Strategic Managerial Incentives in a Two Period Cournot Duopoly[1]

by

Sudesh Mujumdar[2] and Debashis Pal[3]

Abstract: This paper examines the nature of optimal managerial incentives in the context of a duopoly marked by competition between the firm's managers in a dynamic production environment. If the marginal cost of production falls moderately over time or remains unchanged, there exists an equilibrium where one owner requires her manager to maximize profit, whereas the rival-owner requires her manager to maximize sales revenue. The profit-maximizing manager turns his firm into a Stackelberg-leader, while the sales-revenue-maximizing manager turns his firm into a Stackelberg-follower. Further, the profit-maximizing manager may generate a larger firm profit relative to the sales-revenue-maximizing manager.

Keywords: Managerial Incentives, Cournot, Stackelberg, Leader, Follower

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[2]Address: Department of Economics & Finance, University of Southern Indiana, Evansville, Indiana 47712, U.S.A. Email: smujumda@usi.edu

[3]Corresponding author. Address: Department of Economics, University of Cincinnati, Cincinnati, Ohio 45221, U.S.A. Email: debashis.pal@uc.edu

1. Introduction

It is well known in the strategic-managerial-incentives literature that a firm's owner can increase the firm's profit by hiring a manager and assigning him an objective different from profit maximization. It owes its emergence to the seminal endeavors of Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987) which, in turn, draw on the game theoretic work of d'Aspremont and Gérard-Varet (1980), and Schelling's (1960) insight on how the separation between a firm's ownership and control can create strategic advantages for the firm.[4]

One important underlying objective of these endeavors is to furnish a theoretical underpinning for the long-standing practice of tying a manager's compensation directly to his firm's sales.[5] To illustrate the argument that lies at the heart of this theoretical basis, consider the following scenario. There are two firms (Firm x and Firm y), each with an owner and a manager. Suppose the owner of Firm x designs a compensation scheme for her manager that is a weighted average of Firm x's profit and sales revenue, whereas the owner of Firm y specifies a scheme that pays her manager a proportion of Firm y's profit. Each owner-manager pair knows the incentive scheme of the other before any production decisions are taken. Under this scenario, Firm x's owner can set the weights for her manager so that the strategic interaction between the two managers (with production quantity as their choice variable) results in Firm x producing its Stackelberg-leader quantity. The favorable sales incentive makes Firm x's manager more 'aggressive' in production

[4]An excellent introduction to this literature can be found in Basu (1993).

[5]Irwin's (1991) detailed account of the Dutch East India Company engaging in this practice for many years since (and including) 1602 alerts us to its fairly long history.

activity and is responsible for turning his firm into a Stackelberg-leader, yielding it a higher profit.[6, 7]

The force of the above argument has led to two strong conclusions continuing to reside in this strand of the strategic-managerial-incentives literature (that purports to establish a theoretical basis for the practice of tying a manager's compensation directly to sales): (1) a firm with a purely profit-based incentive scheme never earns a larger profit in comparison to the firm that provides a direct incentive for sales. (2) a firm of the former type never emerges as a Stackelberg-leader in the presence of a firm of the latter type.

Now, these conclusions emerge from an environment where decisions regarding the strategic variable are taken simultaneously. For instance, where the strategic variable is production quantity, each manager is committed to his final output before observing the final output of his rival(s). In reality, however, production takes time and a firm can usually find out, at some point before committing to its final output, how much output has thus far been produced by its rival(s). This gives a firm the opportunity to react to such information by adjusting its production in the subsequent periods. As an illustration, consider two rival hotel companies that are in the process of putting up their respective new hotel buildings across the street from each other. Suppose the strategic variable is room capacity. Now, each company, in any period before its rival's hotel is fully built, can observe its rival's

[6] Zábojnik (1998) offers another rationale for tying a manager's compensation directly to sales; it is argued that this can induce more efficient investment in firm-specific human capital by the firm's workers, ultimately benefiting the firm's owner(s).

[7] Irwin (1991) empirically demonstrates how the Dutch East India Company attained the Stackelberg-leadership position vis-à-vis its rival - the British East India Company by offering its managers an incentive scheme with a direct sales component while the managers of the British East India Company were offered a purely profit-based incentive scheme. 'Institutional' factors kept the English East India Company from offering a scheme similar to that offered by its rival.

room capacity and accordingly determine how much capacity it wants to add in the subsequent period.[8] Furthermore, production costs may vary over time due to, say, changing input prices. Even if the nominal cost of production remains unchanged, discounting effectively changes the real cost over time.

The main purpose of this paper is to inquire into the optimal nature of managerial incentives in the context of a duopoly where the firms' managers compete in the (more realistic) 'dynamic' production environment described above. It is worth noting here that there is a body of work examining the form of managerial incentive contracts in 'temporal' settings. In this work, however, the design of the contracts has no strategic aspect to it - in the sense that a firm's owner, in designing the contract for her manager does not pay heed to the nature of the contract for a rival firm's manager. There are different reasons for why this is the case. To mention a couple: in Choi (2004) there is only one firm and in Chevalier and Sharfstein (1996), while there are two rival firms, the demands for their respective products are independent. Now, our paper, by focusing on the strategic aspect in the design of managerial incentive contracts, differs significantly from this body of work.

Our inquiry reveals that on accounting for the temporal dimension of the production process, the indicated long-standing results of the strategic-managerial-incentives literature undergo a drastic change. It is now possible for the profit-maximizing manager to turn his firm into a Stackelberg-leader, even though his rival has a purely sales-based incentive contract. In fact, this rival-manager may very well turn his firm into a Stackelberg-follower. Further, the profit-maximizing

[8]Note that this kind of strategic environment is not confined to only an 'industrial' context. For illustrations from 'non-industrial' environments, see Romano and Yildirim (2005).

manager, by turning his firm into a Stackelberg-leader, may be able to generate a larger firm profit in comparison to his rival (who maximizes sales revenue).

Another striking contrast between the findings that obtain in a static vis-à-vis a dynamic production environment runs as follows. In a static production milieu, the contracts for both firms' managers offer a direct incentive for sales and, consequently, neither firm emerges as a Stackelberg-leader (as shown in Fershtman and Judd (1987), and Sklivas (1987)).[9] In our dynamic production milieu, under many equilibria, one manager is offered a purely profit-based incentive contract while his rival is offered a purely sales-revenue-based incentive contract, and Stackelberg leader-follower outcomes emerge.

One might wonder, here, as to what it is about the temporal dimension of the production process that produces such radically different results. In what follows, we attempt to de-mystify this issue. Note that a direct incentive for sales is equivalent to disregarding a portion of the firm's marginal cost of production in determining its optimal output level. Thus, a sales-based managerial incentive contract shifts out the firm's 'reaction function', making it more 'aggressive' in the output market. This shifting-out of the reaction function leads to an increase in the firm's output, a decrease in the rival's output and, consequently, an increase in the firm's profit. Hence, if production decisions are taken simultaneously (as they

[9] This non-emergence result hinges on an important assumption in the Fershtman-Judd-Sklivas (FJS) framework which states that each owner hires a manager to run her firm. Basu (1995) relaxes this assumption by allowing an owner to choose whether or not she wishes to have a manager. It is shown that, in equilibrium, it is possible that one firm hires a manager and another does not. The result, however, crucially depends on the assumptions that the cost of hiring a manager is positive and it falls within a certain range. In contrast to Basu (1995), we demonstrate that even if both firms hire managers, a purely profit-based incentive scheme for one manager and a purely sales-revenue-based scheme for his rival firm's manager can be supported as an equilibrium pair of incentive schemes.

are in a static production setting), both firms offer their managers contracts that contain a direct incentive for sales.

The temporal dimension of the production process, in contrast, by allowing a firm to make output decisions at multiple time-points in the production process gives it the opportunity to revise its initial output targets. In such a setting it is shown, by Pal (1991), that if the marginal cost of production falls moderately over time, then the firms take their respective optimal production decisions *sequentially*. One firm becomes a *leader* by producing its entire output in the earlier period, while its rival becomes a *follower* by producing its entire output in the later period.[10]

The freedom to make sequential production decisions allows the leader-firm to choose an output combination that lies exclusively on the follower-firm's reaction function. Hence, the follower-firm stands to gain by shifting its reaction function outwards which leads the firm's owner to offer her manager a contract that will maximize the firm's sales revenue. Now, the leader-firm, since it wants to pick a point on the follower-firm's reaction function, does not benefit by shifting its own reaction function. The leader-firm thus, having ascertained the 'optimal' position of the follower-firm's reaction function, will wish to pick the point on it that maximizes its profit. Accordingly, the leader-firm offers its manager an incentive contract that is purely profit-based. The temporal dimension of the production process, therefore, results in the manager of one firm being offered a purely profit-based incentive contract while his rival is offered a purely sales-revenue-based incentive contract. Further, note that since the profit-maximizing manager turns his firm

[10]Such sequential production can also occur when the marginal cost of production does not change over time, as shown by Saloner (1987).

into a Stackelberg-leader, he may generate more firm profit than his rival - the sales-revenue-maximizing manager - who turns his firm into a Stackelberg-follower.

Our work also has a bearing on an intriguing real world phenomenon; the offering of a profit-based incentive scheme (with no direct, sales component) by one firm and a sales-based incentive scheme (with no profit component) by another firm *within the same industry.* One example of where this occurs is the personal-computer (PC) industry; Dell Computers offers its managers a profit-based incentive scheme whereas Hewlett Packard offers its (divisional) managers[11] a sales-based incentive scheme (see Runkle (2000) and Peppers and Rogers (2000)). Another example is from the Mortgage industry, where Lakeland Mortgage Corporation offers a sales (volume)-based incentive scheme and CTX Mortgage offers a profit-based incentive scheme (see the January 1999 issue of Mortgage Banking, pg. 87). The literature, thus far, on strategic-managerial-incentives offers no guidance on why we might confront such polar incentive schemes within the same industry. One explanation is uncovered by our investigation.[12]

The rest of the paper is organized as follows. Section 2 lays out the model. Section 3 explains the second period equilibrium strategies. Sections 4, 5 and 6

[11] Divisional managers are those responsible for day-to-day operational decisions.

[12] One might wonder, here, that while it is easy to see how our production environment could, *in essence*, align itself with that of the PC industry, such alignment is not apparent with respect to the Mortgage industry. Let us attempt to make this explicit. Suppose the strategic variable is the number of (mortgage) loan applications a bank can attract. Currently, there exists technology where a bank can submit a list of names of its customers to one of the big three credit bureaus (Experian, Transunion and Equifax) with a "trigger" that if any of them were to submit a loan application to a rival bank (or to rival banks), then it ought to be alerted (see Lieber (2003)). Thus a bank can determine how aggressive of a 'production' posture its rival(s) is (are) taking based on the extent of 'shopping-around' by its customers and then decide on what 'production' posture it wants to adopt.

characterize the subgame perfect Nash equilibria when the marginal cost of production falls, remains unchanged and rises over time, respectively. Section 7 concludes the paper.

2. The Model

We consider a two stage game involving a quantity-setting duopoly. Each firm has an owner and a manager. In Stage I, the owners simultaneously announce the incentive schemes for their respective managers. The managerial incentive schemes become common knowledge, and then in Stage II, the managers take their production decisions.[13]

One way of modeling our production environment would be to first modify the standard Cournot model so as to incorporate the temporal facet of the production process. To our benefit, Saloner (1987) offers one such modification. Following Saloner, then, we can model Stage II as comprising of two production periods with the market clearing at the end of the second period. In the first production period, managers of firms x and y simultaneously produce outputs q_1^x and q_1^y, respectively. These outputs become common knowledge, and in the second production period, the two managers simultaneously choose non-negative outputs, q_2^x and q_2^y. After the second period, price is determined from the inverse demand function $p(Q)$, where $Q = q_1^x + q_2^x + q_1^y + q_2^y$. Next, we allow the production cost to change over time. Hence, we let the marginal cost of production (which is assumed to be identical for

[13]The incentive schemes, we further maintain, are not renegotiated during any stage of the game. While this is admittedly restrictive, it is certainly not far-fetched given that recontracting can be costly (see Core, Guay and Larcker (2003), p.28, and the relevant references contained therein).

both firms) vary across periods, but maintain that it is constant within a period.[14] Let c_j be the marginal cost of production in period j, where $j = 1, 2$.

An owner's objective is to maximize her firm's profit. To this end, the owner of each firm specifies an incentive scheme for her manager that pays him some combination of the firm's profit and its sales revenue. The incentive scheme for the manager of firm i $(i = x, y)$ is given by:

$$\phi^i = \lambda^i \pi^i + \left(1 - \lambda^i\right) \rho^i$$

where $\lambda^i \in [0, 1]$, is a fixed constant chosen by firm i's owner, π_i and ρ_i denote firm i's profit and revenue, respectively, and are described as follows:

$$\begin{aligned}
\pi^i &= \left(q_1^i + q_2^i\right) p \left(q_1^x + q_2^x + q_1^y + q_2^y\right) - c_1 q_1^i - c_2 q_2^i \\
\rho^i &= \left(q_1^i + q_2^i\right) p \left(q_1^x + q_2^x + q_1^y + q_2^y\right)
\end{aligned}$$

The incentive scheme corresponding to each owner-manager pair becomes known to the rival pair before any production decision is taken in Stage II. The manager of firm i, given his incentive scheme, chooses q_1^i and q_2^i to maximize

$$\begin{aligned}
\phi^i &= \lambda^i \pi^i + \left(1 - \lambda^i\right) \rho^i \\
&= \left(q_1^i + q_2^i\right) p \left(q_1^x + q_2^x + q_1^y + q_2^y\right) - \lambda^i \left(c_1 q_1^i + c_2 q_2^i\right)
\end{aligned}$$

[14]This is a frequently-made assumption in models with a 'dynamic' production environment (see, e.g., Pal (1991) and Chevalier and Scharfstein (1996)). The assumption of constant marginal cost allows us to isolate the *strategic elements* of managerial interactions. If the marginal cost was increasing, each manager would have an additional incentive to distribute production between the periods to minimize total production cost. This phenomenon would make it quite challenging to focus on the strategic interactions that we are interested in. We conjecture that provided the convexity of the cost function is not 'too large', there will exist a SPNE outcome with one profit-maximizing manager and one sales-revenue-maximizing manager.

A strategy for the manager of firm i $(i = x, y)$ specifies an output for period one and an output for period two.[15] Note that second period output is a function of the observed (q_1^x, q_1^y). Let us denote the strategy for the manager of firm i by $\beta^i = \left(\beta_1^i, \beta_2^i \left(q_1^x, q_1^y\right)\right)$, $i = x, y$.

Now, in stage one, the owners of firms x and y simultaneously announce λ^x and λ^y, respectively. The owner of firm i $(i = x, y)$ chooses λ^i to maximize π^i. In stage two, after knowing λ^x and λ^y, the two managers simultaneously choose their production strategies for the two production periods. We use backward induction to characterize the Subgame Perfect Nash Equilibrium (SPNE) outcomes of the game. Below, we establish some notation for expositional clarity/simplicity.

Notation 1. *"an action made by the manager of firm i" is stated as "an action made by manager i", $i = x, y$.*

Notation 2. *For manager i, define his single period reaction function, $R^i(q^k|c$, λ^i, $\lambda^k)$, $(i, k = x, y,\ i \neq k)$ as*

$$R^i(q^k|c, \lambda^i, \lambda^k) = \arg\max_z z\left(p\left(z + q^k\right) - \lambda^i c\right)$$

By a "single period reaction function", we mean the reaction function corresponding to the standard Cournot model that has a single production period. Here, q^i denotes manager i's $(i = x, y)$ output in the single period and c denotes the marginal cost of production in that period. We assume that the single period reaction functions are 'well-behaved' for both managers ensuring, thereby, a unique Cournot-Nash equilibrium for the period.

[15] We confine our attention to 'pure' strategies.

Notation 3. $(N^x(c_j, \lambda^x, \lambda^y), N^y(c_j, \lambda^x, \lambda^y))$ *denotes the unique single-period Cournot-Nash equilibrium outcome given the marginal cost,* c_j*,* $j = 1, 2$*, and the managerial incentive weights of* λ^x*,* λ^y*.*

Below we state a simplifying assumption that is sufficient but not necessary to solve the model.[16] Also, the assumption is satisfied for a wide range of parameter values. For example, the condition holds for all linear demands with 'sufficiently large' demand intercepts.

Condition 1. *Suppose manager* x *produces an output* x^L *and manager* y *produces an output* $R^y\left(x^L|c_2, \lambda^x, \lambda^y\right)$*, where* $x^L = \arg\max_z((zp(z + R^y(z|c_2, \lambda^x, \lambda^y))) - \lambda^x c_1)$*, then (i) For all* $\lambda^x \in [0, 1]$*,* π^y *monotonically increases as* $\lambda^y \to 0$ *and reaches its maximum at* $\lambda^y = 0$*, and (ii) For all* $\lambda^y \in [0, 1]$*,* π^x *monotonically increases as* $\lambda^x \to 1$ *and reaches its maximum at* $\lambda^x = 1$*.*

3. Second-Period Production Equilibrium in Stage II

In this section we consider Stage II and present the second-period equilibrium strategies of the two managers, given their first-period output choices. Saloner (1987) derives the strategies for the case: $c_1 = c_2$. These results also hold for the case: $c_1 \neq c_2$.

Lemma 1. *Given* (q_1^x, q_1^y)*, the second period equilibrium is*

[16] Condition 1 guarantees that the SPNE values of λ^i and λ^k are either $(\lambda^i = 0, \lambda^k = 0)$ or $(\lambda^i = 1, \lambda^k = 0)$. Without the condition, the the SPNE values of λ^i and λ^k will be either $(0 < \lambda^i < 1, 0 < \lambda^k < 1)$ or $(\lambda^i = 1, 0 < \lambda^k < 1)$.

$$\beta_2^i\left(q_1^x, q_1^y\right) = \begin{cases} 0 & if \quad q_1^i \geq R^i\left(q_1^k | c_2, \lambda^i, \lambda^k\right), \\ & \quad\;\; q_1^k \geq R^k\left(q_1^i | c_2, \lambda^i, \lambda^k\right) \\ \\ N^i\left(c_2, \lambda^x, \lambda^y\right) - q_1^i & if \quad q_1^i \leq N^i\left(c_2, \lambda^x, \lambda^y\right), \\ & \quad\;\; q_1^k \leq N^k\left(c_2, \lambda^x, \lambda^y\right) \\ \\ 0 & if \quad q_1^i \geq N^i\left(c_2, \lambda^x, \lambda^y\right), \\ & \quad\;\; q_1^k \leq R^k\left(q_1^i | c_2, \lambda^i, \lambda^k\right) \\ \\ R^i\left(q_1^k | c_2, \lambda^i, \lambda^k\right) - q_1^i & if \quad q_1^i \leq N^i\left(c_2, \lambda^x, \lambda^y\right), \\ & \quad\;\; q_1^k \geq N^k\left(c_2, \lambda^x, \lambda^y\right), \\ & \quad\;\; q_1^i \leq R^i\left(q_1^k | c_2, \lambda^i, \lambda^k\right) \end{cases}$$

$$where\ k \neq i\ and\ k, i = x, y.$$

[Insert Figure 1 here]

The lemma says if (q_1^x, q_1^y) lies on or outside the outer envelope of the second period reaction functions (area “D” in Figure 1), then neither manager produces in the second period. If, in the first period, each manager produces less than his second-period Cournot output (area “A” in Figure 1) then both managers produce up to their respective second-period Cournot output levels. If one manager exceeds his firm’s second-period Cournot output and the other does not (areas “B” and “C” in Figure 1), then the latter manager produces, in the second-period, his best response to the former manager's first-period output and the former manager does not produce. Given a pair of first period outputs (q_1^x, q_1^y), Figure 1 shows the second-period equilibrium outputs with $c_1 > c_2$. Manager x does not produce in the second period, whereas manager y's second-period output equals $R^y(q_1^x|c_2, \lambda^x, \lambda^y) - q_1^y$.

4. The Subgame Perfect Nash Equilibria for $c_2 < c_1$

In this section, we derive the subgame perfect Nash equilibria of our model for $c_2 < c_1$. To do so, we first characterize the equilibria in Stage II, for given λ^x and λ^y.

4.1. Characterization of Stage II Equilibria for $c_2 < c_1$.

LEMMA 2. *If $c_2 < c_1$, $q_1^i > N^i(c_2, \lambda^x, \lambda^y)$ and $\lambda^k \in (0, 1]$, then the optimal $q_1^k = 0$, $i \neq k = x, y$.*

PROOF. Without loss of generality, let $i = x$ and $k = y$. If $q_1^x > N^x(c_2, \lambda^x, \lambda^y)$, then the optimal $q_2^x = 0$ for any q_1^y (from Lemma 1). Since $\lambda^y \in (0, 1]$, manager y maximizes an objective function with a positive weight on profit. Now, since $c_2 < c_1$, it is best for manager y to choose $q_1^y = 0$ and produce his best response in the second period. □

LEMMA 3. *If $c_2 < c_1$, $q_1^i \in [0, N^i(c_2, \lambda^x, \lambda^y)]$ and $\lambda^k \in (0, 1]$, then the optimal $q_1^k \notin (0, N^k(c_2, \lambda^x, \lambda^y)]$.*

PROOF. Without loss of generality, consider $i = x$ and $k = y$. Since $\lambda^y \in (0, 1]$, manager y maximizes an objective function with a positive weight on profit. Now, suppose $q_1^x \in [0, N^i(c_2, \lambda^x, \lambda^y)]$ and $q_1^y \in [0, N^k(c_2, \lambda^x, \lambda^y)]$, then in the second period $q_2^x = N^x(c_2, \lambda^x, \lambda^y) - q_1^x$ and $q_2^y = N^y(c_2, \lambda^x, \lambda^y) - q_1^y$. Consequently, manager y earns $N^y(c_2, \lambda^x, \lambda^y)\, p(N^x(c_2, \lambda^x, \lambda^y) + N^y(c_2, \lambda^x, \lambda^y)) - \lambda^y(c_1 q_1^y + c_2 q_2^y)$. Since, $c_2 < c_1$, manager y is better off by choosing $q_1^y = 0$. □

LEMMA 4. *If* $c_2 < c_1$, $\lambda^i \in (0,1]$, *then manager* i *produces in either of the two periods, but not in both.*

PROOF. Follows from Lemmata 2 and 3. □

4.1.1. *Characterization of Stage II Equilibria for given* $\lambda^x \in (0,1]$ *and* $\lambda^y \in (0,1]$.

LEMMA 5. *If* $c_2 < c_1$, $\lambda^x \in (0,1]$ *and* $\lambda^y \in (0,1]$, *then a second stage SPNE outcome must satisfy one of the following:*

(i) $\{(q_1^x = x^L, q_1^y = 0),\ \left(q_2^x = 0, q_2^y = R^y\left(x^L | c_2, \lambda^x, \lambda^y\right)\right)\}$

(ii) $\{(q_1^x = 0, q_1^y = y^L),\ \left(q_2^x = R^x\left(y^L | c_2, \lambda^x, \lambda^y\right), q_2^y = 0\right)\}$

(iii) $\{(q_1^x = 0, q_1^y = 0),\ (q_2^x = N^x(c_2, \lambda^x, \lambda^y), q_2^y = N^y(c_2, \lambda^x, \lambda^y))\}$

$$\textit{where } k^L = \arg\max_z z\left(p\left(z + R^i\left(z | c_2, \lambda^i, \lambda^k\right)\right) - \lambda^k c_1\right), k \neq i = x, y$$

PROOF. Since $\lambda^x \in (0,1]$ and $\lambda^y \in (0,1]$, it follows from Lemma 4 that each manager produces either in period one or in period two, but not in both. Now, by Lemma 3, both managers cannot produce in period one, completing the proof. □

DEFINITION 1. *Define* $\widehat{c}_2(\lambda^x, \lambda^y)$ *such that*

$$\phi^k\left(N^k(\widehat{c}_2, \lambda^x, \lambda^y), N^i(\widehat{c}_2, \lambda^x, \lambda^y)\right) = \phi^k(k^L, R^i\left(z | \widehat{c}_2, \lambda^i, \lambda^k\right)), k \neq i \ \textit{and}\ k, i = x, y$$

Observe that $\phi^k\left(N^k\left(\widehat{c}_2,\lambda^x,\lambda^y\right),N^i\left(\widehat{c}_2,\lambda^x,\lambda^y\right)\right)$ is manager k's payoff when both managers are producing simultaneously (only) in period II. $\phi^k(k^L,\ R^i\ (z|\widehat{c}_2,\ \lambda^i,\lambda^k))$ is manager k's payoff when manager k is producing only in period I and manager i is producing only in period II. Also, note that for $c_2<\widehat{c}_2\left(\lambda^x,\lambda^y\right)$, $\phi^k(N^k(\widehat{c}_2,\lambda^x,\lambda^y),N^i(\widehat{c}_2,\lambda^x,\lambda^y))>\phi^k(k^L,R^i(z|\widehat{c}_2,\lambda^i,\lambda^k))$ and for $c_2>\widehat{c}_2\left(\lambda^x,\lambda^y\right)$, the converse is true.

Definition 2. *Define* $\widehat{c}_2^{Max}=Maximum_{\lambda^x,\lambda^y}\ \{\widehat{c}_2\left(\lambda^x,\lambda^y\right)\}$

Proposition 1. *If* $c_2<c_1$ *and* $\lambda^x\in(0,1]$, $\lambda^y\in(0,1]$, *then the second stage SPNE outcomes are:*

(i) If $c_2<\widehat{c}_2\left(\lambda^x,\lambda^y\right)$, *then* $\{(q_1^x=0,q_1^y=0),\ (q_2^x=N^x(c_2,\lambda^x,\lambda^y),q_2^y=N^y(c_2,\lambda^x,\lambda^y)\}$ *is the unique equilibrium outcome.*

(ii) If $c_2\in\left(\widehat{c}_2\left(\lambda^x,\lambda^y\right),c_1\right)$, *then there are two equilibrium outcomes, at each, one manager behaves as a leader and produces only in period one, whereas the other behaves as a follower and produces his best response only in period two.*

(iii) If $c_2=\widehat{c}_2\left(\lambda^x,\lambda^y\right)$, *then there are three equilibrium outcomes, as described in parts (i) and (ii) above.*

Proof. Follows directly from Lemma 5 and the definition of $\widehat{c}_2\left(\lambda^x,\lambda^y\right)$. □

Corollary 1. *If* $c_2\in\left(\widehat{c}_2^{Max},c_1\right)$ *and* $\lambda^x\in(0,1]$, $\lambda^y\in(0,1]$, *then in the second stage there are only two equilibrium outcomes, at each, one manager behaves as a leader and produces only in period one, whereas the other behaves as a follower and produces his best response only in period two.*

Proof. Follows from Proposition 1 and the definition of $\widehat{c}_2^{Max}$. □

4.1.2. *Characterization of Stage II Equilibria for given $\lambda^x = 0$ and $\lambda^y = 0$.*

Proposition 2. *If $c_2 < c_1$ and $\lambda^x = 0$, $\lambda^y = 0$, then the only outcomes that can be sustained as SPNE outcomes in the second stage are:*

(i) $\{(q_1^x = x^L, q_1^y = \xi \in [0, R^y\left(x^L|c_2, \lambda^x, \lambda^y\right)]), \left(q_2^x = 0, q_2^y = R^y\left(x^L|c_2, \lambda^x, \lambda^y\right) - \xi\right)\}$

(ii) $\{(q_1^x = \xi \in [0, R^x\left(y^L|c_2, \lambda^x, \lambda^y\right)], q_1^y = y^L), \left(q_2^x = R^x\left(y^L|c_2, \lambda^x, \lambda^y\right) - \xi, q_2^y = 0\right)\}$

(iii) $\{(q_1^x \in [N^x\left(c_1, \lambda^x, \lambda^y\right), x^L), q_1^y = R^y(q_1^x|c_1, \lambda^x, \lambda^y)]), (q_2^x = 0, q_2^y = 0)\}$

(iv) $\{(q_1^x = R^x\left(y^L|c_1, \lambda^x, \lambda^y\right), q_1^y = \xi \in [N^y\left(c_1, \lambda^x, \lambda^y\right), y^L)]), (q_2^x = 0, q_2^y = 0)\}$

$$\text{where } k^L = \arg\max_z z\left(p\left(z + R^i\left(z|c_2, \lambda^i, \lambda^k\right)\right)\right), k \neq i = x, y$$

Proof. With $\lambda^x = 0$ and $\lambda^y = 0$, our strategic setting is identical to the one where each manager maximizes his firm's profit and $c_1 = c_2 = 0$. This scenario has been analyzed by Saloner (1987) who obtains the same set of equilibrium outcomes as those listed in Proposition 2. Hence, his arguments can be employed to support the claims made in the Proposition. □

4.1.3. *Characterization of Stage II Equilibria for given $\lambda^i \in (0, 1]$ and $\lambda^k = 0$, $i \neq k$ and $i, k = x, y$.*

Lemma 6. *If $c_2 < c_1$, $q_1^i = 0$ and $\lambda^k = 0$, then the optimal $q_1^k = k^L > N^k\left(c_2, \lambda^x, \lambda^y\right)$*

Proof. Without loss of generality, let $i = x$ and $k = y$. Since $\lambda^y = 0$, manager y maximizes sales revenue. Now, suppose $q_1^x = 0$ and $q_1^y \leq N^y\left(c_2, \lambda^x, \lambda^y\right)$, then in the second period $q_2^x = N^x\left(c_2, \lambda^x, \lambda^y\right)$ and $q_2^y = N^y\left(c_2, \lambda^x, \lambda^y\right) - q_1^y$. Consequently, manager y earns a revenue of $\rho^y(N^x(c_2, \lambda^x, \lambda^y),\ N^y(c_2, \lambda^x, \lambda^y))$.

Alternatively, if $q_1^x = 0$ and $q_1^y > N^y(c_2, \lambda^x, \lambda^y)$, then in the second period $q_2^x = R^x(q_1^y|c_2)$ and $q_2^y = 0$. For $q_1^y > N^y(c_2, \lambda^x, \lambda^y)$, manager y maximizes sales revenue by choosing $q_1^y = y^L$. This choice of $q_1^y = y^L$ yields a revenue of $\rho^y(R^x(y^L|c_{2,}, \lambda^x, \lambda^y), y^L)$.

Since $\rho^y(R^x(y^L|c_2, \lambda^x, \lambda^y), y^L) > \rho^y(N^x(c_2, \lambda^x, \lambda^y), N^y(c_2, \lambda^x, \lambda^y))$, the proof follows. □

Lemma 7. *If $c_2 < c_1$, $\lambda^x \in (0,1]$ and $\lambda^y = 0$, then $\{(q_1^x = 0, q_1^y = y^L),$ $(q_2^x = R^x(y^L|c_2, \lambda^x, \lambda^y), q_2^y = 0)\}$ is an equilibrium outcome in the second stage.*

Proof. From Lemma 6, if $q_1^x = 0$, then $q_1^y = y^L$. Now, if $q_1^y = y^L > N^y(c_2, \lambda^x, \lambda^y)$, then from Lemma 2, it follows that $q_1^x = 0$. □

Proposition 3. *If $c_2 < c_1$, $\lambda^x \in (0,1]$ and $\lambda^y = 0$, then the only outcomes that can be sustained (locally) as equilibrium outcomes in the second stage are:*

(i) $\{(q_1^x = 0, q_1^y = y^L), (q_2^x = R^x(y^L|c_2, \lambda^x, \lambda^y), q_2^y = 0)\}$

(ii) $\{(q_1^x = x^L, q_1^y = 0), (q_2^x = 0, q_2^y = R^y(x^L|c_2, \lambda^x, \lambda^y))\}$

(iii) $\{(q_1^x = x^L, q_1^y = \xi \in (0, R^y(x^L|c_2, \lambda^x, \lambda^y)]), (q_2^x = 0, q_2^y = R^y(x^L|c_2, \lambda^x, \lambda^y) - \xi)\}$

(iv) $\{(q_1^x \in [N^x(c_2, \lambda^x, \lambda^y), x^L), q_1^y = R^y(x^L|c_2, \lambda^x, \lambda^y)]), (q_2^x = 0, q_2^y = 0)\}$

$$\begin{aligned} \textit{where } x^L &= \arg\max_z z\left(p\left(z + R^y(z|c_2, \lambda^x, \lambda^y)\right) - \lambda^x c_1\right) \\ y^L &= \arg\max_z z\left(p\left(z + R^x(z|c_2, \lambda^x, \lambda^y)\right)\right) \end{aligned}$$

Proof. The proof of part (i) follows from Lemma 7. Now, since $\lambda^y = 0$, manager y's first and second period reaction functions are identical. So, by applying the logic presented in Saloner (1987) to the points on manager y's reaction function lying between (and including) manager x's Cournot-Nash and Stackelberg outcomes, we prove parts (ii) - (iv). Also, it can be checked that no other outcome can be sustained as an equilibrium outcome in the second stage. $\square$

There is a caveat, however, that prevents Proposition 3 from claiming that the outcomes presented in parts $(ii) - (iv)$ are equilibrium outcomes. Observe that instead of lowering q_1^x slightly, manager x may be able to increase his payoff by choosing $q_1^x = 0$. By not producing in the first period, manager x will achieve his Cournot-Nash outcome, but will take advantage of lower second period production cost. As a result if c_2 is small, the outcomes presented in parts $(ii) - (iv)$ of Proposition 3 may not be sustained as SPNE outcomes.

Corollary 2. *If* $c_2 \in \left(\widehat{c}_2^{Max}, c_1\right)$ *and* $\lambda^x \in (0,1]$, $\lambda^y = 0$, *then the outcome* $\{(q_1^x = x^L, q_1^y = 0), \left(q_2^x = 0, q_2^y = R^y\left(x^L|c_2, \lambda^x, \lambda^y\right)\right)\}$ *can be sustained as a SPNE outcome in the second stage.*

Proof. Follows from Proposition 3 and the definition of $\widehat{c}_2^{Max}$. $\square$

4.2. Characterization of Stage I Equilibria for $c_2 < c_1$.

Proposition 4. *If* $c_2 \in (\widehat{c}_2^{Max}, c_1)$ *and* $\lambda^x = 1$, $\lambda^y = 0$, *then* $\{(q_1^x = x^L, q_1^y = 0), \left(q_2^x = 0, q_2^y = R^y\left(x^L|c_2, \lambda^x, \lambda^y\right)\right)\}$ *is sustainable as a SPNE outcome.*

Proof. First note that since $c_2 \in (\widehat{c}_2^{Max}, c_1)$, it follows from Corollary 2 that for $\lambda^x = 1$, $\lambda^y = 0$, $\{(q_1^x = x^L, q_1^y = 0), \left(q_2^x = 0, q_2^y = R^y\left(x^L|c_2, \lambda^x, \lambda^y\right)\right)\}$ is sustainable as a SPNE outcome in Stage II. Now, we need to show that the owner of Firm x cannot do better by choosing any $\lambda^x \in [0,1)$ and the owner of Firm

y cannot do better by choosing any $\lambda^y \in (0,1]$. Consider the stage II outcome $\{(q_1^x = x^L, q_1^y = 0), \left(q_2^x = 0, q_2^y = R^y\left(x^L|c_2, \lambda^x, \lambda^y\right)\right)\}$. Since $c_2 \in (\widehat{c}_2^{Max}, c_1)$, note that from Propositions 1, 2 and Corollary 2 that the outcome can be sustained as a SPNE outcome for all $\lambda^x \in [0,1]$ and $\lambda^y \in [0,1]$. Since manager x acts as a leader and manager y acts as a follower, it follows from Condition 1 that $\lambda^x = 1$ maximizes Firm x's profit and $\lambda^y = 0$ maximizes Firm y's profit. $\square$

Since the profit maximizing manager turns his firm into a Stackelberg leader, he may generate a larger profit for his firm relative to the revenue maximizing manager, who turns his firm into a Stackelberg follower. The example below supports the claim.

Example 1. *Let* $p(Q) = a - Q$, $\lambda^x = 1$, *and* $\lambda^y = 0$. *It can be checked that* $q_1^x = x^L = \frac{a-2c_1}{2}$, $q_2^x = 0$, $q_1^y = 0$ *and* $q_2^y = \frac{a+2c_1}{4}$. *Also,* $\pi^x = \frac{(a-2c_1)^2}{8}$ *and* $\pi^y = \frac{(a+2c_1)^2}{16} - \frac{c_2}{4}(a+2c_1)$. *Observe that* $\pi^x > \pi^y$ *for all* c_2 *that are close to* c_1. *For example, if* $a = 10, c_1 = 1, c_2 = 0.9$, *then* $\pi^x = 8$ *and* $\pi^y = 6.3$.

5. The Subgame Perfect Nash Equilibria for $c_1 = c_2$

In this section, we derive the subgame perfect Nash equilibria of our model for $c_1 = c_2$. To do so, we first characterize the equilibria in Stage II, for given λ^x and λ^y.

5.1. Characterization of Stage II Equilibria for $c_1 = c_2$.

Proposition 5. *If* $c_1 = c_2$, $\lambda^x \in [0,1]$, *and* $\lambda^y \in [0,1]$, *then the only outcomes that can be sustained as equilibrium outcomes in the second stage are:*

(i) $\{(q_1^x = x^L, q_1^y = \xi \in [0, R^y\left(x^L|c_2, \lambda^x, \lambda^y\right)]), \left(q_2^x = 0, q_2^y = R^y\left(x^L|c_2, \lambda^x, \lambda^y\right) - \xi\right)\}$

(ii) $\{(q_1^x = \xi \in [0, R^x\left(y^L|c_2, \lambda^x, \lambda^y\right)], q_1^y = y^L), \left(q_2^x = R^x\left(y^L|c_2, \lambda^x, \lambda^y\right) - \xi, q_2^y = 0\right)\}$

(iii) $\{(q_1^x \in [N^x(c_1,\lambda^x,\lambda^y), x^L), q_1^y = R^y(q_1^x|c_1,\lambda^x,\lambda^y)]), (q_2^x = 0, q_2^y = 0)\}$

(iv) $\{(q_1^x = R^x(\xi|c_1,\lambda^x,\lambda^y), q_1^y = \xi \in [N^y(c_1,\lambda^x,\lambda^y), y^L)]), (q_2^x = 0, q_2^y = 0)\}$

$$\text{where } k^L = \arg\max_z z\left(p\left(z + R^i\left(z|c_2,\lambda^i,\lambda^k\right)\right) - \lambda^k c_1\right), k \neq i = x, y$$

PROOF. Follows directly from Saloner (1987). □

5.2. Characterization of Stage I Equilibria for $c_1 = c_2$.

PROPOSITION 6. *If $c_1 = c_2$, then $(\lambda^x = 0, \lambda^y = 0)$ and $\{(q_1^x = N^x(c_2,\lambda^x,\lambda^y), q_1^y = N^y(c_2,\lambda^x,\lambda^y)), (q_2^x = 0, q_2^y = 0)\}$ is sustainable as a SPNE outcome.*

PROOF. Note that for all $\lambda^x \in [0,1]$ and $\lambda^y \in [0,1]$, $\{(q_1^x = N^x(c_1,\lambda^x,\lambda^y), q_1^y = N^y(c_1,\lambda^x,\lambda^y), (q_2^x = 0, q_2^y = 0)\}$ is sustainable as an equilibrium outcome in Stage II. The rest of the proof follows from Condition 1. □

PROPOSITION 7. *If $c_1 = c_2$, then $(\lambda^x = 1, \lambda^y = 0)$ and $\{(q_1^x = x^L, q_1^y = 0), (q_2^x = 0, q_2^y = R^y(x^L|c_2,\lambda^x,\lambda^y))\}$ is sustainable as a SPNE outcome.*

PROOF. Note that for all $\lambda^x \in [0,1]$ and $\lambda^y \in [0,1]$, $\{(q_1^x = x^L, q_1^y = 0), \left(q_2^x = 0, q_2^y = R^y\left(x^L|c_2,\lambda^x,\lambda^y\right)\right)\}$ is sustainable as an equilibrium outcome in Stage II. The rest of the proof follows from Condition 1. □

From Propositions 6 and 7 we have:

COROLLARY 3. *If $c_1 = c_2$, then $(\lambda^x = 0, \lambda^y = 0)$ and $(\lambda^x = 1, \lambda^y = 0)$ are both sustainable as SPNE outcomes.*

6. The Subgame Perfect Nash Equilibria for $c_1 < c_2$

In this section, we derive the subgame perfect Nash equilibria of our model for $c_1 < c_2$. To do so, we first characterize the equilibria in Stage II, for given λ^x and λ^y.

6.1. Characterization of Stage II Equilibria for $c_1 < c_2$.

NOTATION 4. *Let $x^* = q_1^x + q_2^x$ denote the total output of manager x and $y^* = q_1^y + q_2^y$, the total output of manager y.*

LEMMA 8. *For all $\lambda^x \in [0,1]$ and $\lambda^y \in [0,1]$, any total output pair (x^*, y^*) that lies either inside or outside the outer envelope formed by $R^y(q^x|c_1, \lambda^x, \lambda^y)$ and $R^x(q^y|c_1, \lambda^x, \lambda^y)$, cannot be sustained as a subgame perfect Nash equilibrium outcome in the second stage.*

PROOF. Consider any total output pair (x^*, y^*) that lies inside the outer envelope. Here, at least one manager can gain by producing more in the first period. Hence, such a pair cannot be sustained as a SPNE outcome. Now, consider any total output pair, (x^*, y^*), that lies outside the outer envelope. We know from Lemma 1 that for this output pair, $q_1^x = x^*$, $q_1^y = y^*$, $q_2^x = 0$ and $q_2^y = 0$. Here, a unilateral reduction by either manager of his first period output would make him better off. $\square$

LEMMA 9. *For all $\lambda^x \in [0,1]$ and $\lambda^y \in [0,1]$, any total output pair (x^*, y^*) that lies on the outer envelope formed by $R^y(q^x|c_1, \lambda^x, \lambda^y)$ and $R^x(q^y|c_1, \lambda^x, \lambda^y)$, where $y^* > y^L(c_1)$ or $x^* > x^L(c_1)$, cannot be sustained as a subgame perfect Nash equilibrium outcome in the second stage.*

PROOF. Without loss of generality, suppose $x^* > x^L(c_1)$. We know from Lemma 1 that this x^* is such that $q_2^x = 0$, implying that $q_1^x = x^* > x^L(c_1)$. If manager x were to decrease his first period output to his Stackelberg level then he would be better off. $\square$

6.1.1. *Characterization of Stage II Equilibria for given $\lambda^x \in (0,1]$ and $\lambda^y \in (0,1]$.*

PROPOSITION 8. *If $c_1 < c_2$, then for all $\lambda^x \in (0,1]$ and $\lambda^y \in (0,1]$, $x^* = N^x(c_1, \lambda^x, \lambda^y)$ and $y^* = N^y(c_1, \lambda^x, \lambda^y)$ is the unique outcome that can be sustained as a SPNE outcome in the second stage.*

PROOF. Without loss of generality, consider any total output pair (x^*, y^*) such that $y^* > N^y(c_1, \lambda^x, \lambda^y)$ and $x^* = R^x(y^*|c_1, \lambda^x, \lambda^y)$. From Lemma 1 we know that for this output pair, $q_1^y = y^*$ and $q_2^y = 0$. Here, manager y can gain by producing slightly less in the first period. Let us see why. First, note that the output pair lies on $R^x(q^y|c_1, \lambda^x, \lambda^y)$. Next, since $c_1 < c_2$ and $\lambda^x > 0$, $R^x(q^y|c_2, \lambda^x, \lambda^y) < R^x(q^y|c_1, \lambda^x, \lambda^y)$. Thus, $\exists\ \varepsilon > 0$, such that if $q_1^y = y^* - \varepsilon$, then the optimal q_2^x would still be zero, implying that manager y gains by choosing $q_1^y = y^* - \varepsilon$. $\square$

6.1.2. *Characterization of Stage II Equilibria for given $\lambda^i \in (0,1]$ and $\lambda^k = 0$, $i \neq k$,and $i, k = x, y$.*

LEMMA 10. *For all $\lambda^x \in (0,1]$ and $\lambda^y = 0$, any total output pair (x^*, y^*) that lies on the outer envelope formed by $R^y(q^x|c_1, \lambda^x, \lambda^y)$ and $R^x(q^y|c_1, \lambda^x, \lambda^y)$, where $y^* > N^y(c_1, \lambda^x, \lambda^y)$, cannot be sustained as a subgame perfect Nash equilibrium outcome.*

PROOF. The proof is similar to that of Lemma 9 above. $\square$

Lemma 11. *If $c_1 < c_2$, then for all $\lambda^x \in (0,1]$ and $\lambda^y = 0$, a total output pair (x^*, y^*) that lies on the outer envelope formed by $R^y(q^x|c_1, \lambda^x, \lambda^y)$ and $R^x(q^y|c_1, \lambda^x, \lambda^y)$, where $x^* \in \left[N^x(c_1, \lambda^x, \lambda^y), x^L(c_1)\right]$, can be sustained as a SPNE outcome in the second stage.*

Proof. First note that y^* is the optimal response for manager y, given $x^* = \xi \in \left[N^x(c_1, \lambda^x, \lambda^y), x^L(c_1, \lambda^x, \lambda^y)\right]$. Here, once again, x^* is such that $q_2^x = 0$, implying that $q_1^x = x^* = \xi$. Let us now show that given y^*, it is optimal for manager x to choose a total output of $x^* = \xi$. First note that $q_1^x > \xi$ lowers manager x's profit. Can manager x increase his profit by lowering x^* slightly below ξ? Observe that if there were only one production period, then manager x could increase its profit by choosing $x^* < \xi$. Since there are two production periods, however, if manager x chooses $x^* = \eta < \xi$, manager y would add $[R^y(\eta|c_2, \lambda^x, \lambda^y) - R^y(\xi|c_2, \lambda^x, \lambda^y)]$ in the second period. Since this would be a movement along manager y's reaction function away from manager x's Stackelberg outcome with manager x producing a smaller amount, it would lower manager x's payoff. □

Proposition 9. *If $c_1 < c_2$ and $\lambda^x \in (0,1]$, $\lambda^y = 0$, then $\{(q_1^x \in [N^x(c_1, \lambda^x, \lambda^y), x^L(c_1, \lambda^x, \lambda^y)],\ q_2^x = 0), (q_1^y = R^y(q_1^x|c_1, \lambda^x, \lambda^y), q_2^y = 0)\}$ and $\{(q_1^x = x^L(c_1), q_2^x = 0), (q_1^y \leq R^y(q_1^x|c_1, \lambda^x, \lambda^y), q_2^y = R^y(q_1^x|c_1, \lambda^x, \lambda^y) - q_1^y)\}$ are the only possible SPNE outcomes in the second stage. Therefore, only those total-output pairs that lie on manager y's reaction function between (and including) manager x's Cournot-Nash and Stackelberg leader outcomes are sustainable as SPNE outcomes in the second stage.*

Proof. The proof follows directly from Lemmata 10 and 11. □

COROLLARY 4. *When production cost increases over time, the firm with an incentive scheme involving a strictly positive weight on profit emerges as a leader*[17]*, whereas the revenue maximizing firm emerges as a follower (except at the Cournot-Nash outcome).*

6.1.3. *Characterization of Stage II Equilibria for given $\lambda^x = 0$ and $\lambda^y = 0$.*

PROPOSITION 10. *If $c_1 < c_2$ and $\lambda^x = 0$, $\lambda^y = 0$, then the only outcomes that can be sustained as SPNE outcomes in the second stage are:*

(i) $\{(q_1^x = x^L, q_1^y = \xi \in [0, R^y\left(x^L|c_2, \lambda^x, \lambda^y\right)]), \left(q_2^x = 0, q_2^y = R^y\left(x^L|c_2, \lambda^x, \lambda^y\right) - \xi\right)\}$

(ii) $\{(q_1^x = \xi \in [0, R^x\left(y^L|c_2, \lambda^x, \lambda^y\right)], q_1^y = y^L), \left(q_2^x = R^x\left(y^L|c_2, \lambda^x, \lambda^y\right) - \xi, q_2^y = 0\right)\}$

(iii) $\{(q_1^x \in [N^x(c_1, \lambda^x, \lambda^y), x^L), q_1^y = R^y(q_1^x|c_1, \lambda^x, \lambda^y)]), (q_2^x = 0, q_2^y = 0)\}$

(iv) $\{(q_1^x = R^x(\xi|c_2, \lambda^x, \lambda^y), q_1^y = \xi \in [N^y(c_2, \lambda^x, \lambda^y), y^L)]), (q_2^x = 0, q_2^y = 0)\}$

$$\text{where } k^L = \arg\max_z z\left(p\left(z + R^i\left(z|c_2, \lambda^i, \lambda^k\right)\right)\right), k \neq i = x, y$$

PROOF. Since $\lambda^x = 0$ and $\lambda^y = 0$, our strategic setting is identical to the one where each manager maximizes his firm's profit and $c_2 = c_1 = 0$. As indicated earlier, we can thus invoke Saloner's (1987) arguments to support the claims under the above Proposition. □

6.2. Characterization of Stage I Equilibria for $c_1 < c_2$.

PROPOSITION 11. *If $c_1 < c_2$, then $(\lambda^x = 0,\ \lambda^y = 0)$ and $\{(q_1^x = N^x(c_1, \lambda^x, \lambda^y),$ $q_1^y = N^y(c_1, \lambda^x, \lambda^y)), (q_2^x = 0, q_2^y = 0)\}$ is sustainable as a SPNE outcome.*

[17] A firm is said to be a "leader" if it produces an output that is larger than the Cournot-Nash output, while its rival produces its best response.

PROOF. Follows from the fact that for any $\lambda^x \in [0,1]$ and $\lambda^y \in [0,1]$, the Stage II equilibrium outcome is given by $\{(q_1^x = N^x(c_1, \lambda^x, \lambda^y), q_1^y = N^y(c_1, \lambda^x, \lambda^y)),$ $(q_2^x = 0, q_2^y = 0)\}$. $\square$

LEMMA 12. *A SPNE outcome does not accommodate* $\lambda^x \in (0,1]$ *and* $\lambda^y \in (0,1]$.

PROOF. Without loss of generality, assume $\lambda^y \in (0,1]$. If $\lambda^x \in (0,1]$, then $\exists$ $\epsilon > 0$ such that $\lambda^x - \epsilon \in (0,1]$. It follows from Condition 1 that owner x can increase her profit by choosing $\lambda^x - \epsilon$, instead of λ^x. $\square$

LEMMA 13. *A SPNE outcome does not accommodate* $\lambda^x \in (0,1]$ *and* $\lambda^y = 0$.

PROOF. For $\lambda^x \in (0,1]$ and $\lambda^y = 0$, the Stage II equilibrium outcomes are presented in Proposition 9. If the second stage outcome lies on manager y's reaction function, then $\exists$ $\epsilon > 0$ such that owner y can gain by choosing $\lambda^y + \epsilon$. On the other hand, if the second stage outcome lies at the intersection of owners x and y's reaction functions, then $\exists$ $\epsilon > 0$ such that owner x can gain by choosing $\lambda^x - \epsilon$. $\square$

PROPOSITION 12. *If* $c_1 < c_2$, *then* ($\lambda^x = 0$ *and* $\lambda^y = 0$) *and* $\{(q_1^x = N^x(c_1, \lambda^x, \lambda^y),$ $q_1^y = N^y(c_1, \lambda^x, \lambda^y)), (q_2^x = 0, q_2^y = 0)\}$ *is the unique SPNE outcome.*

PROOF. Follows from Lemmata 12, 13 and Proposition 11 above. $\square$

7. Concluding Remarks

This paper has contributed to the literature on strategic-managerial-incentives by uncovering the optimal nature of managerial incentives that obtains from the strategic interaction of managers in one kind of dynamic production environment. Prior work on strategic-managerial-incentives has only occurred in the context of a static production environment. In such an environment, owners offer the managers symmetric and identical contracts that involve the maximization of either sales revenue or a combination of profit and sales revenue. This stands in stark contrast to the contracts offered in our dynamic production environment. For instance, when the marginal cost of production remains unchanged or falls moderately over time, there exists an equilibrium where the incentive contract for one manager involves purely profit maximization, whereas that for the rival-manager involves only sales-revenue maximization.

Further, in the dynamic production setting, the strategic interaction between the managers resulted in the profit-maximizing manager turning his firm into a Stackelberg-leader, despite the fact that his rival had a sales-based incentive contract. The manager with sales-based incentive contract, it was found, may, in fact, turn his firm into a Stackelberg-follower. Now, with the profit-maximizing manager turning his firm into a Stackelberg-leader, it is very much possible (as we have shown) that he generates a larger firm-profit in comparison to his rival, the sales-revenue-maximizing manager. These findings run counter to what obtains in a static production environment (that has characterized the literature, thus far, on strategic-managerial-incentives)

Also, the strategic-managerial-incentives literature, thus far, has not been capable of furnishing an explanation for an intriguing real-world phenomenon; the offering of a profit-based incentive scheme (with no direct, sales component) by one firm and a sales-based incentive scheme (with no profit component) by another firm *within the same industry.* Our work has made available *one* explanation.

The strategic setting considered in this paper did not make room for the renegotiation of contracts. Given that recontracting is not a costless affair, contract-renegotiation is less likely to be profitable (and hence less likely to occur) for 'short' production-time horizons. For 'long' production-time horizons, however, contract-renegotiation may very well be profitable, making it important to allow for it. The possibility of renegotiation would undermine the "commitment value" of the incentive contracts affecting, in turn, the nature of our findings.[18] It may thus be worthwhile to first obtain a sharper understanding of the conditions under which contract-renegotiation would be profitable - a 'full-fledged' research project in itself.

[18] We are grateful to a referee for bringing this point to our attention.

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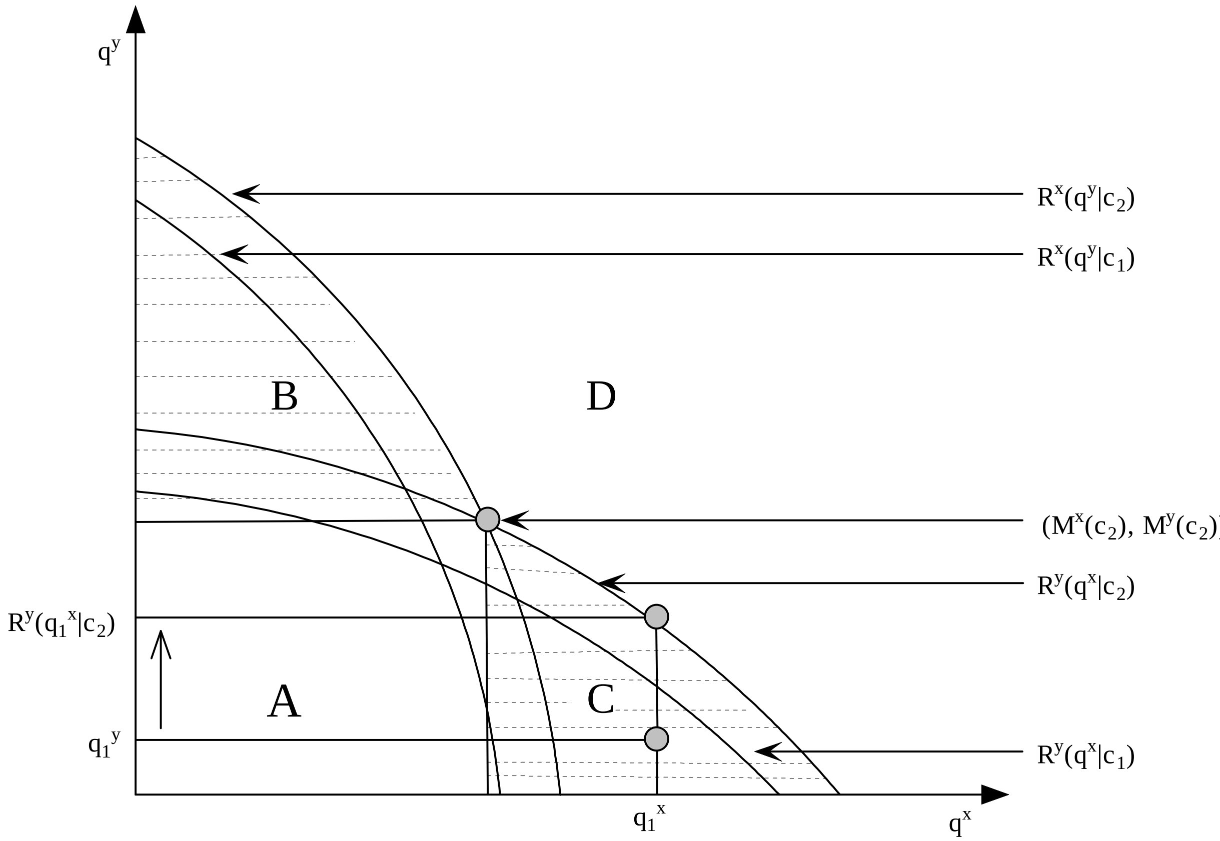

FIGURE 1. Second period equilibrium output for $c_1 > c_2$, given first period output (q_1^x, q_1^y).