Conspicuous Consumption and Resource Exploitation

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Abstract

This paper studies the role of envy in the over-exploitation of natural resources in the context of a dynamic game. We abandon the standard assumption that agents are atomistic. Instead, agents take into account strategic interactions. Envious agents play a dynamic game among themselves, each knowing that, under certain conditions, he can indirectly influence the level of consumption of others by affecting the stock level of a private or a common-property resource. We show that, both in the case of privately owned resources and in the case of common property resources, an open-loop Nash equilibrium under envy can be Pareto optimal, under certain assumptions. In contrast, in the case of Markov-perfect Nash equilibriums, the equilibrium outcome when everyone is status-conscious is inferior to what would be obtained if nobody were status-conscious. In particular, we show that in the case of a common-access resource, there exists a Markov-perfect Nash equilibrium where an increase in status-consciousness leads to a worsening of the common property problem. In a final section, we introduce heterogeneity, and show that social welfare decreases as the degree of heterogeneity in envy becomes more pronounced.

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1 Introduction

The assumption that preferences are independent across households is standard in the economic literature, although it is not particularly appealing. Indeed, social scientists and philosophers have long recognized that status seeking is an important characteristic of human behavior (see Aristotle (1941, Rhetoric, Book II, Chapter 10), Kant (1960, Chapter 6), Rawls (1971, Sections 80-82), Schoeck (1966)). In our discipline, Smith (1759) and Veblen (1899) were among the first to stress the idea that the overall level of satisfaction derived from consumption depends not only on the consumption level itself but also on how it compares to the consumption of other members of society. Veblen (1899) argued that conspicuous consumption stems from the desire to emulate. He defined emulation as "the stimulus of an invidious comparison which prompts us to outdo those with whom we are in the habit of classing ourselves." He claimed that "with the exception of the instinct for self-preservation, the propensity for emulation is probably the strongest and most alert and persistent of economic motives proper."

The Veblen effect was formalised by Duesenberry (1949) and Leibenstein (1950). The subsequent literature has often referred to this type of interdependence as "catching up with the Joneses" as in Abel (1990), "keeping up with the Joneses" as in Gali (1994), "status" as in Fisher and Hof (2000), "jealousy" as in Dupor and Liu (2003), or "envy" as in Eaton and Eswaran (2003).

There is a growing body of empirical evidence that confirms the importance of emulation and envy. Clark and Oswald (1996), using a sample of 5,000 British workers, find that workers' reported satisfaction levels are inversely related to the wage rates of their peers, supporting the hypothesis of positional externalities. Neumark and Postlewaite (1998) propose a model of relative income to rationalize the striking rise in the employment of married women in the U.S. during the past century. Using a sample of married sisters, they find that married women are 16 to 25 percent more likely to work outside the home if their sisters' husbands earn more than their own husbands. Luttmer (2005) matches individual-level panel data on well-being from the U.S. National Survey of Families and Households to census data on local average earnings. After controlling for income and other own characteristics, he finds that local average earnings have a significantly negative effect on self-reported happiness[1].

[1] For discussions of the empirical evidence, see Oswald (1997), Easterlin (2001), Blanchflower and Oswald (2004), Layard (2005). Beyond these studies, status concerns have been introduced to account for observed departures from the neoclassical paradigm in the asset pricing literature (Abel (1990), Gali (1994) and Campbell and Cochrane (1999)), the literature on labor market outcomes (Akerlof and Yellen (1990), the consumption literature (van de Stadt et al. (1985), Kapteyn et al. (1997), Alvarez-Cuadrado and Sutthiphisal

On the other hand, there is a vast literature on the over-exploitation of natural resources, in which economists and other scientists have traditionally focused in the "common property" characteristics of many resources. Gordon (1954) presents a lucid treatment of the economics of common property resources. Hardin (1968) conveys the "tragedy of the commons" to the scientific community. Smith (1968) focuses on the steady-state inefficiency while Plourde (1971), Brown (1974) and Smith (1975) explicitly consider models that exhibit transitional dynamics. Brown (1974) points out that a harvest tax, which must change over time as the stock level evolves, should be introduced to correct for congestion externalities. Smith (1975) reviews the debate on the cause of the extinction of many animal species in prehistoric time, and assesses the role of "over-hunting" by primitive human societies. Kremer and Morcom (2000) analyze multiplicity of equilibria in common property resources. Considering the environment as an international common property, Withagen and van der Ploeg (1991), Dockner and Long (1993), Copeland and Taylor (1995), de Zeeuw and Mäler (1998) show that the environment is over-exploited and analyze the role of coordination and governmental regulation.

There are a few papers that connect these two streams of literature. Ng and Wang (1993) and Howarth (1996) discuss, in a static context, the impact of conspicuous consumption on pollution generation and environmental quality. They argued that a corrective tax would be required. Alvarez-Cuadrado and Long (2007) consider the impact of envy on resource extraction in a model where all resource stocks are privately owned and property rights are respected. They identify conditions under which there are dynamic distortions due to envy, and discuss corrective taxes. They also give a set of sufficient conditions under which envy does not generate distortions. In particular, they show that in the case of costless extraction, if the degree of envy is constant along the symmetric consumption path, then there is no need to introduce corrective taxes[2]. In their model, individuals are price takers, and do not engage in any dynamic game.

The purpose of the present paper is to study the role of envy in the over-exploitation of natural resources in the context of a dynamic game. We abandon the standard assumption that agents are atomistic. Instead, we assume that agents take into account the fact that there are strategic interactions. This is important because in many real world situations, the agents that participate in a dynamic game may each have considerable impact on the

(2006)), the experimental literature (Solnick and Hemenway (1998), Johansson-Stenman et al. (2002) and Alpizar et al. (2005)) and the real business cycle literature (Ravn et al. (2006)).

[2] This condition has been obtained by Fisher and Hof (2000), Liu and Turnovsky (2005) in the context of the standard competitive one-sector growth model. Arrow and Dasgupta (2007) extend the result to a competitive model with many capital goods.

resource stocks. In some cases, "status-conscious agents" may be interpreted as nations rather than individuals. Concern about their relative position in terms of GDP per head may influence their over-exploitation of environmental assets.

In our formulation, envious agents play a dynamic game among themselves, each knowing that, under certain cases, he can indirectly influence the level of consumption of others by affecting the stock level of a private or common-property resource. We consider a differential game, where, for each agent i, the actions he can choose are restricted to a strategy space S_i. If for all agents $i = 1, 2, .., n$,the strategy space is the set of open-loop strategies (i.e. consumption c_i at t is a function of t only), the equilibrium concept is "open-loop Nash equilibrium."[3] In contrast, if the strategy space S_i is the set of feedback strategies (i.e. consumption c_i at t is a function of the currently observed stock levels), the appropriate equilibrium concept is "Markov-perfect Nash equilibrium."[4] We show that an open-loop Nash equilibrium can be Pareto optimal, under the assumption of a constant degree of envy $\varepsilon \in (-1, 0)$. However, if we insists on Markov-perfect Nash equilibriums, the equilibrium outcome when everyone is status-conscious is inferior to what would be obtained if nobody were status-conscious. In particular, we show that in the case of a common-access resource, there exists a Markov-perfect Nash equilibrium where higher status-consciousness leads to a worsening of the common property problem.

In Section 2, in order to focus on the choice between current consumption and future consumption, we assume that exploitation is effortless.We first consider the case where all resource stocks are privately owned. In this case, we show that the social optimum can be supported by an open-loop Nash equilibrium if the degree of envy is constant along the 45-degree line, but in general it cannot be supported by a Markov-perfect Nash equilibrium, unless the utility function is additively separable in private consumption and reference consumption. We next consider the case of a resource stock under common access. It turns out that, even with envy and common-property exploitation, there exist sufficient conditions under which an open-loop Nash equilibrium is socially optimal, and therefore there is no need for intervention. This rather surprising result suggests that the concept of Markov-perfect Nash equilibrium may be more appropriate.

In Section 3, we turn to the case where the exploitation of a resource involves a loss of leisure time. Again, we show that, in a Markov-perfect Nash equilibrium, status-consciousness

[3]The traditional formulation of the market outcome may be thought of as a limiting case of an open-loop Nash equilibrium when the number of agents becomes large.

[4]For detailed explanation of these concepts, see for example, Dockner et al. (2000), Maskin and Tirole (2001). In the Appendix, we provide a simple example of a game with a symmetric open-loop Nash equilibrium, and shows that such an equilibrium is not sub-game perfect.

exacerbates the over-exploitation of the commons. Agents tend to behave more aggressively if they are more concerned about their relative status. Consequently, the social welfare is lower. In addition, the growth rate of the public asset is reduced due to higher extraction rates. We also show that an exogenous technical progress in the resource-extraction sector can reduce welfare, and the magnitude of this welfare-worsening effect is an increasing function of the status-seeking parameter. In a final section, we introduce heterogeneity, and show that social welfare decreases if agents become more heterogeneous in terms of status-seeking, but it increases if they become more heterogeneous in terms of appropriation costs.

2 Exploitation of Natural Resources by Status-conscious Agents

Assume there are n infinitely-lived individuals. Let $c_i(t)$ denote individual i's consumption rate at time t. Let $C_i(t)$ denote the average consumption of his peers:

$$C_i(t) \equiv \frac{1}{n-1} \sum_{j \neq i} c_j(t)$$

Let $U(c_i, C_i)$ be individual i's utility function. In what follows, we will drop the subscript i when there is no confusion. The following notations will be adopted

$$U_c \equiv \frac{\partial U(c, C)}{\partial c}\ , \ U_C \equiv \frac{\partial U(c, C)}{\partial C}\ , \ U_{cc} \equiv \frac{\partial^2 U(c, C)}{\partial c^2}, \text{ etc}$$

An individual is said to be "envious" or "status-conscious" if $U_C < 0$.

We make the following standard assumptions on the function U :

Assumption U1: The utility function $U(c_i, C_i)$ is twice continuously differentiable for all $(c_i, C_i) > (0, 0)$. It is strictly increasing and concave in c_i, and decreasing in C_i. Along the 45-degree line where $c_i = C_i = q$ say, the function U is increasing and strictly concave in q, i.e.,

$$\frac{d}{dq}(U(q, q)) = U_c(q, q) + U_C(q, q) > 0$$

$$\frac{d^2}{dq^2}(U(q, q)) = U_{cc}(q, q) + 2U_{cC}(q, q) + U_{CC}(q, q) < 0$$

Assumption U2: Optimal consumption is strictly positive. This is ensured by assuming

$$\lim_{c \to 0} U_c(c, C) = \infty$$

Definition D1: (Additive envy) A utility function displays additive envy if U_C is negative and independent of c.

Definition D2: (Mutiplicative envy) A utility function displays multiplicative envy if U_C is negative and dependent on c.

Definition D3: (Degree of envy) An individual's *degree of envy* $\varepsilon(q)$ *at a symmetric-consumption point* $(c_i, C_i) = (q, q)$ is his marginal rate of substitution between his own consumption level and the average consumption level of his reference group, evaluated at the same value $c_i = C_i = q$:

$$\varepsilon(q) \equiv \frac{U_C(q,q)}{U_c(q,q)}$$

If ε is independent of q, we say that the degree of envy is constant.

Remark: Assumption U1 implies that

$$-1 < \varepsilon(q) \leq 0$$

i.e., in a graph where C is measured along the horizontal axis, and c along the vertical axis, indifference curves have a positive slope smaller than unity when they cross the 45 degree line. For example, if my $\varepsilon = -0.8$, whenever the average consumption of my peer group increases by 1 unit, I will feel worse off unless my consumption goes up by 0.8 or more.

Example 1: (additive envy)

$$U(c, C) = c^{\sigma} + \gamma C^{\sigma} \text{ where } -1 < \gamma \leq 0 \text{ and } 0 < \sigma < 1$$

In this example, $\varepsilon(q) = \gamma$. If $\gamma = 0$, there is no envy. The restriction $-1 < \gamma$ is imposed so that, starting from a point of equal consumption levels, if each individual's consumption increases by the same amount, everyone will feel better off.

Example 2:(multiplicative envy)

$$U(c, C) = \frac{1}{1-\sigma} c^{1-\sigma+\alpha} C^{-\alpha} \text{ where } 0 < \alpha < \sigma < 1$$

This function may be re-written as

$$U = \frac{1}{1-\sigma} c^{1-\sigma} \left(\frac{c}{C}\right)^{\alpha}$$

where c/C is relative consumption. In this example, $\varepsilon(q) = -\alpha/(1 - \sigma + \alpha) \geq -1$. If $\alpha = 0$, there is no envy.

Example 3: (non-constant envy)

$$U(c, C) = c^{\gamma}C^{-\gamma} + \ln(a + c) \text{ where } a \geq 0 \text{ and } 0 < \gamma < 1$$

In this example, if $\gamma = 0$, there is no envy. If $\gamma > 0$ and $a > 0$, then $\varepsilon(q)$ increases with q

$$0 > \varepsilon(q) = -\frac{\gamma q^{-1}}{(\gamma q^{-1} + (a + q)^{-1})} > -1 \text{ and } \varepsilon'(q) > 0$$

.

2.1 Model 1: Privately-owned Resource Stocks

Consider first the case where there are n identical individuals and n identical stocks of a homogenous natural resource. Individual i owns and exploits the stock x_i. Property rights are respected. The only interaction among individuals is that each observes the levels of consumption of the other $n-1$ individuals, which affect his utility through envy. The utility function $U(c, C)$ satisfies Assumptions U1 and U2.

The welfare of individual i is

$$W_i = \int_0^{\infty} e^{-\rho t} U(c_i(t), C_i(t)) dt, \ \rho > 0.$$

Each individual maximizes his welfare, taking as given the consumption strategy of other individuals, and subject to the equation of motion of the stock

$$\dot{x}_i(t) = G(x_i(t)) - c_i(t)$$

with the initial condition $x_i(0) = x_{i0} > 0$ and the constraint $x_i(t) \geq 0$. The natural growth function $G(x_i)$ is assumed to be strictly concave, with $G(0) = 0$.

We distinguish two types of strategy. An individual i is said to have an open-loop strategy if he is committed to a time path of consumption, $c_i(.)$ which he announces from the outset. We denote an open-loop strategy by ϕ_i. In contrast, a feedback strategy is a consumption rule: it expresses an individual's consumption level at time t as a function of the observed stock levels at time t. We denote feedback strategies by

$$c_i = M_i(x_1, x_2, ..., x_n)$$

where M_i is a function that maps the state-space R_n^+ to the space of consumption $[0, \infty)$.

An open-loop Nash equilibrium is a strategy profile $(\phi_1, \phi_2, ..., \phi_n)$ such that for all i, the (open-loop) strategy ϕ_i is individual i's best reply to the strategies of other individuals.

This means that each individual i takes the entire time paths of consumption of other individuals as given, and ϕ_i is the solution (i.e, the optimal consumption path) of his dynamic optimization problem.

A feedback Nash equilibrium is a strategy profile $(M_1, M_2, ..., M_n)$ such that for all i, the feedback strategy M_i is individual i's best reply to the feedback strategies of other individuals. This means that while each individual i takes other rules M_j as given, he does not take the entire time paths of consumption of other individuals as given: he knows that any change in the stock levels would generally induce changes in consumption levels, and that each individual can at least influence the future level of his stock.

A feedback Nash equilibrium is said to be a Markov-perfect Nash equilibrium if the rules $(M_1, M_2, ..., M_n)$ are independent of the initial stock levels $(x_1(0), x_2(0), ..., x_n(0))$.The concept of Markov-perfect Nash equilibrium is more attractive than open-loop Nash equilibrium, because it corresponds to the concept of subgame perfect Nash equilibrium in discrete-time games. (See, for example, Maskin and Tirole, 2001.)

We wish to compare the outcome under both types of Nash equilibrium (open-loop, and Markov-perfect) with the outcome under a benevolent social planner. We assume that the social planner treats all individuals equally, and maximizes the welfare of the representative individual, taking into account that each individual's utility depends both on his own consumption, c_i, and on the average level of consumption of other individuals, C_i.

We can prove the following propositions.

Proposition 2.1: Assume that resource stocks are privately owned and property rights are respected. The outcome under the social planner can be supported as an open-loop Nash equilibrium if and only if the utility function $U(c, C)$ exhibits *a constant degree of envy,* where $-1 < \varepsilon \leq 0$.

Proof: See the Appendix.

Discussion: Proposition 2.1 indicates that, under the assumption of a constant degree of envy, and if all individuals take the time paths of consumption of others as given, the social optimum and the symmetric laissez-faire outcome coincide. There are no need for government intervention, even if the economy consists of a finite collection of individuals.

In contrast, when individuals use feedback strategies, a constant degree of envy is not sufficient to ensure that the social optimum can be supported by a Markov-perfect Nash equilibrium even though resource stocks are privately owned and property rights are respected. We need stronger restrictions. Our result is reported in Proposition 2.2 below.

Proposition 2.2: Assume that resource stocks are privately owned and property rights are respected. The socially optimal outcome can be supported as a Markov-perfect Nash equilibrium if the following conditions hold simultaneously: (i) the utility function $U(c, C)$ exhibits *a constant degree of envy* ε *at symmetric consumption points* where $-1 < \varepsilon \leq 0$, (ii) $U(c, C)$ is additively separable, and (iii) each M_i is *a function of the stock* x_i *only* (M_i is independent of x_j where $j \neq i$)

Proof: See the Appendix.

Remark: The fact that each player uses (by construction) a policy function that is dependent only on his stock level, and thus independent of the stock levels of others, plays a crucial role in our proof of Proposition 2.2. It is important to note that, in general, it does not seem possible to rule out the existence of other Markov-perfect Nash equilibriums where each individual conditions his consumption on several stocks. For example, a conceivable strategy is

$$c_i = h(x_i, x_j)x_i$$

where $\partial h / \partial x_j \neq 0$. In such cases, it is possible that the resulting Markov-perfect Nash equilibrium (if it exists) does not coincide with the social optimum.

Proposition 2.2 relies heavily on the assumption that $U(c, C)$ is additively separable. In fact, if $U(c, C)$ is not additively separable, in general the social optimum cannot be replicated as a Markov-perfect Nash equilibrium. This is established as Proposition 2.3.

Proposition 2.3: If $U(c, C)$ is not additively separable, in general the social optimum cannot be replicated as a Markov-perfect Nash equilibrium.

Proof: See the Appendix.

The intuition behind Proposition 2.3 is as follows. The socially optimal consumption rule requires that each person's consumption depends only on the current level of the representative stock. In contrast, under individual optimization, each person's marginal utility of consumption depends on the levels of consumption of his peers, which in turn depend on their stocks. It follows each person's consumption strategy depends on all the private stocks. This creates strategic interdependence which tends to result in inferior outcomes.

2.2 Model 2: Exploitation of a Common-property Resource by Envious Agents

Let us turn to the other polar case where there is just one stock of renewable resource, to which all individuals have common access. We will show that when the equilibrium concept is Markov-perfect Nash equilibrium, envy (status-consciousness) creates additional distortion over and above the usual distortion associated with common access. On the other hand, when individuals use open-loop strategies, the symmetric open-loop Nash equilibrium coincides with the outcome under the social planner, if certain assumptions hold.

2.2.1 Common Property Resource: Comparing the social optimum and the open-loop Nash equilibrium under envy

Let us begin with the social planner's problem. The social planner chooses $c(t)$ to maximize

$$\int_0^\infty e^{-\rho t} nU(c,c)dt$$

subject to the transition equation

$$\dot{x} = G(x) - nc, \text{ with } x(0) = x_0 > 0$$

and $x(t) \geq 0$.

Let ψ be the shadow price of the resource stock, and H be the Hamiltonian function for the social planner's problem.

$$H = nU(c,c) + \psi\left(G(x) - nc\right)$$

We use the superscript * to denote the social optimum for the model where there is only one resource stock.The socially optimal path satisfies

$$U_c(c^*, c^*) + U_C(c^*, c^*) = \psi > 0$$

$$\frac{\dot{\psi}}{\psi} = \rho - G'(x^*) \tag{1}$$

$$\dot{x}^* = G(x^*) - nc^*$$

$$\lim_{t\to\infty} e^{-\rho t}\psi(t)x^*(t) = 0, \ \lim_{t\to\infty} e^{-\rho t}\psi(t) \geq 0 \text{ and } \lim_{t\to\infty} x^*(t) \geq 0.$$

Now let us turn to the open-loop Nash equilibrium. Can the socially optimal path of per capita consumption $c^*(t)$ be the same as the symmetric open-loop Nash equilibrium consumption?

The answer to this question is stated as Proposition 2.4:

Proposition 2.4: Under common access, the socially optimal path of per capita consumption $c^*(t)$ coincides with the symmetric open-loop Nash equilibrium consumption if and only if the utility function $U(c,C)$ exhibits *a constant degree of envy* ε, where $-1 < \varepsilon \leq 0$.

Proof:

We must show that the socially optimal consumption path satisfies the necessary conditions of the optimization problem of individual i. Recall that in an open-loop game, individual i takes as given the time path $C_i(t)$ of per-capita consumption of his peer group, and chooses the time path $c_i(t)$ to maximize

$$W_i = \int_0^{\infty} e^{-\rho t} U(c_i, C_i) dt$$

subject to

$$\dot{x} = G(x) - (n-1)C_i - c_i$$

Let $\psi_i(t)$ be shadow price that individual i attaches to the common stock $x(t)$. The Hamiltonian for individual i's optimization problem is

$$H_i = U(c_i, C_i) + \psi_i \left[G(x) - (n-1)C_i - c_i \right]$$

Let the superscript ol in c^{ol} and x^{ol} denote the outcome under the open-loop Nash equilibrium.The necessary conditions are

$$U_c(c_i^{ol}, C_i^{ol}) = \psi_i > 0$$

$$\frac{\dot{\psi}_i}{\psi_i} = \rho - G'(x^{ol}) \tag{2}$$

$$\dot{x}^{ol} = G(x^{ol}) - (n-1)C_i^{ol} - c_i^{ol}$$

$$\lim_{t\to\infty} e^{-\rho t}\psi_i(t)x^{ol}(t) = 0, \ \lim_{t\to\infty} e^{-\rho t}\psi_i(t) \geq 0 \text{ and } \lim_{t\to\infty} x^{ol}(t) \geq 0.$$

We noe derive necessary and sufficient conditions for $x^{ol}(t) = x^*(t)$ and $c^{ol}(t) = c^*(t)$. For $x^*(t)$ to coincide with $x^{ol}(t)$, conditions (1) and (2) require that there exists a positive constant η such that $\psi = \eta\psi_i$ for all t. This implies

$$U_c(c^*(t), c^*(t)) + U_C(c^*(t), c^*(t)) = \eta U_c(c^*(t), c^*(t))$$

for all t. Dividing both sides by $U_c(c^*, c^*)$ we get

$$1 + \frac{U_C(c^*(t), c^*(t))}{U_c(c^*(t), c^*(t))} = \eta > 0$$

This holds iff

$$1 + \varepsilon(c^*(t)) = \eta > 0$$

i.e., ε is independent of $c^*(t)$ and $-1 < \varepsilon \leq 0$. All other necessary conditions of the individual optimization problems are satisfied when $x^{ol}(t) = x^*(t)$ and $c^{ol}(t) = c^*(t)$.

Remark: The result that, in a symmetric open-loop Nash equilibrium, status-consciousness does not matter if and only if the utility function $U(c, C)$ exhibits *a constant degree of envy* extends the result of Proposition 2.1 to the case of a common-property resource. We now proceed to show that it is not true that the social optimum can be supported by a Markov-perfect Nash equilibrium.

2.2.2 Common Property Resource: Comparing the social optimum and the Markov-perfect Nash equilibrium under envy

We now show by an example that (i) the social optimum cannot be supported by a Markov-perfect Nash equilibrium, and (ii) the higher is the degree of envy, the greater is the deviation of the Markov-perfect equilibrium from the social optimum.

To make our point as transparent as possible, we will consider an example where the equilibrium strategies can be explicitly calculated.

We take the utility function

$$U(c, C) = \frac{1}{1-\sigma} c^{1-\sigma+\alpha} C^{-\alpha} \text{ where } 0 < \alpha < \sigma < 1$$

The resource stock $x(t)$ evolves according to the transition equation

$$\dot{x}(t) = Ax(t)^{\gamma} - \delta x(t) - \sum_{i=1}^{n} c_i(t) \text{ with } x(0) = x_0 > 0$$

where $A > 0, \gamma > 0$ and $\delta \geq 0$. In what follows we assume $\sigma = \gamma$. This assumption enables us to obtain closed-form solutions to the social planner's problem and to the game among the individuals.

Consider first the social planner's optimization problem. It can be shown that the solution can be expressed as an optimal control in feedback form:

$$c_i = c(x) = \beta^* x$$

where

$$\beta^* \equiv \frac{\rho + \delta(1-\sigma)}{n\sigma}$$

The resource stock will converge to a steady state $\widehat{x}^*$, where

$$\widehat{x}^* = \left[\frac{A}{b}\right]^{1/(1-\sigma)} \text{ with } b \equiv \frac{\rho + \delta}{\sigma}$$

and the welfare level of the representative individual is

$$W(x_0) = \left[\frac{\rho + \delta(1-\sigma)}{n\sigma}\right]^{1-\sigma} \left(\frac{1}{1-\sigma}\right) \left[\frac{A}{b\rho} + \frac{(x_0)^{1-\sigma} - (A/b)}{\rho + b(1-\sigma)}\right]$$

Notice that the degree of envy has no impact on the socially optimal consumption path.

The socially optimal time path of the resource stock is

$$x^*(t) = \left[\frac{A}{b} + (x_0^{1-\sigma} - \frac{A}{b})e^{-b(1-\sigma)t}\right]^{1-\sigma}$$

and the socially optimal path of consumption is

$$c^*(t) = \beta^* x(t) = \beta^* \left[\frac{A}{b} + (x_0^{1-\sigma} - \frac{A}{b})e^{-b(1-\sigma)t}\right]^{1-\sigma}$$

We now show there exists a Markov-perfect Nash equilibrium where each agent j uses a feedback strategy of the form

$$c_j(t) = \beta^{OA} x(t)$$

where β^{OA} is a positive constant. (The supercript OA indicates that we are dealing with an open-access resource stock.)

Suppose individual i thinks that all $k \neq i$ use the same strategy $c_k(t) = \overline{\beta} x(t)$, for some $\overline{\beta} > 0$.The optimization problem of individual j can then be formulated as follows. Find a time path $\beta_i(t)$ to maximixe

$$\int_0^\infty e^{-\rho t} \frac{1}{1-\sigma} \left[\beta_i(t) x_i(t)\right]^{1-\sigma+\alpha} \left[\overline{\beta} x(t)\right]^{-\alpha} dt$$

subject to

$$\dot{x}(t) = Ax(t)^\sigma - \delta x(t) - (n-1)\overline{\beta} x(t) - \beta_i(t) x_i(t)$$

It can be shown that, provided that $1 > n(1-\sigma+\alpha)$, there is a symmetric Markov-perfect Nash equilibrium, where all individuals use the linear extraction strategy, with

$$\beta^{OA} = K\beta^*,\ K > 1$$

$$K \equiv \frac{n\sigma}{(1-\sigma)\left[\frac{1}{1-\theta+\alpha} - n\right]},\ K'(\alpha) > 0$$

Proposition 2.5: (i) The social optimum cannot be supported by a Markov-perfect Nash equilibrium, and (ii) the higher is the degree of envy, the greater is the deviation of the Markov-perfect equilibrium from the social optimum.

Proof: See the Appendix..

3 A Model with Effort Costs and Amenities

We now modify our model to allow for effort costs and enjoyment of amenity services. We assume there is only one resource stock. All n agents have access to this stock. Let $E_i(t)$ denote agent i's extraction rate from a common-property resource. We assume that the consumption rate $c_i(t)$ is a fraction of the extraction rate $E_i(t)$. Specifically, $E_i(t) = (1+\theta_i)c_i(t)$. Here θ_i is a non-negative number that represents agent i's "wastage rate", which may be interpreted as reflecting his degree of inefficiency in transforming the extracted resource into the consumption good, or perhaps, in a different context, as the bribes or penalties that he must pay to third parties in his illicit resource-appropriation process.

Let $X(t)$ denote the stock level of the common-property resource. We assume that the rate of growth of X is given by the differential equation

$$\dot{X}(t) = AX(t) - \sum_{i=1}^{n} E_i(t)$$

where $A \geq 0$ is a constant.

We define $z_i(t)$ to be agent i's relative consumption level:

$$z_i(t) \equiv \frac{c_i(t)}{C_i(t)}$$

The *net-utility function* of agent i is denoted by $V(z_i, c_i, X, E_i)$ where

$$V = U(z_i, c_i, X) - \kappa_i E_i$$

The variable X appears in the utility function, because the stock X provides a flow of amenities (e.g. recreational uses) that each agent values. The non-negative parameter κ_i represents the "effort cost" of extracting the resource. This parameter may represent (a) a technological coefficient between effort and harvest level, so that a fall in κ_i represents a technological progress in resource extraction, or (b) the difficulty with which the agent hides his illegal activities. Note that we have introduced two separate parameters, θ_i and κ_i, that represent different types of cost of appropriation: κ_i is the "effort cost" which is measured in utility units, while θ_i is the "wastage cost", which acts like an income tax.

We assume that each agent's *gross-utility function* $U(z_i, c_i, X)$ is non-decreasing in relative consumption, z_i, and increasing in absolute consumption, c_i, and in the amenities provided by the stock, X:

$$\frac{\partial U}{\partial z_i} \geq 0, \ \frac{\partial U}{\partial c_i} > 0, \ \frac{\partial U}{\partial X} > 0$$

We assume that, for any given C_i, the individual's utility is strictly increasing and strictly concave in his own consumption level, c_i. Strict concavity is assumed so that the second order condition for individual maximization is satisfied. To proceed further, we make the following specific assumptions:

Assumption A.1: The gross-utility function takes the form

$$U(z_i, c_i, X) = D(z_i)F(c_i, X)$$

where $F(c_i, X)$ is homogeneous of degree one[5], strictly-quasi-concave, and increasing in (c_i, X), and $D(z_i)$ is positive and non-decreasing in z_i.

Without loss of generality, we set $D(1) = 1$. If $D'(.) > 0$, we say that the agents are envious (concerned about relative consumption), while if $D'(.) = 0$ identically, we say that the agents are non-envious.

For given z_i, the marginal rate of substitution of consumption c_i for X is

$$MRS_{c_iX} \equiv \frac{F_{c_i}}{F_X}$$

It is useful to define the ratio of consumption to amenity services by $\beta_i = c_i/X$. Since $F(c_i, X)$ is homogeneous of degree 1, we obtain

$$F(c_i, X) = XF(\beta_i, 1) \equiv Xf(\beta_i)$$

Under Assumption A1, it follows that $f'(\beta_i) = F_c > 0$, $f''(\beta_i) < 0$, $r(\beta_i) \equiv f(\beta_i) - \beta_i f'(\beta_i) = F_X > 0$ and $r'(\beta_i) = -\beta_i f''(\beta_i) > 0$.Hence

$$MRS_{c_iX} \equiv \frac{F_{c_i}}{F_X} = \frac{f'(\beta_i)}{f(\beta_i) - \beta_i f'(\beta_i)} \equiv \omega(\beta_i)$$

Clearly the marginal rate of substitution is diminishing in β_i :

$$\omega'(\beta_i) = \frac{f(\beta_i)f''(\beta_i)}{[f(\beta_i) - \beta_i f'(\beta_i)]^2} < 0$$

Assumption A.2: The function f satisfies the following Inada conditions:

$$\lim_{\beta \to 0} f'(\beta) = \infty, \ \lim_{\beta \to \infty} f'(\beta) = 0$$

Our analysis at a general level does not rely on a specific functional form for F nor D, however at places it will be convenient to specialize in the following Cobb-Douglas case:

$$U(z_i, c_i, X) = z_i^\lambda c_i^\mu X^{1-\mu} \text{ where } \lambda > 0 \text{ and } 0 < \mu < 1 \text{ and } \lambda + \mu < 1$$

Here, the parameter λ is an indicator of the strength of the status-consciousness.

[5] The assumption of homogeneity of degree one in (c_i, X) is borrowed from Long and Sorger (2006). It greatly simplifies the analysis.

3.1 The Cooperative Equilibrium

It is useful to begin with the following benchmark scenario. All agents are identical, and they cooperate by agreeing on a common rate of resource extraction: $E_i(t) = E(t)$. It follows that $c_i(t) = c(t)$ and $z_i(t) = 1$. It is as if there were a social planner seeking to solve the following optimization problem. Choose $c(t)$ to maximize

$$\int_0^\infty e^{-\rho t} \left[D(1)F(c, X) - \kappa(1+\theta)c \right] dt \tag{3}$$

subject to

$$\dot{X} = AX - n(1+\theta)c, \; X(0) = X_0 > 0$$

$$\lim_{t\to\infty} X(t) \geq 0$$

To ensure convergence of the integral, we will assume:

Assumption A.3: The rate of discount exceeds the natural growth rate of the stock: $\rho > A$.

Using the definition $\beta = c/X$, the social planner's problem reduces to finding the time path of the control variable $\beta(t)$ that maximizes the welfare of the representative agent:

$$W^{coop} = \int_0^\infty e^{-\rho t} \left[f(\beta) - \kappa(1+\theta)\beta \right] X dt$$

subject to

$$\dot{X} = X \left[A - n(1+\theta)\beta \right], \; X(0) = X_0 > 0$$

and

$$\lim_{t\to\infty} X(t) \geq 0$$

Let ψ denote the shadow price of the stock X. The Hamiltonian function is

$$H = \left[f(\beta) - \kappa(1+\theta)\beta \right] X + \psi X \left[A - n(1+\theta)\beta \right]$$

The necessary conditions include

$$\frac{\partial H}{\partial \beta} = X \left\{ f'(\beta) - \kappa(1+\theta) - n\psi(1+\theta) \right\} = 0$$

$$\dot{\psi} = (\rho - A)\psi - \left[f(\beta) - (1+\theta)(\kappa + n\psi)\beta \right]$$

and the transversality condition is

$$\lim_{t\to\infty} \psi(t)e^{-\rho t} \geq 0, \; \lim_{t\to\infty} X(t) \geq 0, \; \lim_{t\to\infty} \psi(t)e^{-\rho t} X(t) = 0 \tag{4}$$

Let us consider a candidate solution where $\beta(t) = \overline{\beta}$ (a constant). This yields a corresponding constant $\overline{\psi}$ where

$$f'(\overline{\beta}) = (1+\theta)(\kappa + n\overline{\psi}) \tag{5}$$

or

$$\overline{\psi} = \frac{1}{n}\left[\frac{f'(\overline{\beta})}{(1+\theta)} - \kappa\right] \tag{6}$$

which implies that $\dot{\psi} = 0$, hence

$$(\rho - A)\overline{\psi} = f(\overline{\beta}) - (1+\theta)(\kappa + n\overline{\psi})\overline{\beta} \tag{7}$$

Using (5) and (7),

$$(\rho - A)\overline{\psi} = f(\overline{\beta}) - \overline{\beta} f'(\overline{\beta}) > 0 \tag{8}$$

Substituting (6) into (8), we get the following equation which determines the optimal $\overline{\beta}$, say $\overline{\beta}^*$

$$\left[\frac{f'(\overline{\beta})}{(1+\theta)} - \kappa\right] = \frac{n\left[f(\overline{\beta}) - \overline{\beta} f'(\overline{\beta})\right]}{\rho - A} \tag{9}$$

Proposition 3.1: *Under Assumptions A1, A2 and A3, the cooperative solution consists of following the consumption strategy $c = \overline{\beta}^* X$, where $\overline{\beta}^*$ is the unique positive solution of equation (9).*

Proof: See Appendix

Remark 1: Condition (9) has a straightforward interpretation. Given any $\overline{\beta}$, consider a small decrease in per-capita extraction, say dE at time zero. This will lead to a small decrease in consumption by $dc = dE/(1+\theta)$. The marginal utility loss from reduced consumption (net of reduced extraction cost κ) is thus $\left[f'(\overline{\beta})(1+\theta)^{-1} - \kappa\right] dE$. On the other hand, the impact effect on the stock is an increase by ndE, which leads to a stream of gain in marginal utility of amenities:

$$\int_0^\infty e^{-\rho t}\left\{\left[f(\overline{\beta}) - \overline{\beta} f'(\overline{\beta})\right](ndE)e^{At}\right\} dt = \frac{n\left[f(\overline{\beta}) - \overline{\beta} f'(\overline{\beta})\right]}{\rho - A} dE$$

At the optimal $\overline{\beta}^*$, the marginal utility loss from reduced consumption must equal the marginal utility gain from increased amenity services.

Remark 2: In the Cobb-Douglas case, if $\kappa = 0$, one can obtain an explicit solution:

$$\overline{\beta}^* = \frac{\mu(\rho - A)}{n(1-\mu)(1+\theta)}$$

and thus the growth rate of the public asset is

$$g = A - \frac{\mu(\rho - A)}{1-\mu}$$

which can be negative or positive.

Proposition 3.2: *The welfare of the representative agent under cooperation is*

$$W^{coop} = \overline{\psi}^* X_0$$

where

$$\overline{\psi}^* = \frac{1}{n}\left[\frac{f'(\overline{\beta}^*)}{(1+\theta)} - \kappa\right]$$

An increase in κ or in θ will reduce both $\overline{\beta}^*$ and welfare.

3.2 Non-cooperative resource extraction by envious agents

In this section, we study a differential game involving n identical players. Consider individual i. She faces $n-1$ rival rent-seekers. Suppose she thinks that each rival j adopts a consumption strategy of the form

$$c_j(t) = M_j(X(t)) \text{ where } M_j'(X) > 0$$

That is, at any moment of time, individual j's consumption depends only on the currently observed stock level $X(t)$.

Then

$$C_i(t) = \frac{1}{n-1}\sum_{j\neq i} M_j(X(t)) \equiv M(X(t))$$

The optimization problem for individual i is then to choose a time path of consumption $c_i(t) \geq 0$ that maximizes her life-time utility

$$\int_0^\infty e^{-\rho t}\left\{U\left(\frac{c_i(t)}{M(X(t))}, c_i(t), X(t)\right) - \kappa(1+\theta)c_i\right\} dt$$

subject to

$$\dot{X}(t) = AX(t) - (n-1)(1+\theta)M(X(t)) - (1+\theta)c_i(t)$$

and

$$\lim_{t\to\infty} X(t) \geq 0$$

3.2.1 Markov-perfect Nash equilibrium: the case of identical agents

In this subsection, we prove that under certain conditions, when agents are identical, the game described above has a symmetric Markov-perfect Nash equilibrium, in which all players adopt the same linear strategy

$$c_j(t) = \beta X(t)$$

where β is a positive constant.

Suppose player i knows that all other players use the strategy $c_j(t) = \beta X(t)$.The optimization problem of agent i is to choose a time path of $c_i \geq 0$ that maximizes

$$\int_0^{\infty} e^{-\rho t} \left\{ D\left(\frac{c_i}{\beta X}\right) F(c_i, X) - \kappa(1+\theta)c_i \right\} dt$$

subject to

$$\dot{X} = AX - (n-1)(1+\theta)\beta X - (1+\theta)c_i$$

$$\lim_{t\to\infty} X(t) \geq 0$$

We may interpret $A-(n-1)(1+\theta)\beta$ as player i's net rate of return on holding the asset.

Let ψ_i be the co-state variable. The Hamiltonian is

$$H_i = D\left(\frac{c_i}{\beta X}\right) F(c_i, X) - \kappa(1+\theta)c_i + \psi_i \left[AX - (n-1)(1+\theta)\beta X - (1+\theta)c_i\right]$$

The optimality conditions are

$$\frac{\partial H_i}{\partial c_i} = D'\left(\frac{c_i}{\beta X}\right)\left(\frac{1}{\beta X}\right) F(c_i, X) + D\left(\frac{c_i}{\beta X}\right) F_{c_i}(c_i, X) - (\kappa + \psi_i)(1+\theta) = 0 \tag{10}$$

$$\dot{\psi}_i = \psi_i \left[\rho - A + (n-1)(1+\theta)\beta\right] + D'\left(\frac{c_i}{\beta X}\right)\left(\frac{c_i}{\beta}\right) X^{-2} F - DF_X \tag{11}$$

$$\dot{X} = \frac{\partial H_i}{\partial \psi_i} = AX - (n-1)(1+\theta)\beta X - (1+\theta)c_i \tag{12}$$

$$\lim_{t\to\infty} e^{-\rho t}\psi_i(t) \geq 0 \text{ and } \lim_{t\to\infty} e^{-\rho t}\psi_i(t)X(t) = 0 \tag{13}$$

Let us try a symmetric equilibrium, with

$$\frac{c_i(t)}{X(t)} = \frac{c_j(t)}{X(t)} = \beta \tag{14}$$

We must verify that the optimality conditions (10) to (13) are satisfied when the strategies described by equation (14) are used, for some suitable constant $\beta > 0$.

Using symmetry, equation (10) becomes

$$D'(1)\left(\frac{1}{\beta}\right) f(\beta) + D(1) f'(\beta) - \kappa(1+\theta) - (1+\theta)\psi_i(t) = 0 \tag{15}$$

This equation implies that $\psi_i(t)$ is a constant, i.e. $\dot{\psi}_i = 0$ along the equilibrium play. Hence we must have

$$\psi_i \left[\rho - A + (n-1)(1+\theta)\beta\right] =$$
$$-D'(1)\, f(\beta) + D(1) \left[f(\beta) - f'(\beta)\, \beta\right] \tag{16}$$

These two equations are satisfied iff there exists some $\widehat{\beta} > 0$ which satisfies the following condition

$$\left[\frac{D'(1) f(\beta)\frac{1}{\beta} + D(1) f'(\beta)}{1+\theta} - \kappa\right] \left[(\rho - A) + (n-1)(1+\theta)\beta\right] + D'(1)\, f(\beta) =$$
$$D(1) \left[f(\beta) - f'(\beta)\, \beta\right] \tag{17}$$

Proposition 3.3: *A Markov-perfect Nash equilibrium, where all players play a linear feedback strategy of the form $c = \beta X$, exists iff the equation (17) has a solution $\widehat{\beta} > 0$.*

Example: The Cobb-Douglas Case

$$U = z_i^{\lambda} c_i^{\mu} X^{1-\mu}$$

Eq (17) becomes

$$\left[\frac{\lambda\beta^{\mu-1} + \mu\beta^{\mu-1}}{1+\theta} - \kappa\right] \left[(\rho - A) + (n-1)(1+\theta)\beta\right] =$$
$$-\lambda\beta^{\mu} + (1-\mu)\beta^{\mu}$$

i.e.

$$\left[\frac{\lambda + \mu}{1+\theta} - \kappa\beta^{1-\mu}\right] = \frac{(1-\lambda-\mu)}{(\rho - A)\frac{1}{\beta} + (n-1)(1+\theta)} \tag{18}$$

The LHS of equation (18) is decreasing in β. As β varies from zero to infinity, the LHS falls from $(\lambda+\mu)/(1+\theta)$ to minus infinity if $\kappa > 0$. The RHS is increasing in β, varying from zero to $(1-\lambda-\mu)/\left[(n-1)(1+\theta)\right]$ as β varies from zero to infinity. It follows that if $\kappa > 0$, there exists a unique positive $\widehat{\beta}$ that equates the LHS with the RHS. Furthermore, an increase in κ will lower the curve representing the RHS, resulting in a smaller value of $\widehat{\beta}$. An increase in λ will shift the curve representing the RHS down, and shift the curve representing the LHS up, resulting in a higher value of $\widehat{\beta}$. (If $\kappa = 0$ then a positive $\widehat{\beta}$ exists if and only if $n(\lambda+\mu) < 1$.)

Do these results apply to the general case? The answer is yes, provided the equation (17) has a unique solution $\widehat{\beta} > 0$. Without loss of generality, we set $D(1) = 1$ and treat $D'(1)$

as a parameter: the higher is $D'(1)$, the higher is the degree of status-consciousness of the players. To simplify notation, denote the status consciousness parameter by $\lambda \equiv D'(1)$.

Proposition 3.4: (*The general case*) *Assume* $\widehat{\beta}$ *is unique. Then*

(a) A higher degree of status-consciousness will result in a higher equilibrium rate of extraction and a lower public asset growth rate.

(b) An increase in κ *or* A *will reduce the equilibrium rate of extraction,* $\widehat{\beta}$ *and thus increase the growth rate of the public asset.*

Proof: An increase in $D'(1)$ will shift upwards the curve representing the LHS of (17). Hence the intersection point $\widehat{\beta}$ must move to the right. Similarly, an increase in κ or A shift downwards the curve representing the LHS of (17), thus moving $\widehat{\beta}$ to the left. The growth rate of the public asset in the Markov-perfect equilibrium (MPE) is

$$\frac{\dot{X}}{X} = g^{MPE} = A - n(1+\theta)\widehat{\beta}$$

It follows that an increase in κ or A will increase the growth rate of the public asset. ■

Proposition 3.5: *(comparing the cooperative solution with the non-cooperative equilibrium) The cooperative rate of extraction,* $\overline{\beta}^*$*, is lower than the non-cooperative rate of extraction* $\widehat{\beta}$*.*

Proof: Re-write eq (9) as follows

$$\left[\frac{f'(\beta)}{(1+\theta)} - \kappa\right]\left(\frac{\rho - A}{n}\right) = f(\beta) - \beta f'(\beta) \tag{19}$$

and compare with

$$\left[\frac{\lambda f(\beta)\frac{1}{\beta} + f'(\beta)}{1+\theta} - \kappa\right]\left[\rho - A + (n-1)(1+\theta)\beta\right] + \lambda f(\beta) = f(\beta) - \beta f'(\beta) \tag{20}$$

We first prove that when $\lambda = 0$, $\widehat{\beta}$ must exceed $\overline{\beta}^*$. Both equations have the same right-hand side, which is an increasing function of β; as β varies from 0 to infinity, $f(\beta) - \beta f'(\beta)$ rises continuously. The left-hand side of equation (19) is downward sloping, and is positive for all $\beta < \beta_H$ where by definition $f'(\beta_H) = (1+\theta)\kappa$. For all $\beta < \beta_H$, the value of the LHS of eq (20) is greater than that of equation (19). It follows that $\widehat{\beta}$ exceeds $\overline{\beta}^*$. Now, if $\lambda > 0$, this will make $\widehat{\beta}$ even greater. ■

Proposition 3.6: *(comparing welfare levels) The cooperative solution yields a higher welfare level than that of the Markov perfect equilibrium.*

Proof: See the Appendix

Remark: Since $\widehat{\beta} > \overline{\beta}^*$ as shown in Proposition 3.5, we must have $\psi'(\widehat{\beta}) < 0$, which indicates the welfare in the MPE case is decreasing in β, i.e. $\widehat{\beta}$ always lies to the right of $\overline{\beta}^*$(Fig 1 illustrates this situation).

Combining Propositions 3.5 and 3.6, it is interesting to note that the cooperative equilibrium has both higher welfare level and greater resource growth rate. Let's explore some intuition behind these results. In the cooperative equilibrium or the social planner's problem, the agents know ex ante that their consumption levels will be equal thus the envy parameter λ doesn't play a role in the equilibrium. In the MPE case, however, the agents will observe the resource stock at the beginning of each period and make her own decision about the extraction rate, each trying not to be behind, even though they know that in the symmetric equilibrium their consumption levels will be equal ex post. The "positional externalities" imposed by the status-consciousness can only be eliminated by cooperation.

We have shown in Proposition 3.2 that a fall in κ leads a a higher welfare in the cooperative equilibrium. We now show that, in contrast, in the case of a non-cooperative equilibrium, a fall in κ can decrease the non-cooperative welfare, i.e., technological progress in resource extraction can be welfare-worsening when agents are non-cooperative. Furthermore, the absolute magnitude of the negative impact of technological progress on welfare is an increasing function of the degree of status-consciousness. The next proposition is a formalization of this result.

Proposition 3.7: *A technological progress in resource extraction can reduce welfare in the non-cooperative case. This fall in welfare is an increasing function of the degree of status-consciousness.*

Proof: See the Appendix.

Remark: This result represents the situation that a small increase in κ may be welfare-improving because the benefits from resouce stock preserving outweight the utility loss from less extraction and consumption (see the case in Figure 2, $\hat{\beta}$ reduces to $\hat{\beta}'$ but the welfare is higher than before). However, it won't happen in the cooperative equilibrium since the cooperative equilibrium extraction rate $\overline{\beta}^*$ is always the welfare-maximizing extraction rate.

3.2.2 Heterogeneous agents

So far we have focused on the case of homogeneous players. This sub-section examines the effects of heterogeneity among agents on the properties of Markov-perfect Nash equilibria. To simplify the analysis, we focus on the case where there are only two groups of players. More specifically, let us assume that there are $n_1 \geq 2$ players described by the parameters $(\rho_1, \theta_1, \kappa_1)$ with the utility function D_1 and f_1, and $n_2 \geq 2$ players described by the parameters $(\rho_2, \theta_2, \kappa_2)$ with the utility function D_2 and f_2. The total number of players is $n = n_1 + n_2$. We assume that assumptions A1-A3 hold for both group of players, and the agents in each group compare their consumption with other members in the same group only.

Analysis Note that the transition equations for each group are now different, i.e., for agent i in group 1:

$$\dot{X} = AX - (n_1 - 1)(1 + \theta_1)\beta_1 X - (1 + \theta_1)c_{i1} - n_2(1 + \theta_2)\beta_2 X$$

For agent i in group 2:

$$\dot{X} = AX - (n_2 - 1)(1 + \theta_2)\beta_2 X - (1 + \theta_2)c_{i2} - n_1(1 + \theta_1)\beta_1 X$$

The Hamiltonians become

$$\begin{aligned} H_{i1} &= D_1\left(\frac{c_{i1}}{\beta_1 X}\right) F_1(c_{i1}, X) - \kappa_1(1 + \theta_1)c_{i1} + \\ & \psi_{i1}\left[AX - (n_1 - 1)(1 + \theta_1)\beta_1 X - (1 + \theta_1)c_{i1} - n_2(1 + \theta_2)\beta_2 X\right] \end{aligned} \tag{21}$$

$$\begin{aligned} H_{i2} &= D_2\left(\frac{c_{i2}}{\beta_2 X}\right) F_2(c_{i2}, X) - \kappa_2(1 + \theta_2)c_{i2} + \\ & \psi_{i2}\left[AX - (n_2 - 1)(1 + \theta_2)\beta_2 X - (1 + \theta_2)c_{i2} - n_1(1 + \theta_1)\beta_1 X\right] \end{aligned} \tag{22}$$

We can show that there exist two symmetric linear solutions for these two groups:

$$\frac{c_{i1}(t)}{X(t)} = \beta_1, \frac{c_{i2}(t)}{X(t)} = \beta_2 \text{ where } \beta_1 \text{ and } \beta_2 \text{ are constants} \tag{23}$$

The growth rate of the public asset is therefore given by

$$g = A - n_1(1 + \theta_1)\beta_1 - n_2(1 + \theta_2)\beta_2 \tag{24}$$

For the Cobb-Douglas case with $\kappa_1 = \kappa_2 = 0$ the solutions are:

$$\hat{\beta}_1 = \frac{1}{1 + \theta_1} \frac{(\lambda_1 + \mu_1)[\rho_1 - n_2(\rho_1 - \rho_2)(\lambda_2 + \mu_2) - A]}{1 - n_1(\lambda_1 + \mu_1) - n_2(\lambda_2 + \mu_2)} \tag{25}$$

$$\hat{\beta}_2 = \frac{1}{1 + \theta_2} \frac{(\lambda_2 + \mu_2)[\rho_2 - n_1(\rho_2 - \rho_1)(\lambda_1 + \mu_1) - A]}{1 - n_1(\lambda_1 + \mu_1) - n_2(\lambda_2 + \mu_2)} \tag{26}$$

(Note that if $\theta_1 = \theta_2$, $\lambda_1 + \mu_1 = \lambda_2 + \mu_2 < 1/n$ and $n_1 = n_2 = n/2$, then $\hat{\beta}_1 > \hat{\beta}_2$ if and only if $\rho_1 > \rho_2$, i.e., the more impatient group extracts the resource stock at a faster rate.)

We are especially interested in the effect of heterogeneity in the status-conscious parameter λ on the equilibrium outcome. For example, if we assume there is a mean-preserving spread of λ among agents, i.e., $\lambda_1 = \lambda + \frac{\eta}{n_1}$, $\lambda_2 = \lambda - \frac{\eta}{n_2}$ with $\eta > 0$, how are the growth rate of public assets and welfare affected by an increase in η? The following proposition provides an answer for the Cobb-Douglas case.

Proposition 3. 8 *In the Cobb-Douglas case,*

(a) A mean-preserving spread in the distribution of the status-conscious parameter λ leads to an increase of the public asset growth rate iff $\rho_2 > \rho_1$, i.e., iff the members of the group with stronger status-consciousness are more patient.

(b) If the status-conscious parameter λ is the only source of heterogeneity, a mean-preserving spread in the distribution of λ across agents leads to an decrease of the social welfare.

Proof: See the Appendix.

Simulation results: the joint effects of λ and θ on social welfare In this section the joint effects of λ and θ on social welfare are given by simulation. Again, suppose $\theta_1 = \theta + \frac{\varepsilon}{n_1}$, $\theta_2 = \theta - \frac{\varepsilon}{n_2}$ and $\lambda_1 = \lambda + \frac{\eta}{n_1}$, $\lambda_2 = \lambda - \frac{\eta}{n_2}$. Substituting them into the social welfare function in we can express social welfare as a function of ε and η. The plot of social welfare is given in Fig. 3 (assuming $X_0 = 1$, $\rho = 0.2$, $A = 0.1$, $\lambda = 0.2$, $n_1 = 10$, $n_2 = 10$, $\mu = 0.2$, $\theta = 0.1$).

The saddle-shape diagram allows us to confirm our findings that a mean-preserving spread in the distribution of λ across agents leads to a decrease of social welfare, while a mean-preserving spread in the appropriation cost θ will increase the social welfare, ceteris paribus.

4 Concluding remarks

This paper explores the effect of status-consciousness on resource exploitation in a dynamic setting. The agents in the economy are concerned with both their absolute level of consumption and the relative consumption level within their groups. We show that if agents use open-loop strategies, there are cases under which status-consciousness does not create any distortion, and thus no corrective action is required. If agents use feedback strategies, the social optimum cannot be supported by the Markov-perfect Nash equilibrium, except in the case where resource stocks are privately owned and agents have additively separable utility functions that display a constant envy. Generally, in a Markov-perfect equilibrium, the status-consciousness parameter indeed plays an important role. A higher degree of status-consciousness leads to more aggressive extraction efforts, therefore the social welfare and the growth rate of the public resource are lower. This effect has not been explored in the existing literature on dynamic resource exploitation. We have therefore shown that "positional externalities" worsen the "tragedy of the commons" problem.

Our model with costly extraction can be interpreted as a rent-seeking model with detection-avoidance costs.We introduce two types of cost within the rent-seeking process, a "wastage-cost" θ and an "effort-cost" κ. We show that an increase in κ will reduce the equilibrium rate of extraction and increase the growth rate of the public asset. Thus if the policy maker's primary objective is to protect the public asset from over-extraction, imposing a higher effort-cost (stricter policing of money-laundering) is preferred. We also show that a technological progress, i.e., a smaller κ, can worsen welfare in a Markov-perfect Nash equilibrium. The magnitude of this welfare-worsening effect is an increasing function of the degree of status-consciousness. In the analysis for heterogeneous agents, we show that an increase in the degree of heterogeneity in the status-conscious parameter λ will reduce social welfare while an increase in the degree of heterogeneity in wastage cost θ will increase social welfare.

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APPENDICES

Appendix 1

Example of a symmetric open-loop Nash equilibrium

The following discrete-time example illustrates the non-subgame-perfect property of open-loop Nash equilibria. Two players share a cake which has been cut into 6 identical pieces. They have three days to play the game. Each day, there is diminishing marginal utility of consumption: if $c_i(t) > 0$ is the quantity consumed on day t, the utility is:

$$u_i(t) = \sqrt{c_i(t)}$$

There is no discounting.Each day t, after observing the number of pieces of cake that remain, denoted by $s(t)$, the players simultaneously click each a button on his computer screen, among three possible buttons, marked 0, 1, 2, (i.e., the control variable for player i, denoted by $b_i(t)$, can take on one of the three values, 0 , 1, 2). If the sum of what they select on that day does not exceed $s(t)$, i.e. $b_1(t) + b_2(t) \leq s(t)$ then each gets what he has selected and consumes it (we assume he cannot store it). His utility for that day is then

$$\sqrt{b_i(t)}$$

If $b_1(t) + b_2(t) > s(t)$, then each player will get zero, and his utility is 0.

The transition equation is

$$s(t+1) = s(t) - \min\{b_1(t) + b_2(t), s(t)\}$$

The following pair of open-loop strategies is an open-loop Nash equilibrium: each player chooses 1 (grabs one piece of cake) each day. Along the equilibrium play each will consume one piece every day. His utility over the three days is

$$\sqrt{1} + \sqrt{1} + \sqrt{1} = 3$$

If player 2 uses this strategy ($b_2(t) = 1$ for $t = 1, 2, 3$) and player 1 deviates, say, by choosing $b_1(1) = b_1(2) = 2$ and $b_1(3) = 2$ (or 1 or 0) then player 1's payoff will be

$$\sqrt{2} + \sqrt{2} + \sqrt{0} = 2.8284 < 3$$

Now suppose both players have decided to play the open-loop Nash equilibrium strategies ($b_i(t) = 1$ for $t = 1, 2, 3$ for $i = 1, 2$).But suppose on the second day, player 1 observes that there are only 3 pieces of cake left. Would he still want to continue with his original open-loop strategy? If he does, (and assuming player 2 will play $b_2(t) = 1$ for $t = 2, 3$), he will expect to get

$$\sqrt{1} + \sqrt{1} + \sqrt{0} = 2$$

If he chooses instead $b_2(2) = 2$, and he will get a better payoff

$$\sqrt{1} + \sqrt{2} + \sqrt{0} = 2.4142$$

(assuming player 2 will play $b_2(t) = 1$ for $t = 2, 3$).

This shows that the open-loop Nash equilibrium is, in general, not subgame perfect.

Appendix 2

Proof of Proposition 2.1

The social planner chooses the common consumption rate $c(t)$ to maximize

$$\int_0^\infty e^{-\rho t} n U(c, c) dt$$

subject to n transition equations

$$\dot{x}_k = G(x_k) - c, \text{ with } x_k(0) = x_0 > 0, \; k = 1, 2, ..., n$$

and $x_k(t) \geq 0$.

We use the superscript ^{p} to denote the social planner's optimal choice . Let ψ_k^p be the shadow price of the resource stock, and H be the Hamiltonian function for the social planner's problem.

$$H = nU(c,c) + \sum_k^n \psi_k^p \left(G(x_k) - c\right)$$

The socially optimal path satisfies

$$nU_c(c^p, c^p) + nU_C(c^p, c^p) = \sum_k \psi_k^p > 0$$

$$\frac{\dot{\psi}_k^p}{\psi_k^p} = \rho - G'(x_k^p) \tag{27}$$

$$\dot{x}_k^p = G(x_k) - c^p$$

$$\lim_{t\to\infty} e^{-\rho t}\psi_k^p(t)x_k^p(t) = 0,\ \lim_{t\to\infty} e^{-\rho t}\psi_k^p(t) \geq 0 \text{ and } \lim_{t\to\infty} x_k^p(t) \geq 0.$$

Because of symmetry, we write

$$\psi_k^p = \psi^p, \text{ and } x_k^p = x^p.$$

We now show that the socially optimal consumption path satisfies the necessary conditions of the optimization problem of individual i, where $i = 1, 2, ..., n$. Individual i takes as given the time path $C_i(t) = \frac{1}{n-1}\sum_{k\neq i} c_k^p(t) = c^p(t)$ of per-capita consumption of his peer group, and chooses the time path $c_i(t)$ to maximize

$$V_i = \int_0^\infty e^{-\rho t} U(c_i(t), C_i(t))dt$$

subject to

$$\dot{x}_i(t) = G(x_i(t)) - c_i(t)$$

$$\dot{x}_k(t) = G(x_k(t)) - c^p(t) \text{ where } k \neq i$$

Let $\pi_i(t)$ be shadow price that individual i attaches to his own stock $x_i(t)$. The Hamiltonian for individual i's optimization problem is

$$H_i = U(c_i(t), C_i(t)) + \pi_i(t)\left[G(x_i(t)) - c_i(t)\right]$$

Let the superscript ol in c^{ol} and x^{ol} denote the outcome under the open-loop Nash equilibrium.The necessary conditions are

$$U_c(c_i^{ol}, C_i^{ol}) = \pi_i > 0$$

$$\frac{\dot{\pi}_i}{\pi_i} = \rho - G'(x_i^{ol}) \tag{28}$$
$$\dot{x}_i^{ol} = G(x_i^{ol}) - c_i^{ol}$$
$$\lim_{t\to\infty} e^{-\rho t}\pi_i(t)x_i^{ol}(t) = 0,\ \lim_{t\to\infty} e^{-\rho t}\pi_i(t) \geq 0 \text{ and } \lim_{t\to\infty} x_i^{ol}(t) \geq 0.$$

For $x^p(t)$ to coincide with $x^{ol}(t)$, conditions (27) and (28) require that there exists a positive constant κ such that $\psi^p = \eta\pi_i$ for all t. This would imply

$$U_c(c^p(t), c^p(t)) + U_C(c^p(t), c^p(t)) = \eta U_c(c^p(t), c^p(t))$$

Dividing both sides by $U_c(c^p, c^p)$ we get

$$1 + \frac{U_C(c^p(t), c^p(t))}{U_c(c^p(t), c^p(t))} = \eta > 0$$

This holds iff

$$1 + \varepsilon(c^p(t)) = \eta > 0$$

i.e., ε is independent of c^p and $-1 < \varepsilon \leq 0$. All other necessary conditions of the individual optimization problems are satisfied when $x_i^{ol}(t) = x^p(t)$ and $c_i^{ol}(t) = c^p(t)$.

Proof of Proposition 2.2

Under (i) and (ii) of Proposition 2, the utility function is of the form $U(c, C) = u(c) + \varepsilon u(C)$, where $-1 < \varepsilon < 0$. Since the social planner treats everyone equally, and the initial stocks are identical, we can formulate the social planner's problem as

$$\max_c \int_0^\infty e^{-\rho t}(1+\varepsilon)u(c)dt$$

subject to

$$\dot{x} = G(x) - c,\ x(0) = x_0$$

The optimal solution consists of the (socially optimal) policy function $c = c(x)$, and an associated value function $W(x)$, such that the following Hamilton-Jacobi-Bellman (HJB) equation holds:

$$\rho W(x) = \max_c \left[(1+\varepsilon)u(c) + W'(x)(G(x) - c)\right]$$

Thus

$$u'(c) = \frac{1}{1+\varepsilon}W'(x)$$

i.e. the policy function is

$$c = h\left(\frac{1}{1+\varepsilon}W'(x)\right) \equiv \widehat{h}(x)$$

where $h(.) \equiv (u')^{-1}(.)$.

Substitute the policy function into the HJB equation to get

$$\rho W(x) = \left[(1+\varepsilon)u(\widehat{h}(x)) + W'(x)(G(x) - \widehat{h}(x))\right]$$

Hence

$$\frac{\rho}{1+\varepsilon}W(x) = u(\widehat{h}(x)) + \frac{1}{1+\varepsilon}W'(x)(G(x) - \widehat{h}(x)) \tag{29}$$

Now consider the game between two individuals. (The arguments that follow can be generalized to the case where $n > 2$). Suppose individual i thinks that j uses the socially optimal strategy

$$c_j = h\left(\frac{1}{1+\varepsilon}W'(x_j)\right) \equiv \widehat{h}(x_j)$$

Let $J_i(x_i, x_j)$ be the value function for individual i. Then i's HJB equation is

$$\rho J_i(x_i, x_j) = \max_{c_i}\left\{u(c_i) + \varepsilon u\left(\widehat{h}(x_j)\right) + \frac{\partial J_i}{\partial x_i}\left[G(x_i) - c_i\right] + \frac{\partial J_i}{\partial x_j}\left[G(x_j) - \widehat{h}(x_j)\right]\right\}$$

We now claim that $J_i(x_i, x_j)$ can be constructed as follows:

$$J_i(x_i, x_j) = \frac{1}{1+\varepsilon}W(x_i) + \frac{\varepsilon}{1+\varepsilon}W(x_j)$$

To verify our assertion, note that using this construction, individual i's consumption choice c_i satisfies

$$u'(c_i) = \frac{\partial J_i}{\partial x_i} \equiv \frac{1}{1+\varepsilon}W'(x_i) \text{ i.e. } c_i = h\left(\frac{1}{1+\varepsilon}W'(x_i)\right) \equiv \widehat{h}(x_i)$$

It remains to verify that

$$\rho\left[\frac{1}{1+\varepsilon}W(x_i) + \frac{\varepsilon}{1+\varepsilon}W(x_j)\right] =$$

$$u\left(\widehat{h}(x_i)\right) + \beta u\left(\widehat{h}(x_j)\right) + \frac{W'(x_i)}{1+\varepsilon}(G(x_i) - \widehat{h}(x_i)) + \frac{\varepsilon W'(x_j)}{1+\varepsilon}(G(x_j) - \widehat{h}(x_j))$$

Clearly, this equation is satisfied identically, in view of eq (29).

Proof of Proposition 2.3

To establish the proposition, we offer the following example.

Assume that $U(c, C) = \frac{1}{1-\sigma}c^{1-\sigma+\alpha}C^{-\alpha}$ where $0 < \alpha < \sigma < 1$, and the transition equation is

$$\dot{x}_i(t) = G(x_i) - c_i(t) \text{ with } x_i(0) = x_{i0} > 0$$

where

$$G(x_i) \equiv Ax_i(t)^{\sigma} - \delta x_i(t)$$

Here $A > 0$, $0 < \delta < 1$,$\delta \geq 0$, and $x_{i0} = x_{j0} \equiv x_0$ for all i, j. The social planner, treating all individuals equally, chooses a common time path $c(t)$ to maximize

$$\int_0^\infty e^{-\rho t} \frac{1}{1-\sigma} c^{1-\sigma+\alpha} c^{-\alpha} dt$$

It can be verified that the socially optimal consumption rule is

$$c_i = \beta^{so} x_i \equiv \frac{\rho + \delta(1-\sigma)}{\sigma} x_i$$

where the superscript *so* in β^{so} indicates that this is a socially optimal choice. The welfare of the representative individual under the social planner is

$$W(x_0) = (\beta^{so})^{1-\theta} \left(\frac{1}{1-\sigma} \right) \left[\frac{A}{b\rho} + \frac{(x_0)^{1-\sigma} - (A/b)}{\rho + b(1-\sigma)} \right]$$

where

$$b \equiv \frac{\rho + \delta}{\sigma}$$

The stocks converge to the steady-state level $\widehat{x}^{so}$ where

$$\widehat{x}^{so} \equiv \left(\frac{A}{\delta + \beta^{so}} \right)^{\frac{1}{1-\sigma}} = \left(\frac{\sigma A}{\delta + \rho} \right)^{\frac{1}{1-\sigma}} \tag{30}$$

Now consider the differential game among n players, each exploiting his own resource stock. We want to show that the socially optimal optimal consumption rule cannot support a Markov-perfect Nash equilibrium. We prove this by establishing a contradiction. Suppose that agent i believes that all other agents use the socially optimal strategy

$$c_j = \beta^{so} x_j \equiv \frac{\rho + \delta(1-\sigma)}{\sigma} x_j$$

He will then seek to maximize

$$\int_0^\infty e^{-\rho t} \frac{1}{1-\sigma} c_i^{1-\sigma+\alpha} \left[\frac{1}{n-1} \sum_{j \neq i} \beta^{so} x_j \right]^{-\alpha} dt$$

Take the simple case where $n = 2$. Then player i's HJB equation must satisfy

$$\rho J_i(x_i, x_j) = \max_{c_i} \left\{ \frac{1}{1-\sigma} c_i^{1-\sigma+\alpha} (\beta^{so} x_j)^{-\alpha} + \frac{\partial J_i}{\partial x_i} [G(x_i) - c_i] + \frac{\partial J_i}{\partial x_j} [G(x_j) - \beta^{so} x_j] \right\}$$

Partial differentiation the HJB equation with respect to x_i yields

$$[\rho - G'(x_i)] \frac{\partial J_i}{\partial x_i} = [G(x_i) - c_i] \frac{\partial^2 J_i}{\partial x_i^2} + [G(x_j) - \beta^{so} x_j] \frac{\partial^2 J_i}{\partial x_j \partial x_i} \tag{31}$$

On the other hand, individual i's optimization with respect to c_i gives

$$\frac{(1-\sigma+\alpha)}{1-\sigma} c_i^{-\sigma+\alpha} \left(\beta^{so} x_j\right)^{-\alpha} = \frac{\partial J_i}{\partial x_i}$$

It follows that if the socially optimal optimal consumption rule can be supported as a Markov-perfect Nash equilibrium, the following equation must hold:

$$\frac{(1-\sigma+\alpha)}{1-\sigma} \left[\beta^{so} x_i\right]^{-\sigma+\alpha} \left(\beta^{so} x_j\right)^{-\alpha} = \frac{\partial J_i}{\partial x_i} \tag{32}$$

Differentiating both sides of (32) with respect to x_i, we get

$$\frac{\partial^2 J_i}{\partial x_i^2} = \frac{(1-\sigma+\alpha)(-\sigma+\alpha)\left[\beta^{so} x_i\right]^{-\sigma+\alpha} \left(\beta^{so} x_j\right)^{-\alpha} x_i^{-1}}{1-\sigma}$$

Differentiating both sides of (32) with respect to x_j, we get

$$\frac{\partial^2 J_i}{\partial x_j \partial x_i} = \frac{-\alpha(1-\sigma+\alpha)}{1-\sigma} \left[\beta^{so} x_i\right]^{-\sigma+\alpha} \left(\beta^{so} x_j\right)^{-\alpha} x_j^{-1}$$

Substituting these partial derivatives into equation (31), we get

$$\begin{gathered}
\left[\rho - G'(x_i)\right] \frac{(1-\sigma+\alpha)}{1-\sigma} \left[\beta^{so} x_i\right]^{-\sigma+\alpha} \left(\beta^{so} x_j\right)^{-\alpha} = \\
\left[G(x_i) - \beta^{so} x_i\right] \frac{(1-\sigma+\alpha)(-\sigma+\alpha)\left[\beta^{so} x_i\right]^{-\sigma+\alpha} \left(\beta^{so} x_j\right)^{-\alpha} x_i^{-1}}{1-\sigma} + \\
\left[G(x_j) - \beta^{so} x_j\right] \left[\frac{-\alpha(1-\sigma+\alpha)}{1-\sigma} \left[\beta^{so} x_i\right]^{-\sigma+\alpha} \left(\beta^{so} x_j\right)^{-\alpha} x_j^{-1}\right]
\end{gathered} \tag{33}$$

This must hold for all x_i and all x_j. For the value $x_j = \widehat{x}^{so}$, we obtain from (33) an expression which must hold for all positive values of x_i :

$$\alpha \left[A x_i^{\sigma-1} - \delta - \frac{\rho + \delta(1-\sigma)}{\sigma}\right] = 0$$

But this cannot be true for all $x_i > 0$, unless $\alpha = 0$. This contradiction proves that the socially optimal consumption rule cannot be supported as a Markov-perfect Nash equilibrium, when $\alpha \neq 0$.

Proof of Proposition 2.5

The Hamiltonian for individual i's optimization problem is

$$H_i = \frac{1}{1-\sigma} \left(\beta_i x\right)^{1-\sigma+\alpha} \left(\beta^{OA} x\right)^{-\alpha} + \psi_i \left(A x^\sigma - \delta x - (n-1)\beta^{OA} x - \beta_i x\right)$$

The first order condition with respect to $\beta_i(t)$ is

$$\frac{(1-\sigma+\alpha)}{1-\sigma} \left(\beta_i(t)\right)^{-\sigma+\alpha} (\beta^{OA})^{-\alpha} x(t)^{-\sigma} = \psi_i(t) \tag{34}$$

Let us try a solution where $\beta_i(t)$ is a constant $\beta_i > 0$. Then differentiating equation (34) with respect to t yields

$$\frac{\dot{\psi}_i}{\psi_i} = -\sigma \frac{\dot{x}}{x}$$

Hence

$$\frac{\dot{\psi}_i}{\psi_i} = -\sigma \left(A x^{\sigma - 1} - \delta - (n-1)\beta^{OA} - \beta_i \right) \tag{35}$$

On the other hand, the adjoint equation is

$$\frac{\dot{\psi}_i}{\psi_i} = (\rho - \sigma A x^{\sigma - 1} + \delta + (n-1)\beta^{OA} - \beta_i) - \frac{1}{\psi_i} x^{-\sigma} \left(\beta_i\right)^{1-\sigma+\alpha} \left(\beta^{OA}\right)^{-\alpha} \tag{36}$$

In a symmetric equilibrium, we must have $\beta_i = \beta^{OA}$. Equating the right-hand side of (35) with that of (36) and eliminating $\psi_i^{-1} x^{-\sigma}$ by using (34), we can solve for β^{OA} :

$$\beta^{OA} = \frac{1 - \sigma + \alpha}{(1-\sigma)\left[1 - n(1 - \sigma + \alpha)\right]} \left(\rho + (1-\sigma)\delta\right)$$

For $\beta^{OA} > 0$, we require that $n < 1/(1 - \sigma + \alpha)$.

Comparing β^{OA} with the social optimal β^*,

$$\frac{\beta^{OA}}{\beta^*} = \frac{(1 - \sigma + \alpha) n \sigma}{(1-\sigma)\left[1 - n(1 - \sigma + \alpha)\right]} = \frac{n\sigma}{(1-\sigma)\left[\frac{1}{1-\sigma+\alpha} - n\right]}$$

This ratio is equal to unity if $n = 1$ and $\alpha = 0$. The ratio is increasing in α. Thus, the higher is the degree of envy, the more excessive is the rate of exploitation.

Proof of Proposition 3.1

First, let us show that $\overline{\beta}^*$ is unique. As shown in Fig. 1, the left-hand side (LHS) of equation (9) is decreasing in $\overline{\beta}$, and as $\overline{\beta}$ varies from zero to infinity, the LHS varies from infinity to $-\kappa$. The RHS is positive for all positive $\overline{\beta}$, and increases as $\overline{\beta}$ increases. Thus the curve that represents the LHS must intersect the curve that represents the RHS exactly at one value, say $\overline{\beta}^*$. At $\overline{\beta}^*$, we have

$$\frac{f'(\overline{\beta}^*)}{(1+\theta)} - \kappa > 0 \tag{37}$$

(This is because the numerator of the right-hand side of (9) is positive for all $\beta > 0$, and the denominator is positive because $\rho > A$).

At the constant ratio $\overline{\beta}^*$ of consumption to stock, the growth rate of the stock is

$$g \equiv \frac{\dot{X}}{X} = A - n(1+\theta)\overline{\beta}^* < A < \rho$$

(which may be positive or negative) and thus

$$X(t) = X_0 e^{gt}$$

Next, to show that the strategy $c = \overline{\beta}^* X$ is optimal, we can verify that all the necessary and sufficient conditions are satisfied. The transversality condition (4) is met, because $\psi(t) = \overline{\psi}^* > 0$ by (6) and (37), and because

$$\lim_{t\to\infty} \psi(t) e^{-\rho t} X(t) = 0 = \overline{\psi}^* X_0 \lim_{t\to\infty} e^{-\rho t} e^{gt} = 0$$

Since the objective function (3) is concave in (c, X), and the constraints are linear, the necessary conditions are also sufficient. ■

Proof of Proposition 3.2

Since $X(t) = X_0 e^{gt}$

$$W^{coop} = \int_0^{\infty} e^{-\rho t} \left[f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^* \right] X_0 e^{gt} dt$$

$$W^{coop}(X_0) = \left[f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^* \right] X_0 \frac{1}{\rho - g} = X_0 \frac{f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*}{\rho - A + n(1+\theta)\overline{\beta}^*}$$

where, since $\rho - A > 0$, $\rho - g > 0$.

Now, from (7) and (8),

$$(\rho - A)\overline{\psi} = f(\overline{\beta}) - \overline{\beta} f'(\overline{\beta}) = f(\overline{\beta}) - \overline{\beta}(1+\theta)(\kappa + n\overline{\psi}) \tag{38}$$

we obtain

$$\left(\rho - A + n(1+\theta)\overline{\beta}^*\right) \overline{\psi}^* = f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*$$

It follows that

$$\frac{f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*}{\rho - A + n(1+\theta)\overline{\beta}^*} = \overline{\psi}^* = \frac{1}{n}\left[\frac{f'(\overline{\beta}^*)}{(1+\theta)} - \kappa \right] \tag{39}$$

where the last equality comes from (6). Therefore

$$W^{coop}(X_0) = \overline{\psi}^* X_0 \tag{40}$$

Thus welfare (per person) is the product of the shadow price $\overline{\psi}^*$ and the stock X_0.

An increase in κ or θ will shift down the curve representing the left-hand side (LHS) of equation (9), so the intersection $\overline{\beta}^*$ is moved to the left. Direct computation shows that

$$\frac{\partial \overline{\beta}^*}{\partial \kappa} = \frac{(\rho - A)(1+\theta)}{\left[\rho - A + n(1+\theta)\overline{\beta}^* \right] f''(\overline{\beta}^*)} < 0 \tag{41}$$

Thus

$$\begin{aligned}\frac{\partial W^{coop}}{\partial \kappa} &= \frac{\partial \overline{\psi}^*}{\partial \kappa} X_0 = \frac{1}{(1+\theta)n}\left[f''(\overline{\beta}^*)\frac{\partial \overline{\beta}^*}{\partial \kappa} - (1+\theta)\right] \\ &= \frac{1}{n}\left[\frac{-n(1+\theta)\overline{\beta}^*}{\rho - A + n(1+\theta)\overline{\beta}^*}\right] < 0\end{aligned}$$

A similar calculation shows that welfare falls if θ increases. ■

Proof of Proposition 3.6.

Recall from the cooperative solution that

$$W^{coop} = \overline{\psi}^* X_0$$

$$\overline{\psi}^* = \frac{f(\overline{\beta}^*) - \kappa(1+\theta)\overline{\beta}^*}{\rho - A + n(1+\theta)\overline{\beta}^*} = \frac{1}{(1+\theta)n}\left[f'(\overline{\beta}^*) - \kappa(1+\theta)\right]$$

The welfare of the representative agent in the Markov-perfect equilibrium is

$$W^{MPE} = \int_0^\infty e^{-\rho t}\left[f(\widehat{\beta}) - \kappa(1+\theta)\widehat{\beta}\right] X_0 e^{gt} dt$$

$$= \left[f(\widehat{\beta}) - \kappa(1+\theta)\widehat{\beta}\right] X_0 \frac{1}{\rho - g} = X_0 \frac{f(\widehat{\beta}) - \kappa(1+\theta)\widehat{\beta}}{\left(\rho - A + n(1+\theta)\widehat{\beta}\right)}$$

Now,

$$\begin{aligned}(\rho - A + (1+\theta)(n-1)\widehat{\beta})\widehat{\psi} &= f(\widehat{\beta}) - \widehat{\beta} f'(\widehat{\beta}) - G'(1) f(\widehat{\beta}) \\ &= f(\widehat{\beta}) - \widehat{\beta}\left[f'(\overline{\beta}) + G'(1)\frac{f(\widehat{\beta})}{\widehat{\beta}}\right] \\ &= f(\widehat{\beta}) - \widehat{\beta}(1+\theta)(\kappa + \widehat{\psi})\end{aligned}$$

where the first equality comes from (16) and the third one comes from (15). Therefore

$$\widehat{\psi} = \frac{f(\widehat{\beta}) - \kappa(1+\theta)\widehat{\beta}}{\rho - A + n(1+\theta)\widehat{\beta}}$$

$$W^{MPE} = \widehat{\psi} X_0 \tag{42}$$

Let's denote

$$\psi = \psi(\beta) = \frac{f(\beta) - \kappa(1+\theta)\beta}{\rho - A + n(1+\theta)\beta} \tag{43}$$

We want to show that

$$\overline{\psi}^* > \widehat{\psi}$$

The cooperative equilibrium can be transformed to an equivalent problem:

$$\underset{\beta}{Max}\ W^{coop} = \psi(\beta)X_0$$

Therefore, the first-order condition of the problem above must yield

$$\psi'(\beta) = 0$$

which gives

$$\frac{\partial\psi(\beta)}{\partial\beta} = \frac{[f'(\beta) - \kappa(1+\theta)]\,[\rho - A + n(1+\theta)\beta] - n(1+\theta)\,[f(\beta) - \kappa(1+\theta)\beta]}{(\rho - A + n(1+\theta)\beta)^2} = 0$$

Rearrange terms in the numerator, we have

$$\left[\frac{f'(\beta)}{(1+\theta)} - \kappa\right] = \frac{n\,[f(\beta) - \beta f'(\beta)]}{\rho - A} \tag{44}$$

which is identical to (9) used to determine the cooperative equilibrium strategy $\overline{\beta}^*$. The second order condition is satisfied. This implies that the curve $\psi(\beta)$ defined by (43) reaches its maximum at $\beta = \overline{\beta}^*$. Therefore the MPE solution $\widehat{\beta}$ must yields a smaller ψ, hence a lower welfare. Figure 2 depicts the curve $\psi(\beta)$ ■.

Proof of Proposition 3.7.

By (15), and recall that $D(1) = 1$,

$$\widehat{\psi} = \frac{1}{1+\theta}\left[D'(1)\frac{f(\widehat{\beta})}{\widehat{\beta}} + f'(\widehat{\beta}) - \kappa(1+\theta)\right] \tag{45}$$

Thus, using (45) and (42),

$$\frac{dW^{MPE}}{d\kappa} = X_0\frac{d\widehat{\psi}}{d\kappa} = \frac{X_0}{1+\theta}\left\{\left[D'(1)\left(\frac{\widehat{\beta}f'(\widehat{\beta}) - f(\widehat{\beta})}{\widehat{\beta}^2}\right) + f''(\widehat{\beta})\right]\frac{d\widehat{\beta}}{d\kappa} - (1+\theta)\right\} \tag{46}$$

Now, since the term inside the square brackets is negative, and $\frac{d\widehat{\beta}}{d\kappa}$ is also negative, the sign of the expression inside the curly brackets is ambiguous. Let us explore the special Cobb-Douglas case.

Implicit differentiation of equation (18) shows that, if $\theta = 0$,

$$\frac{d\widehat{\beta}}{d\kappa} = \frac{-\beta^{1-\mu}\,[\rho - A + (n-1)\beta]}{1 - n(\lambda+\mu) + (n-1)\kappa\beta^{1-\mu} + \kappa(1-\mu)\beta^{-\mu}\,[\rho - A + (n-1)\beta]} < 0$$

We evaluate this derivative at $\kappa = 0$:

$$\frac{\partial \widehat{\beta}}{\partial \kappa} = \frac{-\widehat{\beta}^{1-\mu}\left[\rho - A + (n-1)\widehat{\beta}\right]}{1 - n(\lambda + \mu)} < 0$$

Now, from (18), at $\kappa = 0 = \theta$,

$$\rho - A + (n-1)\widehat{\beta} = \frac{(1-\lambda-\mu)\widehat{\beta}}{\lambda + \mu}$$

So, at $\kappa = 0$

$$\frac{\partial \widehat{\beta}}{\partial \kappa} = -\widehat{\beta}^{2-\mu}\left[\frac{(1-\lambda-\mu)}{(\lambda+\mu)\left(1 - n\left(\lambda+\mu\right)\right)}\right] \tag{47}$$

Substituting (47) into (46), we see that the effect of an increase in κ on the equilibrium welfare level is positive if and only if

$$(1-\mu)(1-\mu-\lambda) > \left[1 - n(\mu+\lambda)\right](1+\theta)$$

For $\theta = 0$, this inequality is equivalent to

$$n > \frac{\mu}{\mu + \lambda} + (1-\mu)$$

Since the right-hand side is smaller than 2, it follows that the condition is satisfied if $n \geq 2$. We conclude that for the Cobb-Douglas case, with $\theta = 0$, a marginal increase in κ from a sufficiently small initial value κ_0 will increase the Markov-perfect equilibrium welfare level. The greater is λ, the greater is the magnitude of the increase in welfare, because

$$\frac{d}{d\lambda}\left[\frac{(1-\mu)(1-\mu-\lambda)}{\left[1-n(\mu+\lambda)\right]} - (1+\theta)\right] > 0$$

$\blacksquare$.

Proof of Proposition 3.8.

(a) Substitute $\hat{\beta}_1$ and $\hat{\beta}_2$ into (24) and take derivative with respect to η will yield

$$\frac{\partial g}{\partial \eta} = \frac{\rho_2 - \rho_1}{1 - n_1\left(\lambda_1 + \mu_1\right) - n_2\left(\lambda_2 + \mu_2\right)}$$

by definition, $1 - n_1\left(\lambda_1 + \mu_1\right) - n_2\left(\lambda_2 + \mu_2\right) > 0$, therefore $\frac{\partial g}{\partial \eta} > 0$ iff $\rho_2 > \rho_1$.

(b) The social welfare is the total sum of individual welfare and is given by

$$SW = n_1 W_1 + n_2 W_2 = \frac{n_1 \beta_1^{\mu_1} X_0}{\rho_1 - g} + \frac{n_2 \beta_2^{\mu_2} X_0}{\rho_2 - g}$$

If $\lambda_1 = \lambda + \frac{\eta}{n_1}$, $\lambda_2 = \lambda - \frac{\eta}{n_2}$ and all other parameters are equal across two groups, we have

$$\begin{aligned} \frac{\partial SW}{\partial \eta} &= 0 \Rightarrow \left(\frac{n\lambda - \eta + n\mu}{n(1+\theta)} \right)^{\mu-1} = \left(\frac{\eta + n\lambda + n\mu}{n(1+\theta)} \right)^{\mu-1} \\ &\Rightarrow \eta^* = 0 \end{aligned}$$

$$\frac{\partial^2 SW}{\partial \eta^2} < 0 \text{ at } \eta^* = 0$$

■.

Appendix 8

The effect of heterogeneity in θ on the public asset growth and welfare.

Proposition A.1: *In the Cobb-Douglas case*

(a) The growth rate of the public asset is not related to the production costs, θ_1, θ_2.

(b) If the appropriation cost θ is the only source of heterogeneity, a mean-preserving spread in the distribution of this cost across agents leads to an increase of the social welfare.

Proof:

(a) Denote

$$B_1 = \frac{(\lambda_1 + \mu_1)(\rho_1 - n_2(\rho_1 - \rho_2)(\lambda_2 + \mu_2) - A)}{1 - n_1(\lambda_1 + \mu_1) - n_2(\lambda_2 + \mu_2)}$$

$$B_2 = \frac{(\lambda_2 + \mu_2)(\rho_2 - n_1(\rho_2 - \rho_1)(\lambda_1 + \mu_1) - A)}{1 - n_1(\lambda_1 + \mu_1) - n_2(\lambda_2 + \mu_2)}$$

Substitution yields

$$g = A - n_1(1+\theta_1)\hat{\beta}_1 - n_2(1+\theta_2)\hat{\beta}_2 = A - n_1 B_1 - n_2 B_2$$

where it is clear that g is not affected by θ_1 and θ_2.

(b) Let's consider the social welfare under heterogeneity,

$$SW = n_1 W_1 + n_2 W_2 = \frac{n_1 \beta_1^{\mu_1} X_0}{\rho_1 - g} + \frac{n_2 \beta_2^{\mu_2} X_0}{\rho_2 - g}$$

Suppose $\theta_1 = \theta + \frac{\varepsilon}{n_1}$, $\theta_2 = \theta - \frac{\varepsilon}{n_2}$,

Let's assume that $\mu_1 = \mu_2 = \mu$ and denote $f(\varepsilon) = (\rho_2 - g)n_1\beta_1^{\mu} + (\rho_1 - g)n_2\beta_2^{\mu}$,

We have,

$$f'(\varepsilon) = -\frac{(\rho_2 - g)\mu\beta_1^{\mu-1}}{(1+\theta_1)^2}B_1 + \frac{(\rho_1 - g)\mu\beta_2^{\mu-1}}{(1+\theta_2)^2}B_2 = 0$$

$$\Rightarrow \varepsilon^* = \frac{(\theta+1)(1-C)}{\frac{1}{n_1}C + \frac{1}{n_2}}$$

Where

$$C = \left(\frac{B_2^\mu}{B_1^\mu}\frac{g-\rho_1}{g-\rho_2}\right)^{\frac{1}{\mu+1}}$$

and

$$f''(\varepsilon^*) = \frac{(\mu+1)}{n_1 n_2}\left(\frac{(\rho_1 - g)B_2^\mu n_1}{(1+\theta_2)^{\mu+2}} + \frac{(\rho_2 - g)B_1^\mu n_2}{(1+\theta_1)^{\mu+2}}\right) > 0$$

If $\rho_1 = \rho_2, \lambda_1 = \lambda_2$

$$\varepsilon^* = \frac{(\theta+1)(1-1)}{\frac{1}{n_1} + \frac{1}{n_2}} = 0$$

$\blacksquare$.

Figure 1: The determination of the equilibrium extraction rate

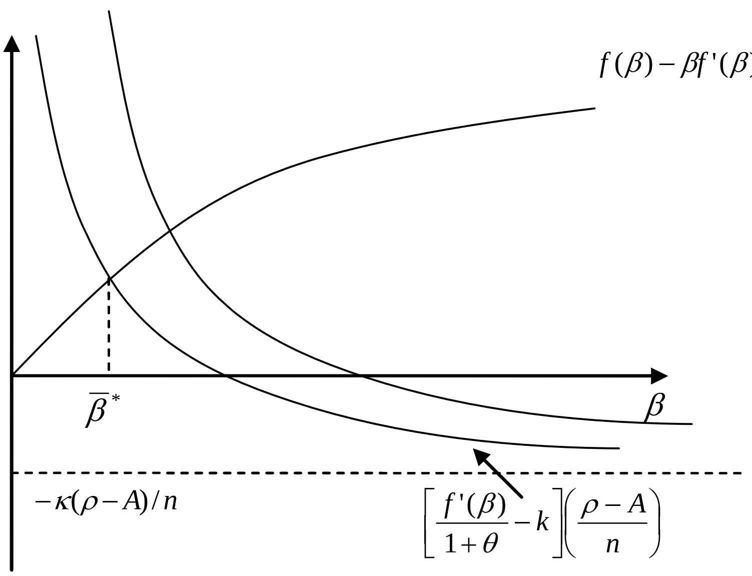

Figure 2: The effect of an increase in k on welfare and extraction rates

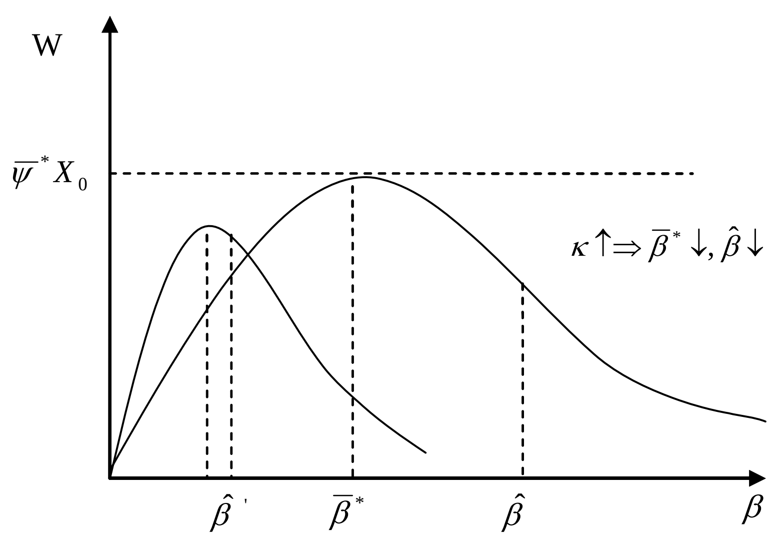

Figure 3: The joint effect of heterogeneity in λ and θ on social welfare