Identity, Incentive Contracts and Organizational Architecture

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November 2014

Abstract

This study focuses on the relationship between junior (agent) and senior employees (principal) in an organizational structure, and on the attitude of an agent toward the objectives in a performance evaluation. The study extends prior management accounting research, with a focus on performance evaluation—namely, signing the contract, determining behavior, measuring performance, and distributing the results. As a result, it becomes possible to add steps that include the principal giving instructions on behavior, which can be observed in actual organizations. To be specific, this study focuses on the psychological stress that agents experience when they diverge from behavioral objectives and provides an "identity coefficient" as a scale measuring the extent to which stress is experienced.

First, this study clarifies that it is beneficial for a principal to enter into a contract with an agent who has a high identity coefficient. At the same time, however, the agent can set the objectives for the contract. Thus, second, this study shows why the principal should reserve the decision rights to set such agent objectives.

In reality, at the time a contract is entered into, it is likely that, in many cases, the principal cannot verify an agent's identity coefficient. Therefore, third, this study investigates a case where identity coefficients is the private information of the agent. This analysis assumes that the principal knows the distribution of the identity coefficients. In this case, an agent has a tendency to over report; therefore, the principal pays an additional rent to motivate the agent to report truthfully.

Consequently, the analysis shows that the influence of the identity coefficient on the incentive coefficient and target value is dependent on the distribution of the identity coefficient within the organization or the labor market.

In addition, there are occasions when mechanisms for truthful reporting do not function, and these occasions are dependent on the distribution of the identity coefficient within the organization or the labor market. This study clarifies that the important problem is how to estimate the distribution of the identity coefficient.

Finally, it is established that if the accuracy of a performance measure can be improved, or an agent's propensity to avoid risk lowered, it is possible to save on the rent of truthful reporting. Therefore, the identity coefficient, introduced in this study, is closely related to discussions that have taken place in prior traditional research, and is worthy of further investigation, alongside these other aspects, as a supplementary scale measuring effects on performance evaluation.

1. Introduction

This paper considers the effects of an agent's individual attributes on performance evaluation. To date, management-accounting research has investigated performance evaluation by focusing only on attributes such as the amount of information available to an agent, the agent's abilities, the agent's attitude toward risk, and the effects that these attributes have on performance evaluation. However, the focus of this study is the attitude of agents toward behavioral objectives provided by the principal, which few researchers have dealt with so far. To be specific, this study focuses on the psychological stress that agents experience when they diverge from behavioral objectives. In this study, psychological stress is set as a variable with operability and introduced into the agency model; a comparative analysis is then carried out with a traditional model.

After concluding a contract, when a low-ranking (junior) person within an organizational hierarchy carries out a task as directed by a higher-ranking (senior) person, the lower-ranking person feels self-inflicted stress (as there is no monitoring or threat from another person to cause such stress). Specifically, there are occasions of high stress when the junior person is aware that the behavior he/she chose and carried out diverged from the instructions given by the senior person. As the junior person agreed to the contract and accepted the instructions of the senior person, the junior person now perceives him/herself as tied to the instructions. Therefore, junior employees can easily find themselves in situations where they experience stress. Additionally, the senior person in the organization, who evaluates performance, has a large amount of discretion; in contrast, the junior person who receives the evaluation has a limited scope of discretion. It is highly likely that such situations exist within current organizations. This is the reason why this study decided to focus on an agent's attitude toward behavioral objectives.

Even when agents (junior employees) believe something that causes them to experience stress when diverging from instructions, the amount of stress experienced is likely to differ from agent to agent. This difference can be said to be dependent on the psychological elements of each individual. In this study, this sort of sensitivity to stress is referred to as an "identity coefficient" and the difference in the extent to which stress is felt is inserted into a model. Furthermore, the agent's belief is reflected in the model and the discussion includes the differences in the extent that agents experience stress when they diverge from a principal's instructions (the senior employee). This again can be considered a point of focus for the agents' attitudes toward behavioral objectives.

By focusing on the identity coefficient, which is an internalized attribute, it becomes possible to expand the interpretability of the traditional agency model with the following three points: First, this analysis considers not only physical disutility, such as physical fatigue resulting from an agent's behavior, but also mental disutility. Second, within the flow of a performance evaluation — namely, signing the contract, determining the behavior, measuring performance, and distributing the results — it becomes possible to add the steps (from observations of actual organizations) of the principal giving instructions around behavioral objectives (or the agent being made aware of the behavioral objectives). Third, a new viewpoint is obtained wherein not only remuneration but also the stress, resulting from the divergence from instructed behavior, becomes means to motivate the agent.

The model in this paper was influenced by identity economics and is based on previous research work[1]. However, the agent's belief addressed in this paper is conceptually different from the identity effect considered in identity economics. Additionally, the specific points of discussion investigated in this paper include the question of who should determine behavioral objectives, and how the equilibrium solution in such a situation makes it unfeasible to verify the identity coefficient.

The structure of this paper is as follows: In Section 2, the model is established. In Section 3, the equilibrium solution that can be obtained when it is possible to verify the agent's identity coefficient is analyzed. In Section 4, the equilibrium solution obtained when it is not possible to verify the agent's identity coefficient is analyzed. In Section 5, analysis is provided after assuming a specific distribution of the identity coefficient, and in Section 6, the qualities of an optimum contract are analyzed based on the general assumptions made in Section 7.

2. The model

This analysis uses the LEN model[2], where $j \in \{A, P\}$, P is the principal (a superior) and A is the agent (a subordinate). The agent is risk-averse and the principal is risk-neutral. To simplify, the analysis assumes that the agent undertakes one type of effort, $e \in \mathbb{R}^{+}$. This paper assumes that $k(e) = 0.5e^2$ represents the personal physical cost incurred in the effort, such as fatigue.

The principal receives stochastic achievement $\tilde{x}$ (e.g., cash flow), $\mathrm{E}[\tilde{x}] = e$. This depends on the agent's effort. However, $\tilde{x}$ is generally regarded as being impossible to verify. $\tilde{x}$ cannot be used in contracts, therefore, the principal uses a verifiable performance measure $\tilde{y} = be + \tilde{\varepsilon}$, $\tilde{\varepsilon} \sim N(0\,, \sigma^2)$. When measuring accounting profit, allowances are made for b (sensitivity to effort) and $\tilde{\varepsilon}$ (the impact of measurement error). At the end of the period, the agent receives compensation w and the agreement is concluded. Compensation is represented by $w(\tilde{y})$ in the compensation contract offered by the principal, and $w(\tilde{y})$ is assumed to be the linear function $w(\tilde{y}) = \alpha + \beta\tilde{y}$, where α is the fixed salary and β is the incentive coefficient. The compensation function is offered by the principal prior to the conclusion of the contract. $U^P \equiv \tilde{x} - w(\tilde{y})$ is the corporate value. The principal gains $\tilde{x} - w(\tilde{y})$ as residuals.

Next, the agent's utility function U^A is

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[1] For example, see Akerlof and Kranton (2000), Fischer and Huddart (2008), and Heinle et al. (2012).

[2] LEN stands for "linear contract," "exponential utility," and "normally distributed performance measure." Refer to Lambert (2001) for further discussions.

$$U^A = -\exp\left[-r\left(w(y) - k(e) + I^A(e, s, \theta)\right)\right]. \tag{1}$$

In this paper, I^A is the belief (dogma) that generates stress for the agent.[3] Since the agent agreed to the contract and accepted the instructions of the principal, he/she is bound by these instructions. . In addition, the principal who evaluates performance has a large amount of discretion; however, in contrast, the agent who receives the evaluation has a limited scope of discretion. Therefore, this study assumes that only the agent has such a belief, and the belief is

$$I^A(e, s, \theta) = -0.5\theta(s - e)^2. \tag{2}$$

Equation (2) is the same as in Heinle et al., (2012, 1314). $s \in \mathbb{R}^+$ represents the level of the behavioral objectives. In this paper, the level of behavioral objectives is regarded as the effort that helps achieve the best utility for the principal. The principal gives instructions on behavioral objectives s after signing the contract, however the agent is aware of s before exerting any effort.

Formula (2) assumes that the agent feels stress when deviating from the best level of effort.[4] The agent's propensity to feel this stress is expressed as $\theta \in [0, \bar{\theta}]$, and this is also the intensity of the psychological factors underlying the agent's desire to try to comply with the instructions. The intensity of the psychological factors that underlie the agent's desire to try to conform to the prescribed behavior can be interpreted as belief and devotion, when the agent falls in line with the prescribed behavior, and as fear and anxiety, when the agent does not conform. However, this study will use the former concept, where θ signifies that the agent falls in line with the prescribed behavior sufficiently positive, where the agent believes he/she is compliant. This study regards θ as the identity coefficient with respect to the instructions of the principal. This assumes a more complex and realistic representation of human behavior than the traditional agency theory. In short, the only scenarios that have been dealt with to date are those where $\theta = 0$.

The principal's utility function U^P is

$$U^P = x - w(y). \tag{3}$$

The principal and the agent make decisions to maximize their own utility toward each other. The agent holds a belief that causes him/her stress and the principal offers a compensation contract and describes an objective, knowingly bringing stress to the agent.

The following diagram shows the timeline for the scenario described in this section.

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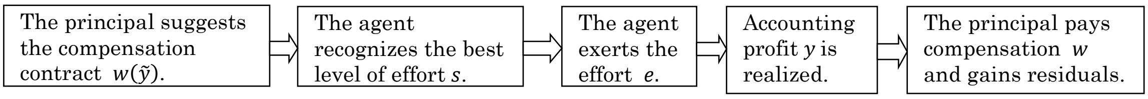
3 So far, the discussion can intuitively surmise that as θ grows, the agent is required to exert more effort to obtain the same utility. It is assumed that the reason why the agent accepts that he/she needs to exert more effort to obtain the same utility as θ is getting larger is that the order of the agent's preferences is different, based on the level of θ.

4 In this paper, the agent feels negative identity preference in the case where both $s > e$ and $s < e$. $s > e$ implies agent laziness. $s < e$ implies that the agent has miscalculated the best level of effort, resulting in great cost to the principal.

Figure 1: Timeline of the contract



3. Case where θ is verifiable

As the agent is the follower in this Stackelberg Model, let us consider aspects using backward induction. The agent's optimal effort, in the face of the contract offered by the principal, can be obtained by solving the incentive compatibility (IC) condition,

$$\text{given } \alpha, \beta \text{ and } s, \quad \max_{e} EU^A = \int -\exp[-r(w(y) - k(e) - 0.5\theta(s-e)^2)]\, f(y)\, dy. \tag{4}$$

$f(y)$ is the density function. (4) is [5]

$$EU^A = -\exp[-r(\alpha + \beta be - 0.5r\beta^2\sigma^2 - k(e) - 0.5\theta(s-e)^2)]. \tag{5}$$

When CE^A is certainty equivalent, $EU^A = U^A(CE^A)$, so

$$\frac{\partial CE^A}{\partial e} = \beta b - e + \theta(s-e) = 0. \tag{6}$$

Consequently, the optimal effort selected by the agent e^* is

$$e^* = \frac{\beta b + \theta s}{1+\theta}. \tag{7}$$

In contrast to the conclusions of traditional agency theory, (7) shows that if we take into account the identity coefficient, we can obtain a positive effort, even when the agent's effort is unobservable and when he/she has a fixed salary contract ($\beta = 0$). Nevertheless, it also indicates that there is more leverage to extract a greater level of effort under an incentive contract ($\beta > 0$). Therefore, this study examines the case of offering incentive contracts even where the identity coefficient is greater than zero. Expected gain of the principal is $EU^P = e - \mathrm{E}[w\,(\tilde{y})]$. Therefore, if the effort is e^*, the problem for the principal is

$$\max_{s,\alpha,\beta} EU^P(e^*) = e^* - E[w(\tilde{y})|e^*]\,, \tag{8}$$

$$\text{subject to} \quad EU^A(e^*) = \int -\exp[-r(w(y) - k(e^*) - 0.5\theta(s-e^*)^2)]\, f(y)\, dy \geq \underline{U}^A. \tag{9}$$

$\underline{U}^A$ is the reservation utility. If we set the reservation wage to zero, individual rationality condition (IR) is $CE^A \geq 0$. However, if $CE^A > 0$, the principal can bring down $w(y)$ to $CE^A = 0$. Therefore, the expected compensation is

$$\alpha + \beta be^* = 0.5(e^*)^2 + 0.5\theta(s-e^*)^2 + 0.5r\beta^2\sigma^2. \tag{10}$$

(10) indicates that the principal pays for the disutility of effort, the disutility of the stress of deviating from instructions, and risk premium. Substituting (7) and (10) with (8), the problem for

[5] See Appendix B.

the principal is

$$\max_{s,\alpha,\beta} EU^P(e^*) = -\frac{\theta}{2(1+\theta)}s^2 + \frac{\theta}{1+\theta}s + \frac{\beta b(1-0.5\beta b)}{1+\theta} - 0.5r\beta^2\sigma^2. \quad (11)$$

$\frac{\theta}{1+\theta} > 0$, and if we partially differentiate for s and rearrange the first order conditions, $s^* = 1$. Moreover, defining α^*, β^* as maximize $EU^P(e^*, s^*)$, we obtain

$$\beta^* = \frac{b}{b^2 + r(1+\theta)\sigma^2}, \alpha^* = \frac{-b^4 + b^2 r\sigma^2 + r^2(1+\theta)\theta\sigma^4}{2(b^2 + r(1+\theta)\sigma^2)^2}. \quad (12)$$

Consequently, the utility for the principal is $EU^P(e^*, s^*, \alpha^*, \beta^*)$, where

$$EU^P(e^*, s^*, \alpha^*, \beta^*) = \frac{0.5b^4 + b^2 r(0.5+\theta)\sigma^2 + 0.5r^2(1+\theta)\theta\sigma^4}{(b^2 + r(1+\theta)\sigma^2)^2}. \quad (13)$$

(See Appendix A)

Lemma 1

(1) $\partial s^* / \partial\theta = 0$ indicates that the level of behavioral objectives has nothing to do with the identity coefficient.

(2) $\partial e^\dagger / \partial\theta > 0$ indicates that the higher the agent's identity coefficient, the higher the optimal level of effort selected by the agent.

(3) $\partial\alpha^\dagger / \partial\theta > 0$ and $\partial\beta^\dagger / \partial\theta < 0$ indicates that, in the optimal contract, the higher the agent's identity coefficient, the higher the fixed salary and the lower the incentive coefficient that the principal is able to offer.

(4) $\partial EU^P(e^*, s^*, \alpha^*, \beta^*) / \partial\theta > 0$ indicates that, in the optimal contract, the higher the agent's identity coefficient, the higher the expected utility that the principal can gain.

Lemma 1 is consistent with the findings of Heinle et al. (2012, 1313-1316). First, Lemma 1 (1), the level of behavioral objectives, is not influenced by the identity coefficient. In addition, Lemma 1 (2) indicates that an agent with a high identity coefficient will select a high level of effort. Moreover, comparing this to a contract with an agent who has a low identity coefficient, Lemma 1 (3) indicates that the principal does not pay a higher remuneration, even if (by chance) the agent has shown a significantly higher performance level. It implies that the principal can save on the risk premium.

In addition, Lemma 1 (4) indicates that the higher the agent's identity coefficient, the higher the expected utility that the principal can gain. Therefore, if the abilities of agents are the same, it is rational for the principal to contract with agents having higher identity coefficient.

Thus far, this study assumes that the principal directs the behavioral objectives. In actuality, however, the agents can set such objectives. Therefore, this study also considers the case where the principal delegates the rights to set own objectives to the agent. This analysis denotes the level of effort, the level of behavioral objectives, fixed salary, and incentive coefficient as e^{**}, s^{**}, α^{**}, β^{**}, respectively. As a result, this analysis obtains Proposition 1 (refer to Appendix 1).

Proposition 1

When the principal delegates the rights of setting s,

(1) the agent selects $I^A = 0$ as s^{**} .

(2) $s^{**} < s^*$ and $e^{**} < e^*$.

(3) $EU^P(e^{**}, s^{**}, \alpha^{**}, \beta^{**}) < EU^P(e^*, s^*, \alpha^*, \beta^*)$.

Proposition 1 (1) may be intuitively obvious. If the agent has the rights to set the objectives, the agent will set them to avoid suffering the psychological cost that comes from deviating from the objectives. In this situation, the level of budget s^{**} is lower than the s^* set by the principal, and effort $e^{**}(s^{**}, \beta^{**})$ is also lower than $e^*(s^*, \beta^*)$. In addition, the principal's expected utility becomes lower. The agent's expected utility is the same as in the case in which the principal has the decision rights to set the objectives, since it satisfies the IR constraint.

Intuitively, it seems desirable that the organization is without stress and the implementer of the organizational objectives sets the optimal objectives voluntarily. However, Proposition 1 indicates that such a structure is not desirable for an efficient organizational architecture[6]. If the principal imposes the right amount of stress on the agent and compensates for the cost of that stress, the agent exerts a higher effort and better performance. This structure is favorable for the organization. Therefore, the principal should reserve the decision rights to set the objectives. In this sense, Proposition 1 is significant and desirable for an organizational architecture.

4. Cases where the principal cannot verify θ

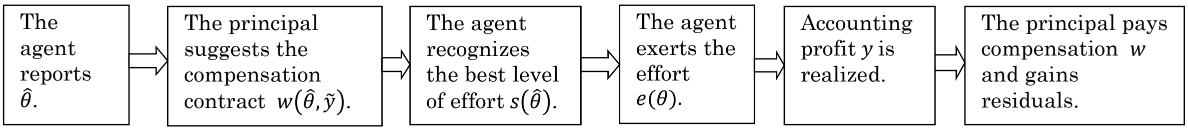
From the discussion so far, it is beneficial for the principal to enter into a contract with an

[6] Brickly et al. (2008) explains that efficient organizational architecture consists of (1) distribution of decision rights, (2) measuring performance, and (3) rewarding performance.

agent who has a high identity coefficient. Therefore, for example, it is a good idea to try to raise agent identity coefficients through education and training, and to employ agents whose identity coefficients are high to begin with. However, in reality, at the time the contract is signed, it is likely that, in many cases, the principal cannot verify the agent's θ. Additionally, even if the results of education and training in raising θ can be observed by some psychological method, it cannot be considered as objective verification when comparing this to financial information. Therefore, its true value is as private information for the agent. Thus, this section will carry out an analysis assuming that the agent is required to provide a self-assessment report.

$f(\theta)$ is set as the probability density function of θ distributed in the interval $[\underline{\theta}, \bar{\theta}]$, and $F(\theta)$ is the cumulative distribution function. This section assumes that the principal knows the distribution of θ. The θ that the agent reports to the principal is $\hat{\theta}$. Based on this, the principal determines and presents a remuneration contract of $s(\hat{\theta})$, $w(\hat{\theta}, y) = \alpha(\hat{\theta}) + \beta(\hat{\theta})y$. In other words, the principal determines $\alpha(\hat{\theta}), \beta(\hat{\theta}),$ and $s(\hat{\theta})$.

Figure 2: Timeline of the contract cases where the principal cannot verify θ



4.1 Incentives for agents to report falsely

To start, this study confirms whether there are incentives for agents to falsely report; if they unconditionally reported the truth, the cases would be the same as the cases for which it is possible to verify the identity coefficient. When the principal maintains the right to determine the target value, the agent's certainty equivalent value turns out to be as follows:

$$CE^A(\theta, \hat{\theta}, e(\theta)) = \alpha(\hat{\theta}) + \beta(\hat{\theta})be(\theta) - 0.5e(\theta)^2 - 0.5\theta(s(\hat{\theta}) - e(\theta))^2 - 0.5r\beta(\hat{\theta})^2\sigma^2. \quad (14)$$

As the agent selects the effort level to maximize this, from

$$\frac{\partial CE^A(\theta, \hat{\theta}, e(\theta))}{\partial e(\theta)} = \beta(\hat{\theta})b - e(\theta) + \theta\left(s(\hat{\theta}) - e(\theta)\right) = 0, \quad (15)$$

the agent, under conditions in which θ is private information, selects an effort level $e^{\dagger}(\theta, \hat{\theta})$ as follows:

$$e^{\dagger}(\theta, \hat{\theta}) = \frac{\beta(\hat{\theta})b + s(\hat{\theta})\theta}{1 + \theta}. \quad (16)$$

When this is substituted into formula (12) and arranged, the certainty equivalent value that assumes an optimum level of effort $CE^A(\theta, \hat{\theta})$ is as follows:

$$CE^A(\theta\,,\hat{\theta}\,) = \alpha(\hat{\theta}) + \frac{\left(\beta(\hat{\theta})b + s(\hat{\theta})\theta\right)^2}{2(1+\theta)} - 0.5\left(r\beta(\hat{\theta})^2\sigma^2 + s(\hat{\theta})^2\theta\right). \tag{17}$$

Here, considering the certainty equivalent values $\alpha^*(\hat{\theta}), \beta^*(\hat{\theta}), s^*(\hat{\theta})$,

$$CE^A\left(\theta\,,\hat{\theta}\middle|\,\alpha^*(\hat{\theta}), \beta^*(\hat{\theta}), s^*(\hat{\theta})\right) = \frac{r^2(1+\hat{\theta})(-\theta+\hat{\theta})\sigma^4}{2(1+\theta)\left(b^2 + r(1+\hat{\theta})\sigma^2\right)^2}. \tag{18}$$

From this, this analysis obtains Lemma 2 (See Appendix A).

Lemma 2

If $\alpha^*(\hat{\theta}), \beta^*(\hat{\theta}), s^*(\hat{\theta})$ is presented, the agent will always report that $\hat{\theta} = \bar{\theta}$.

From Lemma 2, as long as it is not the case that $\theta = \bar{\theta}$, the agent has an incentive to report a higher identity coefficient. Based on this, an agent with a low identity coefficient will report that he/she is an agent with a higher identity coefficient. Moreover, agents who exaggerate their identity coefficients relative to the actual coefficients, after selecting an effort level that is lower than the optimum effort level, are likely to obtain a higher fixed salary to compensate them for their stress. This represents a loss in the principal's utility. Therefore, the principal requires that the truth be reported. Considering this condition, the problem for the principal investigated in this section turns out to be as follows:

$$\max_{s(\theta),\beta(\theta)} E_\theta U^P = E_\theta x\left(\theta\middle|e^\dagger(\theta)\right) - E_\theta w\left(\theta\middle|e^\dagger(\theta)\right), \tag{19}$$

subject to

$$\theta \in \underset{\hat{\theta}}{\operatorname{argmax}}\, CE^A(\theta\,,\hat{\theta}\,) \tag{20}$$

$$CE^A(\theta\,,\hat{\theta}\,) \geq 0 \text{ for all } \theta. \tag{21}$$

E_θ shows that the expected value operation pertains to θ. Formula (20) is the condition for truthful reporting, while formula (21) is the IR condition. The reservation wage is assumed to be zero.

4.2 Motivating agents to report truthfully

First, this section investigates the sufficient condition for truthful reporting. The effects of θ on the certainty equivalent value are as follows:

$$\frac{dCE^A(\theta\,,\hat{\theta}\,)}{d\theta} = \frac{\partial CE^A(\theta\,,\hat{\theta}\,)}{\partial\theta} + \frac{\partial CE^A(\theta\,,\hat{\theta}\,)}{\partial\hat{\theta}} \cdot \frac{d\hat{\theta}}{d\theta}. \tag{22}$$

An agent knows θ will, with θ as a given, be in his/her report, while attempting to maximize the certainty equivalent value, and therefore this first order condition is as follows:

$$\left.\frac{\partial CE^A(\theta,\hat{\theta})}{\partial\hat{\theta}}\right|_{\hat{\theta}=\hat{\theta}^*}=0. \tag{23}$$

Therefore,

$$\frac{dCE^A(\theta,\hat{\theta}^*)}{d\theta}=\left.\frac{\partial CE^A(\theta,\hat{\theta})}{\partial\theta}\right|_{\hat{\theta}=\hat{\theta}^*} \tag{24}$$

is completed. In order to motivate the agent to report truthfully, a requirement is that $\theta=\hat{\theta}^*$.

From formulae (17) and (23), the first order condition is as follows:

$$\begin{aligned}\frac{\partial CE^A(\theta,\hat{\theta})}{\partial\hat{\theta}}=\alpha'(\hat{\theta})+\frac{\left(\beta(\hat{\theta})b+s(\hat{\theta})\theta\right)\left(\beta'(\hat{\theta})b+s'(\hat{\theta})\theta\right)}{1+\theta}-\beta(\hat{\theta})\beta'(\hat{\theta})r\sigma^2\\ -s(\hat{\theta})s'(\hat{\theta})\theta=0.\end{aligned} \tag{25}$$

Therefore, in $\hat{\theta}=\theta$,

$$A\equiv\alpha'(\theta)+\frac{(\beta(\theta)b+s(\theta)\theta)(\beta'(\theta)b+s'(\theta)\theta)}{1+\theta}-\beta(\theta)\beta'(\theta)r\sigma^2-s(\theta)s'(\theta)\theta=0\text{ for all }\theta$$

must be completed. Therefore, when A is differentiated once again in θ,

$$\begin{aligned}\frac{\partial A}{\partial\theta}&=\alpha''(\theta)\\ &+\frac{\left(\beta'(\theta)b+s'(\theta)\theta+s(\theta)\right)(\beta'(\theta)b+s'(\theta)\theta)+(\beta(\theta)b+s(\theta)\theta)\left(\beta''(\theta)b+s''(\theta)\theta+s'(\theta)\right)}{1+\theta}\\ &-\frac{(\beta(\theta)b+s(\theta)\theta)(\beta'(\theta)b+s'(\theta)\theta)}{(1+\theta)^2}-r\sigma^2\left(\beta'(\theta)^2+\beta(\theta)\beta''(\theta)\right)\\ &-s'(\theta)^2\theta-s(\theta)s''(\theta)\theta-s(\theta)s'(\theta)=0\end{aligned} \tag{26}$$

should also be completed. In addition, the second order condition must be as follows:

$$\begin{aligned}\frac{\partial^2 CE^A(\theta,\hat{\theta})}{\partial\hat{\theta}^2}=\alpha''(\hat{\theta})+\frac{\left(\beta'(\hat{\theta})b+s'(\hat{\theta})\theta\right)^2+\left(\beta(\hat{\theta})b+s(\hat{\theta})\theta\right)\left(\beta''(\hat{\theta})b+s''(\hat{\theta})\theta\right)}{1+\theta}\\ -r\sigma^2\left(\beta'(\hat{\theta})^2+\beta(\hat{\theta})\beta''(\hat{\theta})\right)-s'(\hat{\theta})^2\theta-s(\hat{\theta})s''(\hat{\theta})\theta\le 0.\end{aligned} \tag{27}$$

Therefore, when $\hat{\theta}=\theta$, formula (27) is as follows:

$$\begin{aligned}\alpha''(\theta)+\frac{(\beta'(\theta)b+s'(\theta)\theta)^2+(\beta(\theta)b+s(\theta)\theta)(\beta''(\theta)b+s''(\theta)\theta)}{1+\theta}\\ -r\sigma^2\left(\beta'(\theta)^2+\beta(\theta)\beta''(\theta)\right)-s'(\theta)^2\theta-s(\theta)s''(\theta)\theta\le 0.\end{aligned} \tag{28}$$

As in formula (26), $\partial A/\partial\theta=0$, formula (28) can be arranged as follows[7]:

[7] Rearranged

$$\frac{(s(\theta)-\beta(\theta)b)(s'(\theta)-\beta'(\theta)b)}{(1+\theta)^2}\leq 0, \tag{29}$$

$$\text{(the sufficient condition for truthful reporting)} \quad (s(\theta)-\beta(\theta)b)(s'(\theta)-\beta'(\theta)b)\leq 0. \tag{30}$$

Formula (30) is the sufficient condition for truthful reporting.

Next, this study investigates the necessary condition. From formula (17), when the truth is reported $(\hat{\theta}=\theta)$, and the certainty equivalent value is $CE^A(\theta\,,\theta\,)$, the formula becomes:

$$\frac{\partial CE^A(\theta\,,\theta\,)}{\partial\theta}=\alpha'(\theta)+\frac{\big(\beta'(\theta)b+s'(\theta)\theta+s(\theta)\big)(\beta(\theta)b+s(\theta)\theta)}{1+\theta}-\frac{(\beta(\theta)b+s(\theta)\theta)^2}{2(1+\theta)^2}$$
$$-\beta(\theta)\beta'(\theta)r\sigma^2-s(\theta)s'(\theta)\theta-0.5s(\theta)^2. \tag{31}$$

Additionally, as $A=0$, formula (31) can be arranged as follows:

$$\frac{\partial CE^A(\theta\,,\theta\,)}{\partial\theta}=-\frac{(s(\theta)-\beta(\theta)b)^2}{2(1+\theta)^2}\text{ for all }\theta. \tag{32}$$

In other words, the certainty equivalent is the decreasing function of θ. Therefore, when the identity coefficient is low, utility increases. From this, when $\theta=\bar{\theta}$, the IR condition must be completed with an equal sign and the formula becomes:

$$CE^A(\theta\,)=\bar{w}+\int_{\bar{\theta}}^{\theta}\frac{\partial CE^A(t\,)}{\partial t}dt=0.5\int_{\theta}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2dt. \tag{33}$$

Formula (33) is the necessary condition for truthful reporting and for satisfying IR condition. $0.5\int_{\theta}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2dt$ can be interpreted as being the rent of truthful reporting. Even if the agent has an incentive to over report, the possibility to achieve truthful reporting is high because the principal pays a larger rent to an agent with a low identity coefficient than an agent with a high

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$$\alpha''(\theta)+\frac{(\beta'(\theta)b+s'(\theta)\theta)^2+(\beta(\theta)b+s(\theta)\theta)(\beta''(\theta)b+s''(\theta)\theta)}{1+\theta}-r\sigma^2\big(\beta'(\theta)^2+\beta(\theta)\beta''(\theta)\big)-s'(\theta)^2\theta-s(\theta)s''(\theta)\theta$$

$$\leq\alpha''(\theta)+\frac{\big(\beta'(\theta)b+s'(\theta)\theta+s(\theta)\big)(\beta'(\theta)b+s'(\theta)\theta)+(\beta(\theta)b+s(\theta)\theta)\big(\beta''(\theta)b+s''(\theta)\theta+s'(\theta)\big)}{1+\theta}$$

$$-\frac{(\beta(\theta)b+s(\theta)\theta)(\beta'(\theta)b+s'(\theta)\theta)}{(1+\theta)^2}-r\sigma^2\big(\beta'(\theta)^2+\beta(\theta)\beta''(\theta)\big)-s'(\theta)^2\theta-s(\theta)s''(\theta)\theta-s(\theta)s'(\theta)$$

to

$$0\leq-\frac{(\beta(\theta)b+s(\theta)\theta)(\beta'(\theta)b+s'(\theta)\theta)}{(1+\theta)^2}+\frac{(\beta(\theta)b+s(\theta)\theta)s'(\theta)}{1+\theta}+\frac{(\beta'(\theta)b+s'(\theta)\theta)s(\theta)}{1+\theta}-s(\theta)s'(\theta).$$

identity coefficient. In other words, in order to have an agent with a low identity coefficient report the truth, the principal must pay relatively higher rents.

4.3 Deriving the optimum contract

Based on the results of sections 4.1 and 4.2, this study attempts to derive the optimum contract. Based on formula (33), the expected wage for the optimum effort is as follows:

$$Ew\left(\theta \middle| e^{\dagger}(\theta)\right) = 0.5\int_{\theta}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2 dt + 0.5\left(e^{\dagger}(\theta)^2 + \theta\left(s(\theta)-e^{\dagger}(\theta)\right)^2 + \beta(\theta)^2 r\sigma^2\right). \quad (34)$$

From formula (17) and formula (24), the problem for the principal becomes consolidated in the following formula:

$$\max_{s(\theta),\beta(\theta)} E_\theta U^P = \int_{\underline{\theta}}^{\bar{\theta}}\Bigg(-\frac{\theta}{2(1+\theta)}s(\theta)^2 + \frac{\theta}{1+\theta}s(\theta) + \frac{\beta(\theta)b - 0.5\beta(\theta)^2b^2}{1+\theta} - 0.5\beta(\theta)^2 r\sigma^2 - 0.5\int_{\theta}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2 dt\Bigg) f(\theta)d\theta \quad (35)$$

Here, when it is partially integrated to the fifth term, it becomes:

$$\begin{aligned}
&\int_{\underline{\theta}}^{\bar{\theta}}\int_{\theta}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2 dt f(\theta)d\theta \\
&= F(\theta)\int_{\theta}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2 dt\Bigg|_{\underline{\theta}}^{\bar{\theta}} - \int_{\underline{\theta}}^{\bar{\theta}}\frac{d}{d\theta}\int_{\theta}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2 dtF(\theta)d\theta \\
&= F(\bar{\theta})\int_{\bar{\theta}}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2 dt - F(\underline{\theta})\int_{\underline{\theta}}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2 dt \\
&- \int_{\underline{\theta}}^{\bar{\theta}}\frac{d}{d\theta}\int_{\theta}^{\bar{\theta}}\left(\frac{s(t)-\beta(t)b}{1+t}\right)^2 dtF(\theta)d\theta = \int_{\underline{\theta}}^{\bar{\theta}}\left(\frac{s(\theta)-\beta(\theta)b}{1+\theta}\right)^2 F(\theta)d\theta.
\end{aligned} \quad (36)$$

Further, it is defined as follows:

$$H(\theta) \equiv \frac{F(\theta)}{f(\theta)}. \quad (37)$$

From formula (36) and formula (37), formula (35) becomes:

$$\max_{s(\theta),\beta(\theta)} E_\theta U^P \int_{\underline{\theta}}^{\bar{\theta}}\Bigg(-\frac{\theta}{2(1+\theta)}s(\theta)^2 + \frac{\theta}{1+\theta}s(\theta) + \frac{\beta(\theta)b - 0.5\beta(\theta)^2b^2}{1+\theta} - 0.5\beta(\theta)^2 r\sigma^2 - 0.5\left(\frac{s(\theta)-\beta(\theta)b}{1+\theta}\right)^2 H(\theta)\Bigg) f(\theta)d\theta. \quad (38)$$

Here, when

$$I \equiv \left(-\frac{\theta}{2(1+\theta)} s(\theta)^2 + \frac{\theta}{1+\theta} s(\theta) + \frac{\beta(\theta)b - 0.5\beta(\theta)^2 b^2}{1+\theta} - 0.5\beta(\theta)^2 r\sigma^2 - 0.5\left(\frac{s(\theta)-\beta(\theta)b}{1+\theta}\right)^2 H(\theta) \right) f(\theta), \tag{39}$$

to maximize formula (38),

$$\frac{\partial I}{\partial s(\theta)} = 0, \frac{\partial I}{\partial \beta(\theta)} = 0 \tag{40}$$

must be completed. Formula (40) is an Euler equation. Therefore,

$$s(\theta) = \frac{H(\theta)\beta(\theta)b + \theta + \theta^2}{H(\theta) + \theta + \theta^2}, \beta(\theta) = \frac{b(1 + H(\theta)s(\theta) + \theta)}{b^2(1 + H(\theta) + \theta) + r(1+\theta)^2\sigma^2} \tag{41}$$

is obtained. From this, when deriving the explicit solution $s^{\dagger}(\theta), \beta^{\dagger}(\theta)$, it becomes:

$$s^{\dagger}(\theta) = \frac{b^2(H(\theta)+\theta) + r\theta(1+\theta)\sigma^2}{b^2(H(\theta)+\theta) + r(H(\theta)+\theta+\theta^2)\sigma^2} > 0, \tag{42}$$

$$\beta^{\dagger}(\theta) = \frac{b(H(\theta)+\theta)}{b^2(H(\theta)+\theta) + r(H(\theta)+\theta+\theta^2)\sigma^2} > 0. \tag{43}$$

However, (42) and (43) are obtained without considering sufficient condition for truthful reporting $(s(\theta) - \beta(\theta)b)(s'(\theta) - \beta'(\theta)b) \leq 0$. Therefore, Proposition 2 shows that the condition for formulas (42) and (43) satisfy sufficient condition for truthful reporting (See Appendix A).

Proposition 2

When

$$\frac{\partial s^{\dagger}(\theta)}{\partial \theta} - \frac{\partial \beta^{\dagger}(\theta)}{\partial \theta} \cdot b \leq 0,$$

formulas (42) and (43) satisfy sufficient condition for truthful reporting

When $\frac{\partial s^{\dagger}(\theta)}{\partial \theta} - \frac{\partial \beta^{\dagger}(\theta)}{\partial \theta} \cdot b > 0$, (42) and (43) are not necessarily motivate truthful reporting[8]. For example, in the case where verification, Lemma 1 shows $\frac{\partial s^*}{\partial \theta} = 0, \frac{\partial \beta^*}{\partial \theta} < 0$. If it can be obtained same results, in the case where not verification, the necessary and sufficient condition to

[8] To obtain (A) optimal $\beta(\theta)$ and $s(\theta)$ which satisfy necessary and sufficient condition for truthful reporting, maximization problem of this section should be a differential inequality constrained optimization problem. However, in this paper, formulas (42) and (43) are obtained from necessary condition, and (B) formulas (42) and (43) are narrowed by Proposition 2. Strictly, $(\mathrm{A}) \subseteq (\mathrm{B})$, but this paper discusses assuming $(\mathrm{A}) = (\mathrm{B})$.

motivate the agent to report truthfully will not satisfied. Therefore, it should be examined that the mechanism for truthful reporting always works in the optimum contract obtained in this section.

Consequently, in Section 5, after assuming the specific distribution of $f\ (\theta)$, this paper illustrates the range in which the mechanism functions and the range in which it does not based on examples of simple numerical values.

5. The range where the agent is motivated to report truthfully and its probability

For the purpose of simplification, it is assumed that $b = 1, \sigma = 10, r = 0.1$. In Section 5.1, an exponential distribution ($f(\theta) = \lambda\exp[-\lambda\theta]$) is assumed and in Section 5.2, a uniform distribution $\left(f(\theta) = \frac{1}{\bar{\theta}-\underline{\theta}}\right)$ is assumed. This is because it is believed that agents with a large θ are likely to be scarce.

5.1 Case where $f(\theta)$ is an exponential distribution ($\lambda\exp[-\lambda\theta]$)

$$f(\theta) = \lambda\exp[-\lambda\theta]\text{, however, it is assumed that } \lambda \in \mathbb{R}^+, \theta \in [0, \infty).$$

Additionally, $F(\theta) = -\exp[-\lambda\theta] + 1, H(\theta) = \frac{-\exp[-\lambda\theta]+1}{\lambda\exp[-\lambda\theta]} = \frac{-1+\exp[\lambda\theta]}{\lambda}$. Therefore, for satisfying Proposition 2,[9] it becomes:

$$\frac{\partial s^\dagger(\theta)}{\partial\theta} - \frac{\partial\beta^\dagger(\theta)}{\partial\theta} = -\frac{10\lambda\left(11 + \theta(22 - \theta\lambda) + 11\exp[\theta\lambda]\left(-1 + \theta(-2 + \lambda + \theta\lambda)\right)\right)}{(-11 + 11\exp[\theta\lambda] + \theta(11 + 10\theta)\lambda)^2} \leq 0 \qquad (44)$$

Here, when $\lambda = 2$, the range of θ that satisfies formula (44) is $0.063 \leq \theta$. Therefore, even if the optimum contract from formulae (42) and (43) is presented, there will still be instances of truthful and false reporting. The range is divided as shown in Figure 3 below.

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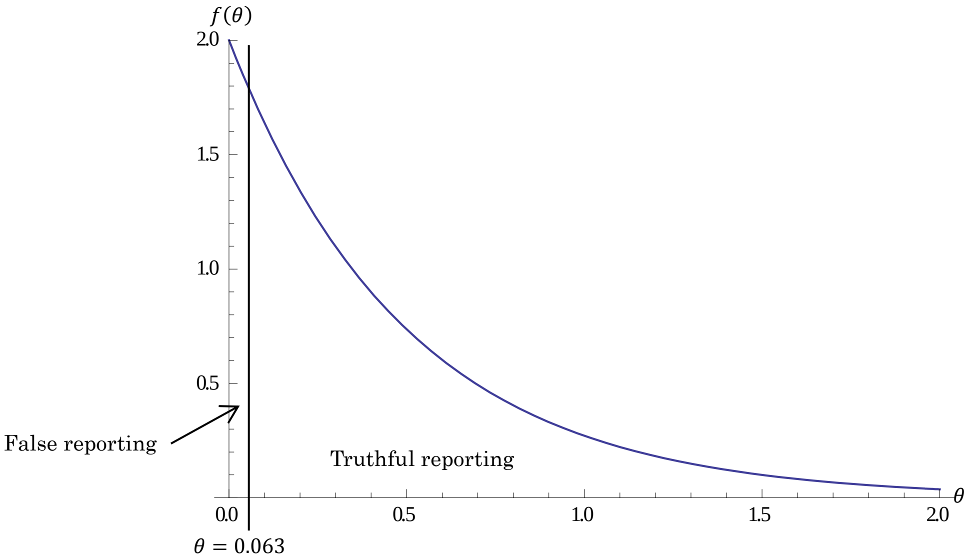
$$\beta^\dagger(\theta) = \frac{\frac{-1+\exp[\lambda\theta]}{\lambda} + \theta}{11\left(\frac{-1+\exp[\lambda\theta]}{\lambda}\right) + 11\theta + 10\theta^2}, \qquad s^\dagger(\theta) = \frac{\frac{-1+\exp[\lambda\theta]}{\lambda} + 11\theta + 10\theta^2}{11\left(\frac{-1+\exp[\lambda\theta]}{\lambda}\right) + 11\theta + 10\theta^2} \qquad \text{(f.1)}$$

$$\frac{\partial\beta^\dagger(\theta)}{\partial\theta} = \frac{10(-1 + \exp[\theta\lambda])\theta\lambda(-2 + \theta\lambda)}{(-11 + 11\exp[\theta\lambda] + \theta(11 + 10\theta)\lambda)^2},$$

$$\frac{\partial s^\dagger(\theta)}{\partial\theta} = \frac{10\lambda(-11 - 20\theta + \exp[\theta\lambda]\,(11 + 20\theta - \theta(11 + 10\theta)\lambda))}{(-11 + 11\exp[\theta\lambda] + \theta(11 + 10\theta)\lambda)^2} \qquad \text{(f.2)}$$

Figure 3: The range in which truthful reporting and false reporting occur



Therefore, the probability $P_{lie}(\theta)$ that false reporting will occur is as follows:

$$P_{lie}(\theta) = \lim_{\underline{\theta} \to 0} \int_{\underline{\theta}}^{0.063} 2\exp[-2\theta]\, d\theta = 0.1184. \tag{45}$$

The range satisfying truthful reporting and the probability that it will occur change according to the constant λ and is shown in the table below.

Table 1: The range in which truthful reporting occurs and its probability

	λ				
	0.1	0.5	1	2	2.519
The range satisfying truthful reporting	$15.6026 \leq \theta$	$2.4814 \leq \theta$	$0.8542 \leq \theta$	$0.063 \leq \theta$	$2.2 \times 10^{-8} \leq \theta$
The probability for truthful reporting to occur	0.2100	0.2892	0.4256	0.8816	0.999 ...
The probability for false reporting to occur	0.7899	0.7108	0.5744	0.1184	5.5×10^{-8}

When $f(\theta)$ is an exponential distribution, the tendency is that the scarcer the agents with a high θ, the higher the probability of motivating the agent for truthful reporting by proposing $s^{\dagger}(\theta)$ and $\beta^{\dagger}(\theta)$.[10] In addition, agents with lower θ tend to resort to false reporting.

5.2 Case where $f(\theta)$ is a uniform distribution $\left(f(\theta) = \frac{1}{\bar{\theta} - \underline{\theta}}\right)$

$$\text{Next, } f(\theta) = \frac{1}{\bar{\theta} - \underline{\theta}} \text{, but it is assumed } \theta \in [\underline{\theta}, \bar{\theta}]. \text{As } F(\theta) = \frac{\theta - \underline{\theta}}{\bar{\theta} - \underline{\theta}}, H(\theta) = \theta - \underline{\theta}.$$

[10] By trial and error, the probability of false reporting becomes less than 1%, when $\lambda \geq 2.17$.

To satisfy Proposition 2,[11]

$$\frac{\partial s^{\dagger}(\theta)}{\partial \theta}-\frac{\partial \beta^{\dagger}(\theta)}{\partial \theta}=-\frac{10\left(-12\theta^2+11\underline{\theta}(1+2\theta)\right)}{\left(11\underline{\theta}-2\theta(11+5\theta)\right)^2}\leq 0 \tag{46}$$

The range of θ that satisfies formula (46) is $0<\theta \leq \frac{1}{12}\left(11\underline{\theta}+\sqrt{11}\sqrt{\underline{\theta}\left(12+11\underline{\theta}\right)}\right)$. In the case of uniform distribution, agents with higher θ tend to falsely report their own θ. Interestingly, it is the converse case in exponential distribution. Therefore, the probability $P_{true}(\theta)$ that false reporting will occur is as follows:

$$P_{true}(\theta)=\int_{\underline{\theta}}^{\frac{1}{12}\left(11\underline{\theta}+\sqrt{11}\sqrt{\underline{\theta}(12+11\underline{\theta})}\right)}\frac{1}{\bar{\theta}-\underline{\theta}}d\theta=\frac{\sqrt{11}\sqrt{\underline{\theta}(12+11\underline{\theta})}-\underline{\theta}}{12(\bar{\theta}-\underline{\theta})}>0. \tag{47}$$

From this, even if the optimum contract from formulae (42) and (43) is presented, there will still be occurrences of truthful and false reporting. Additionally, when we consider $\bar{\theta}$ and $\underline{\theta}$ as variables, we obtain $\frac{\partial P_{true}(\theta,\bar{\theta},\underline{\theta})}{\partial \bar{\theta}}<0, \frac{\partial P_{true}(\theta,\bar{\theta},\underline{\theta})}{\partial \underline{\theta}}>0.$

Therefore, the tendency is that the greater the dispersion of the distribution, the lower the probability of satisfying the necessary and sufficient condition for truthful reporting.

Finally, assuming $\underline{\theta}=0.01$, the tendency is that the greater the dispersion of the distribution, the lower the probability of satisfying the necessary and sufficient condition for truthful reporting, as is shown in Figure 4.

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$$\beta^{\dagger}(\theta)=\frac{\underline{\theta}-2\theta}{11\underline{\theta}-2\theta(11+5\theta)},\qquad s^{\dagger}(\theta)=\frac{\underline{\theta}-2\theta(6+5\theta)}{11\underline{\theta}-2\theta(11+5\theta)} \tag{f.3}$$

$$\frac{\partial \beta^{\dagger}(\theta)}{\partial \theta}=\frac{20(\underline{\theta}-\theta)\theta}{\left(11\underline{\theta}-2\theta(11+5\theta)\right)^2},\qquad \frac{\partial s^{\dagger}(\theta)}{\partial \theta}=\frac{10\left(10\theta^2-\underline{\theta}(11+20\theta)\right)}{\left(11\underline{\theta}-2\theta(11+5\theta)\right)^2} \tag{f.4}$$

Figure 4: The relationship between $\bar{\theta}$ and the probability that truthful reporting will occur

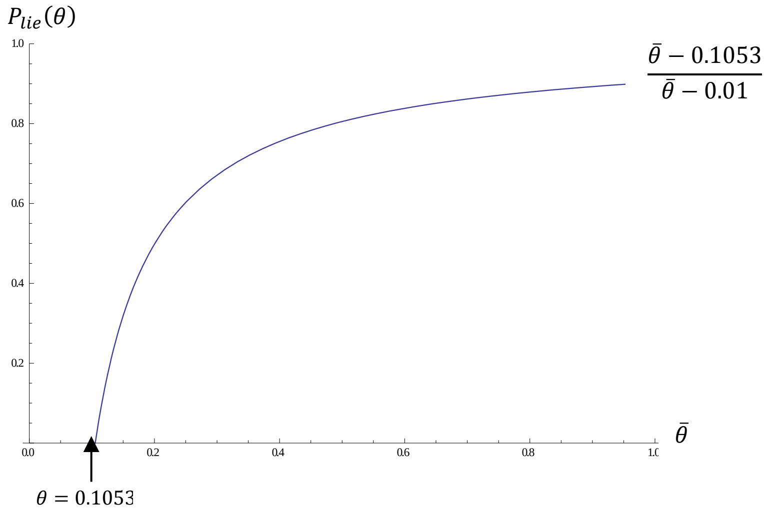


Figure 4 shows that when $\bar{\theta} < 0.1053$, the sufficient condition is always satisfied. If θ becomes higher, the agent report false and $P_{lie}(\theta)$ rises.

6. Interpreting the qualities of the optimum contract

In Section 5, a specific distribution was assumed. However, returning once again to the assumptions considered before Section 4, this section closely examines the general qualities of the optimum contract detailed in this study. First, Proposition 3 is obtained from formulae (42) and (43). (See Appendix A)

Proposition 3

(1)

$$\text{When } \frac{f'(\theta)}{f(\theta)} \geq -\frac{2}{\theta}, \frac{\partial \beta^{\dagger}(\theta)}{\partial \theta} \leq 0, \qquad \text{and when } \frac{f'(\theta)}{f(\theta)} < -\frac{2}{\theta}, \frac{\partial \beta^{\dagger}(\theta)}{\partial \theta} > 0.$$

(2)

$$\text{When } \frac{f'(\theta)}{f(\theta)} < \frac{1}{H(\theta)} - G(\theta), \frac{\partial s^{\dagger}(\theta)}{\partial \theta} < 0, \text{and when } \frac{f'(\theta)}{f(\theta)} \geq \frac{1}{H(\theta)} - G(\theta), \frac{\partial s^{\dagger}(\theta)}{\partial \theta} \geq 0\,.$$

$$\text{However, } G(\theta) \equiv \frac{1}{\theta}\left\{1 + \frac{r\sigma^2}{b^2 + r(1+\theta)\sigma^2}\right\}.$$

According to Lemma 1, when it is possible for the principal to verify θ, the incentive coefficient is the decreasing function for θ. However, Proposition 3 (1) shows that $\beta^{\dagger}(\theta)$ behaves differently with regard to θ, according to the distribution of θ. In companies or labor markets, when the distribution of θ is a uniform distribution, or the distribution is skewed close to $\bar{\theta}$, as was the case with Lemma 1, the higher θ becomes, the lower the incentive coefficient becomes. However, in companies or labor markets with a skew close to $\underline{\theta}$ (for example, exponential distribution) and with a smaller number of agents with a high θ, the higher the reported value of θ, the higher must be $\beta^{\dagger}(\theta)$. However, when $\frac{\partial \beta^{\dagger}(\theta)}{\partial \theta} > 0$, Proposition 2 becomes likely to be satisfied.

One of the implications for considering the identity coefficient of the agent is that the higher θ is, the lower the incentive coefficient becomes. Therefore, it is possible to save on rents, such as for motivation and risk premiums. However when θ cannot be verified, these benefits are sometimes lost.

Additionally, Proposition 3 (2) shows that whether the target value is increased depends on the distribution of θ, as θ becomes higher. In Lemma 1, the target value with regard to θ is a constant, and therefore, this is also a critical difference between the instances when θ can and cannot be verified.

Next, Corollary 1 is obtained from Proposition 2 and Proposition 3. (See Appendix A)

Corollary 1

At the very least, if $\frac{f'(\theta)}{f(\theta)} < -\frac{2}{\theta}$, the agent reports truthfully.

Corollary 1 highlights that if an agent with a high θ is scarce (the distribution of θ within a company or labor market is skewed close to $\underline{\theta}$), the principal can motivate the agent to report truthfully by $\beta^{\dagger}(\theta)$ and $s^{\dagger}(\theta)$. It indicates the importance of estimating the distribution of θ within a company or labor market.

In addition, Corollary 1 shows that $\beta^{\dagger}(\theta)$ and $s^{\dagger}(\theta)$ is effective in the case where adding the agent with high identity to the assumption of traditional agency theory as an exception. Intuitively, it seems rational that assuming that an agent with a high θ is scarce.

However, if the principal aims to influence the shape of the distribution, rather than θ, and it achieves $\frac{f'(\theta)}{f(\theta)} \geq -\frac{2}{\theta}$, ironically, agent with high identity becomes report falsely. Considering such "trade-off" is important.

Furthermore, by comparing $s^{\dagger}(\theta)$ and s^{*}, Proposition 4 can be developed. (See Appendix A)

Proposition 4

When it is not possible to verify the identity coefficient, the principal sets the target value lower than when it is possible to verify the identity coefficient.

Formulae (7) and (16) demonstrate that the higher the target value, the higher the effort level invested by the agent. However, Proposition 3 shows that in the event that it is not possible to verify the identity coefficient, even if, for example, the agent reports $\bar{\theta}$, setting a target value lower than s^* becomes the optimum. This is considered to be due to an attempt to save on the rent of truthful reporting. Related to this, if it is not possible to verify the identity coefficient, the principal must pay the rent for truthful reporting, and in this sense, it is inevitably inefficient compared to when it is possible to verify the identity coefficient.

Last, this section investigates how it might be possible to save on the rent of truthful reporting from the perspective of the accuracy of performance measurement and the agent's attitude toward risk. (See Appendix A)

Proposition 5

If the accuracy of performance measurement can be increased, or the agent's propensity to avoid risk be lowered, it is possible to save on the rent for truthful reporting. In extreme cases, when $r \to 0$ or $\sigma \to 0$, it is possible that no rent will be paid.

In research based on traditional agency theory, attitudes to risk have proven to be important, and the accuracy of performance measurements have had a decisive meaning in terms of assigning weights when preparing management accounting information[12]. For example, supposing that in this study the agent is risk-neutral, then regardless of the size of the identity coefficient, $\beta^\dagger = \frac{1}{b}, s^\dagger = 1$. Therefore, the identity coefficient cannot be substituted for considerations of attitude toward risk. Rather, the identity coefficient introduced in this study is closely related to the discussion that has taken place before in traditional research, and is worthy of investigation alongside other aspects as a supplementary scale regarding effects on performance evaluation.

7. Conclusion and issues for consideration

This study, regarding the attributes of agents, namely psychological attributes, focused on the attitude of agents toward objectives. It assumed that the agents possessed a belief that resulted in experiencing stress when their actions were contrary to behavioral objectives. The label "identity coefficient" was given to the scale measuring the extent of stress agents experienced, and the

[12] For example, refer to Banker and Datar (1989).

effects that this scale on performance evaluations were considered.

When it is possible for the principal to verify the agent's identity coefficient, it was observed that the higher the identity coefficient, the greater the residual the principal could obtain. Therefore, if the agents' abilities are the same, it is rational for principals to enter into contracts with agents with higher identity coefficients (Lemma 1). Additionally, to obtain this outcome, a principal should determine a target-effort level (Proposition 1). In an organization where an agent independently sets the target and feels no stress, although this may seem to be a desirable situation, actually, Proposition 1 shows this is not desirable from the perspective of efficient organizational design.

However, at the time a contract is concluded, it is likely that, in many cases, the principal cannot verify the agent's identity coefficient. Thus, while the principal knows the distribution of the identity coefficient, its true value is to the agent as private information, and therefore the analysis was carried out assuming that the agent was required to self-report this assessment.

When reporting this self-assessment, the agent has an incentive to over report the identify coefficient (Lemma 2). Therefore, the principal pays a rent in order to motivate the agent to report truthfully. $\beta^{\dagger}(\theta)$ and $s^{\dagger}(\theta)$ is one of the optimal solution, which satisfies the necessary condition for truthful reporting, and Proposition 2 shows the conditions for $\beta^{\dagger}(\theta)$ and $s^{\dagger}(\theta)$ satisfying the sufficient condition for truthful reporting. However, the principal must pay a higher rent for an agent with a low identity coefficient than an agent with a high identity coefficient. An equilibrium solution for a rational incentive coefficient and target-effort level is obtained. However, the target-effort level obtained at such time is strictly lower than when it is not possible to verify the identity coefficient (Proposition 4).

The results of Section 5 and Proposition 3 indicate that, depending on the distribution of the identity coefficient within the organization or the labor market, there are occasions when the mechanism for truthful reporting does not function. Additionally, since the identity coefficient effects on the incentive coefficient are different, it was clarified that the important problem is how to estimate the distribution of the identity coefficient. In addition, Corollary 1 shows that $\beta^{\dagger}(\theta)$ and $s^{\dagger}(\theta)$ is effective in the case where adding the agent with high identity to the assumption of traditional agency theory as an exception.

Finally, it was established that if the accuracy of performance measurement can be improved, or the agent's propensity to avoid risk lowered, it is possible to save on the rent of truthful reporting (Proposition 5). In research based on the traditional agency theory, attitude toward risk has been important, and the accuracy of performance measurements have had a decisive meaning in terms of assigning weights when preparing management accounting information. Proposition 5 indicates that the identity coefficient introduced in this paper is closely related to the discussion that has taken place in prior research and is worthy of investigation, alongside these other aspects, as a supplementary scale on effects on performance evaluation.

In terms of future issues to be addressed, the first is that clarify generally optimal solution of $\beta(\theta)$ and $s(\theta)$. The second, in this study, the identity coefficient scale was considered to already exist; however, whether it actually exists needs to be verified based on psychological empirical study and experimental study. The third is the expansion of the model used here. For example, it was assumed that the identity coefficient is positive and finite. However, it is possible that there will be cases where it is negative. In addition, depending on the work, it is likely that the manner in which agents experience stress will not be symmetrical.

The fourth issue is the most important one: investigating the reason why the identity coefficient is generated and additionally through which processes the identity coefficient increases or decreases. For example, will creating a company song and company motto that employees sing and recite be useful in terms of raising identity coefficients? Additionally, if last year's performance was good, and the agent had confidence in the principal's ability to set targets, is his/her identity coefficient likely to rise? Moreover, if we consider the effect the length of time an agent is associated with an organization has on his/her identity coefficient, it seems necessary to investigate a model that reflects the contract period and form of employment. Alternatively, if the agent works actively in a team, this is likely to affect his/her identity coefficient. In such a case, a new idea will be presented in the discussion of the optimum team size. The search for the mechanism that generates the identity coefficient directly connects with new topics of research. These are future issues.

Appendix A

The proof of Lemma 1

(1) From $s^* = 1$, obtained straight forward.

(2) From Equations (7) and (12),

$$\frac{\partial e^*(\beta^*)}{\partial\theta} = \frac{\partial(\beta^* b + \theta)/(1+\theta)}{\partial\theta} = \frac{r^2\sigma^4}{(b^2 + r(1+\theta)\sigma^2)^2} > 0. \tag{A.1}$$

Accordingly, the higher the agent's identity coefficient, the higher the level of optimum effort that he/she will select.

(3) From Equation (12),

$$\frac{\partial\alpha^*}{\partial\theta} = \frac{r\sigma^2(0.25(b^2 - r\sigma^2)^2 + 0.75b^4 + 0.25r^2\sigma^4 + b^2 r\theta\sigma^2 + 0.5r^2\theta\sigma^4)}{(b^2 + r(1+\theta)\sigma^2)^3} > 0 \tag{A.2}$$

holds true. Equation (12) gives $\partial\beta^\dagger/\partial\theta < 0$, and so, in an optimal contract, the higher the agent's identity coefficient, the higher the incentive coefficient.

(4)

$$\frac{\partial EU^P(e^*, s^*, \alpha^*, \beta^*)}{\partial \theta} = \frac{r^2\sigma^4}{2(b^2 + r(1+\theta)\sigma^2)^2} > 0, \tag{A.3}$$

the higher the agent's identity coefficient, the higher the residuals that the principal can gain.

Q. E. D

The proof of Proposition 1

The problem of the agent is

$$\text{given } \alpha \text{ and } \beta, \quad \max_{e,s} EU^A = \int -\exp[-r(w(y) - k(e) - 0.5\theta(s-e)^2)]\, f(y)\, dy. \tag{A.4}$$

The optimal effort e^{**} and the objective s^{**} is obtained by

$$\frac{\partial CE^A}{\partial e} = 0, \frac{\partial CE^A}{\partial s} = 0. \tag{A.5}$$

Accordingly, $e^{**} = s^{**} = \beta b$. Therefore, the agent selects $I^A = 0$ as s^{**}.

Next, the problem for the principal is

$$\max_{\alpha,\beta} EU^P(e^{**}, s^{**}) = e^* - E[w(\tilde{y})|e^{**}, s^{**}], \tag{A.6}$$

$$subject\ to\ \ EU^A(e^{**}, s^{**}) = \int -\exp[-r(w(y) - k(e^*) - 0.5\theta(s^{**} - e^{**})^2)]\, f(y)\, dy \geq \underline{U}^A. \tag{A.7}$$

Therefore, we obtain the optimal incentive coefficient β^{**}

$$\beta^{**} = \frac{b}{b^2 + r\sigma^2}. \tag{A.8}$$

Therefore, $s^{**}(\beta^{**}) < s^*$ holds true. In addition, from equations (7) and (12),

$$e^{**}(\beta^{**}) = \frac{b^2}{b^2 + r\sigma^2}, e^*(\beta^*) = \frac{1}{1+\theta}\left\{\frac{b^2}{b^2 + r(1+\theta)\sigma^2} + \theta\right\}, \tag{A.9}$$

so

$$\frac{\partial e^*(\beta^*)}{\partial \theta} > 0 \text{ and } \lim_{\theta \to 0} e^*(\beta^*) = e^{**}(\beta^{**}). \tag{A.10}$$

The problem for principal is

$$EU^P(e^{**}, s^{**}, \alpha^{**}, \beta^{**}) = \frac{0.5b^2}{b^2 + r\sigma^2}, \tag{A.11}$$

so

$$\frac{\partial EU^P(e^*, s^*, \alpha^*, \beta^*)}{\partial \theta} > 0 \text{ and } \lim_{\theta \to 0} EU^P(e^*, s^*, \alpha^*, \beta^*) = EU^P(e^{**}, s^{**}, \alpha^{**}, \beta^{**}). \tag{A.12}$$

Q. E. D

The proof of Lemma 2

From Equation (18), the principal reports

$$\hat{\theta} > \theta \tag{A.13}$$

and

$$\frac{\partial CE^A\left(\theta, \hat{\theta} \middle| \alpha^*(\hat{\theta}), \beta^*(\hat{\theta}), s^*(\hat{\theta})\right)}{\partial \hat{\theta}} > 0 \tag{A.14}$$

is

$$\frac{r^2\sigma^4\{b^2(1-\theta+2\hat{\theta}) + (1+\theta)r(1+\hat{\theta})\sigma^2\}}{2(1+\theta)\{b^2 + r(1+\hat{\theta})\sigma^2\}^3} > 0. \tag{A.15}$$

Q. E. D

The proof of Proposition 2

$s^\dagger(\theta)$ and $\beta^\dagger(\theta)$ is satisfied necessary condition, but it is not necessarily satisfied sufficient condition. Here, the sufficient condition is Formula (30). $s^\dagger(\theta) - b\beta^\dagger(\theta) \geq 0$, so to be satisfied a necessary and sufficient condition for truthful reporting, it must be

$$\frac{\partial s^\dagger(\theta)}{\partial \theta} - \frac{\partial \beta^\dagger(\theta)}{\partial \theta} \cdot b \leq 0.$$

Q. E. D

The proof of Proposition 3

(1)

$$\frac{\partial \beta^\dagger(\theta)}{\partial \theta}$$
$$= \frac{b(H'(\theta)+1)\{b^2(H(\theta)+\theta) + r(H(\theta)+\theta+\theta^2)\sigma^2\} - b(H(\theta)+\theta)\{b^2(H'(\theta)+1) + r(H'(\theta)+1+2\theta)\sigma^2\}}{\{b^2(H(\theta)+\theta) + r(H(\theta)+\theta+\theta^2)\sigma^2\}^2}. \tag{A.16}$$

When

$$br\sigma^2\{(H'(\theta)+1)(H(\theta)+\theta+\theta^2) - (H(\theta)+\theta)(H'(\theta)+1+2\theta)\} \leq 0$$
$$\Leftrightarrow H'(\theta)\theta^2 - 2\,H(\theta)\theta - \theta^2 \leq 0 \tag{A.17}$$

$\frac{\partial \beta^\dagger(\theta)}{\partial \theta} \leq 0$. Therefore,

$$\frac{f(\theta)^2 - f'(\theta)F(\theta)}{f(\theta)^2} \leq \frac{2}{\theta} \cdot \frac{F(\theta)}{f(\theta)} + 1 \Leftrightarrow \frac{f'(\theta)}{f(\theta)} \geq -\frac{2}{\theta}. \tag{A.18}$$

The case where $\frac{\partial \beta^\dagger(\theta)}{\partial \theta} > 0$ is the same.

(2)

Next, $R(\theta) \equiv b^2(H(\theta)+\theta) + r(\theta+\theta^2)\sigma^2$,

$$\frac{\partial s^\dagger(\theta)}{\partial \theta} = \frac{R'(\theta)(R(\theta)+H(\theta)r\sigma^2) - R(\theta)(R'(\theta)+H'(\theta)r\sigma^2)}{\{R(\theta)+H(\theta)r\sigma^2\}^2} = \frac{r\sigma^2\big(H(\theta)R'(\theta) - R(\theta)H'(\theta)\big)}{\{R(\theta)+H(\theta)r\sigma^2\}^2}. \tag{A.19}$$

Therefore, when $H(\theta)R'(\theta) - R(\theta)H'(\theta) \geq 0$, then $\frac{\partial s^\dagger(\theta)}{\partial \theta} \geq 0$. So

$$R'(\theta) = b^2(H'(\theta) + 1) + r(1 + 2\theta)\sigma^2, \tag{A.20}$$

and

$$H(\theta)\{b^2(H'(\theta) + 1) + r(1 + 2\theta)\sigma^2\} - \{b^2(H(\theta) + \theta) + r(\theta + \theta^2)\sigma^2\}H'(\theta) > 0$$

$$\Leftrightarrow \frac{1}{\theta}\left\{1 + \frac{r\sigma^2}{b^2 + r(1+\theta)\sigma^2}\right\} > \frac{H'(\theta)}{H(\theta)} \tag{A.21}$$

$$\Leftrightarrow \frac{1}{\theta}\left\{1 + \frac{r\sigma^2}{b^2 + r(1+\theta)\sigma^2}\right\} > \frac{1}{H(\theta)} - \frac{f'(\theta)}{f(\theta)}.$$

The case where $\frac{\partial s^{\dagger}(\theta)}{\partial \theta} < 0$ is the same.

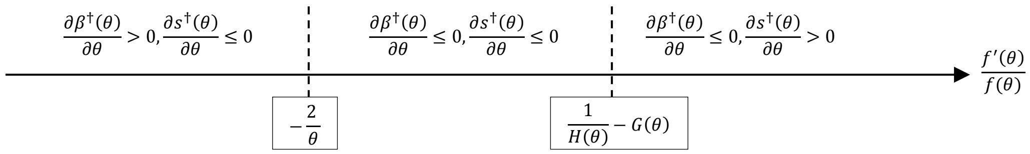
Q. E. D.

The proof of Corollary 1

From Proposition 2, $\frac{\partial s^{\dagger}(\theta)}{\partial \theta} - \frac{\partial \beta^{\dagger}(\theta)}{\partial \theta} \cdot b \leq 0$ is a necessary and sufficient condition for truthful reporting. At least, if $\frac{\partial \beta^{\dagger}(\theta)}{\partial \theta} > 0$ and $\frac{\partial s^{\dagger}(\theta)}{\partial \theta} \leq 0$, the condition is consistent. From Proposition 3(1) and (2), when $\frac{f'(\theta)}{f(\theta)} < -\frac{2}{\theta}$, and when $\frac{f'(\theta)}{f(\theta)} \leq \frac{1}{H(\theta)} - G(\theta)$

$$\frac{1}{H(\theta)} - G(\theta) - \left(-\frac{2}{\theta}\right) = \frac{1}{H(\theta)} + \frac{1}{\theta}\left\{1 - \frac{r\sigma^2}{b^2 + r\theta\sigma^2 + r\sigma^2}\right\} > 0$$

Therefore,



If $\frac{f'(\theta)}{f(\theta)} < -\frac{2}{\theta}$, then $\frac{\partial \beta^{\dagger}(\theta)}{\partial \theta} > 0$ and $\frac{\partial s^{\dagger}(\theta)}{\partial \theta} \leq 0$.

Q. E. D.

The proof of Proposition 4

$$s^{\dagger}(\theta) = \frac{b^2(H(\theta) + \theta) + r\theta(1+\theta)\sigma^2}{b^2(H(\theta) + \theta) + r\theta(1+\theta)\sigma^2 + H(\theta)r\sigma^2}. \tag{A.22}$$

$H(\theta) > 0$, so $s^{\dagger}(\theta) < 1$. Therefore, $s^{\dagger}(\theta) < s^{*}$.

Q. E. D

The proof of Proposition 5

The truthful reporting rent is

$$0.5\int_0^{\bar{\theta}} \left(\frac{s(\theta) - \beta(\theta)b}{1+\theta}\right)^2 H(\theta)f(\theta)d\theta. \tag{A.23}$$

To minimize the truth telling rent, one rational method is to minimize $D(\theta) \equiv s(\theta) - \beta(\theta)b$.

$$D(\theta) = \frac{r\theta(1+\theta)\sigma^2}{b^2(H(\theta)+\theta) + r(H(\theta)+\theta+\theta^2)\sigma^2}, \tag{A.24}$$

so

$$\frac{\partial D(\theta)}{\partial r} = \frac{b^2\theta(1+\theta)(H(\theta)+\theta)\sigma^2}{(b^2(H(\theta)+\theta) + r(H(\theta)+\theta+\theta^2)\sigma^2)^2} > 0,$$

$$\frac{\partial D(\theta)}{\partial \sigma} = \frac{2b^2 r\theta(1+\theta)(H(\theta)+\theta)\sigma}{(b^2(H(\theta)+\theta) + r(H(\theta)+\theta+\theta^2)\sigma^2)^2} > 0. \tag{A.25}$$

In addition, the case where

$$\lim_{\sigma\to 0} D(\theta) = 0, \lim_{r\to 0} D(\theta) = 0, \tag{A.26}$$

The principal does not need to pay a rent.

Q. E. D

Appendix B

$$EU^A = \int -\exp[-r(\alpha + \beta y - k(e) - 0.5\theta(s-e)^2)]\, f(y)\, dy$$

$$= -\exp[-r(\alpha - k(e) - 0.5\theta(s-e)^2)] \int \exp[-r\beta y]\, f(y)\, dy. \tag{B.1}$$

And

$$\int \exp[-r\beta y]\, f(y)\, dy = \int \exp[-r\beta y] \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(y-be)^2}{2\sigma^2}\right] dy$$

$$= \int \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{y^2 - 2ybe + 2yr\beta\sigma^2 + b^2e^2}{2\sigma^2}\right] dy. \tag{B.2}$$

Here, $(y - be + r\beta\sigma^2)^2 = y^2 + b^2e^2 + r^2\beta^2\sigma^4 - 2ybe + 2yr\beta\sigma^2 - 2ber\beta\sigma^2$. Therefore

$$\int \exp[-r\beta y]\, f(y)\, dy = \int \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(y-be+r\beta\sigma^2)^2 + 2ber\beta\sigma^2 - r^2\beta^2\sigma^4}{2\sigma^2}\right] dy$$

$$= \exp[-r\beta be + 0.5r^2\beta^2\sigma^2] \int \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(y-be+r\beta\sigma^2)^2}{2\sigma^2}\right] dy. \tag{B.3}$$

if $\dfrac{y - be + r\beta\sigma^2}{\sigma} = t$, then $\dfrac{dt}{dy} = \dfrac{1}{\sigma}$. When $y\colon -\infty \to \infty, t\colon -\infty \to \infty$. Therefore

$$\int \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{(y-be+r\beta\sigma^2)^2}{2\sigma^2}\right] dy = \int \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{t^2}{2}\right] dt = 1. \tag{B.4}$$

From $\int \exp[-r\beta y]\, f(y)\, dy = \exp[-r\beta be + 0.5r^2\beta^2\sigma^2]$,

$$EU^A = -\exp[-r(\alpha - k(e) - 0.5\theta(s-e)^2)] \exp[-r\beta be + 0.5r^2\beta^2\sigma^2]$$

$$= -\exp[-r(\alpha + \beta be - 0.5r\beta^2\sigma^2 - k(e) - 0.5\theta(s-e)^2)]. \tag{B.5}$$

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