# Referrals in Search Markets* 

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#### Abstract

The paper compares the equilibrium outcomes in search markets with and without referrals. Although consumers would benefit from valuable advice, it is not clear whether firms would unilaterally provide information about competing offers since such information could encourage a consumer to purchase the product elsewhere. In a model of a spatially differentiated product and sequential consumer search, we show that valuable referrals can arise as a part of equilibrium. A firm gives referrals to consumers whose ideal product is sufficiently far from the firm's offering. The effect of referrals on the equilibrium prices is examined, and it is found that prices are higher in markets with referrals. Although consumers can be made worse-off by the existence of referrals, referrals lead to a Pareto improvement for sufficiently high search costs.


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[^0]
## 1 Introduction

There is a number of industries where consumers would have to incur substantial costs to learn the existing product characteristics and available products. The examples include hightech products such as a digital camera or a specialized product such as picture framing, as well as professional services in areas of law, accounting, real estate and healthcare. These products and services are purchased infrequently or their characteristics change quickly and are difficult to assess. Referral systems have developed that capture the economies of scope and scale in information gathering. Whether referrals are made by competing firms or by intermediaries, they are usually subject to moral hazard problems. A referring firm may have an incentives problem when it faces a trade-off between serving a consumer or referring the consumer to another seller, and referrals can be influenced by payments from the sellers of the product.

Referral institutions, rules and regulations address the problems of suboptimal referral intensity and referral biases. Although the pros and cons of regulating referral practices are discussed by professional associations and regulatory authorities, there is not much economics theory on the topic. ${ }^{1}$ We examine referrals among horizontally differentiated sellers (horizontal referrals) ${ }^{2}$ in a model of sequential consumer search. The focus is on the effects referrals have on equilibrium prices, profits, consumer benefits, and overall welfare.

A number of regulations regarding fee-spliting in referrals have been established to reduce the potential opportunistic behavior of sellers. For example, the federal anti-kickback law's main purpose is to protect patients and the federal health care programs from "fraud and

[^1]abuse." ${ }^{3}$ Professional associations in law, accounting, and real estate have established codes of honor that regulate the referral activity in order to guard their reputation and ensure there is no conflict of interest.

The American Bar Association (ABA) forbids payments to non-lawyers and prescribes the division of fees by lawyers in proportion to the actual services performed or responsibility assumed. In particular, the division of fees for "pure" referrals is not to be condoned. ${ }^{4}$ Referral fees between lawyers may be prohibited under state codes of professional responsibility unless certain criteria are met, which may include a provision that the total price is not higher for consumers who follow a referral. ${ }^{5}$ Under the Real Estate Settlement Procedures Act, real estate agents are currently allowed to pay each other referral fees. ${ }^{6}$ The Code of Professional Conduct of the American Institute of Certified Public Accountants (AICPA) has undergone significant changes since 1988. FTC studied the effect of the AICPA's conduct rules on competition among public accountants and found several rules of conduct in violation of Section 5 of the FTC Act. In particular, the FTC argued that a prohibition on referral fees can have anti-competitive effects, and recommended changes in the code. An AICPA member may now pay other CPAs, lawyers, and business professionals to refer potential clients to the CPA, provided it is disclosed.

To better understand the economics of referrals in markets with imperfectly informed consumers, we construct a model of a search market, in which firms can refer consumers to other sellers. In the basic model, referral fees are not allowed or are not enforceable. Firms inside the industry know competitors' product offerings as a part of doing business. It is not clear, however, whether competing firms would ever inform consumers about products

[^2]offered by other stores. Directing consumers to products that match their tastes can result in a loss of business. In the model, firms and consumers simultaneously choose their strategies. Firms set prices and referral sets, i.e. firms decide which consumers are served and which are referred to other sellers. A consumer decides whether to initiate search, given her value for the product, and chooses the first firm at random. Upon the visit to the firm, the consumer learns the location of the firm. Based on this information and on a referral given by the firm (if any), the consumer decides whether to leave the market without a purchase, buy the product, continue random search, or follow the referral.

In the absence of reputation effects, referral fees, and bilateral agreements between firms, referral services do not increase a firm's profit. A firm refers a consumer only if the firm is sure that the consumer would otherwise leave the firm to engage in random search. The firm is indifferent where to refer a consumer, and it might as well provide the best (truthful) referral. It is intuitive that in an equilibrium a firm gives referrals to buyers whose ideal products are sufficiently far from the firm's product. Perhaps surprisingly, the norm of referring consumers to competitors who match consumers' tastes most tends to increase prices and is preferred by sellers. Although referrals provide consumers with valuable information that saves consumer search costs and improves the product match, consumers may be worse-off under referrals. This happens when the benefits from referrals are outweighed by the loss to consumers from a price increase. We show that for sufficiently low search costs, consumers prefer markets without referrals. At the same time, referrals lead to a Pareto improvement in markets with relatively high search costs.

The rest of the paper is organized as follows. Section 2 presents a model of price competition between firms selling a differentiated product under imperfect consumer information. We consider search markets with and without referrals and compare the referral and randomsearch equilibria that arise in such markets. In Section 3, we extend the basic model by examining markets where referral fees are capped at an exogenous level and where they are endogenously chosen by sellers. We also briefly discuss the existing literature on referrals. Section 4 offers concluding comments.

## 2 The Model

We model competition between firms producing a spatially differentiated product. There are $n$ firms located symmetrically on a circle of unit circumference, which produce the product at a zero marginal cost. A unit mass of consumers with unit demand are characterized by their valuation for the product and their preferences over the product brands. Consumers' ideal positions are distributed uniformly over the unit circle. Independently of their spatial preferences, each consumer has a value $v \sim U[0,1]$ for the product ideal to her (the product that is a perfect match with her taste).

Prior to visiting stores, consumers are aware of their value for the product and their ideal positions. Stores know product characteristics of all products available in the market (positions of all stores). When a consumer visits a store, she realizes the store's position and price, while the store realizes her ideal product (taste). Suppose a consumer whose value is $v$ learns that the store's brand is located at distance $x$ from her ideal position. Let the consumer's utility for the store's brand (gross of price and search costs) be $u(x, v)=v-t x$; $t>0$.

Consumer search is sequential, with a marginal cost of search $s>0$ that is common across consumers. Search is with perfect recall and with replacement. When indifferent between searching or not, consumers search. Firms can refer consumers who visit them to other stores. Since search costs come mainly in the form of learning product characteristics, referrals save consumers their search costs. Indeed, we assume that following a referral is costless.

The strategy of a firm is a (nondiscriminatory) price and a referral. The referral can be conditioned on the observed consumer's ideal position. Consumers choose whether to start a random search given her value $v$; after the first search they decide whether to continue random search, follow a referral if it is given, or abandon search and take the best examined item or leave the market without purchasing the product.

The timing of the game is shown on Figure 1.
[Figure 1 HERE]

### 2.1 Random Search

Let us derive the optimal stopping rule for a consumer who is engaged in the random sequential search. In a symmetric price equilibrium, the optimal stopping rule for an actively searching consumer does not depend on the price, and we can first assume zero prices for commodities. If a consumer who has reservation utility $w$ engages in a search once, then her expected utility is

$$
\begin{align*}
& 2 \int_{0}^{\frac{v-w}{t}}(v-t x) d x+2 \int_{\frac{v-w}{t}}^{\frac{1}{2}} w d x-s  \tag{1}\\
= & \frac{(v-w)^{2}}{t}+w-s .
\end{align*}
$$

If she does not search, she gets $w$. Thus, one round of random search pays off in expectation iff $(v-w)^{2} \geq t s$. Thus, critical $w$ is a function of $v, t$, and $s: w^{*}(v)=v-\sqrt{t s}$. This in turn means $v-t x^{*}=v-\sqrt{t s}$, and the critical (reservation) distance is $x^{*}=\sqrt{\frac{s}{t}}$. The optimal stopping rule for the consumer is to stop searching if and only if she is assigned to a position that is less than distance $x^{*}=\sqrt{\frac{s}{t}}$ from her ideal position. The probability to be assigned to a position in this range is $2 \sqrt{\frac{s}{t}}$. When $\sqrt{\frac{s}{t}}>\frac{1}{2}$, she always stops searching, regardless of the assigned position.

Now, we need to see what the equilibrium price is under random search. It is useful to see Wolinsky (1986) for the techniques. We will support a symmetric equilibrium price $p$. The probability of stopping a search is $2 \sqrt{\frac{s}{t}}$ in any other store (they charge the same price $p)$. If store $i$ sets a different price, $p_{i} \neq p$, then it can affect consumers' search behavior a bit. If $x$ satisfies the following, a consumer stops searching at store $i$ :

$$
\begin{equation*}
v-p_{i}-t x \geq v-p-\sqrt{t s} \tag{2}
\end{equation*}
$$

or

$$
\begin{equation*}
\frac{p-p_{i}}{t}+\sqrt{\frac{s}{t}} \geq x . \tag{2'}
\end{equation*}
$$

As a result, demand function per consumer engaged in search activities is

$$
\begin{equation*}
D_{i}\left(p_{i}, p\right)=\lim _{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^{n}\left(1-2 \sqrt{\frac{s}{t}}\right)^{k-1}\left(2 \frac{p-p_{i}}{t}+2 \sqrt{\frac{s}{t}}\right) . \tag{3}
\end{equation*}
$$

Since we assume that consumers engage random search, store $i$ could be visited as the $k$ th store, where $k$ can be any positive integer. By rewriting, we obtain,

$$
D_{i}\left(p_{i}, p\right)=\lim _{n \rightarrow \infty} \frac{1}{n}\left(\frac{1-\left(1-2 \sqrt{\frac{s}{t}}\right)^{n}}{1-1+2 \sqrt{\frac{s}{t}}}\right)\left(2 \frac{p-p_{i}}{t}+2 \sqrt{\frac{s}{t}}\right) .
$$

As the number of firms grows, the demand goes to zero due to $\frac{1}{n}$, if population is kept constant. In order to avoid this problem, we will replicate population of economy $n$ times as the number of stores $n$ grows in order to keep population per number of stores constant. For zero marginal costs, the profit function per searcher is $\tilde{\pi}_{i}\left(p_{i}, p\right)=n p_{i} D_{i}\left(p_{i}, p\right)$. In the limit as $n \rightarrow \infty$,

$$
\begin{equation*}
\tilde{\pi}_{i}\left(p_{i}, p\right)=\frac{p_{i}}{\sqrt{\frac{s}{t}}}\left(\frac{p-p_{i}}{t}+\sqrt{\frac{s}{t}}\right) . \tag{4}
\end{equation*}
$$

Firm $i$ 's profit is equal to the measure of searchers times the profit function per searcher. A firm's price cannot change the measure of consumers who visit the firm. Therefore, firm $i$ chooses $p_{i}$ to maximize its profit per searcher, $\tilde{\pi}_{i}\left(p_{i}, p\right)$. The f.o.c. is

$$
\begin{equation*}
\left(\sqrt{\frac{s}{t}}\right) \frac{\partial \tilde{\pi}_{i}\left(p_{i}, p\right)}{\partial p_{i}}=\frac{p-p_{i}}{t}+\sqrt{\frac{s}{t}}-\frac{p_{i}}{t}=0 \tag{5}
\end{equation*}
$$

Thus, the symmetric equilibrium price $\left(p_{i}=p^{*}\right)$ is

$$
\begin{equation*}
p^{*}=\sqrt{t s} \tag{6}
\end{equation*}
$$

For which parameter values does the random-search equilibrium exist? Since $w^{*}(v)=$ $v-\sqrt{t s}$, in order for some consumers to have a nonnegative equilibrium expected utility and engage in search, price $p^{*}$ has to be no higher than $w^{*}(v)$ for some $v \in[0,1]$, namely $v=1$. Thus, $p^{*}=\sqrt{t s} \leq 1-\sqrt{t s}$, which can be writen as $\sqrt{t s} \leq \frac{1}{2}$ or $s \leq \frac{1}{4 t}$. For some consumers to search beyond the first firm in the equilibrium, we also need $x^{*}=\sqrt{\frac{s}{t}} \leq \frac{1}{2}$, or $s \leq \frac{t}{4}$. (Appendix shows that there does not exist a symmetric pure-strategy equilibrium in which consumers search once-for-all.) Therefore, condition $s \leq \min \left\{\frac{1}{4 t}, \frac{t}{4}\right\}$ on search costs ensures that the random-search equilibrium exists.

Consumers whose willingnesses-to-pay $v$ is greater than or equal to a critical value $\bar{v}=$ $2 \sqrt{t s}$ engage in search. Indeed, if $v \geq \bar{v}$ then $v-p^{*}-\sqrt{t s} \geq 0$, and such a consumer would follow the optimal stopping rule, stopping whenever the distance from the ideal position is
less than $x^{*}$. This implies that a fraction $1-\bar{v}=1-2 \sqrt{t s}$ of consumers engage in search, and each firm's profit can be written as

$$
\begin{align*}
\pi^{*} & =(1-2 \sqrt{t s}) \tilde{\pi}_{i}\left(p^{*}, p^{*}\right)  \tag{7}\\
& =(1-2 \sqrt{t s}) p^{*} \\
& =(1-2 \sqrt{t s}) \sqrt{t s}
\end{align*}
$$

The properties of the random search equilibrium and comparative statics results are stated in Proposition 1.

Proposition 1. When $s \leq \min \left\{\frac{1}{4 t}, \frac{t}{4}\right\}$, there exists a unique symmetric random-search equilibrium with prices $p^{*}=\sqrt{t s}$ and profits $\pi^{*}=(1-2 \sqrt{t s}) \sqrt{t s}$; the critical value of willingness-to-pay for market participation is $\bar{v}=2 \sqrt{t s}$. The equilibrium price increases and consumers' market participation decreases in search cost, s, and product heterogeneity, $t$. Profits can increase or decrease in $s$ and $t$.

Proof. The comparative statics results are as follows: $\partial p^{*} / \partial s>0, \partial p^{*} / \partial t>0, \partial \pi^{*} / \partial t=$ $\frac{1}{2} \sqrt{s / t}-2 s=\frac{1}{2} \sqrt{s / t}(1-4 \sqrt{t s})$, and $\partial \pi^{*} / \partial s=\frac{1}{2} \sqrt{t / s}(1-4 \sqrt{t s})$.
[Figure 2 HERE]

Consumer decisions to engage in search, buy at a firm located at a distance $x$, or engage in sequential search are illustrated in Figure 2. Only consumers whose value for the product is higher than $\bar{v}=2 \sqrt{t s}$ engage in search. Consumers visiting a firm less than $x^{*}=\sqrt{\frac{s}{t}}$ away from her ideal position, buy the product, while others continue to search.

### 2.2 Search With Referrals

The strategy of a firm in search markets with referrals is a uniform price and a referral. We assume that a store person can observe a customer's ideal position $x$ from a conversation with her, but not her willingness-to-pay $v$. We look for a referral equilibrium, in which firms choose symmetric price and referral strategies $\left(p_{R}^{*}, x_{R}\right)$. The symmetric referral rule states that if the distance between a customer's position and a firm is more than $x_{R}$, the firm gives referrals.

Let us start the analysis with a second round search. Suppose that a consumer has been assigned to store $i$ located at distance $x$ from her ideal position. Thus, her utility (gross of search cost) from purchasing commodity there is $v-p-t x$, where $p$ is the symmetric price charged by stores. In the symmetric referral equilibrium, a consumer can get a referral with probability $1-2 x_{R}$. If she gets a referral and follows the suggestion, her utility will be $v-p$. A consumer who visited a store with distance $x<x_{R}$ from her ideal position cannot get a referral. Then, her choice is one of the following three: (i) no purchase and go home, receiving $(-s)$, (ii) purchase and go home, receiving $(v-p-t x-s)$, and (iii) engage in sequential search. If she engages in sequential search, she visits a random firm (firm $j \neq i$ ), which is located at distance $x^{\prime}$ from the consumer and changes price $p$. At firm $j$, the consumer gets and follows a referral with probability $\left(1-2 x_{R}\right)$, recalls firm $i$ 's offer, or buys firm $j$ 's product. Therefore, the expected payoff from engaging in one additional search is

$$
\begin{align*}
& \Delta E U\left(x ; x_{R}\right)  \tag{8}\\
= & \left(1-2 x_{R}\right)(v-p)+2\left(x_{R}-x\right)(v-p-t x)+2 \int_{0}^{x}\left(v-p-t x^{\prime}\right) d x^{\prime}-s-(v-p-t x) \\
= & \left(1-2 x_{R}\right) t x+t x^{2}-s
\end{align*}
$$

It is easy to see that for any $x_{R} \in\left[0, \frac{1}{2}\right]$ and any $x \in\left[0, x_{R}\right]$, we have $\partial \Delta E U\left(x ; x_{R}\right) / \partial x=$ $t\left(1-2 x_{R}+2 x\right) \geq 0$. This means that as long as $\Delta E U\left(x_{R} ; x_{R}\right) \leq 0$, every consumer who has $x<x_{R}$ would not engage in an additional search.

Now, let $\Delta E U\left(x_{R} ; x_{R}\right)=0$ : that is, a consumer who visited a store $x_{R}$ apart from her ideal position is indifferent between searching and not searching given that all other stores are choosing referral rule $x_{R}$ ):

$$
\begin{align*}
\Delta E U\left(x_{R} ; x_{R}\right) & =\left(1-2 x_{R}\right) t x_{R}+t x_{R}^{2}-s  \tag{9}\\
& =t x_{R}-t x_{R}^{2}-s=0
\end{align*}
$$

or

$$
\begin{equation*}
x_{R}=\frac{1}{2}-\frac{1}{2} \sqrt{1-4(s / t)} \tag{10}
\end{equation*}
$$

for $t \geq 4 s$. This value $x_{R}$ describes the symmetric equilibrium referral rule. If a consumer gets a referral and $v \geq p$, then she follows the referral. If she does not get a referral, it means
that sequential search is not beneficial, and as a result, she either purchases or goes home without purchase; she purchases if and only if $v \geq p+t x$. In contrast, if $t<4 s$ then there will be no referral equilibrium (See the Appendix).

In order to calculate the demand function of store $i$, we first need to know which consumers would participate in the market. Since we are analyzing a symmetric equilibrium, we assume that every store is charging price $p$ and its referral rule is described by $x_{R}=\frac{1}{2}-\frac{1}{2} \sqrt{1-4(s / t)}$. Next, we show that a consumer who engages in an initial search activity is willing to purchase any commodities within the distance $x_{R}$ from her ideal position.

Suppose that her willingness-to-pay $v$ satisfies $v-t x_{R}-p \geq 0$, then she has the following expected utility from the initial search (given the optimal stopping rule characterized by $x_{R}$ ):

$$
\begin{align*}
E U_{R}(v, p) & =2 \int_{0}^{x_{R}}(v-t x-p) d x+\left(1-2 x_{R}\right)(v-p)-s  \tag{11}\\
& =(v-p)-t x_{R}^{2}-s
\end{align*}
$$

Recall that a consumer who is assigned to a store that is $x_{R}$ apart from her ideal position is indifferent between engaging or not engaging in an additional search: $t x_{R}-t x_{R}^{2}=s$. Therefore,

$$
\begin{equation*}
E U_{R}(v, p)=v-t x_{R}-p \tag{11'}
\end{equation*}
$$

That is, a consumer whose willingness-to-pay $v$ satisfies $v-t x_{R}-p \geq 0$ obtains a nonnegative expected utility, $E U_{R}(v, p)=v-t x_{R}-p \geq 0$, too. On the other hand, if $v-t x_{R}-p<0$, it is easy to see $E U_{R}(v, p)<0$ for any stopping rule. Thus, given that $p$ is a prevailing symmetric price, a consumer engages in the initial search if and only if her $v$ is not less than $\bar{v}_{R}(p)=t x_{R}+p$.

Note that this observation says that if consumers expect a symmetric equilibrium price $p$, then those who have willingnesses-to-pay below $\bar{v}_{R}(p)$ would never purchase commodities (since they do not enter the market). This fact generates a kinked demand curve in the following analysis. Note also that all consumers engaged in search and assigned to a store within $x_{R}$ distance, will prefer to buy the store's product rather than leave the market. Consumers assigned to a store further than $x_{R}$ away, get a referral and follow it.

In the equilibrium, $p_{i}=p$, and the total demand that a store faces is $\left(\frac{1}{n}\right)$ th of the total
market demand, which is

$$
\begin{equation*}
D(p, p)=2 \int_{0}^{x_{R}}(1-p-t x) d x+\left(1-2 x_{R}\right)(1-p) \tag{12}
\end{equation*}
$$

The first term shows the number of consumers who happen to be assigned to stores within distance $x_{R}$ from her ideal point, and the second term shows the number of consumers who happen to be assigned to stores outside of $x_{R}$ distance from her ideal point and get referrals.

Next, we calculate the demand function of store $i$ assuming that other firms are choosing a symmetric price $p$. As the first step, we will find the consumers' optimal stopping rule when she observe a price $p_{i}$ at store $i$ that is different from $p$. We still assume that every firm including firm $i$ gives truthful referrals to consumers if they think that they have no chance to sell. First, for each price $p_{i}$ (and $p$ ) we calculate the threshold distance $x_{R}\left(p_{i}, p\right)$ for firm $i$ to decide if it gives a referral or not. Firm $i$ gives referrals when it is sure that it cannot sell its product to a customer. Suppose that other firms charge price $p$ and make referrals to customers outside of $x_{R}$. Let $x$ be the distance between a consumer's ideal point and the location of firm $i$. We will next look for the threshold distance for firm $i, x=x_{R}\left(p_{i}, p\right)$, such that a consumer with distance $x_{R}\left(p_{i}, p\right)$ from the ideal point is indifferent between purchasing $i$ 's product and searching once more.

Suppose that an additional search after visiting firm $i$ matches the customer with a product of firm $j \neq i$ with distance $\tilde{x}$ from her ideal point. There are two cases: (i) $\tilde{x} \geq x_{R}$ and she gets a referral, (ii) $\tilde{x}<x_{R}$ and she does not get firm $j$ 's referral (in this case, she will not do further search). A consumer prefers to buy firm $i$ 's rather than firm $j$ 's product if and only if $v-p_{i}-t x<v-p-t \tilde{x}$ (or $\tilde{x}>x+\frac{p_{i}-p}{t}$ ). One additional round of search results in a consumer utility no less than $v-p-t x_{R}$.

First assume that the consumer utility from store $i$ 's product is at least as high as that under the worst realization of an additional search: $v-p_{i}-t x \geq v-p-t x_{R}$ (or $x+\frac{p_{i}-p}{t} \leq x_{R}$, or $\left.p_{i} \leq p+t x_{R}-t x\right)$. In this case, the consumer may recall firm $i$ 's product under some realizations of $\tilde{x}$. If $\tilde{x}>x_{R}$, she gets firm $j$ 's referral and obtains $(v-p)$. If $x+\frac{p_{i}-p}{t} \leq \tilde{x} \leq x_{R}$, she recalls firm $i$ 's product. Finally, if $\tilde{x}<x+\frac{p_{i}-p}{t}$, she buys at firm $j$. Therefore, a customer's gain from engaging in an additional search when firm $i$ charges $p_{i}$ and other firms charge $p$ is as follows

$$
\begin{aligned}
& \left(1-2 x_{R}\right)(v-p)+2\left(x_{R}-x-\frac{p_{i}-p}{t}\right)\left(v-p_{i}-t x\right)+2 \int_{0}^{x+\frac{p_{i}-p}{t}}(v-p-t \tilde{x}) d(\mathfrak{c l a} 3) \\
& -s-\left(v-p_{i}-t x\right) \\
= & t\left[x+\left(\frac{1}{2} \sqrt{1-4(s / t)}+\frac{p_{i}-p}{t}\right)\right]^{2}-\frac{t}{4} \\
= & t\left[x-\frac{1}{2}+\frac{1}{2} \sqrt{1-4(s / t)}+\frac{p_{i}-p}{t}\right]\left[x+\frac{1}{2}+\frac{1}{2} \sqrt{1-4(s / t)}+\frac{p_{i}-p}{t}\right]
\end{aligned}
$$

By setting the above gain to be zero, we obtain

$$
\begin{equation*}
x_{R}\left(p_{i}, p\right)=x=\frac{1}{2}-\frac{1}{2} \sqrt{1-4(s / t)}-\frac{p_{i}-p}{t} . \tag{14}
\end{equation*}
$$

Note that $x_{R}\left(p_{i}, p\right)=x_{R}-\frac{p_{i}-p}{t}$, and the condition we assumed is satisfied for $x=x_{R}\left(p_{i}, p\right)$. Second, suppose that $v-p_{i}-t x<v-p-t x_{R}$ (or $x+\frac{p_{i}-p}{t}>x_{R}$, or $\left.p_{i}>p+t x_{R}-t x\right)$. In that case, the consumer leaves firm $i$ to search further.

Now, we can present firm $i$ 's demand function. First, recall that only consumers whose willingnesses-to-pay are not less than $\bar{v}_{R}(p)=p+t x_{R}$ participate in the market. This implies that the maximum demand for each $x$ is $1-p-t x_{R}$. Consider the following three cases: (i) $p_{i}<p$, (ii) $p_{i} \geq p$ and $p_{i} \leq p+t x_{R}$, and (iii) $p_{i}>p+t x_{R}$. In case (i), all consumers who visit store $i$ buy there since $i$ 's offer is better than engaging in random search.

In case (ii), consumers located at $x<x_{R}\left(p_{i}, p\right)$ buy from firm $i$. With the above threshold value $x_{R}\left(p_{i}, p\right)$, we have demand for commodity $i$

$$
\begin{align*}
D_{i}\left(p_{i}, p\right) & =\left(1-p-t x_{R}\right)\left(2 x_{R}\left(p_{i}, p\right) d x+\left(1-2 x_{R}\right)\right)  \tag{15}\\
& =\left(1-p-t x_{R}\right)\left(1-\frac{2\left(p_{i}-p\right)}{t}\right)
\end{align*}
$$

where $x_{R}=\frac{1}{2}-\frac{1}{2} \sqrt{1-4(s / t)}$ if $p_{i} \in\left[p, p+t x_{R}\right]$. The first term captures demand from consumers who are assigned to store $i$ in their initial search, while the latter term captures demand by referrals from other stores. In case (iii), we have $x_{R}\left(p_{i}, p\right)<0$, and no consumer who is assigned to store $i$ purchases commodity. Moreover, $v-p_{i}<v-p-t x_{R}$ holds for any $v$, and even consumers who got referrals do not purchase at store $i$. Thus, demand for commodity $i$ in this case is zero.

Given this, firm i's profit function is,

$$
\begin{equation*}
\pi_{i}\left(p_{i}, p\right)=\left(1-p-t x_{R}\right) p_{i}\left(1-\frac{2\left(p_{i}-p\right)}{t}\right) \tag{16}
\end{equation*}
$$

if $p_{i} \in\left[p, p+t x_{R}\right], \pi_{i}\left(p_{i}, p\right)=\left(1-p-t x_{R}\right)$ if $p_{i}<p$, and $\pi_{i}\left(p_{i}, p\right)=0$ if $p_{i}>p+t x_{R}$. In Proposition 2 we show that there is a unique symmetric equilibrium in this model.

Proposition 2. When $s \leq \min \left\{\frac{1}{4 t}(2-t)(3 t-2), \frac{t}{4}\right\}$, there exists a unique symmetric referral equilibrium with price $p_{R}^{*}=\frac{t}{2}$; profits $\pi_{R}^{*}=p_{R}^{*}\left(1-p_{R}^{*}-t x_{R}\right)=\frac{t}{2}\left(1-t+\frac{t}{2} \sqrt{1-4(s / t)}\right)$; the critical value of willingness-to-pay for market participation is $\bar{v}_{R}=p_{R}^{*}+t x_{R}$ where $x_{R}=\frac{1}{2}-\frac{1}{2} \sqrt{1-4(s / t)}$. Referral intensity $r=1-2 x_{R}=\sqrt{1-4(s / t)}$ decreases with consumer search costs and increases with product heterogeneity. The equilibrium price is perfectly insensitive to the value of search costs, while it increases as product heterogeneity increases.

Proof. We first show that there is a symmetric equilibrium when $\bar{v}_{R} \leq 1$. Since $\pi_{i}$ is concave in $p_{i}$, the first order condition evaluated at $p_{i}=p$ characterizes symmetric equilibrium. The f.o.c. is $\partial \pi_{i}\left(p_{i}, p\right) / \partial p_{i}=\left(1-p-t x_{R}\right)\left(1-\left(4 p_{i}-2 p\right) / t\right)=0$. Thus, there is a unique symmetric equilibrium price $p_{R}^{*}=\frac{t}{2}$. The value of $\bar{v}_{R}$ can be found by substituting $p_{R}^{*}$ into $v_{R}(p)=p+t x_{R}$. Finally, $\bar{v}_{R} \leq 1$ if and only if $\frac{t}{2}+\frac{t}{2}-\frac{t}{2} \sqrt{1-4(s / t)} \leq 1$. This is equivalent to $t-1 \leq \frac{t}{2} \sqrt{1-4(s / t)}$, or $s \leq \frac{1}{4 t}(2-t)(3 t-2)$. For example, condition $t \leq 1$ is sufficient, provided $s \leq t / 4$.

Figure 4 illustrates how consumer decisions depend on realizations for $v$ and $x$.

## [Figure 4 HERE]

Why does not search cost $s$ matter in determination of price $p_{R}^{*}$ in this case? It is because consumers who are assigned to store $i$ can increase demand only through an increase in the retention rate $2 x_{R}\left(p_{i}, p\right)=2 x_{R}-2\left(p_{i}-p\right) / t$ of consumers who are assigned to store $i$ in their initial search. However, the retention rate increase is not affected by search cost $s$ since sequential search never takes place in the equilibrium. ${ }^{7}$ Thus, the equilibrium price is only determined by heterogeneity parameter $t$ in search markets with referrals.

[^3]
### 2.3 Comparison of Random-Search and Referral Equilibria

Proposition 3. If $t \leq 1$, then both random search and referral equilibria exist under the same parameter restrictions. However, if $1 \leq t \leq \frac{5}{3}$ then a referral equilibrium exists for larger set of parameter values for $s$, while if $\frac{5}{3} \leq t$, then a random search equilibrium exists for larger set of parameter values for $s$ than a referral equilibrium (in particular, a referral equilibrium cannot exist for $t \geq 2$ ).

Proof. For random search, we need $\bar{v} \leq 1$ in order to assure the existence of random-search equilibrium. This is equivalent to (i) $t s \leq \frac{1}{4}$. In contrast, referral equilibrium exists if we have (ii) $t-4 s \geq 0$ and (iii) $t-1 \leq \frac{t}{2} \sqrt{1-4(s / t)}$. First note that (iii) is implied by (ii) when $t \leq 1$. Thus, condition (iii) can bind only when $t \geq 1$. Let us solve these conditions for $s$. The condition for random-search equilibrium is $s \leq \frac{1}{4 t}=g_{1}(t)$. The former condition for referral equilibrium can be written as $s \leq \frac{1}{4} t=g_{2}(t)$, and the latter with $t \geq 1$ is equivalent to $t^{2}-2 t+1 \leq \frac{1}{4}\left(t^{2}-4 s t\right)$, or $s \leq-\frac{3}{4} t+2-\frac{1}{t}=g_{3}(t)$. Note that if $t=1$, $g_{1}(1)=g_{2}(1)=g_{3}(1)=\frac{1}{4}$ holds. It is easy to see that $g_{1}(t)>g_{2}(t)$ for all $t<1$, and $g_{2}(t)>g_{1}(t)$ for all $t>1$. Moreover, $g_{3}(t)>g_{1}(t)$ if and only if $1<t<\frac{5}{3}$. This completes the proof.

Figure 5 illustrates Propostion $3 .{ }^{8}$
[Figure 5 HERE]

Proposition 4. Whenever both random search and referral equilibria exist, we have $p_{R}^{*} \geq p^{*}$ and $x_{R}^{*} \leq x^{*}$. Consumers are better off (thus, market participation is larger or $\bar{v} \geq \bar{v}_{R}$ ) in referral equilibrium if and only if $s \geq 0.09 t$, i.e. search cost is relatively high. Thus, $s \geq 0.09 t$ guarantees that stores earn more profits in referral equilibrium, thus referral equilibrium Pareto-dominates random search equilibrium.

[^4]Proof. Referral equilibrium price is higher than random search equilibrium price if and only if $p^{*}=\sqrt{t s} \leq \frac{t}{2}=p_{R}^{*}$. This is equivalent to $s \leq g_{2}(t)=\frac{t}{4}$. Thus, if both types of equilibria exist, then $p^{*} \leq p_{R}^{*}$. Now, $x_{R}^{*}=\frac{1}{2}-\frac{1}{2} \sqrt{1-4(s / t)}$ and $x^{*}=\sqrt{s / t}$, thus, $x_{R}^{*} \leq x^{*}$ if and only if $2 \sqrt{\frac{s}{t}}-1 \geq \sqrt{1-\frac{4 s}{t}}$, or $4 \frac{s}{t}-4 \sqrt{\frac{s}{t}}+1 \geq 1-\frac{4 s}{t}$. This is equivalent to $2 \sqrt{\frac{s}{t}}-1 \geq 0$, or $s \leq t / 4$, which is necessary for the existence of either type of equilibria.

Next, we compare consumers who are indifferent between participating and not under two equilibria. This determines consumers' welfare. In random search equilibrium, $\bar{v}=2 \sqrt{t s}$ is a threshold value, while in referral equilibrium, $\bar{v}_{R}=t-\frac{t}{2} \sqrt{1-4(s / t)}$. Consumers are better off in referral equilibrium if and only if $\bar{v}_{R} \leq \bar{v}$, or $\sqrt{t}-2 \sqrt{s} \leq \frac{1}{2} \sqrt{t-4 s}$. This is equivalent to $(\sqrt{t}-2 \sqrt{s})^{2} \leq \frac{1}{4}(\sqrt{t}-2 \sqrt{s})(\sqrt{t}+2 \sqrt{s})$, or $3 \sqrt{t}-10 \sqrt{s} \leq 0$, or $s \geq 0.09 t$.

This result is predictable. If search cost $s$ is very low, consumers surely prefer random search equilibrium, since random search equilibrium price $p^{*}$ is low while referral equilibrium price $p_{R}^{*}$ is insensitive to $s$.

## 3 Extensions

## 1. Caps on Pure ("Naked") Referral Fees

Consider pure referral fees - fees paid to the referring firm based on a referral alone. It is often argued that a cap on referral fees would protect consumers. In this section, we analyze how the equilibrium price and participation are affected by referral fees in the presence of a binding cap. If referral fees are decided by stores, then a store that offers the highest referral fee gets all referrals from other firms. Thus, by the standard Bertrand competition, the equilibrium referral fee must be the same as an exogenously determined cap whenever it is less than the equilibrium price.

Let $c>0$ be a cap in specific value (specific cap) for referral fees, and assume that stores making referrals act honestly as long as they are offered the same referral fees. In this case, all stores offer a referral fee $c$ as long as it does not exceed the equilibrium price $\left(p_{R}^{*}(c)>c\right)$. We assume that stores cannot discriminate between consumers arriving by referrals and the ones engaged in random search. Again we look for a symmetric price equilibrium. We have the following proposition.

Proposition 5. Suppose that referral fees are allowed, and that there is an exogenous cap $c>0$ for referral fees such that $c \leq 1-\frac{t}{2}-t x_{R}$, where $s \leq \frac{t}{4}$. Then, there exists a unique symmetric referral equilibrium with caps, in which $p_{R}^{*}(c)=\frac{t}{2}+c, \bar{v}_{R}(c)=p_{R}^{*}(c)+t x_{R}$, and $\pi_{R}^{*}(c)=\left(1-p_{R}^{*}(c)-t x_{R}\right) p_{R}^{*}(c)$. The equilibrium price increases in the referral fee. An increase in c can increase or decrease stores' equilibrium profits.

Proof. With a fixed referral fee $c$, store $i$ 's profit function is written as a sum profits from consumers buying from firm $i$ on their first visit, on their visit by referral, and the payments from other stores for the referrals the store makes. A measure $\left(1-p-t x_{R}\right)$ of consumers participate in the market since only consumers with values exceeding $p+t x_{R}$ initiate search. Store $i$ sells to $2 x_{R}\left(p_{i}, p\right)$ of the searchers who visit store $i$ first and collects referral fee $c$ for the rest of consumers who visit store $i$ first. A proportion $\left(1-2 x_{R}\right)$ of consumers who visit another seller first get referrals to store $i$, and store $i$ receives $p_{i}-c$ for each of the referral customers. Therefore, the profits can be written as

$$
\begin{align*}
\pi_{i}\left(p_{i}, p\right) & =\left(1-p-t x_{R}\right)\left(2 p_{i} x_{R}\left(p_{i}, p\right)+\left(1-2 x_{R}\right)\left(p_{i}-c\right)+c\left(1-2 x_{R}\left(p_{i}, p\right)\right)\right)  \tag{17}\\
& =\left(1-p-t x_{R}\right)\left(p_{i}\left(1-\frac{2\left(p_{i}-p\right)}{t}\right)+c \times \frac{2\left(p_{i}-p\right)}{t}\right) .
\end{align*}
$$

if $p_{i} \leq p+t x_{R}$, and $\pi_{i}\left(p_{i}, p\right)=0$ otherwise. By taking the first order condition, we have

$$
\begin{equation*}
\frac{\partial \pi_{i}}{\partial p_{i}}=\left(1-p-t x_{R}\right)\left(1-\frac{4 p_{i}-2 p}{t}+\frac{2 c}{t}\right) \tag{18}
\end{equation*}
$$

Thus, the equilibrium price, given a referral fee $c$, is $p_{R}^{*}(c)=\frac{t}{2}+c$. Since a consumer who is indifferent between participating and not participating in market has a willingness-to-pay $\bar{v}_{R}(c)=p_{R}^{*}(c)+t x_{R}$, an increase in $c$ reduces market participation and consumers' expected utilities. Thus, the equilibrium profit is described as

$$
\begin{equation*}
\pi_{R}^{*}(c)=\left(1-p_{R}^{*}(c)-t x_{R}\right) p_{R}^{*}(c), \tag{19}
\end{equation*}
$$

where $x_{R}=\frac{1}{2}-\frac{1}{2} \sqrt{1-4(s / t)}$. This implies that a marginal increase in $c$ improves stores' profits if and only if $p_{R}^{*}(c)<\frac{1-t x_{R}}{2}$ (the monopoly price given $t x_{R}$ ), or $c<\frac{1}{2}-\frac{3 t}{4}+$ $\frac{t}{4} \sqrt{1-4(s / t)}$. For the equilibrium to exist, some consumers have to engage in search: $\bar{v}_{R}(c)=c+t-\frac{1}{2} t \sqrt{1-4 \frac{s}{t}} \leq 1$, or $c \leq 1-t+\frac{1}{2} t \sqrt{1-4(s / t)}$. If $t \leq 2$, then $1-$
$t \geq \frac{1}{2}-\frac{3 t}{4}$, and $1-t+\frac{1}{2} t \sqrt{1-4(s / t)} \geq \frac{1}{2}-\frac{3 t}{4}+\frac{t}{4} \sqrt{1-4(s / t)}$. Therefore, for $c \in$ $\left(0, \frac{1}{2}-\frac{3 t}{4}+\frac{t}{4} \sqrt{1-4(s / t)}\right)$, profits increase with the cap on referral fees, while profits decrease with $c$ for $\frac{1}{2}-\frac{3 t}{4}+\frac{t}{4} \sqrt{1-4(s / t)} \leq c \leq 1-t+\frac{t}{2} \sqrt{1-4(s / t)}$.

This proposition is of some interest, since it says that referral costs are $100 \%$ beared by consumers. That is, consumers are clearly worse off by having a high referral fee $c$. The equilibrium price is higher by $c$, implying $\bar{v}_{R}(c)>\bar{v}_{R}$, and the number of market participants shrinks. ${ }^{9}$

## 2. Third Party Referrals

Now, we introduce third party referral agents (brokers), who do not sell products and specialize in referral services by taking $c$ as referral fees. Suppose that there are referral agents of measure $\alpha>0$. Thus, now the total measure of stores and brokers is $1+\alpha$, and consumers visit them randomly. In order to keep symmetry of search costs, here we assume that it costs $s>0$ in the first visit, even if a consumer visits a broker. ${ }^{10}$ If a consumer happens to visit a broker, she gets an honest referral with probability one, while if she visits a store then she does not get a referral with probability $2 x_{R}$ (and she purchase a product at the store). Note that a broker is a passive player in this model. She makes honest referrals to consumers who visit her, and charges referral fees $c$ to stores. In contrast, a store maximizes the following profit function.

$$
\begin{align*}
\pi_{i}\left(p_{i}, p\right) & =\frac{1-p-t x_{R}}{1+\alpha}\left[2 x_{R}\left(p_{i}, p\right) p_{i}+\alpha\left(p_{i}-c\right)+\left(1-2 x_{R}\right)\left(p_{i}-c\right)\right]  \tag{20}\\
& =\frac{1-p-t x_{R}}{1+\alpha}\left[p_{i}\left(1-\frac{2\left(p_{i}-p\right)}{t}\right)+c \times \frac{2\left(p_{i}-p\right)}{t}+\alpha\left(p_{i}-c\right)\right]
\end{align*}
$$

Thus, we have the following proposition.

[^5]Proposition 6. Suppose that referral fees are allowed, there are referral brokers with measure $\alpha$, and there is an exogenous cap $c>0$ for referral fees, binding in the equilibrium and such that $t(1+\alpha) / 2+c \leq 1-t x_{R}$. Then, $p_{R T}^{*}(c)=\frac{t(1+\alpha)}{2}+c, \bar{v}_{R T}(c)=p_{R T}^{*}(c)+t x_{R}$, and $\pi_{R T}^{*}(c)=\left(1-p_{R T}^{*}(c)-t x_{R}\right)\left(p_{R T}^{*}(c)-\frac{\alpha}{1+\alpha} c\right)$. The equilibrium price increases in the measure of brokers, $\alpha$, and referral fees, c. An increase in c can increase or decrease stores' equilibrium profits.

Proof. By taking the first order condition, we have $\partial \pi_{i} / \partial p_{i}=\frac{1-p-t x_{R}}{1+\alpha}\left[1-\frac{4 p_{i}-2 p}{t}+\frac{2 c}{t}+\alpha\right]=$ 0 . Thus, the equilibrium price given a referral fee $c$ is $p_{R T}^{*}(c)=\frac{t(1+\alpha)}{2}+c$. The rest can be shown in the same way as the proof of Proposition 5.

This proposition shows that equilibrium price increases even further by third party referrals. The reason is that a demand curve a store faces becomes steeper, since a store has more perfect-match consumers for whom it does not compete. ${ }^{11}$ The market size, $1-p_{R T}^{*}(c)-t x_{R}$, may shrink a lot due to a high equilibrium price. Members of professional organizations resist paying referral fees to non-members arguing that industry profits are siphoned by outsiders. According to Proposition 6, the existence of brokers can increase the market price, and the effect on

We conclude this section by discussing two extensions. First, suppose there is a cost to giving a referral, and the cost is the same regardless of the quality of referral. Referrals would still be given as long as referral fees can be charged to recover the cost of making a referral. Truthful referrals could arise even when firms bid for referrals as long as firms offering better-matching products are willing to pay more to attract a consumer. High bids can be associated with the high quality of a match or low-cost of seller. This is similar to Pauly (1979), where specialists compete for referrals by offering payments; the welfare can improve under fee-splitting because the highest payments are offered by the lowest-costs firms.

[^6]
## 4 Conclusion

Why would professional lawyers, accountants, physicians and real estate agents favor a prohibition against referrals fees? One explanation could be that these professionals are afraid that referral fees may lead to a conflict of interest and loss of consumer trust. ${ }^{12}$ The argument is that this cannot be to the benefit of consumers and, in the end, is not in the interest of the service provider. This paper shows that when referrals are not trustworthy and consumers rationally discount them, in the resulting random-search equilibrium, prices and profits are lower. Therefore, firms favor the equilibrium with truthful referrals due to the relaxing effect referrals have on price competition. Although referrals may or may not benefit consumers, the referral equilibrium is a Pareto improvement over the random-search equilibrium when consumer search costs are not very low.

[^7]
## Appendix

Here, we show that as long as $s>\frac{t}{4}$, there is no symmetric equilibrium with positive profits. By the discussion on random search equilibrium, we know that if $s>\frac{t}{4}$, consumers have no incentive to search beyond the first firm. Thus, if $s>\frac{t}{4}$, there is no symmetric equilibrium with non-trivial consumer search. Next, we show that there is no symmetric equilibrium in which consumers search once-for-all.

If price $p$ is charged (and is expected), a type $v$ consumer who engaged a search purchases a product if and only if $v-p-t x \geq 0$, or $\frac{v-p}{t} \geq x$. Thus, if $\frac{v-p}{t} \geq \frac{1}{2}\left(v \geq p+\frac{t}{2}\right)$ she always purchases a product at any store of her visit. Now, let us check which types of consumers engage in search. First assume $v \geq p+\frac{t}{2}$. Such a consumer's expected payoff from searching is

$$
\begin{equation*}
2 \int_{0}^{\frac{1}{2}}(v-p-t x) d x-s=v-p-\frac{t}{4}-s \tag{A1}
\end{equation*}
$$

Thus, she engages in the initial search if and only if $v \geq p+\frac{t}{4}+s$. Note that if $s>\frac{t}{4}$ then $p+\frac{t}{4}+s>p+\frac{t}{2}$. This implies that a consumer who engages in search also purchases a product at any store of her visit. On the other hand, if $v<p+\frac{t}{2}$ (or $v-p<\frac{t}{2}$ ) then her expected utility from a search is

$$
\begin{equation*}
2 \int_{0}^{\frac{v-p}{t}}(v-p-t x) d x-s=\frac{(v-p)^{2}}{t}-s<\frac{t}{4}-s<0 \tag{A2}
\end{equation*}
$$

Thus, such a consumer would never engage in the initial search. Hence, given the expected price $p$, consumers who participate in the market are the ones whose willingnesses-to-pay are $p+\frac{t}{2}+s$ or more. Thus, given the expected price $p$, those who have willingnesses-to-pay not less than $p+\frac{t}{4}+s$ participate in the initial search, and no others do.

This leads to a version of the Wernerfelt paradox (Wernerfelt, 1994). Let $\beta=s-\frac{t}{4}>0$. Given that consumers expect price $p$, as long as a charged price (an actual price) at a visited store is less than or equal to $p+\beta$ then the participating consumers who visited the store all purchase the product. Thus, the demand curve is vertical at least between $p$ and $p+\beta$, all stores have incentives to raise their prices from $p$ (as long as $p \leq 1$ ). Therefore, there is no equilibrium price in which consumers engage in once-for-all search.

## References

[1] Simon P. Anderson and Regis Renault, 2000a, Consumer Information and Firm Pricing: Negative Externalities from Improved Information, International Economic Review 41, 721-742.
[2] Simon P. Anderson and Regis Renault, 2000b, Product Characteristics and Price Advertising with Consumer Search, unpublished manuscript.
[3] Dirk Bergemann, Deran Ozmen, 2004, Optimal Pricing Policy with Recommender Systems, unpublished manuscript.
[4] Eyal Biyalogorsky; Eitan Gerstner; Barak Libai, 2001, Customer Referral Management: Optimal Reward Programs, Marketing Science, Vol. 20 (1), pp. 82-95.
[5] Peter F. Colwell and Charles M. Kahn, 2001, The Economic Functions of Referrals and Referral Fees, Journal of Real Estate Finance and Economics, 23 (3), 267-296.
[6] Luis Garicano and Tano Santos, 2004, Referrals, American Economic Review, vol. 94(3), pages 499-525, June.
[7] Mark V. Pauly, 1979, The Ethics and Economics of Kickbacks and Fee Splitting, The Bell Journal of Economics, Vol. 10 (1), pp. 344-352.
[8] Stephen J. Spurr, 1990, The Impact of Advertising and Other Factors on Referral Practices, with Special Reference to Lawyers, The RAND Journal of Economics, Vol. 21 (2), pp. 235-246.
[9] Birger Wernerfelt, 1994, Selling formats for search goods. Marketing Science 13, 298309.
[10] Asher Wolinsky, 1984, Product Differentiation with Imperfect Information, The Review of Economic Studies, Vol. 51 (1), pp. 53-61.
[11] Asher Wolinsky, 1986, True Monopolistic Competition as a Result of Imperfect Information, The Quarterly Journal of Economics, Vol. 101 (3), pp. 493-512.
"...We generally came to the conclusion that if no referral fees were paid, that agents would still refer for no fee to the 'best' agents to service their clients." Karen Ott, a director of the Iowa Association of Realtors.
(Note that firms are not allowed to price-discriminate by $v$ ).
Note that $\partial x_{R}\left(p_{i}, p\right) / \partial p_{i}=-1 / t$.

$$
\begin{aligned}
& x_{R}=\frac{1}{2}-\frac{1}{2} \sqrt{1-4(s / t)} \\
& \pi_{R T}^{*}(c)=\left(1-p_{R T}^{*}(c)-t x_{R}\right)\left(\frac{1}{1+\alpha} p_{R T}^{*}(c)+\frac{\alpha}{1+\alpha}\left(p_{R T}^{*}(c)-c\right)\right) \\
& =\left(1-\frac{t(1+\alpha)}{2}-c-\frac{t}{2}-\frac{t}{2} \sqrt{1-4(s / t)}\right)\left(\frac{1}{1+\alpha}\left(\frac{t(1+\alpha)}{2}+c\right)+\frac{\alpha}{1+\alpha} \frac{t(1+\alpha)}{2}\right) \\
& =\left(1-\frac{t(1+\alpha)}{2}-c-\frac{t}{2}-\frac{t}{2} \sqrt{1-4(s / t)}\right)\left(\frac{t}{2}+\frac{c}{1+\alpha}+\alpha \frac{t}{2}\right) \\
& 1-p_{R T}^{*}(c)-t x_{R} \geq 0 \\
& 1-\frac{t(1+\alpha)}{2}-c-t x_{R} \geq 0 \\
& 1-\frac{t(1+\alpha)}{2}-c-t x_{R}=0 \\
& -\frac{2}{t}\left(c+\frac{1}{2} t+t x_{R}-1\right)=\frac{1}{t}\left(2-t-2 t x_{R}-2 c\right) \\
& \alpha=\frac{2}{t}\left(1-c-\frac{1}{2} t-t x_{R}\right) \\
& \alpha \leq \frac{1}{t}\left(2-t-2 t x_{R}-2 c\right)
\end{aligned}
$$

Comparative statics

$$
\begin{aligned}
& \partial \pi_{R T}^{*}(c) / \partial \alpha=-p_{1}\left(p-\frac{\alpha}{1+\alpha} c\right)+\left(1-p-t x_{R}\right) p_{1}-\left(1-p-t x_{R}\right)\left(\frac{\alpha}{1+\alpha}\right)^{\prime} c \\
& p_{1}=\frac{t}{2} \\
& \frac{\partial\left(\frac{\alpha}{1+\alpha}\right)}{\partial \alpha}=\frac{1}{\alpha+1}-\frac{\alpha}{2 \alpha+\alpha^{2}+1}=(\alpha+1)^{-2} \\
& \partial \pi_{R T}^{*}(c) / \partial \alpha=-\frac{t}{2}\left(p-\frac{\alpha}{1+\alpha} c\right)+\left(1-p-t x_{R}\right) \frac{t}{2}-\left(1-p-t x_{R}\right)(\alpha+1)^{-2} c \\
& p=\frac{t(1+\alpha)}{2}+c \\
& \left(-\frac{1}{2}\right)(\alpha+1)^{-2}\left(2 c-t+c t-2 t \alpha+2 c t \alpha-2 c t x_{R}-2 c^{2}+t^{2}-t \alpha^{2}+3 t^{2} \alpha+c t \alpha^{2}+t^{2} x_{R}+2 t^{2} \alpha x_{R}+3 t^{2}\right.
\end{aligned}
$$

$$
<0
$$

$$
t=1
$$

$$
3 c+\alpha+2 c \alpha+x_{R}-2 c x_{R}+2 \alpha x_{R}-2 c^{2}+2 \alpha^{2}+\alpha^{3}+c \alpha^{2}+\alpha^{2} x_{R}=2 c+2 \alpha+2 c \alpha-2 c^{2}+
$$

$$
\frac{5}{2} \alpha^{2}+\alpha^{3}+c \alpha^{2}+\frac{1}{2}<0=
$$

$$
-2 c^{2}+c\left(2 \alpha+\alpha^{2}+2\right)+\frac{5}{2} \alpha^{2}+\alpha^{3}+2 \alpha+\frac{1}{2}<0
$$

$$
-2 c^{2}+2 c+\frac{1}{2}=0, \text { Solution is: } \frac{1}{2}-\frac{1}{2} \sqrt{2}
$$

$$
\frac{1}{2} \sqrt{2}+\frac{1}{2}=1.2071
$$

true for small $c$-firm' profits decrease with $\alpha$

Suppose $c$ is large, $\alpha$ is small and $x_{R}=1 / 2$
$t(1+\alpha) / 2+c \leq 1-t x_{R}=0.5$

Figure 1. Timing of the Game


Firms choose prices and whether to give a referral to a consumer located at a distance x .

Consumers decide whether to initiate costly search.

Consumers learn the location of the store they visit and the referral (if it is given) and choose to buy the store's product, leave the market, follow the referral, or engage in costly random search.

Figure 2. Consumer Decisions in the Random-Search Equilibrium


Notes: In the region labeled "Buy At Once" consumers get a payoff of $v-t x-p$. In the region labeled "Search" consumers get a payoff of $v-\sqrt{s t}-\mathrm{p}$. In the region labeled "Do Not Search" consumers get a payoff of zero.

Figure 3X. Referrals on a Unit Circle

Firm i


Notes: Firm $i$ refers the consumer located at $x$ to Firm $j$. The referral region of firm $i$ is the arc with locations on the unit circle, which are at least $x_{R}$ away from the location of Firm i.

Figure 3. Consumer Decisions on a Unit Circle: Random Search Vs. Referrals


Notes: A consumer located at x randomly chooses the first firm to visit (Firm i). The consumer learns her most preferred location and the location of Firm i.

## Random Search

The consumer decides whether to buy at Firm i, continue random search, or leave the market. In the equilibrium, the consumer buys at Firm i whenever Firm i is located closer than $\mathrm{x}^{*}$.

## Referrals

1) When Firm $i$ is located further than $x_{R}\left(p_{i}, p\right)$, the consumer receives a referral to the best-matching firm (Firm j ) and decides whether to follow the referral, buy at Firm i, continue random search, or leave the market.
2) When Firm $i$ is located closer than $X_{R}\left(p_{i}, p\right)$, the consumer does not receive a referral and has to either buy at Firm i, continue random search, or leave the market.

In the equilibrium, the consumer follows the referral and buys at Firm j when Firm i is located further than $\mathrm{x}_{\mathrm{R}}$, and the consumer buys immediately otherwise.

Figure 4. Consumer Decisions in the Referral Equilibrium.


Notes: In the region labeled "Buy At Once" consumers get a payoff of $v-1 x-p$. In the region labeled "Best Match" consumers get a payoff of $v-p$. In the region labeled "Do Not Search" consumers get a payoff of zero.

Figure 5. Existence of the Search and Referral Equilibria When Value Is Known


Notes: The search equilibrium (SE) exists for $s \leq \min \left\{\mathrm{g}_{1}(\mathrm{t}), \mathrm{g}_{2}(\mathrm{t})\right\}$, while the referral equilibrium (RE) exists for $s \leq \min \left\{\mathrm{g}_{2}(\mathrm{t}), \mathrm{g}_{3}(\mathrm{t})\right\}$. When the two equilibria exist, referrals lead to a Pareto improvement for $s \geq 0.09 \mathrm{t}$.


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[^1]:    ${ }^{1}$ Few exceptions include Pauly (1979) and Spurr (1990). Search markets with referrals have not yet been studied in the literature. The closest paper on search markets is Wolinsky (1986). On vertical referrals, see Garicano and Santos (2004). An empirical study on referral practices among lawyers by Spurr (1990) examines the proportion of cases referred between lawyers, as dependent on the value and nature of a claim, advertising activity, and other factors.
    ${ }^{2}$ Garicano and Santos (2004) examine referrals between vertically differentiated firms (vertical referrals). Due to complementarity between the value of an opportunity and firms' skills, efficient matching involves assigning more valuable opportunities to high-skill firms. The authors show that flat referral fees can support efficient referrals from high-quality to low-quality firm but not in the opposite direction. The low-quality firm has incentives to keep the best opportunities to itself rather than refer them to a high-quality firm. Income-sharing contracts can solve the incentives problem but the first-best is usually impossible to achieve in their model due to the free-riding problem in team production. The authors also study partnerships that specify the allocation of jobs and income.

[^2]:    ${ }^{3}$ According to the law, anyone who knowingly and willfully receives or pays anything of value to influence the referral of business in federal health care programs such as Medicare and Medicaid, conducts a felony and can be punished by up to five years in prison, criminal fines, penalties, and exclusion from participation in federal health care programs.
    ${ }^{4}$ See The American Bar Association (ABA) Canons of Ethics, the ABA Code of Professional Responsibility, and recently adopted ABA Model Rules of Professional Conduct.
    ${ }^{5}$ For example, a California attorney is not prohibited by the Rules of Professional Conduct from paying a referral fee to another California attorney provided (1) full disclosure and the client's consent; and (2) the total fee is not increased due to the referral and does not exceed reasonable compensation for all services rendered to the client.
    ${ }^{6}$ Real estate agents are concerned that the industry profits are dissipated when intermediaries enter and obtain the licenses for the sole purpose of collecting referral fees. They wonder whether the real estate industry would benefit by outlawing referral fees altogether. Most of real estate agents seem to favor such a change.

[^3]:    ${ }^{7}$ Here, we can see an analogy with the Diamond paradox. In both cases, sequential search does not occur, and equilibrium price is independent of the level of search cost (as long as it is positive). However, here, there is still competition among stores trying to keep customers who visited them initially, and the monopoly price does not prevail as the equilibrium price.

[^4]:    ${ }^{8}$ Readers may wonder what equilibrium shows up when $s>t / 4$. Somewhat unintuitively, there is no symmetric equilibrium with positive profits due to a version of so-called the Wernerfelt paradox. Wernerfelt (1994) considered a monopolist's price setting problem with $t=0$ (homogeneous goods) and $s>0$, and proved that due to self-selection of consumers (according to $v$ ), the lowest willingness-to-pay of market participants always exceeds an expected price by $s$, and there is no equilibrium with positive demand. Condition $s>t / 4$ exactly corresponds to this case in our horizontally differentiated good setting $(t>0)$. See appendix.

[^5]:    ${ }^{9}$ Note simultaneously that a mild referral fees give stores strictly positive incentives to make referrals unlike our basic model.
    ${ }^{10} \mathrm{~A}$ consumer needs to incur a search cost $s$ in order to find the location of a store or a broker by herself anyway.

[^6]:    ${ }^{11}$ This effect is first observed in Anderson and Renault (2000a).

[^7]:    ${ }^{12}$ Another explanation is that tight caps on referral fees discourage entry of brokers, which could benefit the industry. The argument is similar to that in Colwell and Kahn (2001). The authors look at third-party referrals made by a middleman and argue that the prohibition on referral fees discourages the entry of intermediaries. They also find that consumers may prefer non-disclosure of referral fees to full disclosure.

