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REVIEW

35th

ANNUAL REPORT

RESEARCH INSTITUTE FOR ECONOMICS
AND BUSINESS ADMINISTRATION
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CONTENTS

SOME IMPLICATIONS OF THE PRESENT-VALUE-BASED DEPRECIATION WITHIN THE HISTORICAL COST FRAMEWORK

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1. Introduction

The purpose of this paper is to re-examine the present-value-based depreciation concept within the historical cost framework and to report some findings concerning the selection of a depreciation method. One is that if the profitability pattern (as defined later) over time of a fixed asset should remain constant the adoption of the declining-balance method is consistent within our theory. The other result is that if the profitability should decline linearly then the straight-line depreciation is a logical choice in our model.

2. The Depreciation Concept as the Historical Cost of Each Year's Expired Service

The original acquisition cost of a plant asset is usually measured by the amount of expenditure made to obtain the good. From the view-point of its user, it may be said to consist of a series of physical services which are planned to be extracted from the asset.

If we consider a fixed asset as a lump sum purchase of such a bundle of services, then the annual depreciation expense is nothing but that part of the expenditure which can naturally be considered as paid for the service being consumed in this year. How much was paid for the service expired in each financial year? The answer is our measure of depreciation strictly based on the historical cost. The amount paid for the consumed service is equivalent to what could have been dispensed with if that service layer had been unnecessary.

In the first year, one year's service of the brand new asset was consumed. After consumption of this service layer, it becomes a one-year-used asset, which contains everything except that particular service. Accordingly, if we can retrospectively find the acquisition price of such a one-year-used good as of the actual purchase date of the new asset, then the historical cost of the first-year-service

can be determined by taking the difference between the past prices of the new and one-year-used goods.

Stated another way, if that new service layer had been unnecessary, we could have bought a one-year-used asset, not a new one, and saved the difference, which of course is equal to the acquisition price of the first year service from a brand new asset.

This reasoning can be easily generalized, so that we can propose that the i th year depreciation expense is equal to the difference between the historical costs of an i-1 year-used asset and of an i year-used one, both at the date the firm actually bought the good. It is almost self-evident that this difference is the money additionally paid to obtain the i th year plant service [7].

For later use, symbolic expressions of the above will be made. Let

A^i : the acquisition price of the i year-used fixed asset as of the date the firm actually bought the good.

S^i : the service actually consumed during the i th year.

$E(S^i)$: the acquisition price of the service consumed during the i th year.

S^p : the remaining service capacity of the asset when it reached the end of the useful life.

$E(S^p)$: the acquisition price of the service capacity at end of the life.

$D(i)$: the historical-cost-based depreciation expense for the i th year.

The historical cost principle dictates that

$$A^0 = E(S^1) + E(S^2) + \text{----------} + E(S^n) + E(S^p) \tag{1}$$

Obviously,

$$A^1 = E(S^2) + E(S^3) + \text{----------} + E(S^n) + E(S^p) \tag{2}$$

Taking the difference between (1) and (2),

$$A^0 - A^1 = E(S^1) = D(1). \tag{3}$$

Generalizing,

$$A^i = E(S^{i+1}) + E(S^{i+2}) + \text{-------} + E(S^n) + E(S^p) \tag{4}$$

and

$$A^{i-1} = E(S^i) + E(S^{i+1}) + \text{------} + E(S^n) + E(S^p) \tag{5}$$

Comparing (4) and (5).

$$A^{i-1} - A^i = E(S^i) = D(i). \tag{6}$$

3. On Application of a Present Value Formula
to Estimate Used Asset Costs

The past price data of the used assets are often unavailable. This is particularly the case with such peculiar assets as buildings, chemical plants, etc. How can we implement our depreciation concept in these situations?

To approach this problem, a new perspective in the relationship between asset value and depreciation seems to be required. Under the economic depreciation concept, for example, the asset value is first defined as a discounted present value of the future net cash-inflow series emanating from the asset operation. Since this concept seems to happily correspond to the "probable future economic benefit" conception of assets [5], we will continue with this definition, Then, as a first-order difference of the beginning and end year present values, the depreciation expense is defined and quantified [2].

Symbolically,

$$PV(i) = \frac{C(i)}{(1+r)^1} + \frac{C(i+1)}{(1+r)^2} + \text{------} + \frac{C(n)}{(1+r)^{n+1-i}} = \sum_{k=1}^{n+1-i} \frac{C(k+i-1)}{(1+r)^k} \qquad (7)$$

and

$$\begin{aligned} D(i) &= PV(i) - PV(i+1) \\ &= C(i) - PV(i)\, r, \end{aligned} \qquad (8)$$

where $PV(i)$: the discounted present value of the future net cash-inflows from the asset at the beginning of year i,

$\qquad C(i)$: the estimated net cash-inflow for year i,

$\qquad r$: the discount rate which equalizes the discounted value of a new asset to its acquisition cost,

$\qquad n$: the useful life of the asset,

$D(i)$ and A^i : as defined above.

In short, the prior established asset values determine the depreciation expense in the conventional theory (for example, [1], [3], [4], [6], [8], [9]).

Suppose that under the condition of certainty we have succeeded in finding a relation like (7) above between the acquisition cost of a new fixed asset and its estimated discounted value.

$$A^0 = \sum_{k=1}^{n} \frac{C(k)}{(1+r)^k} \qquad (9)$$

Then, we will be justified in constructing the equation

$$A^i = PV(i+1)_0 = \sum_{k=1}^{n-i} \frac{C(k+i)}{(1+r)^k} \quad (i = 0, 1, 2, ----, n), \tag{10}$$

where $PV(i+1)_0$: the discounted present value of the future net cash-inflows
from an i year-used asset as estimated at the beginning of year 1.

and using this formula to estimate the (supposedly) lacking historical acquisition price of an i year-used asset, because there exists logical consistency between (9) and (10).

But everyone knows various difficulties facing implementation of this kind of estimation. They include impossibility of accurately predicting future cash-inflow values as well as theoretical difficulty of attributing the cash-flow amounts (or any other amounts) to the fixed asset without arbitrariness because they are joint products of all input goods ([10], [11]).

We will show below, however, that the present value formula can still be used if we reconsider it from a new point-of-view.

4. The Fixed Asset Cost as the Discounted Present Value of the Future Depreciation Values

All writers on the economic depreciation so far have assumed that annual cash-inflow values $C(i)$ are completely exogenous variables and that the costs (present values) derived therefrom determine depreciation expenses. But this is only partially true. As adequately expressed by Ladelle, the object of discounting in the present value formula consists in the 'value of the enjoyment of the asset' during a year [8]. Hence, the use of net cash-inflow values is only a gross, inaccurate approximation, for they include other inputs' contributions, too. The enjoyment value more closely relates to the quantity and cost of the service from the fixed asset for each year. Whereas the depreciation expense is the original acquisition cost of the service $E(Si)$ provided each year, the value of the service enjoyment — denoted as $V(Si)$ hereafter — may be considered as a current market valuation of the economic value of the same service extracted from the asset and hence as a function of the annual service Si or of the depreciation expense $D(i) = E(Si)$.
That is,

$$V(Si) = f(E(Si)) = f(D(i)) \tag{11}$$

We can introduce a revaluation or service profitability factor M and assume $V(Si)$ is the product of M and the depreciation expense. M will temporarily be regarded as constant during the asset's life. Some effects of its change over time will be considered later. We can reconstruct the asset cost formula by applying this relationship (12).

$$V(Si) = f(D(i))$$ (12)

$$= M \times D(i).$$

$$A^i = \sum_{k=1}^{n-i} \frac{V(Sk+i)}{(1+s)^k}$$

$$= \sum_{k=1}^{n-i} \frac{M \times D(k+i)}{(1+s)^k} \quad (i = 0, 1, 2, ----, n)$$ (13)

where s : the discount rate which equalizes the present value to the original
cost A^i of an i year-used asset at time 0.

D(i) is the usual depreciation expense, while $V(Si)$ can be called 'depreciation value', for that denotes the enjoyment value of each year's fixed asset service. Hence, by this formula (13), we are proposing a new fixed asset concept as a 'discounted present value of the future depreciation values'.

What is the exact meaning of formula (13), especially the term $M \times D(k+i)$? We are assuming here (1) that the valid object of discounting must be that amount of annual cash-inflow which reflects the true contribution of the fixed asset for each year; (2) that this object is the depreciation value $M \times D(k+i)$ and not the usually adopted annual net cash-inflow $C(k+i)$ per se because this reflects other inputs' contributions too; (3) that this depreciation value as the annual service value does exist even though not accurately measurable; and (4) that this value does not exceed annual cash-inflow amount, being usually equal or less. This last requirement means that we are positing each year's depreciation value can be totally recovered from annual revenue, whatever amount it may be. In the last analysis, therefore, we admit we are assuming an oligopolistic or monopolistic firm.

We also note that $D(k+i)$ in (13) is not intended to solely represent annual 'technical' operation service (i.e., the quantity of annual output products) of a fixed asset alone. As will be made clear in next section, $D(i+k)$ in the present value model does include — in addition to that operating service cost — the financial cost of having the future year service layers wait for this year, because such financial service is necessary to realize annual operating service. So, in formula (13), we in effect are assuming that this sum total of annual operating and financial

service costs can be wholly recoverable from revenue (and a proportional amount of net profit obtained).

Even though service enjoyment value V(Si) usually is unmeasurable, some operations on equation (13) will be shown to enable us to reach an annual depreciation expense formula. In this sense, we may say that our making the annual service value variable partially endogenous and our interpreting a fixed asset cost as a discounted present value of the future depreciation values do have some theoretical merit.

5. Logical Consistency of the Declining-Balance Method When the Profitability of the Annual Service Remains Constant Over Time

Just as an annual decrease in the volume of water in a reservoir is equal to the amount flowing out during the same period, so can the depreciation expense as the service flow be measured as a difference between the beginning and end year present values of the asset involved. Accordingly, from (13),

$$A^{i-1} = \frac{M \times D(i)}{(1+s)^1} + \frac{M \times D(i+1)}{(1+s)^2} + \text{--------------} + \frac{M \times D(n)}{(1+s)^{n+1-i}}$$

$$A^i = \frac{M \times D(i+1)}{(1+s)^1} + \frac{M \times D(i+2)}{(1+s)^2} + \text{--------------} + \frac{M \times D(n)}{(1+s)^{n-i}} \qquad (14)$$

Hence, $A^{i-1}(1+s) - A^i = M \times D(i)$.

That is, $(A^{i-1} - A^i) + s A^{i-1} = M \times D(i)$.

Since $A^{i-1} - A^i$ is the depreciation expense,

$$D(i) + s A^{i-1} = M \times D(i).$$

Therefore, $D(i) = \frac{s}{M-1} A^{i-1}$ (i = 1, 2, ----, n). $\qquad (15)$

Thus, we have proven that a particular depreciation expense pattern as indicated by (15) is consistent with our 'discounted depreciation value' concept of fixed assets. If we continue with the assumption that the discount rate s and the service revaluation factor M remain constant during the whole useful life of the asset, then a decreasing asset cost balance A^{i-1} is multiplied by the constant factor

(depreciation rate) $s/(M-1)$. So, a sort of decreasing charge depreciation has been derived as logically consistent within our theory. In the following, we will prove that this in fact is the declining-balance method. The depreciation rate $s/(M-1)$ must be quantified as $1-(S/A)^{1/n}$ where S is the salvage value.

(Theorem 1)

Let the depreciation be defined as the difference between the beginning and end year discounted present values of the future service values of the fixed asset. Further, assume that those service values are not the year-end cash-inflows per se but some values proportional to the annual depreciation expense measures. If, in this case, the revaluation factor M of the annual asset service should remain constant every year, then the declining-balance method is consistent with those assumptions.

(Proof)

The characteristics of the declining-balance method consist in (1) that each year's depreciation amount is determined by multiplying a constant rate by the beginning year undepreciated fixed asset cost and (2) that that rate equals $1-(S/A^0)^{1/n}$, where S is the salvage value. Of these two points, the former has been shown to hold in (15). The constant rate — labelled DR hereafter — is $s/(M-1)$.

The latter point can be easily proven as follows.

$$DR = s/(M-1). \text{ Since } D(1) = A^0 - A^1,$$

$$A^1 = A^0 - D(1) = A^0 - A^0 s/(M-1) = (1-DR) A^0.$$

$$A^2 = A^1 - D(2) = A^1 - A^1 DR = (1-DR) A^1 = (1-DR)^2 A^0.$$

$$A^n = S = (1-DR) A^{n-1} = (1-DR)^2 A^{n-2} = \text{----} = (1-DR)^n A^0.$$

Therefore,

$$(1-DR)^n = S/A^0.$$

$$DR = 1 - (S/A^0)^{1/n}.$$

$$\text{(Q. E. D.)}$$

In conclusion, $DR = s/(1-M) = 1-(S/A^0)^{1/n}$. As stated above, we cannot determine s or M directly. In practice and also in theory too, we must rely on this formula to determine the depreciation rate.

An implication of our conclusion here is the rigidity of depreciation method choice, i.e., the proposition that the declining-balance method is always logical for any consumption pattern of the fixed asset service during the useful life, as far

as the service profitability factor M should remain constant every year. This may appear absurd and unacceptable. But it is not so. If very intensive use of the asset is planned in a year, then, the estimated remaining amount of its service capability for all future years is correspondingly lower, so that the cost for some relatively greater consumption of service in a year tends to be offset by the correspondingly lower 'stocking cost' of the remaining service layers with respect to the good so as to generate an unchanged depreciation pattern (i.e., the declining-balance method pattern) in every case, if the depreciation expense is defined as the sum of those two costs.

6. Depreciation as a Measure of the Annual Aggregate Operating and Stocking Service Costs.

What is the semantic content of such depreciation information? To see this, we will expand the depreciation formula (15) as follows.

$$D(i) = \frac{s\ A^{i-1}}{M-1} = \frac{1}{M-1} \times s \times (\frac{V(Si)}{(1+s)^1} + \frac{V(Si+1)}{(1+s)^2} + \text{---------}$$

$$+ \frac{V(Sn)}{(1+s)^{n-i+1}})$$

$$= \frac{V(Si)}{(1+s)^1} \times s \times \frac{1}{M-1} + (\frac{V((Si+1)}{(1+s)^2} + \text{---------}$$

$$+ \frac{V(Sn)}{(1+s)^{n-i+1}}) \times s \times \frac{1}{M-1} \tag{16}$$

In considering the service enjoyment value of a fixed asset, we can devide 'operating' and 'stocking' service values. The former concerns actual annual operation of the fixed asset, while the latter involves the necessity of stocking future years' plant service layers simultaneously, which is indispensable, in case of a multi-year good, for this year's asset operation. Therefore, a realistic depreciation measurement must include both service costs in its annual expense.

The first term in (16) represents the cost for the i th year operating service value from the asset. $V(Si)/(1+s)$ shows the acquisition cost of the i th year net service value and hence when multiplied by s (the rate of return) the profit from the operating service. Since M is, as defined above, the ratio of the annual service value $V(Si)$ from the fixed asset to the depreciation expense $D(i)$, the

value M $-$ 1 represents the ratio of V(Si) $-$ D(i) (i.e., net service profit for year i) to the depreciation expense. Consequently, $1/(M-1)$ may be said to denote the ratio of the i th year depreciation expense to the annual net service profit. By multiplying this to the estimated net operating service profit s, V(Si)/(1 + s), we get the operating service cost component of the depreciation expense.

The other element, that is, the annual stocking service cost is precisely reflected by the second term of (16). V(Si + 1)/(1 + s)^2 ---- + V(Sn)/(1 + s)$^{n-i+1}$ is the total cost of the yet unused operating service layers as of the end of year i, so that when multiplied by s this produces the required annual net profit for justifying this financial investment. Hence, when this is further multiplied by $1/(M-1)$, the depreciation-profit ratio, then we see that the stocking service cost portion of the annual depreciation expense results.

In consequence, we have proven that our depreciation formula has the semantic content of 'annual aggregate operating and stocking service costs' for using the fixed asset each year. Even if the operating service value should remain level every year, the stocking service cost declines corresponding to the decreasing undepreciated cost balance. So, even in this case, too, we see that the decreasing charge method (the declining balance method) will be judged as consistent within our theory.

If M should vary over time, some other depreciation method than the declining-balance method may possibly be derived. Some consideration will be given to this problem in next section.

7. Validity of the Straight-line Method When the Asset Service Profitability Declines Linearly over Time

Just in contrast to our assertion on appropriateness of the declining-balance method in a constant net service value situation, we will show that the straight-line method is logically consistent within our theory only if the annual net service value declines in a specific periodic pattern.

If the straight-line method is to be supported in our theory,

D(i) = D (i+1)

for i = 1, 2, -----, n$-$1.

That is,

$$\frac{s}{Mi-1} A^{i-1} = \frac{s}{Mi+1-1} A^i$$ (17)

where Mi and Mi+1 are the variable revaluation factors of the i th and i+1 th year depreciation expenses.

After some arrangements.

$$\frac{Mi+1 - 1}{Mi - 1} = \frac{A^i}{A^{i-1}} < 1. \tag{18}$$

Equation (24) demonstrates that Mi+1 must be smaller than Mi if the straight-line method is to be consistent. That is, Mi must decrease over time. Since Mi is the net service value to depreciation ratio, we may restate that the straight-line method is justified only if the asset service profitability in the above sense decreases over time and all other conditions remain the same.

More specifically, we will prove below that a linearly decreasing pattern of Mi just suffices to generate the straight-line depreciation.

(Theorem 2)

Under the same depreciation concept as stated in theorem 1 above, if the service profitability factor M of the fixed asset service should decline linearly every year, then, the straight-line method follows within our theory.

(Proof)

From (17) above,

$$\frac{A^{i-1}}{Mi - 1} = \frac{A^i}{Mi+1 - 1} \ .$$

$$A^{i-1} Mi+1 - A^{i-1} = A^i Mi - A^i \ .$$

$$Mi+1 - \frac{A^i}{A^{i-1}} Mi = \frac{A^{i-1} - A^i}{A^{i-1}} = \frac{D(i)}{A^{i-1}} \ .$$

This is a first-order linear difference equation with respect to Mi. D(i) on its right side can be set as K (a constant), as we are referring to the straight-line depreciation. So,

$$Mi+1 - \frac{A^i}{A^{i-1}} Mi = \frac{K}{A^{i-1}} \ . \tag{19}$$

We will solve equation (19).

$$M2 = \frac{A^1}{A^0} M1 + \frac{K}{A^0} \ .$$

$$M3 = \frac{A^2}{A^1} M2 + \frac{K}{A^1} = \frac{A^2 A^1}{A^1 A^0} M1 + \frac{A^2}{A^1 A^0} K + \frac{K}{A^1}$$

$$= \frac{A^2}{A^0} \, M1 + \frac{A^2}{A^1 \, A^0} \, K + \frac{K}{A^1} \, .$$

$$M4 = \frac{A^3}{A^2} \, M3 + \frac{K}{A^2} = \frac{A^3 \, A^2 \, A^1}{A^2 \, A^1 \, A^0} \, M1 + \frac{A^3 \, A^2}{A^2 \, A^1 \, A^0} \, K + \frac{A^3}{A^2 \, A^1} \, K$$

$$+ \frac{K}{A^2} \, .$$

$$= \frac{A^3}{A^0} \, M1 + \frac{A^3}{A^1 \, A^0} \, K + \frac{A^3}{A^2 \, A^1} \, K + \frac{K}{A^2} \, .$$

$$Mi = \frac{A^{i-1}}{A^0} \, M1 + \frac{A^{i-1}}{A^1 \, A^0} \, K + \frac{A^{i-1}}{A^2 \, A^1} \, K + \text{-----}$$

$$+ \frac{A^{i-1}}{A^{i-2} \, A^{i-3}} \, K + \frac{K}{A^{i-2}} \, . \tag{20}$$

Accordingly, we also get the following.

$$Mi+1 = \frac{A^i}{A^0} \, M1 + \frac{A^i}{A^1 \, A^0} \, K + \frac{A^i}{A^2 \, A^1} \, K + \text{-----} + \frac{A^i}{A^{i-1} A^{i-2}} \, K + \frac{K}{A^{i-1}} \, . \tag{21}$$

Taking the difference between Mi and Mi+1,

$$Mi - Mi+1 = \frac{A^{i-1}-A^i}{A^0} \, M1 + \frac{A^{i-1}-A^i}{A^1 \, A^0} \, K + \frac{A^{i-1}-A^i}{A^2 \, A^1} \, K + \text{.....}$$

$$+ \frac{A^{i-1}-A^i}{A^{i-2} \, A^{i-1}} \, K - \frac{K}{A^{i-1}}$$

$$= \frac{K}{A^0} \, M1 + \frac{K^2}{A^1 \, A^0} + \frac{K^2}{A^2 \, A^1} + \text{-----} + \frac{K^2}{A^{i-2} A^{i-1}}$$

$$- \frac{K}{A^{i-1}} \tag{22}$$

To calculate the difference between Mi−1 and Mi, we will replace i in (28) with i−1. Then,

$$Mi-1 - Mi = \frac{K}{A^0} \, M1 + \frac{K^2}{A^1 \, A^0} + \frac{K^2}{A^2 \, A^1} + \text{-----} + \frac{K^2}{A^{i-3} A^{i-2}}$$

$$- \frac{K}{A^{i-2}} \tag{23}$$

Taking the difference between (22) and (23), we will compare the adjacent two pairs of Mi's difference.

$$(Mi - Mi+1) - (Mi-1 - Mi) = \frac{K^2}{A^{i-2} A^{i-1}} - \frac{K}{A^{i-1}} + \frac{K}{A^{i-2}}$$

$$= \frac{K^2}{A^{i-2} A^{i-1}} - \frac{K(A^{i-2} - A^{i-1})}{A^{i-2} A^{i-1}}$$

$$= \frac{K^2}{A^{i-2} A^{i-1}} - \frac{K^2}{A^{i-2} A^{i-1}} = 0.$$

Hence, any two consecutive pairs of Mi has been shown to decrease by the same amount. So, we have proven that Mi must decrease linearly so as to justify the adoption of the straight-line depreciation within our theory.

<div align="right">(Q. E. D.)</div>

(Example)

A fixed asset has been acquired at $220, with its estimated useful life of 4 years and its salvage value of $20. Its future net service revenues are assumed to be as follows:

Year	amount
1	60.0
2	72.0
3	86.4
4	145.15

The discount rate s which equates the present value of these future values to its acquisition cost $220 is 0.20. In this case, the following, linearly decreasing service profitability series is just sufficient to generate a set of straight-line depreciation amounts.

Year	Mi values	Depreciation
1	1.88 *	50.0
2	1.68 * *	50.0
3	1.48	50.0
4	1.28	50.0

$$* \quad 220 \times \frac{0.2}{1.88 - 1} = 50.$$

$$* * \quad (220 - 50) \times \frac{0.2}{1.68 - 1} = 50.$$

8. Conclusion

The purpose of this paper is to advance a new depreciation theory within the confine of the historical cost principle which can provide some perspective into the problem of selecting an appropriate deprecation method in each business situation.

We have started by adopting the economic depreciation concept where the historical cost of a fixed asset is regarded as the discounted present value of the future net service values (or the future net cash-inflows) to be generated by the good. In this theory, the authors have so far considered those future service values (or net cash-inflows) as a pure 'exogenous variable'. From this, a serious doubt about the predictability of those future values has arisen and the feasibility of this depreciation model has been questioned.

In opposition to this view, I have taken the standpoint that the future service value of a fixed asset is to be considered as a partially 'endogenous' variable. That is, the depreciation is those annually consumed service quantity as valued by its historical cost, whereas the service value is the same provided service as measured by its current market value. Hence, we have proposed the idea that multiplying a revaluation factor by the annual depreciation expense will produce the annual net service value from a fixed asset, which is the object of discounting in the present value model.

By inserting this idea into the model, we have derived a depreciation expense formula. According to this, if the revaluation factor should remain constant every year, then — contrary to the commonsence prediction — the declining-balance method has been obtained as consistent within our theory. And if this factor should fall linearly (by the same amount) every year, then — again contrary to the usual presumption — the straight-line depreciation has been shown to be logical in our specific model.

These apparently paradoxical results may be explained by the fact that a fixed asset will annually provide two kinds of services: operating service and stocking service. The former service can only be realized by the support of the latter. In this sense, there is some rationale for the depreciation concept which reflects the annual costs for both kinds of services. This is our proposed depreciation concept. Since the stocking service cost tend to decline every year because of diminishing unused asset service layers, a flat operating service pattern over time does produce a decreasing-charge depreciation.

Another paradoxical result about straight-line method choice in case of a linearly decreasing profitability (i.e., revaluation factor) comes from the increased proportion

of the depreciation expense in the annual service revenue V(Si) in this situation.

If this should stay flat over time, then, an increase in Mi will result in a tendency to raise depreciation. Hence, at a certain decreasing pattern (i.e., a linealy declining case) of Mi, this tendency will be just offset by the above stated element of decreasing stocking cost so as to generate a constant annual depreciation expense every year.

No consideration has been given to the utility of the accounting information our depreciation measure will produce. So, we cannot as yet recommend our depreciation method choice to the practice. All we have done is a reporting of some implications of the present-value-based depreciation within the historical cost framework.

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FOREIGN SUBSIDIARIES AS CONTRIBUTORS TO PARENT COMPANIES

– A NEW PARADIGM OF MULTINATIONAL ENTERPRISES –

Hideki YOSIHARA

1. Technological Superiority of Parent Companies

Foreign Companies in Prewar Japan

The first foreign company that expanded its operations to Japan in prewar days was the American firm Western Electric, whose Japanese subsidiary, Nippon Electric, was established on July 17th, 1899.[1]

Nippon Electric was in the telephone business. Backed up by the overwhelming technological superiority of its American parent company, Nippon Electric experienced rapid growth in Japan, and was able to achieve excellent financial performance. Before long, competing firms were founded in Japan, and they grew to be competitors of Nippon Electric. In 1931 the Manchurian Incident occurred, and, following this event, the Japanese government adopted a negative attitude towards foreign companies. As a consequence, a difficult period started for Nippon Electric. Later, when the Pacific War broke out, Western Electric was compelled to withdraw from Japan.

During the Meiji era, many foreign experts had been invited to work in Japan, so that this country could acquire Western knowledge and technology. After this acquisition was completed, the foreigners, having fulfilled their function, were fired and returned to their countries of origin. In a similar way, while there exists technological disparity between Japanese and foreign firms, the latter are a useful and strong presence, but, as the Japanese side – such as the Japanese partner in a joint venture or Japanese competitors – catches up, the usefulness of the foreign company decreases and its competitive position loses strength. Since, sooner or later, the technological gap is closed, the success of foreign companies is unavoidably temporary. M. Mason, who studied the expansion of foreign companies (Western

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1) Regarding my considerations on foreign companies in Japan during the pre-war years, I benefited from the papers – and subsequent discussions – presented at the 16th Fuji Conference of the Society for the Sudy of Business History, in which I participated (Jan., 1-4, 1989). The conference topic was "Foreign Businesses in Prewar Japan."

Electric and other companies) in Japan in the prewar years, concludes that foreign companies in Japan during this period could be compared to the oyatoi gaikokujin (hired foreign experts).[2]

The oyatoi-gaikokujin theory is built around two variables — i.e., heavy involvement of the Japanese government and technological superiority of the parent company. In this paper I will concentrate exclusively on the second aspect, technological superiority.

The variable "technological advantage" may be considered from two aspects. The first is the idea that the technological gap between the foreign company and its Japanese competitors is a key factor in the former's success. That is, whether a foreign company in Japan will succeed is determined by whether there is a technological gap between the foreign firm and its Japanese competitors.

The second idea is that the superior technology is unidirectionally transferred from the parent company in the country of origin to the subsidiary in Japan. The parent company has superior technology which, when transferred to the subsidiary in Japan, becomes a source of competitive strength.

The Traditional Paradigm of Multinational Companies

The two points mentioned above — i.e., that the technological superiority of the foreign company over the indigenous firms is a key success factor for the former, and that the superior technology flows from the parent company in the country of origin to the subsidiary in foreign countries — are ideas shared by many theories of multinational firms developed up to the present.

The basic idea of Vernon's product cycle model is that the driving force for American multinational companies to enter foreign markets is the superior resources (capacity for product development, marketing know-how, managerial and administrative know-how, etc.) developed in the U.S. These superior resources are transferred from the parent company in the U.S. to the foreign subsidiary. Subsidiaries in each foreign country take this advanced technology and marketing know-how and use them as competitive weapons against indigenous firms.[3]

One of the features of S. Hymer's theory of multinational firms is its emphasis on the hierarchical relationship existing between the parent company in the country of origin and its foreign subsidiaries. Excellent resources, which are the foundation of competitive strength, are transferred from the parent to the foreign subsidiaries. In order to make good use of these resources at every foreign subsidiary, the

2) M. Mason, "Temporary Guests: Western Electric and ITT in Prewar Japan." (Paper presented at the Fuji Conference mentioned above.)
3) R. Vernon, Sovereignty at Bay, Basic Books, 1971, p. 65-77.

parent has to build a global organizational structure, and develop an adequate management control system.[4]

Multinational corporations are also treated in international economics as a topic in the theory of direct foreign investment. Direct foreign investment is considered as an international transfer of corporate resources. International transfer actually means movement of resources from the parent in the country of origin to the foreign countries where subsidiaries are located. Transfer of resources in other direction, such as from the foreign subsidiary to the parent or mutual transfers among foreign subsidiaries, are not considered.

The following three points can be indicated as features of existing theories of multinational enterprises. The first point is the idea that the driving force that leads multinational corporations to expand their businesses to foreign markets is the technological superiority of the parent firm. (Here and in the following paragraphs I use the term "technological superiority" in a broad sense, which includes not only product and production technology, but also marketing and managerial know-how.) The second point is the idea that this technological superiority is transferred from the parent company to the foreign subsidiaries. The third point is a hierarchical relationship between the parent in the country of origin and the subsidiaries abroad. Foreign subsidiaries are considered to be controlled by the parent company.

2. Transfer of Technology from the Subsidiary to the Parent

The Two Objectives of Japanese Investment by Foreign Companies

It is said that history repeats itself. But a detailed examination will show that things which occurred in the past are not reproduced with precision in the future. The same can be said about foreign companies in Japan.

The number of foreign companies expanding their businesses into Japan is growing steadily. In spite of the abrupt appreciation in the value of the yen that occurred after the autumn of 1985 and the stabilization of the new high value of the currency, this trend has not changed. What then, are the objectives pursued by foreign companies in coming to Japan?

President K. R. Whitmore of the U.S. firm Eastman Kodak, talking about the objectives of entering Japan, says.

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4) Y. Miyazaki, *Gendai no Nihon Kigyo wo Kangaeru* (Thoughts on Contemporary Japanese Enterprises), Iwanami Shoten, 1974, p. 188-208.

Japan is not only an important market, but it is also a source of technology. We are aiming at gathering superior resources in Japan, such as advanced technology and excellent human resources. Putting to work our company's resources and the Japanese technology and market information, we will be able to display a powerful strength." (Nihon Keizai Shimbun, Oct. 30, 1985.)

As this statement makes clear, foreign companies have two basic reasons to enter Japan. The first reason is the large size of the Japanese market. Japan is the second largest market in the world after that of the U.S., and due to the recent appreciation in the value of the yen the scale of the Japanese market has become even wider. Foreign firms cannot afford to ignore the Japanese market.

The second major reason that foreign firms have to expand operations into Japan is their eagerness to acquire Japanese technology and know-how.

According to executives of European firms, since Japan is ahead of other countries in a number of advanced technologies such as electronics, companies need a base in Japan if they are to rank among the world's leading enterprises in the future. European companies want to become fully fledged players in the Japanese market, and acquire the advanced technology of the Japanese.[5]

Recently, American and European companies, mainly in the electrical and chemical fields, have successively began to establish product development centers and research laboratories in Japan. The purpose of expanding R&D activities into Japan is not to transfer the technology of the parent company to this country, but rather to employ Japanese engineers and researchers to further develop a technology in Japan, then to use that technology in the Japanese subsidiary and transfer it back to the parent company.

For those companies which expand to Japan for the first reason mentioned above, i.e., to take advantage of a very big market, the technological superiority of the parent company becomes a competitive weapon. Regarding this point, the approaches that have been taken until now in the theory of multinational enterprises may be applicable. But with regard to the second reason, i.e., the acquisition of superior Japanese technology, the existing theories seem to be unwarranted. Technological superiority is not in the country of the parent, but in Japan. Foreign companies establish themselves in Japan in order to obtain valuable Japanese technology and not to develop in Japan the superior technology of their parent companies.

The Case Of Fuji-Xerox

Fuji-Xerox was established in 1962 as a joint venture of the British Rank Xerox, a subsidiary of the U.S.'s Xerox, and Japan's Fuji Film. In other words, Fuji-Xerox is

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5) H. Mirza, P. J. Buckley and J. R. Sparkes, "Swimming against the Tide?: The Strategy of European Manufacturing Investors in Japan," presented at the Fourth Japanese-European Conference on Management (Tokyo, Sep. 29 and 30, 1987).

a "grandchild" company (subsidiary of a subsidiary) of the U.S.'s Xerox.

At the beginning, Fuji-Xerox was able to develop smoothly and obtained good business results in Japan thanks to the overwhelming technological superiority of its parent company. The company could be counted − together with IBM Japan, Coca Cola Japan and a handful of other foreign firms − as one which was very successful in this country.

In 1970, the selenium drum patent and other basic patents expired, and, due to the gradual strengthening of Japanese firms (Ricoh, Canon, and others), Fuji-Xerox's technological superiority quickly disappeared, resulting in a fierce competitive situation.

It was during this competitive process that Fuji-Xerox started to develop their own product, such as the Xerox 2200, suited to the Japanese market. Nowadays, Fuji-Xerox is on a par with its parent company in its capacity to develop photo-copier products, specially compact copiers, and in some areas the company has even surpassed its American parent. In fact, not only are products developed by Fuji-Xerox being exported to the U.S. and England, but the product development system, and the methods of production management and quality control, are being transferred to the parent company.[6]

Now, Xerox is competing in the U.S. and European countries with Japanese copier manufacturers. According to some observers, Xerox is able to cope with Japanese competition because of Fuji-Xerox's product development capabilities, and production and quality control systems. This view maintains that, assuming the U.S.'s Xerox was not backed up by Fuji-Xerox, its competitive strength would be reduced.

Fuji-Xerox's case is important in two senses. First, it shows that technology is not only transferred from the parent company to the foregin subsidiary, but also from the foreign subsidiary to the parent in the country of origin. Second, it also shows that even after the original technological superiority diminishes or disappears, the foreign subsidiary, by developing its own strength, can grow and achieve good performance.

3. The Sources of the Foreign Subsidiary's Superiority

Geographical Advantage

Foreign subsidiaries can, even if partially, occupy a place of technological superiority

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6) *Fuji-Xerox 20 Nen no Ayumi* (Fuji-Xerox: 20 Years of History), Fuji-Xerox, 1983, p. 203-205. *Fuji-Xerox,* Nomura Management School, 1983, p. 39.

vis-a-vis their parent companies. Fuji-Xerox is one such example. What are the reasons that allow a foreign subsidiary to obtain a position of technological superiority over the parent company?

The first is the geographical advantage of the foreign subsidiary.

The first reason why Fuji-Xerox achieved technological superiority is that the company had to satisfy Japanese users. Japanese users, private individuals or institutions (firms, public offices, etc.), when compared with their American or European counterparts, are more discriminating, and moreover products must operate in more severe conditions.

In the early days of copying technology, during rainy days or very humid weather, printing (copying) conditions were not good. Japanese users were unsatisfied with this situation, and strongly demanded an improvement in the quality of the products. Likewise, maintenance was not sufficient, and users demanded quick response from the service station in case trouble developed during usage. If the copier manufacturer could not eliminate the source of the user's dissatisfaction, the user would shift to competitor's brands, because in Japan there was not just one supplier of copying machines, but several. This resulted in a strong competitive situation.

This severe competition is the second reason contributing to the development of a competitive superiority on the part of Fuji-Xerox.

In the U.S. or Europe, Xerox's position is overwhelming, and the intensity of the competition is not as high as it is in Japan. Backed up by its technological superiority, Xerox was able to achieve a position of quasimonopoly in those markets.

The third reason is the development of surrounding industries in Japan. Specially important is the development of the electronics industry. Copiers are making increasing use of electronic devices, and having operations in Japan, where the electronics industry is well developed, is an important advantage for manufacturers of copying machines.

I have been talking about the geographical advantage of being in a foreign country (in Japan, in the case of Fuji-Xerox), In fact, it is the geographical advantage that American and European companies enjoy in their own countries that allows them to develop original technology, and, based on this original technology, expand operations into foreign markets.

The U.S. market demanded goods for high-income consumers and articles that could contribute to labor-saving (labor-efficient technologies). American firms, in order to meet these market needs, developed American style goods such as big cars, electric home appliances, and machines for factory automation. These firms subsequently expanded to the different world markets, using their original products

and technologies as competitive weapons, and became multinational enterprises.[7]

The Japanese market had several features that contrasted with the American market. The income level was relatively lower, and the country's area was smaller and much more densely populated. In order to adapt to these Japanese conditions, motor vehicles, for example, had to be smaller, fuel-efficient, and strong to endure road conditions. If, as a further example, we take electrical home appliances such as color TV sets, refrigerators, and microwave ovens, the Japanese market demanded appliances which were sturdy, compact and quiet. Since Japanese homes are usually small, refrigerators that make noise during the night are not accepted.

Nowadays, Japanese cars, color TVs, VCRs and office machines are displaying great competitive strength in the world markets. These products are further examples of the response given by the manufacturers to the special needs that characterized the Japanese market.

As I pointed out earlier in this paper, one of the reasons why American and European firms are expanding operations into Japan is their desire to acquire superior technology from this country. If so, what kinds of technology are they aiming at?

One of them is Japanese production technology. American semiconductor manufacturers such as Texas Instruments and Motorola have factories in Japan. It is said that the productivity of these Japanese factories is the highest in the company, and at the same time the rate of defective items is prominently the lowest.[8] American semiconductor manufacturers take home the production technology of Japanese factories, and introduce it in their home factories. Also, these manufacturers are making efforts to expand the application of Japanese production technology to their other foreign plants as well.

The second is product development capability, in particular the ability to repeatedly develop new products and launch them into the market. One of the characteristics of the Japanese market, compared with the American or European markets, is that in Japan products have a short life cycle, and new products are continuously being introduced. In consumer products such as color TVs, VCRs and automobiles, or in OA machines such as word-processors or personal computers, there are few cases of products that enjoy a life longer than one year.

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7) R. Vernon, *op. cit.,* p. 77-81.
8) See "Motorola no Keiei Rinen to Global Senryaku" (Motorola's Managerial Philosophy and Global Strategy), *Business Review,* Vol. 34, No. 2, p. 83.

In Japan, having a new-product development capability sufficient to respond competitively to the rush of new products is a must if a company is to keep its competitive strength. This new-product development system, which shortens development time, is one of the technologies that American and European firms want to learn.

Electronics technology is the third technology that American and European companies are trying to acquire in Japan. Japan is the most advanced country with regard to electronic products (with the probable exception of electronics for military use). The civilian Japanese electronics industry is the world leader not only in color TVs, VCRs, personal computers, and wordprocessors, but also in electronic parts and semiconductors.

This electronics technology is valuable not only for the electronics industry, but also for firms in other industries. In industrial fields such as chemistry, machinery and optics, the application of electronics technology, such as mechatronics, chemical-electronics and opto-electronics, is becoming an important managerial consideration. American and European companies compete to expand their operations to Japan and strive to acquire electronics technology.

Entrepreneurship at Foreign Subsidiaries

As pointed out earlier, the geographical advantage was one of the reasons why Fuji-Xerox could achieve a technological level superior in some areas to the level of its parent firm. But we must not forget the importance of the efforts made by Fuji-Xerox executives, managers, and engineers.

Fuji-Xerox is a 50-50 joint venture between England's Rank Xerox and Japan's Fuji Film. From the establishment of the company, a policy of all-Japanese management was adopted.[9] All the full-time directors, including the presidents, have been Japanese nationals.[10]

The fact that from the time the company was established all executives were Japanese, eager to develop their own technologies, is another important reason for the high level of technological capability achieved by Fuji-Xerox. Fuji-Xerox was originally a sales company. Manufacturing of copiers and expendables was performed by Fuji Film (including its subsidiary).[11] But ever since its birth as a sales company,

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9) *Fuji-Xerox 20 Nen no Ayumi*, P. 36-37.
10) *Ibid.*, p. 46.
11) The integration of production and sales was materialized in April, 1971 and thereafter Fuji-Xerox had production capabilities. *Fuji-Xerox 20 Nen no Ayumi*, p. 124-126.

engineers and technicians were trained inhouse, accumulating independent technological capabilities with which they tried to develop products. Behind this development stands the attitude of Japanese managers and engineers, who are aware of the importance of possessing independent technological capability if the company is to grow in the future, and, accordingly, try to persuade the parent company's management (Fuji Film, Rank Xerox and Xerox in the U.S.A.) regarding the need to enhance these independent technological capabilities.[12]

Thus, there are two important factors that explain why Fuji-Xerox has now achieved such a high technological level. The first factor is Japan's geographical advantage, and the second is the initiative of Japanese managers and engineers, who strive to develop an independent technology. The second factor could also be called "entrepreneurship of the Japanese subsidiaries."

It is sometimes said that innovation emerges at the periphery rather than at the center. If this is so, the causes for this phenomenon to occur in the context we are talking about are the geographical advantages of the foreign subsidiary and the initiative (entrepreneurship) possessed by this subsidiary. In order to nurture foreign subsidiaries able to contribute to the parent firm, efforts are necessary to take advantage of the geographical situation, and to promote entrepreneurship in the management of subsidiaries.

4. Barriers of pride

Let's assume that a foreign subsidiary makes the most out of its geographical advantage and surpasses the parent company in its technological capability. For this technology developed at the foreign subsidiary to be transferred to the parent company, the latter has first to recognize the value of the new technology, and make the decision to adopt it. But this decision is not easily made.

The first product independently developed by Fuji-Xerox was the Xerox 2200, a small-sized copier.[13] This machine, when compared with the desk-top machine on the market at that time (developed by the U.S.'s Xerox) was somewhat slower in copying speed, but it had the capability to copy from books and magazines, its size was substantially smaller (by 20 cm in depth and 11 cm in height), and it was also lighter, by 18 kg. Manufacturing cost was approximately one half of the cost of

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12) *Ibid.,* p. 143—144.
13) The following description of the development of the Xerox 2200 is based on *Fuji-Xerox 20 Nen no Ayumi,* p. 145, and Atsuo Miyamoto, "Shashi Sowa, Fuji-Xerox" (Fuji-Xerox: History and Episodes), 31, *Nihon Kogyo Shimbun,* Oct., 22, 1982.

functionally equivalent copiers.

Since Fuji-Xerox succeeded in its trial development of the machine, it informed its parent companies (England's Rank-Xerox and U.S.'s Xerox) about this success and requested formal permission to develop and manufacture. The answer was negative. The head of the engineering department at the U.S.'s Xerox, Mr. W. Becker is quoted as saying: "It is not likely that they can get so advanced functions at such a low cost."

At that point, the person responsible for the development at Fuji-Xerox took the prototype to Rank-Xerox in England and put it to work. Mr. Becker, who was in England, saw the prototype himself and changed his mind: "I was opposed to independent developments, but I must admit that I have to rectify my opinion."

After the small-size copying machine developed by Fuji-Xerox was seen and its excellence recognized by both engineers and managers at the parent company, the product was formally approved and began to be exported to many countries around the world. After that, the U.S.'s Xerox's R&D strategy was modified, and there was a tendency to gradually entrust to Fuji-Xerox the development of small-size copiers.

From the point of view of the U.S.'s Xerox, it was very difficult to imagine that Fuji-Xerox, a "grandchild" company, would be able to develop a better product. For this reason, reports from Fuji-Xerox on the new machine were not taken seriously until people from the parent company actually saw the prototype. However, after seeing the newly-developed product once, the parent company took a fair and open attitude, recognized the value of the new machine, and not only legitimized Fuji-Xerox's product development activities, but positively supported them.

The transfer of superior technology from a foreign subsidiary to the parent company does not take place when the parent does not recognize the value of the technology developed by the foreign subsidiary. In practice this recognition is something very difficult to achieve. The parent company usually has a sense of pride and superiority. It is because of these feelings that the technical superiority of the foreign subsidiary is not easily admitted. But if innovations are to be born at a foreign subsidiary and then transferred to the parent company and used by it, the parent company must overcome these feelings of superiority and pride.

If the reverse transfer (from subsidiary to parent) of technology is to occur, the foreign subsidiary should persuade the parent company. In doing so, it is necessary to avoid damaging the parent company's pride or openly challenging

the feelings of superiority of the parent company. As we saw in the case of Fuji-Xerox, "the proof of the pudding is in the eating," and actually demonstrating products incorporating superior technology is an effective method to gain recognition from the parent firm.

5. Paradigm Change

Up until now, both executives of multinational firms and scholars working in the field of international business have been sharing the following way of thinking. First, the success of the foreign subsidiary, vis-a-vis competition from indigenous firms, depends on whether or not the subsidiary is in a superior technological position. Second, superior technology (in a broad sense which includes not only product and production technology but also marketing know-how and brand, managerial and administrative systems and know-how, etc.) is tranferred from the parent to the subsidiary. In the third place is the idea that the parent in the country of origin controls the foreign subsidiary.

This way of thinking may be called the "traditional paradigm of multinational enterprises." This traditional paradigm has been applied to the study of foreign companies in Japan during the prewar years, as we saw at the beginning of this paper. However, the behavior of foreign companies that came to Japan in the post-war years has some features that challenge the traditional paradigm. Fuji-Xerox is only one example. When we look at the recent behavior of foreign firms in Japan, the need for a new way of thinking about multinational companies becomes a necessity.

The new paradigm of multinational enterprises which would complement the traditional paradigm has the following characteristics.

First, the foreign subsidiary is considered as a source of innovation. The foreign subsidiary, based on its geographical advantage and entrepreneurship, develops new products, and creates new production technologies or managerial systems. Foreign subsidiaries have the potential to become a source of technology.

The second idea is that new technology born at the foreign subsidiary is transferred to the parent firm. Up to now it was thought that technology flows from the parent to the subsidiary, but in reality there is also a flow of technology in the opposite direction. So far, the foreign subsidiary has been considered the beneficiary of the transfer of superior technology from the parent firm. According to the new paradigm, the reverse phenomenon also occurs, and the parent becomes the beneficiary that adopts innovations originated in the foreign subsidiary. The foreign subsidiary

makes contributions to the parent firm.

In order for multinational companies to develop according to this new paradigm, the following two conditions have to be met.

The first relates to the existence of management initiative or entrepreneurship at the foreign subsidiary. According to the traditional paradigm, the foreign subsidiary is controlled by the parent company. On the contrary, in the new paradigm the subsidiary is encouraged to pursue an independent style of management.

Concretely speaking, the following requirements have to be satisfied. The president of the foreign subsidiary should be a local national rather than a foreign expatriate. The foreign subsidiary should not be considered by the parent simply as a branch factory located abroad, but as an independent firm. It should be equipped with a whole range of functions, from development and production to marketing and sales. Particularly important is the capability of the subsidiary to develop its products.

Another condition is that the parent company should overcome feelings of superiority over the foreign subsidiary, and humbly learn from it. The parent should be able to recognize the innovations made at the subsidiary. Then, the parent company should try to adopt superior innovations made at a foreign subsidiary and make efforts so that synergetic effects emerge from the interaction of the foreign subsidiary's resources and those at the parent company. Global synergy is one of the goals that multinational firms should pursue from now on.

THE GAINS FROM FREE TRADE FOR A MONETARY ECONOMY [1]

Murray C. KEMP [2]

1. Introduction

The classical gains-from-trade proposition has been proved in a barter context only. It has been taken for granted that the same proposition is valid for monetary economies.

In the present note it is shown that, for economies of a standard monetary type, free trade may be disadvantageous; the traditional production and consumption gains may be swamped by a loss of satisfaction from holding cash when relative prices change.

2. Analysis

Consider a small country which produces, consumes and trades two commodities and which derives satisfaction from its holding of cash balances. To avoid inessential complications, suppose that all households are alike in preferences, endowments and family composition. Let p_i denote the domestic price of the ith commodity, c_i the domestic demand for the ith commodity, $I(p_1, p_2)$ the maximum value of output at prices p_1 and p_2, $\pi(p_1, p_2)$ a suitable price index (positive, increasing, concave and homogeneous of degree one), m the stock of money demanded, and $\overline{m}$ the given stock of money. Then the community can be viewed as maximizing a utility function

$$u[c_1, c_2, \ m/\pi(p_1, p_2)] \tag{1}$$

subject to a budget constraint

$$p_1 c_1 + p_2 c_2 + m = I(p_1, p_2) + \overline{m} \tag{2}$$

The solution to the problem may be written

$$c_i = c_i(p_1, p_2, \bar{m}) \qquad i = 1, 2 \tag{3a}$$

$$m = m(p_1, p_2, \bar{m}) \tag{3b}$$

In balance-of-payments equilibrium,

$$m(p_1, p_2, \bar{m}) = \bar{m} \tag{4}$$

Moreover, since the country has no influence on the terms of international trade,

$$p_1 = \lambda p_2 \qquad (\lambda > 0, \ \text{constant}) \tag{5}$$

Eqs (4) and (5) form a complete system in p_1 and p_2. Let us assume that it has a unique solution

$$p_i = p_i(\lambda, \bar{m}) \qquad i = 1, 2 \tag{6}$$

By varying the terms of trade parametrically, one can generate alternative feasible output, consumption and export pairs. Presumably there is a critical value of λ, say λ^*, at which trade is extinguished. Suppose that, initially, $\lambda = \lambda^*$.

The initial no-trade equilibrium is disturbed by a small increase in λ. What happens to the wellbeing of the typical household? From (1),

$$du = u_1 dc_1 + u_2 dc_2 + u_{m/\pi} d(m/\pi) \tag{7}$$

where $u_i \equiv \partial u/\partial c_i$ and $u_{m/\pi} \equiv \partial u/\partial(m/\pi)$. However, from the first-order conditions associated with (1) and (2),

$$u_i/u_{m/\pi} = p_i/\pi(p_1, p_2) \tag{8}$$

Hence

$$\pi \frac{du}{u_{m/\pi}} = (p_1 dc_1 + p_2 dc_2) - \frac{m}{\pi} (\pi_1 dp_1 + \pi_2 dp_2) \tag{9}$$

where $\pi_i \equiv \partial\pi/\partial p_i$. Finally, from constraint (2) and equilibrium condition (4),

$$p_1 dc_1 + p_2 dc_2 = 0, \tag{10}$$

implying that

$$\pi \frac{du}{u_{m/\pi}} = - \frac{m}{\pi} (\pi_1 dp_1 + \pi_2 dp_2) \tag{11}$$

Evidently expression (11) may be of either sign. If (11) does not vanish, there can be found a world terms of trade, say $\hat{\lambda}$, which is near but not equal to λ^* and such that trade is harmful, and there can be found a world terms of trade, say $\bar{\lambda}$ such that trade is beneficial. If $|\lambda - \lambda^*|$ is sufficiently large, trade is necessarily gainful.

It might be thought that the possibility of harmful trade flows from a failure to make use of all relevant properties of the price index π. In particular, it might be thought that the possibility will go away if π is required to be a true cost of living index, so that

$$\pi_i = \gamma c_i \qquad (\gamma > 0, \text{ constant}) \qquad (12)$$

However it is easy to show that there is no escape along that route. Thus, availing ourselves of (6), $\sum \pi_i dp_i = 0$ if and only if

$$\frac{d}{d\lambda} \ \pi(\lambda p_2(\lambda, \bar{m}), \ p_2(\lambda, \bar{m}) \ ) = 0 \qquad (13)$$

or, in view of (12), if and only if

$$(\lambda c_1 + c_2)(dp_2/d\lambda) \ = \ -c_1 p_2 \qquad (14)$$

which may or may not be satisfied.

To remove the troublesome finding, it suffices to assume, in addition, that the income velocity of circulation is a constant:

$$I(\lambda p_2, \ p_2)/\bar{m} \ = \ k \qquad (k > 0, \text{ constant}) \qquad (15)$$

For then

$$\frac{dp_2}{d\lambda} \ = \ \frac{-p_2 I_1}{\lambda I_1 + I_2} \ \ \frac{-p_2 x_1}{\lambda x_1 + x_2} \qquad (16)$$

where x_i is the output of the ith commodity. Since $x_i = c_i$ in the initial equilibrium, the righthand side of (16) reduces to $-p_2 c_1/(\lambda c_1 + c_2)$ and (14) is satisfied. Evidently (15) is a very strong and implausible assumption, since it rules out the possibility that consumption of one good requires larger cash balances, per unit of value, than consumption of the other good.

Suppose that $\lambda = \hat{\lambda} > \lambda^*$, as in Figure 1. What can the government do to cancel or reverse the loss associated with $\hat{\lambda}$? Tinkering with the nominal stock of money will be ineffective. Given any λ, the homogeneity of $\pi(p_1, \ p_2)$ ensures that real cash balances (and therefore u) are independent of $\bar{m}$. On the other hand, the government

could kill all trade by imposing a sufficiently heavy duty on imports (of the second commodity). That would eliminate the loss but it would not convert the loss into a gain. To achieve the latter objective it will be necessary, I suppose, to destroy the motive for holding cash. How that is to be done cannot be determined without laying the monetary foundations much more carefully.
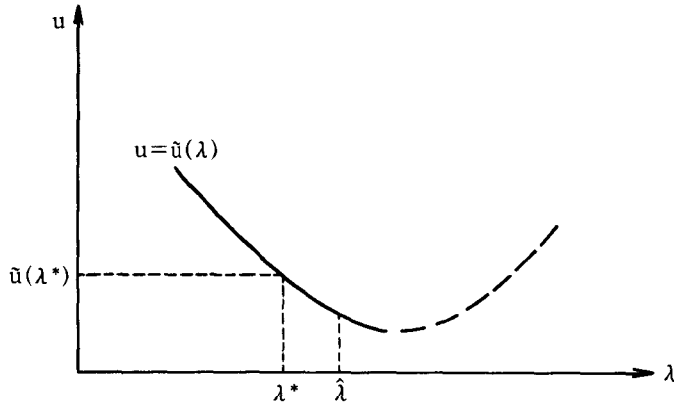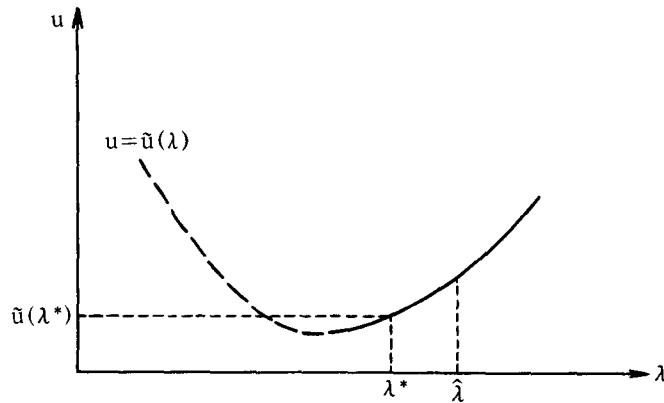

Figure 1(a)

Figure 1(b)

3. Final remark

How seriously one takes the possibility of harmful trade depends on one's confidence in the utility function (1). In the analysis of trade gains and losses one compares alternative steady states. But in a context of uncertainty-free steady-state analysis the case for (1) is at its weakest. In a more thorough analysis one would relate the demand for money to uncertainty — uncertainty about the scope for barter trades and about future household incomes.

ON THE OPTIMAL TIMING OF FOREIGN AID

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Abstract We offer a dynamic formulation of the aid process, in which questions of timing are central. It is shown that in most cases it is optimal to give the aid partly in a lump, at the outset, and partly as a flow, over time. However, if the donor and recipient cooperate, stock and flow aid may be of opposite sign. It is shown also that when marginal utility is of constant elasticity, the rate of flow of aid increases (decreases) if and only if the initial world stock of capital is less than (greater than) its steady-state value.

1. Introduction

The available formal analysis of foreign aid is static, with no provision for the passage of time. Its chief weakness is its inability to address questions of phasing or timing. In particular, it has nothing to say about the *optimal timing* of aid, including its division into stock and flow components.

A similar criticism can be directed to the analysis of reparations, indeed of all forms of unrequited transfer between governments. However we shall employ throughout the concrete terminology of aid.

Here we offer a dynamic formulation in which questions of timing are central. The formulation has the virtue and defect of being very simple. Thus, for the most part, we recognize just two countries, each producing a single commodity (the same in each country) by means of homogeneous capital and labour, and differing only in their initial factor endowments. Of course, in such a world there is no scope for conventional trade; in the absence of private international investment and of transfers between governments, each economy would be isolated. However, at the end, we do broaden our analysis by introducing a second traded commodity.

The country offering aid will be referred to as the donor, the other country as the recipient. The donor seeks that combination of initial stock transfer Ω and continuous flow transfer $<\omega(t)>$ which achieves a given improvement in the recipient's welfare at least welfare cost to the donor. In its search for the optimal pair $(\Omega, <\omega(t)>)$ the donor may or may not enjoy the cooperation of the recipient. If the recipient cooperates, Ω and $\omega(t)$ are unrestricted in sign; in the absence of cooperation, Ω and $\omega(t)$ are constrained to be non-negative. The optimal pair will

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prove to be sensitive to changes in assumptions about cooperation and to changes in the assumed degree of capital mobility.

It is shown that in most cases it is optimal to combine stock and flow aid, that is, to give the aid partly in a lump, at the outset, and partly as a flow, over time. However, if the two countries cooperate, stock and flow aid may be of opposite sign. It is shown also that when marginal utility is of constant elasticity, the rate of flow of aid increases (decreases) if and only if the initial world stock of capital is less than (greater than) its steady-state value.

2. Private Capital Immobile Between Countries

In many real-world situations the case for foreign aid rests in part on the reluctance of private capital to migrate. Indeed, "for most of the least developed countries, foreign aid has become more important than borrowing as a source of funds from abroad" (Levy (1988), p.152). We therefore begin our analysis under the assumption that private capital is internationally immobile.

Without loss, it is assumed that the labour force in each country is initially equal to one. The following standard notation will be employed.

ϱ the rate of time preference in each country

n the rate of growth of the labour force in each country

k_i the capital: labour ratio and capital stock in the i th country, $i=1, 2$

$f(k_i)$ the common production function, assumed to be increasing, strictly concave and to satisfy the Inada conditions

c_i the level of consumption per capita in the i th country

$u(c_i)$ the common individual utility function, assumed to be increasing, strictly concave and such that $\lim_{c_i \to 0} u'(c_i) = \infty$

In a state of isolation, the i th country solves the problem

(P.1) $\max_{c_i} \int_0^\infty u(c_i) \exp(-\delta t) dt$ $(\delta \equiv \varrho - n > 0)$

       s.t. $\dot{k}_i = f(k_i) - nk_i - c_i$

           $k_i(0) = k_{i0}$, given

Let the maximum value of the integral be denoted by $v(k_{i0})$.

The first country (the donor) wants to (or is required to) increase $v(k_{20})$ to $v(k_{20}) + \Delta$, $\Delta > 0$. The problem facing the donor then is to find the least utility-cost way of doing this. Formally, the donor's problem is to find

(P.2) $\quad \max\limits_{c_1,c_2,\omega,\Omega} \int_0^\infty u(c_1)\exp(-\delta t)dt$

$$\text{s.t.}\quad \dot{k}_1 = f(k_1) - nk_1 - c_1 - \omega$$

$$\dot{k}_2 = f(k_2) - nk_2 - c_2 + \omega$$

$$k_1(0) + \Omega = k_{10}, \text{ given}$$

$$k_2(0) - \Omega = k_{20}, \text{ given}$$

$$\int_0^\infty u(c_2)\exp(-\delta t)dt \geqq v(k_{20}) + \Delta$$

The reasoning behind this formulation is as follows: for any Ω and $<\omega(t)>$ there is an optimal consumption path $\{<c_2(t)>:\Omega, <\omega(t)>\}$ which the recipient would choose; however the donor would select the *same* path if, as we have assumed, the donor controls $<c_2(t)>$, for it is in the donor's interest that any aid be used efficiently.

Alternatively, the donor's task is that of finding, for *some* $\lambda > 0$,

(P.2′) $\quad \max\limits_{c_1,c_2,\omega,k_1(0),k_2(0)} \left[\int_0^\infty u(c_1)\exp(-pt)\,dt + \lambda\int_0^\infty u(c_2)\exp(-pt)dt\right]$

$$\text{s.t.}\quad \dot{k}_1 = f(k_1) - nk_1 - c_1 - \omega \tag{1}$$

$$\dot{k}_2 = f(k_2) - nk_2 - c_2 + \omega \tag{2}$$

$$k_1(0) + k_2(0) \leqq k_{10} + k_{20}, \text{ given} \tag{3}$$

Evidently $\lambda \gtreqqless 1$ if and only if, in the optimum,

$$\int_0^\infty u(c_1)\exp(-\delta t)dt \lesseqqgtr \int_0^\infty u(c_2)\exp(-\delta t)dt$$

In (P.2) and (P.2′) the controls ω and Ω are unrestricted in sign. This implies a high degree of cooperativeness between donor and recipient. In some situations it may be more realistic to require both ω and Ω to be non-negative, so that to the constraints of (P.2′) are added

$$\omega \geqq 0 \tag{4}$$

$$k_2(0) \geqq k_{20} \tag{5}$$

In the analysis to follow we shall consider both the case in which (4) and (5) are imposed (the non-cooperative case) and the case in which they are not imposed (the cooperative case).

The current-value Hamiltonian associated with (P.2′) is

$$\begin{aligned}
H = u(c_1) + \lambda u(c_2) &+ \psi_1 [f(k_1) - nk_1 - c_1 - \omega] \\
&+ \psi_2 [f(k_2) - nk_2 - c_2 + \omega]
\end{aligned} \tag{6}$$

and, if the full complement of constraints (1) − (5) is recognized, we have as necessary conditions

$$\partial H/\partial c_1 = u'(c_1) - \psi_1 = 0 \tag{7}$$

$$\partial H/\partial c_2 = \lambda u'(c_2) - \psi_2 = 0 \tag{8}$$

$$\partial H/\partial \omega = -\psi_1 + \psi_2 \leqq 0, \quad \omega \geqq 0, \ \omega(-\psi_1 + \psi_2) = 0 \tag{9}$$

$$\dot{\psi}_1 = \delta \psi_1 - \psi_1 [f'(k_1) - n] \tag{10}$$

$$\dot{\psi}_2 = \delta \psi_2 - \psi_2 [f'(k_2) - n] \tag{11}$$

It follows from (9) that

$$\psi_1(0) \geqq \psi_2(0) \tag{12}$$

Suppose, realistically in the case of foreign aid, that, before and after aid, the donor is the wealthier of the two countries so that $\lambda < 1$. In the absence of aid, $k_1(t) > k_2(t)$ for all $t \geqq 0$ and the marginal product of capital is always greater in the second country. The same is true if all aid is given in a lump, at $t = 0$. It is intuitively clear, then, that all aid should be given in a lump and that the optimal $\omega(t)$ is identically zero. More formally, suppose that there is a non-degenerate interval $[t_1, t_2]$ on which $\omega(t) > 0$. Then, from (9),

$$\psi_1(t) = \psi_2(t) \text{ for } t \in [t_1, t_2] \tag{13}$$

so that, bearing in mind (7) and (8),

$$u'(c_1) = \lambda u'(c_2) \tag{14}$$

Since $\lambda < 1$, $u'(c_1(t)) < u'(c_2(t))$ and

$$c_1(t) > c_2(t) \text{ for } t \in [t_1, t_2] \tag{15}$$

From $(9) - (11)$, on the other hand, if $\omega(t) > 0$ then $k_1(t) = k_2(t)$ and $\dot{k}_1(t) = \dot{k}_2(t)$ so that, from (1) and (2), $c_1 + \omega = c_2 - \omega$ or

$$c_1(t) = c_2(t) - 2\omega(t) < c_2(t) \text{ for } t \in [t_1, t_2] \tag{16}$$

Evidently (15) and (16) cannot both be true. Hence $\omega(t)$ cannot be positive on a non-degenerate interval.

Thus our intuitive conclusion, that the whole of the aid should be given in a lump, is justified. However in arriving at that conclusion it was assumed that constraints (4) and (5) are satisfied. If those constraints are not imposed then, instead of (9), we have $\psi_1(t) = \psi_2(t)$ for all t. It then follows from (10) and (11) that $k_1(t) = k_2(t)$ for all t.
Hence

$$\Omega = \frac{1}{2} (k_{10} - k_{20}) \tag{17}$$

and, since the donor remains the wealthier country, $\omega(t)$ must be negative on some interval. In fact the optimal $\omega(t)$ is negative always. For suppose that $\omega(t) = 0$ at $t = t_1$, so that $c_1(t_1) = c_2(t_1)$. From (7) and (10),

$$\alpha(c_1(t_1))(c_1(t_1))/c_1(t) = f'(k_1(t_1)) - \delta \tag{18}$$

where

$$\alpha(c) \equiv -u''(c)c/u'(c) > 0$$

is the elasticity of marginal utility. Similarly, from (8) and (13),

$$\alpha c_2(t_1)\dot{c}_2(t_1)/c_2(t_1) = f'(k_2(t_1)) - \delta \tag{19}$$

Hence

$$\dot{c}_1(t_1)/c_1(t_1) = \dot{c}_2(t_1)/c_2(t_1) \tag{20}$$

It follows that $c_1(t) = c_2(t)$ for all $t \geq t_1$; and, running time backwards, that $c_1(t) = c_2(t)$ for all $t \in [0, t_1]$. This in turn implies that the two countries enjoy the same level of wellbeing, a contradiction.

If marginal utility is of constant elasticity these results can be considerably sharpened. Thus if $\alpha(c) = \bar{\alpha}$, (18) and (19) imply that c_1 and c_2 change at the same

proportional rate, increasing (decreasing) if and only if the initial world capital stock is smaller than (greater than) its steady-state value, that is, if and only if $\frac{1}{2}$ $(k_{10} + k_{20})$ is less than (greater than) $\hat{k}$, where $\hat{k}$ is defined by $f'(\hat{k}) = \delta$; it then follows that $\omega(t)$ steadily rises (falls) under the same necessary and sufficient condition.

Let us summarize our conclusions to this point. Suppose that, before and after aid, the donor is the wealthier of the two countries. In the non-cooperative solution to (P.2′), all aid is offered at the outset. In the cooperative solution there is an "excessive" initial pulse of aid equal to $\frac{1}{2}$ $(k_{10} - k_{20})$, followed by a flow of aid which is always negative; if marginal utility is of constant elasticity then consumption in each country, as well as $-\omega(t)$, rises (falls) if and only if the initial world capital stock is less than (greater than) its steady-state value.

Other cases can now be treated relatively briefly. Thus suppose that before aid the donor is the wealthier country but that after aid the recipient is wealthier, so that $\lambda > 1$. Then, whether or not the recipient is cooperative, it is optimal to set $\Omega = \frac{1}{2}$ $(k_{10} - k_{20}) > 0$ and to supplement this lump of aid with a flow which is always positive. If marginal utility is of constant elasticity then optimal consumption changes at the same proportional rate in each country; if and only if $\frac{1}{2}$ $(k_{10} + k_{20})$ is less than (greater than) $\hat{k}$ then $c_1(t)$, $c_2(t)$ and $\omega(t)$ are increasing (decreasing), and if and only if $\frac{1}{2}$ $(k_{10} + k_{20}) = \hat{k}$ then all three variables are positive and constant.

Suppose, finally, that, before and after aid, the recipient is the wealthier country, as it might be in the event of post-war reparations. In the non-cooperative (cooperative) case, $\Omega = 0$ $(\Omega = \frac{1}{2}$ $(k_{10} - k_{20}) < 0)$ and $\omega(t)$ is positive for all $t \geqq 0$. If the elasticity of marginal utility is constant then optimal consumption changes at the same proportional rate in each country, with $c_1(t)$, $c_2(t)$ and $\omega(t)$ increasing (decreasing, constant) if and only if $\frac{1}{2}$ $(k_{10} + k_{20})$ is less than (greater than, equal to) $\hat{k}$.

3. Private Capital Mobile Between Countries

If capital is completely mobile between countries, so that the marginal product of capital is always and everywhere the same, then it is a matter of indifference to the donor whether aid is given in a lump or as a flow or as a mixture of both; moreover, if some or all of the aid is given as a flow, it is of no importance to the donor whether it is increasing or decreasing, positive or negative, for the difference between one aid profile and another will be continuously offset by an equal and opposite difference between the associated profiles of capital flow. Nevertheless the many optimal patterns of aid have one thing in common — they are all associated with the same pair of

optimal consumption paths $<c_1(t), c_2(t)>$.

If marginal utility is of constant elasticity then $c_1(t)$ and $c_2(t)$ change at the same proportional rate, increasing or decreasing according as $\frac{1}{2}(k_{10}+k_{20})$ is smaller or greater than $\hat{k}$.

These conclusions hold whether or not the donor and recipient cooperate; however the extent of the indeterminacy is less in the absence of cooperation, for then Ω and $\omega(t)$ are constrained to be non-negative.

4. Final Remarks

Our analysis can be extended in several directions. Thus the donor and recipient might be allowed to differ not only in initial endowments but also in technology and rate of time preference; and they might be allowed to produce more than one good, so that trade and aid could be seen to interact. Finally one might recognize that the absorption of aid is costly, with the cost increasing with the rate of absorption; then the sharp distinction between stock and flow would disappear.

Here we briefly indicate how the analysis must be modified to accommodate the possibility that the donor is more productive than the recipient and the possibility that there are two commodities, one a pure consumption good, the other a pure investment good. In addition we record a fact which will come as no surprise to the attentive reader of Sections 2 and 3. Problem (P.2) contains the a priori restriction that a stock transfer can occur only at $t=0$. If the problem is formulated more flexibly, with k_1 and k_2 as control variables, it can be shown that $t=0$ is the only date at which a stock transfer would occur in an optimal plan.

The donor more productive than the recipient. We confine our attention to the case in which $k_{10}>k_{20}$ and private capital is immobile.

Let the donor's production function be denoted by $f(k_1)$ and the recipient's by $\beta f(k_2)$, with $0<\beta<1$. In isolation k_1 approaches $\hat{k}_1$, defined by $f'(\hat{k}_1) = \varrho$, and k_2 approaches $\hat{k}_2$, defined by $\beta f'(\hat{k}_2) = \varrho$. Since $\varrho/\beta > \varrho$, $\hat{k}_1 > \hat{k}_2$. The donor seeks the solution to (P.2'), with $\beta f(k_2)$ replacing $f(k_2)$ in constraint (2); and the new necessary conditions consist of (7) − (11), with (11) replaced by

$$\dot{\psi}_2 = \varrho\psi_2 - \psi_2\beta f'(k_2) \tag{11'}$$

If the constraints $\omega \geqq 0$, $\Omega \geqq 0$ are ignored (the cooperative case), the solution is characterized by

$u'(c_1(t)) < u'(c_2(t))$, hence $c_1(t) > c_2(t)$ for all t

$f'(k_1(t)) = \beta f'(k_1(t))$ for all t

$\lim_{t \to \infty} k_1(t) = \hat{k}_1$

$\lim_{t \to \infty} k_2(t) = \hat{k}_2$

It follows that if $f'(k_{10}) > \beta f'(k_{20})$ then it is optimal to set $\Omega < 0$ and $\omega(t) > 0$ for all t, that is, it is optimal to shift capital to the more productive donor country; and that if $f'(k_{10}) < \beta f'(k_{20})$ then it is optimal to set Ω equal to the (positive) solution to the equation

$$f'(k_{10} - \Omega) - \beta f'(k_{20} + \Omega) = 0$$

and to then set $\omega(t) > 0$ for all t or $\omega(t) < 0$ for all t, depending on the values of β and λ.

If the constraints $\omega \geq 0$, $\Omega \geq 0$ are recognized (the non-cooperative case), capital cannot be shifted to the donor country. It follows that if $f'(k_{10}) > \beta f'(k_{20})$ then it is optimal to set $\Omega = 0$ and $\omega(t)$ always positive but smaller than in the cooperative case.

A second commodity when a second commodity is recognized, we have the additional complication of variable terms of trade. However, if the two countries cooperate, so that $k_1(t) = k_2(t)$ for all t, the relevant analysis remains straightforward and yields conclusions not unlike those of Section 2. The appendix contains a complete examination of the case in which private capital is internationally immobile. Here we merely list the main conclusions.

(i)   Except when the initial world capital: labour ratio greatly exceeds its steady-state value, production is always and everywhere incompletely specialized.

(ii)   Suppose that production is incompletely specialized in each country.
       Then $c_1(t) \gtrless c_2(t)$ and $\omega \lessgtr 0$ if and only if $\lambda \lessgtr 1$. If the consumption good is relatively capital-intensive, c_1 monotonely increases (decreases) if the world capital: labour ratio is less than (greater than) its steady-state value. If in addition the elasticity of marginal utility is constant then $|\omega|$ monotonely increases.

APPENDIX

In this appendix we extend the analysis of Section 2 to accommodate a second commodity. One of the two commodities is a pure consumption good, the other a pure investment good. Both can be traded internationally. With each commodity there is associated a production function with the same properties as f; however the two commodities differ uniformly in their relative factor intensities. As in earlier sections, the two countries are distinguished only by their initial wealth.

The following additional notation is needed.

p the relative price of the consumption good in terms of the investment good

$g(p, k_i)$ the per capita revenue or GNP function of the ith country, in terms of the investment good

The flow of aid is recorded in terms of the consumption good.

Throughout the appendix it is assumed that *private* capital is internationally immobile but, of course, intersectorally mobile. Moreover it is assumed that the donor and recipient cooperate, so that both flow aid and stock aid may be of either sign. Then, for some $\lambda > 0$, the two countries seek

$$(P.3) \quad \max_{c_i, k_i(0), \omega, p} \int_0^\infty [u(c_1) + \lambda u(c_2)] \exp(-\delta t) dt$$

$$\text{s.t.} \quad \dot{k}_1 = g(p, k_1) - nk_1 - pc_1 - p\omega$$

$$\dot{k}_2 = g(p, k_2) - nk_2 - pc_2 + p\omega$$

$$g_p(p, k_1) + g_p(p, k_2) \geq c_1 + c_2$$

$$k_1(0) + k_2(0) \leq k_{10} + k_{20}, \text{ given}$$

where, from the familiar properties of the revenue function when commodities differ uniformly in factor intensity, $g_p(p, k_i)$ is the per capita output of the consumption good in the ith country. Defining $\bar{c}_1 \equiv c_1 + \omega$ and $\bar{c}_2 \equiv c_2 - \omega$, (P.3) can be reformulated as

(P.3′) $\quad \max_{\bar{c}_i, k_i(0), \omega, p} \int_0^\infty [u(\bar{c}_1 - \omega) + \lambda u(\bar{c}_2 + \omega)] \exp(-\delta t)dt$

s.t. $\quad \dot{k}_1 = g(p, k_1) - nk_1 - p\bar{c}_1$ $\qquad\qquad$ (21)

$\qquad \dot{k}_2 = g(p, k_2) - nk_2 - p\bar{c}_2$ $\qquad\qquad$ (22)

$\qquad g_p(p, k_1) + g_p(p, k_2) = \bar{c}_1 + \bar{c}_2$ $\qquad\qquad$ (23)

$\qquad k_1(0) + k_2(0) \leq k_{10} + k_{20}$, given $\qquad\qquad$ (24)

The current-value Hamiltonian associated with (P.3′) is

$$H = u(\bar{c}_1 - \omega) + \lambda u(\bar{c}_2 + \omega) + \sum_i \psi_i[g(p, k_i) - nk_i - p\bar{c}_i]$$

and the Lagrangean is

$$L = H + \psi[g_p(p, k_1) + g_p(p, k_2) - \bar{c}_1 - \bar{c}_2]$$

so that we have, as necessary conditions,

$\quad \partial L/\partial \bar{c}_1 = u'(\bar{c}_1 - \omega) - p\psi_1 - \psi = 0$ $\qquad\qquad$ (25)

$\quad \partial L/\partial \bar{c}_2 = \lambda u'(\bar{c}_2 + \omega) - p\psi_2 - \psi = 0$ $\qquad\qquad$ (26)

$\quad \partial L/\partial \omega = -u'(\bar{c}_1 - \omega) + \lambda u'(\bar{c}_2 + \omega) = 0$ $\qquad\qquad$ (27)

$\quad \partial L/\partial p = \psi_1[g_p(p, k_1) - \bar{c}_1] + \psi_2[g_p(p, k_2) - \bar{c}_2]$

$\qquad\qquad + \psi[g_{pp}(p, k_1) + g_{pp}(p, k_2)] = 0$ $\qquad\qquad$ (28)

$\quad \dot{\psi}_1 = \delta\psi_1 - \delta L/\delta k_1 = \delta\psi_1 - \psi_1[g_k(p, k_1) - n] - \psi g_{pk}(p, k_1)$ $\qquad$ (29)

$\quad \dot{\psi}_2 = \delta\psi_2 - \delta L/\delta k_2 = \delta\psi_2 - \psi_2[g_k(p, k_2) - n] - \psi g_{pk}(p, k_2)$ $\qquad$ (30)

We begin our study of these conditions by establishing a useful preliminary proposition.

LEMMA: On the optimal path, $p > 0$.

Proof Suppose that $p(t) = 0$ for some t, say t'. From the assumed properties of the production functions, $g_p(0, k_i) = 0$. Hence, from (23), $\bar{c}_1 + \bar{c}_2 = 0$. From this result, and the non-negativity of c_i, $c_1 = c_2 = 0$, contradicting the assumed properties of the utility function.   □

From $(25) - (27)$, with $p > 0$,

$$\psi_1 = \psi_2 \equiv \psi_0 \tag{31}$$

which, with (29) and (30), implies that

$$\psi_0 g_k(p, k_1) + \psi g_{pk}(p, k_1) = \psi_0 g_k(p, k_2) + \psi g_{pk}(p, k_2) \tag{32}$$

It is convenient to consider separately the two cases, $\psi > 0$ and $\psi = 0$.

Case A: $\psi > 0$ In this case,

$$g_p(p, k_1) + g_p(p, k_2) - \bar{c}_1 - \bar{c}_2 = 0 \tag{33}$$

Together with (28) and (31), this implies that

$$g_{pp}(p, k_1) + g_{pp}(p, k_2) = 0 \tag{34}$$

which, since $g_{pp}(p, k_1) \geq 0$, implies that

$$g_{pp}(p, k_i) = 0 \qquad i = 1,2 \tag{35}$$

Thus production must be completely specialized in each country.

Given the restrictions imposed on the utility function, it is impossible that both countries produce only the investment good. And since capital can be cooperatively shifted internationally and the same production functions prevail everywhere, it is inefficient for the countries to specialize in producing different goods.

Thus we are left with just one possibility: both countries produce the consumption good only. Then

$$\dot{k}_i = -nk_i \tag{36}$$

and

$$f_c(k_i) = \bar{c}_i \qquad i = 1,2 \tag{37}$$

where f_c is the per capita production function for the consumption good. Eq. (32) therefore reduces to

$$(p\psi_0 + \psi)f_c'(k_1) = (p\psi_0 + \psi)f_c'(k_2) \tag{38}$$

which, since $p\psi_0 + \psi > 0$, implies that $k_1 = k_2 \equiv k$. From (37), therefore, $\bar{c}_1 = \bar{c}_2 \equiv \bar{c}$. In the absence of a switch of regime, $\bar{c}$ and k, and therefore $c_1 = \bar{c}_1 - \omega$ and $c_2 = \bar{c}_2 + \omega$, converge monotonely to zero.

Case B: $\psi = 0$ From (25) and (26), $\psi_0 > 0$. Hence, from (28),

$$g_p(p, k_1) - \bar{c}_1 + g_p(p, k_2) - \bar{c}_2 = 0 \tag{39}$$

Bearing in mind (29) and (30), with $\psi_0 > 0$, (32) can be seen to reduce to

$$g_k(p, k_1) = g_k(p, k_2) \tag{40}$$

As we have noted, it would be inefficient for the two countries to specialize in producing different commodities. We therefore focus on the remaining possibilities: that each country produces only the consumption good and that each country produces both commodities.

(Bi) Suppose that each country specializes in producing the consumption good. Then (40) reduces to

$$pf_c'(k_1) = pf_c'(k_2) \tag{41}$$

Since p is positive, (41) implies that

$$k_1 = k_2 = k \tag{42}$$

Thus

$$\bar{c}_1 = \bar{c}_2 \equiv \bar{c} = f_c(k) \tag{43}$$

where k follows $\dot{k} = -nk$. Evidently the trajectory is the same as in Case A.

(Bii) If neither country specializes, we can write

$$g(p, k_i) = r(p)k_i + w(p) \tag{44}$$

where $r(p)$ and $w(p)$ are the price-dependent rental rate and wage rate, respectively. From (21) and (22), defining $\tilde{k} \equiv k_1 + k_2$,

$$\dot{\tilde{k}} = r(p)\tilde{k} + 2w(p) - n\tilde{k} - p(\bar{c}_1 + \bar{c}_2) \tag{45}$$

Furthermore, from (39),

$$r'(p)\tilde{k} + 2w'(p) = \bar{c}_1 + \bar{c}_2 \tag{46}$$

On the other hand, from (25) and (26),

$$\bar{c}_1 + \bar{c}_2 = \phi(p\psi_0) + \phi(p\psi_0/\lambda) \tag{47}$$

Hence, from (46) and (47),

$$r'(p)\tilde{k} + 2w'(p) = \phi(p\psi_0) + \phi(p\psi_0/\lambda) \tag{48}$$

which can be viewed as determining p as a function of $\tilde{k}$ and $\psi_0 : p = p(\tilde{k}, \psi_0)$. Thus (45) can be rewritten as

$$\dot{\tilde{k}} = [r(p(\tilde{k}, \psi_0)) - p(\tilde{k}, \psi_0)\,r'(p(\tilde{k}, \psi_0))]\tilde{k}$$
$$+ 2[w(p(\tilde{k}, \psi_0)) - p(\tilde{k}, \psi_0)w'(p(\tilde{k}, \psi_0))] - n\tilde{k} \tag{49}$$

On the other hand, from (29) and (30),

$$\dot{\psi}_0 = \psi_0[\varrho - r(p(\tilde{k}, \psi_0))] \tag{50}$$
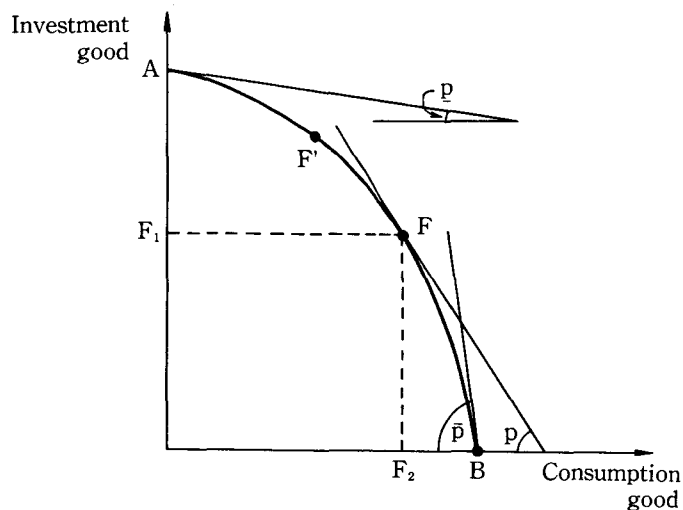
$OF_1 : [\pi(p) - pr'(p)]\,\tilde{K} + 2\,[w(p) - pw'(p)]$
$OF_2 : r'(p)\tilde{K} + 2w'(p)$

Figure 1

We now have all the pieces needed for the construction of the phase diagram associated with (P.3'). Let us begin with Case (Bii). In Figure 1, AFB is the world per capita production possibility curve for a given value of $\tilde{k}$. Given ψ_0 and $\tilde{k}$, (48) determines the price ratio p and therefore the production point F. Differentiating (48) with respect to p and ψ_0, we obtain

$$\frac{dp}{d\psi_0} = \frac{p[\phi'(p\psi_0) + (1/\lambda)\phi'(p\psi_0/\lambda)]}{r''(p)\tilde{k} + 2w''(p) - [\phi'(p\psi_0) + (1/\lambda)\phi'(p\psi_0/\lambda)]\psi_0} < 0 \qquad (51)$$

Now, from (44), $r(p)\tilde{k} + 2w(p) = g(p, \tilde{k})$. Hence

$$r''(p)\tilde{k} + 2w''(p) = g_{pp}(p, \tilde{k}) \qquad (52)$$

which, given the incomplete specialization of production, is positive. It follows that the denominator of (51), and therefore $dp/d\psi_0$, are negative. Thus, as ψ_0 increases, p declines and the production point moves to the north-west from F. However there is no finite value of ψ_0 such that production occurs at A, for there is no finite value of ψ_0 such that

$$0 = \phi(\underline{p}\psi_0) + \phi(\underline{p}\psi_0/\lambda)$$

where $\underline{p}$ is defined by Figure 1. On the other hand, when ψ_0 takes on a sufficiently small positive value, the production point reaches B. At that point, $p = \bar{p}$ and we have the pair of equations

$$r(\bar{p})\tilde{k} + 2w(\bar{p}) = \bar{p}[r'(\bar{p})\tilde{k} + 2w'(\bar{p})] \qquad (53)$$

$$r'(\bar{p})\tilde{k} + 2w'(\bar{p}) = \phi(\bar{p}\psi_0) + \phi(\bar{p}\psi_0/\lambda) \qquad (54)$$
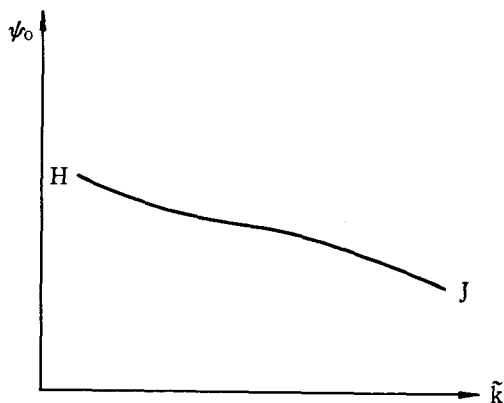
Figure 2(a) : r'(p) > o
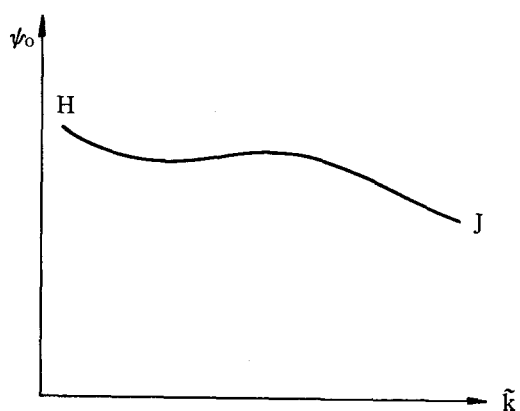
Figure 2(b) : r'(p) < o

so that, differentiating with respect to $\bar{p}$, ψ_0 and $\tilde{k}$, and eliminating $d\bar{p}$,

$$\frac{d\psi_0}{d\tilde{k}} = \frac{r(\bar{p})[r''(\bar{p})\tilde{k} + 2w''(\bar{p})] - [\phi'(\bar{p}\psi_0) + (1/\lambda)\phi'(\bar{p}\psi_0/\lambda)][r(\bar{p}) - \bar{p}r'(\bar{p})]}{\bar{p}[\phi'(\bar{p}\psi_0) + (1/\lambda)\phi'(\bar{p}\psi_0/\lambda)][r''(\bar{p})\tilde{k} + 2w''(\bar{p})]} \tag{55}$$

If the consumption good is relatively capital-intensive, so that $r' > 0$, then $d\psi_0/d\tilde{k} < 0$; otherwise, the sign of (55) is ambiguous. Thus we arrive at Figure 2; above HJ both commodities are produced, below HJ only the consumption good is produced.

Returning to (50) and (51), we seek the loci $\dot{k}|_{(50)} = 0$ and $\dot{\psi}_0|_{(51)} = 0$. Differentiating (49) with respect to p, $\tilde{k}$ and ψ_0, we obtain

$$dp = -(1/\Delta)r'(p)d\tilde{k} + (p/\Delta)[\phi'(p\psi_0) + (1/\lambda)\phi'(p\psi_0/\lambda)]d\psi_0 \tag{56}$$

where

$$\Delta \equiv r''\tilde{k} + 2w'' - [\phi'(p\psi_0) + (1/\lambda)\phi'(p\psi_0/\lambda)]\psi_0 > 0$$

Differentiating (50) with respect to p and $\tilde{k}$, we obtain

$$d\dot{\tilde{k}} = -p[r''(p)\tilde{k} + 2w''(p)]dp + [r(p) - pr'(p) - n]d\tilde{k} \tag{57}$$

And, finally, differentiating (51) with respect to p and ψ_0, with p satisfying $\varrho = pr(p)$, we obtain

$$d\dot{\psi}_0 = -\psi_0 r'(p)dp \tag{58}$$

Substituting from (56) into (57) and (58),

$$d\dot{\tilde{k}} = \left[\frac{pr'}{\Delta}(r''\tilde{k} + 2w'') + (r - pr' - n)\right]d\tilde{k}$$
$$- (1/\Delta)p^2[\phi'(p\psi_0) + (1/\lambda)\phi'(p\psi_0/\lambda)][r''\tilde{k} + 2w'']d\psi_0 \tag{59}$$

and

$$d\dot{\psi}_0 = -\psi_0 r'\{-(1/\Delta)r'd\tilde{k} + (1/\Delta)p[\phi'(p\psi_0) + (1/\lambda)\phi'(p\psi_0/\lambda)]d\psi_0\} \tag{60}$$

Now consider the function

$$h(x) \equiv \det \begin{bmatrix} x - A & B \\ -\psi_0(r')^2/\Delta & \psi_0 r'p/\Delta \end{bmatrix} \tag{61}$$

where

$$A \equiv \frac{pr'}{\Delta}(r''\tilde{k} + 2w'') + (r - pr' - n)$$

and

$$B \equiv \frac{p^2}{\Delta}[\phi'(p\psi_0) + \phi'(p\psi_0/\lambda)/\lambda][r''\tilde{k} + 2w'']$$

It can be verified that

$$h(0) = -(p\psi_0 r'/\Delta)[\phi'(p\psi_0) + \phi'(p\psi_0/\lambda)/\lambda][r-pr'-n] \tag{62}$$

is negative if n is sufficiently close to zero. It follows that, for sufficiently small n, the stationary state of the system [(50), (51)] is a saddlepoint: If the initial $(\psi_0, \tilde{k})$ is chosen on one of the stable arms of the saddle, (ψ_0, k) converges to the stationary state.
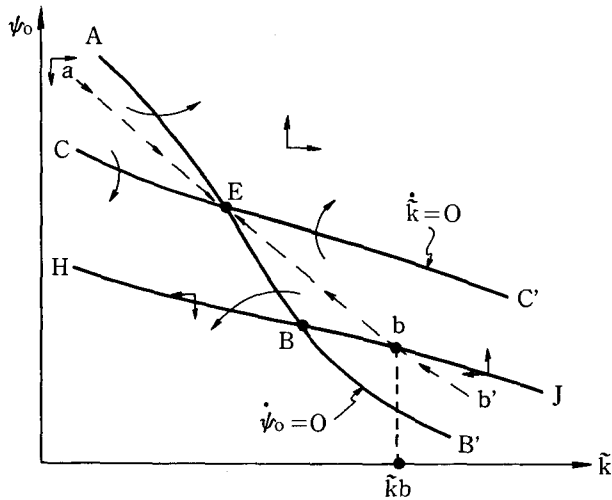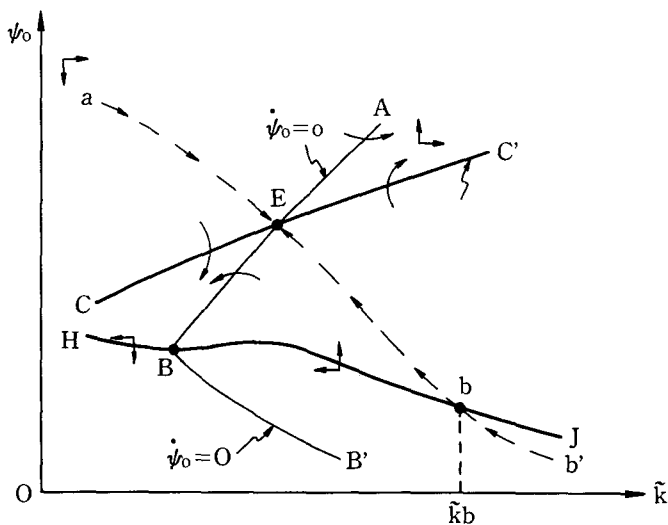

Figure 3 : r'(p)>o

Figure 4 : r'(p)<o

Figures 3 and 4 are the phase diagrams associated with (P.3') when n is sufficiently small but positive. In each figure the optimal trajectory is represented by the dotted locus aEbb'. On bb' only the consumption good is produced; on aEb both goods are produced in each country. Moreover, $k_1 = k_2$ on bb', implying that, if $k_{10} + k_{20} > \tilde{k}_b$, there is an initial stock transfer such that $k_1(0) = k_2(0) = \frac{1}{2}(k_{10} + k_{20})$.

While the phase diagram makes clear the existence and saddlepoint stability of a steady state, it contains no information about the manner in which aid and consumption behave through time. However, we know from our earlier discussion of Case A, that, when both countries produce only the consumption good, the rate of flow of aid and the two rates of consumption go monotonely to zero. If $\lambda < 1$ then $c_1 > c_2$ for all t, and $\omega < 0$ for all t. It remains to study the behaviour of aid and consumption above HJ, where each country produces both commodities.

From (27) we know that, if and only if $\lambda \lesseqgtr 1$, $c_1 \gtreqless c_2$ for all t. What more can be said? Differentiating (59) with respect to time, we find that

$$\{(1/\psi_0)(r''\tilde{k} + 2w'') - [\phi'(p\psi_0) + (1/\lambda)\phi'(p\psi_0/\lambda)]\} p\dot{\psi}_0$$

$$= (p/\psi_0)(r''\tilde{k} + 2w'')\dot{\psi}_0 - r'\dot{\tilde{k}} \tag{63}$$

where both the left side of the equation and the coefficient of $\dot{\psi}_0$ are positive. On the other hand, the stable arms of Figures 3 and 4 are negatively sloped, implying that $\dot{\psi}_0$ and $\dot{\tilde{k}}$ are of opposite sign, with $\dot{\psi}_0 < 0$, $\dot{\tilde{k}} > 0$ on Ea and $\dot{\psi}_0 > 0$, $\dot{\tilde{k}} < 0$ on Eb. Together with (63), these facts imply that, if the consumption good is relatively capital-intensive, so that $r'(p) > 0$, then $d(p\psi_0)/dt$ is negative on Ea and positive on Eb. Since $u'(c_1) = p\psi_0$ and $u'(c_2) = p\psi_0/\lambda$, we can infer that, if the consumption good is relatively capital-intensive, c_i monotonely increases on Ea and monotonely decreases on Eb. Thus we have shown that if the consumption good is relatively capital-intensive then c_i increases everywhere on the left stable arm of the saddle and decreases everywhere on the right stable arm.

Let us now consider the behaviour of aid above HJ. Suppose that $\tilde{k}(0) > \tilde{k}_b$ and $\lambda < 1$, so that $c_1(t) > c_2(t)$ for all t. When $\tilde{k}$ reaches $\tilde{k}_b$ at $t = \hat{t}$, $k_1 = k_2$ and $\omega < 0$. At $\hat{t}$, each country produces the same amount of the consumption good, and c_1 exceeds c_2 by ω, the amount of the transfer. After $\hat{t}$, each country

produces both goods. However the transfer will continue to be effected by a net shipment of the consumption good only; if the recipient were to ship both goods, the donor would simply sell back all investment goods received. Since $c_1(t) > c_2(t)$ for all t, the flow of aid is negative for all t. Moreover, from the equations of accumulation

$$\dot{k}_1 = r(p)k_1 + w(p) - nk_1 - p[\phi(p\psi_0) - \omega] \tag{64a}$$

$$\dot{k}_2 = r(p)k_2 + w(p) - nk_2 - p[\phi(p\psi_0/\lambda) + \omega] \tag{64b}$$

and the equality of k_1 and k_2, we deduce that

$$\phi(p\psi_0) - \omega = \phi(\psi_0/\lambda) + \omega$$

so that

$$\omega = \frac{1}{2}[\phi(p\psi_0) - \phi(p\psi_0/\lambda)] \tag{65}$$

Differentiating (65) with respect to time,

$$\dot{\omega} = \frac{1}{2}[\phi'(p\psi_0) - (1/\lambda)\phi'(p\psi_0/\lambda)]\frac{d}{dt}(p\psi_0)$$

$$= \frac{1}{2}\left[\phi(p\psi_0)\frac{p\psi_0\phi'(p\psi_0)}{\phi(p\psi_0)} - \phi(p\psi_0/\lambda)\frac{(p\psi_0/\lambda)\phi'(p\psi_0/\lambda)}{\phi(p\psi_0/\lambda)}\right]\frac{d(p\psi_0)/dt}{p\psi_0} \tag{66}$$

If the elasticity of marginal utility is constant, (66) reduces to

$$\dot{\omega} = \frac{1}{2\alpha}[\phi(p\psi_0/\lambda) - \phi(p\psi_0)]\frac{d(p\psi_0)/dt}{p\psi_0}$$

Since ϕ is a decreasing function,

$$\text{sign}\,[\phi(p\psi_0/\lambda) - \phi(p\psi_0)] = \text{sign}\,(\lambda - 1)$$

Thus, if $r' > 0$ then on Ea ω monotonely increases (decreases) and on Eb ω monotonely decreases (increases) if $\lambda < 1$ ($\lambda > 1$). In short, if the consumption good is relatively capital-intensive we reach conclusions very similar to those reached in the one-good case.

It remains to consider the possibility that the population is declining or stationary. If the population is declining, there is only the steady state with $\tilde{k} = 0$. If the population is stationary we have a more interesting case. For the locus $\dot{\tilde{k}} = 0$ is determined by (49) and (50); and the locus HJ is determined by (53) and (54). Comparing the two pairs of equations, one sees that, if $n = 0$, the locus $\dot{\tilde{k}} = 0$ coincides with HJ. Moreover, differentiating (49) and (50) with respect to p, ψ_0 and n, we find that

$$\frac{d\psi_0}{dn} = -\frac{r''\tilde{k} + 2w'' - [\phi'(p\psi_0) + \phi'(p\psi_0/\lambda)/\lambda][r''\tilde{k} + 2w'']^{-1}\tilde{k}}{[\phi'(p\psi_0) + \phi'(p\psi_0/\lambda)/\lambda]p} > 0$$

thus confirming that the locus $\dot{\tilde{k}} = 0$ lies above HJ if $n > 0$.

References

[1] Levy, V. (1988), Does concessionary aid lead to higher investment rates in low-income countries? Review of Economics and Statistics 69, 152-156.

A SIMPLE UNION-RIDDEN GENERAL EQUILIBRIUM MODEL

Kazuo SHIMOMURA

1. Introduction

In this paper we present an equilibrium model involving a maximizing labour union. It is not the main purpose of this paper to construct an union-ridden model which is realistic in such a sense that it can well explain some important facts in labour markets. Instead, we shall construct a simple, abstract and yet general equilibrium model from which more complicated union-ridden general equilibrium models could develope.

Before starting formal analysis, the following points may be worth mentioning.

First, with respect to the labour union objectives, we shall basically follow a traditional approach: A labour union maximizes the expected utility of a union member under the assumptions that all member are identical and that employed members are randomly chosen: See McDonald and Solow (1981), Oswald (1982, 1985), Sampson (1983) and Farber (1986). Although that approach generally does not consider the possibility of redistribution of wage income and working hours over all union members, there may be no reason, at least from a theoretical point of view, to exclude that possibility from our analysis, if such redistribution can make any member better-off. To our knowledge, the redistribution activities were first discussed by Pencavel (1985). He virtually showed that if all union members pool their income and the labour union redistributes this total income over all members in such a way as to maximize the expected utility of each member then maximizing the expected utility of each member is equivalent to maximizing the rents from unionization. Although he argued that the income redistribution within the union has some relevancy to unionism in the real world, he did not examine the theoretical implication of the maximization of the rents from unionization for union-ridden equilibria. That attempt was made by Kemp, Long and Shimomura (1990) which investigated how the pure theory of international trade and investment may be affected by introducing maximizing labour unions. However, they did not take into account the redistribution of working hours; they assumed that each worker can inelastically provide a fixed amount of working hours. Formulating the objective function for a labour union later in this paper, we shall take into

account the intra-union redistribution of both wage income and working hours.

Second, we shall make the production-side of our model consistent with the standard theory of production. Thus, it will be assumed that there are many competitive firms which, taking factor and commodity prices as fixed, determine the demands for capital services and working hours and the supply of their commodity in such a way as to maximize their profit. Entry into and exit from the market for their commodity are free for them. Hence, the commodity price always tends to be equal to its unit cost.

Third, just as workers can form a labour union, so may capitalists be able to form their union, say capitalists' club, which appears in the market for capital service as a monopolist. This possibility will be also taken into account in this paper.

The structure of this paper is as follows. In Section 2 our model is presented. In Section 3 the pre-union equilibrium of our model is examined. In Section 4 the reaction functions of workers and capitalists are derived. In Section 5 and 6 several types of equilibrium solutions are obtained and the relationship among them is visually shown.

2. The Assumptions

Let us state the main formal assumptions underlying our model.

A.1   There are N households who earn their income only from providing labour services measured by time, say hours. Let us call the households workers. Each worker has a system of preference ordering over consumption and leisure which is common among workers and represented by the utility function

$$u = u(c_w, 1 - l) \tag{1}$$

where c_w is the amount of a worker's consumption and l is working hours. The maximum working hours are assumed to be unity for any worker. The utility function is strictly qusi-concave and has the following properties; for any $(c_w, h) > 0$

$$u_c(c_w, h) \equiv \frac{\partial}{\partial c_w} u(c_w, h) > 0$$

$$u_h(c_w, h) \equiv \frac{\partial}{\partial h} u(c_w, h) > 0 \tag{2}$$

$$u_{cc}(c_w, h) \equiv \frac{\partial^2}{\partial c_w{}^2} u(c_w, h) < 0$$

$$u_{ch}(c_w,\ h) \equiv \frac{\partial^2}{\partial c_w\,\partial_h}\ u(c_w,\ h)\ >\ 0$$

$$(2)$$

$$u_{hh}(c_w,\ h) \equiv \frac{\partial^2}{\partial_h{}^2}\ u(c_w,\ h)\ <\ 0$$

A.2 There are K households who earn their income only from renting capital equipments which they possess. Let us call the households capitalists. It is assumed that each capitalist possesses just one unit of capital equipment and that the maximum renting hours for each equipment are equal to be unity. The utility of each capitalist depends only on his/her consumption.

$$v\ =\ v(c_r)$$

$$(3)$$

The shape of the function $v(.)$ is common among capitalists. $v(c_r)$ has the following properties; for any $c_r > 0$

$$v'(c_r) \equiv \frac{d}{dc_r}\ v(c_r)\ >\ 0$$

$$(4)$$

$$v''(c_r) \equiv \frac{d^2}{dc_r{}^2}\ v(c_r)\ <\ 0$$

A.3 There are no "middle class" people who earn their income from providing both capital and labour services.

A.4 There is a single consumption good which is produced by competitive firms. The production function is denoted by

$$Y\ =\ F(X,\ L)$$

$$(5)$$

where X and L are the renting hours of capital and the working hours of labour, respectively. From A.1 and A.2 $X \leqq K$ and $L \leqq N$. The production function is strictly quasi-concave, homogeneous of degree on in X and L, and satisfies Inada condition. The marginal productivities of X and L are decreasing and both of the factors of production are indispensable.

Denoting by w, r and $c(w,\ r)$ the wage rate per working hour, the rental rate per renting hour and the unit-cost function of the consumption good corresponding to (5), each firm maximizes, for given w and r, its profit

$$\pi \equiv (1 - c(w, r))y \tag{6}$$

where y is the output of the firm and the price of the commodity serves as numerare.

A.5 Workers can form a labour union which appears as a monopolist in labour market. The union sets the minimum wage rate, say w_m, leaving competitive firms to determine the volume of output and, therefore, the volume of employment. The wage rate w, which possibly differs from w_m, is determined through market mechanism. Suppose that as a result of choosing a certain w_m the labour union obtains a pair of the wage rate and the volume of employment (w, L) where $L \leqq N$. As stated at the previous section, the union offers a scheme of the redistribution of wage income and working hours, $(c_w(\tau), l(\tau))$, $0 \leqq \tau \leqq N$, randomly to all union members in such a way as to maximize the expected utility of each member. That is, the union seeks to maximize

$$U \equiv \max_{c_w, l} \frac{1}{N} \int_0^N u(c_w(\tau), 1 - l(\tau)) \, d\tau \tag{7}$$

$$\text{sub. to} \quad wL - \int_0^N c_w(\tau) \, d\tau \geqq 0$$

$$\int_0^N l(\tau) \, d\tau - L \geqq 0$$

$$w, L, N \text{ given and } L \leqq N$$

Associated with (7) is the Lagrangean

$$J \equiv \frac{1}{N} u(c_w(\tau), 1 - l(\tau)) - \lambda_c c_w(\tau) + \lambda_l l(\tau)$$

and the necessary conditions for optimality are

$$\frac{\partial J}{\partial c_w(\tau)} = \frac{1}{N} u_c(c_w(\tau), 1 - l(\tau)) - \lambda_c = 0$$

$$\frac{\partial J}{\partial l(\tau)} = -\frac{1}{N} u_h(c_w(\tau), 1 - l(\tau)) + \lambda_l = 0$$

$$\lambda_C \geqq 0, \ \lambda_C \cdot \ [wL - \int_0^N c_w(\tau)d\tau] \ = \ 0 \tag{8}$$

$$\lambda_l \geqq 0, \ \lambda_l \cdot \ [\int_0^N l(\tau)d\tau - L] \ = \ 0$$

and λ_C and λ_l are independent of τ.[1] One may easily verify from (8) that the optimal solution to (7) is

$$c_w(\tau) \ = \ \frac{wL}{N} \ \text{and} \ l(\tau) \ = \ \frac{L}{N} \ \text{for any} \ \tau \epsilon [0, \ N] \tag{9}$$

Substituting (9) to (7), we see

$$U \ = \ u(wl, \ 1-l) \tag{10}$$

It may be natural to assume that (10) is the objective function of the labour union.

A.6 If workers can form a labour union, there may be no reason to assume away that capitalists can form a union, say capitalists' club. By a parallel argument to A.5, we can obtain the objective function of the capital union as follows.

$$V \ = \ v(r\delta) \tag{11}$$

where $\delta \equiv \dfrac{X}{K}$, which may be called the rate of capital utilization.

A.7 Workers and capitalists decide to unionize themselves respectively, only if it is to their interest to do so. Thus, the unionization of workers takes place, only if U is greater than the level of utility which may be obtained by each worker if workers give up forming a labour union and behave as price takers in labour market.

3. A Preliminary Analysis

First of all, let us consider pre-union equilibria. The convex curve AEA′ in Figure 1 depicts 1 = c(r, w), which is supposed to hold through market mechanism. The point E represents a pre-union equilibrium, i.e., (r*, w*) satisfies

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1) See Long and Vousden (1977, p. 15).

$$1 = c(w, r) \tag{12}$$

and the factor market clearing condition

$$\frac{N}{K} l(w) = \frac{c_w(w, r)}{c_r(w, r)} \tag{13}$$

where $l(w) \equiv \arg \max \{u(wl, 1-l) \text{ sub. to } 1-l \geqq 0\}$, $c_w(w, r) \equiv \frac{\partial}{\partial w} c(w, r)$ and $c_r(w, r) \equiv \frac{\partial}{\partial r} c(w, r)$. In what follows it is assumed that the function $l(w)$ can be defined for any $w > 0$ and that $l'(w) \equiv \frac{d}{dw} l(w)$ is always positive. It follows that for any $w > 0$ we have

$$wu_c(wl(w), 1-l(w)) - u_h(wl(w), 1-l(w)) = 0 \tag{14}$$

and

$$u_c(wl(w), 1-l(w)) + l(w) \{wu_{cc}(wl(w), \tag{15}$$
$$1-l(w)) - u_{hc}(wl(w), 1-l(w))\} > 0$$

The latter inequality is established if, for example, $u(c_w, h)$ is homogeneous of degree one in c_w and h. As is well known, (13) at (w^*, r^*) is equal to the slope of BEB'.
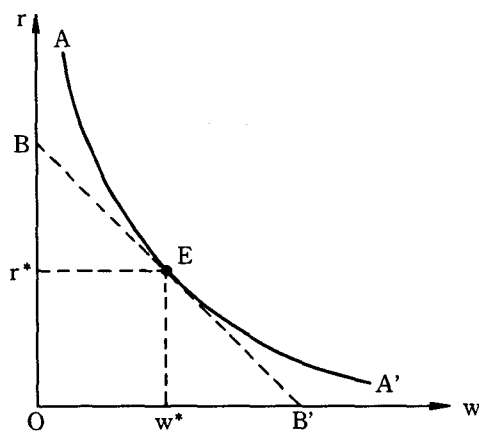
Figure 1 factor price flontier

For the convenience of later argument, let us obtain a sort of the supply curve of the output. From (12) r can be regarded as a decreasing and convex function of w. Let us denote it by η (w). Then the functions of w

$$Y_K(w) \equiv \frac{K}{c_r(w, \eta(w))}$$

and

$$Y_N(w) \equiv \frac{Nl(w)}{c_w(w, \eta(w))}$$

express the volumes of the output for given w in the case that the volumes of capital and labour become finding respectively. Since

$$\frac{d}{dw} Y_K(w) = -\frac{1}{c_r^3} (c_r c_{rw} - c_w c_{rr}) < 0$$

and

$$\frac{d}{dw} Y_N(w) = \frac{N}{c_w^3} \{l' c_w c_r - l \cdot (c_r c_{ww} - c_w c_{wr})\} > 0$$

, the graphs of $Y_K(w)$ and $Y_N(w)$ are like in Figure 2, which tells that for $w < w^*$ labour is finding while for $w > w^*$ capital is finding. Therefore, the kinky curve AEB expresses the supply curve of the output and if w is smaller (resp. greater) than w^* then r declines (resp. rises) and w rises (resp. declines) until w coincides with w^*.
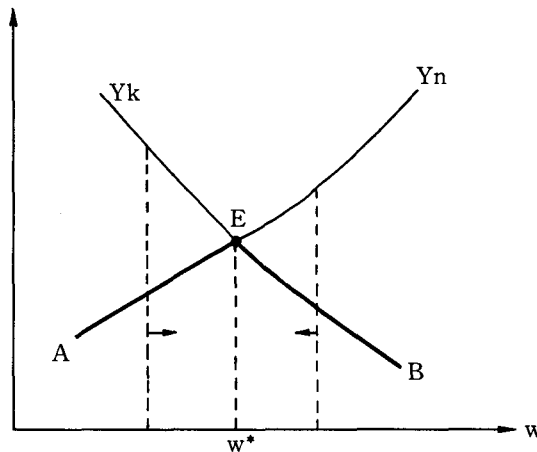
Figure2   supply curve AE B

4. Deriving the Reaction "functions"

At this section we shall derive the reaction "functions",

$$\underline{w}(r_m) \equiv \arg\max_{w_m} U \text{ for given } r_m \tag{16}$$

$$\underline{r}(w_m) \equiv \arg\max_{r_m} V \text{ for given } w_m \tag{17}$$

The double quotation marks are used above because $\underline{w}(r_m)$ and $\underline{r}(w_m)$ are, as will be shown, correspondences for some r_m and w_m, respectively.

The derivation of $\underline{w}(r_m)$

First, Suppose that r_m is set r^*. If w_m is set above $\phi(r_m)$ where $\phi \equiv \eta^{-1}$, then for any $(r, w) \geqq (r_m, w_m)$ $1 < c (r, w)$, which implies that no firm produces the output. Therefore $U = u(0, 1)$. If w_m is set below $\phi(r_m)$, then (r, w) becomes equal to $(r_m, \phi(r_m))$. For, at any other point on QR in Figure 3A r must decline to r_m, since on QR labour is finding, as was shown at the previous section. Therefore, for any $w_m \leqq \phi(r_m)$

$$U\big|_{(18)} \equiv u(\phi(r_m)l(\phi(r_m)), \ 1-l(\phi(r_m))) \tag{18}$$

See Figure 3B, which depicts the graph of U for $r_m > r^*$. From this figure it may be clean that when $r_m \geqq r^*$

$$\underline{w}(r_m) = \{w_m \mid 0 \leqq w_m \leqq \phi(r_m)\} \tag{19}$$
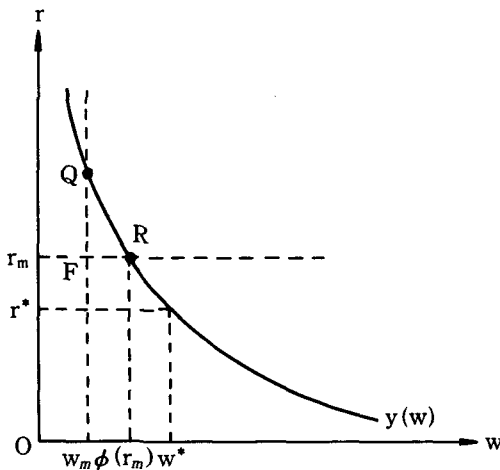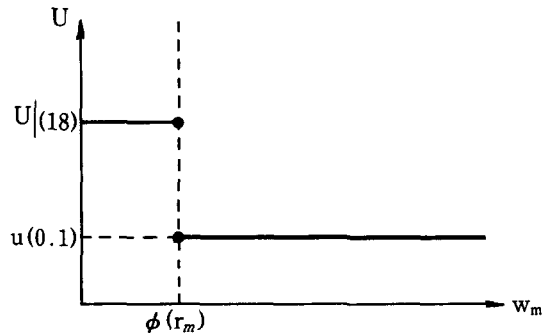
Figure 3A $\pi m \geqq \pi^*$ Figure 3B

Second, let us examine the case in which r_m is set below r^*. Before starting our main analysis on this case, let us consider the function of w

$$\tilde{U}(w) \equiv u\left(\frac{wc_w(w,\ \eta(w))K}{c_r(w,\ \eta(w))N},\ 1 - \frac{c_w(w,\ \eta(w)K}{c_r(w,\ \eta(w))N} \right)$$

, which is equivalent to

$$\tilde{U}(w) \equiv u\left(\frac{wL(w)}{N},\ 1 - \frac{L(w)}{N} \right) \tag{20}$$

where $L(w)$ is the solution to the equation $w = F_L(L,\ K) \equiv \dfrac{\partial}{\partial L} F(L,\ K)$. In what follows, we assume that (20) has a positive, unique and global maximum solution, say $\tilde{w}$, and that there exists no local maximum solution. These conditions are certainly satisfied if, for example, both the utility function and the unit-cost function are of Cobb-Douglas type. Differentiating (20) with w, we see

$$\tilde{U}'(w) \equiv \frac{d\tilde{U}(w)}{dw} = \frac{L(w)}{N} u_c + \frac{L'(w)}{N} (u_c w - u_h) \tag{21}$$

Therefore, from (14) $\tilde{U}'(w^*) = \dfrac{L(w^*)}{N} u_c \left(\dfrac{wL(w^*)}{N},\ 1 - \dfrac{L(w^*)}{N} \right) > 0,$

which implies that $w^* < \tilde{w}$. Figure 4 is a possible graph of $\tilde{U}(w)$, where w° is the solution to $N = L(w)$.
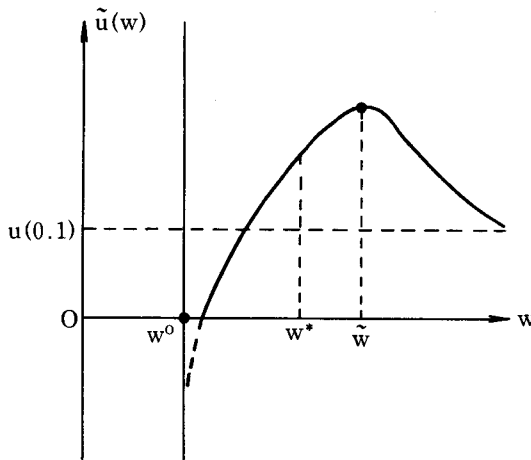
Figure 4 the graph ob $\tilde{U}(w)$

Now let us return our main business. Suppose that r_m is set between r^* and $\eta(\tilde{w})$. If w_m is set above $\phi(r_m)$, then for any $(r, w) \geqq (r_m, w_m)$ $1 < c(r, w)$, which implies that $U = u(0, 1)$. If w_m is set between w^* and $\phi(r_m)$, then w must decline to w_m along the locus of $\eta(w)$, since capital is finding for $w > w^*$. Then $U = \tilde{U}(w_m)$.

If w_m is set below w^*, then the area $\{(w, r)/(w, r) \geqq (w_m, r_m)\}$ contains (w^*, r^*). Therefore (w, r) must be equal to (w^*, r^*), and $U = \tilde{U}(w^*)$ for any $w_m \leqq w^*$. We now obtain Figure 5B, from which it may be clear that when $r^* > r_m \geqq \eta(\tilde{w})$

$$\underline{w}(r_m) = \phi(r_m) \tag{22}$$
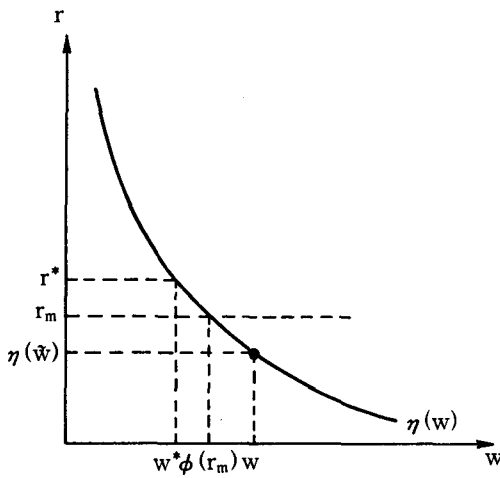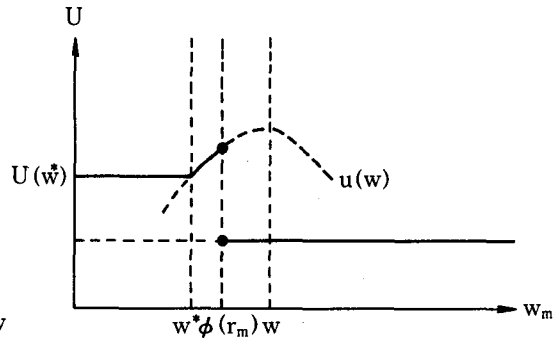
Figure 5A $r^* > r_m \geqq \eta(\tilde{w})$ Fibure 5B

What remains is the case in which r_m is set below $\eta(\tilde{w})$. By making a similar argument to above, we can obtain the graph of U as in Figure 6, from which we see that when $\eta(\tilde{w}) > r_m$
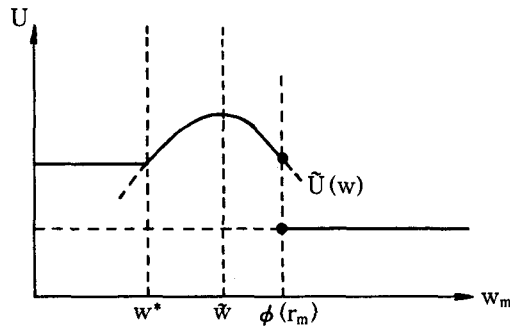
Figure 6 $\eta(\tilde{w}) > r_m$

$$\underline{w}(r_m) = \tilde{w} \tag{23}$$

Taking (19), (22) and (23) together, we obtain the graph of $\underline{w}(r_m)$ as depicted in Figure 7. Note that $\underline{w}(r_m)$ is a correspondence for $r_m \geqq r^*$. Moreover, we can easily verify that
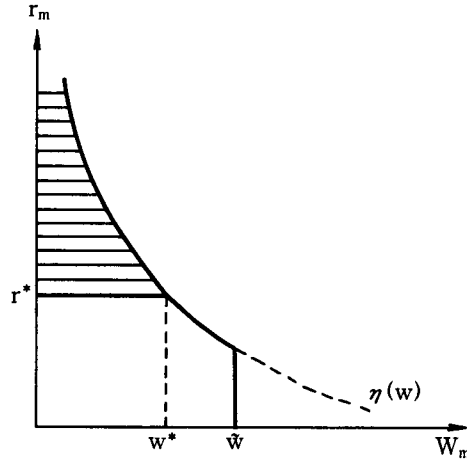
Figure 7 $\underline{W}(r_m)$

$$(r, w) = \left\{ \begin{array}{l} (r_m, \phi(r_m)) \text{ for } r_m \geqq \eta(\tilde{w}) \\ (\eta(\tilde{w}), \tilde{w}) \text{ for } r_m < \eta(\tilde{w}) \end{array} \right\} \tag{24}$$

The derivation of $\underline{r}(w_m)$

A similar argument to what we have made brings Figures 8A and 8B. The difference between these figures comes from whether the counterpart of $\tilde{U}(w)$,

$$\tilde{V}(r) \equiv \frac{rc_r(\phi(r), r)Nl(\phi(r))}{c_w(\phi(r), r)}$$

, has a peak at some r, say $\tilde{r}$, which is greater than r^*. For example, if the rate of factor substitution is not smaller than one, then $\tilde{V}(r)$ is decreasing. When $\tilde{V}(r)$ is decreasing, we have, from Figure 8A,
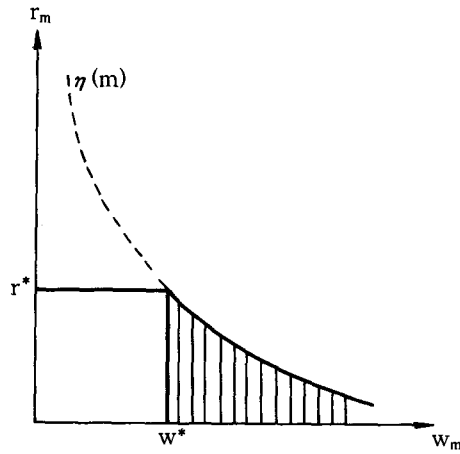
Figure 8A $\tilde{V}(r)$ is decreasing

$$\underline{r}\ (w_m) = \begin{cases} \{r_m \mid 0 \leqq r_m \leqq \eta(w_m)\} & \text{for } r_m > r^* \\ r^* & \text{for } r_m \leqq r^* \end{cases} \tag{25}$$
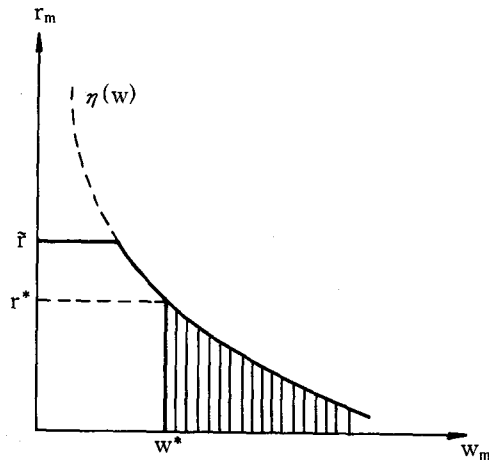
Figure 8B $\tilde{V}(r)$ has a peak

Moreover we can easily see that

$$(r, w) = \begin{cases} (\eta(w_m),\ w_m) & \text{for } w_m \geqq w^* \\ (r^*,\ w^*) & \text{for } w_m < w^* \end{cases} \tag{26}$$

5. Union-ridden Equilibria

Now take a look at Figure 9 which is obtained by putting Figures 7 and 8 together. Inspecting Figure 9, we find the following things.
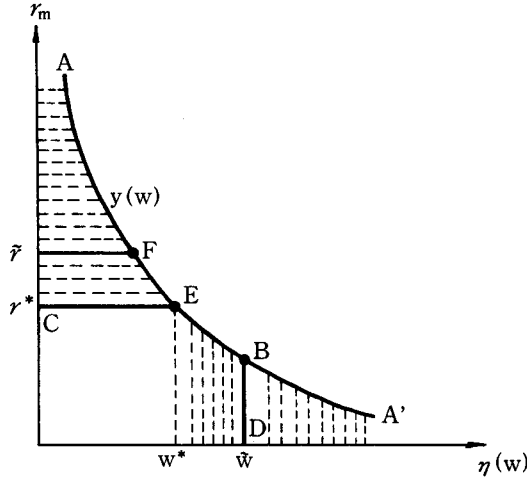
Figure 9 union-ridden equilibria

(i) The Nash noncooperative equilibria exist infinitely. For example, in the case where the graph of $\underline{r}$ (w_m) is like Figure 8A, any point on CEBD represents a Nash equilibrium. However, (r, w) to be realized through market mechanism is common among (r_m, w_m) expressed by the points on CE and BD, respectively: If (r_m, w_m) is on CE (resp. BD), then $(r, w) = (r^*, w^*)$ (resp. $(\eta(\tilde{w}), \tilde{w})$). Thus, if (r_m, w_m) is on CE, unionization does not bring any substantial profit to either capitalists or workers.

(ii) The points on BD represent the Stackelberg equilibria with the labour union acting as the leader. As we discussed at the previous section, the equilibrium wage rate $\tilde{w}$ is determined by the following system of equations.

$$Q \equiv [F_L(x, 1) + xF_{LL}(x, 1)] \, U_c(\frac{K}{N} xF_L(x, 1), 1 - \frac{K}{N} x) \tag{27}$$
$$- U_h(\frac{K}{N} xF_L(x, 1), 1 - \frac{K}{N} x) = 0$$

$$\tilde{w} = F_L(x, 1) \tag{28}$$

where $x \equiv \dfrac{L}{K} = \dfrac{lN}{K}$. Differentiating (27) with x and K/N, we

see

$$(\frac{\partial Q}{\partial x}) \, dx + x \, N \frac{U_h}{U_c} \, (u_{cc}F_L - uc_h) - (u_{hc}F_L - u_{hh}) \} \, d(K/N) = 0$$

where $(\dfrac{\partial Q}{\partial x})$ is negative. Therefore,

$$\frac{dx}{d(K/N)} = - \frac{x}{(\frac{\partial Q}{\partial x})} \, \{\frac{U_h}{U_c} \, (u_{cc}F_L - uc_h) - (u_{hc}F_L - u_{hh})\} < 0$$

It follows from (28) that the increase in K/N lowers w̃: We obtain the following statements:

Capital accumulation and/or decrease in membership lower the working hours per unit of capital and the wage rate, and raise the rental rate. Capitalists are better-off, while workers worse-off.

The last statement may need a proof. Considering (27), we see

$$\frac{du}{d(K/N)} = \frac{\partial u}{\partial(K/N)} = x \cdot [u_c F_L - u_h] \, \frac{dx}{d(K/N)}$$

$$= - x^2 F_{LL} \cdot \frac{dx}{d(K/N)} < 0$$

6. A Pareto-optimum Locus

In this section we shall obtain a Pareto-optimum locus on the (r, w)-plane. Consider the maximum problem,

$$\max U = u \, (wl, 1-l)$$

$$\text{Sub to } F \, (lN, \delta K) \geqq wlN + r\delta K$$

$$v(r\delta) \geqq V$$

$$1 - l \geqq 0 \quad l \geqq 0$$

$$1 - \delta \geqq 0 \quad \delta \geqq 0$$

$$V \text{ given}$$

It may be intutively obvious that δ is equal to 1. Resorting to the theorem on Kuhn-Tucker conditions[2], we have the system of equations

$$F(lN, K) = wlN + rK$$

$$u_c(wl, 1-l)F_L(lN, K) = u_h(wl, 1-l) \qquad (28)$$

$$v(r) = V$$

, which determines l, w and r.

Recall (14). We can easily verify that the pre-union equilibrium point satisfies (28) if $r = r^*$. Now, differentiating (28) with l, w, r and V, we have

$$\frac{dr}{dw} = -\frac{lN}{K}\left[+\frac{[F_L-w][u_{cc}F_L-u_{hc}]}{\frac{\partial^2}{\partial l^2}u(wl, 1-l)+Nu_cF_{LL}}\right] \qquad (29)$$

Since $F_L - w = 0$ at the pre-union equilibrium point, we see

$$\frac{d_r}{d_w}\bigg|_{(29)} = -\frac{lN}{K} = -\frac{c_w}{c_r} = \frac{dr}{dw}\bigg|_{l=c(w,r)} \qquad (30)$$

at the pre-union equilibrium. Moreover, differentiating (30) with w and initially evaluating it at the pre-union equilibrium,

$$\frac{d^2r}{dw^2}\bigg|_{(29)} - \frac{d^2r}{dw^2}\bigg|_{l=c(w,r)} = \left(-\frac{lN}{K}\right)\cdot\frac{[NF_{LL}\frac{dl}{dw}-1][u_{cc}F_L-u_{hc}]}{[\frac{\partial^2u}{\partial l^2}+N_{uc}F_{LL}]} \qquad (31)$$

, where

$$NF_{LL}\cdot\frac{dl}{dw} - 1 = -\frac{\frac{\partial^2u}{\partial l^2}+NF_{LL}[u_c+l\{u_{cc}F_L-u_{hc}\}]}{\frac{\partial^2u}{\partial l^2}+u_cNF_{LL}} > 0$$

For, from (15), $u_c + l(u_{cc}F_L-u_{hc}) > 0$. Hence, (31) must be negative. We now obtain a Pareto-optimum locus PEP″P′ as is depicted in Figure 10. Moving toward the point P′ along the Pareto-optimum locus, V declines and U rises.

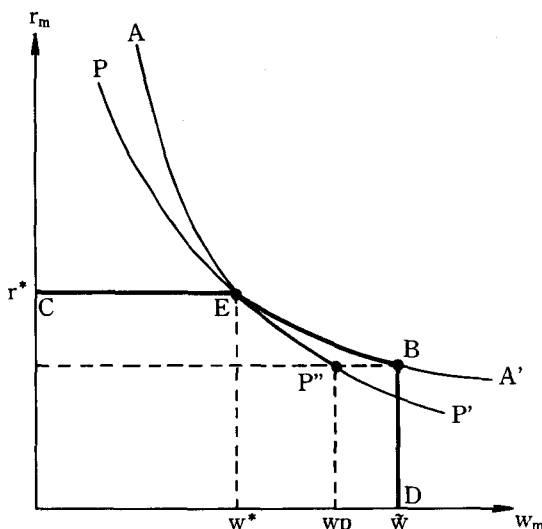2) See, for example, Intriligator (1981, pp. 66-69).

Figure 10   several types of union-ridden equilibria

AEA'	:	$1 = c(w,r)$
PEP"P'	:	the pareto-optimum locus
E	:	the pre-union equi bigium
BD	:	the Stackelberg eguilibia with the leboun
		union acting as the leader
CE	:	the Stackelbeng equilibia with the capitalists'
		club acting as the leader
CEBD	:	the Nash incouperative equilibia
EP"	:	the candidetes for the Nash cooperative
		epuilibia

Figure 10 is the final product in this paper, where several types of union-ridden equlibriu as well as the pre-union equilibrium are shown together. Note that only a portion of the Pareto-optimum locus, $P''E$, represents the condidates for the cooperative equilibria: Any point on either portion of the locus, PE or $P''P'$, cannot be a cooperative equilibrium. For, at any point on PE (resp. $P''P'$) except E (resp. P'') capitalists (resp. workers) would give up forming a capital (resp. labour) union.

7. A Concluding Remark

How can we single out a cooperative equilibrium point on EP'' in Figure 10? Following the literature on cooperative game, one can apply a Cobb-Douglas function defined over pay-off space and obtain the cooperative equilibrium point as the maximizer of the function on the pay-off frontier like $PEP''P'$ in Figure 10: The constant elasticities of the function represent the degree of the differentials in bargaining skills between

two players. See, for example, Ohyama (1990). Using his term, the greater the measure of bargaining skill[3] of the labour union (resp. the capitalists' club) relative to that of the capitalists' club (resp. labour union) the closer to the point P" (resp. E) in Figure 10 the Nash cooperative equilibrium is. In fact, if the measure of bargaining skill of the labour union (resp. the capitalists' club) is sufficiently great, capitalists (resp. workers) give up unionizing themselves and the Stackelberg equilibrium with the labour union (resp. the capitalists' club) acting as the leader is established: The difference in bargaining power between the two groups determines whether our model is the monopoly model, the efficient bargain model or the pre-union model.[4]

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3) Ohyama (1990, p. 3).
4) The first two terms are from Oswald (1985, p. 166).

INDUSTRIAL POLICY UNDER INTERNATIONAL OLIGOPOLY

Kenji KOJIMA

Abstract

This paper considers the industrial policy such as product development subsidy in international oligopolistic market. A three-stage game in which the government is the first mover to influence equilibrium outcomes of oligopolistic firms is constructed. It is found that subsidization of the government crucially depends on the slopes of reaction curves in outputs and product development. It is also shown that the government with the multinational firm could bring the asymmetric equilibrium outcomes.

1. Introduction

Governments in a number of countries subsidize research and development activities of their domestic firms which are in competition with other countries' firms in international markets.[1] This paper presents a theory to explain industrial policies in the international oligopolistic market where the product development rivalry between firms plays an important role.

We focus on subsidization of product development such as, for example, Japanese and the U.S.A. subsidization of semiconductors. Product development is assumed to be undertaken before the output is produced, with firms anticipating the effect of product development on their demands. The government is assumed to commit credibly itself to product development subsidies before the product development decisions are made by the oligopolistic firms. The government becomes the first mover in multi-stage game and can influence the equilibrium outcome of the game played by private firms by changing the set of credible actions taken by them.

The model of firm behavior is based on a two-stage game played by two competing firms, one is multinational firm, the other domestic firm. The multinational firm produces for two national markets. They are assumed to be registered in different countries. In the first stage firms choose product development levels, and in the second stage, output levels. The second stage equilibrium is a Nash equilibrium in outputs, taking

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1) Trade and industrial policy under oligopoly has recently studied by the followings; Dixit (1984), Eaton and Grossman (1986), Spencer and Brander (1983), and others.

product development levels as given by the preceding stage. This gives rise to a subgame perfect equilibrium in the two-stage game.

This two-stage game is extended in the following ways. The both governments are allowed to simultaneously set such product development subsidies. In the extended game, we examine the subgame perfect equilibrium.

The main results are as follows. The government with multinational firm will subsidize if increase in other firm's output lowers the marginal profitability of its own country's firm. On the other hand, the government with domestic firm will subsidize if increase in other firm's output raises the marginal profitability of its own country's firm. Thus subsidization of the government crucially depends on the slopes of reaction curves in outputs and product development. The government with the multinational firm could bring the asymmetric equilibrium outcomes.

Section 2 contains the model, the government is introduce and the main results are developed in Section 3, and Section 4 contains the concluding remarks.

2. The Model

We begin by analyzing the last stage in the firms' rivalry. Each firm i produces output y^i at variable cost c^i, which includes all cost except product development and earns revenue R^i. The product development level of firm i is denoted x^i and costs r^i per unit. A subsidy per unit of product development is denoted s_i. It is assumed that each variable belongs to the real line and R^i and c^i are twice continuously differentiable in their arguements. Profit of multinational firm m and domestic firm d are then

$$\pi^m (y^m_1, y^m_2, y^d; x^m, s_1) = R^m_1 (y^m_1; x^m) +$$
$$R^m_2 (y^m_2, y^d; x^m) - c^m (y^m_1, y^m_2) - (r^m - s_1) x^m, \tag{1.a}$$

$$\pi^d (y^m_2, y^d; x^d, s_2) = R^d_2 (y^m_2, y^d; x^d) - c^d (y^d) - (r^d - s_2) x^d, \tag{1.b}$$

where the subscript denotes the market $j=1, 2$.
Each profit function is also assumed to be strictly concave in its arguements everywhere.

Outputs y^m_2 and y^d are substitutes. Using subscripts to denote derivatives, this implies

$$\pi^m_3 < 0, \ \pi^d_1 < 0. \tag{2}$$

The effect of an increase in product development level is to induce R^i given y^i, and the rate of increase declines as x^i increases:

$$R^i_x > 0; \ R^i_{xx} < 0. \tag{3}$$

The Nash equilibrium in output is characterized by first order conditions

$$\pi^m_1 = \partial R^m_1/\partial y^m_1 - \partial c^m/\partial y^m_1 = 0, \tag{4.a}$$

$$\pi^m_2 = \partial R^m_2/\partial y^m_2 - \partial c^m/\partial y^m_2 = 0, \tag{4.b}$$

$$\pi^d_2 = \partial R^d_2/\partial y^d - \partial c^d/\partial y^d = 0, \tag{4.c}$$

and second order conditions

$$\pi^m_{11} = \partial^2 R^m_1/\partial y^m_1 \partial y^m_1 - \partial^2 c^m/\partial y^m_1 \partial y^m_1 < 0, \tag{5.a}$$

$$\pi^m_{22} = \partial^2 R^m_2/\partial y^m_2 \partial y^m_2 - \partial^2 c^m/\partial y^m_2 \partial y^m_2 < 0, \tag{5.b}$$

$$\pi^d_{22} = \partial^2 R^d_2/\partial y^d \partial y^d - \partial^2 c^d/\partial y^d \partial y^d < 0. \tag{5.c}$$

We also assume that own effects of output on marginal profit dominate cross effects, giving rise to following condition:

$$A = \pi^m_{11} \pi^d_{22} - \pi^m_{13} \pi^d_{12} > 0, \tag{6.a}$$

$$B = \pi^m_{22} \pi^d_{22} - \pi^m_{23} \pi^d_{12} > 0, \tag{6.b}$$

$$C = \pi^m_{11} \pi^m_{22} - \pi^m_{21} \pi^m_{12} > 0, \tag{6.c}$$

$$D = \pi^m_{11}(\pi^m_{22} \pi^d_{22} - \pi^m_{23} \pi^d_{21}) - \pi^m_{21}(\pi^m_{12} \pi^d_{22} - \pi^m_{13} \pi^d_{21}) < 0. \tag{6.d}$$

These conditions ensure uniqueness and locally strict stability of equilibrium.

The solutions $y^m_1{}^*$, $y^m_2{}^*$, and y^{d*} to (4) depend on x^m, x^d and can be written as

$$y^m_1{}^* = q^m_1(x^m, x^d); \ y^m_2{}^* = q^m_2(x^m, x^d); \ y^{d*} = q^d(x^m, x^d). \tag{7}$$

The reaction functions $G^m_j(y^d)$, $G^d(y^m_2)$, $j = 1, 2$ is characterized:

$$\pi^m_1 (G^m_1 (y^d), y^d) = 0, \tag{8.a}$$

$$\pi^m_2 (G^m_2 (y^d), y^d) = 0, \tag{8.b}$$

$$\pi^d_2 (G^d (y^m_2), y^m_2) = 0. \tag{8.c}$$

The slope is obtained by differentiating (8):

$$G^{m}_{1}{}'(y^{d}) = -\pi^{m}_{13}(G^{m}_{1}(y^{d}), y^{d})/\pi^{m}_{11}(G^{m}_{1}(y^{d}), y^{d}), \qquad (9.a)$$

$$G^{m}_{2}{}'(y^{d}) = -\pi^{m}_{23}(G^{m}_{2}(y^{d}), y^{d})/\pi^{m}_{22}(G^{m}_{2}(y^{d}), y^{d}), \qquad (9.b)$$

$$G^{d'}(y^{m}_{2}) = -\pi^{d}_{12}(G^{d}(y^{m}_{2}), y^{m}_{2})/\pi^{d}_{22}(G^{d}(y^{m}_{2}), y^{m}_{2}). \qquad (9.c)$$

We thus have sign $(G^{m}_{1}{}') = $ sign (π^{m}_{13}); sign $(G^{m}_{2}{}') = $ sign (π^{m}_{23}); sign $(G^{d'}) = $ sign (π^{d}_{12}).

The reaction curve is upward sloping if π^{m}_{13}; π^{m}_{23}; $\pi^{d}_{12} > 0$ and downward sloping if π^{m}_{13}; π^{m}_{23}; $\pi^{d}_{12} < 0$. Following Bulow, Geanakoplos, and Klemperer (1985), we will also consider the output of the two firms to be strategic complements if π^{m}_{13}; π^{m}_{23}; $\pi^{d}_{12} > 0$ and strategic substitutes if π^{m}_{13}; π^{m}_{23}; $\pi^{d}_{12} < 0$.

We totally differentiate the first order conditions:

$$\begin{pmatrix} \pi^{m}_{11} & \pi^{m}_{12} & \pi^{m}_{13} \\ \pi^{m}_{21} & \pi^{m}_{22} & \pi^{m}_{23} \\ 0 & \pi^{d}_{21} & \pi^{d}_{22} \end{pmatrix} \begin{pmatrix} dy^{m}_{1} \\ dy^{m}_{2} \\ dy^{d} \end{pmatrix} = - \begin{pmatrix} \pi^{m}_{14}\,dx^{m} \\ \pi^{m}_{24}\,dx^{m} \\ \pi^{d}_{23}\,dx^{d} \end{pmatrix} \qquad (10)$$

We assume that there are joint economies across markets, that is $\pi^{m}_{12} > 0$ and demands in the two markets are unrelated, that is $\pi^{m}_{13} = 0$. Condition (3) implies that π^{m}_{14}; π^{m}_{24}; $\pi^{d}_{23} > 0$. Then we obtain that

$$dy^{m}_{1}/dx^{m} = \{-\pi^{m}_{14}B + \pi^{m}_{24}\pi^{m}_{12}\pi^{d}_{22} - \pi^{d}_{23}\pi^{m}_{12}\pi^{m}_{23}\,dx^{d}/dx^{m}\}/D > 0$$
$$\text{if sign } (\pi^{m}_{23}) = \text{sign } (dx^{d}/dx^{m}), \qquad (11.a)$$

$$dy^{m}_{2}/dx^{m} = \{(-\pi^{m}_{11}\pi^{m}_{24} + \pi^{m}_{14}\pi^{m}_{21})\pi^{d}_{22} - \pi^{m}_{23}\pi^{d}_{23}\,dx^{d}/dx^{m}\}/D > 0$$
$$\text{if sign } (\pi^{m}_{23}) = \text{sign } (dx^{d}/dx^{m}), \qquad (11.b)$$

$$dy^{d}/dx^{m} = \{-C\pi^{d}_{23}\,dx^{d}/dx^{m} + (\pi^{m}_{11}\pi^{m}_{24} - \pi^{m}_{14}\pi^{m}_{21})\pi^{d}_{21}\}/D > 0$$
$$\text{if sign } (\pi^{d}_{21}) = \text{sign } (dx^{d}/dx^{m}) > 0,$$

$$dy^{d}/dx^{m} < 0$$
$$\text{if sign } (\pi^{d}_{21}) = \text{sign } (dx^{d}/dx^{m}) < 0, \qquad (11.c)$$

$$dy^{m}_{1}/dx^{d} = \{-(\pi^{m}_{14}B - \pi^{m}_{24}\pi^{m}_{12}\pi^{d}_{22})\,dx^{m}/dx^{d} - \pi^{d}_{23}\pi^{m}_{12}\pi^{m}_{23}\}/D > 0$$
$$\text{if sign } (\pi^{m}_{23}) = \text{sign } (dx^{m}/dx^{d}) > 0,$$

$$dy^{m}_{1}/dx^{d} < 0$$
$$\text{if sign } (\pi^{m}_{23}) = \text{sign } (dx^{m}/dx^{d}) < 0, \qquad (11.d)$$

$$dy^m_2/dx^d = \{ -(\pi^m_{11}\,\pi^m_{24} - \pi^m_{21}\,\pi^m_{14})\,\pi^d_{22}\,dx^m/dx^d + \pi^m_{11}\,\pi^m_{23}\,\pi^d_{23}\}/D > 0$$
$$\text{if sign } (\pi^m_{23}) = \text{sign } (dx^m/dx^d) > 0,$$

$$dy^m_2/dx^d < 0$$
$$\text{if sign } (\pi^m_{23}) = \text{sign } (dx^m/dx^d) < 0, \tag{11.e}$$

$$dy^d/dx^d = \{ -\pi^d_{23} + (\pi^m_{11}\,\pi^m_{24} - \pi^m_{21}\,\pi^m_{14})\,\pi^d_{21}\,dx^m_{21}\,dx^d\}/D > 0$$
$$\text{if sign } (\pi^d_{21}) = \text{sign } (dx^m/dx^d). \tag{11.f}$$

The comparative static results show that a firm's equilibrium level of output is increasing in own product development level if the slopes of reaction curves of the product development and outputs are the same. It is decreasing in the other firm's product development if the slopes of reaction curves of the product development and outputs are downward sloping, that is, strategic substitutes and increasing if they are strategic complements.

We now proceed to the first stage, in which firms choose product development levels. Firms are aware of the dependence of output on product development levels. Thus, profit can be written as functions of x^m, x^d, ϕ^i represent the profit function for firm i.

$$\phi^m (x^m,\ x^d\,;s_1) \equiv \pi^m (q^m_1 (x^m,\ x^d),\ q^m_2 (x^m,\ x^d),$$
$$q^d (x^m,\ x^d)\,;x^m,\ s_1) \tag{12.a}$$

$$\phi^d (x^m,\ x^d\,;s_2) \equiv \pi^d (q^m_2 (x^m,\ x^d),\ q^d (x^m,\ x^d)\,;x^d,\ s_2) \tag{12.b}$$

The Nash equilibrium in product development levels is characterized by the first order conditions for each firm using (1), (4), (12).

$$\phi^m_1 = \partial R^m_2/\partial q^d \cdot \partial q^d/\partial x^m - (r^m - s_1) = 0, \tag{13.a}$$

$$\phi^d_2 = \partial R^d_2/\partial q^m_2 \cdot \partial q^m_2/\partial x^d - (r^d - s_2) = 0, \tag{13.b}$$

and second order conditions

$$\phi^m_{11} < 0, \tag{14.a}$$

$$\phi^d_{22} < 0. \tag{14.b}$$

We also assume the condition

$$E = \phi^m_{11}\,\phi^d_{22} - \phi^m_{12}\,\phi^d_{21} > 0. \tag{15}$$

The reaction functions $H^m(x^d)$, $H^d(x^m)$ is characterized:

$$\phi^m_1(H^m(x^d), x^d) = 0, \tag{16.a}$$

$$\phi^d_2(H^d(x^m), x^m) = 0. \tag{16.b}$$

The slope is obtained by differentiating (16):

$$H^{m'}(x^d) = -\phi^m_{12}/\phi^m_{11}, \tag{17.a}$$

$$H^{d'}(x^m) = -\phi^d_{21}/\phi^d_{22}. \tag{17.b}$$

We thus have sign $(H^{m'})$ = sign (ϕ^m_{12}); sign $(H^{d'})$ = sign (ϕ^d_{21}).

The reaction curve is upward sloping if $\phi^m_{12} > 0$; $\phi^d_{21} > 0$ and downward sloping if ϕ^m_{12} ; $\phi^d_{21} < 0$. We will also consider product development level of the two firms to be strategic complements if $\phi^m_{12} > 0$; $\phi^d_{21} > 0$ and strategic substitutes if $\phi^m_{12} < 0$; $\phi^d_{21} < 0$.

We totally differentiate the first order conditions:

$$\begin{bmatrix} \phi^m_{11} & \phi^m_{12} \\ \phi^d_{21} & \phi^d_{22} \end{bmatrix} \begin{bmatrix} dx^m \\ dx^d \end{bmatrix} = \begin{bmatrix} -ds_1 \\ -ds_2 \end{bmatrix}. \tag{18.a} \tag{18.b}$$

ϕ^m_{11}, $\phi^d_{22} < 0$ is implied by condition (3). Then we obtain that

$$dx^m/ds_1 > 0 \text{ if sign } (ds_2/ds_1) = \text{ sign } (\phi^m_{12}), \tag{19.a}$$

$$dx^d/ds_1 > 0 \text{ if sign } (ds_2/ds_1) = \text{ sign } (\phi^d_{21}) > 0,$$

$$dx^d/ds_1 < 0 \text{ if sign } (ds_2/ds_1) = \text{ sign } (\phi^d_{21}) < 0, \tag{19.b}$$

$$dx^m/ds_2 > 0 \text{ if sign } (ds_2/ds_1) = \text{ sign } (\phi^m_{12}) > 0,$$

$$dx^m/ds_2 < 0 \text{ if sign } (ds_2/ds_1) = \text{ sign } (\phi^m_{12}) < 0, \tag{19.c}$$

$$dx^d/ds_2 > 0 \text{ if sign } (ds_2/ds_1) = \text{ sign } (\phi^d_{21}). \tag{19.d}$$

This results show that a firm's equilibrium level of product development level is increasing in the subsidy of its own national government if the slope of reaction curves of subsidy and product development are the same. It is increasing in the other country's subsidy if the slopes of those reaction curves are upward sloping, that is, strategic complements and decreasing if they are strategic substitutes.

3. Industrial Policy

We characterize the subsidies that would maximize the national welfare. The optimal subsidy is found by maximizing net national welfare W^i, which is consumer surplus U^i and the profit of the firm less the cost of the subsidy. W^i and and U^i are assumed to be twice continuously differentiable in their arguements and stricly concave in its arguements everywhere. Each country subsidizes product development of its registered firm independently given the subsidy imposed by the other. The national welfare of each country is

$$W^1 (s_1, s_2) = \{U^1 (y^m_1) - R^m_1\} + \pi^m - s_1 x^m, \qquad (20.a)$$

$$W^2 (s_1, s_2) = \{U^2 (y_2) - R^m_2 - R^d_2\} + \pi^d - s_2 x^d, \qquad (20.b)$$

where $U^1(y^m_1) = \int_0^{y^m_1} p^1 (u) \, du \, ; dU^1/dy^m_1 = p^1 (y^m_1) \, ; U^2 (y_2) = \int_0^{y_2} p^2 (v) \, dv,$
where $y_2 = y^m_2 + y^d \, ; \, dU^2/dy_2 = P^2 (y_2)$
earned by each country when both subsidize product development.
We assume that demand curves are downward sloping, that is, $p_1^1 (y^m_1) < 0;$ $p_1^2 (y_2) < 0.$

The Nash equilibrium gives rise to where the first order conditions

$$W^1_1 (s_1, s_2) = 0, \qquad (21.a)$$

$$W^2_2 (s_1, s_2) = 0. \qquad (21.b)$$

Optimal subsidy is

$$s_1 > 0$$
$$\text{if sign} (\pi^d_{21}) = \text{sign} (\pi^m_{23}) = \text{sign} (dx^m/dx^d) < 0, \qquad (22.a)$$

$$s_2 > 0,$$
$$\text{if sign} (\pi^d_{21}) = \text{sign} (dx^m/dx^d) > 0^{[2]}. \qquad (22.b)$$

Positive subsidies of country 1 are called for if both output and product development level are strategic substitutes. On the other hand, country 2 is required to subsidize if they are strategic complements.

The comparative static results of the two-stage game played by the competing firms and the optimal subsidy imply how the subsidy of the government influences the equilibrium outcome of the game. The increase of subsidy provided by the government with the multinational firm increases product development level of its own,

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2) The detail proof is available from the author.

and reduces that of the domestic firm. If the product development of the multinational firm raises, the output of its own increases and that of the other decreases. If the product development level of the domestic firm lowers, the output of the other increases and that of its own decreases. The optimal product development subsidy of the government with the multinational firm maximizes its national welfare by influencing its own firm to gain more market share. On the other hand, the increase of subsidy by the government with the domestic firm increases product development level of both firms. If the product development of both firms raises, the output of the both increases. The optimal product development subsidy of the government with the domestic firm maximizes its national welfare by bringing neutral effect to the both firms.

4. Concluding remarks

Governments play an important role in certain international industries. Such industries consist of a few firms which are involved in a strategic game. This paper considers the optimal industrial policy maximizing the national welfare under the international oligopolistic setting. We have found that subsidization of the government crucially depends on the slopes of reaction curves in outputs and product development. The optimal product development subsidy of the government with the multinational firm maximizes its national welfare by influencing its own firm to gain more market share. On the other hand, the optimal product development subsidy of the government with the domestic firm brings neutral effect to the both firms.

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REALTIME DISTRIBUTED CONTROL OF WORLD LINK MODELS IN THE INTERNATIONAL COMPUTER NETWORKS

Ko MIYAZAKI

1. Introductions

In recent years, the technology of international data communications has made a remarkable progress, so that we can now make computer-to-computer calls at anytime from any place in the world. This trend has been accelerated further by the fact that powerful portable personal computers have come into wider use as intelligent terminals.

For example, Toshiba's laptop personal computer, J-3100SGT/101 weighing only 6.8 kilograms, is a 32-bits MPU (intel 80386/20MHz) machine with co-processor (intel 80387/20MHz), 4MB RAM and 100MB hard disk. This laptop computer has proven to be capable of solving and simulating large-scale econometric models. More important, this very portability of laptops makes it possible to carry out econometric model simulations in the audience's presence at a conference held in any country.

It might be no wonder that just one laptop may not be able to solve such a huge econometric model as the PROJECT-LINK Model[1] with 70 countries involved, but a combination of two or more laptops together could make it well. This can be done in reality by building a kind of parallel processing system on a local area network (LAN) of laptops. In addition, if international data communication functions are equipped with in this system, econometricians can get the most up-to-date models or data easily and directly from various econometric database centers around the world.

With this system, realtime model simulation could be realized at any place at any time — at the conference room of "PROJECT-LINK", "G7" or "Summit", for example. The aim of this paper[2] is to present the concept of a realtime econometric model simulation system[3].

2. The concept of a ECONET

"ECONET" stands for "ECOnomic model simulation NETwork system." It is an international information network system to enable econometricians to carry out realtime econometric model simulations on personal computers at any place in the world.

The essence of ECONET is as follows:

1) Distributed processing system to solve large-scale econometric models by combining several laptops together.

2) Econometric database system to provide econometric models and macro economic data.

3) International data communication system to connect those systems mentioned above.

The concept of "ECONET" is shown in Figure 1.
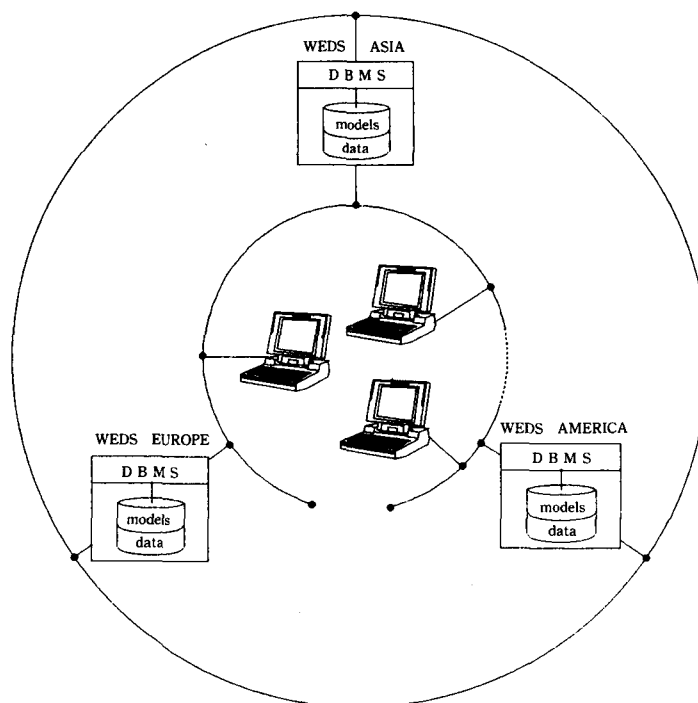
Figure 1 The concept of "ECONET"

1) Distributed processing system:

This system is at the core of ECONET. It makes it much easier for econometricians of each country to control their country's models and data during simulation of the WORLD LINK MODEL, because, unlike the centralized processing where the equations of all countries are treated as those of one big country, the models of each country are processed in parallel and independently of those of the other.

The advantage of this distributed model simulation lies not only on controllability of models but also on costs. Running costs and communication costs are much cheaper in this distributed processing method on laptops than in the centralized processing way on a very large scale computer.

2) Econometric database system:

This is a database service system to provide econometricians with models and data whenever the need arises. This system should be equipped with a function of transferring text files or binary files stored in the database directly into the disk storage of terminals.

It is also desirable that every country or area have a database system centering on econometric models as well as economic data of its own country or area.

3) International data communication system:

This system may be part of the econometric database system mentioned above. The data stored in the database system should be easily accessible from terminals of the distributed processing system through telecommunication lines such as public telephones or public data communication networks.

3. Distributed Link Model Simulation System

The distributed link simulation method was proposed by Professor Ippei Sugiura (Osaka Sangyo University)[4] and the system has been developed by him together with Professors Mamoru Shibayama (Osaka International University) and Yasuo Nunokami (Konan University).

This system was originally designed for the purpose of transplanting the PROJECT-LINK model simulation programs developed for large-scale computers to much less expensive smaller machines.

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4) Ippei Sugiura, Distributed Control of Econometric Data Bases and Models, in Duncan Ironmonger et al.(ed.), National Income and Economic Progress, Macmillan Press, GBR, 1988

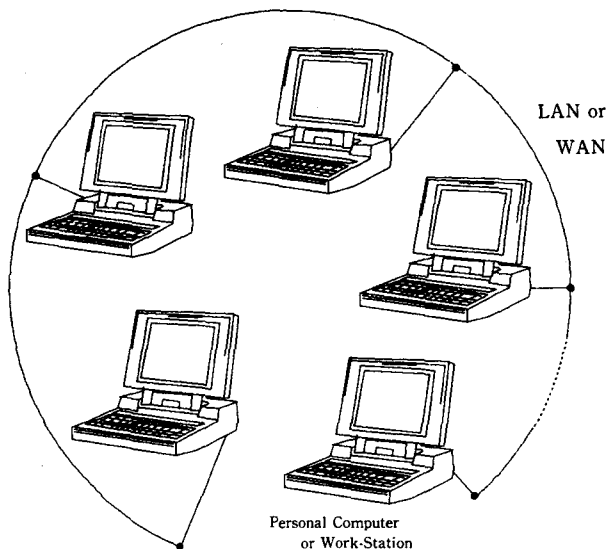The concept of the Distributed Link Model Simulation System is shown in Figure 2.

Figure 2 The concept of the Distributed Link Model Simulation System

The system is composed of two or more personal computers (or work-stations), combined with one another to form a LAN (Local Area Network) system. The distributed link simulation method is to allocate one or more models to each personal computer and to run the world link model on the LAN of several personal computers in the way of parallel processing. The personal computer assigned the trade matrix model plays a supervisory role and controls the flow of the simulation process.

4. World Economic Database System

This Distributed Link Model Simulation System made up of several personal laptop computers, easy to take along with, would be able to be brought in the conference room wherever it may be held, and a world link model simulation could be conducted on the spot.

What would happen if new models or data should be needed for continued simulation? To prepare against this situation and to provide all necessary information whenever needed from overseas or at home econometric database centers should be established in major countries in the world. Then, econometricians can access the database system to obtain models or data from any place at any time whenever needed.

In other words, any econometric database system should be accessible from any place in the world through telecommunication lines.

KOBE university World Economic Database System (KOBE-WEDS) aims to be one of such econometric database centers. A network of "KOBE-WEDS" is shown in Figure 3.
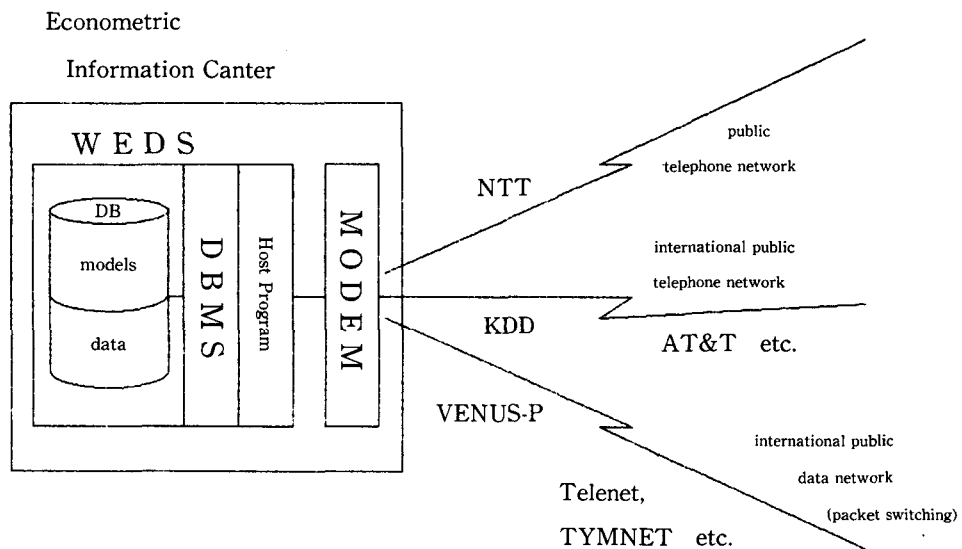
Figure 3 A network of "KOBE-WEDS"

There are three ways to communicate with KOBE-WEDS. Most commonly used are the public telephone networks for domestic calls and the international public telephone networks for international long distance calls. The third way is the international public data networks for international packet-switching digital data communications — "Telenet" provided by GTE and "TYMNET" by McDonnell Douglas, for example.

A sample of the operational procedure of "KOBE-WEDS" is shown in Figure 4.

```
+--------------------------------------------------------------------------+
|              - Welcome to KOBE WORLD ECONOMIC DATABASE SYSTEM! -          |
+--------------------------------------------------------------------------+
                                    ↓
+--------------------------------------------------------------------------+
| Menu No.000 - KOBE WORLD ECONOMIC DATABASE SYSTEM [KOBE-WEDS] -           |
|          1  HOW TO USE KOBE-WEDS                                          |
|          2  WORLD ECONOMIC DATA & ECONOMETRIC MODELS                      |
|          3  COMMUNICATION AMONG USERS                                     |
|          4  PROGRAMS & DATA TRANSFER AMONG USERS                          |
|          5  UTILITIES                                                     |
|          6  LOGOUT (qq)                                                   |
| YOU MAY COME BACK TO THIS TOP MENU ANYTIME BY TYPING '/'.                 |
| ENTER CHOICE NUMBER OR COMMAND: 2                                         |
+--------------------------------------------------------------------------+
                                    ↓
+--------------------------------------------------------------------------+
|  Menu No.200 - WORLD ECONOMIC DATA & ECONOMETRIC MODELS -                 |
|          1  WRITE A BULLETIN OR SAVE DATA/MODELS IN WEDS                  |
|          2  READ BULLETINS OR DOWNLOAD DATA/MODELS FROM WEDS              |
|          3  DELETE BULLETINS OR DATA/MODELS IN WEDS                       |
|          4  CHANGE PROTOCOLS (XMODEM, YMODEM OR YMODEM-BATCH)             |
|          5  QUIT (q)                                                      |
| ENTER CHOICE NUMBER OR COMMAND: 2                                         |
+--------------------------------------------------------------------------+
                                    ↓
+--------------------------------------------------------------------------+
|    - READ BULLETINS OR DOWNLOAD DATA/MODELS FROM WEDS -                   |
| 1. BULLETIN         2. ASIAN LINK MODEL   3. US-CHINA-JAPAN MODEL         |
| 4. US-JAPAN MODEL   5. TOOLS                                              |
| H: HELP  Q: QUIT                                                          |
| ENTER CHOICE NUMBER OR COMMAND: 2                                         |
+--------------------------------------------------------------------------+
                                    ↓
+--------------------------------------------------------------------------+
|                      - ASIAN LINK MODEL -                                 |
|        IP              DATE         TITLE                                 |
|    1.  KOBE  WEDS      89/11/24     Asian Link        (Model/Data)        |
|    2.  KOBE  WEDS      89/11/24     Trade             (Model/Data)        |
|    3.  KOBE  WEDS      89/11/24     Rest Of the World (Model/Data)        |
|    4.  KOBE  WEDS      89/11/24     Thailand          (Model/Data)        |
|    5.  KOBE  WEDS      89/11/24     Singapore         (Model/Data)        |
|    6.  KOBE  WEDS      89/11/24     Philippines       (Model/Data)        |
|    7.  KOBE  WEDS      89/11/24     Malaysia          (Model/Data)        |
|    8.  KOBE  WEDS      89/11/24     Indonesia         (Model/Data)        |
|    9.  KOBE  WEDS      89/11/24     Hongkong          (Model/Data)        |
|   10.  KOBE  WEDS      89/11/24     Taiwan            (Model/Data)        |
|   11.  KOBE  WEDS      89/11/24     Korea             (Model/Data)        |
|   12.  KOBE  WEDS      89/11/24     USA               (Model/Data)        |
|   13.  KOBE  WEDS      89/11/24     Japan             (Model/Data)        |
| H: HELP  Q:QUIT                                                           |
| ENTER CHOICE NUMBER OR COMMAND:                                          |
+--------------------------------------------------------------------------+
                                    ⋮
```

Figure 4   A sample of the operational procedure of "KOBE-WEDS"

5. Realtime Model Simulation System

This is an integrated system combining the Distributed Link Model Simulation System and the World Economic Database System with the telecommunication system described earlier. Using this system the user can carry out simulations of world link models, however large they might be, at any place in the world while models and/or data necessary for simulation are being provided through telecommunication lines whenever needed.

It is to be noted that this realtime simulation system should have such facilities in the distributed model simulation system that the old models or data may be replaced by the new ones downloaded from some database centers easily to continue the simulation. This is a very important point of the "realtime" model simulation.

The concept of the Realtime Model Simulation System is shown in Figure 5.
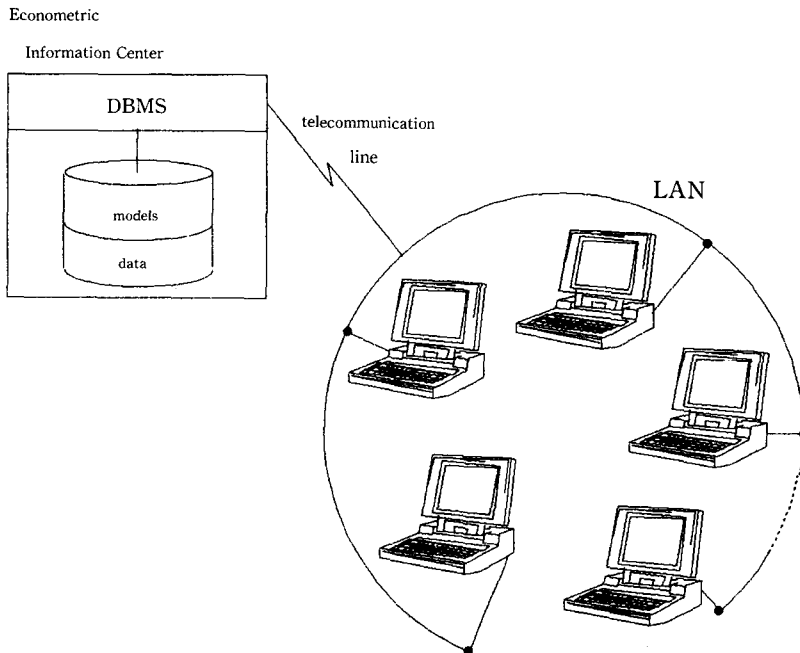
Figure 5 The concept of the Realtime Model Simulation System

6. Experiments of Realtime Model Simulation System
− In case of the "Econometric Model of Asian Link" −

The "Econometric Model of Asian link"[5] is an econometric link system for 11 areas − Japan, the United States, eight of the East and Southeast Asian countries, and the rest of the world. This model consists of 550 equations in all including the trade models.

The details of equations are listed in Table 1.

Table 1 Econometric Model of Asian Link, Number of Equations

Model	Number of Equations
Japan	61
U.S.A.	33
Korea	36
Taiwan	38
Hongkong	46
Indonesia	85
Malaysia	62
Philippines	25
Singapore	49
Thailand	55
Rest Of the World	26
Trade	34
Total	550

The procedure flow of the Realtime Model Simulation is shown in Figure 6.

---

5) Shinichi Ichimura and Mitsuo Ezaki ed., Economic Models of Asian Link, Springer-verlag, 1985. Mitsuo Ezaki, Mamoru Shibayama and Shinichi Ichimura, An Economic Link System for the East and Southeast Asian Countries, Japan and the United States, Southeast Asian Studies, Vol.22, No.3, 1984.

Start

↓

Step 1 Connect to Econometric Information Center.
Start the terminal program. Select a communication network. (If data network was selected, Login to the network and set communication parameters.) Login to an Econometric Information Center.

↓

Step 2 Download models and/or data from Econometric Information Center.
Select models and/or data items. (Change communication parameters, if necessary.) Send data to the terminal. Receive data from the center.

↓

Step 3 Terminate communication process.
Logout from the Econometric Information Center. (Logout from the network, if necessary.) Quit the terminal program.

↓

Step 4 Decompress the downloaded file.
Decompress the downloaded file. Transfer the data to simulation system.

↓

Step 5 Initialize Distributed Model Simulation System.
Initialize the LAN system. Initialize a machine including trade models first. Initialize the other machines.

↓

Step 6 Start Simulation.
Start the machine with the trade models. Start the other machines.

↓

End

Figure 6   The flow of the Realtime Model Simulation

Two different experiments were undertaken. The first experiment was given at Stanford University on November 3, 1988 and the second at East West Center in Hawaii on January 12, 1990.

At Stanford University, we gave a demonstration of the "Distributed Link Model Simulation System" (shown at step 5 and step 6 in Figure 6). Three laptop computers (Toshiba's model J-3100SGT/041) were connected by LAN (Simple Corporation's 10-NET) and the link model is divided into 3 groups — Japan, U.S.A. and the others — each of which was assigned to one laptop computer. Then the final test was simulated for 9 years from 1972 to 1980. In this case, the time needed to simulate was about 3 minutes[6]. The experiment system is shown in Figure 7.
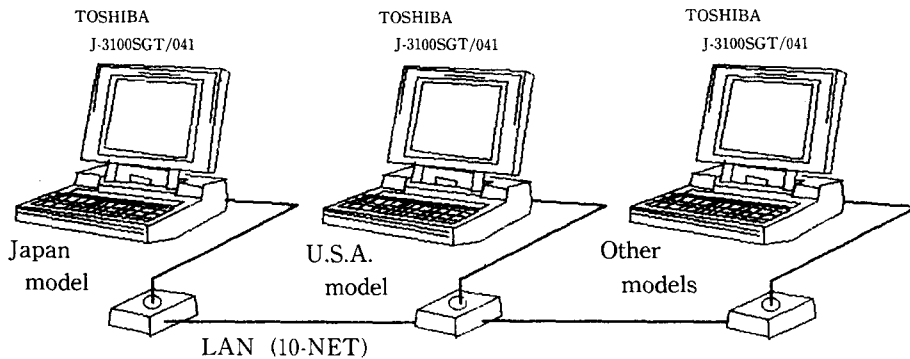
Figure 7 The experiment system at Stanford University

At East West Center, we showed an experiment of the Realtime Model Simulation System. The experiment system is shown in Figure 8.

---

6) The time needed to carry out this simulation by using just one laptop computer was about 40 seconds.

HITACHI

Work-Station 2050/32
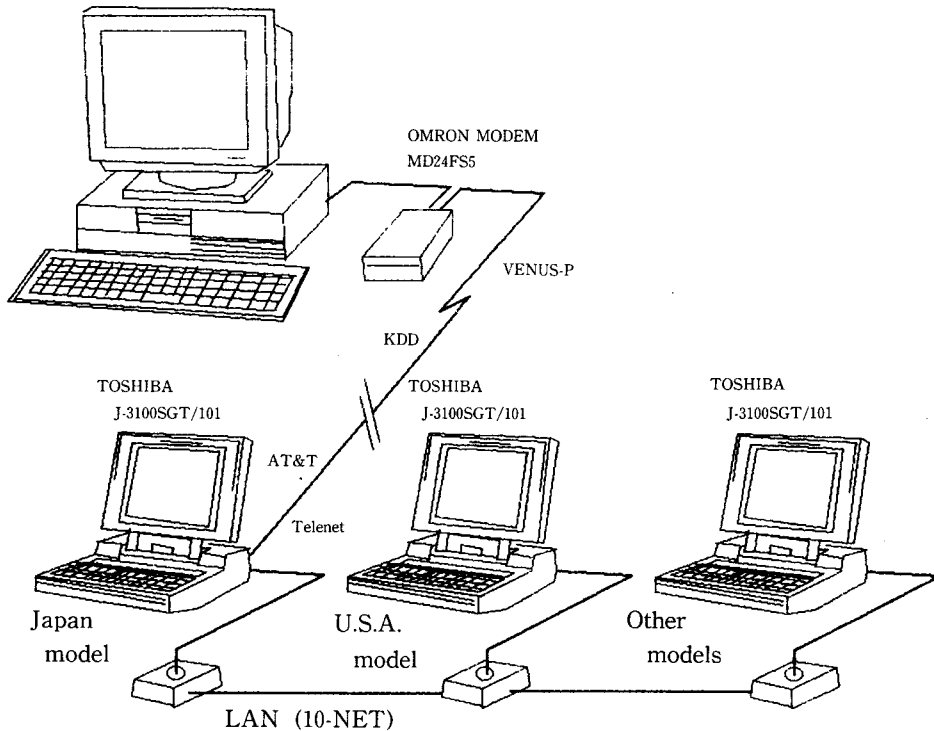
Figure 8 The experiment system at East West Center

The order of the demonstration is as follows:

1) Installing the models except Japan in every laptop computer.

2) Downloading a Japan model from KOBE-WEDS into the disk files of the laptop computer through a international public telephone network.

3) Simulating the entire Asian Link Model.

In this case, the time needed to download the Japan model (file size: 6,656 bytes[7]) was about 1 minute, that is to say, about 1 second needed to transfer one data block (128 bytes) in the XMODEM protocol.

We also made the same experiment with success by using "Telenet", one of the international public data network services.

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7) This file is compressed. The original file size is 81,408 bytes.

KOBE ECONOMIC & BUSINESS REVIEW

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KOBE UNIVERSITY ECONOMIC REVIEW 35, 1989

Edited by Faculty of Economics,
Kobe University, Kobe, Japan

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Edited by the School of Business Administration,
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RESEARCH INSTITUTE FOR ECONOMICS AND BUSINESS ADMINISTRATION, KOBE UNIVERSITY

HISTORICAL SKETCH

In 1919, a research organization named the Institute for Commerce was founded in Kobe Higher Commercial School, one of the chief predecessors of Kobe University, with a gift made by F. Kanematsu & Company, a leading mercantile firm in Kobe. The organization was designed to carry on and facilitate integrated research on business and commerce and to formulate and publish the results of these studies and investigations in such form as to make them available to the business community.

With the founding of Kobe University of Commerce, successor of Kobe Higher Commercial School, in 1929, the Institute extended its research activities by adding several divisions. One was the famous Latin-American Library, which soon became the center of research in this field in Japan. A room for statistics equipped with various computing machines was established and began publication of *Jūyo Keizai Tōkei* and *Sekai Bōeki Tōkei* annually. A filing room was prepared to deposit press clipping files systematically arranged by topics and dates. Another room was designed to become the center of all possible original records and data having to do with the beginning and progress of Japanese business.

On the campus of Kobe University of Commerce, another organization named the Institute for Business Mechanization was founded in 1941 utilizing business machines donated by the IBM Corporation and others. With Professor Yasutaro Hirai as its head a broad and forward-looking plan for business mechanization in Japan was developed.

In 1944, Kobe University of Commerce changed its name to Kobe University of Economics. After the War, however, the University was consolidated with three other colleges in Hyogo Prefecture to become Kobe University. With this development, the two Institutes were also amalgamated into the Research Institute for Economics and Business Administration, Kobe University. At present, the Institute, with its twenty-one full-time professional staff members, carries on studies and investigations in international economics, international environment, international comparative economics, international business, management information systems and international cooperation.

ORGANIZATION

The Institute consists of six sections. Each section and its research subjects are as follows:
  1. **International Economic Studies**
     International Economics, International Monetary Economics,
     Maritime Economics, International Labor Relations
  2. **International Environmental Studies**
     Resource Development, International Organizations,
     International Industrial Adjustment
  3. **International Comparative Economic Studies**
     Pacific Basin I (Oceanian Economy),
     Pacific Basin II (Latin American Economy),
     Pacific Basin III (North American Economy)
  4. **International Business Studies**
     Comparative Business, Multinational Enterprise,
     International Business Finance
  5. **Management Information Systems**
     Business and Accounting Information, Information Processing System,
     International Comparative Statistics
  6. **International Cooperation**
     In addition to the ordinary work of each section, several research committees,

whose members are not limited to the Institute staffs, are regularily held to carry on joint studies. At present, there are ten standing research committees, as follows: Committee of International Economics, Committee of Maritime Economics, Committee of Labor Market, Committee of International Studies on Economic and Industrial Structure, Committee of World Link Model, Committee of International Comparative Economics, Committee of International Comparative Monetary Systems, Committee of International Business Finance, Committee of Management and Accounting Information Systems, and Committee of International Comparative Statistics.

For convenience and greater efficiency in carrying out its research activities, the Institute has a general office which is responsible for 1) the collection and preservation of a comprehensive collection of books, periodicals, pamphlets, and original records and data of finance, trade, commerce, industry and business generally; 2) the classification, cataloguing, indexing, arranging, annotation and compilation of these research materials; and 3) the formulation and publication of the results of the investigations and studies accomplished by the professional staff members of the Institute.

As an affiliated institute, the Documentation Center for Business Analysis has been established in 1964. It is the first systematic information facilities in the field of business administration in Japan that has been recognized and authorized by the Ministry of Education. The purpose is to collect and to make intensive control of all kinds of materials on business administration and to make them available to scholars, universities, governments, and business world with the aid of modern documentation techniques.