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Online Appendix of
Does a Leading Indicator Related to a Customer Improve a Firm's Profit?

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Proof of Parametric Assumption

In symmetric cases (*i.e.*, $(PI, PI) = (L, L), (NL, NL)$), regardless of the relationship between α and γ, we can obtain the positive outcomes under $\alpha < 1/4$, which is the condition of $\pi_{it}^{(L,L)} > 0$. Additionally, we obtain $\alpha < \gamma$ from $q_{i1}^{(NL,L)} > 0$. □

Proof of Observation 3 and 4.

We consider the decision-making of the owner. We identify the optimal strategies in each case where the combinations of performance indicators are $(PI, PI) = (L, L), (NL, L)$ to consider the equilibrium strategy of the performance indicator. First, by backward induction, the optimal strategy in $(PI, PI) = (L, L)$ is as follows:

$$
\begin{aligned}
q_{i1}^{(L,L)} &= \frac{1}{2 - 2\alpha - \gamma}, \\
q_{i2}^{(L,L)} &= \frac{2 - \gamma - 2\alpha + r(1 + \gamma)}{(3 - 2\alpha)(2 - 2\alpha - \gamma)}, \\
I_i^{(L,L)} &= \frac{1}{2 - 2\alpha - \gamma}, \\
p_{i1}^{(L,L)} &= \frac{1 - 2\alpha}{2 - 2\alpha - \gamma}, \\
p_{i2}^{(L,L)} &= \frac{(1 - 2\alpha)\big(2 - \gamma - 2\alpha + r(1 + \gamma)\big)}{(3 - 2\alpha)(2 - 2\alpha - \gamma)}, \\
\pi_{i1}^{(L,L)} &= \frac{1 - 4\alpha}{2(2 - 2\alpha - \gamma)^2}, \\
\pi_{i2}^{(L,L)} &= \frac{(1 - 2\alpha)\big(2 - \gamma - 2\alpha + r(1 + \gamma)\big)^2}{(3 - 2\alpha)^2(2 - 2\alpha - \gamma)^2}.
\end{aligned}
\tag{A.1}
$$

When the Parametric Assumption is satisfied, all outcomes are positive.

Next, the optimal strategies and profit in $(PI, PI) = (NL, L)$ are as follows:

$$
\begin{aligned}
q_{i1}^{(NL,L)} &= \frac{\gamma-\alpha}{\gamma(2-\alpha-\gamma)}, \\
q_{j1}^{(NL,L)} &= \frac{\gamma+\alpha}{\gamma(2-\alpha-\gamma)}, \\
q_{i2}^{(NL,L)} &= \frac{(1-\alpha)\gamma(2-\alpha-\gamma)+r\left(\gamma(1+\gamma)+\alpha^2(1-\gamma)-\alpha(3-2\gamma+\gamma^2)\right)}{(3-\alpha)\gamma(2-\alpha-\gamma)}, \\
q_{j2}^{(NL,L)} &= \frac{(1+\alpha)\gamma(2-\alpha-\gamma)+r\left(\gamma(1+\gamma)-\alpha^2(1-\gamma)+\alpha(3-2\gamma+\gamma^2)\right)}{(3-\alpha)\gamma(2-\alpha-\gamma)}, \\
I_{i}^{(NL,L)} &= \frac{\gamma-\alpha}{\gamma(2-\alpha-\gamma)}, \\
I_{j}^{(NL,L)} &= \frac{\gamma+\alpha}{\gamma(2-\alpha-\gamma)}, \\
p_{i1}^{(NL,L)} &= \frac{\gamma-\alpha}{\gamma(2-\alpha-\gamma)}, \\
p_{j1}^{(NL,L)} &= \frac{\gamma(1-2\alpha)+\alpha}{\gamma(2-\alpha-\gamma)}, \\
p_{i2}^{(NL,L)} &= \frac{(1-\alpha)\gamma(2-\alpha-\gamma)+r\left(\gamma(1+\gamma)+\alpha^2(1-\gamma)-\alpha(3-2\gamma+\gamma^2)\right)}{(3-\alpha)\gamma(2-\alpha-\gamma)}, \\
p_{j2}^{(NL,L)} &= \frac{(1-\alpha)\gamma(2-\alpha-\gamma)+r(\gamma(1+\gamma)-\alpha^2(1-\gamma)+\alpha(3-4\gamma-\gamma^2))}{(3-\alpha)\gamma(2-\alpha-\gamma)}, \\
\pi_{i1}^{(NL,L)} &= \frac{(\gamma-\alpha)^2}{2\gamma^2(2-\alpha-\gamma)^2}, \\
\pi_{j1}^{(NL,L)} &= \frac{(\gamma+\alpha)(\gamma(1-4\alpha)+\alpha)}{2\gamma^2(2-\alpha-\gamma)^2}, \\
\pi_{i2}^{(NL,L)} &= p_{i2}^{(NL,L)} q_{i2}^{(NL,L)}, \\
\pi_{j2}^{(NL,L)} &= p_{j2}^{(NL,L)} q_{j2}^{(NL,L)}.
\end{aligned}
\tag{A.2}
$$

When the Parametric Assumption is satisfied, all outcomes are positive.

From Eqs. (11), (A.1), and (A2), we obtain $\Pi_i^{(NL,L)} - \Pi_i^{(L,L)}$ and $\Pi_i^{(NL,NL)} - \Pi_i^{(L,NL)}$ as follows:

$$\Pi_i^{(NL,L)} - \Pi_i^{(L,L)} = -\frac{2\alpha D}{(3-2\alpha)^2(3-\alpha)^2\gamma^2(2-\alpha-\gamma)^2(2-2\alpha-\gamma)^2},$$
$$\Pi_i^{(NL,NL)} - \Pi_i^{(L,NL)} = -\frac{2\alpha E}{9(3-\alpha)^2(2-\gamma)^2\gamma^2(2-\alpha-\gamma)^2}, \tag{A.3}$$

where $D \equiv 3(2-\gamma)\gamma\big(27-46\gamma+15\gamma^2+6\gamma^3-\gamma^4+r^2(18-4\gamma-9\gamma^2+12\gamma^3-\gamma^4)+2r(18-29\gamma+24\gamma^2-9\gamma^3+\gamma^4)\big)-4\alpha^7(1+2r^2(1-\gamma)^2+4r(1-\gamma)\gamma+2\gamma^2)+4\alpha^6\big(11+\gamma+20\gamma^2-6\gamma^3+4r\gamma(11-12\gamma+3\gamma^2)+r^2(22-42\gamma+26\gamma^2-6\gamma^3)\big)-\alpha^5\big(193+48\gamma+312\gamma^2-204\gamma^3+26\gamma^4+4r\gamma(201-242\gamma+108\gamma^2-13\gamma^3)+r^2(386-708\gamma+540\gamma^2-212\gamma^3+26\gamma^4)\big)-\Big(432+233\gamma+547\gamma^2-668\gamma^3+185\gamma^4-12\gamma^5+2r\gamma(992-1321\gamma+798\gamma^2-189\gamma^3+12\gamma^4)+r^2(864-1518\gamma+1421\gamma^2-752\gamma^3+185\gamma^4-12\gamma^5)\Big)-\alpha^3\Big(522+585\gamma+233\gamma^2-994\gamma^3+492\gamma^4-70\gamma^5+2\gamma^6+2r\gamma(1432-2111\gamma+1546\gamma^2-536\gamma^3+70\gamma^4-2\gamma^5)+2r^2(522-847\gamma+991\gamma^2-671\gamma^3+254\gamma^4-35\gamma^5+\gamma^6)\Big)-3\alpha\Big(27+189\gamma-293\gamma^2+70\gamma^3+78\gamma^4-38\gamma^5+4\gamma^6+2r\gamma(186-343\gamma+317\gamma^2-162\gamma^3+41\gamma^4-4\gamma^5)+2r^2(27+3\gamma+44\gamma^2-82\gamma^3+69\gamma^4-22\gamma^5+2\gamma^6)\Big)+\alpha^2(324+801\gamma-600\gamma^2-516\gamma^3+570\gamma^4-144\gamma^5+9\gamma^6+2r\gamma(1212-1987\gamma+1662\gamma^2-735\gamma^3+146\gamma^4-9\gamma^5)+r^2(648-822\gamma+1343\gamma^2-1224\gamma^3+666\gamma^4-148\gamma^5+9\gamma^6))$, and $E \equiv 3(2-\gamma)\gamma(27-46\gamma+15\gamma^2+6\gamma^3-\gamma^4+r^2(18-4\gamma-9\gamma^2+12\gamma^3-\gamma^4)+2r(18-29\gamma+24\gamma^2-9\gamma^3+\gamma^4))+\alpha^3(9-45\gamma+16\gamma^2+11\gamma^3-5\gamma^4-r\gamma^2(2+\gamma-\gamma^2)+2r^2(9-27\gamma+29\gamma^2-14\gamma^3+2\gamma^4))-\alpha^2(54-288\gamma+169\gamma^2+42\gamma^3-54\gamma^4+10\gamma^5-r\gamma(36-16\gamma+51\gamma^2-48\gamma^3+11\gamma^4)+r^2(108-324\gamma+346\gamma^2-171\gamma^3+24\gamma^4+\gamma^5))+\alpha(81-513\gamma+466\gamma^2-65\gamma^3-102\gamma^4+46\gamma^5-5\gamma^6-r\gamma(180-250\gamma+266\gamma^2-195\gamma^3+71\gamma^4+10\gamma^5)+r^2(162-522\gamma+526\gamma^2-263\gamma^3+15\gamma^4+25\gamma^5-5\gamma^6))$. Based on the numerical example ($r=0.1$ and $\gamma=0.2$), we consider $\Pi_i^{(NL,L)}-\Pi_i^{(L,L)}$ and $\Pi_i^{(NL,NL)}-\Pi_i^{(L,NL)}$. In this case, we obtain $D \simeq 22.932-178.4489\alpha+497.241\alpha^2-691.5768\alpha^3+531.872\alpha^4-228.678\alpha^5+51.182\alpha^6-4.627\alpha^7>0$ and $E \simeq 22.932-5.6469\alpha-3.2804\alpha^2+0.804\alpha^3>0$. From this analysis, we obtain $\Pi_i^{(NL,L)}<\Pi_i^{(L,L)}$ and $\Pi_i^{(NL,NL)}<\Pi_i^{(L,NL)}$. This implies that both firms adopt the leading indicator under specific conditions.

In addition, we consider $\Pi_i^{(NL,NL)}-\Pi_i^{(L,L)}$ as follows:

$$\Pi_i^{(NL,NL)} - \Pi_i^{(L,L)} = \frac{2\alpha F}{9(3-2\alpha)^2(2-\gamma)^2(2-2\alpha-\gamma)^2}, \tag{A.4}$$

where $F \equiv 3(70-113\gamma+51\gamma^2-8\gamma^3+\gamma^4)+4\alpha^3(17-8\gamma+2\gamma^2)-\alpha^2(52+60\gamma-8\gamma^3)-\alpha(199-404\gamma+132\gamma^2+4\gamma^3-2\gamma^4)-2r(2+\gamma-\gamma^2)(8\alpha^3+8\alpha^2(5-\gamma)+2\alpha(19-7\gamma+\gamma^2)-3(2+\gamma-\gamma^2))+r^2(1+\gamma)^2(8\alpha^3-8\alpha^2(5-\gamma)+2\alpha(37-16\gamma+\gamma^2)-3(8-2\gamma-\gamma^2))$. When we specify r and γ as $r=0.1$ and $\gamma=0.2$, we obtain $F \simeq 145.0068-107.2932\alpha-81.07776\alpha^2+65.49\alpha^3>0$ under $0<\alpha<1/4$. From this outcome, there exists the case in which $\Pi_i^{(NL,L)}<\Pi_i^{(L,L)}$, $\Pi_i^{(NL,NL)}<\Pi_i^{(L,NL)}$, and $\Pi_i^{(NL,NL)}>\Pi_i^{(L,L)}$ hold under the specific condition. □