Public Infrastructure, Employment and Sustainable Growth in a Small Open Economy With and Without Foreign Direct Investment

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Abstract

The paper builds a theoretical model of endogenous growth motivated by the recent Indian paradox of an improving GDP growth rate in the face of unsatisfactory *employment* growth rate. The source of the problem is believed to be inadequate growth of $manufacture$ for the absorption of unskilled or semi-skilled labour in rural sectors. The paper studies the impact of free trade on *employment* and GDP growth in a small, developing economy in the absence as well as presence of $foreign\ direct\ investment$. The model also recognizes the importance of *public* $infrastructure$ accumulation to support the growth process. The results indicate that free trade with or without a corresponding free inflow of foreign capital into the manufacturing sector has a positive impact on *employment* and GDP growth. However, the beneficial effect is stronger in the presence of foreign capital. Foreign and domestic capital grow at equal rates in equilibrium.

JEL Classification: F43, H23, H41, H54, O11

Key Words: Endogenous growth, Direct foreign investment, Employment, Free trade, Infrastructure, Public goods, Tax.

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by

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1 Introduction

A somewhat puzzling feature of the Indian economy and one that has recently drawn the attention of economists and policy makers is that while its GDP has been growing at a healthy rate, the rate of growth of employment has registered a fall. Bose (2005) presents us with a tell tale set of figures that corroborate this fact. His analysis relates to the growth performance of India's GDP and employment since the 1950's. Till around the early 1980's, India's average GDP growth was of the order of 3.5 per cent, with per capita GDP growing at a meagre rate of 1.5 per cent. The nineties, on the other hand, made significant progress, with GDP growth at factor cost and the corresponding growth of per capita GDP registering rates as high as 6.1 per cent and 4 per cent respectively.

Paradoxically enough, however, employment growth, according to NSS data, had recorded a downswing to 1.02 per cent per annum in the second half of the 1990's when compared with the 2.72 per cent per annum that marked the decade stretching from the mid-1980's. The rural sector in fact sustained the brunt of the decline, 0.01 per cent, as opposed to the 2.59 per cent pertaining to the earlier period. Bose's finding is that unemployment increased from 20 million to 27 million in the second half of the nineties. He goes on to point out that 60 per cent of India's labour force is engaged in agriculture, where the possibility of employment generating innovations has pretty much dried up. The associated scenario in the industrial sector too does not inspire confidence. Thus, while India had a higher proportion of

[1]This author acknowledges comments and suggestions from Bidisha Chakrabarty, Manash R. Gupta, Debasis Mondal, Mihir Rakshit and Abhirup Sarkar.

labour force in organized manufacture compared to countries like Thailand and Indonesia in the 1970's, the picture now has reversed, with both countries' labour force in organized manufacture significantly dominating India.

Much of India's improved growth performance in recent years is explained by the services sector, led primarily by its success in information technology. However, while this has helped to raise the *GDP*, it has not made much of a dent on the employment scenario. This is explained by the need for relatively skilled labour in service related enterprises. Moderately or highly skilled workers constitute a relatively small share of India's work force. Consequently, growth of employment in services has not ensured overall improvements in the employment scenario. India's relative success in services is brought out clearly by the following table also.

Cross-country data 2002

	GDP (US $ m)	Per capita GDP (US $)	Agriculture share in GDP (%)	Industry share in GDP (%)	Services share in GDP (%)
India	5,10,177	470	23	27	51
China	12,66,052	960	15	51	34
Ethiopia	7 6,059	100	40	12	48
Japan	39,93,433	34,010	1	31	68
Uganda	5,803	240	32	22	46
UK	15,66,283	25,510	1	26	73
US	1,03,83,100	35,400	2	23	75

Source: *World Development Indicators, 2004*

A share of 23 per cent for Indian agriculture translates to an aggregate value of approximately $ 1,17,340 million. By striking contrast, the estimated produce of Japan and the UK were lower at $ 39,934 million and $ 15,662 million respectively. By the early 1990s, India was self-sufficient in foodgrain production, which increased from 50.8 million tons in 1950 to 176.3

million tons in 1990 (at a compound growth rate of 2.7 percent per annum from 1949 through 1987). Per capita availability of cereals went up from 334 grams per day in 1951 to 470 grams per day in 1990. Availability of edible oils increased from 3.2 kilograms per year per capita in 1960 to 5.4 kilograms in 1990. Similarly, the supply of sugar per capita increased from 4.7 to 12.5 kilograms per year during the same period.

Yet, India is still miles away from the rich countries in terms of the share that agriculture *should* enjoy in its GDP. Clearly, one cannot suggest an absolute decline in the size of India's agriculture. What is called for is a rise of the other sectors *relative* to agriculture. The Chinese case cannot but capture our attention in this context. Considering China to be notches higher than India in the scale of economic development, the composition of its sectoral shares too would appear to exhibit a higher echelon in economic evolution compared to India. Going by this hypothesis, what is called for in India is a rise in the share, not of services, but of industry.

It is clear then that the manufacturing sector calls for more serious attention. A strategy for bringing about sustained manufacture growth, however, cannot consist of a programme of demand management alone. In India in particular, while manufacture has often been viewed as suffering from excess capacities, the fact remains at the same time that there are major supply constraints too which inhibit industrial development. Perhaps the most significant of these constraints takes the shape of inadequate infrastructure. Shortage of infrastructure causes congestion and, as a result, a strong tendency for diminishing returns to capital in industry. The low rate of return so generated acts as a disincentive to investment. This implies a low rate of labour absorption, thus causing the vicious circle of poverty to perpetuate.

Infrastructure though is a bulky commodity, such as an airport, that calls for large investments of capital and long gestation lags to be created. Moreover, the service flows generated by infrastructure are often characterized by public good features, viz., non-rivalry and non-excludability, though the extent to which these characteristics are present could vary across services. Both the bulkiness as well as the public good properties act on the other hand as disincentives to private participation in its provision. Thus, while private capital is averse to penetrating industrial activities on account of insufficient infrastructure, it is wary of creating the necessary infrastructure

for itself at the same time. It is precisely for this reason that one expects the government, which is not guided by any obvious (economic) profit motive, to play a major role in the accumulation of infrastructure. The strategy for development suggested by this scenario requires manufacture to grow to absorb surplus labour and raise the rate of growth of employment simultaneously with the rate of growth of GDP. While private capital ought to bring about this transformation, the government needs to offer a helping hand for supporting development of infrastructure.

This provides the broad outline of the theoretical problem to be discussed below. The model to be used will of course represent an abstraction, emphasizing certain features of reality and ignoring others. So, it is best not to interpret it as *the* representative model for India, even though it will be utilized to arrive at analytical conclusions about its growth strategies. The major characteristic of the approach will be that we shall "think dynamically", to quote from Harrod (1939), in terms of *sustained* growth rates rather than the short-term concept of GDP changes from one year to the next. This implies a parting of company with standard Keynesian multiplier policies, which calculate the effect of increases in exogenous expenditures on employment and output, assuming the existence of excess capacities in manufacture. We have already argued above that this approach could have limited usefulness in contemporary India, given clear evidence of supply constraints faced by industry on account of inadequate availability of infrastructure. A comprehensive balanced growth strategy, therefore, is what seems to define the need of the day.

The paper attempts to analyse the employment potential of liberal economic policies, especially of free trade. The benefits derived from opening up a developing economy are judged mainly by the effect on sustainable growth rates of capital and GDP, which in turn have implications for the size as well as growth rate of employment. This means that traditional gains from trade results will assume a back seat in the exercise, particularly so since an improvement in welfare (measured by the size of utility over time) has dubious significance when accompanied by large scale unemloyment. Unemployed workers will not have the purchasing power necessary to access the aggregate utility gains. Thus, in what follows, we shall have more than one instance of open economy policies causing rises in employment and GDP growth, but which may or may not increase the value of the utility func-

tional. This phenomenon will have important significance for democratically elected governments in developing societies, which are judged by the electorate on the basis of tangible indices of success. Employment is clearly one such index.

The paper constitutes an exercise in endogenous growth theory. The literature on development models based on the techniques of endogenous growth is not too large. The exercises that do exist in the area of growth in an open economy, such as Cheng *et al* (2005), Feenstra (1996), Lucas (1993), Rivera-Batiz, Romer (1991-a, 1991-b), Stokey (1996), Trindale (2005) etc., do not address the questions posed in this paper. In an indirect way, the strategy underlying our work bears a resemblance to Grossman & Helpman (1991), who looked into the effects of free trade on resource allocation for research and knowledge accumulation, and hence growth, in a small open economy. We are concerned with a similar allocational problem for private and public capital, or infrastructure, to determine the growth rate of the economy.[2]

There have been a few contributions to infrastructure-based growth. The seminal contribution by Barro (1990) viewed infrastructure as a non-accumulable, pure private good. Futagami *et al* (1993) extended the model to accommodate infrastructure accumulation. Barro and Sala-i-Martin (2004) retain the flow dimension of Barro's original work, but allow it to be a pure public good, which may or may not be subject to congestion. Dasgupta (1999, 2001, 2003-04, 2004) studies the stock aspect of infrastructure whose services may be pure or impure public goods. Turnovsky (1997) discusses the role of fiscal policy for infrastructure development in a growing economy.

The point of departure of the present work from the ones quoted lies in its attempt to address issues relating to open economy growth in the face of labour unemployment. The analytical structure of the contribution revolves around interactions between free trade, foreign direct investment, infrastructure development and growth of manufacture.

[2]To avoid possible misinterpretation, it is best to clarify at the outset that apart from the resource allocation problem, the model of the paper is substantially different from the Grossman-Helpman exercise. In particular, it is not concerned with questions of brand proliferation or quality ladders. See Chapter 6, Grossman & Helpman (1991).

The paper is organized as follows. Section 2 describes the analytical features of the model, while Section 3 works out the details of balanced growth equilibrium under autarky. Section 4 deals with results for the small open economy in the absence of international capital flows. Section 5 establishes the efficient balanced growth path and compares it to the market economy's growth path. Section 6 brings in foreign direct investment and discusses its beneficial effects. The Appendix derives a scheme for decentralizing the efficient path of Section 5.

2 The Model of a Mixed Economy

The model economy has three sectors of production, denoted Y, Z and G. Sector Y produces a pure consumption good (denoted Y), the Z-sector produces a (Solow (1956) type) consumption-cum-investment good Z. The output of sector G, written $\dot{G}$, is identically the same as investment in public infrastructure. Commodities Y and Z are produced under competitive conditions and investment in G is under government control. In this sense, the model below represents a Mixed Economy. All outputs are produced with the help of the services of private and public capital as well as semi-skilled labour, denoted by K, G and L respectively. The use of the same notation for the stocks as well as flows of private and public capital implies constant stock-flow ratios for both forms of capital. Neither the stocks nor the service flows of G are marketed. (See the Appendix, however, for a decentralization result which views all commodities as marketed products.)

There is a surplus of semi-skilled workers available in unlimited amounts at a subsistence wage rate $\bar{w}$, *á la* Lewis (1954). Thus, by assumption, the absolute size of employable labour is very large. It is this fact rather than the precise rate of population growth that will motivate the modelling strategy to follow. Hence, the analysis will emphasize growth of absolute magnitudes in preference to per capita values of variables. The government views accumulation of industrial capital as an important vehicle for employment generation. This is captured by assuming labour to be complementary with private capital. Thus, we may suppose that $L_i/K_i = \lambda = constant,\ i = y,\ z,\ g$ where y, z and g index the Y, Z and G-sectors respectively. (The coefficient λ can

be assumed to vary across sectors at the cost of extra algebra, but without the benefit of additional insight.)

Commodity Z acts as the *numéraire*. The price of commodity Y is p, the rate of interest r and the services of G are supplied free of user charge. Commodities Y and Z are produced under competitive conditions, the rate of interest r being equated to the private marginal productivity of capital services. The services of G are supplied free of user charge. The government finances its purchases of private capital by imposing lump-sum taxes T_y and T_z on sectors Y and Z, which could vary across time points in the spirit of the discussion of such levies in Barro (1990) and Barro & Sala-i-Martin (2004). The entire tax revenue is spent on purchasing K-services at the market rate of interest. Thus, the government does not directly organize the production of infrastructure stocks. Instead, as is often the case for both developed as well as developing economies, it floats tenders to contract out production of infrastructure stocks (such as underground cable networking, hospital buildings or accommodation for university professors) to private capitalists. Note, however, that the demand for capital is restricted by the government's budget constraint. Hence, capital may not earn its marginal product in the G-sector, a potential source of inefficiency in the economy. Another inefficiency will arise from the shadow price for G underlying the lump-sum tax for the private sector.

Labour and capital being complementary, the single notation K may be employed to denote the joint input of the two factor services. Given this convention, technologies in the two sectors are represented by neoclassical production functions satisfying the Inada conditions. In particular, we write

$$\begin{aligned} Y &= Gf_y(k_y) \\ &= GA_y k_y^{\alpha},\ 1 > \alpha > 0; \qquad (1) \\ Z &= Gf_z(k_z) \\ &= A_z k_z^{\beta},\ 1 > \beta > 0; \qquad (2) \\ \dot{G} &= Gf_z(k_z) \end{aligned}$$

$$= \quad GA_g k_g^{\gamma},\ 1 > \gamma > 0, \tag{3}$$

where $k_i = K_i/G$ is the factor intensity and K_i the capital allocation in the i-th sector, $i = y, z, g$.

Profits in the Y and Z-sectors are given by

$$\left.\begin{array}{rcl} \Pi_y & = & pY - (r + \bar{w}\lambda)K_y - T_y \\ \Pi_z & = & Z - (r + \bar{w}\lambda)K_z - T_z \end{array}\right\}. \tag{4}$$

As with the joint input K, let us use $r\ wlog$ to denote the term $r + \bar{w}\ \lambda$. Given the meaning assigned to r, the necessary foc's for profit maximization are

$$r = pf_y'(k_y), \tag{5}$$

$$= f_z'(k_z). \tag{6}$$

The form of the foc's rule out corner solutions. This will by guaranteed under autarky by the choice of the preference function to be used below. The result will hold for an open economy too as Proposition 4 will demonstrate.

The ratios T_y/pY and T_z/Z stand for the sizes of the government in sectors Y and Z. It will be assumed that at each point of time the government fixes T_y and T_z at levels consistent with competitive shares. In other words,

$$\frac{T_y}{pY} = 1 - \alpha$$

$$\frac{T_z}{Z} = 1 - \beta.$$

Since T_y and T_z are lump sum taxes, profit maximization ensures that irrespective of the quantum of K_y and K_z, the entire existing supply of the free

input G will be used up by the two private sectors. The G-sector, though, is not a profit maximizer and is *assumed* to employ G to capacity.

The values of marginal products of G in sectors Y and Z are

$$\begin{aligned} q_y &= (1-\alpha)pf_y(k_y), \\ q_z &= (1-\beta)f_z(k_z). \end{aligned}$$

Thus, it follows from Euler's Theorem that

$$\begin{aligned} T_y &= q_y G \\ T_z &= q_z G. \end{aligned}$$

However, q_y and q_z are merely effective prices underlying T_y and T_z. It may appear that q_y and q_z guarantee productive efficiency. This is not the case, since efficiency calls for effective prices equal to the *social* marginal productivity of the public good G. This matter will be taken up in Section 5 below.

Full employment of capital services implies that at each point of time t,

$$k(t) = k_y(t) + k_z(t) + k_g(t), \tag{7}$$

where $k = K/G$. The entire tax revenue is spent on purchasing K at the market rate of interest. Thus, capital may not earn its marginal product in the G-sector. The government's budget constraint is written

$$\left.\begin{aligned} rk_g &= T_y/G + T_z/G \\ &= q_y + q_z \\ &= (1-\alpha)pf_y(k_y) + (1-\beta)f_z(k_z) \end{aligned}\right\}. \tag{8}$$

Dividing (8) by r, substituting from (5), (6) and adding $k_y + k_z$ to both sides, we have

$$k = \frac{k_y}{\alpha} + \frac{k_z}{\beta} \ . \tag{9}$$

The savings rate at each t is chosen optimally by a dynastic household. The latter is endowed with an instantaneous felicity function

$$u(Y_c(t), Z_c(t)) = ln[v(Y_c(t), Z_c(t))],$$

where $Y_c(t)$ and $Z_c(t)$ are the consumptions of Y and Z by the household at time t and v is an increasing, linearly homogeneous, strictly quasi-concave function in Y_c and Z_c. The household's demand for the two commodities at any t is found by maximizing u subject to

$$\left.\begin{array}{rcl} E(t) + \dot{K}(t) & = & r(t)K(t) \\ p(t)Y_c(t) + Z_c(t) & = & E(t) \end{array}\right\}, \tag{10}$$

where $E(t)$ represents the consumption budget at t. In particular, assuming v to have the Cobb-Douglas form

$$v(Y_c, Z_c) = Y_c^{\delta} Z_c^{1-\delta}, \ 0 < \delta < 1,$$

the demand function for $Y_c(t)$ is

$$Y_c(t) = \frac{\delta E(t)}{p(t)}. \tag{11}$$

Under autarky, Y market clearance at each instant requires $Y_c = Y = Gf_y(k_y)$, dropping t for simplicity.[3]

[3]This implies that the Z market clears also. To see this, note that

The model is completed by noting that the household maximizes

$$U = \int_0^\infty ln[Y_c^\delta Z_c^{1-\delta}]\ e^{-\rho t}\ dt,$$

subject to (10) at each t. The solution to the problem yields

$$\dot{E} = E(r - \rho). \tag{12}$$

3 Balanced Growth under Autarky

We may set out a reduced system of equations based on the specifications of Section 2. Define a new variable $x = E/G$. Then, (12) and (3) reduce to

$$\dot{x} = x(r - \rho - f_g(k_g)). \tag{13}$$

Next, (10) and (3) yield

$$\begin{aligned} pY_c + Z_c + \dot{K} &= r(K_y + K_z + K_g) \\ &= r(K_y + K_z) + rK_g \\ &= r(K_y + K_z) - rK_g + q_yG + q_zG + rK_g, \\ &\quad \text{using (8)}, \\ &= (rK_y + q_yG) + (rK_z + q_zG), \\ &\quad \text{using Euler's Theorem}, \\ &= pY + Z. \end{aligned}$$

Now, $Y_c = Y = Gf_y(k_y)$ implies that $Z_c = Z - \dot{K}$, or, $Z_c + \dot{K} = Z = Gf_z(k_z)$. In an open economy, of course, market clearance for each commodity will not hold.

$$\dot{k} = k\left\{r - \frac{x}{k} - f_g(k_g)\right\}. \tag{14}$$

Instantaneous market equilibrium in sector Y gives

$$\begin{aligned} Gf_y(k_y) &= \frac{\delta E}{p} \\ \text{or,} \quad f_y(k_y) &= \frac{\delta x}{p}. \end{aligned} \tag{15}$$

Equations (5), (6), (7), (9), (13), (14) and (15) represent a set of 7 equations in the seven unknowns $k_y(t),\ k_z(t),\ k_g(t),\ p(t),\ r(t),\ \dot{x}(t)$ and $\dot{k}(t)$ given $k(t)$ and $x(t)$.

The following proposition establishes the balanced growth property of the economy under autarky.

Proposition 1 *Under autarky, the economy is characterized by a unique balanced growth path.*

Proof: For balanced growth, $\dot{x} = 0$ and $\dot{k} = 0$ and the 7 equations determine time invariant values of the 7 unknowns $k_y,\ k_z,\ k_g,\ p,\ r,\ x$ and k. Substituting (7), (9) and (6) into (13), we see that

$$\begin{aligned} f_z'(k_z) &= \rho + f_g(k - k_y - k_z) \\ &= \rho + f_g\left(\left(\frac{1}{\alpha} - 1\right)k_y + \left(\frac{1}{\beta} - 1\right)k_z\right). \end{aligned}$$

Denote the inverse of the function f_g by ϕ. Using this inverse function, the last equation reduces to

$$
\begin{aligned}
k_y &= \frac{\alpha}{1-\alpha}\left[\phi(f_z' - \rho) - \frac{1-\beta}{\beta}k_z\right] \\
&\equiv \Omega(k_z). \qquad (16)
\end{aligned}
$$

Similarly, (5), (6), (15), (9), (1), (13) and (14) yield

$$
\begin{aligned}
\rho &= \frac{x}{k} \\
&= \frac{pf_y}{\delta[k_y/\alpha + k_z/\beta]} \\
&= \frac{f_z'/f_y')f_y}{\delta[k_y/\alpha + k_z/\beta]} \\
&= \frac{k_y f_z'/\alpha}{\delta[k_y/\alpha + k_z/\beta]},
\end{aligned}
$$

or,

$$
k_y f_z' = \alpha\delta\rho\left(\frac{k_y}{\alpha} + \frac{k_z}{\beta}\right).
$$

This is written

$$
\begin{aligned}
k_y &= \frac{(\alpha\delta\rho/\beta)k_z}{f_z' - \delta\rho} \\
&\equiv \Phi(k_z). \qquad (17)
\end{aligned}
$$

In view of the Inada conditions, $f_z' \downarrow 0$ as $k_z \uparrow \infty$, $f_z' \uparrow \infty$ as $k_z \downarrow 0$. Also, $\phi' > 0,\ \phi(0) = 0,\ \phi(\infty) = \infty$. Hence, $\Omega(k_z)$ is monotonically decreasing

in k_z, with $\Omega(0) = \infty$. Further, $\exists$ a $\bar{k}_z > 0$ such that $\Omega(\bar{k}_z) = 0$. On the other hand, $\Phi' > 0$, $\Phi(0) = 0$ and $lim_{k_z \to \tilde{k}_z} \Phi(k_z) = \infty$, where $f'_z(\tilde{k}_z) = \delta\rho$. Thus, the functions $\Omega(k_z)$ and $\Phi(k_z)$ have a unique intersection, guaranteeing a unique solution for the values of k_y and k_z associated with the balanced growth path. ■

Figure 1 illustrates the result.

Figure 1 here.

The autarkic balanced growth equilibrium can be shown to be locally saddle point stable. However, for the purpose of this paper, it is the stability of the open economy equilibrium that is more relevant. Hence, we shall postpone the stability exercise to the next section.

4 Balanced Growth in a Small Open Economy

We move on now to the free trade scenario. The economy is small relative to total world trade and is unable to affect the international price ratio p^f. This section will consider trade in commodities alone and assume away free international flow of capital. As opposed to the autarky situation, the equilibrium condition (15) drops out and the instantaneous equilibrium of the system is represented by the 6 equations (5), (6), (7), (9), (13), (14), which determine the variables $k_y(t)$, $k_z(t)$, $k_g(t)$, $r(t)$, $\dot{x}(t)$ and $\dot{k}(t)$ given $k(t)$, $x(t)$ and the world price p^f.

4.1 Incomplete Specialization and Existence of Balanced Growth Path

Under balanced growth once again, $\dot{x} = \dot{k} = 0$, and the set of unknowns changes to k_y, k_z, k_g, r, x and k given the world price p^f. An interesting

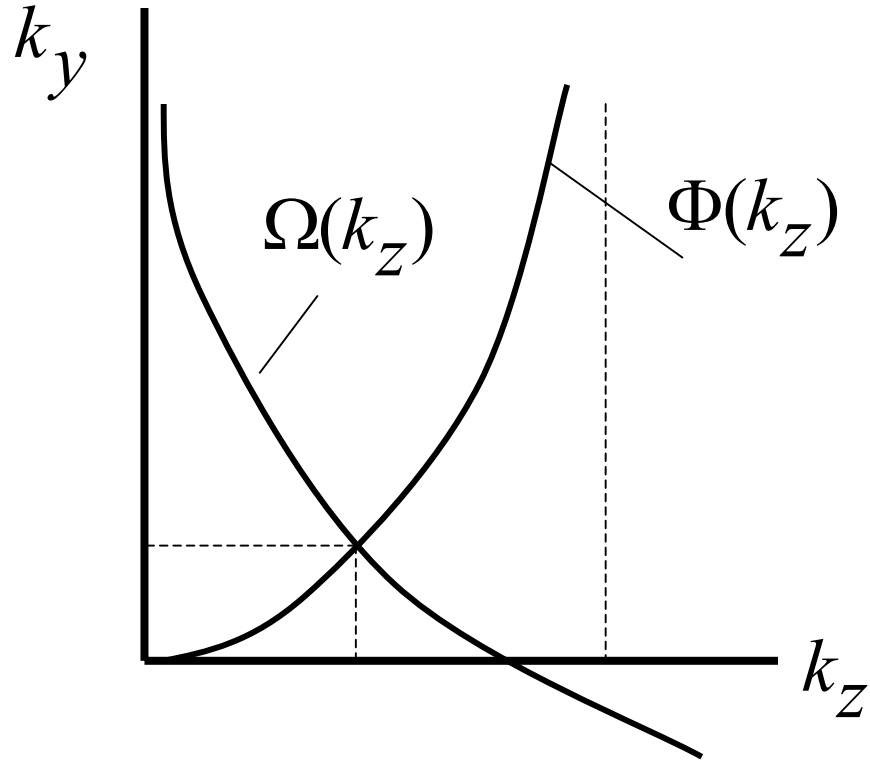


Figure 1 Equilibrium under Autarky

feature of the balanced growth equilibrium follows.

Proposition 2 *For any given world price ratio, the small open economy will necessarily be incompletely specialized.*

Proof: Given any allocation k_g for sector G, the residue $\tilde{k} = k - k_g$ will be allocated to sectors Y and Z. For the open economy, incomplete specialization calls for (5) and (6) to be satisfied. In other words,

$$\begin{aligned} h(k_y) &= p^f f'_y(k_y) - f'_z(k_z) \\ &= p^f f'_y(k_y) - f'_z(\tilde{k} - k_y) = 0 \end{aligned} \tag{18}$$

needs to hold for $0 < k_y < \tilde{k}$. Strict concavity implies $h'(k_y) < 0$. Further, the Inada conditions imply that $h(k_y) \to \infty$ as $k_y \to 0$ and $h(k_y) \to -\infty$ as $k_y \to \tilde{k}$. Hence, there exists a unique k_y satisfying (18) such that $0 < k_y < \tilde{k}$, as was to be proved. This implies that $0 < k_z < \tilde{k}$ also. ■

Comment 1 The result is not surprising. In the presence of the pure public good, the factor allocation problem reduces to the allocation of the single factor K. Hence, the Inada conditions along with concavity rule out corner solutions.

We prove next the existence of a balanced growth path for the open economy. The proposition to follow establishes an important comparative statics property also.

Proposition 3 *There exists a unique free trade balanced growth path for each specification of the world price ratio. Moreover, a rise (fall) in the world relative price of the pure consumption good leads to an increase (decrease) in the balanced growth rate of the economy.*

Proof: Following the steps in Section 2, we observe that equation (16) is valid for the open economy also and the properties of $\Omega(k_z)$ remain unaltered. Equation (17), however, calls for changes, since its derivation used (15), the condition of autarkic market clearance. This needs now to be replaced by the equation of trade balance, i.e.,

$$p^f Y_c + Z_c + \dot{K} = p^f Y + Z.$$

Equations (13) and (14) imply, given balanced growth, that $\rho = x/k$, a familiar relationship between expenditure and asset size for logarithmic utility functions. Moreover, balanced growth also requires that $\dot{G}/G = \dot{K}/K = \dot{E}/E = r - \rho$. Our proof of existence will search for values of k_y and k_z consistent with this condition.

$$\begin{aligned}
x &= \frac{E}{G} \\
&= \frac{p^f Y_c + Z_c}{G} \\
&= \frac{p^f Y + Z - \dot{K}}{G}, \quad \text{(using trade balance)}, \\
&= p^f f_y(k_y) + f_z(k_z) - \frac{(r-\rho)K}{G} \\
&\qquad \text{(using the condition of balanced growth)}, \\
&= p^f f_y(k_y) + f_z(k_z) - (f_z'(k_z) - \rho)k.
\end{aligned}$$

Hence,

$$\begin{aligned}
\rho &= \frac{x}{k} \\
&= \frac{p^f f_y(k_y) + f_z(k_z)}{[k_y/\alpha + k_z/\beta]} - (f_z'(k_z) - \rho),
\end{aligned}$$

or,

$$\frac{k_y}{\alpha} + \frac{k_z}{\beta} = \frac{p^f f_y(k_y)}{f_z'(k_z)} + \frac{f_z(k_z)}{f_z'(k_z)}.$$

We shall argue that this implicit function of k_y and k_z gives rise to an explicit function $k_y = \Gamma(k_z)$ such that $\Gamma(k_z) \to 0$ as $k_z \to 0$ and $\Gamma'(k_z) > 0$. To establish the first property, rewrite the implicit function as

$$\frac{1}{\alpha} = \frac{p^f f_y(k_y)/k_y}{f_z'(k_z)} + \frac{1}{k_y}\left[\frac{f_z(k_z)}{f_z'(k_z)} - \frac{k_z}{\beta}\right],$$

to see that $k_y \to 0$ as $k_z \to 0$. Again differentiate the LHS of the implicit relation to get

$$\begin{aligned}\frac{1}{\alpha} + \frac{1}{\beta} &= \frac{p^f f_y'(k_y)(dk_y/dk_z) f_z'(k_z) - p^f f_y(k_y) f_z''(k_z)}{(f_z'(k_z))^2} \\ &\quad + \frac{d}{dk_z}\left(\frac{f_z(k_z)}{f_z'(k_z)}\right)\end{aligned}$$

or,

$$\begin{aligned}\left(\frac{1}{\alpha} - 1\right)\frac{dk_y}{dk_z} &= -\frac{p^f f_y(k_y) f_z''(k_z)}{(f_z'(k_z))^2} - \frac{1}{\beta} + \frac{d}{dk_z}\frac{f_z(k_z)}{f_z'(k_z)}, \\ &\qquad \text{(using (5), (6))} \\ &= -\frac{p^f f_y(k_y) f_z''(k_z)}{(f_z'(k_z))^2} \\ &> 0 \text{ using (2) .}\end{aligned}$$

The properties of $\Omega(k_z)$ and $\Gamma(k_z)$ imply, corresponding to each exogenously specified value of p^f, the existence of a unique pair (k_z, k_y) which

keeps the economy in balanced growth equilibrium. Further, it is easy to see from the expression for $k_y/\alpha + k_z/\beta$ that a rise in p^f leads to an upward shift in $\Gamma(k_z)$, leaving $\Omega(k_z)$ unaltered. Hence, a rise in p^f causes the balanced growth equilibrium value of k_y to rise and that of k_z to fall. Hence, (6) and (12) imply a rise in the rate of balanced growth for the economy. ■

The idea underlying the proof is captured by Figure 2.

Figure 2 here.

Comment 2 The comparative statics exercise has an intuitive appeal. In order to raise the rate of growth, a first requirement is to raise the rate of interest, or else $\dot{E}/E$ will not increase. Since the rate of interest equals the marginal product of the Z-sector, k_z needs to fall. A rise in the price of the pure consumption good relative to that of the consumption cum capital good supports this fall in k_z and a corresponding rise in k_y. Note that a rise in k_g is required simultaneously for balanced growth to be maintained. This could not have come about in the face of a rise in r had the G-sector been engaged in profit maximization. The result therefore depends on the fact that the government is not a profit maximizer.

Comment 3 It is reasonable to expect that a developing economy has a comparative advantage in producing the pure consumption good over the consumption cum capital good. As a result, p ought to rise rather than fall as trade opens up. Under the circumstances, our results so far point out that the goal of employment generation and growth are better served under free trade than under autarky.

4.2 Stability

The balanced growth path has a strong stability property. To see this, define a new variable $\omega = x/k$, so that (13) and (14) are combined to yield

$$\dot{\omega} = \omega(\omega - \rho).$$

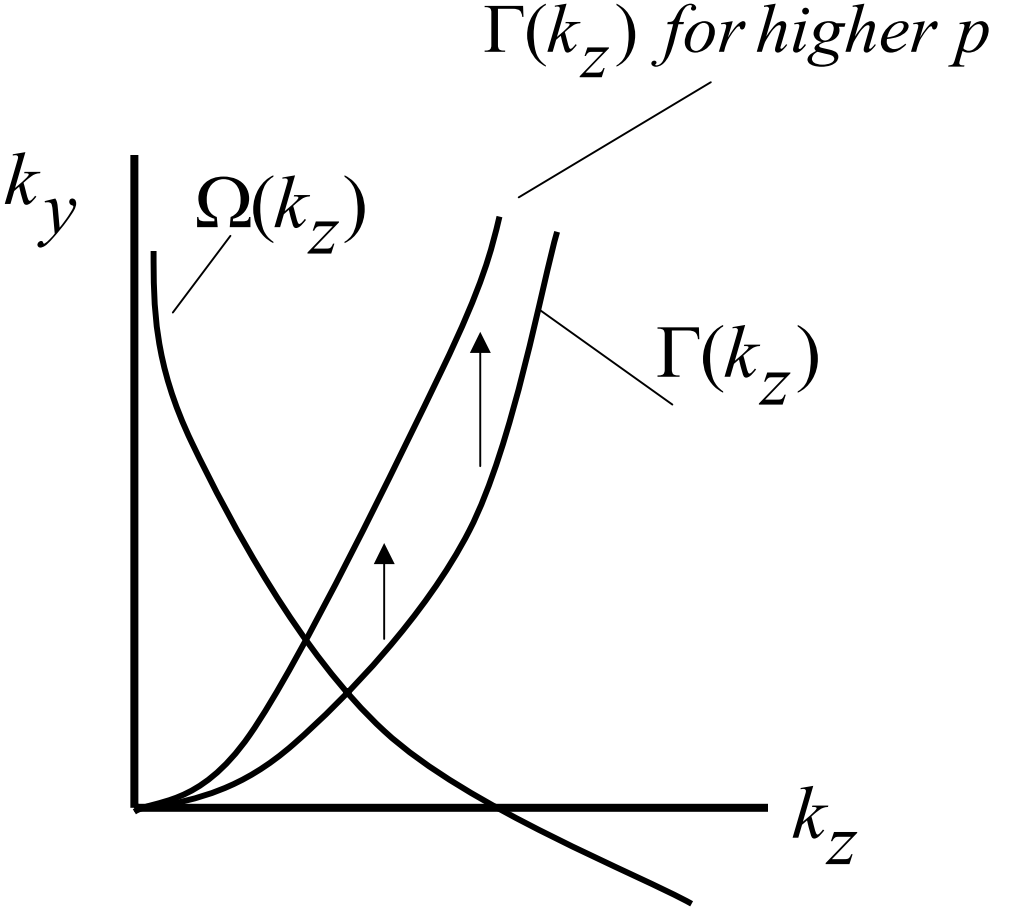


Figure 2 Equilibrium under Free Trade without FDI

Proposition 3 established that balanced growth requires $x = \rho k$, or, what amounts to the same thing, $\omega = \rho$. The above differential equation shows, moreover, that for any other choice of ω, the system veers away from balanced growth. On the other hand, the choice of $\omega = \rho k$ keeps $x/k = \ constant$ forever. However, although it is feasible to choose $\omega = \rho k$ at each point of time, this does not ensure that the system as a whole is in balanced growth equilibrium, for the choice does not imply that x and k are constants, as balanced growth requires. Given the constancy of x/k, however, analysing the stability of the balanced growth path boils down to studying any one of the two equations (13) and (14). We confine our attention to the latter.

Substituting from (6) and (7) into (14), we get

$$\dot{k} = k\{f_z'(k_z) - \omega - f_g(k - k_y - k_z)\}$$

Differentiating this equation,

$$\frac{d\dot{k}}{dk} = k\left(f_z''\frac{dk_z}{dk} - f_g'\left(1 - \frac{dk_y}{dk} - \frac{dk_z}{dk}\right)\right).$$

Stability is proved by noting that $d\dot{k}/dk < 0$.

Proposition 4 *Under free trade, the unique balanced growth equilibrium is globally saddle point stable.*

Proof: Differentiate the system (9) and (18) totally to get

$$\begin{pmatrix} 1/\alpha & 1/\beta \\ p^f f_y'' & -f''z(k_z) \end{pmatrix}\begin{pmatrix} dk_y \\ dk_z \end{pmatrix} = \begin{pmatrix} dk \\ 0 \end{pmatrix}.$$

Solving,

$$\frac{dk_y}{dk} = \frac{f_z''}{p^f f_y''/\beta + f_z''/\alpha} > 0$$

and

$$\frac{dk_z}{dk} = \frac{p^f f_y''}{p^f f_y''/\beta + f_z''/\alpha} > 0.$$

Finally,

$$\begin{aligned} 1 - \frac{dk_y}{dk} - \frac{dk_z}{dk} &= \frac{\alpha(1-\beta)p^f f_y'' + \beta(1-\alpha)f_z''}{\alpha p^f f_y'' + \beta f_z''} \\ &> 0. \end{aligned}$$

Substituting for the signs of dk_z/dk and $1 - dk_y/dk - dk_z/dk$ in the expression for $\dot{dk}/dk$, the result follows. ■

Figure 3 illustrates the stability result.

Figure 3 here.

4.3 Aggregate Private-Public Capital Ratio and Pattern of Trade

Proposition 3 derived a condition under which free trade will lead to a rise in the rate of growth of the economy and, in particular, of employment, since K and L are complementary. However, we should be able to forecast a level effect also if we can establish the direction of movement of k. A

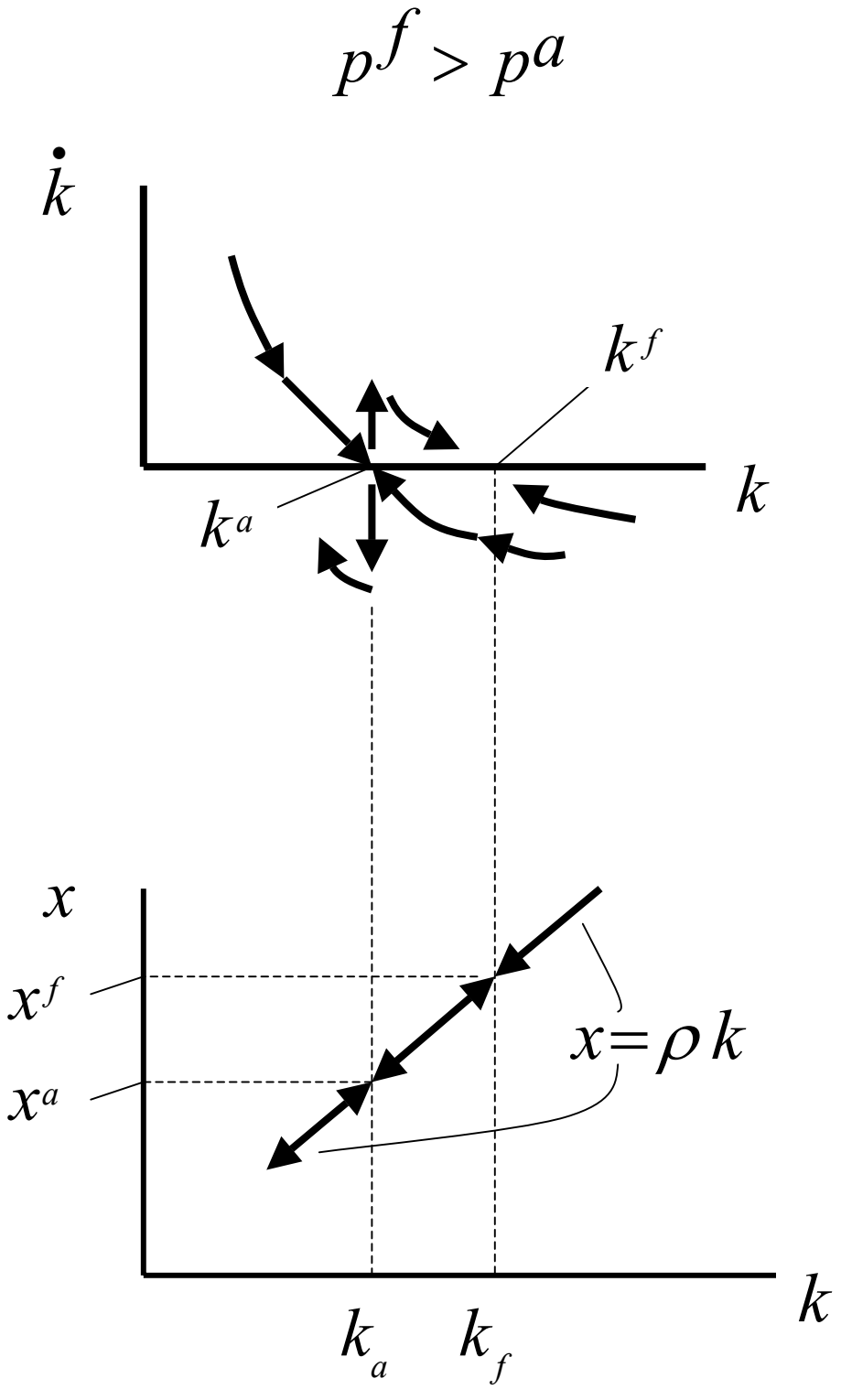


Figure 3 Stability of Equilibrium and Effect of Price Rise

balanced growth path involving a high value of k will generate a higher level of employment per unit of G compared to one involving a lower k. We proceed now to study whether the higher rate of growth goes hand in hand with a higher level of k. Towards this end, we construct an alternative presentation of the balanced growth equilibrium to compare the equilibria under autarky and free trade.

Let $k^i,\ i = a, f$ denote the overall K/G ratios under autarky and free trade. Similarly, let $k^i_j,\ i = a, f;\ j = y, z, g$ be the factor intensities under the two systems for sectors Y, Z and G. Finally, let $\tilde{k}^i = k^i - k^i_g = k^i_y + k^i_z,\ i = a, f$. Then, according to (18),

$$\left.\begin{array}{rcccl} r^a &=& p^a f'_y(k^a_y) &=& f'_z(\tilde{k}^a - k^a_y) \\ r^f &=& p^f f'_y(k^f_y) &=& f'_z(\tilde{k}^f - k^f_y) \end{array}\right\}, \tag{19}$$

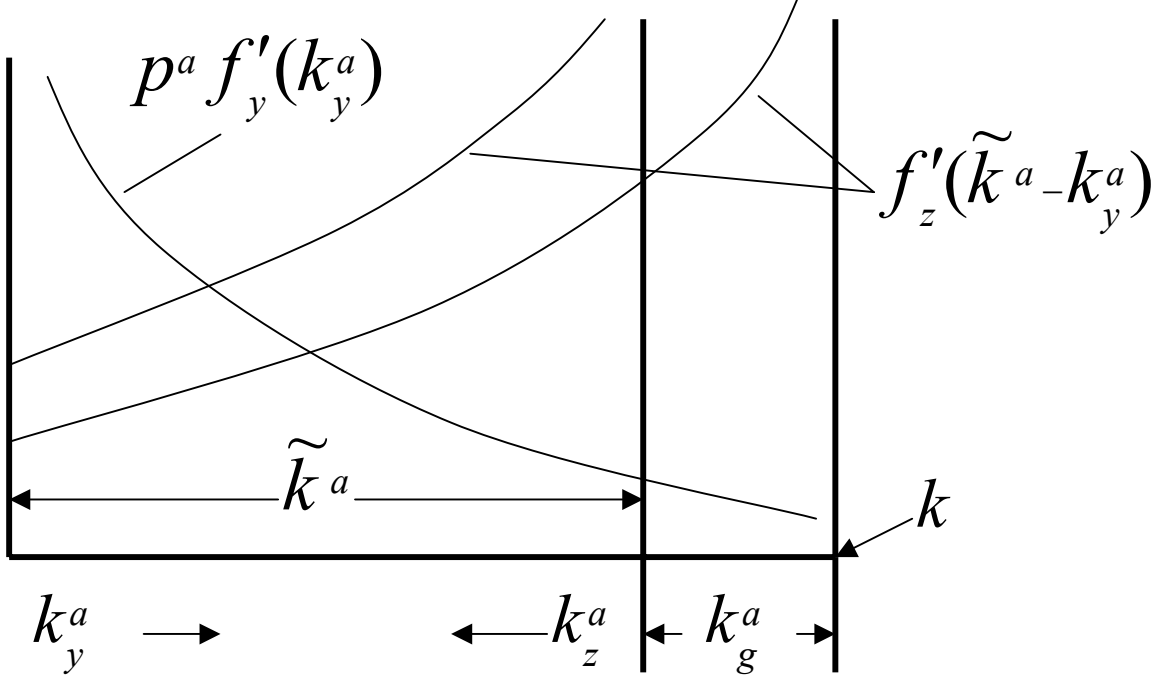
where r^a and r^f stand for the balanced growth equilibrium rates of interest under autarky and free trade. See Figure 4. Given k^a, it follows from concavity and Inada conditions that r^a is monotone increasing in k^a_g. Also, $lim_{k^a_g \to 0} = lim_{k^a_g \to 0} f'(\tilde{k}^a - k^a_y) = \underline{r}^a > 0$, with $r^a \downarrow \underline{r}^a > 0$ as $k^a_g \downarrow 0$ and $r^a \uparrow \infty$ as $k^a_g \uparrow k^a$. We shall call this relationship $\Psi^a(k^a_g, k^a, p^a)$.

Figure 4 here.

Under balanced growth, on the other hand, $\dot{G}/G = \dot{E}/E$. Hence, the two systems will also satisfy the conditions

$$\left.\begin{array}{rcl} r^a &=& \rho + f_g(k^a_g) \\ r^f &=& \rho + f_g(k^f_g) \end{array}\right\}, \tag{20}$$

using (3) and (12). This gives us a second relationship $r^a = \Delta^a(k^a_g)$. It is monotone increasing with $\Delta^a(k^a_g) \downarrow \rho$ as $k^a_g \downarrow 0$ and $\Delta^a(k^a_g) \uparrow \rho + f_g(k^a)$ as $k^a_g \uparrow k^a$. The functions Ψ^a and Δ^a intersect at a unique point in the



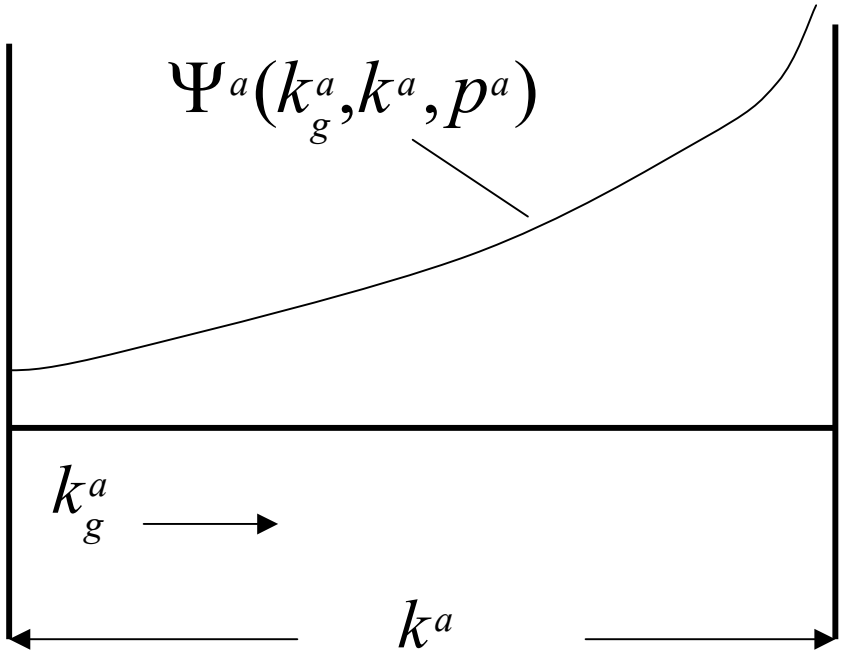


Figure 4 *Construction of* Ψ^a

open interval $(0, k^a)$, since there exists a unique balanced growth equilibrium according to Proposition 1. The same argument implies that $\rho > \underline{r}^a$.

Figure 5 here.

A parallel set of observations can be made about the equilibrium under free trade, with the corresponding curves $\Psi^f(k_g^f, k^f, p^f)$ and $\Delta^f(k_g^f)$ sustaining a unique intersection in the open interval $(0, k^f)$, as guaranteed by Proposition 3. Note that $\Delta^f(k_g^f) = \Delta^f(k_g^a)$ whenever $k_g^f = k_g^a$. In view of this, we shall drop the superscripts and indicate both functions by $\Delta(\cdot)$. We wish to compare the two equilibria to arrive at a conclusion about the relative magnitudes of k^a and k^f.

Proposition 5 *The aggregate ratio of private to public capital for free trade is higher than the one for autarky if the free trade price of the pure consumption good is higher than the one for autarky.*

Proof: Suppose to the contrary that $k^f \leq k^a$. Since $p^f > p^a$, it is clear from (19) that $\Psi^f(k_g, k^f, p^f) > \Psi^a(k_g, k^a, p^a)\ \forall\ k_g \leq k^f$. But $\Delta^f(k_g) = \Delta^a(k_g), \forall\ k_g \leq k^f$. Hence, it follows that $r^f < r^a$, contradicting Proposition 3. ■

Comment 4 Proposition 5 therefore establishes that that there is a positive level effect over and above the rate of growth effect. With K going up per unit of G, the size of employment too rises per unit of G.

Comment 2 has already argued why the rate of growth improves under free trade if $p^f > p^a$. In particular, it pointed out that the output of Z should fall and that of Y should increase. On the other hand, the rise in K/G implies that K rises relative to G at the same time that Z falls. This suggests that the country builds it capital stock K through imports and exports the pure consumption good. The next result proves this result on the pattern of trade.

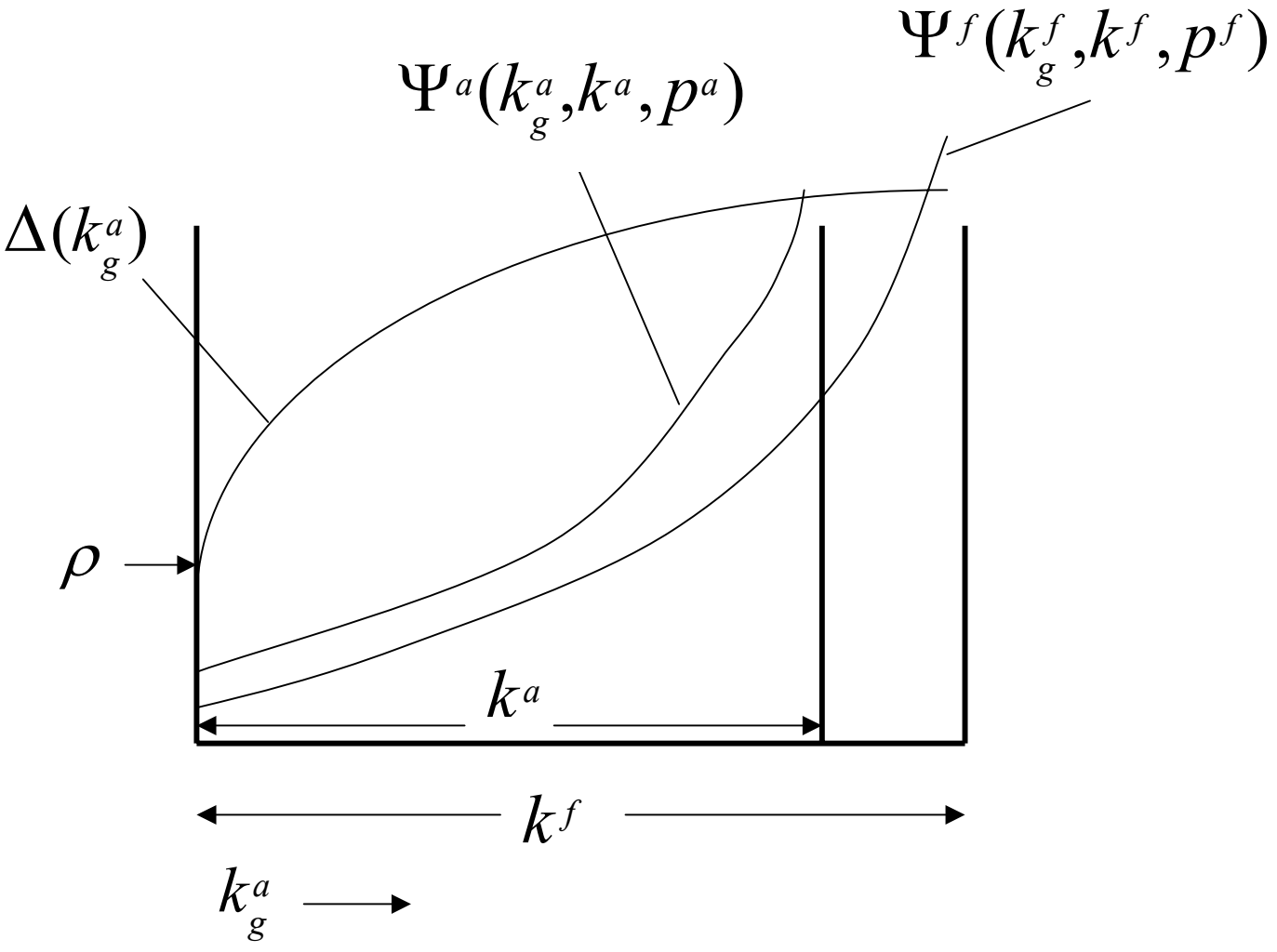


Figure 5 Comparing Autarky and Free Trade, $p^f > p^a$

Proposition 6 *The economy exports (imports) the pure consumption good and imports (exports) the consumption cum capital good along the balanced growth path if free trade leads to a higher (lower) world relative price of the pure consumption good compared to that under autarky.*

Proof: Let us denote the excess demand for Z relative to G by e_z. We have,

$$\begin{aligned} e_z &= \frac{Z_c}{G} + \frac{\dot{K}}{G} - \frac{Z}{G} \\ &= (1-\delta)\frac{E}{G} + \frac{\dot{K}}{G} - f_z(k_z), \text{ (using (11))} \\ &= (1-\delta)x + (r-\rho)k - f_z(k_z), \text{ (using the assumption of balanced growth)} \\ &= (1-\delta)\rho k + (f_z'(k_z) - \rho)k - f_z(k_z), \text{ (using (6))} \\ &= -\delta\rho k + f_z'(k_z)k - f_z(k_z). \end{aligned} \tag{21}$$

Differentiating with respect to p,

$$\begin{aligned} \frac{de_z}{dp} &= (f_z' - \delta\rho)\frac{dk}{dp} + (kf_z'' - f_z')\frac{dk_z}{dp} \\ &> 0, \end{aligned} \tag{22}$$

using Propositions 3 and 5 and the fact that $r = f_z' > \rho > \delta$ ρ for positive growth. ■

5 Optimality Questions

The open economy can lead to higher growth compared to autarky. Yet, as Section 2 pointed out, there are two sources of economic inefficiency in

the model. First, the G-sector may not equate r to the marginal product of capital. Secondly, the effective prices underlying the taxes charged to sectors Y and Z do not reflect the social marginal productivity of G. It is of normative interest therefore to ask if a socially optimal growth path exists and, if it does, to compare it with the equilibrium of Section 4.1.

The socially optimal path is the one to be chosen by a Command Economy. In what follows, we shall restrict ourselves to balanced growth paths alone. The paper is not concerned with the stability properties of a full-fledged Command Economy's growth path, since, in our view, centralized planning has little practical relevance in contemporary world. The Command Economy solution nonetheless allows us to emphasize the sub-optimality of the Mixed Economy path and creates room for investigating whether the Command path is sustainable under free competition, supported by appropriate taxes and subsidies. We shall face up to this question in the Appendix, which will demonstrate that the balanced growth path is decentralizable. The reason why the issue is relegated to the Appendix is that, quite apart from the difficulties of centralized monitoring of the decentralized solution (in the spirit of the second fundamental theorem of welfare economics), the Command Economy growth rate will be seen to fall short of the Mixed Economy growth rate. This poses the policy problem mentioned in Section 1, because it indicates that growth enhancement is not necessarily equivalent to welfare improvement.

A Socially Optimal Balanced Growth Path

We shall prove three results in this section. First we shall demonstrate the existence of a socially optimal balanced growth path. Secondly, we will argue that the balanced growth rate for the Command Economy falls short of that for the Mixed Economy. Finally, we will investigate the intuition underlying the apparently perverse behaviour of the growth rate and trace the phenomenon to the fact that the Command Economy internalises all externalities and avoids misallocations arising from the government's budget constraint. The internalisation of externalities caused by G leads the economy to choose a higher G/K compared to the Mixed Economy and a higher G/K can be

sustained, as we shall see, only at the cost of a lower growth rate.

Proposition 7 *A unique socially optimal balanced growth rate exists for the small economy under free trade without foreign direct investment.*

Proof: In order to discover the best growth path, we solve a social planner's exercise. This is done by maximizing the current value Hamiltonian

$$\begin{aligned}\mathcal{H} &= ln\ (Y_c^{\delta} Z_c^{1-\delta}) + \eta\ [p^f A_y(\phi_y K)^{\alpha} G^{1-\alpha} + A_z(\phi_z K)^{\beta} G^{1-\beta} - p^f Y_c - Z_c] \\ &+ \xi\ ((1-\phi_y-\phi_z)K)^{1-\gamma} G^{1-\gamma},\end{aligned}$$

where ϕ_y and ϕ_z are the shares of physical capital used in the Y and Z-sectors and η and ξ are the co-state variables associated with K and G respectively. The foc's for a unique optimum path are

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial Y_c} &= 0, \\ \frac{\partial \mathcal{H}}{\partial Z_c} &= 0, \\ \frac{\partial \mathcal{H}}{\partial \phi_y} &= 0, \\ \frac{\partial \mathcal{H}}{\partial \phi_z} &= 0,\end{aligned}$$

$$\begin{aligned}\dot{\eta} &= -\frac{\partial \mathcal{H}}{\partial K} + \eta\ \rho, \\ \dot{\xi} &= -\frac{\partial \mathcal{H}}{\partial G} + \xi\ \rho\end{aligned}$$

and

$$\eta(t)\ K(t)\ e^{-\rho t} \to 0 \quad \text{as} \quad t \to \infty,$$
$$\xi(t)\ G(t)\ e^{-\rho t} \to 0 \quad \text{as} \quad t \to \infty.$$

These reduce to

$$\frac{\delta Y_c^{\delta-1}}{Y_c^{\delta} Z_c^{1-\delta}} = p^f \eta, \tag{23}$$
$$\frac{(1-\delta) Z_c^{-\delta}}{Y_c^{\delta} Z_c^{1-\delta}} = \eta,$$
$$\eta \alpha p^f A_y (\phi_y K)^{\alpha-1} G^{1-\alpha} = \xi \gamma A_g ((1-\phi_y-\phi_z)K)^{\gamma-1} G^{1-\gamma}, \tag{24}$$
$$\eta \beta A_z (\phi_y K)^{\beta-1} G^{1-\beta} = \xi \gamma A_g ((1-\phi_y-\phi_z)K)^{\gamma-1} G^{1-\gamma},$$

$$\dot{\eta} = -\eta \alpha p^f A_y p^f (\phi_y K)^{\alpha-1} G^{1-\alpha} + \eta\rho,$$
$$\dot{\xi} = -\eta(1-\alpha) p^f A_y (\phi_y K)^{\alpha} G^{-\alpha} - \eta(1-\beta) A_z (\phi_z K)^{\beta} G^{-\beta}$$
$$-\xi(1-\gamma) A_g ((1-\phi_y-\phi_z)K)^{\gamma} G^{-\gamma} + \xi\rho. \tag{25}$$

Denote the rate of balanced growth by g. Then,

$$g = \frac{\dot{Y}_c}{Y_c} = \frac{\dot{Z}_c}{Z_c} = \frac{\dot{K}}{K} = \frac{\dot{G}}{G}$$

and

$$\frac{\dot{Y}_c}{Y_c} = p^f A_y \alpha (\phi_y K)^{\alpha-1} G^{1-\alpha} - \rho$$

$$
\begin{aligned}
&= A_z\beta(\phi_z K)^{\beta-1}G^{1-\beta} - \rho \\
&= \frac{\dot{Z}_c}{Z_c}. \qquad (26)
\end{aligned}
$$

Equation (3) yields

$$
g = A_g\left(\frac{(1-\phi_y-\phi_z)K}{G}\right)^{\gamma}. \qquad (27)
$$

Differentiating (23) and (24),

$$
\frac{\dot{\eta}}{\eta} = \frac{\dot{\xi}}{\xi} = -(2-\delta)g.
$$

Given these details, (25) is manipulated to give

$$
\frac{1-\alpha}{\alpha}\left(\frac{\phi_y}{1-\phi_y-\phi_z}\right) + \frac{1-\beta}{\beta}\left(\frac{\phi_z}{1-\phi_y-\phi_z}\right) = \frac{\rho + ((1-\delta)+\gamma)g}{\gamma g}. \qquad (28)
$$

Moreover using (26) and (27),

$$
\begin{aligned}
\frac{\phi_y}{1-\phi_y-\phi_z} &= \left(\frac{g+\rho}{p^f A_y \alpha}\right)^{1/(\alpha-1)}\left(\frac{A_g}{g}\right)^{1/\gamma}, \\
\frac{\phi_z}{1-\phi_y-\phi_z} &= \left(\frac{g+\rho}{A_z\beta}\right)^{1/(\beta-1)}\left(\frac{A_g}{g}\right)^{1/\gamma}. \qquad (29)
\end{aligned}
$$

Hence, (28) gives rise to

$$\frac{1-\alpha}{\alpha}\left(\frac{g+\rho}{p^f A_y \alpha}\right)^{1/(\alpha-1)} + \frac{1-\beta}{\beta}\left(\frac{g+\rho}{A_z \beta}\right)^{1/(\beta-1)} = \frac{\rho + ((1-\delta)+\gamma)g}{\gamma g} \times \left(\frac{g}{A_g}\right)^{1/\gamma}. \quad (30)$$

Equation (30) solves for the socially optimal growth rate for the economy. Representing the LHS and RHS of (30) by $\Lambda(g)$ and $\Theta_c(g)$ respectively, it is easy to conclude that they have a unique intersection in the positive orthant. ■

Figure 6 at the end of the next proposition combines Propositions 7 and 8. The latter proposition compares the growth rate for the socially optimal path with the one under free trade.

Proposition 8 *The free trade equilibrium leads to a higher growth rate in comparison to the socially optimal balanced growth path.*

Proof: Using (12), (5) and (6), we have

$$k_y = \left(\frac{g+\rho}{p^f A_y \alpha}\right)^{1/(\alpha-1)},$$

$$k_z = \left(\frac{g+\rho}{A_z \beta}\right)^{1/(\alpha-1)},$$

which means that the equations determining factor intensities in the Y and Z-sectors are the same as the ones for the Command Economy, viz. (29). Equation (3) implies

$$k_g = \left(\frac{g}{A_g}\right)^{1/\gamma}.$$

Next, substitute for k_y, k_z and k_g in (7) and (9) to obtain

$$\left.\begin{array}{rcl} k & = & ((g+\rho)/p^f A_y\alpha)^{1/(\alpha-1)} + ((g+\rho)/A_y\beta)^{1/(\beta-1)} + (g/A_g)^{1/\gamma}, \\ k & = & 1/\alpha((g+\rho)/p^f A_y\alpha)^{1/(\alpha-1)} + 1/\beta((g+\rho)/A_y\beta)^{1/(\beta-1)} \end{array}\right\}. \tag{31}$$

Equating these two

$$\frac{1-\alpha}{\alpha}\left(\frac{g+\rho}{p^f A_y\alpha}\right)^{1/(\alpha-1)} + \frac{1-\beta}{\beta}\left(\frac{g+\rho}{A_z\beta}\right)^{1/(\beta-1)} = \left(\frac{g}{A_g}\right)^{1/\gamma}. \tag{32}$$

The LHS is identical to the curve $\Lambda(g)$ in (30). The RHS may be denoted by $\Theta_f(g)$. It is easy to see that $\Theta_f(g)$ is higher than $\Theta_c(g)$ for all positive g. Consequently, the balanced growth rate for the free trade equilibrium must be larger than the socially optimal growth rate. ■

Figure 6 here.

The ideas underlying Proposition 5 can be used to demonstrate that a higher growth rate for the Command Economy can come about only if the balanced growth value of K/G falls from the Mixed Economy level.

Proposition 9 *In balanced growth equilibrium, the Command Economy employs a lower ratio of private to public capital compared to the Mixed Economy.*

Proof: Let k^c denote the overall K/G ratio and $k^c_j,\ j = y, z, g$ the factor intensities for the Command Economy. We know from Proposition 5 that the balanced growth equilibrium under free trade satisfies

$$\Psi^f(k^f_g, k^f, p^f) = \Delta^f(k^f_g)$$

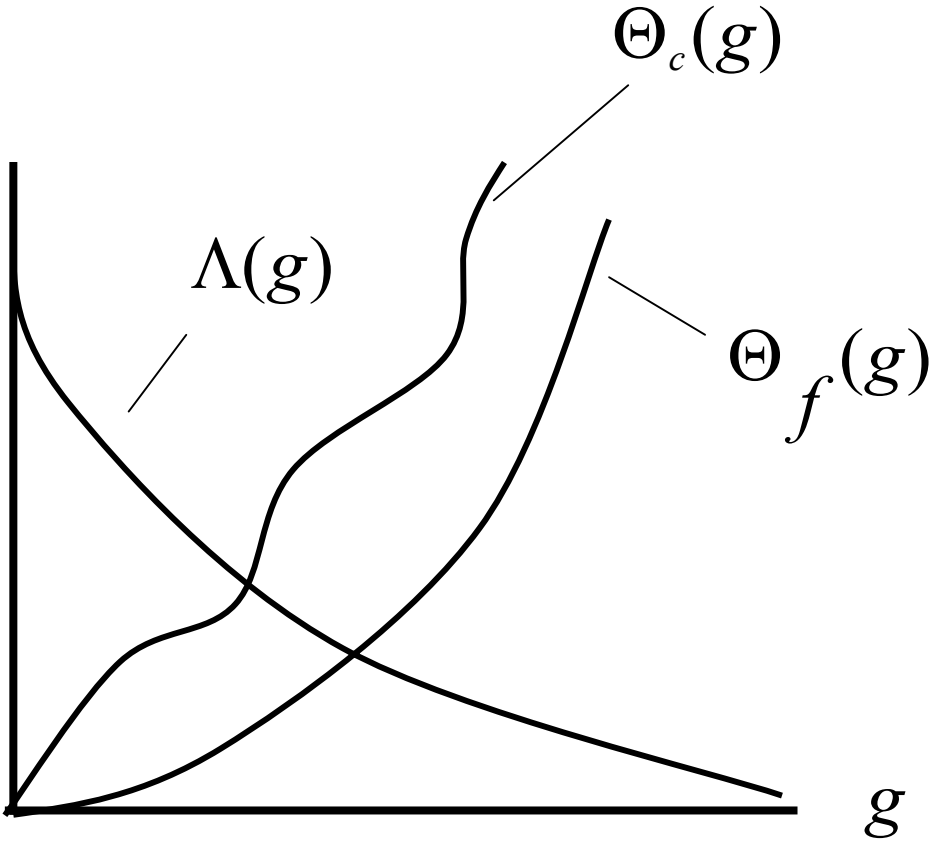


Figure 6 *Comparison with Social Optimum*

for a unique $k_g^f \in (0, k^f)$. Efficiency requires that the Command Economy too satisfy a condition similar to (18). Hence, there exists a curve $\Psi^c(k_g^c, k^c, p^f)$ along which the pair (k_y^c, k_z^c) is allocated efficiently between sectors Y and Z. Moreover, since both economies face the same p^f, the functions $\Psi^f(k_g^f, k^f p^f)$ and $\Psi^c(k_g^c, k^c, p^f)$ are identical except for their domains of definition, viz. $[0, k^f]$ and $[0, k^c]$. To emphasize this fact, we shall write $\Psi(k_g, k^f)$ for $\Psi^f(k_g^f, k^f, p^f)$ and $\Psi(k_g, k^c)$ for $\Psi^c(k_g^c, k^c, p^f)$. Further, (26) and the assumption of balanced growth imply that for the Command Economy $k_g = \Delta(k_g)$. Thus, the condition for balanced growth for the two economies are

$$\begin{aligned} \Psi(k_g, k^f) &= \Delta(k_g), \\ \Psi(k_g, k^c) &= \Delta(k_g). \end{aligned}$$

Suppose now that $k_c \geq k_f$. Then, it is easy to check (See Figure 7) that under free trade, the rate of growth for the Command Economy is higher than that for the Mixed Economy. This contradicts Proposition 8. ■

Figure 7 here.

Comment 5 The intuition for Propositions 8 is as follows. The Command Economy will internalise all externalities and end up employing G according to its social marginal productivity. This should cause it to choose a higher value of G/K, or a lower value of K/G, compared to the Mixed Economy, i.e., $k^c < k^f$. In this case, for every choice of k_g, the residual left over for allocation between the Y and Z-sectors is smaller for the Command Economy than for the Mixed economy. With less capital per unit of G available for allocation, the equilibrium value of marginal product goes up, i.e., the value of the function $\Psi(k_g, k^c)$ exceeds the value of $\Psi(k_g, k^f)$ for every k_g and in particular for the value of $k_g = \bar{k}_g$ (say) such that $\Psi(\bar{k}_g, k^f) = \Delta(\bar{k}_g)$. Thus, at $\bar{k}_g$, we have $\Psi(\bar{k}_g, k^c) > \Psi(\bar{k}_g, k^f) = \Delta(\bar{k}_g)$. Hence, with a monotone increasing Ψ, the value of k_g that equates Ψ and Δ must fall below $\bar{k}_g$, thereby giving rise to a smaller rate of growth than in the Mixed Economy.

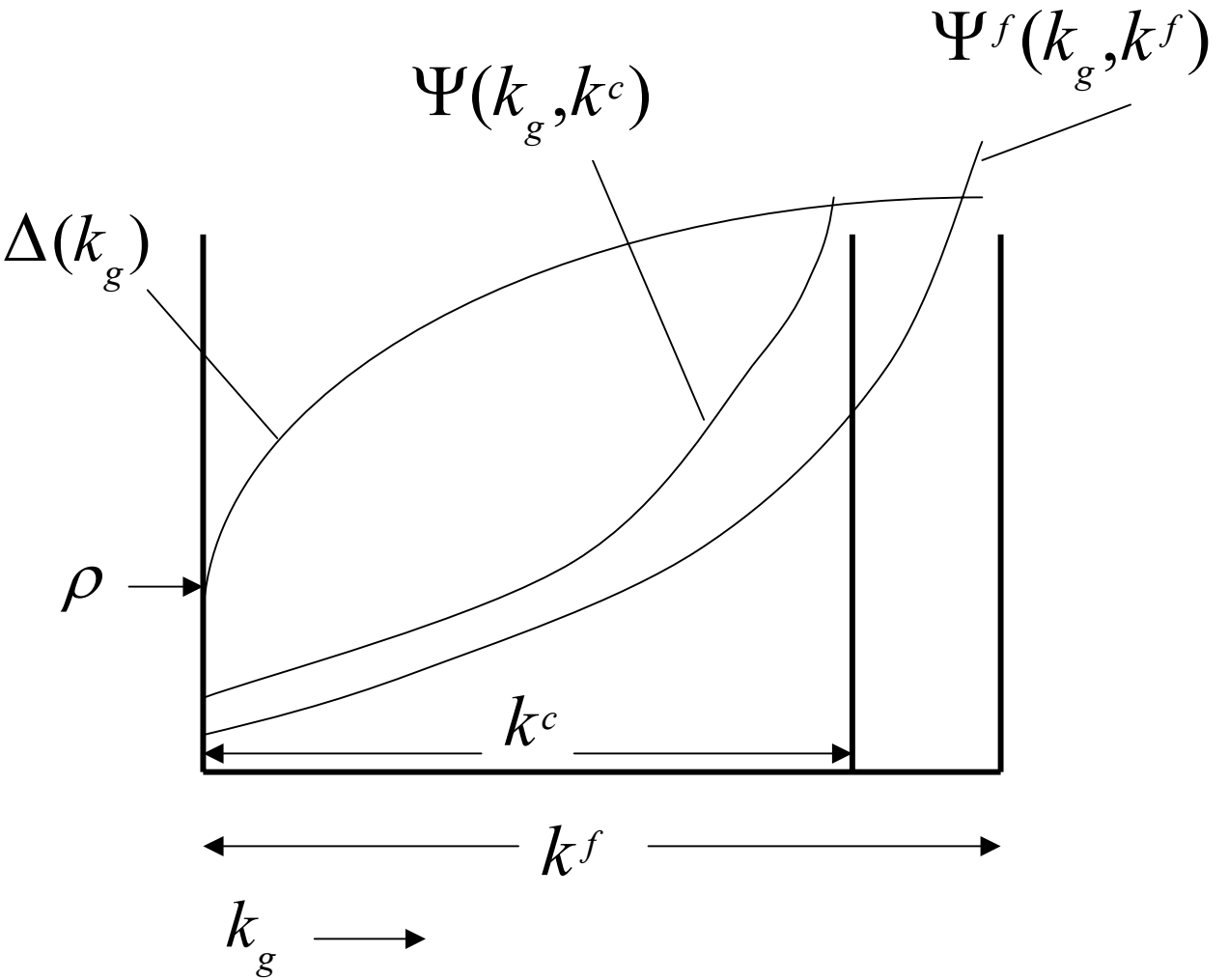


Figure 7 Comparing Free Trade and Command Solution

Since both the rate of growth as well as level of employment per unit of G is lower under social optimality, it follows that the "inefficient" Mixed Economy path dominates the "efficient" Command Economy path from the point of view of employment policy, though the latter obviously dominates in welfare. This suggests that the government in power could possibly opt for a policy of smaller welfare if, in exchange, it achieves a higher level and rate of growth of employment. The level of unemployment being a crucial index for judging a government's success or failure in a democratic society, it may not be paradoxical for the government to choose the inefficient path. On the other hand, even if it wished to adopt the Command Economy path, the degree of centralization could be a cost that far outweighs the benefits.

6 Foreign Direct Investment

The recent spate of interest amongst policy planners in the links between foreign direct investment (fdi) and growth in a developing economy suggests that we end up this paper with a few observations on the issue. We shall demonstrate that fdi augments the beneficial effect of free trade on employment and growth.

The superscript "*df*" will denote equilibrium magnitudes in the presence of fdi. Clearly, a necessary condition for fdi flows to occur is that the world rate of interest r^{df} fall short of r^f, the rate of interest prevailing under free trade without fdi . Free arbitrage ensures on the other hand that there is instantaneous adjustment of the domestic rate of interest to r^{df}. We shall argue that the variables k_y, k_z, k_g and k too adjust to the new equilibrium instantaneously. Similarly, Moreover, we shall show that it is feasible for domestic and foreign capital to adjust without lag to the equilibrium rate of growth of aggregate capital.

Proposition 10 *Under a flexible rate of interest, the inflow of fdi leads the small open economy to adjust instantaneously to its long term growth rates of aggregate capital, infrastructure, employment and consumption expenditure. Capital, employment and infrastructure grow at a common rate, higher than*

the one prevailing under balanced growth prior to the inflow of foreign capital. Household expenditure grows at a lower rate. The absolute levels of capital, employment and infrastructure are higher for all future. Finally, it is feasible for domestic and foreign capital to adjust instantaneously to the rate of growth of aggregate capital.

Proof: For an equilibrium to occur at r^{df}, equations (6), (7) and (9) are rewritten as

$$\left.\begin{array}{rcl} r^{df} & = & p^f f'_y(k_y^{df}), \\ & = & f'_z(k_z^{df}), \end{array}\right\} \tag{33}$$

$$k^{df} = k_y^{df} + k_z^{df} + k_g^{df}, \tag{34}$$

$$\text{and} \quad k_g^{df} = k_y^{df} \frac{1-\alpha}{\alpha} + k_z^{df} \frac{1-\beta}{\beta}, \tag{35}$$

Since $r^{df} < r^f$, concavity of the f_y and f_z implies, using (33), that $k_i^{df} > k_i^f$, $i = y, z$. Moreover, (35) yields $k_g^{df} > k_g^f$. Finally, (34) establishes that $k^{df} > k^f$. In other words, at the new balanced growth equilibrium, the overall as well as sectoral K/G ratios are higher.

We may assume *wlog* that foreign capital flows are infinitely elastic at r^{df}. Suppose the economy is experiencing steady growth without foreign capital and fdi is allowed from $t = t_0$ onwards. Then, the gap $k_i^{df} - k_i^f, i = y, z, g$ can be filled up by foreign capital without any lag. In this sense, there is instantaneous adjustment to the new equilibrium.

At this equilibrium, the constancy of k^{df} implies that aggregate K and G grow at the same rate. However, using $k_g^{df} > k_g^f$ and (3), we see that $(\dot{G}/G)^{df} > (\dot{G}/G)^f$. Hence, both aggregate private capital (inclusive of foreign capital) and public capital grow faster in the balanced growth state with fdi. Moreover, at t_0, $K_{t_0}^{df}$ is larger than $K_{t_0}^f$, which follows from the

fact that $k^{df} > k^f$ and $G^{df}_{t_0} = G^f_{t_0}$, G being non-traded. Consequently, not only does aggregate private capital grow faster in the presence of fdi, the absolute size of K is higher. This implies in turn that both the aggregate size of employment as well as its rate of growth is higher with free inflow of foreign capital. Thus, all the positive expectations from fdi outlined above are fulfilled. The rate of growth of household expenditure is still governed by (12). Since $r = r^{df} < r^f$, we conclude that $\dot{E}/E$ must fall under the regime of direct foreign investment.

To see how domestic and foreign capital can adjust immediately to the higher rate of growth of aggregate capital, consider the identity

$$\frac{K_{dm}}{K}\frac{\dot{K}_{dm}}{K_{dm}} + \frac{K_{fr}}{K}\frac{\dot{K}_{fr}}{K_{fr}} = \frac{\dot{K}}{K},$$

where K_{dm} and K_{fr} stand for the absolute sizes of domestic and foreign capital and $K_{dm} + K_{fr} = K$. Given the infinite elasticity of foreign capital supply, it is feasible for it to choose the growth rate $\dot{K}_{fr}/K_{fr} = \dot{K}/K \; \forall \, t \geq t_0$. Since $K_{dm}/K + K_{fr}/K = 1$, this would imply that $\dot{K}_{dm}/K_{dm} = \dot{K}/K \; \forall \, t \geq t_0$ also. ■

Comment 6 As Proposition 10 indicates, there is no obvious guarantee that the *level* of E under an fdi regime is higher than the one in its absence. Recalling that r stands for the return to the composite factor K and L, consider again (10) at t_0, which we rewrite as

$$E^f(t_0) + \dot{K}^f_{dm}(t_0) = r^f K^f_{dm}(t_0) + \lambda \bar{w} \; K^f_{dm}(t_0),$$

to emphasize the fact that $K = K_{dm}$ before fdi flows in. Now suppose, for the sake of argument, that fdi occurs creating additional employment, but that the interest rate as well as the domestic investment remain unaltered. Assuming that all profits from foreign capital are repatriated back, the above equation changes to

$$E^f(t_0) + \dot{K}^f_{dm}(t_0) = r^f K^f_{dm}(t_0) + \lambda\bar{w}\ K^f_{dm}(t_0) + \lambda\bar{w}\ K^{df}_{fr}(t_0), \qquad (36)$$

In this situation, it is clear that $E^f(t_0) > E^{df}(t_0)$. However, adjustments being instantaneous, it is not clear how the equation changes along with the drop of r^f to r^{df}. Thus, as in the last section we are not guaranteed whether welfare and the rate of growth of GDP improve simultaneously.

7 Conclusions

The paper has attempted to develop a model for studying the effect of free trade on the sustainable growth rate of a small open economy in the presence of a pure public good that acts as a vital input into the production process. The input in question is accumulable and is best viewed as infrastructure, which is widely recognized as a primary bottleneck for growth in developing economies.

We have studied three alternative regimes. The first involves growth in a closed economy, the second considers an open economy without foreign capital inflows and the third introduces the possibility of fdi inflows. We saw that the introduction of free trade without fdi led to growth effects that depended on the direction of movement of the commodity price ratios as the economy shifted from autarky to free trade. We argued, however, that the price movement that is most likely to occur would improve the growth rate for the economy. The growth path was found to be socially inefficient and, paradoxically enough, the optimal growth path involved a lower growth rate as well as a lower level of employment. This seemed to give rise to a choice issue between welfare and employment.

In the presence of fdi, there is an unconditional rise in the rates of growth, both of the aggregate private and aggregate public capital stocks, *vis-à-vis* the rate corresponding to free trade without fdi. At the same time though the rate of growth of household expenditure declined. Hence, once again, the tension between growth and welfare appeared to crop up.

The apparent paradox disappears however once we recall that we are concerned here with a model of surplus labour. The rise in aggregate utility that welfare improvement entails does not percolate down to the population in the presence of unemployment, since large masses of the work force are deprived of the purchasing power necessary to partake in the enjoyment of improved welfare. On the other hand, a policy of higher employment has the advantage of equipping more people with the means of claiming a share in the GDP. Consequently, a democratically elected government would necessarily consider the policy of employment improvement to be more attractive to the one that raises aggregate utility over time without delivering the goods produced to the multitudes.

The paper therefore suggests that in a small developing economy, a policy of free trade with fdi is unambiguously superior to autarky or trade without fdi. The claimed superiority of the policy, however, is subject to the applicability of the model used in the paper. The crucial analytical concept for the paper was the unlimited availability of unskilled labour at a subsistence wage rate. Under balanced growth of course, this feature will disappear sooner or later. Once it does, labour markets would move in to determine wages and destroy the simple structure of our model. On the other hand, once this happens, the developing economy would for all practical purposes be developed. Thus, our results apply to a developing economy on its way to development.

We shall end the paper with a suggestion for further extension. In our model, commodity Z was a hybrid consumption-cum-capital good. One might wish to make a more rigid distinction between Y and Z by assuming it to be a pure capital good as in Oniki and Uzawa (1965) and Bond *et al* (2003). This exercise wil enrich the conclusions reached for the flexible approach adopted here.

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Appendix: A Decentralization Scheme

The Decentralized Economy is assumed to be made up of five agents, viz. the representative household, three aggregative firms and the government. The first four agents have well-defined objective functions that they maximize at parametrically specified prices. The government provides the initial stock of G and finances its accumulation over time from the revenue generated by the

sale of G-services to the firms. The household has a dynastic structure and maximizes (2) subject to an instantaneous budget constraint. Its budgetary resources fall into two parts. First, it has an income $rK(t)$ from private capital holdings. Second, it receives a lump-sum subsidy equal to $\Delta(t)$ from the government. The amount of subsidy will be specified later. Hence, the household's budget constraint is

$$p^* Y_c(t) + Z_c(t) + \dot{K} = rK(t) + \Delta(t). \tag{A. 1}$$

Firm I produces Y and is assumed to maximize the instantaneous profit function

$$\Pi^y = p^* A_y K_y^{\alpha} G^{1-\alpha} - rK_y - \bar{T}_y, \tag{A. 2}$$

where $\bar{T}_y$ represents a lump-sum tax charged to Firm I. Next, Firm II produces Z (the numéraire good) and maximizes

$$\Pi^z = A_z K_z^{\beta} G^{1-\beta} - rK_z - \bar{T}_z, \tag{A. 3}$$

where $\bar{T}_z$ is a lump-sum tax paid by the firm. Finally, Firm III produces increments in infrastructure stocks and maximizes

$$\Pi^g = \mu A_g K_g^{\gamma} G^{1-\gamma} - rK_g - \bar{T}_g, \tag{A. 4}$$

where μ is the price of the firm's product and $\bar{T}_g$ is a lump-sum tax.

The government accumulates G by purchasing $\dot{G}$ from Firm III.[4] The net revenue accruing to the government on the infrastructural account is the difference between its tax revenue from the firms and its investment cost for creating additional infrastructure:

[4]Without loss of generality, Firm III could be under government operation, in which case the relevant prices are used for book-keeping.

$$D = \bar{T}_y + \bar{T}_z + \bar{T}_g - \mu A_z K_g^{\gamma} G^{1-\gamma}. \tag{A. 5}$$

The following proposition derives the values of r, μ, $\bar{T}_i(t)$, $i = y, z, g$ and $\Delta(t)$ that will support the socially optimal path at each t.

Proposition A *There exist time dependent values of the household subsidy and taxes on firms and time invariant values of the rate of interest and the price of output of Firm III which induce the decentralized economy to choose the socially optimal growth path.*

Proof: Dropping t for convenience, let K^*, G^* be the optimal path under balanced growth. Similarly, denote the optimal value of ϕ_y by ϕ_y^*. Choose

$$r = r^* = p^* \alpha A_y (\phi_y^* K^*)^{\alpha} (G^*)^{1-\alpha}. \tag{A. 6}$$

Since K and G grow at the same rate and ϕ_y is a constant, r^* is a constant for all t. Similarly, let

$$\mu = \frac{\xi(t)}{\eta(t)} \ \forall \ t$$

and

$$\frac{1}{G} \ (T_y + T_z + T_g) = ((2 - \delta) g^* + \rho) \ \mu,$$

where g^* is the socially optimal balanced growth rate. From (5), μ is a constant. Hence, $(T_y + T_z + T_g)/G$ is a constant also.

The household maximises the Hamiltonian

$$\mathcal{H}^h = ln \ (Y_c^{\alpha} Z_c^{1-\alpha}) + \eta_h (r^* K + \Delta - p^* Y_c - Z_c),$$

where η_h is the relevant costate variable. The necessary conditions for the optimum solution to the problem are

$$\frac{\delta Y_c^{\delta-1}}{Y_c^{\delta} Z_c^{1-\delta}} = p^* \eta_h, \tag{A. 7}$$

$$\frac{(1-\delta) Z_c^{-\delta}}{Y_c^{\delta} Z_c^{1-\delta}} = \eta_h, \tag{A. 8}$$

$$\tag{A. 9}$$

$$\dot{\eta}_h = -\eta_h r^* + \eta_h \rho \tag{A. 10}$$

$$= \alpha p^* A_y p^* (\phi_y^* K^*)^{\alpha-1} (G^*)^{1-\alpha} + \eta_h \rho, \tag{A. 11}$$

The last leg of (A. 11) follows from (A. 6).

The instantaneous profit maximization problems of the three firms lead to

$$\begin{aligned} p^* \alpha A_y K_y^{\alpha-1} G^{1-\alpha} &= \beta A_z K_z^{\beta-1} G^{1-\beta} \\ &= \mu\gamma A_g K_g^{\gamma-1} G^{1-\gamma} \\ &= r^*. \end{aligned} \tag{A. 12}$$

Further, choose

$$\frac{T_y}{G} = p^*(1-\alpha) A_y K_y^{\alpha} G^{-\alpha} \tag{A. 13}$$

$$\frac{T_z}{G} = (1-\beta) A_z K_z^{\beta} G^{-\beta} \tag{A. 14}$$

$$\frac{T_g}{G} = \mu(1-\gamma) A_g K_g^{\gamma} G^{-\gamma} \tag{A. 15}$$

Adding the last three equations, using the choice of μ and $(T_y + T_z + T_g)/G$ and appealing to (5), we get

$$\begin{aligned}\dot{\xi} &= -\eta(1-\alpha)p^* A_y (K_y)^\alpha G^{-\alpha} - \eta(1-\beta)A_z (K_z)^\beta G^{-\beta} \\ &\quad -\xi(1-\gamma)A_g (K_g)^\gamma G^{-\gamma} + \xi\rho. \qquad \text{(A. 16)}\end{aligned}$$

From the socially optimal solution, we see that $K_y = \phi_y^* K^*$, $K_z = \phi_z^* K^*$, $K_g = (1 - \phi_y^* - \phi_z^*)K^*$ satisfy (A. 16) and (A. 12).

It is easy to see that the household chooses $\dot{E}/E = r^* - \rho$, which is the same as the socially optimum rate of growth. However, this does not fix the level of E. Towards this end, choose $\Delta(t) = D(t)\ \forall t$, so that the government's budget is balanced. Further,

$$\begin{aligned}\Delta &= \bar{T}_y + \bar{T}_z + \bar{T}_g - \mu A_z K_\gamma G^{1-\gamma} \\ &= \mu((2-\delta)g^* + \rho - A_z K_\gamma G^{1-\gamma} \\ &= \mu((1-\delta)g^* + \rho) \\ &> 0.\end{aligned}$$

Next, in the household's problem,

$$p^* Y_c + Z_C + \dot{K} = r^* K^* + \Delta.$$

The RHS of the last equation reduces to

$r^*(K_y * + K_z^* + K_g^*) + (1-\alpha)p^* Y^* + (1-\beta)Z^* + (1-\gamma)\dot{G}^*$,

using (A. 13), (A. 14) and (A. 15)

$= \alpha p^* Y^* + \beta Z^* + \gamma \dot{G}^* + (1-\alpha)p^* Y^* + (1-\beta)Z^* + (1-\gamma)\dot{G}^*$, using (A. 12)

$= p^*Y^* + Z^*$.

Thus, the household's choice variables satisfy the trade balance equation and can be chosen to be identically the same as the ones for the socially optimal solution.

Finally, for the chosen values of G^* and K_i^*'s, (A. 13), (A. 14) and (A. 15) determine the levels of the T_i's. ■