KOBE ECONOMIC & BUSINESS REVIEW

24th

ANNUAL REPORT

RESEARCH INSTITUTE FOR ECONOMICS
AND BUSINESS ADMINISTRATION
KOBE UNIVERSITY

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THE DEMAND FOR INTERNATIONAL CURRENCIES AND RESERVES

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We can not find the economy without money in an actual world. In an international economy, therefore, we should start with the analysis of holding international currencies. According to the IMF's definition of liquid foreign reserves, we will define the holdings of international currencies as the sum of the official gold and gold tranche positions, the SDR position, and the holdings of convertible foreign currencies. The rates of foreign exchanges are supposed to be fixed throughout our analysis.

The motives for holding of international currencies, which owe to Keynesian classification, can be classified into three heads: the transactions motive, the precautionary motive, and the speculative motive. In the international economy, however, these motives have to be modified, where we have less developed institutions as compared to our national economy. (a) The transactions motive—one reason for holding of foreign currencies is to bridge the interval between external receipts and payments. The strength of this demand depends on the international balance of payments, and on the discrepancy (the time interval) between the hour we will have external payments and the hour we will receive from foreign countries. (b) The precautionary motive—foreign currencies are held to provide for contingencies requiring urgent imports to meet a bad harvest or other disasters. (c) The speculative motive—we should like to deal with only autonomous international long-term capital movements in this head, because they are related to long-term interest rates as Keynesian speculative demand for domestic money is. Exchange arbitrage or speculation and short-term capital movements are excluded. Especially, hot money is excluded because of its political instability.

1. The Transactions Motive

International currencies are held by a country to bridge the interval between its external receipts and payments. The determinants of this demand, as shown in the following paragraphs, are the time interval between its external payments and receipts, and the mean value and the variance of its daily, statistically independent balance of payments. The international currencies held by the monetary authority are only a reflex of international transactions in the private sectors. Therefore, the monetary authority does not assume to play an important role on the holdings of foreign reserves except in the case of the precautionary motive.

Time is divided into discrete, uniform intervals called "week". The markets, where all contracts are made, are open only on Monday of each week. And the fulfilments of the contracts are carried out on any other days during the week. The net payments due,[1] $\sum(m_\tau - l_\tau)$, consists of the excess payments due for a given hour of the week and the sum of excess payments due for the preceeding hours. Let us assume that k is a maximum random variable representing the net payments due at any given time (T) in the week. If k is bigger than the initial amount of foreign reserves, M, the country cannot fulfil the payment obligations during the week. Therefore, a cumulative possibility function for k, $\pi = P_b(k \leq M)$, is an indicator of security level against possible illiquidity. The level of security depends on the initial holdings of foreign reserves, M, the daily balance of payments, $m_\tau - l_\tau$, and the time interval. The smaller the initial holdings are, the higher the possibility of insolvency will be. The bigger the daily balance of payments or the time interval is, the greater the value of k, and then the risk against insolvency, will be.

All contracts on Monday involve our expectations for daily receipts and our daily payment plans in the following six days during the week. Therefore, the daily balances of payments can be divided into two parts: the statistically independent part and the part affected by the preceeding days' balances of payments on which our expectations and plans are reflected. Figure 1 shows an example of a country's balances of payments

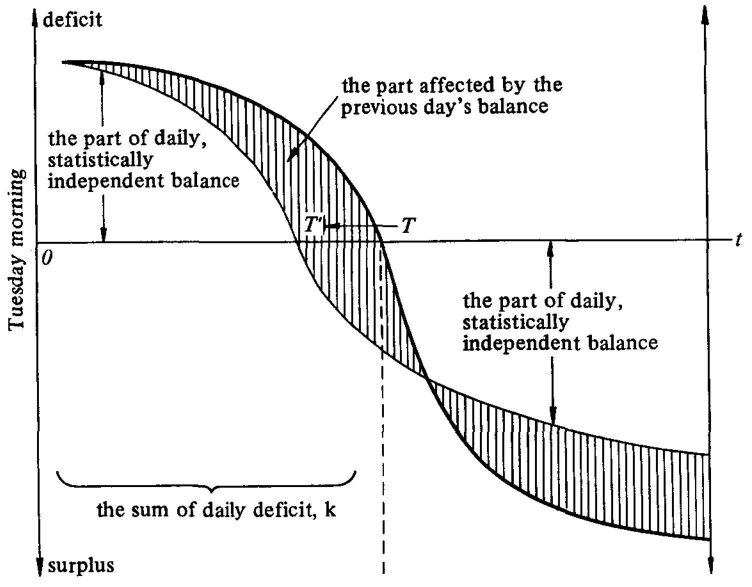

Fig 1 An Example of Daily Balance of Payments in a Week

1) Meinich, P., *A Monetary General Equilibribrium Theory for an International Economy,* 1971.

which are indicated by a thick continuous line. The daily balances of payments start with a deficit on Tuesday and are reversed into a surplus at time T. They are the sum of the statistically independent part and the part (shaded part) affected by the preceeding days' balances of payments. The maximum net payment due, k, which should be paid by initial holdings, M, is equal to the integration of the dialy balances of payments from 0 to T. Suppose the shaded part is proportionate to the previous day's balance of payments at the rate of γ. If this ratio γ is equal to zero, the shaded part will disappear and the reverse point T will be moved to T'. The increase in γ, on the contrary, will make the reverse point delay forward, and the period of deficit balances will be longer. Therefore, the ratio γ can be taken as an effective indicator to show the time interval between the external payments and receipts.

Let us divide the statistically independent part of the daily balances of payments into its expectation and variance, and focus only on the expectation under the usual assumption of the constant variance. The public holdings of foreign reserves are supposed to be a reflex of individuals' holdings in the private sectors. And the distributions of wealth and income in the country are also supposed to be given. On Monday when all contracts are made, an individual or the country can change only the total transactions, m, which will be carried out on the following days during the week. The total amount of receipts, l, is constant because it means the total export value of the country. The reduction of the total amount of payments, m, will decrease its expectation, $E(m)$. And the decrease in $E(m)$ will reduce the expectation of daily transactions, $E(m_\tau)$ or $E(m_\tau - l_\tau)$. If γ is constant, the reduction of $E(m_\tau - l_\tau)$ will decrease the statistically independent part of daily balances of payments. (If the statistically independent part is constant, the ratio γ will decrease.) Consequently, the reduction of total transactions, m, will make the maximum value, $k = \sum (m_\tau - l_\tau)$, be smaller and will improve the level of security, π. Then, the security level may be defined by $\pi = \pi\,(M/P,\ \sigma_u,\ \gamma,\ m/P)$, where P is the price level and σ_u is the standard deviation of daily, statistically independent balances of payments.

The higher the level of security is, the greater the social utility will be increased. Accordingly, π may be inserted into the social utility function, $U = U\,(\pi,\ L/P)$, where L/P is the real holdings of foreign reserves at the end of the week. Maximizing the social utility function, subject to the budget restraint, eq. (3),

$$\frac{1}{P}\ \frac{\partial U}{\partial L/P} - \lambda = 0, \tag{1}$$

$$\frac{1}{P}\ \frac{\partial U}{\partial \pi}\ \frac{\partial \pi}{\partial m/P} - \lambda = 0, \tag{2}$$

$$M = m - l + L. \tag{3}$$

From eqs. (1)–(3), we can get the demand function of international currencies,

$$L = L(\sigma_u, \gamma, M/P). \tag{4}$$

Eq. (1) is related to the Meinich's stock of value which represents the speculative demand for international currencies. Eq. (2), on the other hand, is related to the transactions demand. Therefore, eq. (4) involves two types of demands, speculative and transactions. However, we may conclude that eq. (4) represents explicitly the transactions demand for international currencies because it has the parameters, σ_u, and γ.

An example is found in the Kenen and Yudin's article,[2] which estimated the transactions demand. Kenen and Yudin tested the normality of foreign reserve change ΔR_t, by using a Markov process,

$$\Delta R_t = e + p\Delta R_{t-1} + u_t. \tag{5}$$

The variance of u_t, σ_u, is closely related to the variance of ΔR_t. If the value of $\sigma^2{}_{\Delta R}$ is high, the riskness against insolvency, and then the demand for reserves, will be increased. Therefore, we can say that they estimated our security level, π, in the reverse form, $1-\pi$. In addition, e and σ_u, in eq. (5) are completely identical to our mean and variance of daily, statistically independent balances of payments, because they are the mean and the variance of independent residuals, u_t. On the parameter, p, in eq. (5), however, Kenen and Yudin imposed the constraint, $0 \leq p \leq 1$, and defined it as the "carry-forward" of all past disturbances embodied in $\rho\Delta R_{t-1}$. It is identical to Clark's[3] speed of adjustment for external disequilibrium which must not have any negative value. On the contrary, however, our parameter, γ, in eq. (4) is not the carry-forward, but only a coefficient of response to the previous day's balance of payments which may have negative value. All liabilities occured on Tuesday, for example, must be paid on Tuesday in our model. They must not be transfered to the next day, Wednesday, for we have not any credit organization. Therefore, our γ is only the ratio of today's balance of payments affected to the previous day's balance. When we estimate p in eq. (5), the value of p can take a negative sign. Therefore, we had better not impose the restriction on the sign of p and recognize that it is the estimated value of γ in eq. (4).

Kenen and Yudin estimated the regression coefficients for eq. (6) which depended on the estimated parameters, e, p, and σ_u, per capita income, y, and liquid liabilities, L_t, or domestic money supply, M_t, of 14 countries.

$$R_{it} = \beta_0 + \beta_1 e_i + \beta_2 p_i + \beta_3 \sigma_{ui} + \beta_4 y_i + \beta_5 L_{it} + e_{it}, \tag{6}$$

where i denotes each country and t denotes 1957 or 1962. In this cross-section analysis, p was arbitrarily fixed at zero when the estimated p was negative or not singnificantly

2) Kenen, P. B. and F. B. Yudin, The Demand for International Reserves, *R. E. Stat.*, Vol. 47, Aug. 1965.

3) Clark, P. B., Optimum International Reserves and the Speed of Adjustment, *J. P. E.*, Mar./Apr. 1970.

different from zero in eq. (5). Consequently, they concluded that y and L_t should be eliminated from eq. (6), because the estimated values of β_4 and β_5 were not significantly different from zero. Then, they estimated again for eq. (6′).

$$R_{it} = \beta_0 + \beta_1 e_i + \beta_2 p_i + \beta_3 \sigma_{ui} + e_{it}'. \tag{6'}$$

The variables, y and L_t, which Kenen and Yudin eliminated from eq. (6), are closely related to the store of value described in eq. (1) above. And the estimated regression coefficients, e, p, and σ_u, for the second equation (6′) are related to transactions motive only. Therefore, we can say definitely that they estimated the transactions demand for foreign reserves only.

In their article, the parameters, e and p, were ultimately eliminated from eq. (6′). And the demand for reserves was defined as a function of only σ_u. This elimination of e can be greatly appreciated, for the demand function is similar to our equation (4). But we cannot agree with the elimination of the parameter p. In eq. (4), the parameter γ can take a positive or negative sign. But it had better not be set at zero, because it represents the time interval between the external payments and receipts in the country concerned. We consider, therefore, that the elimination of p is misleading.

We estimated the holdings of foreign reserves at the end of 1972 in Table 1. All data were taken from IMF: *International Financial Statistics.* The normal equation (5), and all other cross-sectional models used in Kenen and Yudin's article were tested again. In estmating e, p, and σ_u, we deleted outliers from the series data by applying Anscombe's procedure[4)] of rejection of outliers to the liquid reserve changes, ΔR_t, for we could not get any available information about political or institutional changes. Consequently, we obtained the similar results about the estimated parameters, e, p, and σ_u, to their table. Table 2–Table 6 show these least-squares estimates e, p, and σ_u, for the various kinds of periods. In Table 2 and Table 3, we used the 181 monthly and 61 quarterly reserve data for the period, December 1957–December 1972. The t-test was satisfied at the 0.05 or 0.1 level of significance in the majority of countries. The similar result was also obtained in Table 4 and Table 6, in which we used the 61 monthly reserve data for two periods, December 1957–December 1962 and December 1967–December 1972. But Table 5, which related to the period, December 1962–December 1967, gave us a different result. The estimated p was significantly different from zero only in five countries such as France, Italy, New Zealand, Sweden and Switzerland. All other countries did not satisfy the t-test.

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4) Anscombe, F. J., The Rejection of Outliers, *Technometrics*, Vol. 2, 1960. Let n be the number of observations and k be the number of independent variables in a normal equation. If the least-squares error is $|u_M| \geq c\sigma_u$' the M^{th} observation, R_M, will be deleted as a outlier, where $c=3.06(1\text{-}1.85/n\text{-}k)$. And the least-squares method will be applied again to the remaining observations, ΔR_1, $\Delta R_2, \ldots$, ΔR_{M-1}, $\Delta R_{M+1}, \ldots$, ΔR_N. If the inequality is still maintained, a new observation numbered M will be deleted. This procedure will be used iteratively until $|u_M| < c\sigma_u$ results.

Table 1 Foreign Reserve Holdings in 1972, Computed from Kenen and Yudin's Best Equation: $R_t = \beta_0 + \beta_3\ \sigma_{ui}$
(millions of dollars)

	Actual Reserves	Full Period Analysis: Monthly Data, Dec. 1957 – Dec. 1972.		Full Period Analysis: Quarterly Data, Dec. 1957 – Dec. 1972.		Short Period Analysis: Monthly Data, Dec. 1967 – Dec. 1972.	
		R^2 = 0.72		R^2 = 0.78		R^2 = 0.79	
		Computed Reserves	Excess or Short fall	Computed Reserves	Excess or Short fall	Computed Reserves	Excess or Short fall
Austria	2,719.00	4,271.67	−1,552.67	3,002.94	−283.94	3,973.69	−1,254.69
Belgium	3,870.00	4,136.64	−266.64	4,355.71	−485.71	3,991.80	−121.80
Canada	6,049.80	5,060.72	989.08	4,302.86	1,746.94	4,535.98	1,513.82
Denmark	854.80	3,941.62	−3,086.81	3,515.21	−2,660.41	3.586.09	−2,731.29
Finland	757.80	3,403.69	−2,645.89	2,526.31	−1,768.51	3,047.48	−2,289.68
France	10,015.00	5,618.01	4,396.99	7,055.82	2,959.18	5,428.35	4,589.65
Germany	23,785.00	22,413.15	1,371.85	23,365.42	419.58	23,699.18	85.82
Italy	6,079.00	6,074.97	4.03	7,896.90	−1,817.90	6,066.38	12.62
Japan	18,365.00	4,431.20	13,933.80	4,987.36	13,377.64	6,731.02	11,633.98
Netherlands	4,785.00	5,535.97	−750.97	5,126.90	−341.90	5,336.69	−551.69
New Zealand	578.00	3,331.52	−2,753.52	2,755.16	−2,177.16	2,928.47	−2,350.47
Norway	1,325.00	3,787.91	−2,462.91	2,733.58	−1,408.58	3,467.71	−2,142.71
Sweden	1,575.00	3,478.91	−1,903.91	3,311.68	−1,736.68	3,286.74	−1,711.74
Switzerland	7,488.00	12,486.11	−4,998.11	10,848.81	−3,360.81	12,959.19	−5,471.19
United Kingdom	5,647.00	5,921.32	−274.32	8,108.74	−2,460.74	4,854.62	792.38

Table 2 Autoregressive Equations: Full Period Analysis (Monthly Changes in Reserves, Dec. 1957 – Dec. 1972)

	e (t – value)	p (t – value)	R^2	D_W	σ_u	χ^2 ($\nu = 19$)	χ^2 ($\nu = 9$)
Austria	8.97224	–0.05370	0.07030	2.11502	32.96990	a 9.70175	a 5.08187
	* (3.46142)	(–0.91611)					
Belgium	7.91894	0.28966	0.35655	1.86783	41.95734	a 14.39884	a 11.21965
	* (2.39524)	* (5.10398)					
Canada	7.50254	0.44817	0.47448	2.16185	70.74265	50.23699	44.28324
	(1.34674)	* (7.13945)					
Denmark	2.40251	0.00787	0.00875	2.00308	31.24738	a 25.80000	a 12.14286
	(1.01474)	(0.11514)					
Finland	2.00725	0.08939	0.06285	2.07274	16.07173	c 35.85714	c 22.54286
	(1.65056)	(1.30003)					
France	19.63516	0.40249	0.61190	2.18296	88.10248	c 34.43195	c 21.00000
	*(2.79550)	* (10.07723)					
Germany	50.69834	0.05047	0.09281	1.80896	183.07280	a 18.98701	25.00780
	* (3.42943)	(1.52622)					
Italy	19.75753	0.38299	0.42931	1.84021	102.33703	a 25.26590	a 15.72832
	* (2.49088)	* (6.31378)					
Japan	15.16611	0.20713	0.82836	1.69286	51.13308	a 22.97436	a 15.15385
	* (3.66131)	* (18.43698)					
Netherlands	11.26659	–0.00759	0.01214	1.93973	49.39032	43.15205	c 20.28655
	* (2.91229)	(–0.15778)					
New Zealand	1.69591	0.35571	0.37955	2.02419	16.87762	a 11.36158	a 5.31638
	(1.32908)	* (5.53362)					
Norway	5.95320	–0.05086	0.07256	1.63812	20.82276	c 32.71429	c 17.47619
	* (3.66617)	(–0.93737)					
Sweden	2.74945	0.38361	0.46596	2.29876	21.46902	c 31.95349	c 18.00000
	(1.64528)	* (6.95873)					
Switzerland	15.46668	–0.08450	0.11531	2.54804	162.76618	116.23353	91.02395
	(1.22595)	* (–1.79913)					
United Kingdom	6.10998	0.11684	0.24363	1.76400	97.55070	c 30.80952	c 20.80952
	(0.80863)	* (3.39673)					

* Significantly different from zero at 0.05 level.
a Distributions marked "a" are those that satisfied the χ^2 test for normality at the 0.05 level of significance.
c Wound satisfy the χ^2 test for normality at the 0.01 level of significance.

Table 3 Autoregressive Equations: Full Period Analysis (Quarterly Changes in Reserves, Dec. 1957 – Dec. 1972)

	e (t – value)	p (t – value)	R^2	D_W	σ_u	χ^2 (ν = 19)	χ^2 (ν = 9)
Austria	23.07445 *(3.04553)	0.18271 * (1.75310)	0.19060	1.93588	51.22457	a 12.57143	a 3.64286
Belgium	34.31622 *(2.30927)	0.21623 (1.60561)	0.16275	1.96002	102.26305	a 16.59322	a 5.91525
Canada	20.88914 (1.39762)	0.20049 * (2.84425)	0.34935	2.13997	100.80634	a 18.76923	a 6.07692
Denmark	8.58355 (1.14733)	0.14689 (1.35521)	0.12132	1.98659	56.13697	a 12.82456	a 4.92982
Finland	6.53081 (1.43953)	0.12614 * (1.90234)	0.21137	2.06969	33.04479	a 11.42105	a 1.77193
France	71.84228 * (2.42794)	0.45114 * (6.70181)	0.66631	2.58895	205.81047	a 21.14286	a 9.71429
Germany	322.89605 * (2.86534)	0.28474 * (2.86374)	0.33491	1.46841	827.89520	48.20690	44.75862
Italy	33.24857 (1.00951)	0.34773 * (2.92225)	0.34179	2.00121	237.89095	a 13.03448	a 6.82759
Japan	37.89330 *(2.08121)	0.35258 * (15.43281)	0.90878	1.31036	126.91478	a 25.47059	a 14.29412
Nethearlands	41.90470 * (2.88564)	0.01966 (0.19741)	0.02661	1.59111	98.60653	a 25.45614	a 16.85965
New Zealand	5.94387 (1.07765)	0.26007 * (2.05182)	0.22901	1.76637	41.77332	a 17.27119	a 11.33898
Norway	17.44196 * (3.24392)	–0.05032 (–0.41082)	0.05482	1.97819	37.17291	a 16.48276	c 10.27586
Sweden	17.82988 * (2.10102)	0.29763 * (2.43889)	0.28262	1.68931	63.00026	a 24.75862	a 17.17241
Switzerland	105.08042 * (3.24704)	–0.62988 * (–5.99109)	0.61959	1.97057	239.18382	a 28.96491	21.77193
United Kingdom	21.42356 (0.15029)	0.23913 * (2.82122)	0.33514	2.26674	245.97099	a 20.42857	a 11.50000

Note: See note for Table 2.

Table 4 Autoregressive Equations: Short Period Analysis (Monthly Changes in Reserves, Dec. 1957–Dec. 1962)

	e (t – value)	p (t – value)	R^2	Dw	σ_u	χ^2 ($\nu = 19$)	χ^2 ($\nu = 9$)
Austria	5.27950 * (1.90988)	0.47055 * (3.90143)	0.44375	1.79698	19.67149	a 13.20339	a 7.94915
Belgium	12.83129 * (3.16515)	0.32442 * (3.80171)	0.44011	1.91030	30.01676	a 17.73684	a 11.59649
Canada	–7.12391 (–1.11666)	0.52848 * (6.23377)	0.63852	2.12258	47.16180	39.71429	c 18.64286
Denmark	0.72577 (0.35963)	–0.18654 (–1.51120)	0.14841	2.11593	15.33403	a 13.72414	a 5.10345
Finland	0.67766 (0.58338)	0.28064 * (2.25131)	0.25811	1.73966	8.49908	a 20.62069	a 10.96552
France	45.96968 * (3.98969)	0.17465 (1.55888)	0.15780	1.68654	69.62757	a 9.31579	a 3.52632
Germany	63.39336 * (3.08950)	0.14035 * (1.71259)	0.18266	1.96820	154.26269	a 26.15789	a 13.70175
Italy	22.87583 * (2.38857)	0.50427 * (4.25680)	0.47739	2.06848	63.39325	a 22.01695	a 7.27119
Japan	16.95919 * (2.69801)	0.33643 * (2.99313)	0.35003	1.92095	45.10822	a 24.75862	c 18.20690
Netherlands	12.56985 * (2.15585)	0.09829 (0.75159)	0.09906	1.94378	43.15037	a 22.69492	a 13.03390
New Zealand	0.04729 (0.02419)	0.40120 * (3.70048)	0.42677	1.66572	14.86866	a 21.31034	a 7.86207
Norway	2.73141 * (2.14211)	–0.11338 (–0.93813)	0.12439	2.21413	9.62094	a 17.17241	a 8.55172
Sweden	6.05203 * (3.37117)	0.29013 * (3.06544)	0.36110	1.71061	12.90287	a 17.03509	a 2.12281
Switzerland	10.09240 (1.42940)	0.00969 (0.08139)	0.01097	1.96224	51.79931	a 23.35088	a 8.08772
United Kingdom	0.26771 (0.01895)	0.26937 *(2.30001)	0.26457	1.96750	105.30127	a 20.62069	a 13.03448

Note: See note for Table 2.

Table 5 Autoregressive Equations: Short Period Analysis (Monthly Changes in Reserves, Dec. 1962 – Dec. 1967)

	e (t – value)	p (t – value)	R^2	D_W	σ_u	χ^2 (ν = 19)	χ^2 (ν = 9)
Austria	7.55214 * (1.83484)	−0.08040 (−0.59716)	0.07885	1.97255	31.03847	a 17.94915	a 7.27119
Belgium	10.95090 * (2.80799)	0.18630 (1.39518)	0.12672	1.97596	25.86742	a 15.23729	a 3.88136
Canada	2.20894 (0.36594)	0.05466 (1.33690)	0.11880	1.23876	44.67247	a 24.71429	a 11.14286
Denmark	5.55023 (1.38663)	−0.05256 (−0.39701)	0.05251	1.99662	30.48537	a 17.94915	a 8.96610
Finland	−2.30424 (−1.16034)	−0.07709 (−0.58553)	0.07732	1.99509	15.12309	a 18.62712	a 14.72881
France	22.22081 * (2.59339)	0.48923 * (4.26508)	0.48464	2.09660	50.00635	a 21.24561	a 9.84211
Germany	26.85933 (1.54917)	−0.18143 (−1.41346)	0.13005	2.04499	131.54228	a 14.55932	a 5.23729
Italy	22.68864 * (1.87172)	0.67497 * (6.43863)	0.64763	1.65853	90.67355	a 19.14035	a 5.63158
Japan	−3.78398 (−1.10303)	0.01621 (0.1447)	0.01948	2.06348	25.87203	a 26.15789	c 17.56140
New Zealand	11.26806 * (1.91744)	0.02233 (0.16863)	0.02233	1.99717	43.59033	a 19.98305	a 8.28814
Newzealand	−1.18019 (−0.62798)	0.38712 * (3.40487)	0.39588	2.04087	14.30308	a 15.10345	a 6.13793
Norway	7.48439 * (2.21633)	−0.18540 (−1.32173)	0.11276	1.92854	25.37963	a 15.91525	a 7.61017
Sweden	2.57401 (1.03524)	0.20745 * (2.13348)	0.24219	2.08570	18.89689	c 31.65517	a 15.10345
Switzerland	19.31829 (1.04880)	−0.40382 * (−3.16537)	0.36685	2.14630	141.35469	53.88136	29.64407
United Kingdom	−15.06550 (−1.26742)	0.02251 (0.30884)	0.04238	1.71487	88.03785	a 15.18182	a 10.27273

Note: See note for Table 2.

Table 6 Autoregressive Equations: Short period Analysis (Monthly Changes in Reserves, Dec. 1967 – Dec. 1972)

	e (t – value)	p (t – value)	R^2	Dw	σ_u	χ^2 ($\nu = 19$)	χ^2 ($\nu = 9$)
Austria	19.93472	0.15439	0.09267	1.98392	67.59316	a 23.37288	a 11.67797
	* (2.16417)	(1.22571)					
Belgium	7.97304	0.21304	0.21378	2.09535	71.08975	a 19.14035	a 9.14035
	(0.81026)	* (1.91884)					
Canada	33.70466	0.33573	0.35058	1.87432	96.95633	a 17.86207	a 9.58621
	* (2.39203)	* (2.99790)					
Denmark	5.31208	0.14894	0.06894	1.98198	51.90013	a 17.94915	a 9.30508
	(0.78333)	(1.13005)					
Finland	6.29522	0.29867	0.26287	2.00292	26.20361	a 24.72881	a 8.96610
	* (1.71773)	* (2.30330)					
France	7.26962	0.40718	0.63612	2.41780	139.37353	a 28.13208	a 15.49057
	(0.37871)	* (6.02858)					
Germany	235.79829	0.12423	0.12431	1.98448	1015.24339	a 24.72881	a 19.47458
	* (1.72412)	(0.94586)					
Italy	9.43737	0.23108	0.19254	2.07426	169.70114	a 23.37288	a 9.98305
	(0.42636)	* (1.79801)					
Japan	98.03417	0.26415	0.64101	1.46182	201.29348	a 23.28571	a 14.71429
	* (3.33579)	* (6.27394)					
Netherlands	19.72521	–0.12745	0.15631	1.67434	85.21820	a 26.85714	a 13.64286
	* (1.67475)	(–1.54188)					
New Zealand	4.09779	0.29306	0.29663	2.14237	20.54660	a 8.20690	a 1.65517
	(1.45316)	* (2.54939)					
Norway	13.84478	–0.13901	0.05894	2.05607	44.95277	a 15.91525	a 7.27119
	* (2.30162)	(–1.09646)					
Sweden	6.66831	0.44882	0.42753	2.11107	37.57625	a 20.66102	a 8.12712
	(1.31350)	* (3.73796)					
Switzerland	49.43356	–0.29461	0.30959	2.50607	402.51282	a 16.48276	a 9.58621
	(0.92842)	* (–2.65378)					
United Kingdom	25.90548	0.13130	0.33365	1.41825	112.10249	a 43.54545	a 23.72727
	* (1.69421)	* (2.78649)					

Note: See note for Table 2.

Table 7 Tests for Stability of the Standard Error (σ_u): Full Period Analysis (Monthly Changes in Reserves, Dec. 1957 – Dec. 1972)

	p_2 (t – value)	R^2	D_W	Bartlett test (χ^2, ν=4)	λ (t – value)	R^2	D_W
Austria	0.02904 (0.37328)	0.09749	1.99785	40.94922	11.63957 * (4.61)	0.32625	1.68485
Belgium	–0.02484 (–0.23597)	0.10752	1.99513	44.94836	20.47824 * (4.88)	0.34198	1.59491
Canada	0.04260 (0.56043)	0.10039	2.00565	60.91871	36.97845 * (2.86)	0.20017	1.50118
Denmark	–0.01795 (–0.23211)	0.06494	1.97969	38.25488	11.36010 * (5.98)	0.40784	2.17129
Finland	0.10706 (1.40752)	0.25123	1.97867	60.40965	3.20577 * (5.66)	0.38934	1.90241
France	–0.02910 (–0.38503)	0.31912	1.99510	24.95130	55.84945 * (2.57)	0.17986	1.67450
Germany	–0.00615 (–0.01819)	0.10035	2.00009	25.00780	353.40758 * (3.89)	0.29076	1.92355
Italy	0.10703 (1.40964)	0.32038	1.98736	36.23989	105.65941 * (4.26)	0.30104	1.58146
Japan	0.25158 * (3.41199)	0.40341	2.10481	38.67709	24.37720 * (3.28)	0.24314	1.83025
Netherlands	–0.11304 (–1.50832)	0.07340	1.95633	a 11.11838	14.37353 * (2.31)	0.15783	1.94251
New Zealand	0.10847 (1.46783)	0.36460	2.01112	2.78472	0.84660 (1.32)	0.06551	1.99479
Norway	–0.09722 (–1.29333)	0.15786	1.99070	52.15086	5.99989 * (5.56)	0.38999	2.34971
Switzerland	0.09212 (1.19296)	0.40077	2.01280	27.78106	3.20726 * (2.70)	0.18826	1.50072
Switzerland	–0.35005 * (–4.76367)	0.47788	1.87928	88.87491	470.11557 * (6.48)	0.44520	1.93578
United Kingdom	0.03965 (0.52315)	0.19137	1.99269	a 7.33976	28.88486 (1.16)	0.04568	1.92850

* Significantly different from zero at the 0.05 level.
a Distributions marked "a" are those that satisfied the Bartlett χ^2 test at the 0.05 or 0.01 level of significance.
λ is the coefficient of linear trend, $u_t^2 = u_0^2 + \lambda T$, when u_t^2 is treated as a point-estimate of σ_u^2.
p_2 is the coefficient estimated from the equation, $\Delta R_t = e + p_1 \Delta R_{t-1} + p_2 \Delta R_{t-2} + u'_t$.

Table 8 Tests for Stability of the Standard Error (σ_u): Full Period Analysis (Quarterly Changes in Reserves, Dec. 1957 – Dec. 1972)

	p^2 (t – value)	R^2	D_W	Bartlett test (χ^2, $\nu = 1$)	λ (t – value)	R^2	D_W
Austria	−0.21223 (−1.08845)	0.14598	1.75341	6.65669	33.12399 (1.26)	0.10195	2.25369
Belgium	0.18372 (1.29102)	0.19175	1.92690	a 1.24713	144.18707 (1.22)	0.09121	1.85066
Canada	0.03161 (0.23187)	0.18082	1.97350	29.17799	358.47155 * (2.58)	0.31597	2.31561
Denmark	−0.49776 * (−4.01933)	0.44948	1.95554	9.89321	144.56102 * (4.82)	0.53317	2.13362
Finland	−0.06817 (−0.46507)	0.13791	1.81021	9.79218	34.05889 * (2.77)	0.32674	2.28664
France	0.03105 (0.22531)	0.46509	1.99183	36.60453	1448.13642 * (2.89)	0.34341	1.48889
Germany	−0.09238 (−0.67283)	0.16961	1.95566	33.66134	28245.39480 * (3.01)	0.35208	1.64892
Italy	−0.29197 * (−2.21466)	0.36018	1.81678	a 2.49897	883.29965 * (1.92)	0.21216	2.33690
Japan	−0.02402 (−0.17243)	0.52308	1.94697	4.51828	592.33357 * (2.41)	0.29674	1.23926
Netherlands	0.37534 * (2.56355)	0.27979	1.92413	a 3.41080	302.79068 * (2.75)	0.32344	1.41359
New Zealand	−0.39019 * (−3.18073)	0.43426	1.64673	a 0.11903	20.61918 (1.17)	0.08011	1.73308
Norway	0.38331 * (3.00657)	0.33517	1.97660	27.24932	59.03942 * (4.35)	0.48899	2.18586
Sweden	−0.01001 (−0.07084)	0.21387	1.90300	16.27569	169.65402 * (3.36)	0.39097	2.28668
Switzerland	0.01574 (0.11609)	0.25507	1.99249	10.93168	2120.23960 * (2.86)	0.33765	2.28666
United Kingdom	0.04169 (0.26584)	0.25643	1.98364	a 5.31583	27.10611 (0.03)	0.00448	2.00638

Note: See note for Table 7.

Table 9 Tests for Stability of the Standard Error (σ_u): Short Period Analysis (Monthy Changes in Reserves, Dec. 1957 – Dec. 1962)

	p^2 (t – value)	R^2	Dw	Bartlett test (χ^2 , ν = 1)	λ (t – value)	R^2	Dw
Austrai	−0.08731 (−0.58414)	0.43182	1.87312	a 4.10948	12.01134 * (2.64)	0.30528	1.48050
Belgium	−0.01964 (−0.14559)	0.19287	2.00860	a 0.38614	−3.53462 (−0.37)	0.04988	2.27570
Canada	−0.14664 (−1.08948)	0.51972	2.01831	33.89680	112.83583 * (4.06)	0.46906	2.03804
Denmark	−0.30771 * (−2.31480)	0.32592	1.63488	a 0.22015	3.06694 (1.25)	0.09809	1.79852
Finland	0.01667 (0.12030)	0.06543	1.90209	a 1.00160	−1.16417 (−1.52)	0.14980	1.54811
France	−0.08748 (−0.63136)	0.09374	1.91405	a 0.30532	57.76032 (1.06)	0.04507	1.73059
Germany	0.12393 (0.92601)	0.12833	2.06643	a 2.69559	440.67710 (1.44)	0.13791	1.88611
Italy	0.11270 (0.81905)	0.47174	1.95426	a 5.43531	111.81832 * (2.55)	0.29449	1.88189
Japan	0.26052 * (1.99091)	0.34064	2.08573	a 0.00192	−12.75816 (−0.52)	0.06880	1.45167
New Zealand	−0.07407 (−0.59725)	0.09483	1.86686	a 0.05936	−18.02780 (−0.70)	0.09273	1.69176
Newzealand	0.02776 (0.21918)	0.34793	2.01596	a 0.99250	−0.93175 (−0.43)	0.05716	2.30256
Norway	−0.15417 (−1.12520)	0.11464	1.90520	a 1.28170	0.36869 (0.48)	0.06354	2.01058
Sweden	0.13387 (1.01107)	0.32260	1.99161	a 0.12194	0.06103 (0.03)	0.00418	1.22781
Switzerland	0.08134 (0.53084)	0.09389	1.79669	a 0.74087	36.37331 (0.93)	0.12466	1.77023
United Kingdom	0.10655 (0.80122)	0.11472	1.99916	a 2.69507	−15.57176 (−0.11)	0.01461	2.08777

Note: See note for Table 7.

Table 10 Tests for Stability of the Standard Error (σ_u): Short Period Analysis (Monthly Changes in Reserves, Dec. 1962 – Dec. 1967)

	p_2 (t – value)	R^2	Dw	Bartlett test (χ^2, $\nu = 1$)	λ (t – value)	R^2	Dw
Austria	−0.12844 (−0.94164)	0.14783	1.93212	a 0.61282	5.43104 (0.49)	0.06433	1.84595
Belgium	0.15739 (1.09284)	0.13287	1.95278	a 1.03559	12.62952 (1.74)	0.18391	1.65670
Canada	−0.22309 * (−1.70209)	0.42361	2.05330	a 3.31385	−37.19301 (−1.53)	0.15377	1.76686
Denmark	−0.13454 (−1.00726)	0.14739	1.79091	a 0.06779	6.12788 (0.92)	0.12106	2.31485
Finland	0.03641 (0.27041)	0.08575	2.00349	a 1.46505	−0.62581 (−0.22)	0.02920	2.07931
France	0.17512 (1.08384)	0.17281	1.94686	a 0.43960	−17.95687 (−0.61)	0.08242	1.99411
Germany	−0.09800 (−0.73766)	0.09896	1.95953	a 1.43059	−45.37687 (−0.23)	0.03111	1.72880
Italy	−0.07143 (−0.53581)	0.41275	1.98373	a 1.03912	47.18809 (0.50)	0.06709	2.04089
Japan	−0.04719 (−0.34969)	0.13280	1.92215	a 0.47299	10.56544 * (2.16)	0.24774	1.91468
Netherlands	−0.06129 (−0.45553)	0.06456	2.01976	a 0.02335	−2.85779 (−0.13)	0.01722	2.24967
New Zealand	0.06769 (0.49705)	0.33548	1.99643	a 0.66934	0.90484 (0.38)	0.05101	1.97349
Norway	−0.21776 (−1.54700)	0.18588	1.89808	a 5.99068	18.10717 * (2.05)	0.22871	1.80302
Sweden	0.13686 (0.74115)	0.27726	2.01053	a 2.42830	0.19795 (0.04)	0.00585	1.88831
Switzerland	−0.34401 * (−2.58135)	0.47208	1.91525	a 4.35448	515.19091 * (1.93)	0.21150	1.60098
United Kingdom	−0.08204 (−0.59713)	0.08519	1.96463	a 5.09515	168.50225 * (1.88)	0.21159	1.40225

Note: See note for Table 7.

Table 11 Tests for Stability of the Standard Error (σ_u): Short Period Analysis (Monthly Changes in Reserves, Dec. 1967 – Dec. 1972)

	p_2 (t – value)	R^2	D_W	Bartlett test (χ^2, $\nu = 1$)	λ (t – value)	R^2	D_W
Austria	–0.00117 (–0.00864)	0.15905	1.99599	11.88297	127.62578 *(2.61)	0.30189	1.94289
Belgium	–0.05522 (–0.40628)	0.10649	1.98524	a 3.24584	81.63847 (1.52)	0.15152	1.53088
Canada	–0.15163 (–1.12378)	0.36467	1.95625	a 0.30542	–217.24940 * (–2.17)	0.24782	1.16932
Denmark	0.02304 (0.16666)	0.15381	1.98147	a 0.04305	10.02843 (0.33)	0.04319	1.77390
Finland	0.07025 (0.51240)	0.23249	1.96210	a 4.59079	19.79732 * (2.71)	0.31357	1.94008
France	–0.03519 (–0.26129)	0.31000	1.99480	a 3.03121	–391.50309 (–1.45)	0.14396	1.84693
Germany	–0.04539 (–0.33631)	0.13289	2.00757	a 0.05885	2586.60975 (0.19)	0.02519	1.34717
Italy	0.16248 (1.20580)	0.20944	1.97105	9.09663	583.50065 * (1.70)	0.17752	2.17008
Japan	0.20441 (1.54253)	0.27074	2.07129	82.91669	1950.12411 *(4.32)	0.49317	2.09061
Netherlands	–0.16443 (–1.24595)	0.07972	1.94167	20.47546	96.00222 (0.96)	0.12921	2.04005
New Zealand	0.14345 (1.08149)	0.27961	1.91490	a 0.05269	–6.06610 (–1.26)	0.10077	2.14588
Norway	–0.09564 (–0.73820)	0.18373	1.98222	a 0.87333	12.64055 (0.50)	0.06587	1.99840
Sweden	0.15122 (1.09810)	0.43342	2.02482	a 0.61253	1.90851 (0.11)	0.01459	1.96630
Switzerland	–0.36918 * (–2.85862)	0.47988	1.89099	a 6.15918	2845.68422 * (1.72)	0.18172	1.28644
United Kingdom	0.02745 (0.20206)	0.21211	1.98437	16.60390	612.81294 * (3.72)	0.43828	1.80187

Note: See note for Table 7.

The value of p was negative, but significantly different from zero in Switzerland in all tables (except for Table 4). Therefore, it was theoretically wrong in Kenen and Yudin's article. However, we should not neglect it, even if it was only one country.

The χ^2 test for normality was tabulated on the last two columns in Table 2–Table 6. We split a normal distribution into 10 or 20 cells whose frequencies were equally 10 or 5%. And we recorded the observed frequencies of outcomes in the corresponding cells. The last two columns, therefore, involved two kinds of χ^2 values; one was with $\nu=9$ and other was $\nu=19$. The most countries passed the χ^2 test for normality. In order to test the statistical independence of residuals, u_t, on the other hand, we calculated the Durbin-Watson's ratio, Dw, and an additional term, p_2, for the autoregressive equation. They were arrayed in Table 2–Table 6 and Table 7–Table 11 respectively. They suggested that the residuals, u_t, were statistically independent. The Bartlett χ^2 and λ in Table 7–Table 11 provided the homogeneity test for variance, σ_u. They, however, did not give us any satisfactory results.

The calcutated parameters, e, p, and σ_u, were used as the input data into the cross-

Table 12 Cross-Sectional Models (millions of dollars)

	$R_{it} = \beta_0 + \beta_1 e + \beta_2 p_i + \beta_3 \sigma_{ui} + \epsilon_{it}$				Dw	R^2
	β_0	β_1	β_2	β_3		
Full Period Analysis (Monthly Changes in Reserves)						
(1) R_{57}	131.484 (0.51)	−3.543 (−0.51)	1,215.306 (1.40)	8.619 * (2.27)	1.92	0.92
(1) R_{72}	1,195.520 (0.64)	333.422 * (1.98)	6,719.201 (1.07)	−17.553 (−0.64)	2.27	0.79
Full Period Analysis (Quanterly Changes in Reserves)						
(2) R_{57}	187.081 (0.80)	−3.697 (−0.65)	−470.199 (−0.51)	7.457 * (3.35)	2.18	0.92
(2) R_{72}	79.183 (0.04)	52.430 (1.21)	13,457.393 * (1.92)	4.905 (0.29)	2.10	0.82
Shot Period Analyses						
(3) R_{57}	−338.881 (−0.97)	−8.494 (−0.65)	637.017 (0.69)	31.591 * (5.17)	2.66	0.87
(3) R_{62}	−272.103 (−1.11)	27.998 * (3.07)	1,589.752 * (2.47)	34.608 * (8.11)	2.68	0.97
(4) R_{62}	67.510 (0.13)	104.862 * (2.88)	342.432 (0.22)	18.744 * (3.68)	1.90	0.79
(4) R_{67}	17.664 (0.03)	177.390 * (4.38)	998.581 (0.57)	18.497 * (3.27)	1.82	0.85
(5) R_{67}	435.971 (0.53)	−24.706 (−1.32)	4,458.568 (1.54)	13.901 * (2.81)	2.14	0.74
(5) R_{72}	827.417 (0.53)	98.593 * (2.78)	9,168.065 (1.68)	−0.495 (−0.05)	2.26	0.89

* Significantly different from zero at the 0.05 level.

sectional models. According to Kenen and Yudin's procedure, p was fixed at zero and the corresponding estimates of e and σ_u were respectively replaced by the mean change in reserves and the simple deviation of ΔR_t, when p was negative or not significantly different from zero. Table 12–Table 14 present the regression coefficients for the various cross-sectional models generated from possible combinations of e, p, and σ_u. The least-sequares estimates in these tables were generally characterized by the high values of R^2 and Dw. The values on row (4) in Table 13 and Table 14, however, were comparatively low, which related to the period, December 1962–December 1967. As shown in Table 5, the value of p was fixed at zero in more than ten countries for this period. Accordingly, there was not any time interval between external payments and receipts in almost countries.

In Table 12, the coefficient, β_1, attached to e was negative when e was not significant, while it was positive when e was significant. This fact is contradictory to the theoretical presumption: the country that anticipates a positive change in reserves in the near future will take small reserves as being sufficient to finance its deficit balance.

Table 13 Cross-Sectional Models (millions of dollars)

	$R_{it} = \beta_0 + \beta_2 p_i + \beta_3 \sigma_{ui} + \epsilon_{it}$			Dw	R^2
	β_0	β_2	β_3		
Full Period Analysis (Monthly Changes in Reserves)					
(1) R_{57}	131.679 (0.53)	1,205.947 (1.46)	8.062 * (8.22)	1.87	0.91
(1) R_{72}	1,177.172 (0.57)	7,599.862 (1.09)	34.896 * (4.21)	1.95	0.73
Full Period Analysis (Quarterly Changes in Reserves)					
(2) R_{57}	213.020 (0.95)	−357.753 (−0.41)	6.075 * (9.17)	2.04	0.93
(2) R_{72}	−288.718 (−0.16)	11,862.534 (1.69)	24.512 * (4.62)	2.21	0.81
Short Period Analyses					
(3) R_{57}	−367.475 (−1.08)	841.875 (1.00)	28.673 * (7.07)	2.75	0.88
(3) R_{62}	−177.852 (−0.56)	914.519 (1.16)	44.229 * (11.67)	1.68	0.95
(4) R_{62}	494.454 (0.78)	2,414.905 (1.34)	20.457 * (3.19)	1.43	0.63
(4) R_{67}	739.907 (0.84)	4,504.493 (1.80)	21.396 * (2.40)	1.52	0.55
(5) R_{67}	586.211 (0.69)	4,154.705 (1.40)	7.870 * (4.13)	2.68	0.72
(5) R_{72}	227.870 (0.12)	10,380.657 (1.52)	23.572 * (5.40)	1.87	0.81

Note: See note for Table 12.

Table 14 Cross-Sectional Models (millions of dollars)

	$R_{it}=\beta_0+\beta_1 e_i+\epsilon_{it}$	Dw	R^2	$R_{it}=\beta_0+\beta_2 P_i+\epsilon_{it}$	Dw	R^2	$R_{it}=\beta_0+\beta_3 \sigma_{ui}$	Dw	R^2
	β_1			β_2			β_3		
Full Period Analysis (Monthly Changes in Reserves)									
(1) R_{57}	45.363 *(6.54)	1.30	0.87	−901.067 (−0.46)	1.65	0.13	7.619 *(7.84)	1.91	0.90
R_{72}	214.841 *(4.96)	2.24	0.79	−1520.485 (−0.15)	1.58	0.04	32.102 *(4.04)	1.89	0.72
Full Period Analysis (Quarterly Changes in Reserves)									
(2) R_{57}	14.451 *(6.43)	1.42	0.86	1170.770 (0.50)	1.96	0.14	6.024 *(9.57)	1.89	0.93
R_{72}	67.684 *(4.75)	1.99	0.78	18030.063 (1.63)	2.31	0.33	26.218 *(4.71)	2.09	0.78
Short Period Analyses									
(3) R_{57}	36.412 *(2.49)	2.18	0.52	−64.234 (−0.04)	1.77	0.01	28.061 *(7.00)	2.62	0.88
R_{62}	74.237 *(4.82)	1.53	0.78	−483.208 (−0.18)	0.98	0.05	43.564 *(11.48)	1.45	0.95
(4) R_{62}	114.166 *(2.55)	1.39	0.53	1686.696 (0.72)	1.22	0.20	19.364 *(2.96)	0.86	0.60
R_{67}	193.321 *(4.15)	1.33	0.73	3742.886 (1.29)	1.38	0.21	19.357 *(2.02)	1.00	0.62
(5) R_{67}	22.048 *(2.69)	2.18	0.56	−520.562 (−0.13)	1.04	0.04	6.855 *(3.76)	2.33	0.70
R_{72}	89.276 *(6.87)	2.11	0.88	−3623.098 (−0.32)	1.57	0.09	21.038 *(4.97)	1.98	0.79

Note: See note for Table 12.

β_1 must be negative if e is significant. Accordingly, the variable, e, should be deleted from the normal equation. After deleting, we estimated the coefficients for the normal equation which contained per capita income, y, and domestic money supply, M_t, in Table 15. The estimated coefficient, β_3, attached to σ_u, was significantly different from zero for all periods. But the coefficients, β_2 and β_4, which were attached to p and y, were not. Although p was not significant in all periods, we ought not to forget that the estimate, p, was fixed at zero in more than ten countries for the period, December 1962–December 1967 and in about five countries on the average for other periods. The supply of domestic money, M_t, on the other hand, was significant only in 1972 for each period.

Finally, we calculated the holdings of foreign reserves in 1957 and 1962 to compare our estimations with Kenen and Yudin's. They were tabulated in Table 16. On the last two columns, we made two pairs of rankings derived from columns (1) and (2) in Table 16 and the corresponding columns in Kenen and Yudin's article. The rank correlation coefficients for two columns were $r=0.742$ in 1957 and $r=0.728$ in

Table 15 Cross-Sectional Models (millions of dollars)

	$R_{it} = \beta_0 + \beta_2 p_i + \beta_3 \sigma_{ui} + \beta_4 y_i + \beta_5 M_{it} + \epsilon_{it}$					D_W	R^2
	β_0	β_2	β_3	β_4	β_5		
Full Period Analysis (Monthly Changes in Reserves)							
(1) R_{57}	−90.00 (−0.20)	717.972 (0.77)	7.853 * (7.83)	0.167 (0.41)	0.020 (1.33)	2.32	0.91
(1) R_{72}	−3,319.447 (−0.96)	−3,551.828 (−0.76)	24.307 * (4.41)	1.061 (1.20)	0.138 * (5.06)	2.56	0.93
Full Period Analysis (Quanterly Changes in Reserves)							
(2) R_{57}	−459.767 (−0.96)	−0.075 (−0.00)	6.015 * (9.32)	0.494 (1.42)	0.012 (0.97)	2.26	0.93
(2) R_{72}	−3,750.436 (−0.96)	−630.206 (−0.10)	19.981 * (4.80)	0.971 (1.00)	0.115 * (3.62)	2.68	0.91
Short Period Analyses							
(3) R_{57}	−893.083 (−1.47)	896.645 (1.02)	29.511 * (6.75)	0.492 (1.09)	−0.007 (−0.39)	2.67	0.87
(3) R_{62}	159.512 (0.20)	878.115 (1.01)	44.360 * (10.70)	−0.215 (−0.46)	−0.001 (−0.21)	1.68	0.94
(4) R_{62}	1,539.864 (0.82)	2,437.732 (1.32)	23.327 * (3.35)	−0.923 (−0.78)	0.05 (0.45)	1.70	0.62
(4) R_{67}	2,073.787 (0.92)	4,321.465 (1.61)	21.984 * (1.98)	−0.705 (−0.66)	0.05 (0.24)	1.71	0.45
(5) R_{67}	1,644.720 (0.94)	3,035.941 (0.91)	7.657 * (3.95)	−0.626 (−0.77)	0.021 (1.26)	2.99	0.72
(5) R_{72}	−17776.203 (−0.55)	1,269.975 (0.26)	17.410 * (5.16)	0.486 (0.54)	0.109 * (4.19)	2.52	0.94

Note: see note for Table 12.

1962. Therefore, the hypothesis of independence was rejected.

2. The Precautionary Motive

Foreign reserves are held to provide for contingencies requring urgent imports to meet a bad harvest or other disasters. The strength of this demand depends not only on the population or the productivity of the country, but also on her ability of borrowing from foreign governments or international financial institutions. If the liquid reserves fell below a certain minimum level, the country would feel impelled to adopt severe readjustment measures, and short or medium private loans abroad would be reduced to zero or negative values. And also official financial institutions would hesitate to favour the deficit country with official loans because of her unhealthy economic policy. The minimum reserve level, therefore, can be regarded as an effective indicator to the precautionary demand for international currencies. The 20 or 30% ratio or reserves

Table 16 **Foreign Reserve Holdings in 1957 and 1962, Computed from Kenen and Yudin's Best Equation; Dec. 1957 – Dec. 1962 (millions of dollars)**

	1957			1962			Rank of			
	Actual Reserves	Computed Reserves	Excess or Shortfall (1)	Actual Reserves	Computed Reserves	Excess or Shortfall (2)	(1) (1957)	Kenen and Yudin's Table 3 (1957)	(2) (1962)	Kenen and Yudin's Table 3 (1962)
Austria	522.00	412.97	109.03	1,077.00	927.28	149.72	6	5	6	7
Belgium	1,148.00	703.26	444.74	1,753.00	1,377.97	375.03	3	3	3	4
Canada	1,936.00	1,184.36	751.74	2,560.50	2,124.88	435.62	2	1	2	5
Denmark	172.00	333.57	−161.37	255.60	804.02	−548.42	12	13	13	13
Finland	180.10	99.46	80.64	304.00	440.56	−136.56	7	9	10	10
France	645.00	2,271.06	−1,626.06	4,049.00	3,811.98	273.02	–	–	–	–
Germany	5,197.00	4,189.69	1,007.31	6,957.00	6,790.67	166.33	1	7	5	9
Italy	1,479.00	1,639.83	−160.83	4,068.00	2,832.00	1,236.00	11	6	1	1
Japan	828.00	1,126.74	−298.74	2,022.00	2,035.42	−13.42	13	11	9	2
Netherlands	1,056.00	1,040.06	15.94	1,943.00	1,900.86	42.14	9	8	7	6
New Zealand	152.00	270.20	−126.20	172.00	718.05	−546.05	10	12	12	14
Norway	196.90	156.84	40.06	303.00	529.65	−225.75	8	10	11	11
Sweden	481.00	223.03	257.97	802.00	632.41	169.59	4	4	4	8
Switzerland	1,898.00	1,690.43	207.58	2,919.00	2,910.55	8.45	5	2	8	3
United Kingdom	2,274.00	2,815.80	−541.80	3,308.00	4,657.70	−1,349.70	14	14	14	12

to imports which was proposed by Triffin has been widely accepted as the minimum reserve level.[5] Table 17 shows the mean values of the reserve ratios calculated from yearly data (1957–1971) for 40 countries. The most major countries maintained a reserve ratio of not less than 30%. The low ratio which was less than 20% was found only in a few countries. Complementarily, we tested whether these means were kept constant during ten years. The test introduced to solve this type of problem was the rank-sum test.[6] The results were arrayed in Table 18. The yearly data were split into two groups. The number of data in two gorups were N and $N-K$. We calculated the median for each group, for it was natural to test the difference of two medians rather than the difference of two means in nonparametric situations. The hypothesis H_0: $e_1 = e_2$ was rejected only in six countries. All other countries passed the test.

Before introducing the rank-sum test, we checked the randomness of the reserve-import ratio data. The test based on runs was used in Table 19. Each reserve-import ratio was replaced by the letter a if it was larger than the median and by the letter b if it was smaller than the median. N_1 and N_2 denoted the number of b's and a's in the set. U denoted the total number of runs. We obtained $N_1 = 7$, $N_2 = 8$ and $U = 2$ or 3 in the most of countries. Therefore, the hypothesis of randomeness was rejected in almost countries. The fact that the size of U was small and N_1 was nearly equal to N_2

Table 17 Means and Variances of Reserve Ratios to Imports

1957-1971	Mean	Variance		Mean	Variance
Indust. Count.	0.484	0.208	Australia	0.467	0.232
U.S.	0.853	2.774	New Sealand	0.169	0.075
United Kingdom	0.214	0.071	Latin America	0.343	0.020
Indust. Europe	0.453	0.069	Argentina	0.292	0.437
Austria	0.594	0.086	Brazil	0.277	0.174
Belgium	0.349	0.072	Mexico	0.369	0.025
Denmark	0.168	0.025	Peru	0.229	0.170
France	0.405	0.387	Venezuela	0.559	0.111
Germany	0.549	0.220	Other Asia	0.287	0.012
Italy	0.576	0.336	Burma	0.677	0.746
Netherlands	0.321	0.056	Ceylon	0.227	0.228
Norway	0.215	0.014	China, Rep.	0.489	0.186
Sweden	0.197	0.029	India	0.323	0.183
Switzerland	0.963	0.138	Indonesia	0.192	0.556
Canada	0.325	0.044	Korea	0.359	0.175
Japan	0.315	0.311	Malaysia	0.436	0.098
Other Dep. Areas	0.388	0.039	Pakistan	0.354	0.540
Finland	0.220	0.065	Philippines	0.160	0.042
Spain	0.451	0.731	Singapore	0.125	0.021
Turkey	0.380	0.647	Thailand	0.869	0.186

All data are taken from IMF: IFS; 1972 Supplement.

5) Triffin, R., *Gold and the Dollar Crisis*, 1962.
6) Hoel, P. G., *Elementary Statistics*, 2nd ed., 1967, 252–255.

Table 18 Rank-Sun Test for Reserve Ratios to Imports.

	1957-1971 mean	1957-1971 median	*K* years median 1	*N-K* years median 2	(*K*, *N-K*)	rank-sum test *R*
Indust. Count.	0.484	0.470	0.590	0.390	(8, 7)	29 *
U.S.	0.853	0.820	1.180	0.450	(8, 7)	29 *
United Kingdom	0.214	0.200	0.255	0.150	(8, 7)	40
Indust. Europe	0.453	0.470	0.510	0.455	(5,10)	54
Austria	0.594	0.600	0.620	0.570	(8, 7)	48
Belgium	0.349	0.360	0.380	0.345	(5,10)	54
Denmark	0.168	0.160	0.165	0.160	(8, 7)	51
France	0.405	0.390	0.430	0.390	(8, 7)	61
Germany	0.549	0.540	0.580	0.470	(10, 5)	25
Italy	0.576	0.560	0.680	0.520	(8, 7)	41
Netherlands	0.321	0.320	0.365	0.260	(8, 7)	32 *
Norway	0.215	0.210	0.195	0.240	(8, 7)	87 *
Sweden	0.197	0.210	0.220	0.160	(8, 7)	41
Swizerland	0.963	0.960	0.970	0.900	(10, 5)	26
Canada	0.325	0.330	0.360	0.295	(7, 8)	70
Japan	0.315	0.260	0.325	0.240	(8, 7)	43
Other Dev. Areas	0.388	0.390	0.430	0.340	(7, 8)	71
Finland	0.220	0.220	0.240	0.180	(7, 8)	71
Spain	0.451	0.390	0.630	0.350	(8, 7)	51
Turkey	0.380	0.310	0.345	0.240	(8, 7)	46
Australia	0.467	0.500	0.540	0.350	(8, 7)	41
New Zealand	0.169	0.160	0.180	0.105	(9, 6)	33
Latin America	0.343	0.350	0.340	0.350	(8, 7)	63
Argentina	0.292	0.260	0.240	0.340	(8, 7)	67
Brazil	0.277	0.280	0.255	0.290	(8, 7)	64
Mexico	0.369	0.360	0.370	0.345	(7, 8)	73
Peru	0.229	0.220	0.210	0.240	(8, 7)	67
Venezuela	0.559	0.530	0.555	0.530	(8, 7)	58
Other Asia	0.287	0.280	0.295	0.270	(8, 7)	53
Burma	0.677	0.710	0.565	0.780	(8, 7)	71
Ceylon	0.227	0.240	0.245	0.140	(10, 5)	27
China, Rep.	0.489	0.520	0.565	0.420	(8, 7)	43
India	0.323	0.290	0.290	0.260	(8, 7)	58
Indonesia	0.192	0.100	0.170	0.100	(9, 6)	32
Korea	0.359	0.340	0.395	0.310	(8, 7)	43
Malaysia	0.436	0.420	0.380	0.450	(8, 7)	85 *
Pakistan	0.354	0.310	0.450	0.210	(8, 7)	29 *
Philippines	0.160	0.150	0.145	0.200	(8, 7)	71
Singapore	0.125	0.120	0.110	0.130	(10, 5)	53
Thailand	0.869	0.890	0.880	0.890	(8, 7)	58

* means that the mean value does not pass the two sided test with $\alpha = 0.05$.
All data are taken from IMF, *IFS;* 1972 Supplement.

meant that a sequence of identical letters which was proceeded was not easily followed by a different letter. In other words, the initial ratio of reserves to imports was maintained for several years. The sign test for trend offers strong support for this fact. The results of the sign test were tabulated in Table 20.

Another reason for holding reserves is to keep the confidence of domestic money. This kind of holdings can be classified into the precautionary demand. Reserve ratio to domestic money supply or foreign liabilities is utilized as a significant indicator, because the reduction of this ratio looses in the confidence of domestic money. An exam-

Table 19 Test for Randomness

	median	N_1	N_2	U	test
Indust. Count.	0.470	7	8	2	rejected
U.S.	0.820	7	8	2	rejected
United Kingdom	0.200	7	8	4	rejected
Indust, Europe	0.470	7	8	3	rejected
Austria	0.600	7	8	7	accepted
Belgium	0.360	7	8	3	rejected
Denmark	0.160	7	8	5	accepted
France	0.390	7	8	3	rejected
Germany	0.540	7	8	2	rejected
Italy	0.560	7	8	5	accepted
Netherlands	0.320	7	8	3	rejected
Norway	0.210	7	8	2	rejected
Sweden	0.210	7	8	5	accepted
Switzerland	0.960	7	8	3	rejected
Canada	0.330	7	8	6	accepted
Japan	0.260	7	8	4	rejected
Other Dec. Areas	0.390	7	8	7	accepted
Finland	0.220	7	8	6	accepted
Spain	0.390	7	8	4	rejected
Turkey	0.310	6	9	5	accepted
Australia	0.500	7	8	7	accepted
New Zealand	0.160	7	8	7	accepted
Latin America	0.350	5	10	3	rejected
Argentina	0.260	7	8	7	accepted
Brazil	0.280	7	8	7	accepted
Mexico	0.360	7	8	7	accepted
Peru	0.220	7	8	6	accepted
Venezuela	0.530	7	8	7	accepted
Other Asia	0.280	5	10	4	accepted
Burma	0.710	6	9	5	accepted
Ceylon	0.240	5	10	2	rejected
China, Rep.	0.520	7	8	4	rejected
India	0.290	6	9	3	rejected
Indonesia	0.100	7	8	2	rejected
Korea	0.340	6	9	5	accepted
Malaysia	0.420	7	8	6	accepted
Pakistan	0.310	7	8	2	rejected
Philippines	0.150	7	8	8	accepted
Singapore	0.120	7	8	6	accepted
Thailand	0.890	7	8	3	rejected

ple is found in Courchene and Youssef's article,[7] in which the reserve ratio to domestic money supply was called the country's ability of domestic money to spend externally. Table 21–Table 24 represent the regression coefficients estimated by using the same method as Courchene and Youssef. The demand for reserves was a function of domestic money supply modified by the long-term interest rate, r_l, in their article. Our data used were consisted of 56 quarterly time series observations during the period, 1959(I)–1972(IV). We divided all data into 26 quarterly observations such as 1959(I)–1965(II), 1962(II)–1969(I), and 1966(III)–1972(IV). And we tried to estimate the

7) Courchene, T. J. and G. M. Youssef, The Demand for International Reserves, *J. P. E.*, Vol. 75, Aug. 1967.

Table 20 Sign Test for Trend

	S			S	
Indust. Count.	3.031	*	Australia	1.299	
U.S.	5.629	*	New Zealand	0.433	
United Kingdom	2.165	*	Latin America	0.433	
Indust. Europe	0.433		Argentina	0.433	
Austria	0.433		Brazil	0.433	
Belgium	1.299		Mexico	2.165	*
Denmark	1.299		Peru	0.433	
France	2.165	*	Venezuela	1.299	
Germany	0.433		Other Asia	0.433	
Italy	0.433		Burma	1.299	
Netherlands	2.165	*	Ceylon	2.165	*
Norway	0.433		China, Rep.	2.165	*
Sweden	1.299		India	0.433	
Switzerland	2.165	*	Indonesia	2.165	*
Canada	1.299		Korea	0.433	
Japan	1.299		Malaysia	0.433	
Other Dev. Areas	0.433		Pakistan	1.299	
Finland	1.299		Philippines	0.433	
Spain	0.433		Singapore	0.433	
Turkey	1.299		Thailand	0.433	

The countries marked "*" do not have any trend.

regression coefficients in each period. If the Durbin-Watson's ratio was ill-conditioned, we used the iterative procedure[8] to remove auto-correlation in regression residuals. (Table 22 corresponds to Table 1 in the Courchene and Youssef's article.) In all tables, the coefficients, b, listed on columns (1) and (2) were significantly different from zero. The coefficients, b_2, on columns (4) and (7), however, were significantly different from zero only in a few countries. Especially, the estimate, b_2, on column (10) was not significantly different from zero in almost countries. Therefore, it can be said that the variable, r_t, should be deleted from the regression equation. In addition, the coefficient, b_1, on column (9) was negative or not significantly different from zero in about four countries in each table. It must not be negative, because an increase in domestic money supply in circulation will produce theoretically an increase in the holdings of foreign reserves. Therefore, we had better use the estimated coefficient on column (2) as an indicator to explain the confidence of domestic money, which represents the average reserve ratio to domestic money supply. In Table 25, we arrayed the reserve ratios estimated on column (2) in Table 21–Table 24 again.

We often find a wrong idea that the demand for reserves should be consisted entirely of the precautionary demand, for our reserve data available are only official data and the monetary authority does not engaged in foreign trade or speculative activities. For example, Thorn[9] defined the demand for reserves as a function of imports, $R_t=$

8) Johnston, J., *Econometric Method*, 1963, 192–195

9) Thorn, R. S., The Demand for International Demand: A Note in Behalf of the Rejected Hypothesis, *R. E. Stat.*, Vol. 49, Nov. 1967.

Table 21 Regression Estimates for Reserves: Quarterly Data, 1959 (I) – 1972 (IV) (nominal terms)

	$R_t = a + bX_t + e_t$, where X_t equals		$R_t = a + b_1X_t + b_2r_t + e'_t$ where X_t equals						Iterative Procedure for X_t = Money Supply		
	Imports	Money Supply	Imports			Money Supply					
	b (1)	b (2)	b_1 (3)	b_2 (4)	R^2 Dw (5)	b_1 (6)	b_2 (7)	R^2 Dw (8)	b_1 (9)	b_2 (10)	R^2 Dw (11)
Switzerland	2.873	0.446	3.974	−2374.239	0.972	0.551	−1226.170	0.985	0.551	−1226.170	0.985
	*(24.12)	*(35.99)	*(18.13)	*(−5.58)	1.340	*(25.88)	*(−4.69)	1.583	*(25.88)	*(−4.69)	1.583
Netherlands	0.731	0.333	0.563	−880.890	0.955	0.297	−394.849	0.966	0.299	−210.950	0.863
	*(19.43)	*(26.69)	*(12.20)	*(−4.95)	0.702	*(14.68)	*(−2.24)	0.702	*(8.99)	(−0.91)	1.980
Sweden	0.408	0.100	0.786	−2.060	0.696	0.196	−443.133	0.696	0.087	−17.344	0.337
	*(7.02)	*(7.39)	*(3.37)	(−1.67)	0.486	*(4.05)	*(−2.06)	0.482	(1.52)	(−0.06)	1.732
Dweden	0.312	0.305	0.247	140.272	0.513	0.296	23.341	0.664	−0.009	19.365	0.055
	*(4.68)	*(6.79)	(1.33)	(0.37)	0.163	*(4.38)	(0.18)	0.583	(−0.39)	(0.08)	1.639
Germany	1.816	0.557	2.208	−3649.642	0.858	0.654	3194.945	0.890	0.572	−185.168	0.656
	*(11.94)	*(13.92)	*(9.25)	*(−2.09)	0.465	*(11.09)	*(2.18)	0.580	*(5.97)	(−1.04)	1.877
Belgium	0.705	0.327	0.699	317.458	0.920	0.327	−54.560	0.955	0.260	264.626	0.768
	*(17.61)	*(24.27)	*(11.60)	(0.14)	0.587	*(16.67)	(−0.03)	0.482	*(8.60)	(0.23)	1.916
Italy	0.963	0.058	1.067	−70736.434	0.859	0.059	−18166.679	0.863	0.021	−77781.571	0.175
	*(12.45)	*(12.88)	*(8.36)	(−1.03)	0.420	*(8.56)	(−0.29)	0.302	(1.71)	(−1.14)	1.776
Japan	2.465	0.155	5.401	•−1059.053	0.856	0.259	•−763.926	0.962	0.220	•−536.040	0.897
	*(8.47)	*(13.51)	*(9.93)	*(−5.93)	0.430	*(21.99)	*(−10.65)	0.812	*(13.23)	*(−4.93)	1.642
Australia	2.196	0.761	2.130	24.235	0.518	1.071	−489.132	0.814	0.002	160.172	0.219
	*(4.76)	*(9.16)	*(2.72)	(0.11)	0.114	*(8.83)	*(−3.29)	0.371	(0.03)	*(2.05)	1.874

* Significantly different from zero at the 0.05 level.

Table 22 Regression Estimates for Reserves: Quarterly Data, 1959 (I) – 1965 (II) (nominal terms)

	$R_t = a + bX_t + e_t$, where X_t equals		$R_t = a + b_1X_t + b_2r_t + e'_t$ where X_t equals						Iterative Procedure for X_t = Money Supply		
	Imports	Money Supply	Imports			Money Supply					
	b (1)	b (2)	b_1 (3)	b_2 (4)	R^2 Dw (5)	b_1 (6)	b_2 (7)	R^2 Dw (8)	b_1 (9)	b_2 (10)	R^2 Dw (11)
Switzerland	2.617	0.432	2.801	–416.306	0.935	0.511	–1033.193	0.954	0.546	–1088.486	0.893
	*(13.26)	*(14.46)	*(9.65)	(–0.87)	1.390	*(11.82)	*(–2.36)	1.016	*(8.24)	(–1.81)	2.022
Netherlands	0.943	0.452	0.914	–75.087	0.883	0.507	261.855	0.912	0.414	186.856	0.701
	*(9.68)	*(10.95)	*(7.28)	(–0.37)	0.876	*(8.74)	(1.32)	0.779	*(4.43)	(0.88)	1.549
Denmark	1.070	0.399	1.545	–788.943	0.881	0.575	–1187.758	0.930	0.519	–812.141	0.769
	*(7.73)	*(7.95)	*(7.81)	*(–3.02)	1.352	*(10.72)	*(–5.26)	0.515	*(5.79)	*(–2.99)	1.847
Sweden	0.990	0.317	0.909	148.189	0.794	0.221	693.209	0.757	–0.028	–192.051	0.409
	*(6.78)	*(4.79)	*(3.90)	(0.45)	0.957	*(3.18)	*(2.64)	0.935	*(–2.25)	(–1.11)	1.950
Germany	1.532	0.397	1.677	–1728.780	0.718	0.396	45.047	0.739	0.102	593.964	0.106
	*(5.36)	*(5.79)	*(4.74)	(–0.71)	0.360	*(5.06)	(0.02)	0.260	(1.06)	(0.54)	1.838
Belgium	1.243	0.465	1.286	–1376.628	0.928	0.462	390.089	0.964	0.444	–244.938	0.905
	*(12.49)	*(18.56)	*(11.98)	(–1.06)	1.251	*(17.47)	(0.44)	0.972	*(10.36)	(–0.30)	1.928
Italy	0.861	0.089	1.106	–91443.986	0.633	0.195	•–328.494	0.879	0.147	•–131.583	0.591
	*(4.05)	*(4.72)	*(3.98)	(–1.34)	0.697	*(8.89)	*(–5.82)	1.032	*(3.63)	(–1.83)	1.657
Japan	0.665	0.042	0.902	•–106.572	0.768	0.070	•–174.358	0.816	0.009	–55997.983	0.204
	*(5.38)	*(5.25)	*(5.67)	*(–2.16)	0.567	*(6.69)	*(–3.45)	0.516	(0.95)	(–1.71)	1.536
Australia	1.652	0.916	1.551	–519.166	0.783	0.784	–360.580	0.783	–0.031	–262.248	0.200
	*(4.07)	*(5.32)	*(4.81)	*(–3.90)	0.579	*(4.81)	*(–2.58)	0.872	(–0.24)	(–1.69)	2.042

Note: See note for Table 21.

Table 23 Regression Estimates for Reserves: Quarterly Data, 1962 (II) – 1969 (I) (nominal terms)

	$R_t = a + bX_t + e_t$, where X_t equals		$R_t = a + b_1X_t + b_2r_t + e'_t$ where X_t equals						Iterative Procedure for X_t = Money Supply		
	Imports	Money Supply	Imports			Money Supply					
	b (1)	b (2)	b_1 (3)	b_2 (4)	R^2 Dw (5)	b_1 (6)	b_2 (7)	R^2 Dw (8)	b_1 (9)	b_2 (10)	R^2 Dw (11)
Switzerland	2.694	0.397	2.948	−312.166	0.870	0.395	16.686	0.895	0.395	16.686	0.895
	*(9.04)	*(10.35)	*(5.19)	(−0.53)	2.787	*(6.18)	(0.04)	3.016	*(6.18)	(0.04)	3.016
Netherlands	0.546	0.228	0.475	−494.963	0.872	0.201	−390.479	0.879	0.137	−263.164	0.552
	*(7.53)	*(8.38)	*(6.93)	*(−2.84)	0.827	*(7.18)	*(−2.22)	0.855	*(3.02)	(−1.29)	1.571
Denmark	0.395	0.061	0.485	−101.869	0.374	0.025	177.869	0.212	−0.091	−253.387	0.270
	*(2.48)	(1.75)	(1.63)	(−0.36)	0.320	(0.29)	(0.45)	0.307	(−1.20)	(−0.81)	1.580
Sweden	0.196	−0.083	−0.476	1125.696	0.731	−0.134	665.776	0.696	−0.070	−216.816	0.387
	(1.44)	(−1.07)	*(−2.94)	*(5.14)	0.904	*(−2.36)	*(4.82)	0.813	*(−2.42)	(−0.49)	1.753
Germany	0.671	0.237	0.830	−1606.534	0.767	0.274	−1445.127	0.826	0.247	−1366.175	0.755
	*(4.45)	*(5.57)	*(6.04)	*(−3.19)	1.222	*(7.35)	*(−3.56)	1.494	*(5.63)	*(−2.72)	1.684
Belgium	0.532	0.252	0.377	3647.950	0.668	0.214	2327.775	0.782	0.038	218.104	0.215
	*(4.21)	*(6.19)	*(2.50)	(1.73)	0.603	*(4.32)	(1.32)	0.486	(0.81)	(0.42)	1.836
Italy	1.763	0.121	1.700	•−232.991	0.748	0.121	•−341.887	0.927	0.097	•−307.295	0.723
	*(5.39)	*(8.92)	*(5.33)	(−1.63)	0.625	*(11.78)	*(−4.28)	0.689	*(4.22)	*(−2.59)	1.506
Japan	0.319	0.023	0.391	−18557.167	0.504	0.044	−78383.006	0.614	0.015	−36107.534	0.372
	*(2.27)	*(3.65)	*(2.09)	(−0.46)	0.400	*(3.13)	(−1.68)	0.456	*(2.21)	(−1.24)	1.629
Australia	−0.204	−0.060	0.516	−340.328	0.360	0.044	−224.026	0.279	−0.051	−8.976	0.144
	(−0.74)	(−0.74)	(1.26)	*(−2.26)	0.457	(0.46)	(−1.87)	0.326	(−0.67)	(−0.08)	1.690

Note: See note for Table 21.

Table 24 Regression Estimates for Reserves: Quarterly Data, 1966 (III) – 1972 (IV) (nominal terms)

	$R_t = a + bX_t + e_t$, where X_t equals		$R_t = a + b_1X_t + b_2r_t + e'_t$ where X_t equals						Iterative Procedure for X_t = Money Supply		
	Imports	Money Supply	Imports			Money Supply					
	b (1)	b (2)	b_1 (3)	b_2 (4)	R^2 Dw (5)	b_1 (6)	b_2 (7)	R^2 Dw (8)	b_1 (9)	b_2 (10)	R^2 Dw (11)
Switzerland	3.263	0.539	4.247	−3833.639	0.951	0.560	−773.436	0.971	0.560	−773.436	0.971
	*(10.66)	*(19.69)	*(14.00)	*(−4.76)	1.771	*(18.40)	(−1.48)	1.730	*(18.40)	(−1.48)	1.730
Netherlands	0.799	0.385	0.600	−1705.530	0.957	0.321	−832.856	0.957	0.318	−743.345	0.903
	*(8.34)	*(14.21)	*(9.92)	*(−7.27)	1.652	*(9.96)	*(−2.93)	1.104	*(5.97)	(−1.79)	1.964
Denmark	0.244	0.086	0.604	−480.409	0.286	0.161	−42.248	0.454	0.094	−78.045	0.285
	(1.71)	*(2.43)	(1.73)	(−1.13)	0.627	*(2.66)	(−1.51)	0.864	(1.15)	(−0.20)	1.591
Sweden	0.157	0.385	0.715	−1710.805	0.353	0.451	652.416	0.623	0.016	−130.389	0.083
	(0.93)	*(3.68)	*(2.34)	*(−2.14)	0.378	*(4.21)	(−1.76)	0.989	(0.31)	(−0.14)	1.739
Germany	2.614	0.842	2.768	−1891.328	0.890	0.872	−1424.745	0.890	0.850	−2620.601	0.778
	*(7.01)	*(9.93)	*(6.30)	(−0.68)	0.602	*(9.07)	(−0.69)	1.110	*(5.53)	(−0.85)	1.870
Belgium	0.578	0.330	0.673	−11136.029	0.795	0.334	−1637.058	0.861	0.211	2226.176	0.522
	*(6.33)	*(8.75)	*(6.28)	(−1.58)	0.791	*(8.12)	(−0.31)	0.560	*(3.29)	(0.28)	1.522
Italy	0.400	0.026	0.558	−117.666	0.532	0.032	−98875.508	0.653	0.006	•−134.715	0.252
	*(3.09)	*(4.20)	*(3.27)	(−1.38)	0.582	*(4.37)	(−1.44)	0.601	(0.38)	(−1.17)	1.819
Japan	4.483	0.251	6.447	•−1482.312	0.843	0.271	•−578.187	0.961	0.258	•−476.837	0.925
	*(5.70)	*(14.31)	*(7.78)	*(−3.70)	0.613	*(17.21)	*(−3.40)	1.064	*(11.96)	*(−2.13)	1.760
Australia	4.247	1.297	6.412	−393.386	0.355	1.584	−485.988	0.862	−0.049	193.621	0.584
	*(2.29)	*(7.48)	(1.77)	(−0.69)	0.237	*(8.09)	*(−2.47)	0.811	(−0.51)	*(3.20)	1.785

Note: See note for Table 21.

Table 25 Regression Coefficients Estimated from Courchene and Youssef's Equation

	1959(I) ~ 1972(IV)	1959(I) ~ 1965(II)	1962(II) ~ 1969(I)	1966(III) ~ 1972(IV)
Switzerland	0.466	0.432	0.397	0.539
Netherlands	0.333	0.452	0.228	0.385
Denmark	0.100	0.339	0.061	0.086
Sweden	0.305	0.317	−0.083	0.385
Germany	0.557	0.397	0.237	0.842
Belgium	0.327	0.465	0.252	0.330
Italy	0.058	0.089	0.121	0.026
Japan	0.155	0.042	0.023	0.251
Austria	0.761	0.916	−0.060	1.298

Table 26 Growth Rates of R_t and I_t

	$\log R_t = a + b_t + \epsilon_{it}$		D_w	R^2	$\log I_t = \alpha + \beta_t + \epsilon_t$		D_w	R^2
	$\hat{a}$	$\hat{b}$			$\hat{\alpha}$	$\hat{\beta}$		
Austria	6.9253 (52.97)	0.0428 (2.97)	1.19	0.60	7.4809 (233.07)	0.0737 (20.87)	2.22	0.98
Belgium	7.1005 (127.81)	0.0633 (10.36)	1.77	0.94	7.8790 (216.57)	0.1001 (25.02)	1.11	0.99
Canada	7.4334 (98.10)	0.0610 (7.32)	0.80	0.98	8.4223 (134.49)	0.0802 (11.64)	0.43	0.95
Denmark	5.3723 (38.22)	0.0773 (5.00)	0.91	0.79	7.1025 (281.56)	0.0888 (32.01)	2.17	0.99
Finland	5.2709 (32.15)	0.0513 (2.84)	0.93	0.58	6.5696 (112.28)	0.0853 (13.25)	1.55	0.96
France	7.1240 (29.21)	0.1323 (4.93)	0.38	0.79	8.3806 (157.36)	0.1012 (17.28)	0.99	0.98
Germany	8.4574 (80.45)	0.0639 (5.53)	1.60	0.82	8.7600 (226.98)	0.1050 (24.73)	1.06	0.99
Italy	7.6574 (82.80)	0.0783 (7.64)	0.80	0.90	7.9656 (151.16)	0.1133 (19.55)	1.51	0.98
Japan	6.8609 (63.24)	0.0985 (7.73)	0.81	0.90	7.9550 (219.85)	0.1401 (32.98)	1.77	0.99
Netherlands	7.1388 (116.10)	0.0669 (9.89)	1.40	0.93	8.0376 (186.58)	0.0989 (20.89)	0.99	0.98
New Zealand	5.1443 (26.37)	−0.0206 (−0.96)	1.24	0.26	6.5552 (106.63)	0.0356 (5.27)	1.53	0.81
Norway	5.1977 (91.47)	0.1110 (17.76)	1.08	0.97	6.9681 (217.91)	0.0834 (23.73)	1.03	0.99
Sweden	6.2475 (60.50)	0.0451 (3.97)	0.79	0.72	7.6011 (249.38)	0.0829 (24.71)	1.39	0.99
Switzerland	7.4334 (169.96)	0.0793 (16.44)	1.02	0.97	7.3626 (166.18)	0.0961 (19.72)	1.09	0.98
United Kingdom	8.0233 (95.90)	−0.0084 (−0.85)	1.55	0.24	9.3481 (165.91)	0.0886 (14.36)	0.52	0.97

The yearly data (1957-1971) were taken from IMF, *IFS*, 1972 supplement.

$\gamma_0 I_t$. He wrote it in the logarithmic form,

$$\log R_{it} = \alpha_0 + \alpha_1 \log I_{it} + \alpha_2 \log \gamma_{1960} + e_{it}, \tag{7}$$

and estimated the regression coefficients. Though he obtained higher coefficients of determination and significant t-values for the regression coefficients, his model was a tautology. If R_{it} and I_{it} can be written as $R_{it} = R_{i0} \exp(b_{1it})$ and $I_{it} = I_{i0} \exp(b_{2it})$, the definition of the reserve ratio to imports becomes $\gamma_{i0} = (R_{it}/I_{it}) \exp[b_{2i} - b_{1i})t]$, where b_{1i} and b_{2i} are the rates of growth of i^{th} country reserves and imports between the base data for γ_{i0} and the current period in which eq. (7) applies. Rewriting logarithmically,

$$\log R_{it} = (b_{1i} - b_{2i})\, t + \log I_{it} + \log \gamma_{i0}. \tag{8}$$

This equation is identical to Thorn's eq. (7) when $\alpha_0 = (b_{1i} - b_{2i})\, t$ and $\alpha_1 = \alpha_2 = 1$. The conditions which α_1 and α_2 were nearly equal to unity and α_0 was increasing with the passage of time t, were confirmed by Kenen and Yudin.[10] In order to make sure of the validity of the equations, $R_{it} = R_{i0} \exp(b_{1i}\, t)$ and $I_{it} = I_{i0} \exp(b_{2i}\, t)$, we estimated the regression coefficients for them in Table 26.

The precautionary demand for foreign reserves is mainly controlled by the government or the monetary authority of the country concerned. Our eq. (4), however, was only a reflex of the private sectors' behaviors. Accordingly, it has to be modified by the precautionary demand. Namely, the demand for reserves will be shifted by the reserve ratio to imports and the reserve ratio to domestic money supply.

3. The Speculative Motive

As described in section 1, the speculative motive is related to our eq. (1). In the models of portfolio selection applied to the explanation of long-term foreign asset holdings, investors have the opportunity to choose from a whole range of combinations of expected rates of returns and variances by picking up the appropriate size of domestic and foreign assets. Suppose that a world consisting of two countries has only three types of wealth: real assets, money, and bonds. Populations, income, and wealth are constant. Grubel,[11] for example, established the model of internationally diversified portfolios. The expected rate of returns, $E(i_{1,2})$, and risk, $V(i_{1,2})$, on the average investor's portfolio in Country 1 are,

$$E(i_{1,2}) = P_1\, i_1 + P_2\, i_2, \tag{9}$$
$$V(i_{1,2}) = P_1\, \sigma_1 + 2P_1\, P_2\, \sigma_{1,2} + P_2\, \sigma_2{}^2, \tag{10}$$

10) Kenen, P. B. and E. B. Yudin, The Demand for International Reserves: Reply, *R. E. Stat.*, Vol. 49, Nov. 1967.

11) Grubel, H. G., Internationally Diversified Portfolio: Welfare Gains and Capital Flows, *A. E. R.*, Dec. 1968.

$$P_1 + P_2 = 1, \tag{11}$$

where subscripts 1 and 2 refer to the two countries, i_1 and i_2 are the rates of interest on bonds, σ_1^2, σ_2^2, and $\sigma_{1,2}$ variances and covariance of returns, and P_1 and P_2 the proportions of bonds of Country 1 and County 2 respectively held in the average portfolio of Country 1. Eqs. (9)–(11) altogether will depict an efficient frontier curve on the E—V space. The optimal value of the rate of returns and variances will be obtained at the point of the efficient frontier tangent to one of indifference curves, which draw the social expected utility on the same E—V space. Therefore, the optimal holdings of foreign assets (including foreign reserves) depend on the size of total wealth assets held by the public, the size of interest differential, the size of risk differential,the degree of correlation of returns on domestic and foreign assets ($\rho = \sigma_{1,2}/\sigma_1 \sigma_2$), and tastes of the public.

In the models of portfolio balance, money is an asset bearing expected yield in the form of opportunity cost and the least risk. Accordingly, the demand for foreign reserves can be defined as the function of the opportunity cost if the risk is constant or zero. The opportunity cost is usually measured by the rate of returns on long-term government bonds, i, or per capita income, y. Heller's application[12] of the Baumol-Tobin model to foreign reserves is assiciated with this kind of opportunity cost. π stands for the cumulative probability of financing external imbalances by using reserves, R_n. The opportunity cost of holding reserves is equal to iR_n, and the loss of national income is $(1/m)R_n$ if the external imbalances, R_n, is adjusted by expediture-changing policies, where m is the marginal propensity to import. Then the net gain of holding reserves is defined by the equation of $\pi(1/m)R_n - iR_n$. Maximizing this equation, we get $\pi_n = iR_n$, where π_n is the probability with which the country will have to use the n^{th} dollar. Suppose that the country has a consecutive series of deficit balance followed by a consecutive series of surplus balance, the sizes of which are equal to the average absolute change in reserve, h. If the probability of each deficit is assumeed to be fifty per cent, the probability π_n is given by the probability of the occcurrence of x consecutive deficits of size h. Then, $\pi_n = (0.5)^{n/h}$, where $x = n/h$. Optimal holdings of reserves are;

$$R_{opt} = h \frac{\log(im)}{\log 0.5} \,. \tag{12}$$

Heller estimated the optimal holdings of reserves in 1963 for eq. (12).

Table 27 shows the optimal reserves at the end of 1972 estimated for the same equation as Heller. These optimal reserves are the mixture of the precautionary demand with the speculative demand, though Heller claimed that they were the precautionary demand only. He instisted that the government action of adjusting imbalances due to

12) Heller, H. R., Optimal International Reserves, *E. J.*, June 1966.

Table 27 Optional Reserve Holdings in 1972, Computed from Heller's Equation (millions of dollars)

	r	m	h	R_{opt}	R_{1972}	R_{1972}/R_{opt}
Belgium	0.0704	0.418	194.201	987.911	3870.000	3.917
Canada	0.0723	0.186	303.370	1886.023	6050.000	3.208
France	0.0735	0.127	788.707	5318.780	10015.000	1.883
Germany	0.0790	0.145	1644.004	10601.060	23785.000	2.244
Italy	0.0747	0.146	427.663	2788.000	6079.000	2.180
Japan	0.0700	0.068	2362.769	10380.102	18365.000	1.769
Netherlands	0.0688	0.359	205.441	1097.005	4785.000	4.362
Sweden	0.0729	0.180	124.019	775.403	1575.000	2.031
Switzerland	0.0497	0.270	267.708	1665.144	7488.000	4.497
United Kingdom	0.0891	0.161	690.078	4225.841	5647.000	1.336
United States	0.0563	0.048	812.245	6931.175	13150.000	1.897

the precautionary motive would have an dominating influence on holding a certain stock of reserves. In his eq. (12), however, the government action depends on our observation period. Suppose that a country experienced 20 million dollars negative reserve change in 1960. If the observation period is over 1951–1970, the deficit balance to be adjusted by the country is one million dollars per year. If it is over 1955–1964, however, the deficit balance to be adjusted is doubled per year. Accordingly, the government action of adjusting imbalance is determined arbitrarily by our observers. There is not any guide line to the government action. If the observation period is shorter, the government action of adjusting imbalance will more drastic. Therfore, his model does not have any sufficient indicator to the precautionary demand. His opportunity cost of holding reserves, on the other hand, was given directly by the yield forgone if these assets would have been invested. And it was compared to the loss of national income. Who would compare these costs, $\pi(1/m)R_n - iR_n$, ? He neglected that the reserves were the reflex of the transactions and speculative demands in the private sectors. Is it not necessary to make any assumption about investors' maximizing behaviors in his model? If his model has implicitly these investors' behaviors, his R_{opt} will involve the speculative demand for foreign reserves. In short, it can be sai dthat his optimal reserves calculated from eq. (12) were the specialized precautionary demand, in which the speculative demand was unconsciously included.

In Heller's model, the total deficit balance was supposed to be canceled out by the total surplus balance in the observation period. And the yearly reserve changes were corrected for the trend estimated by a least-squares regression, which might have prevailed during the period. The size of yearly deficit or surplus with the probability of fifty per cent, therefore, was h calculated as the mean of the absolute least-squares residuals. Suppose that the yearly reserve changes in a country are observed as shown in Fig. 2. The simple sum of these changes is not equal to zero. The regression residuals, however, are canceled by each other during the period. The typical year-to-year changes in reserves can be graphed hypothetically in Fig. 3, for the absolute least-

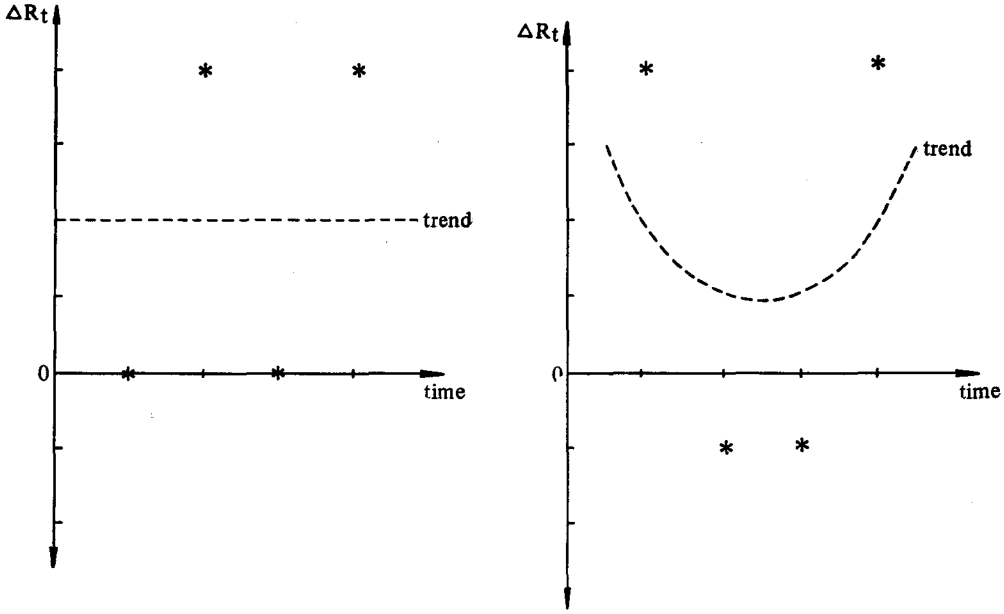

Fig 2 Observed Yearly Reserve Changes (millions of dollars)

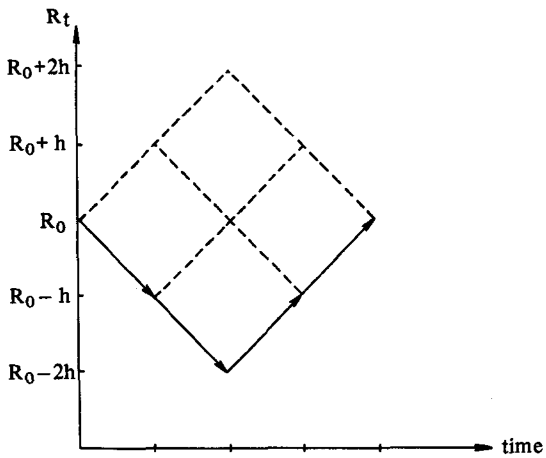

Fig 3 Average Absolute Changes in Reserves ($h = 3$)

squares residuals, h, is two million dollars per year. What kind of the regression equation was used to calculate h in Fig. 2? Heller's purpose was to estimate a "static optimal reserves" which the country held. Therefore, it might be a horizontal line or an ordinaly U-type sine curve, for these two equations had a stable level of reserve changes, a displacement from which would result in a backward movement. To make sure of it, we calculated the values of h from various kinds of regression equations and compared them with the values of Heller's h in Table 28. The last two columns gave

Table 28 The Values of *h* Computed from the Various Kinds of Regression Equations (Yearly data: 1949 – 1963)

	Heller's *h*	Linear Equation	Quatratic Equation	Cubic Equation	U-type Sine curve	Horizontal Line
Belgium	151.0	139.686	138.362	138.821	151.353	152.837
Canada	122.7	137.811	125.125	107.881	123.645	134.755
France	425.1	296.652	310.763	309.247	431.547	419.408
Germany	527.0	524.283	464.043	421.780	463.289	527.071
Italy	275.2	213.443	235.948	158.589	240.632	239.980
Japan	235.8	226.240	229.064	195.085	230.276	226.878
Netherlands	166.4	168.101	165.025	167.280	165.811	169.510
Sweden	58.4	63.056	58.726	56.405	57.111	62.051
Switzerland	100.1	78.286	79.232	79.234	98.916	101.898
United Kingdom	553.8	547.167	550.349	528.162	572.428	568.867
United States	862.0	847.189	845.083	733.346	849.387	869.327

Table 29 Discrepancies and Stable Levels

	Discrepancies		Stable Levels		
			U-type Sine Curve		Horizontal Line
	U-type Sine Curve	Horizontal Line	Initial Level	Final Level	Initial Level = Final Level
Belgium	0.353	1.837	92.174	92.170	70.857
Canada	0.905	12.055	298.875	298.844	101.143
France	6.447	–5.692	589.257	589.213	309.143
Germany	–63.711	0.071	197.853	197.906	532.500
Italy	–24.528	–35.220	–86.162	–86.115	206.857
Japan	–5.524	–8.922	254.971	254.952	130.857
Netherlands	–0.689	3.110	168.996	168.988	122.571
Sweden	–1.289	4.651	80.111	80.103	34.929
Switzerland	–1.184	1.798	123.405	123.402	102.143
United Kingdom	18.628	15.067	203.602	203.585	99.786
United States	–12.613	7.327	–1068.501	–1068.436	–655.786

Table 30 Optional Reserves Holdings in 1960 (millions of dollars)

	R_{opt}	R_{opt}
Belgium	869	872.6
Canada	866	868.8
France	3192	3284.5
Germany	3773	3783.5
Italy	1893	1656.5
Japan	1770	1719.9
Netherlands	933	927.5
Sweden	379	372.3
Switzerland	616	610.5
United Kingdom	3810	3935.8
United States	8085	8176.9

Table 31 Indices of Adequancy of Reserves, Calculated from Heller's Equation

	R_{1963}/R_{opt}	R_{1972}/R_{opt}
Belgium	2.23	3.571
Canada	3.01	2.955
France	1.54	1.738
Germany	2.03	2.035
Italy	1.73	2.001
Japan	1.16	0.948
Netherlands	2.25	4.016
Sweden	2.01	1.868
Switzerland	5.00	4.504
United Kingdom	0.83	1.176
United States	2.08	1.861

us closer values to Heller's h than other columns, except in the case of Italy and the United Kingdom. Which is suitable for the trend, a horizontal line or an ordinary U-type curve? Heller did not make any suggestion about this problem. We made the discrepancies between Heller's h and our values calculated from these two equations in Table 29. The corresponding stable levels of reserve changes were also listed in the same table. Suppose that it is desirable for the countries to maintain a higher level of reserve inflows. It will be easily shown that the best equation to calculate h is the horizontal line for Germany, Italy and the United States, and the U-type sine curve for other countries. Complementarily, we calculated the optimal reserve levels in Table 30, so as to compare them with Heller's R_{opt} in 1963.

Our close inspection just described suggests that the main defect of Heller's model is to be found in the absence of a structual disequilibrium in international accounts. The consecutive imbalance in the country concerned made the least-squares trend shift up or down in his model. But it was not reflected in the optimal reserve level, R_{opt}, because it was eliminated from the calculation of h. This elimination reveals that the country suffering from the consecutive deficit balance may take unexpectedly a high index of adequancy of reserves, $R_{actual}/R_{optimal}$. For example, see the case of the United States in Table 31. In addition, the structual trend in his model was a horizontal line or an ordinary U-type sine curve. These two types of curves were guessed from Heller's aim of study; the measurement of a static reserve level. Why should the structual trend be expressed only in these two curves? It is difficult for us to find a more persuasive answer about this question.

Now, we have to complete our model. As described in section 1, our eq. (4) included the store of values which related implicitly to the speculative motive. Let us modify it to be able to deal with bond market clearly. Suppose that individuals will maximize their utilities obtained from expected interest on bonds, subject to the budget restraint including interest on bonds held on Monday and the purchases of new bonds during the week.[13] D stands for the number of bonds outstanding, each paying \$1 of interest per year perpetually. They are homogeneous and gilt-edged securities. The holdings of bonds on Monday are D/i. Individuals buy $(D^*-D)/i$ of new bonds, receive interest on bonds, D, and carry forward D^*/i in nominal value to the next week. If other things are equal and the effect of bond holdings, D^*/i, on utility is neglected, we get

$$\xi \frac{i}{P} \frac{\partial U}{\partial (D^*/P)} = \lambda, \tag{13}$$

where ξ is the rate at which the holdings of bonds are converted into in dollar terms. If D^*/i is measured in dollars, ξ is equal to one. The variances and covariance of returns are assumed to be constant. Therefore, we will get a new function of reserves which involves explicitly new parameters such as the rates of interest on bonds, and variances and covariance of returns;

$$L = L(\sigma_u, \gamma, \delta, i_j, \sigma_j^2, \sigma_{1,2}, \frac{M}{P}, \xi \frac{D_j}{Pi_j}), \tag{14}$$

where δ is the indicator to the precautionary demand and subscript j is each country.

We have studied the demand for international currencies. The motives for holding them were classified into three heads: the transactions motive, the precautionary motive, and the speculative motive. Our conclusion is eq. (14). When we wish to estimate the demand for international currencies, the Kenen and Yudin's procedures are better than Heller's . But setting p at zero will be wrong, because it means the neglection of time interval between external payments and receipts. Therefore, we will have to devise any other parameter when p is not significantly different from zero. The reason why we reject the Heller's equation can be ascribable to the fact that it eliminates the trend of reserve changes. The reserve level calculated from eq. (12) may be different far from the demand for reserves we wish to estimate.

The foreign exchange rate has been fixed throughout our analysis. In the case of the flexible exchange rate system, however, all parameters in eq. (14) will be changed freely. μ stands for one of parameters, σ_u and γ, to explain the transactions motive. All interactive effects between the transactions motive and the speculative or precautionary motive on marginal utilities, $\partial(\partial U/\partial\pi\ \partial\pi/\partial(L/P))/\partial(L/P)$, $\partial(\partial U/\partial(L/P)/\partial\pi\ \partial\pi/\partial(L/P)$ and $\partial(\partial U/\partial(L/P))/\partial\pi$, are supposed to be zero. Using eqs. (1)–(3), we obtain

$$\frac{d(L/P)}{d\mu} = \frac{\dfrac{\partial\left(\dfrac{\partial U}{\partial \pi}\dfrac{\partial \pi}{\partial (L/P)}\right)}{\partial \pi}\dfrac{d\pi}{d\mu}}{\dfrac{\partial\left(\dfrac{\partial U}{\partial \pi}\dfrac{\partial \pi}{\partial (L/P)}\right)}{\partial \pi}\dfrac{\partial \pi}{\partial (L/P)} + \dfrac{\partial\left(\dfrac{\partial U}{\partial (L/P)}\right)}{\partial (L/P)}} < 0, \tag{15}$$

13) Cf. Gurley, J. G. and E. S. Shaw, *Money in a Theory of Finance*, 1960, chaps. 2 and 3, and mathematical appendix.

where $d\pi/d\mu < 0$, $\partial\pi/\partial(L/P) > 0$ from $\partial\pi/\partial(m/P) < 0$, and $\partial(\partial U/\partial\pi\ \partial\pi/\partial(L/P))/\partial\pi\ \partial\pi/\partial(L/P) < 0$ and $\partial(\partial U/\partial(L/P))/\partial(L/P) < 0$ from diminishing marginal utility. Therefore, an increase in σ_u^2 (the variance of statistically independent part of daily balance of payments) or γ (time interval), which is induced by the variable foreign exchange rate, will reduce the holdings of international currencies.

Eq. (15) will have to be modified by the bond market. A decrease in the rate of interest on foreign bonds, i_2, will increase the foreign reserve holdings. If the risk attached to foreign assets, σ_2^2, or the degree of correlation of returns on domestic and foreign assets, ρ, is greater, on the other hand, the holdings of international currencies will be decreased. The convertor, ξ, will be also changed. Therefore, the sign of eq. (15) will not be easily determined. The parameter δ to explain the precautionary motive, however, will be expected to be a stabilizing factor in the demand for international currencies.

STUDIES FOR THE PREDICTION OF SHIPPING MARKETS —A SURVEY AND THEIR APPLICATIONS—

Tetsuji SHIMOJO

CONTENTS

I Preface

The desire to predict the future of shipping markets has led to many types of effort by those, not only who are concerned in the industry, but also who are interested in the movements of the markets. It has been a common objective since the birth of mankind to know the future situations as early and exactly as may be possible. In their long history, innumerable methods have been proposed and attempted.

Limiting them within prediction in shipping markets, there remain various inheritances from propositions handed down by word of mouth, for example 'A peak a decade' or 'High in winter low in summer' about business cycles in the shipping markets, to propositions of experts, for example 'Charter hire rate index increases before and decreases after the freight rate index.' Though not so explicit and apparent, there are many sorts of experimental rules to predict the future of the shipping markets corresponding to the complexity of the cases. And some of them became more reliable and some were forgotten through the tests of experiences.

In fact, the methods actually being used to predict the markets and to forecast future movements of cargo can hardly be other than such experimental rules. Futhermore, most of them are merely assumptions deduced from the past experiences of the forecasters, rather than propositions examined by the real data. And many of them are so vague that they can not be expressed in any forms of propositions. Nevertheless, it can not be denied that they are used most frequently in the real world and that they have been giving actual results of their own. This type of prediction, generally known as intuitive prediction, is apt to be thought as a very simple method, but, in fact, the procedures employed use very many experimental rules and very complex calculations. That is the reason why the method had never been described in terms of literal deduction, nor attested theoretically.[1)]

Another type of prediction, far simpler compared with the intuitive prediction, is actually used. Though it can not be compared with the intuitive prediction in accuracy and delicacy, it is peculiar in view of its objectivity and repetitiveness. Because of the fact, this method is used most commonly in the practical world to complement the intuitive prediction, and is calculated repeatedly till the user is satisfied. This is the easiest method of prediction as neither the experimental rules nor the strict theory are necessary, and everybody can use this method with a little ability to collect data and to calculate them. We can call it the clerical prediction.

The intuitive prediction, assisted by the clerical prediction, is now the dominant practical method of prediction in the real world. In contrast with it, the journalistic prediction, bureaucratic prediction, or academic prediction can hardly satisfy the practical requirements, because the objects of these methods are too general to match practical needs, though it is natural in view of their inherent characteristics.

In this paper, we will survey the various methods of prediction and the fundamental studies advocating them, limiting them within the forecast of shipping markerts, and consider what efforts are needed in order to build more practical methods of prediction. This objective would seem to be an intention to improve the practicability of bureaucratic and academic prediction, but, in our consciousness, it is rather an intention to raise the level of the clerical prediction which can be combined with the intuitive prediction.

II Collection and Calculation of Data

The most important task for the clerical prediction is collecting data. Some kinds of prediction can be finalized with only appropriate data. Bross says that the simplest method of prediction, persistency prediction, can often give us the most correct prediction.[2)] Port charges and turn-round times in the port at which they are making their

1) Shimojo [50]. By the way, Shimojo [47] classified preditions into (1) intuitive, (2) journalistic, (3) bureaucratic, and (4) academic predictions.

2) Bross, Irwin D. J., 'Design for Decision', New York, 1953, Chap. 3. He listed as simple methods of prediction, (1) persistency, (2) trejectory, (3) cyclic, (4) associative, (5) analogue, and (6) hindsight predictions.

ship call can be predicted by the data of other ships which called most recently at the port.

Prediction of the future situations should begin with an investigation on the same objects at the times closest to the present. Predictions utilizing this information as it is, or some modification of the data involving the rates of change or smoothing depend upon a premise that the circumstances at the times closest to the present will remain unchanged in the near future. As far as the premise would be accepted, these methods of prediction are the simplest ones.

Table 1 shows some results of these very simple methods of prediction, namely, the correlation coefficients and the percentages of errors to the mean values of their original data when (A) the persistency prediction and (B) the trajectory prediction are used for the indexes and monthly average freight rates in the several routes. The correlation coefficients are calculated between the predicted value p_t and the actual value A_t, and percentages of the standard diviations of (P_t-A_t) to the mean value $\sum_{t=1}^{n} A_t/n$ are added to the table. Needless to say, the persistency prediction means

$$P_t = A_{t-1},$$

and the trajectory prediction is

$$P_t = A_{t-1}+(A_{t-1}-A_{t-2}).$$

Table 1 Correlation and errors of very simple methods

Name of Data	Period	Monthly Data				Quarterly Data			
		A		B		A		B	
Norwegian Voyage Charter Index	50–75	.9833	6.3%	.9828	6.6%	.9167	14.1%	.9368	14.0%
Norwegian Tanker Freight Index	50–74	.9436	24.8	.9437	26.3	.7731	48.1	.7690	56.1
Gulf/Japan Grain Rates	56–72	.9577	10.9	.8763	20.3	.8858	17.6	.8390	27.3
H. R./Japan Coal Rates	53–74	.9642	13.2	.9326	18.9	.9128	20.6	.8837	26.6
Gulf/U.K. Grain Rates	54–74	.9327	19.6	.8654	29.5	.8778	26.0	.7914	38.0
P. G./Japan Tanker Rates	52–73	.8910	33.1	.8120	48.1	.7445	47.3	.6720	65.5
P. G./U.K. & C. Tanker Rates	52–73	.9189	33.7	.8517	49.5	.7329	56.7	.6613	76.6

Observing the table, we can pick up some problems important for our present investigation. In Table 1, we took up only the freight rates that may be the most interesting object for the shipping markets, but it is worth trying to take up the amount of international trade, the sea-borne cargo movements, the tonnage of fleet, etc. as the objects of prediction.

(1) Prediction period: The first problem is about the period between the time of prediction and the object time of the prediction. We will call this period the prediction period. The accuracy of prediction is likely to be higher, the shorter the prediction period. This matter has a close connection with the problem of how strong a confidence the predicter can have in his own prediction.[3] In Table 1, compare the precision in the prediction of monthly data with that of quarterly data. There can be found apparent differences.
(2) Nature of data: Another kind of difference in the prescision is found in the Table 1. This difference depends on the natures of indicators. In general, predictions of tramp rates are more precise than that of tanker. Are they caused by the differences between the nature of tramp market and that of tanker market, or between the methods used to make up these indicators?
(3) Coverage of data: The most important nature of data is the coverage, which means how many components the data are representing. Even if every individual event is random, the more events the data may include, the less the randomness of data will be. We can see in Table 1 that freight indexes are more precise than the freight rates as the former have bigger coverage than the latter.

Through the observation of Table 1, we can list up the three problems so as to select the method of prediction. In addition to them, we must consider the following problems;
(4) Divisible and indivisible indicators: Freight rates and their indexes are indivisible indicators in the sense that no part of the value corresponds to any real things. Tonnages of fleet and cargo are, on the contrary, in this sense, the divisible indicators, although they are distinguished into the stock and flow quantities respectively in another classification of statistics. If two ways of classification of statistical data were applied to the shipping market data, there are defined four types of statistics; stock divisible (fleet), stock indivisible. (rate level), flow divisible (cargo movement), and flow indivisible (rate index). Such peculiarities of the data will be important problems at our processing the data and selecting the method of prediction.
(5) Change of natures by calculation: The problems mentioned above will make new problems during the calculation of data. Monthly data will become improper for the persistency prediction when they have been processed into quarterly data, and they will lose their seasonal characteristics when they have been processed into annual data. Annual rates of change of the tonnages may be indivisible data, and an average of several indicators may have larger coverage and less variableness.

Most of the published indicators have been processed by some definite intensions, so that we must be aware of the methods by which they were processed, in order to use

3) Shimojo [56] Appendix I.

them properly for our purposes. Many of the published data, however, can not fit our purposes as they are. And modification of data to fit our purposes will require us to prepare much more information. Otherwise, we must overcome the difference between the data and our intention by means of the alteration of our model.

In a sense of fitness to our purposes, the raw data are the most favorable as they have less definite intention. There remain, however, some difficulties in dealing with the raw data, the difficulty of collecting them and the difficulty of processing them because of their massive volume.

In case where raw data of less volume are sufficient, for example in statistical inference from sampling, we can more easily deal with raw data by means of classification and summation for the purpose of prediction. Considering the change of natures by calculation, however, we must always ask ourselves which should be done before others, prediction, classification or summation.

"Decomposition" is one of the most popular method of the clerical prediction. Dividing the object of prediction properly, some simple prediction will be proceeded in each of the division, and finally the results of each division will be consolidated into a conclusion. In this method, peculiar consideration of each division can be made, so that this may be a thoroughgoing prediction.

We have discussed some problems in the stage of clerical prediction, where a single indicator is the object of simple prediction. For this popular method, there is no outstanding work we can quote, so that we can not help making an abstract discussion. For the sake of our objective, however, the following chapters shall compensate for the lack of concreteness of this chapter.

III Analyses of Relations Between Indicators

No indicator can exist independently of the others. Especially, economic indicators, always have some sort of relationship with other economic indicators or past values of their own. Knowledge about these relations between various indicators, therefore, will be an important piece of information for prediction purposes. For this reason, analyses of relationship between indicators are very important tasks.

As far as prediction is concerned, the following four types analyses of relationship between the indicators have been made;

1. Relation to its own past value

Putting a time series back for some periods and calculating correlation coefficients with its original time series for every periods put back, we can obtain a peculiar curve named a correlogram. Similarly, we can also estimate parameters for a multivariate equation in which a time series is explained by itself with time lags of 1, 2, 3, etc. Correlation coefficients and parameters obtained in such methods will offer useful information about cycles of the indicators.[4] We can predict their future value with some significant information we obtained.

There is a study to attempt prediction by a transient probability matrix of freight rates during a period.[5] This matrix will be almost a unitary matrix as far as we can observe in Table 1. While, there were observed the relationships between the rate level and the changes of freight rates. The higher the rate level may be, the bigger the changes of freight rates, so that it will be more difficult to predict the future in the prosperous times of the market.

The most apparent cyclical movement in the shipping market indicators may be the seasonal fluctuations, of which a lot of studies have been made.[6] Some other studies have employed the Census Method or the spectral analysis method to analyze the other fluctuations of the shipping market indicators.[7]

The common intention of these studies is to attempt to find some cyclic patterns out of the shipping market indicators and to discover the peculiarity in their movements through pursuing the causes of the cyclic patterns. It may not be intended to utilize the cyclic pattern directly in the prediction of the shipping markets. We have to bear in mind that these studies offered very much information useful for the later analyses.[8]

2. Sequential relations between indicators

When we can observe an indicator always preceding another, we can predict the future movement of the latter through observation of the preceding indicator. Though such an apparent relation can hardly seen, it is often said that charter hire increases before and decreases after freight rate, and we can find not a few studies to prove the saying.[9] There are also some important indications that the freight rates and charter hire rates may have close relationships with the length of charter periods or frequency of future contracts, or with the balance between rates for outward and homeward voyages.[10]

3. Economic causal relations

Since the econometric methods came to be used in the shipping market forecasting, many attempts have been made to build models consisting of some equations which express the economic causal relations between variables in the shipping markets. Starting from the basic assumption that freight levels are determined by the

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4) Shimojo [49] shows correlogram, and distributed lag model is used in Miyashita [24].

5) Hiramoto [6].

6) Sawa [35], Matsumoto [21] and Lewis [17] are especially important works.

7) Yamada [41] utilized Census Method, and Moridaira [28] utilized the spectral analysis on freight rates.

8) Isserlis [11], Wogemam [44], Tinbergen [39], Einarsen [5], Meuldijk [22] and Shimojo [48].

9) Lewis [17], Maeda [18], Weston [45] etc.

10) Lewis [17], Shimojo [54], Zannetos [46].

demand and supply of the shipping services, the freight level function has traditionally been explained by the quantity of cargo movements and the tonnage of fleet,[11] though each of these variables are occasionally replaced by the rate of change of each variable.[12]

Freight level has been considered to be related the prices of cargo carried, the distance carried, the quantity of fleet operated or the laid-up tonnage, etc., and many studies have adopted these variables as their explanatory variables.[13] Some studies use the compound variables processed from demand and supply variables to explain the freight levels.[14]

Most of the models quoted above center around the freight level function, in which many kinds of explanatory variables are used. Meanwhile, these explanatory variables have to be explained by other factors, so that the models of several equations came to be seen gradually often along with the possibility of utilizing the computers. We have, however, to postpone these models to the next chapter.

4. Statistical causal relations

It is the fundamental relation that the demand and supply reflect the prices, and that the prices give influences to the quantities of demand and supply. However, even if the changes of various quantities are observed in the shipping markets, we cannot state how much changes they will finally give to the freight markets, before the relation has been measured between the change in freight rates and the changes in the quantities of demand and supply.

Poor harvest of grain or strikes in coal mine or ports are reflected in the shipping markets in different way according to times and places they may take place. Many events are reported daily, but most of them are neither quantitative nor measurable, so that we can not use them for our prediction. Even if they are quantitative, their relation to the shipping markets must have been known.

There is a study that classifies the various events occurred around the shipping markets, and that deduces the statistical relations between the frequencies of each class of events and the changes of shipping markets.[15] This study aims the statistical causal relations rather than the economic causal relations. These types of studies seem more to be contributing in searching the factors which have close relation to the shipping markets than to be useful for the prediction.

By the similar methods, it is possible in the future to evaluate the non-numerical data in relation to the shipping markets, and to deduce the behavioural patterns of the

11) Matsumoto [20], Tinbergen [40] and Koopmans [15] are predecessors of the series.

12) Kogane [14], Mori [26], Osawa [33], Suguki [38], etc.

13) Matsumoto [19], Bates [1], Deakin [3], Miyashita [25], etc.

14) Quantity carried per carrying capacity in Koopmans [16], Shipping market estimating index in Yoshimura [43], degree of busyness in Shimojo [49], etc.

15) Shimojo & Yoshida [52].

ship-owners and charterers from the contracts they have concluded in the past. We expect the more convenient methods and the more appropriate applications of them will be developed in the near future.

* * *

The analyses of relations between shipping indicators can not always be used directly in prediction of the markets. Their contribution to the prediction was to supply us with information on such things as parameters in behavioural equation, adoptability of the form of equation in, say, the freight function, effects of processing and combining of variables, cycles of freight rates and their index, changes of seasonal fluctuation patterns, and so on. And it must be the function of clerical prediction to use this informations.

To improve the levels of clerical prediction with this information, another important consideration must be given. It is usually so long time since the information has been found out or estimated that it can not be employed in our present purpose. We must estimate new parameters with the latest data. Not only that, but we may have to modify the models and processing method of variables. That is, we need to update this information including the methods of estimation.

IV Models for Prediction of Shipping Markets

Based on information obtained through the analyses of relations among various indicators in the shipping markets, we can build models for prediction of the shipping markets in a wider sense. Models having only one equation are also included, but we will hereafter discuss the common problems relating to models consisting of multiple equations. One of the purposes for which we use the multi-equation systems is to make the models dynamic and appropriate for prediction.

There are not so many studies in model building and prediction of shipping markets, and we can find only a few examples which consist of several equations including a freight determining function. That may be not only because of difficulty in making a function determining the freight level, but also because most of such models are intended to predict the demand on shipbuilding, in which the variable of freight level stand merely as an exogeneous variable.[16)]

Meanwhile, the models taking the shipping markets as their centre and explaining the processes of determining the freight levels and chartering hire rate levels can be classified into several types—the econometric models, the system dynamic models, the models in dynamic programming, the behavioural distribution models, and so on.

16) Chapter IV of Shimojo [51]. There appeared thereafter some models, e.g. Japanese Shipbuilders Association Models (one of which was Suganuma [37]), Ishiwata-Moridaira [9], and Norman [31].

Of these, we can now discuss the following three types of simulating models.[17]

1. Econometric models

There are a few econometric models which include the freight level indicators as endogenous variables, such as Ishiguro model and Yamada model both of which relate to the world shipping markets, Kagoike's tramp market model, Niida model, Suzuki model and Ishiwata-Moridaira model, the last three of which are Tanker market models.[18] We must not fail to refer to the Charemza's tanker market model of disequilibrium.[19] In recent years, we heard about models that were actually used for prediction of the shipping market, but we do not know these models in detail.

All of these models are built around three most important variables, the freight level, the quantity of demand and quantity of supply, and place other variables to explain them, in almost similar way which the freight level reflect the tonnage of ships supplied through the orders for new building of ships and the laying-up of ships. And it is a common idea in these models that the freight level doesn't feed back to the quantity of demand because its reflection can be negligible in the macro-scopic models.

2. System dynamic models

The only model is presented by Raff who utilizes the techniques of Industrial Dynamics developed by Forrester.[20] This model traces a logical series from the world demand for petroleum oil to the tanker markets, though the behavioural decisions of oil companies to operate their own and chartered fleet of tankers, and of independent owners to coordinate their fleet. This model contains about 350 equations and enable us to observe the movement under definite conditions. These types of models are expressed in the language named DYNAMO and simulated under the program of the same name.

The Ruff's model, in general with this type of model, is a system of the popular or assumptional logics described in each of the equations. Parameters describing the relations between variables are given by experience or as selectable assumption. This is a peculiarity of this type of model, in comparison with the econometric models in which the estimation of parameters is one of the most important tasks.

This peculiarity makes the type of model able not only to determine the values of variates for any period but also to create any sort of variables regardless of data available. Raff did not intend to predict but proposed a set of assumptions concerning the structure of tanker markets. This model, however, shows us the possibility that we can use these types of models in prediction of the market if we would model the freight

17) Dynamic Programming Model of Devanny [4] Part II is neglected here, as it is an optimizing model of operating plans in the shipping firms.

18) Ishiguro [7], Yamada [42], Kagoike [13], Niida [30], and Ishiwata-Moridaira [10].

19) Charemza [2].

20) Raff [34] uses the technique developed by Jay W. Forrester of M. I. T.

determining processes in detail and supply them with accurate data.

3. Behavioural distribution model

Johansen's production function can be applied in the shipping markets.21) We can use the supply schedule which is made by summation from the individual production function of each ship. By giving the cost informations for the world merchant fleet by flag, age, kind, size, etc., we may have a list of assumed or nearly actual world merchant fleet. Under some various conditions, classification and summation will produce respective cost distribution of the fleet, or supply schedule.22)

We can generate a lot of cargo offers in similar way by route and by item. Simulating the processes these cargo offers would select appropriate ships, we could build a model determining the freight and charter hire rate level in any submarkets. Needless to say, these types of models can not reproduce the reality even if the same distributions as reality were given to them. These can simulate not more than stochastically same situation of the market. But they can be useful in such a prediction as, say, what changes will take place on the share of Japanese fleet in case of an increase in the seamen's wages.23)

* * *

These three types of models have their own peculiarities and problems. In our view, their respective peculiarities are complementary to others, not substitutional for each other. To this point we will discuss below.24)

The econometric models are the most popular for us, and the most advanced theoretically and experimentally. They are, especially treating with macroscopic structures, the most powerful technique, as far as the necessary data can be furnished. As we have stated, however, data around the shipping markets are very poor, not only that, but it is almost impossible to obtain immediate data necessary for the prediction, as many of them are covering world-wide matters. This point can not be recovered by only the methods in econometrics.

In contrast, the system dynamic models and the behavioural distribution models do not depend so much on data. These models are built more freely by means of the parameters and distributions which can be measured mainly from the cross sectional data at the latest point of time. They can be modified and improved by the frequent simulations. If needed, they can contain the equation that changes its form according to changes of situations, and be designed to select one of more submodels accordingly.

The shipping markets are excellently dependent, and one of the most sensible

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21) Johansen [12].

22) Devanny [4] Part I seems to us to be a model of similar idea, but we can not obtain any detailed information.

23) Shimojo [55] is an attempt of the kind.

24) See Appendix for the detailed discussion of these three types of models.

objects, to environmental changes, so that the prediction of them must utilize the most elastic models. Placing the econometric models in the center, and preserving the economic and statistical strictness, it will be necessary as a minimum to use the system dynamic models and the behavioural distribution models in the role that they can play most effectively for their respective characters.

V Tentative Conclusion for the Prediction System of the Shipping Markets

It might not be true that the prediction of shipping markets was the most important problem for the shipping firms and the bureaus concerning to the shipping industry. Moreover, even if it can be an important problem, the one they think so may not be the predictions which we have been discussing above. Prediction needed by the practical interests would be more concrete, e.g. a freight rate appropriate for a certain item of cargo to be carried through a certain route, or a charter hire rate for a certain newly built ship, etc.

There are many more uncontrollable factors, for the shipping interests than those dealt with by most of the models discussed above. For the uncontrollable factors, we must as firms decide our countermeasures through the prediction of them.

Limiting our objective the prediction of a freight rate for a certain route, we will face many more difficulties than we have discussed above. One of the biggest problem is the data. The data around the shipping markets are limited. For this reason, the shipping market models will be very small ones which contain, at most, only ten or several more equations, including the shipbuilding markets.

The shipping market is a truely complex market. It is said to be divided into innumerable submarkets, by routes, items of cargo, forms of transport, forms of contracts, etc. Nevertheless, the data is scarce for these individual submarkets. Although we can find a study for relations among these submarkets placing an overburden on the data,[25] the scarcity of data seems to be preventing the development of the study.

We must start immediately to furnish the data, but such efforts cannot make fruits in short time. It may be impossible for the shipping market models of econometric type to respond immediately the practical needs. Whatever the structual changes mean to be, in order to include it into our model, we can not help inducing somewhat different types of techniques. For a prediction method which is able to use any pieces of data very immediately, is there nothing but the intuitive prediction traditionally having been employed?

The intuitive prediction method has many strong points that the other methods do not have, and at the same time, it has many vital weak points. In order to recover

25) Miyashita [24].

the weak points, the clerical prediction method has been used in the practical world. We have surveyed the techniques to be induced in the clerical prediction. We believe that the mechanization of prediction must be necessary in order to make the clerical prediction exert its ability with all of these effective weapons.

The ideal prediction system of shipping markets must contain at least the following abilities in addition to the functions the present clerical prediction system already has;

(1) Abilities to compile, estimate, test, update and simulate the econometric, system dynamic and behavioural distribution models.

(2) Abilities easily to register, update and retrieve the data of any types; time series, cross section, surveyed and individual.

(3) Abilities to numericize the non-numeric data through dialogues with the decision maker, in order to give an explicit basis to his intuitive prediction.

Our tentative problem is (1) of the above, to build a new consolidated model. Firstly we have to pick up many problems in the course of examining various types of models proposed in the past, to investigate the causes of the problems, and to search for new techniques to delete or overcome the causes. Such series of efforts and ideas will, we believe, be the foundation of more effective prediction systems. We can only refer to a fact that a voluntary group has started its activity for the purpose.

Nevertheless, the probelm of (2) is still the most difficult problem for us, for which we have nothing to offer. There must be much of data, concrete and precise, being used in the practical activities of shipping firms. Even the data not used any longer by firms may contain much informations useful for us. That is the reason why we would like to expect the conprehension and cooperation of the shipping firms to our efforts.

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APPENDIX THREE TYPES OF SIMULATION MODELS IN SHIPPING MARKETS

Foreword

The shipping market is an excellently competitive market. Demanders as well as suppliers of shipping services should decide their own behaviours, by observing the movements of shipping markets. For the suppliers of the shipping services, above all, the prices are far beyond their control, and only to be expected and predicted, in accordance with N. Wiener's words; "There are two sorts of variates. We can control the one. But we cannot control the other variate so that we must predict its movement in the future."

The shipping markets that determine the ocean freight rates, are not only of very complex structures, but also reflect very delicately various events in the world and change their structures bit by bit. Attempts to simulate the market movement by some models, therefore, in spite of their frequent and long time efforts, will be successful for only a very short time.[1)]

In this paper, we have described these efforts in short, dividing them into three types of simulation models. Here, we will discuss three models out of respective types, in order to pursue for their peculiarities in more detail, and investigate to the causes in the shipping market that have made it difficult for us to build such simulating models.

1) See Shimojo [51].

I Econometric Models

Grasping the shipping markets in macro-scopic way, it is a peculiarity of this type of models to explain the processes of determining some general freight market indicator in the world. The tramp freight rate index or tanker freight index are used as the freight market indicator according to their objective.

We will take up Ishiwata-Moridaira Model as an example of the type of model.[2] This was developed as a short-term prediction model of the tanker markets, and consisted of four sectors, cargo movement, tonnage of fleet, shipbuilding and freight markets. This included 17 variables, apart from 6 dummy variables, constituting 8 behavioural functions and 5 definitional equations in quarterly basis. This is rather small in comparison with the general econometric models, but the largest one for the shipping markets. That may be caused by one of characteristics of the shipping market models.

In this model, the freight market indicator is explained by the rate of change in cargo movement, freight market indicator of the last period, and rate of change in the freight market indicator during the last period. The tonnage of fleet is not used to determine the freight market indicator, but, instead, it is explained by the past freight market indicator through the quantity of shipbuilding.

Generally speaking, the model closely follows the theoretical structure. The theoretical structure itself, however, is a system of assumptions and the one described very abstractly. It must, therefore, be a job of 'high degree of freedom' to specify a model suitable for the theoretical structure corresponding the variables to those data which are available actually. Even though it can be done rather freely, the econometric model could not be free from their inherent restrictions.

One of the restrictions is that a sufficient length of time series must be obtained for every variable in the model. Every variate in the theoretical structure can not always be observed in reality as it is. We must find something else as similar as possible, or make ourselves some substitutes by means of processing other data.

For the quantity demanded for shipping services, the seaborne cargo movements have been used traditionally. They are, however, the quantity transported or supplied ex-post rather than demanded. Meanwhile, the tonnage of fleet used as the quantity supplied is not more than the capacity supplied, which is very inelastic in short term. Therefore, for a macro-model of shipping market, a form of explaining the rate of changes of freight rates by means of the rates of changes of quantity transported might be the most reasonable. The Ishiwata-Moridaira model includes an effect of expectation in addition to the above.

2) Ishiwata-Moridaira [10] is the newest, in which many experiences in the past are employed.

Shipping market is not so simple as can be expressed by only ten equations or such. Nevertheless, the reason why no larger models had been made may be due to the fact that only a limited number of variates can be furnished in forms of time series data. In comparison with tramp market, there have been more examples of analysis and modeling for tanker market, because of availability of data and less complexity in cargo items.

Tramp market can not been expressed satisfactorily in such a simple model. The number of variables will be multiplied according to variety of cargo items, and, consequently, will need much more equations. We can never find any data for sufficient length of times to support such a complex theoretical structure. This is the one of the obstructions in the way of the econometric analysis of shipping markets.

Another restriction to them is a problem of time lags. The relations between variates appeared in the theoretical structure are sometimes expressed with some time lags. These time lags are included in model by means of difference equations or distributed lag models. Apart from time intervals of actually available data, the restriction of a fixed 'difference' of time is unavoidable for this type of model[3].

In order to build up more precise models according to the theoretical structure, we must resolve these two problems. We must, therefore, develop a model of a new type which can be effective to both whole and part; i.e. on the one hand, the model may divide the market into various routes, items of cargo, types of contracts, etc. or the fleet into several types, sizes, flags, etc., and on the other hand the model may sum them up to the total demand classified by each behavioural pattern and to the total supply sorted by each rank of their operating costs.

Moreover, in order to trace the processes of determining and varying the freight rates or charter hire rates, which had been united into a concept of freight level, we must employ a more elastic way of expression. Elasticity of expectation and periodical substitution in future transactions must be included in the model in order to express the bargaining processes of contracts by means of various lead times and durations. We should overcome the restrictions on the data and techniques with all effort for these purposes.

II Technological Sequential Models

Even the most complex technological structure in a macroscopic view can be divided into the simplest physical rules. The shipping market, if it be grasped as an accumulation of individual behaviours, could be expressed as a series of comparatively simple decision making as well. Raff proposed a tanker market model of this type by

3) For the variety in the shipping business cycles, see Moridaira [28] and Shimojo [48].

means of System Dynamics.[4)]

In this model, when the demand of oil is given as an exogeneous variable, oil companies will decide their orders to build new tankers and to charter tankers according to their operating fleet of tankers. Tanker brokers consequently decide the charter hire rates through negotiations with the tanker owners, who decide to increase or decrease their own fleet of tankers. Shipbuilders receiving orders to build new tankers, will deliver new tankers to oil companies and tanker owners with some time lags according to availability of their building docks. The tanker market will develop in accordance with the movement of demand and supply of tanker services, conserving these connections.

Raff's model consists of seven sectors, i.e., four sectors in oil company, oil supplying, chartering, operating and coordinating sectors, in addition to three environmental sectors, tanker broker, tanker owner, and shipbuilder. Each sector, utilizing informations input from other sectors, decides his behaviour and behaves it. These behaviours are described in about 350 equations.

These 350 equations can be sorted into four types of equations. Level equations define changes of variates with a certain time lag. Rate equations define the rates of changes. Auxiliary equations are to define relations between variables And initial equations define initial values of the variables. In addition to that, there are various types of functions which express some definite distribution of values or some time lags. In any way, these equations do not always need steps that estimate their parameters by means of data in the past.

Each equation represents a very simple relation between the system variables. Informations apriori or parameters in these equations seem to be deduced by a bit of empirical observations, so that we will not need much detailed informations in the past. This may be a peculiarity of the models of this type.

As an aspect that determines freight rates or charter hire rates we can trace the model as follows: Given the total tonnages of tanker fleet existing and being operated, the tanker broker sector calculates the ratio of operating fleet, and deduces a charter hire rate level immediately from a static supply scheme which is prepared to indicate correspondences between the charter hire rates and the ratio of operating fleet. At the same time, the tanker broker sector supplies an expectation of the charter hire rates, which will be employed by the rest of sectors for their future planning.

The supply scheme in this model is a definite relation between the two variables, and is based on an assumption that the scheme will not be affected by the variation of operating costs of tankers or other factors in the shipping markets. The scheme shapes a curve with a minimum level up to 90% of the ratio of operating fleet, steep

4) Raff [34].

upward up to 97%, and with constant level onward.

The supply curve of shipping services, in general, having the ratio of operating fleet as abscissa, seems to converge to a point, $x = 1$, $y =$ lay-up point freight rate of the most economical ship, and looks like a hyperbola in the senond quadrant. Raff gives a similar curve as his supply scheme, but gives no idea how he has deduced it. We can consider it to be arbitrary assumption, as well as the other parameters of the model.

In contrast to the macroscopic inclination of the econometric models, the technological sequential model seems to have a rather microscopic inclination. These peculiarities might be caused by the fact that the former depended upon the time series data while the latter intended to approach to individual logical processes. We can attempt, therefore, to combine these two types of models. Operations to summarize the results obtained from the simple logical models, to deliver the summation to the macro-model, and to divide the results obtained by the macro-model, will not cause serious errors if the time intervals will be small enough.

In the clerical systems for prediction of shipping markets which are very frequently utilized in shipping firms, there can be frequently seen the decomposition method in which the value of variable is predicted for each subdivision of the system and then the conclusion will be deduced from them by means of summation or other simple consolidation. A simulation model of the shipping markets can be built up of such forms of operations.

The behaviour of each subdivision can be described as comparatively simple logical process in this type of model, but, if we assume that their behaviour is decided by taking the whole behaviour of the market into their consideration, the simulation will need a large number of repetition to summarize and to divide. Such a process will be disposed as a simultaneous equation system in most of econometric models, but this is not always possible.

No simultaneous equation system is accepted by the System Dynamics, but instead, the system is recommended to be expressed as a sequential system with much smaller time intervals. Such iterative solution is employed by the econometric models containing non-linear equations as well. If we do not place any severe restrictions on the time intervals in the model, we will be able to express all kind of models as sequential systems, and simulation will be much easier.

III Behavioural Distribution Models

The supply scheme in Raff model was given as a static and definite one. But, in general, the scheme should be varied from time to time according to various environmental changes. A supply scheme placing the supply capacity on the abscissa seems to be given its shape by the distribution of operation costs of the fleet. Above all, to know position of a certain ship group in the scheme will require us to draw a total supply scheme by means of the cost distribution of the fleet.

Johansen using the data of 377 Norwegian tankers supplied by Eide,[5] deduced a macroscopic and short-term production function from their capacity distribution. Carrying capacity per ton-mile of each tanker was firstly classified by both amounts of fuel consumption and labour necessity, and the function was deduced from the distribution.

Starting at this idea, we drew supply schemes for each type of Japanese fleet,[6] assuming some levels of unit prices of fuel and labour. We can draw such a scheme of full cost including capital costs of the ships if needed. However, the supply scheme of only Japanese fleet will be useless for any shipping market models. We could draw a world total supply scheme for any type of ships, if we have detailed specifications for the world fleet.

As an initial attempt, we collected world tanker data classified by flag, age and size, and gave each rank of ships the same specifications as the corresponding rank of Japanese ships, but we had to give some estimated values to the number of crew and wage rates according to the flag. From these data we could calculate the operating costs in a certain route for each ships, and could get a supply scheme for the route, and positions of Japanese ships in the scheme.[7]

Such an idea seems to be employed widely in the simulating models of shipping markets. From these data we could obtain informations in regard of behaviour no longer of individual tankers but of tanker group classified by flag, age, and size. This may be based on an assumption that every tanker within a group will be have in the same way as the other tanker within the group. This assumption can be accepted the more easily, if the more detailed classification has been done.

New building of ships, scrapping, casualties, or changes of flags may modify gradually the constitutions of each group, and consequently delicate changes of supply scheme of the world tanker may take place. We can simulate these changes by means of handling the individual data from a group to another, and get the supply schemes of any time points.

The demand for shipping services can be explained by the accumulation of demander's behaviour as well. If we can classify world total cargo movement into groups by cargo items, routes, sizes of cargo lots, etc., we might obtain a demand scheme which could be corresponding to the supply schemes.

To investigate these possibilities, we have dealt with the following attempts in

5) Johansen [12].
6) Shimojo [55].
7) ibid.

our tanker market simulation: Firstly a number of oil transportation offers were generated randomly according to the real distribution of them. Each offer, then, was confronted with each tanker, and the most convenient and cheapest tanker was found out. This was a fixture. But the freight rate for the fixture was given at the end of the week by the highest rate in the fixtures fixed during the week. For the market price of the week could be thought as the supply price of marginal supplier.

Assuming a certain level of demand, we have repeated such simulations for some seventy weeks. We can observe the results on profit of each tankers, and find out which types of tankers were the most favourable at that conditions of markets. Furthermore we can see the facts that the most favourable types of tankers will depend considerably upon the market conditions.

This type of simulation was a sort of Monte Carlo experiments. We can find some inclination of the results only after numerous trials have been done under a certain condition. The result of each trial is merely the one seen by chance, so that many trials have to be done. But for some purposes this type of simulation can supply useful information.

If we could prepare much more detailed data for the ships and cargo, we could employ them in this type of simulation under various conditions using the distributions in which ships and cargo offers had been realized. And we would like to say that the demand and supply schemes in the shipping model should be produced in this method.

A NOTE ON THE INTERNATIONAL TRANSMISSION OF POLICY EFFECTS UNDER FLEXIBLE EXCHANGE RATES

Kazuhiro IGAWA

1. Introduction*

In a famous paper [3], R. A. Mundell demonstrated the following conclusions about the international transmission of the effects of fiscal and monetary policies under a flexible exchange rate system. Under flexible exchange rates;
(Conclusion 1): Expansive fiscal policy increases income both at home and abroad
(Conclusion 2): Monetary expansion increases income at home and decreases it abroad.

These rather surprising conclusions are derived from a model with capital mobility. If interest rates are affected by economic policies, the capital account, which is sensitive to changes in the level of interest rates, changes. Under a flexible exchange rate system, the trade balance should adjust to keep the balance of payments equal to zero. This change in the trade balance plays the role of transmitter.

His conclusions are intended to replace the traditional theory that, under the flexible rates, there is no change in trade balance and therefore no effects from business cycle disturbances originating abroad.

As a device for a simple demonstration of the role of capital mobility, Mundell's model is an interesting one. However, the assumption in his capital flow function that net capital flow is a function only of the levels of domestic and foreign interest rates, is unrealistic. In the short-run, it is well known that capital flows also depend on the foreign exchange rate, and this fact is emphasized by Niehans [4]. Dornbusch [1] attempted to rehabilitate Mundell's model by arguing that the level of the foreign exchange rate looses the power to affect capital flows in the long-run. However, it is reasonable that, in long-run full stock-flow equilibrium, interest rates also loose this power (or have effects in the opposite direction) as Tsiang explained in his paper [5]. Therefore, it seems that Mundell's model is insufficient to explain the real world phenomena.

In this paper, we reinvestigate the above Conclusions 1 and 2 by using more realistic models which will be expressed in the following sections. Our conclusions, which

* I wish to acknowledge helpful discussion with Mr. Hara of Osaka University and his interesting paper [2], which treats the same problem from a different point of view. In his paper, he stresses the facts that demand for money and the level of saving (expenditure) depend on the exchange rate. These are factors that Mundell neglected for simplification.

will be explained in Sections 3 and 4, are summarized as Statements 1 and 2, corresponding to Conclusions 1 and 2 respectively. Under flexible exchange rates;
(Statement 1): In the short-run, expansive fiscal policy increases income both at home and abroad and this transmission effect is larger or smaller than that obtained in Mundell's model, depending on the degree to which capital flows are sensitive or insensitive to interest rates. In the long-run, however, expansive fiscal policy will increase income at home and normally decrease it abroad.
(Statement 2): In the short-run, monetary expansion increases income both at home and abroad. In the long-run, it increases income abroad but the direction of income change at home is ambiguous.

These statements should not be surprising if one is not committed to the applicability of Mundell's model to real world phenomena but believes on the basis of past experience in a positively interrelated international transmission mechanism.

2. An Alternative Short-run Model

It is convenient to show first the Mundell-type model, expressed by the following set of equations.

$$(\mathrm{M}) \quad \begin{cases} Y = E(i, Y) + T(Y, Y^*, \pi) + G \\ Y^* = E^*(i^*, Y^*) - T \\ M = L(i, Y) \\ M^* = L^*(i^*, Y^*) \\ T + C(i, i^*) = 0 \end{cases}$$

We call this the M-model. Here, Y is output, E is expenditure, i is the level of interest rates, G is government expenditure on home goods, M is the money supply and L is the demand for money, all for the home country. The variables with asterisk refer to the foreign country. T is the trade balance and C is net capital inflow (outflow if it is negative) of the home country. π is the foreign exchange rate (defined as the domestic currency price of foreign exchange).

The price level of home goods (or foreign goods) is constant in terms of home (or foreign) currency and is expressed as unity by choosing the appropriate unit. In his paper Mundell assumed perfect capital mobility for simplicity and for the merit of posing a stereotype. However, we use a more general model where we can treat the case of imperfect capital mobility. Besides the above features, the above model is fundamentally the same as the one by Mundell.

The partial derivatives in this model and the following two models have the following usual properties:

$$\partial E/\partial i = E_i < 0, \ (\partial E^*/\partial i^* = E^*{}_i < 0)$$

$$\partial E/\partial Y = E_y > 0, \ (\partial E^*/\partial Y^* = E^*_y > 0)$$
$$\partial T/\partial Y = T_y < 0, \ (\partial T/\partial Y^* = T^*_y > 0)$$
$$E_y + T_y > 0, \ (E^*_y - T^*_y > 0)$$
$$\partial L/\partial Y = L_y > 0, \ (\partial L^*/\partial Y^* = L^*_y > 0)$$
$$\partial L/\partial i = L_i < 0 \ (\partial L^*/\partial i^* = L^*_i < 0)$$

The following properties are assumed for the M-model.

(A-1) $\partial C/\partial i = C_i > 0, \ \partial C/\partial i^* = C^*_i < 0$

(A-2) $\partial T/\partial \pi = T_\pi > 0$

In our alternative short-run model, we assume a different role of the foreign exchange rate. It is empirically well known that the effects of a change in foreign exchange rate on trade balance can be expressed as "J curve" effects. That is, exchange rate depreciation causes the trade balance to deteriorate in the very short-run and then gradually improves it from that level. In the long-run, the trade balance will improve from the initial level under Assumption (A-2).

This implies that depreciation must produce capital inflow in the short-run, under a flexible exchange rate system. As we are no interested in the short-run dynamics but the properties of short-run comparative statics, the above role of the exchange rate may be specified as follows. We assume (A-2)′ instead of (A-2) and introduce a further assumption (A-3) in addition to (A-1)

(A-2)′ $\quad T_\pi = 0$

(A-3) $\quad \partial C/\partial \pi = C_\pi > 0$

These mean that the trade balance does not move, but the capital account improves with a depreciation in foreign exchange rate. Obviously, if T_π is negative our main conclusion in this paper is strengthened.

Whether the trade balance deteriorates (as in "J curve" theory) or improves (as Mundell assumed) will depend on the situation of the economy and the time period concerned. In this paper, however, we are interested only in the fact recently observed in the real world; namely that the foreign exchange rate is not effective for short-run adjustments in the trade balance. Thus we believe (A-2)′ as a realistic assumption. With this assumption and the assumption of foreign exchange stability, C_π must be positive as shown in (A-3).

Our short-run model can now be expressed as

$$\text{(S)} \quad \begin{cases} Y = E(i, Y) + T(Y, Y^*) + G \\ Y^* = E^*(i^*, Y^*) - T(Y, Y^*) \\ M = L(i, Y) \\ M^* = L^*(i^*, Y^*) \\ T + C(i, i^*, \pi) = 0 \end{cases}$$

We call this the S-model.

3. Short-run Policy Effects

Let us investigate and compare the policy effects of those two types of model. From the M-model, we can derive the following results by total differentiation.

$$\begin{pmatrix} 1-E_y-T_y, & -T^*_y, & -E_i, & 0, & -T_\pi \\ T_y, & 1-E^*_y+T^*_y, & 0, & -E^*_i, & T_\pi \\ L_y & 0 & L_i & 0 & 0 \\ 0 & L^*_y & 0 & L^*_i & 0 \\ T_y & T^*_y & C_i & C^*_i & T_\pi \end{pmatrix} \begin{pmatrix} dY \\ dY^* \\ di \\ di^* \\ d\pi \end{pmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} dG + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} dM$$

We then get the following policy effects:

(1) $(dY/dG)_M = T_\pi L_i \{L^*_y(C^*_i+E^*_i)+L^*_i(1-E^*_y)\}/\Delta$

where, Δ is the determinant of the above (5×5) square matrix and is positive from stability conditions. The sign of $(dY/dG)_M$ is positive by the assumed properties of partial derivatives.

(2) $(dY^*/dG)_M = T_\pi L^*_i L_y C_i/\Delta>0$

(3) $(dY/dM)_M = T_\pi \{L^*_y(E^*_iE_i+E_iC^*_i-E^*_iC_i) -(1-E^*_y)(C_i-E_i)L^*_i\}/\Delta>0$

(4) $(dY^*/dM)_M = L^*_iT_\pi(1-E_y)C_i/\Delta<0$

By the same procedure we get the following from the S-model.

$$\begin{pmatrix} . & . & . & . & 0 \\ . & . & . & . & 0 \\ . & . & . & . & 0 \\ . & . & . & . & 0 \\ . & . & . & . & C_\pi \end{pmatrix} \begin{pmatrix} dY \\ dY^* \\ di \\ di^* \\ d\pi \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} dG + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} dM$$

where points in this square matrix express the same elements as in the previous square matrix.

(5) $(dY/dG)_S = C_\pi L_i \{(1-E^*_y+T^*_y)L^*_i+E^*_iL^*_y\}/\Delta_S$

where Δ_S is the determinant of the above square matrix and is positive from stability conditions. $(dY/dG)_S$ will be positive.

(6) $(dY^*/dG)_S = -C_\pi L^*_i L_i T_y/\Delta_S > 0$

(7) $(dy/dM)_S = C_\pi E_i\{(1-E^*_y+T^*_y)L^*_i+E^*_i L^*_y\}/\Delta_S > 0$

(8) $(dY^*/dM)_S = -C_\pi E_i T_y L^*_i/\Delta_S > 0$

For the M-model, intuitive explanations of Equations (1) and (2) are as follows. An increase in government expenditure on home goods produces an increase in effective demand for home goods and this increases output. As the output level increases, the interest rate in the home country must increase to keep the demand for home money constant. This increase in the interest rate induces capital inflow. To maintain the balance of payments in equilibrium, the trade balance should be deteriorated. The trade balance deterioration in the home country implies an increase in the effective demand for foreign goods and this in turn increases output in the foreign country. So far about Mundell's model.

The above explanations are also applicable to the corresponding equations (5) and (6) in our S-model. The differences in the results are not qualitative but quantitative. If, in the absence of exchange rate adjustment, the trade balance deterioration effect is larger than the capital account improvement effect, then exchange rate adjustment produces a greater increase in Y and a smaller increase in Y^* in the M-model than in the S-model. Conversely if the trade balance deterioration is initially smaller than the capital account improvement, then, following exchange-rate adjustment, the increase in Y is smaller and the increase in Y^* is larger in the M-model than in the S-model. This is because a depreciation of the exchange rate improves the trade balance in the M-model but improves only the capital account in the S-model.

On the other hand, the intuitive explanations for monetary policy in the M-model —Equations (3) and (4)—, are as follows. The increase in home money supply decreases the home interest rate and induces capital outflows. This must be matched by trade balance improvement through changes in the exchange rate. This in turn increases output in the home country and decreases it in the foreign country. The interest rate deceases in the foreign country. So far about Mundell's model.

In the case of our S-model, —Equations (7) and (8) —to maintain the balance of payments the trade balance is not affected by exchange rate adjustment. Therefore, the capital outflow induced by the fall in the home interest rate must be balanced by capital inflow through a depreciation in foreign exchange rate. Therefore, the only channel to affect effective demand in the home country is a change in home expenditure induced by the interest rate change. As the interest rate decreases in the home country, the home expenditure level increases. This increases home output and induces a trade balance deterioration. This deterioration in turn induces an increase in output level in the foreign country. This type of transmission mechanism has been well known since the Keynesian revolution.

It must be noted that the interest rate in the foreign country increases notwithstanding the decrease in the home country. This is explained as follows. The relative decrease in the home interest rate shifts demand from home securities to foreign securities. However, this demand shift is counter balanced by a demand shift in the opposite direction caused by the depreciation of the foreign exchange rate. This point expresses one of the main features of our S-model.

4. Long-run Policy Effects

As Dornbusch [1] mentioned, it might be more appropriate to interpret Mundell's model as a specification of longer-run phenomena. We agree with this so far as the specification of the long-run role of the exchange rate is concerned. In the long-run, however, we must interpret the capital flow function as relating adjustments in capital flows to an increase in wealth. That is, division of an increment of wealth depends on the level of interest rates in both countries. This point is strongly expressed by Tsiang. However, to treat flow capital movements properly, it is necessary to consider them in a growth model in which intertemporal relations are important.

In this paper, we are not interested in a growing economy but rather in treating a stationary economy. Therefore, stock adjustment capital movements have an important role, and the appropriate long-run stationary model is different from both the M- and S-models. This is because, after portfolio adjustments are completed, the capital account should be zero in the case of stock adjustment. Furthermore, it now becomes necessary to treat explicitly international net interest payments (services account), which is not explicitly included in the short-run M- or S-models. The reason for this is that, if the capital account is zero, the balance of trade plus service accounts should be zero in the stationary state.

We can set up the following model for the long-run economy. We shall call it the L-model.

$$\text{(L)} \quad \begin{cases} Y = E(i,\ Y) + T(Y,\ Y^*,\ \pi) + G \\ Y^* = E^*(i^*,\ Y^*) - T \\ M = L(i,\ Y) \\ M^* = L^*(i^*,\ Y^*) \\ T + S(i,\ i^*,\ \pi) = 0 \end{cases}$$

This L-model has three properties:

(i) The trade balance depends on π assuming Marshall-Lerner conditions are satisfied in the long-run. Therefore (A-2) is assumed.

(ii) As flows adjustments of capital are neglected, the capital account is zero in the long-run.

(iii) Net interest payments in terms of foreign currency, S, is

$$S = i^*W - iW^*/\pi$$

where, W is holdings of foreign securities by home residents and W^* is holdings of home securities by foreign residents. As W (W^*) is positively related to i^* (i) and negatively related to i (i^*), through portfolio adjustmets, we can assume the following properties.

(A-4) $\partial S/\partial i = S_i < 0, \quad \partial S/\partial i^* = S_i^* > 0$

(A-5) $\partial S/\partial \pi = S_\pi > 0$

From the above L-model we obtain the following relations and policy effects.

$$\begin{pmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ T_y & T_y^* & S_i & S_i^* & T_\pi + S_\pi \end{pmatrix} \begin{pmatrix} dY \\ dY^* \\ di \\ di^* \\ d\pi \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} dG + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} dM$$

where points in the square matrix show the same elements as in the case of the M-model

(9) $(dY/dG)_L = (\Delta_L)_{11}/\Delta_L$

where Δ_L is the determinant of the above square matrix and $(\Delta_L)_{11}$ is the cofactor of the element in first row and first column. The Hicksian stability condition gives us the sign of $(dY/dG)_L > 0$

(10) $(dY^*/dG)_L = -L^*_i(T_\pi L_i S_\pi + L_y S_i T_\pi)/\Delta_L$

(11) $(dY/dM)_L = T_\pi[S_i\{(1-E^*_y+T^*_y)L^*_i + L^*_y E^*_i + T^*_y L^*_i\} - E_i(T^*_y L^*_i - L^*_y S^*_i)]/\Delta_L$

(12) $(dY^*/dM)_L = -L^*_i\{-T_\pi(1-E_y)S_i + S_\pi T_y E_i\}/\Delta_L > 0$

The intuitive explanation is as follows. For Equations (9) and (10), an increase in government expenditure on home goods will increase output of home goods. This increase in Y produces both deterioration of the trade balance and, through an increase in the home interest rate, deterioration of the services account. Therefore, the foreign exchange rate must depreciate. If the services account is relatively insensitive to depreciation of π, the trade balance must improve after the adjustment in π to maintain the total balance of payments. In this case Y^* will decrease. On the other hand, if S_π is large enough, the services account will improve after the adjustment in π. This implies that the trade balance deteriorates from its initial level and thus increases Y^*. This, however, will be a rare case.

For Equations (11) and (12), the increase in home money supply reduces the home interest rate and this reduction improves the services account and increases home expenditure. Therefore, the trade balance should deteriorate, which will bring about a change in π. Whether Y increases or not depends on the relative magnitude of the changes in E and T. If the change in E is larger than that in T, Y will increase and vice-versa. Y^*, on the other hand, increases unambiguously because of the decrease in T.

5. Concluding Remarks

The pioneering work of Mundell has stimulated many to his works. This paper is one such attempt and our main modifications come from recent experience in the foreign exchange market.

Our demonstration used extremely simple models, following Mundell's example. It is possible, however, to extend the analysis to more complex cases. In this paper we have not analyzed the dynamic process of transmission which seems very important. Analyses of short-run dynamics related to the foreign exchange market, long-run dynamics of market adjustments and economic growth will be necessary extensions. If we introduces flow capital adjustments in a long-run growth model, we will get different conclusions from those in our long-run stationary model.

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A CONTROL ESTIMATION APPROACH TO TIME-VARYING PARAMETER MODELS*

Yukio UCHIDA

This paper presents an effective control estimation approach to the adaptive nonlinear econometric models in time-varying structures by using small sample time-series data. We can observe that the estimated errors of endogenous variables are very small based on the final test, and that the proposed method finds a satisfactory solution to both the under-size problem and the structural change problem in the econometric literature.

1. Introduction

Econometricians have for some time been aware of the fact that the well-behaved assumptions imposed on the estimation of time-invariant parameter models frequently appear invalid under realistic situations. More recently, a great deal of attention has been paid to the problem of time-varying parameters.[1] When considering the structural changes, for example, statistical data for long time periods are often not available. As a result, we encounter the so-called under-size problem in the estimation of simultaneous econometric relationships. Such problems may be reduced by an effective method for estimation of time-varying parameters.

Current research on the time-varying parameter problem has been developed in the three areas of (i) the time-varying parameter problem for adjusting the constant term in regression equations, (ii) random-parameter models or systematic (nonrandom) variation models and (iii) the applications of Kalman filtering and smoothing techniques to econometric models.[2]

In this paper we present a control estimation method for nonlinear econometric models represented by adaptive form in time-varying structures. This method is developed based on the small sample time-series data. The state variable representation of an adaptive econometric model is given in Section 2. Section 3 through Section 5 discuss a method of estimating time-varying parameters by using the state variable form. In Section 6, the proposed method is numerically experimented, and the results

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1) For recent detailed discussion of time-varying parameters, see the special issue of the Annals of Economic and Social Measurement [6] and Sant [5].

2) For the Kalman filtering and smoothing techniques, refer to Kalman [4], Arimoto [1] and Bryson and Ho [3].

are compared with those of the least squares method. Some concluding remarks are given in Section 7.

2. State Variable Representation of an Adaptive Econometric Model

Consider the general adaptive econometric model described by

$$y_t = \alpha(y_t, p_t, \theta_t) + u_t \tag{1}$$

where y, p, θ and u are the vectors of the endogenous variables, predetermined variables, time-varying parameters, and random disturbances, respectively; all the vectors have compatible dimensions. Additionaly, it is assumed that

$$E(u_t) = 0,\ E(u_t u'_\tau) = \delta_{t\tau}\, \Omega_t \tag{2}$$

where $\delta_{t\tau}$ is the Kronecker delta. Let the vector of the exogenous variables be denoted by x. Then the predetermined variables are classified as

$$v_{t-1} \in \{x_t, x_{t-1}, x_{t-2}, \ldots\ldots\} \tag{3}$$

$$z_{t-1} \in \{y_{t-1}, y_{t-2}, y_{t-3}, \ldots\ldots\ldots\} \tag{4}$$

$$p_t = \begin{pmatrix} v_t \\ z_{t-1} \end{pmatrix}. \tag{5}$$

From (4) the vector of predetermined endogenous variables can be represented by the following first-order difference equation

$$z_t = \Delta z_{t-1} + \Lambda y_t \tag{6}$$

where the elements in the matrices Δ and Λ consist of zero and unity only.[3]
It follows from (1) and (5) that

$$y_t = \alpha(y_t, z_{t-1}, \theta_t, v_t) + u_t. \tag{7}$$

Define

$$s_t = \begin{pmatrix} y_t \\ z_t \end{pmatrix} \tag{8}$$

$$C = (I, 0). \tag{9}$$

Then (6) and (7) can be expressed as

$$s_t = \beta(s_t, s_{t-1}, \theta_t, v_t) + C'u_t \tag{10}$$

$$y_t = Cs_t \tag{11}$$

which stand for the state variable form of the adaptive econometric model.

3) For the description of Equation (6), see also the example given in Section 6.

3. The Control Estimation Method

The estimation problem of time-varying parameters consists of minimizing the sum of the residual square

$$\sum_{t=1}^{T} u'_t u_t. \tag{12}$$

However, it is not possible in general to implement this method. In this paper, therefore, we present an alternative method that is well suited to the estimation of time-varying parameter models.

Before proceeding to the discussion, some notations are defined as follows: $\{s_t^0, y_t^0, z_t^0, \theta_t^0\}_{t=1}^T$, $\{s_t^*, y_t^*, z_t^*, \theta_t^*\}_{t=1}^T$ and $\{s_0, s_t, y_t, v_t\}_{t=1}^T$ denote the sequences of prior estimates, posterior estimetes, and data, respectively. For the sake of convenience, it is supposed that $s_0=s_0^0=s_0^*$, and that $z_0=z_0^0=z_0^*$. Thus, in the sequence $\{s_0, s_t^0, y_t^0, z_t^0, \theta_t^0, v_t\}_{t=1}^T$, it follows that

$$y_t^0 = \alpha(y_t^0, z_{t-1}^0, \theta_t^0, v_t) \tag{13}$$

$$z_t^0 = \Delta\, z_{t-1}^0 + \Lambda\, y_t^0 \tag{14}$$

$$s_t^0 = \beta(s_t^0, s_{t-1}^0, \theta_t^0, v_t) \tag{15}$$

$$y_t^0 = C s_t^0. \tag{16}$$

Similarly, it is also observed in $\{s_0, s_t^*, y_t^*, z_t^*, \theta_t^*, v_t\}$ that

$$y_t^* = \alpha(y_t^*, z_{t-1}^*, \theta_t^*, v_t) \tag{17}$$

$$z_t^* = \Delta\, z_{t-1}^* + \Lambda\, y_t^* \tag{18}$$

$$s_t^* = \beta(s_t^*, s_{t-1}^*, \theta_t^*, v_t) \tag{19}$$

$$y_t^* = C s_t^*. \tag{20}$$

Assume that performance measure is given by

$$\phi = \sum_{t=1}^{T} \phi_t \tag{21}$$

$$\phi_t = (y_t - y_t^*)'\, Q_t(y_t - y_t^*) + (\theta_t^0 - \theta_t^*)'\, R_t(\theta_t^0 - \theta_t^*) \tag{22}$$

where the i-jth elements in the weight matrices Q_t and R_t are defined as

$$Q_t(i,j) = \frac{\delta_{ij}}{y_t(i)^2} \tag{23}$$

$$R_t(i,j) = k \cdot \frac{\delta_{ij}}{\theta_t^0(i)^2}, \qquad k > 0, \tag{24}$$

The problem to be considered is to find θ_t^*, the posterior estimates of parameters so as to minimize the performance measure (21) subject to the constraints (8), (9) and (13) through (20). This method is replaced by the optimal control problem when we regard θ_t and y_t as the vectors of the control variable and the target variable, respectively.

Thus we call this method *control estimation.*

4. The Linearized Approximation and Control Estimation

First, let us approximate the nonlinear model (10) in the neighborhood of $\{s_o, s_t^0, \theta_t^0, v_t\}_{t=1}^T$ as

$$s_t \cong \beta(s_t^0, s_{t-1}^0, \theta_t^0, v_t) + \frac{\partial\beta^0}{\partial s_t}(s_t - s_t^0) + \frac{\partial\beta^0}{\partial s_{t-1}}(s_{t-1} - s_{t-1}^0)$$

$$+ \frac{\partial\beta^0}{\partial\theta_t}(\theta_t - \theta_t^0) + C'u \qquad (25)$$

where terms higher than the second term are ignored, and where

$$\frac{\partial\beta^0}{\partial s_t} = \frac{\partial}{\partial s_t}\beta(s_t^0, s_{t-1}^0, \theta_t^0, v_t). \qquad (26)$$

Similarly, both $\frac{\partial\beta^0}{\partial s_{t-1}}$ and $\frac{\partial\beta^0}{\partial\theta_t}$ are defined as in (26).

By substituting (15) in (25), we have the solution to s_t as

$$s_t = A_t s_{t-1}^0 + B_t\theta_t + b_t + C_t u_t \qquad (27)$$

where

$$(A_t, B_t, C_t) = (I - \frac{\partial\beta^0}{\partial s_t})^{-1}(\frac{\partial\beta^0}{\partial s_{t-1}}, \frac{\partial\beta^0}{\partial\theta_t}, C') \qquad (28)$$

$$b_t = s_t^0 - (A_t s_{t-1} + B_t\theta_t^0). \qquad (29)$$

Second, we explain the control estimation of the linearized model. Estimators of time-varying parameters θ_t in (27) are given by[4)]

$$\theta_t^* = \theta_t^0 - \theta_t^m \qquad (30)$$

$$\theta_t^m = g_t + G_t s_{t-1}^* \qquad (31)$$

where

$$g_t = [R_t + B_t'(H_t + C'Q_tC)B_t']^{-1}B_t'[h_t - C'Q_ty_t + (H_t + C'Q_tC)$$

$$(b_t + B_t\theta_t^0)] \qquad (32)$$

$$G_t = [R_t + B_t'(H_t + C'Q_tC)B_t]^{-1}B_t'(H_t + C'Q_tC)A_t \qquad (33)$$

$$h_{t-1} = (A_t - B_tG_t)'[h_t - C'Q_ty_t + (H_t + C'Q_tC)(b_t + B_t\theta_t^0)] \qquad (34)$$

$$H_{t-1} = (A_t - B_tG_t)'(H_t + C'Q_tC)A_t \qquad (35)$$

and $\qquad (t = 1, 2, \ldots\ldots, T)$

$$h_T = 0, H_T = 0. \qquad (36)$$

$\{g_t, G_t, h_t, H_t, h_0, H_0\}_{t=1}^T$ as shown above are obtained by solving backward

4) The derivation of (30) through (39) is given in the Appendix.

period by period. Thus the minimal value ϕ_L of ϕ is calculated as

$$\phi_L = s_0' H_0 s_0 + 2h_0' s_0 + q_0 \tag{37}$$

where

$$q_T = 0 \tag{38}$$

$$q_{t-1} = g_t + y_t'Q_t y_t + g_t' R_t g_t + (b_t + B_t^0\theta_t^0 - B_t g_t)' [2(h_t - C'Q_t y_t) + (H_t + C'Q_t C)(b_t + B_t\theta_t^0 - B_t g_t)]. \tag{39}$$
$$(t = 1, 2, \ldots\ldots, T)$$

5. The Control Estimation of a Nonlinear Model

In this section we use the results obtained in the previous section to give the control estimation algorithm of a nonlinear model. The dimensions of y_t and θ_t are NY and $N\theta$, respectively.

START:

STEP 1: Determine the prior estimates $\{\theta_t^0\}_{t=1}^T$ of time-varying parameters and the weight k. Find the prior estimates $\{s_t^0\}_{t=1}^T$ of state variables as the solution to (13) and (14). Go to STEP 6, where $l=0$, $\{s_t^*=s_t^0\}_{t=1}^T$.

STEP 2: Perform the linearized approximation of the model in the neighborhood of $\{s_0, s_t^0, v_t, \theta_t^0\}_{t=1}^T$ and obtain $\{A_t, B_t, b_t\}_{t=1}^T$.

STEP 3: Obtain $\{g_t, G_t\}_{t=1}^T$ from (32) through (39), and set $t=1$.

STEP 4: Find the revised values θ_t^m and the posterior estimates θ_t^* of parameters from (30) and (31) and also obtain the posterior estimates s_t^* of state variables as the solution to (17) and (18). Go to STEP 5 when $t=T$. Otherwise, repeat this step, where $t=t+1$.

STEP 5: The average revised value π_θ^l (%) at each period for each parameter is calculated by

$$\pi_\theta^l = 100 \times \sqrt{\frac{1}{N\theta \cdot T} \sum_{t=1}^T \sum_{i=1}^{N\theta} \left(\frac{\theta_t^m(i)}{\theta_t^0(i)}\right)^2}. \tag{40}$$

STEP 6: The average error of estimate π_Y^l(%) at each period for each endogenous variable is calculated by

$$\pi_Y^l = 100 \times \sqrt{\frac{\sum_{t=1}^T (y_t - y_t^*)' Q_t (y_t - y_t^*)}{NY \cdot T}}. \tag{41}$$

Go to STEP 2 when $l=0$. Otherwise, go to STEP 7.

STEP 7: Go to STEP 8, when for any i-th component of parameter

$$\left| \frac{\theta_t^m(i)}{\theta_t^0(i)} \right| < EPS. \tag{42}$$

Otherwise, go to STEP 2, where $l=l+1$, $\{s_t^0=s_t^*, \theta_t^0=\theta_t^*\}_{t=1}^T$.

STEP 8: Obtain the performance value ϕ from (20) and (21) and calculate

$$\pi_N = 100 \times \left| \frac{\phi - \phi_L}{\phi} \right| . \tag{43}$$

STOP:

Several remarks are called for on the above algorithm.

Remark 1.1: π_N stands for the error value (%) due to the linearized approximation of the nonlinear model. The above algorithm using the linearized model is not desirable when π_N is large.

Remark 1.2: The prior estimates of time-varying parameters may be replaced by estimates of time-invariant parameters $\{\theta_t = \theta\}_{t=1}^{T}$ based on the existing estimation methods.

Remark 1.3: It can be told by experience that k is less than 0.1. In some cases, it is desirable that it be about 0.01.

Remark 1.4: $\pi_Y{}^0$ stands for the average error of estimate in the final test of the prior estimate. The prior estimate of parameter is not desirable when $\pi_Y{}^0$ is large, since the above algorithm gives no guarantee that the posterior estimate of a parameter converges to the true value of the parameter.

Remark 1.5: We can observe the convergent properties of this algorithm by finding the locus of $\pi_Y{}^0$, $\pi_Y{}^l$ and $\pi_\theta{}^l$, $l=1,2,$ ......

Remark 1.6: $\pi_Y{}^l$ denotes the average error of estimate in the final test of the posterior estimate.

6. An Illustrative Example

As an illustrative example we estimate the time-varying parameters in a simple model of wage-price relationships.

The Model:

$$W_t = \alpha_{0,t} + \alpha_{1,t} W_{t-2} + \alpha_{2,t} Y_t + \alpha_{3,t} ETA_{t-1} \tag{44}$$

$$P_t = \beta_{0,t} + \beta_{1,t} P_{t-2} + \beta_{2,t} W_{t-2} \tag{45}$$

Where

W: real wage index in industry as a whole; calendar year 1970=100.

P: price index (GNP deflator), calendar year 1970=100.

Y: GNP at market prices; in constant 1970 prices (billion yen).

ETA: productivity of labor; in constant 1970 prices (billion yen per ten thousand workers).

Data: half-yearly data from the first half of 1956 to the second half of 1973.

Estimation Period: from the first half of 1957 to the second half of 1973.

Prior Estimates of Parameters: ordinary least squares estimates.

Weight: $k=0.01$.

Each variable is defined as

$$y_t = (W_t, P_t)'$$
$$v_t = (Y_t, ETA_{t-1})'$$
$$z_t = (W_t, W_{t-1}, P_t, P_{t-1})'$$
$$s_t = (y_t', z_t')'$$
$$\theta_t = (\alpha_{0,t}, \alpha_{1,t}, \alpha_{2,t}, \alpha_{3,t}, \beta_{0,t}, \beta_{1,t}, \beta_{2,t})'$$

$$\Delta = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}.$$

All the results obtained are summarized in the Tables 1 and 2 and Figures 1 through 11.

Remarks on the results given in the tables and figures are as follows.

Remark 2.1: The estimated errors of W and P using the control estimation method are extremely small. (See Table 2, and Figures 1, 2 and 11.)

Remark 2.2: Speed of the convergence of the control estimation algorithm is very fast. (See Table 2, Figure 10 and Figure 11.)

Remark 2.3: The transient patterns of $\alpha_{0,t}{}^*$ through $\alpha_{3,t}{}^*$ appear similar. (See Figures 3 through 6.) Also, the transient patterns of $\beta_{0,t}{}^*$ through $\beta_{2,t}{}^*$ appear similar. (See Figures 7 through 9.)

7. Concluding Remarks

Despite the simple model used in the experiments, the estimated errors of the endogenous variables are substantially improved. The characteristic features of the control estimation method are as follows.

Remark 3.1: The control estimation method is simultaneous for the whole model, unlike the single estimation method.

Remark 3.2: The usual simultaneous estimation method is applied to the case where the number of observations is larger than the number of parameters. On the other hand, the proposed method can also be applied to the under-size case.

Remark 3.3: Structural changes, which are unobservable in time-invariant structures, are identified by estimates of time-varying parameters. Thus by the use of a time-varying model, long period data can be analysed even where structural changes are present.

Remark 3.4: It may be possible to predict correctly the parameters in the future by using the up-to-date value of parameters.

Remark 3.5: The effects of seasonal change about parameters are observable.

Remark 3.6: The final test of the model can be implemented in connection with the estimation.

Remark 3.7: The stability of parameters are observed by finding the locus the estimates of time-varying parameters.

Remark 3.8: The Kalman filter in the engineering control literature has been devised as a method of estimating unobservable variables. The control estimation method proposed here is applicable to estimation of both time-varying parameters and unobservable variables.[5)]

APPENDIX

As far as we are concerned with the derivation of the control estimates of a linearized model, we now assume that $\{s_t^*, y_t^*\}_{t=1}^{T}$ denotes the solution to the linearized model. Then we have

$$s_t^* = A_t s_{t-1}^* + B_t \theta_t^* + b_t \qquad (A,1)$$

$$y_t^* = C s_t^*. \qquad (A,2)$$

Next we define the following function as

$$\lambda_{t-1} = \min_{\theta_\tau^* \; t<\tau<T} \sum_{t=1}^{T} \phi_\tau \qquad (A,3)$$

(subject to (22), (A,1) and (A,2))

By the principle of optimality in dynamic programming ([2]), λ_{t-1} can be expressed by the following successive equation.

$$\lambda_{t-1} = \min_{\theta_t^*} (\phi_t + \lambda_t). \qquad (A,4)$$

Accordingly, θ_t^* is calculated, step by step like T, T-1, ..., 1. For the sake of convenience, we assume:

$$h_T = 0,\ H_T = 0,\ q_T = 0 \qquad (A,5)$$

$$\lambda_T = s_T^{*\prime} H_T s_T^* + 2h_T' s_T^* + q_T = 0. \qquad (A,6)$$

It follows from (A,4), (A,6) and (22) that the optimization at period T is

$$\begin{aligned}\lambda_{T-1} &= \min\,(\phi_T + \lambda_T) \\ &= \min\,[(y_T - y_T^*)' Q_T (y_T - y_T^*) + (\theta_T^0 - \theta_T^*)' R_T (\theta_T^0 - \theta_T^*) \\ &\quad + s_T^{*\prime} H_T s_T^* + 2h_T' s_T^* + q_T]. \end{aligned} \qquad (A,7)$$

From (A,1), (A,2) and (A,7) we obtain

5) For the estimation of unobservable variables using the control estimation method, see Uchida [7].

$$\lambda_{T-1} = \min_{\theta_T^*} [(A_T s_{T-1}^* + B_T\theta_T^* + b_T)' (H_T + C'Q_T C) (A_T s_{T-1}^* + B_T\theta_T^* + b_T) + 2(h_T - C'Q_T y_T)' (A_T s_{T-1}^* + B_T\theta_T^* + b_T) + (\theta_T^0 - \theta_T^*)' R_T(\theta_T^0 - \theta_T^*) + q_T + y_T Q_T y_T. \quad (A,8)$$

Now calculate $\frac{\partial\lambda_{T-1}}{\partial\theta_T^*}=0$, and using the following identity equation

$$[R_T + B_T(H_T+C'Q_TC)B_T]^{-1} R_T = I - [R_T+B_T (H_T+C'Q_TC)B_T]^{-1} B_T'(H_T+C'Q_TC)B_T, \quad (A,9)$$

then θ_T^* is given as

$$\theta_T^* = \theta_T^0 - [R_T+B_T'(H_T+C'Q_TC)B_T]^{-1} B_T'[h_T - C'Q_T y_T + (H_T+C'Q_TC) (b_T+B_T\theta_T^0+A_T s_{T-1}^*)]. \quad (A,10)$$

By defining θ_T^m, g_T and G_T as (31), (32) and (33), we obtain (30) at period $t=T$. Subsitution of θ_T^* in (A,10) for (A,8) yields

$$\lambda_{T-1} = s_{T-1}^{*\prime} H_{T-1} s_{T-1}^* + 2h_{T-1}' s_{T-1}^* + q_{T-1} \quad (A,11)$$

where h_{T-1}, H_{T-1} and q_{T-1} are defined as (34), (35) and (39). Notice that we use

$$G_T' R_T G_T = (A_T - B_T G_T)' (H_T + C'Q_T C) B_T G_T \quad (A,12)$$

to calculate H_{T-1}. In similar procedure, the optimization at periods T-1, T-2, ..., 1 is done with replacing the above T by T-1 T-2, ..., 1. Therefore the minimal value ϕ_L of ϕ is

$$\theta_L = \min_{\theta, 1<t<T} \phi \text{ (Subject to (21), (22), (A,1) and (A,2))} = \lambda_0 = s_0' H_0 s_0 + h_0' s_0 + q_0. \quad (A,13)$$

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Table 1 The Prior Estimates (the Least Squares Estimates) and the Posterior Estimates (the Control Estimates) of Parameters

	t	α_0, t	α_1, t	α_2, t	α_3, t	β_0, t	β_1, t	β_2, t
Control Estimates	1957.1	4.809	0.7737	5.1630E-04	0.8950	−7.9847E-02	0.7530	0.3639
	.2	4.816	0.7834	5.1723E-04	0.8955	−7.9860E-02	0.6962	0.3511
	1958.1	4.828	0.7979	5.1841E-04	0.8967	−7.9866E-02	0.6694	0.3456
	.2	4.821	0.7895	5.1790E-04	0.8959	−7.9862E-02	0.6870	0.3493
	1959.1	4.828	0.7988	5.1869E-04	0.8969	−7.9858E-02	0.7060	0.3535
	.2	4.824	0.7940	5.1858E-04	0.8962	−7.9857E-02	0.7094	0.3543
	1960.1	4.813	0.7786	5.1685E-04	0.8954	−7.9853E-02	0.7269	0.3580
	.2	4.813	0.7804	5.1721E-04	0.8954	−7.9855E-02	0.7186	0.3563
	1961.1	4.825	0.7952	5.1883E-04	0.8969	−7.9849E-02	0.7468	0.3623
	.2	4.812	0.7784	5.1702E-04	0.8953	−7.9850E-02	0.7461	0.3622
	1962.1	4.804	0.7678	5.1569E-04	0.8945	−7.9855E-02	0.7169	0.3559
	.2	4.793	0.7495	5.1329E-04	0.8933	−7.9859E-02	0.7002	0.3522
	1963.1	4.786	0.7404	5.1232E-04	0.8921	−7.9854E-02	0.7261	0.3579
	.2	4.805	0.7681	5.1566E-04	0.8946	−7.9853E-02	0.7293	0.3586
	1964.1	4.808	0.7737	5.1644E-04	0.8951	−7.9852E-02	0.7349	0.3599
	.2	4.796	0.7534	5.1356E-04	0.8934	−7.9853E-02	0.7285	0.3585
	1965.1	4.783	0.7307	5.1070E-04	0.8910	−7.9853E-02	0.7286	0.3585
	.2	4.790	0.7407	5.1175E-04	0.8925	−7.9854E-02	0.7275	0.3583
	1966.1	4.798	0.7563	5.1404E-04	0.8935	−7.9851E-02	0.7428	0.3618
	.2	4.803	0.7633	5.1492E-04	0.8942	−7.9852E-02	0.7370	0.3604
	1967.1	4.809	0.7747	5.1663E-04	0.8952	−7.9853E-02	0.7308	0.3590
	.2	4.800	0.7578	5.1394E-04	0.8937	−7.9854E-02	0.7265	0.3581
	1968.1	4.801	0.7587	5.1415E-04	0.8936	−7.9854E-02	0.7248	0.3577
	.2	4.805	0.7680	5.1557E-04	0.8946	−7.9855E-02	0.7172	0.3559
	1969.1	4.807	0.7725	5.1632E-04	0.8950	−7.9855E-02	0.7167	0.3557
	.2	4.806	0.7698	5.1586E-04	0.8947	−7.9855E-02	0.7203	0.3566
	1970.1	4.809	0.7760	5.1691E-04	0.8953	−7.9854E-02	0.7244	0.3576
	.2	4.810	0.7800	5.1763E-04	0.8957	−7.9856E-02	0.7112	0.3544
	1971.1	4.806	0.7696	5.1585E-04	0.8947	−7.9858E-02	0.6981	0.3512
	.2	4.807	0.7727	5.1637E-04	0.8950	−7.9859E-02	0.6838	0.3477
	1972.1	4.816	0.7961	5.2026E-04	0.8974	−7.9858E-02	0.6894	0.3490
	.2	4.815	0.7969	5.2068E-04	0.8972	−7,9857E-02	0.6964	0.3507
	1973.1	4.808	0.7758	5.1690E-04	0.8953	−7.9855E-02	0.7225	0.3572
	.2	4.798	0.7448	5.1168E-04	0.8924	−7.9849E-02	0.7781	0.3708
L. S. E. (t-value)		4.807 ——	0.7710 10.4985	5.1607E-04 3.6783	0.8948 1.6454	−7.9855E-02 ——	0.7198 6.2934	0.3565 3.7119

Table 2 The Average Revised Value π_θ and the Average Error of Estimate π_Y.

ℓ	π_θ^ℓ (%)	π_Y^ℓ (%)
0	————	3.310
1	0.1485	7.8462E-02
2	6.3607E-03	2.1786E-03
3	2.1119E-04	7.8481E-05
4	8.3189E-06	3.3382E-06
5	3.7484E-07	1.5800E-07

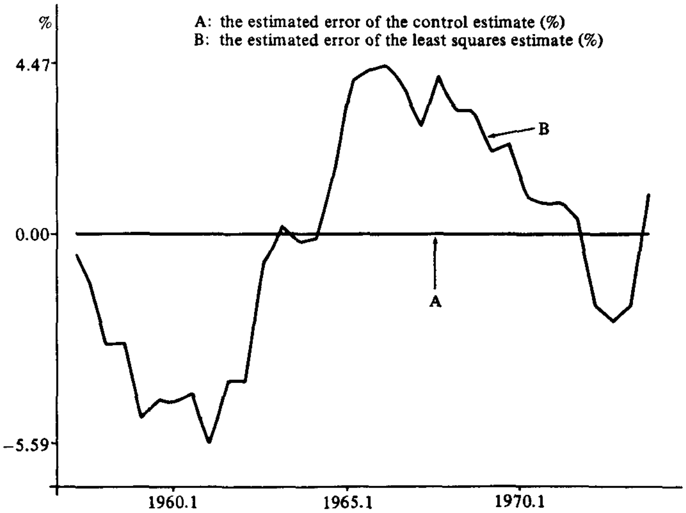

Figure 1 The Error of Estimate of the Real Wage Index (W) in the Final Test

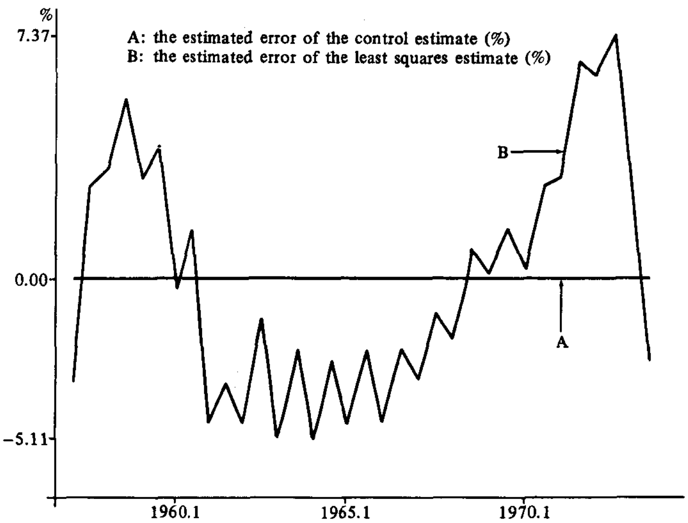

Figure 2 The Error of Estimate of GNP Deflator (P) in the Final Test

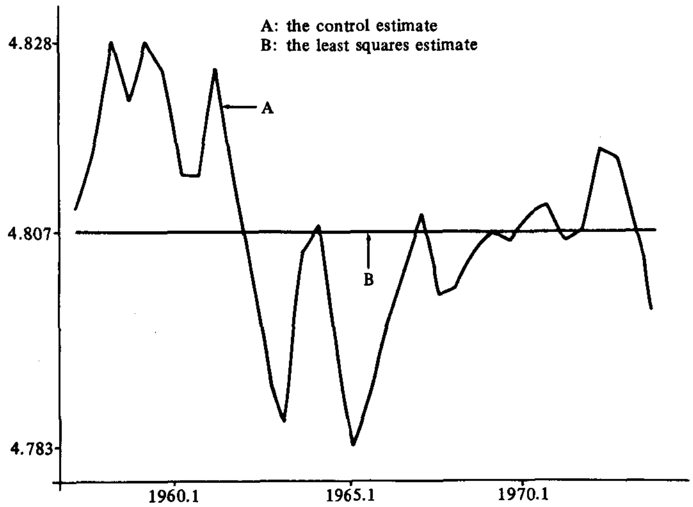

Figure 3 The Estimate of Parameter $\alpha_{0,t}$

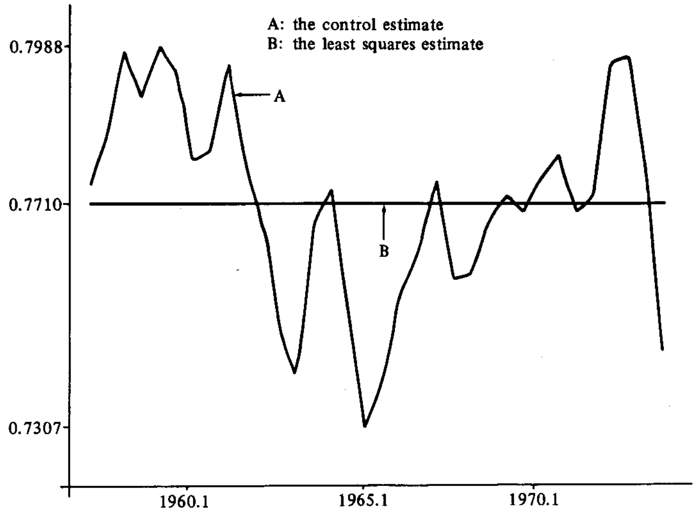

Figure 4 The Estimate of Parameter $\alpha_{1,t}$

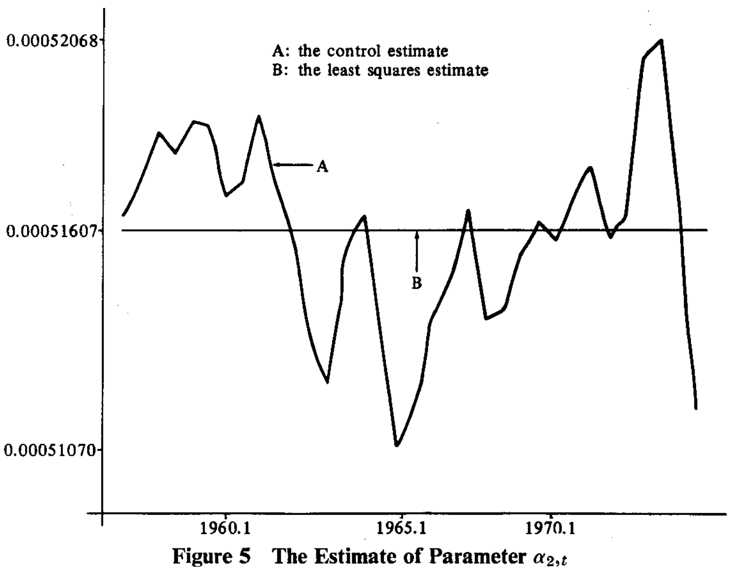

Figure 5 The Estimate of Parameter $\alpha_{2,t}$

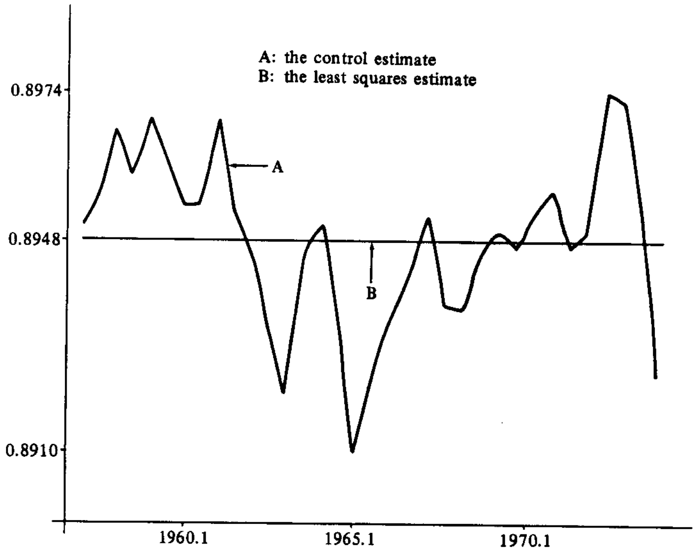

Figure 6 The Estimate of Parameter $\alpha_{3,t}$

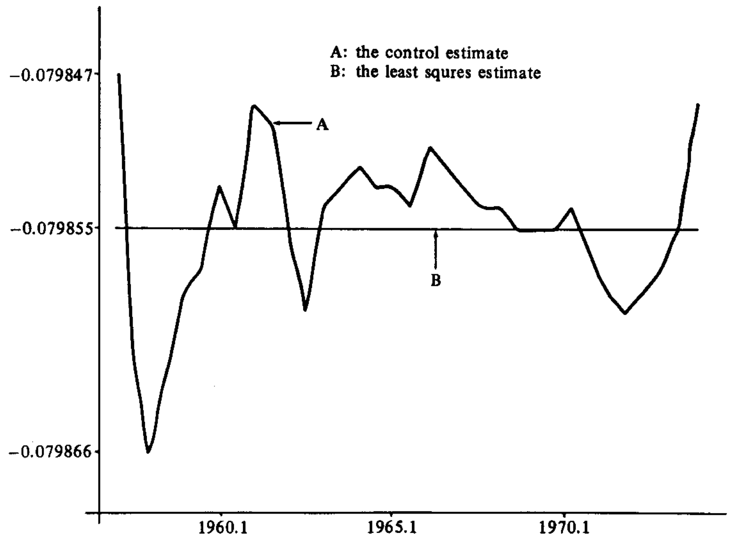

Figure 7 The Estimate of Parameter $\beta_{0,t}$

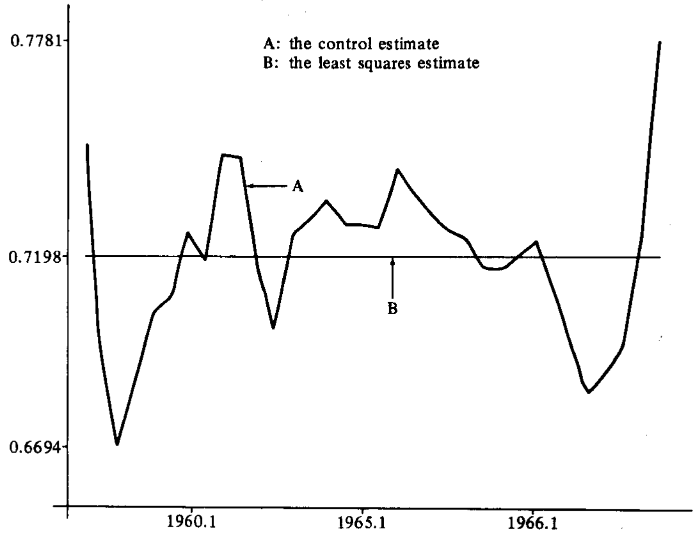

Figure 8 The Estimate of Parameter $\beta_{1,t}$

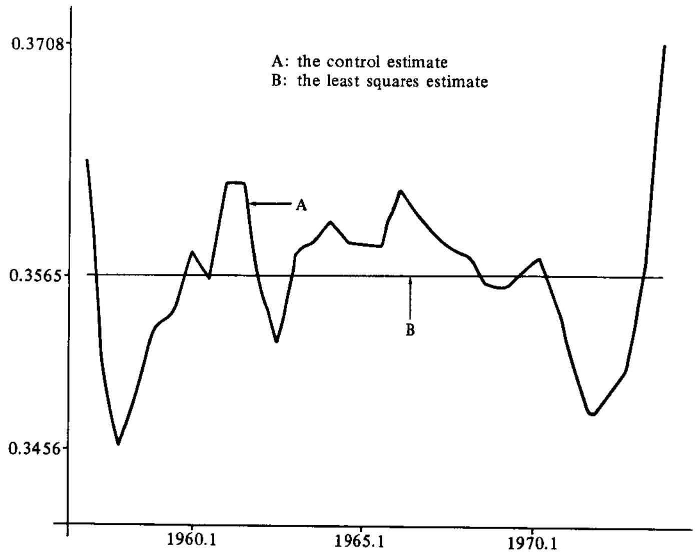

Figure 9 The Estimate of Parameter $\beta_{2,t}$

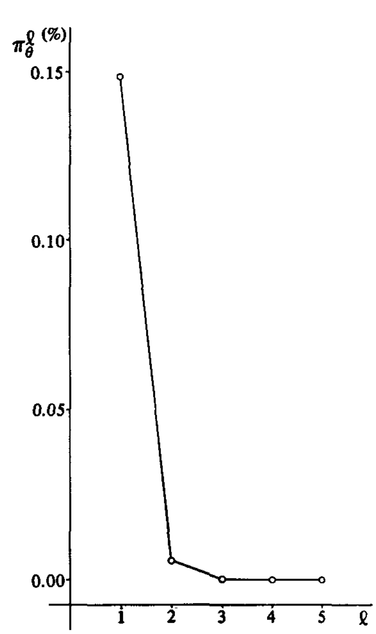

Figure 10 The Average Revised Value π_θ

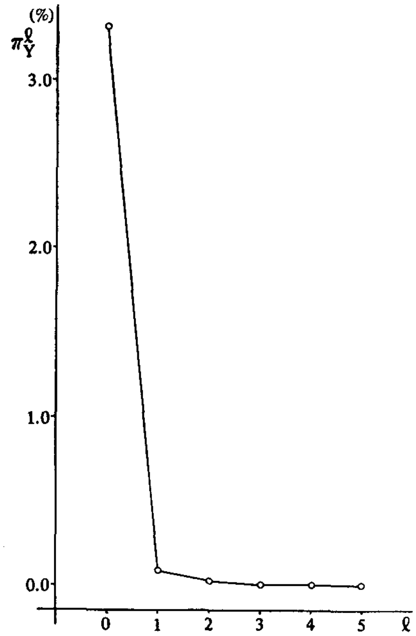

Figure 11 The Average Error of Estimate π_y

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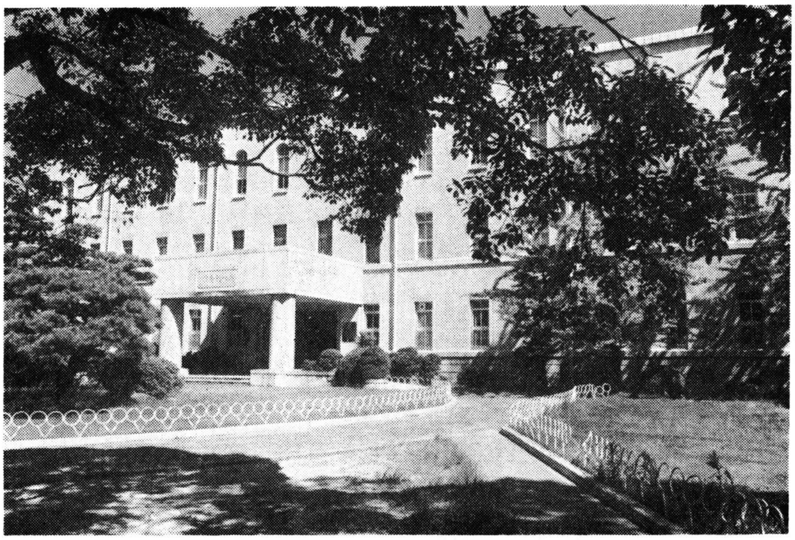

HISTORICAL SKETCH

In 1919, a research organization named the Institute for Commerce was founded in Kobe Higher Commercial School, one of the chief predecessors of Kobe University, with a gift made by F. Kanematsu & Company, a leading mercantile firm in Kobe. The organization was designed to carry on and facilitate integrated research on business and commerce and to formulate and publish the results of these studies and investigations in such form as to make them available to the business community.

With the founding of Kobe University of Commerce, successor of Kobe Higher Commercial School, in 1929, the Institute extended its research activities by adding several divisions. One was the famous Latin-American Library, which soon became the center of research in this field in Japan. A room for statistics equipped with various computing machines was established and began publication of *Jūyō Tōkei Keizai* monthly and *Sekai Bōeki Tōkei* annually. A filing room was prepared to deposit press clipping files systematically arranged by topics and dates. Another room was designed to become the center of all possible original records and data having to do with the beginning and progress of Japanese business.

On the campus of Kobe University of Commerce, another organization named the Institute for Business Mechanization was founded in 1941 utilizing business machines donated by the IBM Corporation and others. With Professor Yasutaro

Hirai as its head a broad and forward-looking plan for business mechanization in Japan was developed.

In 1944, Kobe University of Commerce changed its name to Kobe University of Economics. After the War, however, the University was consolidated with three other colleges in Hyōgo Prefecture to become Kobe University. With this development, the two Institutes were also amalgamated into the Research Institute for Economics and Business Administration, Kobe University. At present, the Institute, with its twenty five full-time professional staff members, carries on studies and investigations in international economy, business administration, and information systems in Japan.

LOCATION AND BUILDINGS

The Research Institute for Economics and Business Administration is located on the campus of Kobe University, Rokko, Kobe. It consists of two three-storied buildings. One is named the Kanematsu Kinenkan and has a floor space of about 2,900 square meters, which includes a president's room, forty-one offices, six rooms used as a library, a room for statistics, etc. Another is built in 1964. It has a floor space of about 1,900 square meters, which is chiefly used as the Documentation Center for Business Analysis, a library and a conference room.

ORGANIZATION

Under the directorship of the president, the Institute operates with two research groups. Each research group and its sections are as follows:

A Group of International Economy

(1) International Trade

(2) International Finance

(3) Maritime Economics

(4) Latin-American Economy

(5) Oceanian Economy

(6) International Labour Relations

B Group of Business Administration

(1) International Management

(2) Business Administration and Information Systems

(3) Accounting

(4) Business Statistics

Besides the regular work of the Institute, research committees may be created to carry on any special work requiring the joint study of academic and business circles. At present, there are five standing research committees, as follows: Experts Group on the World Trade Structure, Committee of International Finance, Committee of Information Systems, Committee of Oceanian Economy and Committee of Maritime Labour.

For convenience and greater efficiency in carrying out its research activities, the Institute has a general office which is responsible for 1) the collection and preservation of a comprehensive collection of books, periodicals, pamphlets, and original records and data of finance, trade, commerce, industry and business generally; 2) the classification, cataloguing, indexing arranging, annotation and compilation of these research materials; and 3) the formulation and publication of the results of the investigations and studies accomplished by the professional staff members of the Institute.

As an affiliated institute, the Documentation Center for Business Analysis has been established in 1964. It is the first systematic information facilities in the field of business administration in Japan that has been recognized and authorized by the Ministry of Education. The purpose is to collect and to make intensive control of all kinds of materials on business administration and to make them availabe to scholars, universities, governments, and business world with the aid of modern documentation techniques.

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