \beta^* (1 - \tau_t^*) R_t^* \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma}.$$  \hfill (A21)

Maximizing the lifetime utility (A18) with respect to $Y_t^*$ (and $C_t^*$) subject to the budget constraint (A19) (and considering the equivalent functional forms to (A1), (A2), Eqs. (A9), and (A15)), we also obtain

$$\frac{p_t^F}{q_t} C_t^{*-\sigma} = A_t^{-x} Y_t^{*x}.$$  \hfill (A22)

From the households’ preferences, we can derive the demand for domestic and foreign goods:

$$Y_t = \left( p_t^H \right)^{-\sigma} \left[ \nu C_t + \frac{1-n}{n} \nu^* \left( \frac{1}{q_t} \right)^{-\sigma} C_t^* \right].$$  \hfill (A23)

and

$$Y_t^* = \left( p_t^F \right)^{-\sigma} \left[ \nu \left( \frac{1-n}{1-n} \right) C_t + (1-\nu^*) \left( \frac{1}{q_t} \right)^{-\sigma} C_t^* \right].$$  \hfill (A24)
The equilibrium in the asset market requires that

\[ D_t + D^*_t = 0. \]  \hfill (A25)

The productivity shocks in countries \( H \) and \( F \) are, respectively,

\[ \log A_t = \rho \log A_{t-1} + \epsilon_t, \]  \hfill (A26)

and

\[ \log A^*_t = \rho \log A^*_t - 1 + \epsilon_t^*. \]  \hfill (A27)

The equilibrium of this economy is a set of stationary stochastic processes:

\( \{ C_t, C^*_t, Y_t, Y^*_t, L_t, L^*_t, R_t, D_t, D^*_t, q_t, p^H_t, p^F_t, T_oT_t, T_t, T^*_t, gap_t, SDF_t, SDF^*_t \} \) satisfying Eqs. (3), (4), (5), (A9), (A23), (A24), (A12), (A13), (A20), (A14), (A15), (A16), (A21), (A17), (A22), (A23), (A24), (A25) ) Given the exogenous stochastic processes \( A_t, A^*_t \), and an initial value for \( D_{-1} \).

\section*{A2. Parameterization}

We choose standard parameter values from the relevant literature, which we summarize in Table A1.

\begin{center}
\textbf{Table A1. Parameterization}
\end{center}

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Discount factor</td>
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</tr>
<tr>
<td>Inverse intertemporal elasticity of substitution</td>
<td>1</td>
</tr>
<tr>
<td>Inverse elasticity of labor supply</td>
<td>0.47</td>
</tr>
<tr>
<td>Bond adjustment cost parameter</td>
<td>0.01; [0.001, 0.1]</td>
</tr>
<tr>
<td>Degree of openness</td>
<td>0.5</td>
</tr>
<tr>
<td>Home country size</td>
<td>0.5</td>
</tr>
<tr>
<td>Elasticity of substitution between domestic and imported goods</td>
<td>3</td>
</tr>
<tr>
<td>Persistence of productivity shock</td>
<td>0.95</td>
</tr>
<tr>
<td>Standard deviation of productivity shock</td>
<td>0.0071</td>
</tr>
</tbody>
</table>

\section*{A3. Optimal global policy}

In Section 4, we examine the optimal global capital controls policy. Under the optimal global policy, we include \( \tau_i \) (or \( \tau^*_i \)) as a policy instrument.\(^6\) We let \( x_t \) denote the \( N \times 1 \) vector of endogenous variables. Except for the policy instrument, the remaining \( N - 1 \) endogenous variables in \( x_t \) satisfy the \( N - 1 \) structural conditions, which are

\[ E_t f(x_t, x_{t+1}, \zeta_t) = 0, \]  \hfill (A28)

where the vector \( \zeta_t \) denotes the exogenous variables. As we argue in Section 4, we obtain the global policy maker’s first-order conditions by setting up a Lagrangian problem in which the global policy maker maximizes global welfare subject to the
first-order conditions of the private agents and the market-clearing conditions of the two economies. More specifically, we derive the optimal Ramsey policy from the maximization problem:

$$\max_{\{x_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t U^W(x_t, \zeta_t)$$

s.t. \( E_t f(x_t, x_{t+1}, \zeta_t) = 0. \)

(A29)

We set up the Lagrangian problem:

$$\mathcal{L}_0 = E_0 \sum_{t=0}^{\infty} \beta^t \{ U^W(x_t, \zeta_t) + \xi'_t f(x_t, x_{t+1}, \zeta_t) \},$$

(A30)

where \( \xi_t \) denotes the Lagrange multipliers associated with the first-order conditions of the private agents and the market-clearing conditions of the economy in (A28). Taking the derivatives of \( \mathcal{L}_0 \) with respect to the \( N \) endogenous variables, we obtain the \( N \) first-order conditions, which are characterized by the following equation:

$$U_1^W(x_t, \zeta_t) + E_t \xi'_t f_1(x_t, x_{t+1}, \zeta_t) + \beta^{-1} \xi'_{t-1} f_2(x_{t-1}, x_t, \zeta_{t-1}) = 0.$$

(A31)

Taking the derivatives of \( \mathcal{L}_0 \) with respect to \( \xi_t \), we obtain the \( N \) equilibrium conditions in the private sector in (A28). The first-order conditions of the private agents, the market-clearing conditions of the two economies, and the global policy maker’s first-order conditions characterize the cooperation equilibrium. The Ramsey equilibrium process is therefore characterized by the \( N \) equations (A28) and the \( N \) equations (A31). For the variables, we have \( N \) elements of \( x \) and \( N \) multipliers of \( \lambda \). In total, we therefore have \( 2N - 1 \) variables and \( 2N - 1 \) equations.

A4. Nash equilibrium

In Section 4, we also examine a Nash equilibrium in which the Home policy maker optimizes \( \tau_t \) to maximize country H’s welfare level, while the Foreign policy maker optimizes \( \tau^*_t \) to maximize country F’s welfare level. Specifically, we consider an open-loop Nash equilibrium in which each country’s policy maker chooses the optimal allocation given the evolution of the other country’s policy instrument. We let \( x_t \) denote the \( N \times 1 \) vector of endogenous variables in this case. Except for the two policy instruments, the remaining \( N - 2 \) endogenous variables in \( x_t \) satisfy the \( N - 2 \) structural conditions, which we again express with

$$E_t f(x_t, x_{t+1}, \zeta_t) = 0.$$

(A32)

As we argue in Section 4, we obtain the first-order conditions for the each country’s policy maker by setting up two Lagrangian problems in which the Home and Foreign policy makers maximize their own welfare subject to the first-order conditions of the private agents and the market-clearing conditions of the two economies. More
specifically, we set up a Lagrangian problem for country $i$ ($1$ or $2$):  

$$L_{i,0} = E_0 \sum_{t=0}^{\infty} \beta^t \{U_i(x_t, \zeta_t) + \lambda_{1,t}^i f(x_t, x_{t+1}, \zeta_t)\}. \tag{A33}$$

Taking the derivatives of $L_{i,0}$ with respect to the $N - 1$ variables (except for the policy instrument of the other country), we obtain the $2N - 2$ first-order conditions characterized by  

$$U_1^i(x_t, \zeta_t) + E_t \lambda_{1,t}^i f_1(x_t, x_{t+1}, \zeta_t) + \beta^{-1} \lambda_{1,t-1}^i f_2(x_{t-1}, x_t, \zeta_{t-1}) = 0. \tag{A34}$$

Taking the derivatives of $L_{i,0}$ with respect to $\lambda_{i,t}$, we obtain the $N - 2$ equilibrium conditions in the private sector in (A32), which are common for both countries. The first-order conditions for each country’s policy maker, the first-order conditions of each country’s private agents, and the market-clearing conditions of the two economies characterize an open-loop Nash equilibrium. The Nash equilibrium process is therefore characterized by the $N - 2$ equations (A32) and the $2N - 2$ equations (A34). For the variables, we have $N - 2$ elements of $x$ and $2N - 2$ multipliers of $\lambda$. In total, we therefore have $3N - 4$ equations and $3N - 4$ variables.

A5. Robustness check for the case in which the home and foreign goods are complements ($\theta = 0.5$)

In Section 4, we show our results for the case in which the home and foreign goods are imperfect substitutes ($\theta = 3$), except for Figure 1 (a). In this section, we show that our main results remain unchanged when the home and foreign goods are complements ($\theta = 0.5$).

As Figure A1 shows, we obtain similar figures of the standard deviation of the risk-sharing gap and conditional global welfare level for different values of $\delta$ corresponding to Figure 2 in Section 4.

We also obtain similar figures in Figure A2 corresponding to Figure 3. We should note that when $\theta = 0.5$, the difference in the conditional global welfare levels under
the optimal global policy and the Nash equilibrium can reach more than 0.14%, up to 0.43%, which is higher than that in Figure 3.

Notes

1If we do not restrict our attention to policy coordination, there is a vast literature on capital controls. See, for example, Kitano and Takaku (2018) and the references therein.
2The two-country models in Benigno (2009), Corsetti et al. (2010), and Rabitsch (2012) incorporate nominal rigidities into their models. The two-country model in Kim and Kim (2018) incorporates capital.
3Kim and Kim (2003) show that second-order solutions are necessary because conventional linearization may generate spurious welfare reversals when long-run distortions exist in the model.
4Bodenstein et al. (2018)’s program reads a Dynare model file and generates the first-order conditions of policymakers under the optimal global policy and the Nash equilibrium. See Adjemian et al. (2011) for details on Dynare.
5The case where \( \theta = 1 \) corresponds to a special case in which an automatic form of risk insurance is provided (Cole and Obstfeld 1991).
6Following De Paoli and Lipinska (2013), we include only one policy instrument.
7The first-order necessary condition for optimality at \( t = 0 \) is (A31) with \( \lambda_{-1} = 0 \).
8\( N \) in A3 is not identical to \( N \) in A4.

References


