A Spurious Regression Approach to Estimating Structural Parameters*

Chi-Young Choi
Department of Economics
University of New Hampshire

Ling Hu
Department of Economics
Ohio State University

Masao Ogaki
Department of Economics
Ohio State University

June 11, 2004

Ohio State University Department of Economics Working Paper #04-01

Abstract

Economic models often imply that certain variables are cointegrated. However, tests often fail to reject the null hypothesis of no cointegration for these variables. One possible explanation of these test results is that the error is unit root nonstationary due to a nonstationary measurement error in one variable. For example, currency held by the domestic economic agents for legitimate transactions is very hard to measure due to currency held by foreign residents and black market transactions. Therefore, money may be measured with a nonstationary error. If the money demand function is stable in the long-run, we have a cointegrating regression when money is measured with a stationary measurement error, but have a spurious regression when money is measured with a nonstationary measurement error. We can still recover structural parameters under certain conditions for the nonstationary measurement error. This paper proposes econometric methods based on asymptotic theory to estimate structural parameters with spurious regressions involving unit root nonstationary variables. We also develop a test for the null hypothesis of cointegration for dynamic Ordinary Least Squares estimation, using one of our estimators for spurious regressions.

Keywords: Spurious regression, GLS correction method, Dynamic regression, Test for cointegration.

JEL Classification: C10, C15

*We thank Kotaro Hitomi, Hide Ichimura, Yoshihiko Nishiyama, Peter Phillips, and seminar participants at the Bank of Japan, Kyoto University, Ohio State University, and University of Tokyo for helpful comments.

1 Introduction

Economic models often imply that certain variables are cointegrated. However, tests often fail to reject the null hypothesis of no cointegration for these variables. One possible explanation of these test results is that the error is unit root nonstationary due to a nonstationary measurement error in one variable. A nonstationary error in one variable leads to a spurious regression when the true value of the variable and the other variables are cointegrated. In the unit root literature, when the stochastic error of a regression is unit root nonstationary, the regression is called a spurious regression. This is because the standard t test tends to be spuriously significant even when the regressor is statistically independent of the regressand in Ordinary Least Squares. Monte Carlo simulations have often been used to show that the spurious regression phenomenon occurs with regressions involving unit root nonstationary variables (see, e.g., Granger and Newbold (1974), Nelson and Kang (1981, 1983)). Asymptotic properties of estimators and test statistics for regression coefficients of these spurious regressions have been studied by Phillips (1986, 1998) and Durlauf and Phillips (1988) among others. For example, currency held by the domestic economic agents for legitimate transactions is very hard to measure due to currency held by foreign residents and black market transactions. Therefore, money may be measured with a nonstationary error. As shown by Stock and Watson (1993) among others, if the money demand function is stable in the long-run, we have a cointegrating regression when all variables are measured without error. If the variables are measured with stationary measurement errors, we still have a cointegrating regression. However, if money is measured with a nonstationary measurement error, we have a spurious regression. We can still recover structural parameters under certain conditions for the nonstationary measurement error.

This paper proposes a new approach to estimating structural parameters with spurious regressions. Our approach is based on the Generalized Least Squares (GLS) solution of the spurious regression problem analyzed by Ogaki and Choi (2001), who use an exact small sample analysis based on the conditional probability version of the Gauss-Markov Theorem. We develop asymptotic theory for two estimators motivated by the GLS correction: GLS corrected dynamic regression and FGLS corrected dynamic regression estimators. These estimators will be shown to be consistent and asymptotically normally distributed in spurious regressions.

We also develop a Hausman type test for the null hypothesis of cointegration against the alternative hypothesis of no cointegration (or a spurious regression). We construct this test as we note that both the dynamic OLS and GLS corrected dynamic regression estimators are consistent in cointegration estimation while the dynamic OLS estimator is more efficient. On the other hand, when the regression is spurious, only the GLS corrected dynamic regression estimator is consistent. Hence we could do a cointegration test based on the specifications on the error. We show that under the null hypothesis of cointegration, the test statistics has a usual χ^2 limit distribution; while under the alternative hypothesis of a spurious regression, the test statistic diverges. Dynamic OLS is often used in many applications for cointegration. However, no test for cointegration has been developed for dynamic OLS. As in Phillips and Ouliaris (1990), the popular Augmented Dickey-Fuller test for the null hypothesis of no cointegration is designed to be applied for the residual from static OLS rather than the residual from dynamic OLS. Because the OLS and dynamic OLS estimates are often substantially different, it is desirable to have a test for cointegration applied to dynamic OLS. Another aspect of our Hausman type test is that it is for the null hypothesis of cointegration. Ogaki and Park (1998) argued that it is desirable to test the null hypothesis of cointegration rather than that of no cointegration in many applications.

In the unit root literature, asymptotic theory and Monte Carlo simulations have been the main tools to analyze econometric methods. The exact small sample analysis based on the Gauss-Markov Theorem has not been used in general. There seem at least two reasons for this. First, in the unit root literature, most applications involve stochastic regressors, and the conditional expectation version of the Gauss-Markov Theorem is necessary. The standard measure theory definition of the conditional expectation assumes that the random vector's unconditional expectation exists and is finite. As the textbook of Judge et al. (1985) explains, this severely limits the usefulness of the conditional expectation version of the Gauss-Markov Theorem, because it is not possible to prove the existence of the unconditional expectation of the OLS estimator in most applications and simulations (due to the fact that the inverse of $X'X$ is involved in the OLS estimator where X is the design matrix). Second, the strict exogeneity assumption is usually violated in time series applications.

Ogaki and Choi (2001) propose to overcome the first difficulty by considering a definition of the conditional expectation based on the conditional probability measure. The conditional expectation based on the conditional probability measure can be defined even when the unconditional expectation does not exist as in Billingsley (1986). The Law of Iterated Expectations may not be satisfied when this definition is employed, but this does not cause problems for our particular application for spurious regressions. The second difficulty can be dealt with by adding leads and lags of the first difference of the stochastic regressors, leading to dynamic regressions proposed by Phillips and Loretan (1991) and Stock and Watson (1993) among others. Ogaki and Choi's analysis shows that the idea of endogeneity correction by the dynamic regressions can also be used for spurious regressions. The GLS corrected dynamic regression estimator is basically an estimator obtained from the first differenced version of a dynamic OLS regression. It is surprising that the endogeneity correction of dynamic regression works even when all variables in the regression are stationary because of first differencing.

Using the conditional probability version of the Gauss-Markov Theorem, Ogaki and Choi (2001) study the exact small sample properties of spurious regressions. For the case of a classic spurious regression of a random walk onto a random walk that is independent of the regressand, they find that only the spherical variance assumption is violated. Therefore, they propose a GLS correction for the spurious regression. This solution is essentially the same as the well known solution of taking the first difference of all variables in this case, but the solution can also be used for the case with endogeneity, as long as the dynamic regression technique solves the endogeneity problem.

However, the stringent assumptions such as known covariance matrices employed by Ogaki and Choi (2001) for the exact small sample analysis are not satisfied in applications. For this reason, in order to apply the GLS correction, it is necessary to relax some of their assumptions. Because the exact small sample properties cannot be analyzed when these assumptions are relaxed, we use asymptotic theory to analyze large sample properties of estimators and test statistics based on the GLS correction. The GLS corrected dynamic regression estimator is basically an estimator obtained from the first differenced version of a dynamic OLS regression. These methods are applied to revisit the widely known issues in macroeconomics: estimation of parameters in the money demand function, the long-run implications of the consumption-leisure choice, output convergence, and purchasing power parity (PPP).

The rest of the paper is organized as follows. Section 2 presents asymptotic theory. Section 3 gives empirical results. Section 4 contains concluding remarks.

2 The econometric model

We consider the following data generating process for observations $\{x_t, y_t\}$,

$$y_t = \beta x_t + g_t + e_t \tag{1}$$

$$\Delta x_t = v_t \tag{2}$$

where g_t is generated by finite number of leads and lags of Δx_t,

$$g_t = C(L^{-1})\Delta x_t + D(L)\Delta x_t.$$

In our following analysis, to simplify notation, we usually take $g_t = \alpha\Delta x_t = \alpha v_t$.

Suppose that β in (1) is the structural parameter of interest. The inference procedure about β differs according to different assumptions on the error term e_t in (1). When e_t is stationary, the regression is a cointegration regression; when e_t is a unit root nonstationary process, the regression is spurious. The latter case is motivated by our empirical studies in Macroeconomic modeling and it is the main interest in this project.

Assumption 1 *Let both v_t and u_t be zero mean stationary processes with $E|v_t|^\gamma < \infty$, $E|u_t|^\gamma < \infty$ for some $\gamma > 2$. Also assume that v_t and u_t are statistically independent, and they are both strong mixing with size $-\gamma/(\gamma-2)$. Finally, e_t is assumed to be generated as $\Delta e_t = u_t$.*

Assumption 2 *Let both v_t and u_t be specified as in Assumption 1, and assume that $e_t = u_t$.*

The conditions on v_t and u_t ensure the invariance principles: for $r \in [0,1]$, $n^{-1/2}\sum_{t=1}^{[nr]} v_t \rightarrow_d \sigma_1 V(r)$, $n^{-1/2}\sum_{t=1}^{[nr]} u_t \rightarrow_d \sigma_2 U(r)$ where $V(r)$ and $U(r)$ are independent standard Brownian motions and σ_1^2 and σ_2^2 are long run variances of the sequences $\{v_t\}$ and $\{u_t\}$ respectively. The functional central limit theorem holds for weaker assumptions than assumed here (de Jong and Davidson (2000)), but the conditions assumed above are general enough to include many stationary Gaussian or non-Gaussian ARMA processes that are commonly assumed in empirical modeling.

In the next two sections, we will summarize the asymptotic properties of different estimation procedures under these two assumptions. Under assumption 1, the regression is spurious and in this situation, OLS is not consistent while both GLS correction and feasible GLS correction will give consistent and asymptotically equivalent estimators. Under assumption 2, GLS corrected estimator is not efficient as it is $\sqrt{n}$ convergent, but the FGLS corrected estimator is n convergent and asymptotically equivalent to the OLS estimator. Hence FGLS corrected estimator is robust with respect to the error specifications given in assumption 1 and assumption 2. It is asymptotically equivalent to GLS corrected estimator in spurious regressions and it is asymptotically equivalent to OLS estimator in cointegration regressions.

2.1 Regressions with I(1) error

In this section, we consider the situation when the error term is I(1), i.e., $\Delta e_t = u_t$. The estimation methods we consider are dynamic OLS, GLS corrected dynamic regression estimator, and FGLS corrected dynamic regression estimator.

2.1.1 The dynamic OLS spurious estimation

Consider the OLS estimation of the regression

$$y_t = \beta x_t + \alpha v_t + e_t. \tag{3}$$

This is a spurious regression since for any value of β, the error term is always I(1). The OLS estimator $\hat{\beta}_n$ has the following limiting distribution

$$\hat{\beta}_n - \beta \rightarrow \frac{\sigma_2 \int_0^1 V(r)U(r)dr}{\sigma_1 \int_0^1 V(r)^2 dr} \equiv \xi \tag{4}$$

which can be written as a mixture of normal distributions centered at zero (Phillips, 1989). As discussed in Phillips (1986), in spurious regressions the noise is as strong as the signal, hence uncertainty about β persists in the limiting distributions.

2.1.2 GLS corrected dynamic regression estimation

When we think of the problem with spurious regressions, it is the persistence in the error. In this problem, the error can be written as

$$e_t = \rho e_{t-1} + u_t,$$

where $\rho = 1$. Then we can filter all the variables by taking full difference, and use OLS to estimate

$$\Delta y_t = \beta \Delta x_t + \alpha \Delta v_t + u_t. \tag{5}$$

This procedure can be viewed as a GLS corrected estimation. Note that in regression (3), the estimator of β and α are asymptotically independent, hence we don't have to include v_t in the regression if we are only interested in β. But here we need to include the differences of the leads and lags (which are I(-1)) that are correlated with v_t to produce a consistent estimate for β.

Define $\theta = (\beta, \alpha)'$, and let $\tilde{\theta}$ denote the GLS corrected estimator, then we can show that

$$\sqrt{n}(\tilde{\theta}_n - \theta) \to_d (\sigma_v^2 Q)^{-1} N(0, \sigma_v^2 \sigma_u^2 Q) = N\left(0, \frac{\sigma_u^2}{\sigma_v^2} Q^{-1}\right) \tag{6}$$

with

$$Q = \begin{bmatrix} 1 & 1-\psi_v \\ 1-\psi_v & 2(1-\psi_v) \end{bmatrix}, \tag{7}$$

where $\sigma_v^2 = E(v_t^2)$, $\sigma_u^2 = E(u_t^2)$, and ψ_v is the first order autocorrelation coefficient of sequence $\{v_t\}$. We can see that β can be consistently estimated (jointly with α) and the estimator is asymptotically normal.

2.1.3 The Cochrane-Orcutt feasible GLS corrected dynamic regression estimation

To use GLS to estimate a regression with serial correlation in empirical work, usually a Cochrane-Orcutt feasible GLS procedure is adopted. This procedure also works for spurious regressions and this has been shown by Blough (1992) and Phillips and Hodgson (1994). They show that this procedure gives asymptotically equivalent estimator as in the differenced regression. Below we give the results in our framework.

Let the residual from OLS regression (3) denoted by $\hat{R}_t$, i.e.

$$\hat{R}_t = y_t - \hat{\beta}_n x_t - \hat{\alpha}_n v_t.$$

To conduct the Cochrane-Orcutt GLS estimation, first we run an AR(1) regression of $\hat{R}_t$,

$$\hat{R}_t = \rho \hat{R}_{t-1} + error. \tag{8}$$

It can be shown that

$$\hat{\rho}_n - 1 = o_p(1) \quad \text{and} \quad n(\hat{\rho}_n - 1) = O_p(1).$$

Next, consider the following Cochrane-Orcutt transformation of the data:

$$\tilde{y}_t = y_t - \hat{\rho}_n y_{t-1}, \quad \tilde{x}_t = x_t - \hat{\rho}_n x_{t-1}, \quad \tilde{v}_t = v_t - \hat{\rho}_n v_{t-1}. \tag{9}$$

Now consider OLS estimation in the regression

$$\tilde{y}_t = \tilde{\beta}\tilde{x}_t + \tilde{\alpha}\tilde{v}_t + error = z_t'\theta + error. \tag{10}$$

The OLS estimator of θ in (10) is

$$\tilde{\theta}_n = \left[\sum_{t=1}^{n} z_t z_t'\right]^{-1}\left[\sum_{t=1}^{n} z_t \tilde{y}_t\right]. \tag{11}$$

The limiting distribution for $\tilde{\theta}$ can be shown to be the same as in (6):

$$\sqrt{n}(\tilde{\theta}_n - \theta) \to_d (\sigma_v^2 Q)^{-1} N(0, \sigma_v^2\sigma_u^2 Q) = N\left(0, \frac{\sigma_u^2}{\sigma_v^2}Q^{-1}\right), \tag{12}$$

If in regression (3) v_t is not included, we still have the same limiting distribution based on the residual from the OLS estimation.

In Appendix B, we describe some extensions of the model. We show that if a constant is included in the data generating process, the GLS or FGLS corrected estimation give results that are asymptotically equivalent as given in (6).

2.2 Regressions with I(0) error

In this section, we will consider the following problem. When the error term in (1) is I(0) instead of I(1), so that the regression is a cointegration rather than a spurious regression, while we apply the same procedure as under the spurious assumption, how the estimator behaves asymptotically.

2.2.1 The dynamic OLS estimation

Under assumption 2, the DGP of y_t is

$$y_t = \beta x_t + \alpha v_t + u_t. \tag{13}$$

Clearly, this is a cointegration regression, and the limiting distribution of the OLS estimator of β can be written as

$$n(\hat{\beta}_n - \beta) \to_d \frac{\sigma_2 \int_0^1 V(r)dU(r)}{\sigma_1 \int_0^1 V(r)^2 dr}. \tag{14}$$

2.2.2 GLS corrected dynamic regression estimation

Now, if we take a full difference as we did in the I(1) case, the regression becomes

$$\Delta y_t = \beta\Delta x_t + \alpha\Delta v_t + u_t - u_{t-1} = z_t'\theta + u_t - u_{t-1}.$$

Note that we lose efficiency in this transformation as the estimator $\tilde{\beta}$ is now $\sqrt{n}$ convergent rather than n convergent as in the cointegration. With some minor revision of equation (6), we get results for the limiting distribution of the estimator in this case:

$$\sqrt{n}(\tilde{\theta}_n - \theta) \to_d N\left(\mathbf{0}, \frac{2\sigma_u^2(1-\psi_u)}{\sigma_v^2}Q^{-1}\right), \tag{15}$$

where ψ_u is the first order autocorrelation coefficient of sequence $\{u_t\}$.

Therefore, the GLS correction or differencing is not efficient if the variables are actually cointegrated.

2.2.3 The Cochrane-Orcutt feasible GLS corrected dynamic regression estimation

Instead of taking full difference, if we estimate the autoregression coefficient in the error and use this estimator to filter all sequences, we will obtain an estimator that is asymptotically equivalent to the OLS estimator. Intuitively, in the case that the error is serially uncorrelated, then the AR(1) coefficient $\hat{\rho}_n$ will converge to zero, hence the transformed regression will be asymptotically equivalent to the original regression. Or, if the error is stationary and serially correlated, then the AR(1) coefficient will be less than unit, and as has been shown in Park and Phillips (1988), the GLS estimator and OLS estimator in a cointegration regression are asymptotically equivalent.

First run OLS estimation of

$$y_t = \hat{\beta}_n x_t + \hat{\alpha}_n v_t + \hat{u}_t = z_t' \hat{\theta}_n + \hat{u}_t.$$

Then run an AR(1) regression of $\hat{u}_t$,

$$\hat{u}_t = \hat{\rho}_n \hat{u}_{t-1} + error.$$

Write

$$\hat{u}_t = y_t - z_t'\theta + z_t'(\theta - \hat{\theta}_n) = u_t + z_t'(\theta - \hat{\theta}_n).$$

Now, consider the Cochrane-Orcutt transformation (9) and estimate

$$\tilde{y}_t = \tilde{\beta}_n \tilde{x}_t + \tilde{\alpha}\tilde{v}_t + error. \tag{16}$$

The limiting distribution of $\tilde{\beta}_n$ can be shown to be

$$n(\tilde{\beta}_n - \beta) \to_d \frac{(1-\psi_u)^2 \sigma_1 \sigma_2 \int_0^1 V(r)dU(r)}{(1-\psi_u)^2 \sigma_2^2 \int_0^1 V(r)^2 dr} = \frac{\sigma_2 \int_0^1 V(r)dU(r)}{\sigma_1 \int_0^1 V(r)^2 dr}. \tag{17}$$

which is exactly the same as the limit of the OLS estimator given in (14). In summary, the feasible GLS is not only valid in spurious regression, but also harmless to the estimator in the limit when the regression is actually a cointegration.

2.3 Simulations

From above analysis, we note that the FGLS corrected estimator is a robust procedure with respect to error specifications: it is asymptotically equivalent to the GLS corrected estimator in spurious regressions and it is asymptotically equivalent to the OLS estimator in cointegration regressions. In this section, we use simulations to study its finite sample performances compared to the other two estimators. In the simulation, we generate v_t and ϵ_t from independent standard normal distribution and let $u_t = \epsilon_t + 0.5\epsilon_t$. The parameters are set to be $\beta = 2, \alpha = 0.5$. Figure 1 shows the finite sample distribution when the error term is unit root nonstationary. The left figure plots the distribution of the GLS corrected estimator and the right figure plots that of the FGLS corrected estimator. We can see that although the FGLS corrected estimator is consistent, it has larger variance even when the sample is relatively large. Figure 2 shows the finite sample distribution when the error term is I(0). The left figure plots the distribution of the OLS estimator and the right figure plots that of the FGLS corrected estimator. These two estimators both converge very fast and the difference between them is almost invisible.

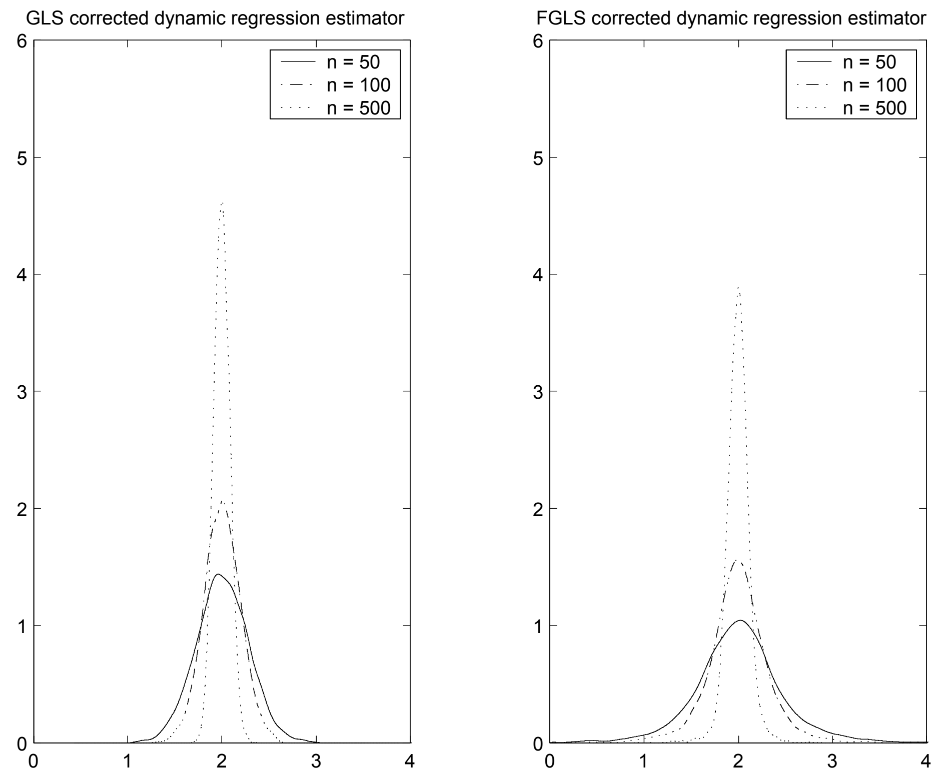

Figure 1: Distributions of GLS and FGLS estimators when the error is I(1)

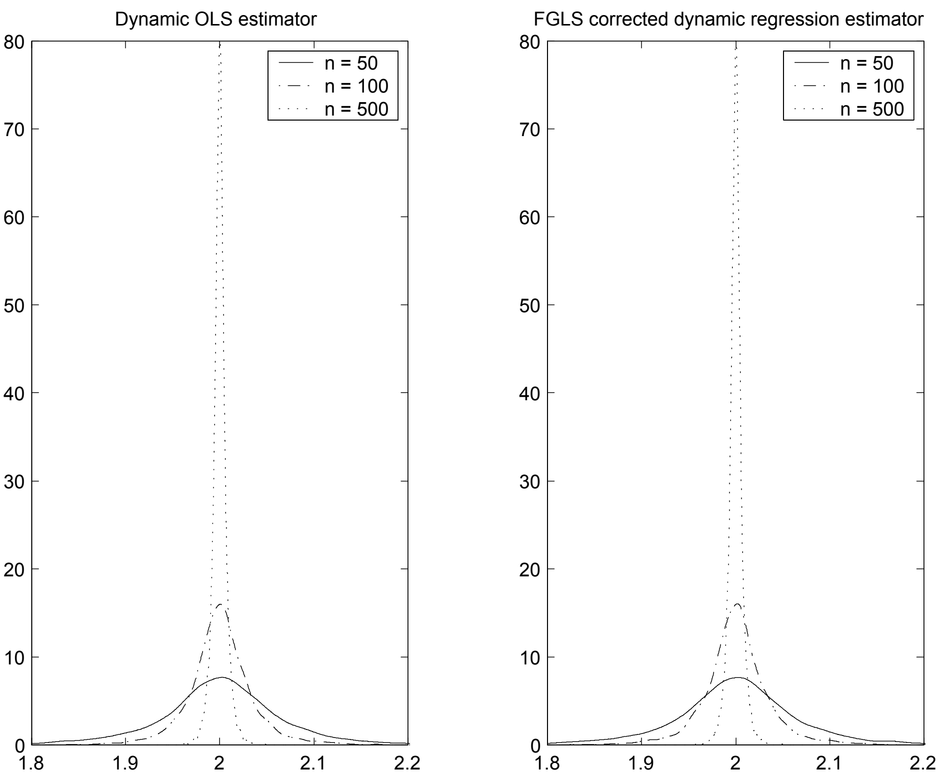

Figure 2: Distributions of OLS and FGLS estimators when the error is I(0)

2.4 Hausman-type cointegration test

In this section, we construct a Hausman-type cointegration test based on the difference of two estimators: an OLS estimator ($\hat{\beta}_n$) and a GLS corrected estimator ($\tilde{\beta}_n$) corresponding to $\rho = 1$. This is equivalent to compare estimators in a level regression and in a differenced regression. We let the error be I(0) under the null and the error be I(1) under the alternative. Our discussions above show that under the null, both OLS and GLS corrected are consistent but OLS estimator is more efficient; while under the alternative, which corresponds to a spurious regression, only GLS corrected estimator is consistent. The DGP under the null is

$$y_t = \beta x_t + \alpha v_t + u_t, \tag{18}$$

and under the alternative the error is unit root nonstationary as given in assumption 1.

Let q_n denote the difference between these two estimators

$$\sqrt{n} q_n = \sqrt{n}(\tilde{\beta}_n - \hat{\beta}_n).$$

Under the null hypothesis of cointegration, the limiting distribution of $\sqrt{n} q_n$ is dominated by the GLS corrected estimator $\tilde{\beta}$, since $\sqrt{n}\hat{\beta}_n$ is $o_p(1)$. In particular, we can write

$$\sqrt{n} q_n \to_p N(0, \tau^2), \quad \tau^2 = \frac{4\sigma_u^2(1 - \psi_u)}{\sigma_v^2(1 + \psi_v)}$$

which is derived from (15). To estimate τ, define

$$\hat{\tau}_n^2 = \frac{4\hat{\sigma}_u^2(1 - \hat{\psi}_u)}{\hat{\sigma}_v^2(1 + \hat{\psi}_v)},$$

where $\hat{\sigma}$ and $\hat{\psi}$ denote the sample counterparts of the variance and autocorrelation coefficients. Under the assumptions on series u_t, v_t, it is clear that τ_n is a consistent estimator for τ.

Define the test statistics $h_n = n q_n^2 / \hat{\tau}_n^2$, then under the null hypothesis,

$$h_n = \frac{n q_n^2}{\hat{\tau}_n^2} \to \frac{[N(0, \tau^2)]^2}{\tau^2} \sim \chi^2(1). \tag{19}$$

Hence h_n has a limiting $\chi^2(1)$ distribution under the null hypothesis. Next, under the alternative of I(1) errors, the inconsistent OLS estimator dominates.

$$q_n = \tilde{\beta}_n - \hat{\beta}_n \to \xi \tag{20}$$

where ξ is bounded in probability.

So under the alternative, for the statistics defined in (19), $q_n = O_p(1)$, $\hat{\tau}_n^2$ still converges to τ, hence h_n diverges. In summary, the Hausman-type test statistics has a limiting $\chi^2(1)$ distribution under the null and diverges under the alternative. By a similar argument, the Hausman-type test statistics has a limiting $\chi^2(p)$ distribution under the null and diverges under the alternative when there are p cointegrating vectors to estimate.

Note that in this test the null hypothesis is cointegration, while usually the null of a cointegration test is that no cointegrating relationship presents. Both type of tests will be useful to empirical researchers. For instance, if the results reject the null of no cointegration and also reject the null of cointegration, we may need to seek alternative model specifications.

3 Empirical applications

In this section we apply the GLS-type correction methods and the Hausman-type cointegration test to analyze four macroeconomic issues: (i) long-run money demand in the U.S.; (ii) long-run implications of the consumption-leisure choice; (iii) output convergence among industrial and developing countries; (iv) Purchasing Power Parity (PPP) for traded and non-traded goods.

3.1 U.S. money demand

The long-run money demand function has often been estimated under the cointegrating restriction among real balances, real income, and interest rate. The restriction is legitimate if the money demand function is stable in the long-run and if all variables are measured without nonstationary error. Indeed Stock and Watson (1993) found supportive evidence of stable long-run M1 demand by estimating cointegrating vectors. However, if money is measured with a nonstationary measurement error, we have a spurious regression and the estimation results based on a cointegration regression become questionable. We apply our GLS correction methods to estimate long-run income and interest elasticities of M1 demand. To this end, regression equations are set up with real money balance ($\frac{M}{P}$) as regressand and income (y) and interest (i) as regressors. Following Stock and Watson (1993), the annual time series for M1 deflated by the net national product price deflator is used for $\frac{M}{P}$, real net national product for y and the six month commercial paper rate in percentage for i. $\frac{M}{P}$ and y are in logarithms while three different regression equations are considered depending on the measures of interest. We have tried the following three functional forms. Equation 1 has been studied by Stock and Watson (1993).

$$\begin{aligned}
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma i_t + u_t, \quad \text{(equation 1)} \\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln(i_t) + u_t, \quad \text{(equation 2)} \\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln\left[\frac{1+i_t}{i_t}\right] + u_t. \quad \text{(equation 3)}
\end{aligned}$$

It is worth noting that the liquidity trap is possible for the latter two functional forms. When the data contain periods with very low nominal interest rates, the latter two functional forms may be more appropriate.

Table 1 presents the point estimates for β (income elasticity of money demand) and γ based on the three estimators under scrutiny: dynamic OLS, GLS corrected dynamic regression estimator, and FGLS corrected dynamic regression estimator.[1] Several features emerge from the table. First, all the estimated coefficients have theoretically 'correct' signs: positive signs for income elasticities and negative signs for γ for the first two functional forms and positive signs for γ for the first third functional form. Second, GLS corrected regression estimates of the income elasticity are implausibly low for all three functional forms for low values of k, and increase to more plausible values near one as k increases. The fact that the results become more plausible as k increases suggests that the endogeneity correction of dynamic regressions works in this application for moderately large values of k such as 3 and 4. The results for low values of k are consistent with those of low income elasticity estimates of first differenced regressions before researchers started to apply the cointegration methods to estimate money demand. Therefore, the estimators in the old literature of first differenced regressions are likely to be downward

[1] In FGLS corrected dynamic regression estimator, the serial correlation coefficient in error term is estimated before being applied to the Cochrane-Orcutt transformation while it is assumed to be unity in GLS corrected dynamic regression estimator which is equivalent to regressing the first difference of variables without constant term.

biased because of the endogeneity problem. Third, all point estimates of the three estimators are very similar, and the Hausman test fails to reject the null hypothesis of cointegration for large enough values of k. Hence there is little evidence against cointegration. However, it should be noted that a small random walk component is very hard to detect by any test for cointegration. Therefore, it is assuring to know that all three estimators are similar for large enough values of k, and the estimates are robust with respect to whether the regression error is I(0) or I(1).

We report the value of k chosen by the Bayesian Information Criterion (BIC) rule throughout our empirical applications in order to give some guidance in interpreting results. It is beyond the scope of this paper to study detailed analysis of how k should be chosen because this issue has not been settled in the literature of dynamic cointegrating regressions.

3.2 Long-run implications of the consumption-leisure choice

Consider a simplified version of Cooley and Ogaki's (1996) model of consumption and leisure in which representative household maximizes

$$U = E_0[\sum_{t=0}^{\infty} \delta^t u(t)]$$

where E_t denotes the expectation conditioned on the information available at t. We adopt a simple intraperiod utility function that is assumed to be time- and state-separable and separable in nondurable consumption, durable consumption, and leisure

$$u(t) = \frac{C(t)^{1-\beta} - 1}{1-\beta} + v(l(t))$$

where $v(\cdot)$ represents a continuously differentiable concave function, $C(t)$ is nondurable consumption, and $l(t)$ is leisure.

The usual first order condition for a household that equates the real wage rate with the marginal rate of substitution between leisure and consumption is given as:

$$W(t) = \frac{v'(l(t))}{C(t)^{-\beta}}$$

where $W(t)$ is the real wage rate. We assume that the stochastic process of leisure is (strictly) stationary in the equilibrium as in Eichenbaum, Hansen, and Singleton (1988). Then an implication of the first order condition is that $\ln(W(t)) - \beta \ln(C(t)) = \ln(v'(l(t)))$ is stationary. When we assume that the log of consumption is difference stationary, this implies that the log of the real wage rate and the log of consumption are cointegrated with a cointegrating vector $(1, -\beta)'$. Now assume that $\ln(W(t))$ and $\ln(C(t))$ are measured with errors. Imagine that the $\ln(C(t))$ is measured with a stationary measurement error, $\xi(t)$, and that $\ln(W(t))$ is measured with a difference stationary measurement error, $\epsilon(t)$ (perhaps because of the difficulty in measuring fringe benefits). Assume that $\epsilon(t)$ is independent of $\ln(C(t))$ and $\xi(t)$ at all leads and lags. Consider a regression

$$\ln(W^m(t)) = a + \beta \ln(C^m(t)) + u(t), \tag{21}$$

where $W^m(t)$ is the measured real wage rate, $C^m(t)$ is measured consumption, and $u(t) = -\epsilon(t) + \beta\xi(t) + \ln(v'(l(t))) - a$. If $\epsilon(t)$ is stationary, then $u(t)$ is stationary, and Regression (21) is a cointegrating regression as in Cooley and Ogaki. In this simple version, the preference parameter β is the Relative Risk Aversion (RRA) coefficient, which is equal to the reciprocal of the intertemporal elasticity of substitution (IES). Cooley and Ogaki show that the same regression can be used to estimate the reciprocal of the long-run IES when preferences for consumption is subjected to time nonseparability such as habit formation. For simplicity, we interpret β as the RRA coefficient in this paper.

If $\epsilon(t)$ is unit root nonstationary, then Regression (21) is a spurious regression because $u(t)$ is nonstationary in this case. Hence the standard methods for cointegrating regressions cannot be used. However, the preference parameter β can still be estimated by the spurious regression method.

Table 2 presents the estimation results for the RRA coefficient (β) based on various estimators. We used the same data set that Cooley and Ogaki used.[2] The results in Table 2 illustrate several points. First, all point estimates for β have theoretically correct positive sign. Second, for nondurables (ND), GLS-corrected dynamic regression estimates of β are much lower than Dynamic OLS estimates for all values of k. As a result, the Hausman-type cointegration test rejects the null hypothesis of cointegration for all values of k at the 1 percent level. Therefore, the evidence supports the view that Regression (21) is a spurious regression, and the true value of the RRA coefficient is likely to be much lower than the dynamic OLS estimates. Both the GLS corrected and the robust FGLS corrected dynamic regression estimation results are consistent with the view that the RRA coefficient is about one for the value of k chosen by BIC. For nondurables plus services (NDS), GLS corrected dynamic regression estimates of β are much lower than Dynamic OLS estimates for small values of k. As a result, the Hausman-type cointegration test rejects the null hypothesis of cointegration at the 5 percent level when k is 0, 1, and 2. It still rejects the null hypothesis of cointegration at the 5 percent level when k is 3. It does not reject the null hypothesis when k is 4 and 5. According to the BIC rule, k is chosen to be 3, and there is some evidence against cointegration. However, because the GLS corrected dynamic regression estimates get closer to dynamic OLS estimates as k increases, the evidence is not very strong. It is likely that a small random walk component exists for the error term of the regression for NDS, making it a spurious regression. The robust FGLS corrected dynamic regression estimates are close to both GLS corrected dynamic regression estimates and dynamic OLS estimates as long as k is 3 or greater.

Thus, we have fairly strong evidence that we have a spurious regression for ND, and some evidence that we have spurious regression for NDS. The true value of RRA is likely to be about one for both ND and NDS.

3.3 Output convergence across national economies

In this section, we apply the techniques to reexamine a long standing issue of macroeconomics, the hypothesis of output convergence. For this application, our main purpose is not to estimate unknown structural parameters, but to test the null hypothesis of cointegration with the Hausman-type test. As a key proposition of the neoclassical growth model, the hypothesis has been one of the popular subjects in macroeconomics and has attracted considerable attention in the empirical field particularly during the last decade. Besides its important policy implications, the convergence hypothesis has been used as a criterion to discern the two main growth theories, the exogenous growth theory and the endogenous growth theory. However, it remains the subject of continuing debate mainly because the empirical evidence supporting the hypothesis is mixed. Nevertheless the established literature based on popular international dataset such as the Summers-Heston (1991) suggests a stylized fact in output convergence among various national economies: convergence among industrialized countries but not among developing countries and not between industrialized and developing countries.

Given that a mean stationary stochastic process of output disparities between two economies is interpreted as supportive evidence of stochastic convergence, unit-root or cointegration testing procedures are often used by empirical researchers to evaluate the convergence hypothesis. In this vein, our

[2] See Cooley and Ogaki (1996, page 127) for the detailed description of data.

techniques proposed here fits in the study of output convergence. We consider four developing countries (Columbia, Ecuador, Egypt, and Pakistan) along with four industrial countries (Germany, Luxemburg, New Zealand, Switzerland). The raw data are extracted from the *Penn World Tables* of Summers-Heston (1991) and consist of annual real GDP per capita (RGDPCH) over the period of 1950-1992. The following two regression equations are considered with regard to the cointegration relation.

$$y_t^D = \alpha + \beta y_t^I + \varepsilon_t, \quad (22)$$
$$y_t^I = \alpha + \beta y_t^I + \varepsilon_t, \quad (23)$$

where y_t^{DEV} and y_t^{IND} denote log real GDP per capita for developing and industrial countries, respectively.

Tables 3-1 and 3-2 report the results which exhibit a large variation in estimated coefficients. Recall that our interest in this application lies in the cointegration test based on the Hausman-type test. As can be seen from Table 3-1, irrespective of country combinations, the null hypothesis of cointegration can be rejected when developing countries are regressed onto industrial countries, indicating that there is little evidence of output convergence between developing countries and industrial countries. The picture changes dramatically when industrial countries are regressed onto industrial countries as in (23). Table 3-2 displays that the Hausman test fail to reject the null of cointegration in all cases considered. Our finding is therefore consistent with the stylized fact in the literature of the so-called *convergence clubs*.

3.4 PPP for traded and non-traded goods

As a major building block for many models of exchange rate determination, PPP has been one of the most heavily studied subjects in international macroeconomics. Despite extensive research, however, the empirical evidence on PPP remains inconclusive, largely due to econometric challenges involved in determining its validity. As is generally agreed, most real exchange rates show very slow convergence which makes estimating long-run relationships difficult with existing statistical tools. The literature suggests a number of potential explanations for the very slow adjustment of relative price: volatility of nominal exchange-rate; market frictions such as trade barriers and transportation costs; imperfect competition in product markets; and the presence of non-traded goods in the price basket. According to the commodity-arbitrage view of PPP, the law of one price holds only in tradable goods and the departures from PPP are primarily attributed to the large weight placed on nontraded goods in the CPI. This view has obtained support from many empirical studies based on disaggregated price indices. They tend to provide ample evidence that prices in non-traded goods are much more disperse than traded goods and consequently non-traded goods exhibit far larger deviations from PPP than traded goods. Given that general price indices involve a mix of both traded and non-traded goods, highly persistent deviations of non-traded goods from PPP can lead to the lack of conclusive evidence on the long run PPP relationship. As in the previous application, our main purpose for this application is not to estimate unknown structural parameters, but to test the null hypothesis of cointegration with the Hausman-type test.

Let p_t and p_t^* denote the logarithms of the consumer price indices in the base country and foreign country respectively, and s_t be the logarithm of the price of foreign country's currency in terms of the base country's currency. Long-run PPP requires that a linear combination of these three variables be stationary. To be more specific, long-run PPP is said to hold if $f_t = s_t + p_t^*$ is cointegrated with p_t such that $\epsilon_t \sim I(0)$ in

$$f_t^T = \alpha + \beta p_t^T + \epsilon_t,$$
$$f_t^N = \alpha + \beta p_t^N + \epsilon_t,$$

where the superscripts T and N denote the price levels of traded goods and non-traded goods, respectively.

Following the method of Stockman and Tesar (1995), Kim (2004) recently used the real exchange rate for total consumption using the general price deflator, and the real exchange rate for traded and non-traded goods using implicit deflators for non-service consumption and service consumption, respectively.[3] We use Kim's dataset to apply our techniques to the linear combination of sectorally decomposed variables. Table 4 presents the results using quarterly price and exchange rate data for six countries, Canada, France, Italy, Japan, U.K., and U.S. for the period of 1974 Q1 through 1998 Q4. With the Canadian dollar used as numeraire, Table 4 presents the estimates for β which should be close to unity according to long-run PPP. For traded goods, estimates are above unity in most cases but the variation across estimators does not seem substantial, resulting in non-rejection of the null of cointegration in all cases considered. By sharp contrast, the Hausman-type cointegration test rejects the null hypothesis in every country when the price for non-traded goods is used. It is noteworthy that there exists considerable difference between GLS-corrected estimates for β and their DOLS and FGLS counterparts which are far greater than unity. That is, supportive evidence of PPP is found for traded goods but not for non-traded goods, congruent with the general intuition as well as the findings by other studies in the literature such as Kakkar and Ogaki (1999) and Kim (2004).[4]

4 Concluding remarks

In this paper, we developed two estimators to estimate structural parameters in spurious regressions: GLS corrected dynamic regression and FGLS corrected dynamic regression estimators. A GLS corrected dynamic regression estimator is basically a first difference version of a dynamic OLS regression estimator. The asymptotic theory showed that, under some regularity conditions, the endogeneity correction of the dynamic regression works for the first differenced regressions for both cointegrating and spurious regressions. This result is useful because it is not intuitively clear that the endogeneity correction works even in regressions with stationary first differenced variables.

We also developed the Hausman-type cointegration test by comparing the dynamic OLS regression and GLS corrected dynamic regression estimators. As noted in the Introduction, this task is important because no test for cointegration has been developed for dynamic OLS, and because tests for the null hypothesis of cointegration are useful in many applications.

We applied our estimation and testing methods to four types of applications. In the first two applications, we estimated unknown structural parameters. The main purpose of the last two applications was to test for cointegration.

In the first application of estimating the money demand function, the results suggest that the endogeneity correction of the dynamic regression works with a moderately large number of leads and lags for the GLS corrected dynamic regression estimator. The GLS corrected dynamic regression estimates are very low with low orders of leads and lags, and then increase to more plausible values as the order

[3] For details, see the Appendix for the description of data. We thank JB Kim for sharing the dataset.

[4] Engel (1999) finds little evidence for long-run PPP for traded goods with his variance decomposition method. However, it should be noted that his method is designed to study variations of real exchange rates over relatively shorter periods compared with coingegration-type methods that are designed to study long-run relationships.

of leads and lags increases. Dynamic OLS estimates are close to the GLS corrected dynamic regression estimates for large enough order of leads and lags, and we find little evidence against cointegration with the Hausman-type cointegration test. The FGLS corrected dynamic regression estimates are very close to the GLS corrected dynamic regression and the dynamic OLS estimates for large enough order of leads and lags.

In the second application of the long-run implications of the consumption-leisure choice, we found strong evidence against cointegration when nondurables are used as the measure of consumption with the Hausman-type cointegration test. We also found some evidence against cointegration when nondurables plus services are used as the measure of consumption. We estimate the RRA coefficient to be about one with both GLS corrected and FGLS corrected dynamic regression estimators.

Hence, in these first two applications, the FGLS corrected dynamic regression estimator worked well in the sense that it yielded estimates that are close to those of the estimator that seems to be correctly specified. This is consistent with our simulation results in Section 2 that the small sample efficiency loss from using FGLS corrected dynamic regression estimator is negligible for reasonable sample sizes. Therefore, we recommend the robust FGLS corrected dynamic regression estimator when the researcher is unsure about whether or not the regression error is I(0) or I(1). This is important because it is difficult to detect a small random walk component in the error term when the error is actually I(1) and to detect a small deviation from a unit root when the dominant autoregressive root is very close to one when the error is actually I(0).

We applied the Hausman-type cointegration test to log real output of pairs of countries to study output convergence across national economies. Our test results are consistent with the stylized fact of convergence clubs in that we reject the null hypothesis of cointegration between developing and developed countries while failing to reject the null hypothesis of cointegration between two developed countries. We also applied the Hausman-type cointegration test to study long-run PPP. Our test results support the view that long-run PPP holds for traded goods but not for non-traded goods.

Appendix

Appendix A: Proof of results in section 2.1

To show the distribution of the OLS estimator in regression (3), define

$$H_n = \begin{bmatrix} n & 0 \\ 0 & n^{1/2} \end{bmatrix}. \tag{24}$$

The OLS estimator for β and α can be written as

$$\begin{bmatrix} \hat{\beta}_n - \beta \\ \hat{\alpha} - \alpha \end{bmatrix} = \left[H_n^{-1} \begin{bmatrix} \sum_{t=1}^n x_t^2 & \sum_{t=1}^n x_t v_t \\ \sum_{t=1}^n x_t v_t & \sum_{t=1}^n v_t^2 \end{bmatrix} H_n^{-1} \right]^{-1} \begin{bmatrix} n^{-2} \sum_{t=1}^n x_t e_t \\ n^{-1} \sum_{t=1}^n v_t e_t \end{bmatrix}$$

For the first term,

$$\begin{bmatrix} n^{-2} \sum_{t=1}^n x_t^2 & n^{-3/2} \sum_{t=1}^n x_t v_t \\ n^{-3/2} \sum_{t=1}^n x_t v_t & n^{-1} \sum_{t=1}^n v_t^2 \end{bmatrix} \to \begin{bmatrix} \sigma_1^2 \int_0^1 V(r)^2 dr & 0 \\ 0 & \sigma_v^2 \end{bmatrix}. \tag{25}$$

where $\sigma_v^2 = E(v_t^2)$. For the second term,

$$\begin{bmatrix} n^{-2} \sum_{t=1}^n x_t e_t \\ n^{-1} \sum_{t=1}^n v_t e_t \end{bmatrix} \to_d \begin{bmatrix} \sigma_1 \sigma_2 \int_0^1 V(r) U(r) dr \\ \sigma_1 \sigma_2 \int_0^1 U(r) dV(r) \end{bmatrix}.$$

Equation (4) then follows.

To show the limit distribution of the GLS corrected estimator in regression (5), let $z_t = (\Delta x_t, \Delta v_t)$[5], then

$$\sqrt{n}(\tilde{\theta}_n - \theta) = \left[n^{-1} \sum_{t=1}^n z_t z_t' \right]^{-1} \left[n^{-1/2} \sum_{t=1}^n z_t u_t \right]. \tag{26}$$

For the first term,

$$n^{-1} \sum_{t=1}^n z_t z_t' = \begin{bmatrix} n^{-1} \sum v_t^2 & n^{-1} \sum v_t \Delta v_t \\ n^{-1} \sum v_t \Delta v_t & n^{-1} \sum \Delta v_t^2 \end{bmatrix} \to \sigma_v^2 \begin{bmatrix} 1 & 1 - \psi_v \\ 1 - \psi_v & 2(1 - \psi_v) \end{bmatrix} = \sigma_v^2 Q, \quad \textit{say},$$

where ψ_v is the first order autocorrelation coefficient of $\{v_t\}$.

For the second term, we want to show that

$$n^{-1/2} \sum_{t=1}^n z_t u_t = \begin{bmatrix} n^{-1/2} \sum v_t u_t \\ n^{-1/2} \sum \Delta v_t u_t \end{bmatrix} \to N(0, \sigma_v^2 \sigma_u^2 Q).$$

To show this, let $\boldsymbol{\lambda} = (\lambda_1, \lambda_2)'$ be an arbitrary vector of real numbers.

$$\begin{aligned} n^{-1/2} \sum_{t=1}^n \boldsymbol{\lambda}' z_t u_t &= n^{-1/2} \sum_{t=1}^n (\lambda_1 v_t + \lambda_2 \Delta v_t) u_t \\ &= \lambda_1 n^{-1/2} \sum_{t=1}^n v_t u_t + \lambda_2 n^{-1/2} \sum_{t=1}^n \Delta v_t u_t \\ &\to N(0, \sigma_v^2 \sigma_u^2 (\lambda_1^2 + 2\lambda_1 \lambda_2 (1 - \psi_v) + 2\lambda_2^2 (1 - \psi_v)) \\ &= N(0, \sigma_v^2 \sigma_u^2 \boldsymbol{\lambda}' Q \boldsymbol{\lambda}) \end{aligned}$$

[5]Through this paper, we always use z_t to denote the vector of independent variables and let θ to denote the vector of parameters (but keep in mind that we are mostly interested in β). Note that in different regressions, those symbols denote different variables.

Hence, for the quantity defined in (26), we have the limiting distribution given in (6).

To derive the limiting distribution for the FGLS corrected estimator, we first derive the limiting distribution for $\hat{\rho}_n$ in regression (8). Write the process of $\hat{R}_t$ as

$$\begin{aligned}\hat{R}_t &= y_t - \hat{\beta}x_t - \hat{\alpha}v_t \\ &= y_{t-1} - \hat{\beta}x_{t-1} - \hat{\alpha}v_{t-1} + [(y_t - y_{t-1}) - \hat{\beta}_n(x_t - x_{t-1}) - \hat{\alpha}(v_t - v_{t-1})] \\ &= \hat{R}_{t-1} + [(\beta - \hat{\beta}_n)v_t + (\alpha - \hat{\alpha})(v_t - v_{t-1}) + u_t] \\ &= \hat{R}_{t-1} + h_t, \quad say.\end{aligned}$$

>From this expression, we can see that $\hat{R}_t$ is a unit root process with serially correlated error h_t. Then the OLS estimator $\hat{\rho}_n$ can be written as

$$\hat{\rho}_n = \frac{\sum_{t=1}^n \hat{R}_t\hat{R}_{t-1}}{\sum_{t=1}^n \hat{R}_{t-1}^2} = 1 + \frac{\sum_{t=1}^n \hat{R}_{t-1}h_t}{\sum_{t=1}^n \hat{R}_{t-1}^2}$$

To derive the limit of $\hat{\rho}_n$, write

$$\hat{R}_t = y_t - \hat{\beta}_n x_t - \hat{\alpha}_n v_t = (\beta - \hat{\beta}_n)x_t + (\alpha - \hat{\alpha}_n)v_t + e_t. \tag{27}$$

Hence the denominator

$$\begin{aligned}\hat{R}_t^2 &= (\beta - \hat{\beta}_n)x_t^2 + e_t^2 + 2(\beta - \hat{\beta}_n)x_te_t \\ &\quad +(\alpha - \hat{\alpha}_n)^2v_t^2 + 2(\alpha - \hat{\alpha}_n)(\beta - \hat{\beta}_n)x_tv_t + 2(\alpha - \hat{\alpha}_n)e_tv_t,\end{aligned}$$

where we can see that the sum of the first line diverges faster as they are products of I(1) variables. In particular,

$$\begin{aligned}n^{-2}\sum_{t=1}^n \hat{R}_t^2 &= (\beta - \hat{\beta}_n)^2 n^{-2}\sum_{t=1}^n x_t^2 + n^{-2}\sum_{t=1}^n e_t^2 + 2(\beta - \hat{\beta}_n)n^{-2}\sum_{t=1}^n x_te_t + o_p(1) \\ &\to_d \xi^2\sigma_1^2\int_0^1 V(r)^2dr + \sigma_2^2\int_0^1 U(r)^2dr + 2\xi\sigma_1\sigma_2\int_0^1 V(r)U(r)dr \equiv \zeta.\end{aligned}$$

For the numerator,

$$\hat{R}_{t-1}h_t = [(\beta - \hat{\beta}_n)x_{t-1} + (\alpha - \hat{\alpha})v_{t-1} + e_{t-1}][(\beta - \hat{\beta}_n + \alpha - \hat{\alpha})v_t - (\alpha - \hat{\alpha})v_{t-1} + u_t]$$

The sum of all the terms of products in this expression converges when normed with n^{-1}. We omit the details here as we will not make use of the exact distribution of $\hat{\rho}_n$. Plug in the limits of all the terms, we can write

$$n^{-1}\sum_{t=1}^n \hat{R}_{t-1}h_t \to \eta, \quad say.$$

Hence,

$$n(\hat{\rho}_n - 1) = \frac{n^{-1}\sum_{t=1}^n \hat{R}_{t-1}h_t}{n^{-2}\sum_{t=1}^n \hat{R}_{t-1}^2} \to_d \frac{\eta}{\zeta}. \tag{28}$$

Actually, in our following computations, all we need to know is that

$$\hat{\rho}_n - 1 = o_p(1) \quad \text{and} \quad n(\hat{\rho}_n - 1) = O_p(1).$$

Below, we show how to derive the limit distribution for $\tilde{\theta}$. For the sequence of $\tilde{y}_t$, we can write it as

$$
\begin{aligned}
\tilde{y}_t &= y_t - \hat{\rho}_n y_{t-1} \\
&= \beta x_t + \alpha v_t + e_t - \hat{\rho}_n(\beta x_{t-1} + \alpha v_{t-1} + e_{t-1}) \\
&= \beta(x_t - \hat{\rho}_n x_{t-1}) + \alpha(v_t - \hat{\rho}_n v_{t-1}) + (e_t - e_{t-1}) + (1-\hat{\rho}_n)e_{t-1} \\
&= \beta\tilde{x}_t + \alpha\tilde{v}_t + u_t + (1-\hat{\rho}_n)e_{t-1} \\
&= z_t'\theta + u_t + (1-\hat{\rho}_n)e_{t-1}
\end{aligned}
$$

Now, we can write

$$
\hat{\theta} - \theta = \left[\sum_{t=1}^n z_t z_t'\right]^{-1}\left[\sum_{t=1}^n z_t[u_t + (1-\hat{\rho}_n)e_{t-1}]\right]. \tag{29}
$$

For the first term:

$$
\sum_{t=1}^n z_t z_t' = \begin{bmatrix} \sum \tilde{x}_t^2 & \sum \tilde{x}_t\tilde{v}_t \\ \sum \tilde{x}_t\tilde{v}_t & \sum \tilde{v}_t^2 \end{bmatrix}
$$

The asymptotics of each term follows. First,

$$
\begin{aligned}
\sum_{t=1}^n \tilde{x}_t^2 &= \sum_{t=1}^n (x_t - \hat{\rho}_n x_{t-1})^2 \\
&= \sum_{t=1}^n [(1-\hat{\rho}_n)x_{t-1} + v_t]^2 \\
&= (1-\hat{\rho}_n)^2 \sum_{t=1}^n x_{t-1}^2 + 2(1-\hat{\rho}_n)\sum_{t=1}^n x_{t-1}v_t + \sum_{t=1}^n v_t^2.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
n^{-1}\sum_{t=1}^n \tilde{x}_t^2 &= n(1-\hat{\rho}_n)^2\left(n^{-2}\sum_{t=1}^n x_{t-1}^2\right) + 2(1-\hat{\rho}_n)\left(n^{-1}\sum_{t=1}^n x_{t-1}v_t\right) + n^{-1}\sum_{t=1}^n v_t^2 \\
&= n^{-1}\sum_{t=1}^n v_t^2 + o_p(1) \\
&\to \sigma_v^2
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
n^{-1}\sum \tilde{x}_t\tilde{v}_t &= n^{-1}\sum_{t=1}^n v_t^2 - \hat{\rho}_n\left(n^{-1}\sum_{t=1}^n v_t v_{t-1}\right) + o_p(1) \\
&\to_p \sigma_v^2(1-\psi_v).
\end{aligned}
$$

Finally, $n^{-1}\sum_{t=1}^n \tilde{v}_t^2 \to 2\sigma_v^2(1-\psi_v)$. Hence,

$$
n^{-1}\sum_{t=1}^n z_t z_t' \to_p \sigma_v^2 \begin{bmatrix} 1 & 1-\psi_v \\ 1-\psi_v & 2(1-\psi_v) \end{bmatrix} = \sigma^2 Q. \tag{30}
$$

Now, consider the second term in (29)

$$
\sum_{t=1}^n z_t[u_t + (1-\hat{\rho}_n)e_{t-1}] = \begin{bmatrix} \sum_{t=1}^n \tilde{x}_t[u_t + (1-\hat{\rho}_n)e_{t-1}] \\ \sum_{t=1}^n \tilde{v}_t[u_t + (1-\hat{\rho}_n)e_{t-1}] \end{bmatrix}.
$$

It is not hard to see that $n^{-1}\sum_{t=1}^{n} z_t[u_t+(1-\hat{\rho}_n)e_{t-1}] \to_p 0$. Intuitively, $\tilde{x}_t$ behaves asymptotically like v_t, while u and v are independent by assumption. Again, our remaining task is to show that

$$n^{-1/2}\sum_{t=1}^{n} z_t[u_t+(1-\hat{\rho}_n)e_{t-1}] \to N(\mathbf{0}, \sigma_v^2\sigma_u^2 Q). \tag{31}$$

This can be shown in the same way as in the proof for (6). Combine (31) with (30), we obtain the limit distribution for $\tilde{\theta}$, as given in (11).

Appendix B: Some extensions

So far we have assumed that there is no constant term or deterministic time trends in the DGP of y_t. If there is a constant term, e.g.

$$y_t = \delta + \beta x_t + \alpha v_t + u_t.$$

Correspondingly, in the OLS estimation, we also include a constant.

The limit of $\hat{\beta}_n$ and $\hat{\alpha}_n$ are similar as in the case without constant, except that we have demeaned Brownian motions instead of standard Brownian motions in the limit. Since this is still a spurious regression, the estimator of the constant term diverges as was shown in Phillips (1986). Here

$$\begin{aligned}\hat{\delta}_n &= \bar{y} - \hat{\beta}_n\bar{x} - \hat{\alpha}_n\bar{v}_t \\ &= \delta_0 + (\hat{\beta}_n - \beta)\bar{x} + (\hat{\alpha}_n - \alpha)\bar{v} + \bar{e}\end{aligned}$$

Hence

$$\begin{aligned}n^{-1/2}\hat{\delta}_n &= n^{-1/2}\delta_0 + (\hat{\beta}_n - \beta)n^{-3/2}\sum_{t=1}^{n} x_t + (\hat{\alpha}_n - \alpha n^{-3/2}\sum_{t=1}^{n} v_t + n^{-3/2}\sum_{t=1}^{n} e_t \\ &\to_d \bar{\xi}\sigma_1\int_0^1 V(r)dr + \sigma_2\int_0^1 U(r)dr.\end{aligned}$$

where we let $\bar{\xi}$ to denote the limit of $\hat{\beta}_n - \beta$. Next, if we do GLS or the differenced regression, the constant is canceled so we could have the same limit result as is given by (6). Finally, consider the Cochrane-Orcutt feasible GLS estimation. Still let $\hat{R}_t$ denote the OLS residual

$$\hat{R}_t = y_t - \hat{\delta}_n - \hat{\beta}_n x_t - \hat{\alpha}_n v_t.$$

Then do another OLS estimation in

$$\hat{R}_t = \hat{\rho}_n\hat{R}_{t-1} + error.$$

Write

$$\begin{aligned}\hat{R}_t &= y_t - \hat{\delta}_n - \hat{\beta}_n x_t - \hat{\alpha}_n v_t \\ &= \hat{R}_{t-1} + [(\beta - \hat{\beta}_n)v_t + (\alpha - \hat{\alpha}_n)(v_t - v_{t-1}) + u_t] \\ &= \hat{R}_{t-1} + h_t\end{aligned}$$

which takes the same form as in the previous section where no constant is included. Hence we still have

$$\hat{\rho}_n - 1 = \frac{\sum_{t=1}^{n}\hat{R}_{t-1}h_t}{\sum_{t=1}^{n}\hat{R}_{t-1}^2}.$$

Write the process of $\hat{R}_t$ as:

$$\hat{R}_t = y_t - \hat{\delta}_n - \hat{\beta}_n x_t - \hat{\alpha}_n v_t = (\beta - \hat{\beta}_n)(x_t - \bar{x}) + (\alpha - \hat{\alpha}_n)(v_t - \bar{v}) + (e_t - \bar{e}). \quad (32)$$

Comparing equation (32) with (27), the only difference in (32) is that all terms are subtracted by their sample means. This will correspond to demeaned Brownian motions instead of standard Brownian motions in the limit of the distribution of $\hat{\rho}_n$. Using similar methods as in the previous section, we can show that

$$\hat{\rho}_n - 1 = o_p(1) \quad \text{and} \quad n(\hat{\rho}_n - 1) = O_p(1).$$

Next, conduct the Cochrane-Orcutt transformation as in (9), and consider the OLS estimator in the regression

$$\tilde{y}_t = \tilde{\beta}_n \tilde{x}_t + \tilde{\alpha}_n \tilde{v}_t + error.$$

Define $z_t = (\tilde{x}_t, \tilde{v}_t)'$ and $\theta = (\beta, \alpha)'$, then

$$\tilde{\theta}_n = \left[\sum_{t=1}^{n} z_t z_t'\right]^{-1} \left[\sum_{t=1}^{n} z_t \tilde{y}_t\right].$$

For $\tilde{y}_t$, write

$$\begin{aligned} \tilde{y}_t &= y_t - \hat{\rho}_n y_{t-1} \\ &= (1 - \hat{\rho}_n)\delta + \beta\tilde{x}_t + \alpha\tilde{v}_t + u_t + (1 - \hat{\rho}_n)e_{t-1} \\ &= z_t'\theta + (1 - \hat{\rho}_n)\delta + u_t + (1 - \hat{\rho}_n)e_{t-1} \end{aligned}$$

Hence we can write

$$\hat{\theta}_n - \theta = \left[\sum_{t=1}^{n} z_t z_t'\right]^{-1} \left[\sum_{t=1}^{n} z_t[(1 - \hat{\rho}_n)\delta + u_t + (1 - \hat{\rho}_n)e_{t-1}]\right]. \quad (33)$$

The only difference of (33) with (29) is that we have a term $(1 - \hat{\rho}_n)\delta$ here. However, since δ is just a finite constant and $n(1 - \hat{\rho}_n) = O_p(1)$, this term disappears in the limit. Therefore, using the Cochrane-Orcutt transformation, the limiting distribution of the estimators are the same no matter whether we have or have not a constant in the data generating process of the data. So we have the same result as given by (6).

Appendix C: Proof of results in section 2.2

To show the limit distribution of the dynamic OLS estimator in the cointegration, using the matrix H_n defined in (24), we can write

$$\begin{bmatrix} n(\hat{\beta}_n - \beta_0) \\ n^{1/2}(\hat{\alpha}_n - \alpha_0) \end{bmatrix} = \left[H_n^{-1} \begin{bmatrix} \sum_{t=1}^n x_t^2 & \sum_{t=1}^n x_t v_t \\ \sum_{t=1}^n x_t v_t & \sum_{t=1}^n v_t^2 \end{bmatrix} H_n^{-1}\right]^{-1} \begin{bmatrix} n^{-1}\sum_{t=1}^n x_t u_t \\ n^{-1/2}\sum_{t=1}^n v_t u_t \end{bmatrix}$$

For the first term on the right hand side,

$$\begin{bmatrix} n^{-2}\sum_{t=1}^n x_t^2 & n^{-3/2}\sum_{t=1}^n x_t v_t \\ n^{-3/2}\sum_{t=1}^n x_t v_t & n^{-1}\sum_{t=1}^n v_t^2 \end{bmatrix} \to \begin{bmatrix} \sigma_1^2 \int_0^1 V(r)^2 dr & 0 \\ 0 & \sigma_v^2 \end{bmatrix}. \quad (34)$$

so the estimator of the I(1) and I(0) components are asymptotically independent. For the second term on the right hand side,

$$\begin{bmatrix} n^{-1}\sum_{t=1}^{n} x_t u_t \\ n^{-1/2}\sum_{t=1}^{n} v_t u_t \end{bmatrix} \to_d \begin{bmatrix} \sigma_1\sigma_2\int_0^1 V(r)dU(r) \\ N(0,\sigma_v^2\sigma_u^2) \end{bmatrix}. \qquad (35)$$

Equation (14) then follows.

To show the limit distribution for FGLS corrected estimator in regression (16), write

$$\begin{aligned} & n^{-1}\sum_{t=1}^{n}\hat{u}_t^2 \\ = \;& n^{-1}\sum_{t=1}^{n}u_t^2 + 2\left(n^{-1}H_n^{-1}\sum_{t=1}^{n}u_t z_t'\right)H_n(\theta-\hat{\theta}) + H_n(\theta-\hat{\theta})'\left(n^{-1}H_n^{-1}\sum_{t=1}^{n}z_t z_t'\right)H_n(\theta-\hat{\theta}) \\ = \;& n^{-1}\sum_{t=1}^{n}u_t^2 + o_p(1) \to \sigma_u^2 \end{aligned}$$

Similarly, we can show that

$$n^{-1}\sum_{t=1}^{n}\hat{u}_t\hat{u}_{t-1} = n^{-1}\sum_{t=1}^{n}u_t u_{t-1} + o_p(1) \to \psi_u\sigma_u^2.$$

$$\hat{\rho}_n = \frac{n^{-1}\sum_{t=1}^{n}\hat{u}_t u_{t-1}}{n^{-1}\sum_{t=1}^{n}\hat{u}_t^2} \to_p \psi_u$$

Conduct the Cochrane-Orcutt transformation (9) and estimate

$$\tilde{y}_t = \tilde{\beta}\tilde{x}_t + \tilde{\alpha}\tilde{v}_t + error.$$

For the sequence of $\tilde{y}_t$, we can write it as

$$\tilde{y}_t = \beta\tilde{x}_t + \alpha\tilde{v}_t + u_t - \hat{\rho}_n u_{t-1}.$$

Using the same weight matrix H_n, write

$$\begin{bmatrix} n(\tilde{\beta}-\beta) \\ n^{1/2}(\tilde{\alpha}-\alpha) \end{bmatrix} = \left[H_n^{-1}\begin{bmatrix} \sum_{t=1}^{n}\tilde{x}_t^2 & \sum_{t=1}^{n}\tilde{x}_t\tilde{v}_t \\ \sum_{t=1}^{n}\tilde{x}_t\tilde{v}_t & \sum_{t=1}^{n}\tilde{v}_t^2 \end{bmatrix}H_n^{-1}\right]^{-1}\begin{bmatrix} n^{-1}\sum_{t=1}^{n}\tilde{x}_t(u_t-\hat{\rho}_n u_{t-1}) \\ n^{-1/2}\sum_{t=1}^{n}\tilde{v}_t(u_t-\hat{\rho}_n u_{t-1}) \end{bmatrix} \qquad (36)$$

There are three different elements in the first term. Using similar methods as in the earlier proofs, we can show that

$$\begin{aligned} n^{-2}\sum_{t=1}^{n}\tilde{x}_t^2 &\to_d (1-\psi_u)^2\sigma_1^2\int_0^1 V(r)^2dr \\ n^{-2/3}\sum_{t=1}^{n}\tilde{x}_t\tilde{v}_t &\to_p 0 \\ n^{-1}\sum_{t=1}^{n}\tilde{v}^2 &\to (1-2\psi_u\psi_v+\psi_u^2)\sigma_v^2 \end{aligned}$$

Hence, the limit of the first item in (36) is

$$H_n^{-1}\left[\sum_{t=1}^{n} z_t z_t'\right] H_n^{-1} = \begin{bmatrix} n^{-2}\sum_{t=1}^{n}\tilde{x}^2 & n^{-3/2}\sum_{t=1}^{n}\tilde{x}\tilde{v} \\ n^{-3/2}\sum_{t=1}^{n}\tilde{x}\tilde{v} & n^{-1}\sum_{t=1}^{n}\tilde{v}^2 \end{bmatrix}$$
$$\rightarrow_d \begin{bmatrix} (1-\psi_u)^2\sigma_1^2\int_0^1 V(r)^2 dr & 0 \\ 0 & (1-2\psi_u\psi_v+\psi_u^2)\sigma_v^2 \end{bmatrix}.$$

Next, consider the second term in (36). Actually, we are only interested in the first element,

$$\begin{aligned} & n^{-1}\sum_{t=1}^{n}\tilde{x}(u_t-\hat{\rho}_n u_{t-1}) \\ = \; & n^{-1}\sum_{t=1}^{n}(v_t+(1-\hat{\rho}_n)x_{t-1})(u_t-\hat{\rho}_n u_{t-1}) \\ = \; & n^{-1}\sum_{t=1}^{n}v_t u_t-\hat{\rho}_n\sum_{t=1}^{n}v_t u_{t-1}+(1-\hat{\rho}_n)n^{-1}\sum_{t=1}^{n}x_{t-1}u_t-\hat{\rho}_n(1-\hat{\rho}_n)n^{-1}\sum_{t=1}^{n}x_{t-1}u_{t-1} \\ \rightarrow \; & (1-\psi_u)^2\sigma_1\sigma_2\int_0^1 V(r)dU(r). \end{aligned}$$

Therefore, we obtain the limit distribution for $\tilde{\beta}_n$ as given in (17).

Appendix D: Data descriptions

In the first two empirical analyses, we use the same data set as in Stock and Watson (1993, page 817) for the U.S. money demand, and the data set of Cooley and Ogaki (1996, page 127) for the long-run intertemporal elasticity of substitution. Readers are referred to the original work for further details on data.

Per capita output series are extracted from the *Penn World Tables* of Robert Summers and Alan Heston (1991). They are annual data on real GDP per capita (RGDPCH) for four developing countries (Columbia, Ecuador, Egypt, and Pakistan) along with four industrial countries (Germany, Luxemburg, New Zealand, and Switzerland) over the period of 1950-1992.

In the PPP application, we borrow the dataset from Kim (2004) who constructed the real exchange rate for total consumption using the general price deflator, and the real exchange rate for traded and non-traded goods using implicit deflators for non-service consumption and service consumption, respectively. Data are quarterly observations spanning from 1974 Q1 to 1998 Q4. The exchange rates for Canada, France, Italy, Japan, the United Kingdom, and the United States are taken from the International Financial Statistics (IFS) CD-ROM, and bilateral real exchange rates of traded and non-traded goods classified by type and total consumption deflators from the Quarterly National Accounts and Data Stream are studied.

Table 1: Application to Long Run U.S. Money Demand

Estimator	k	Equation 1		Equation 2		Equation 3	
		$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\beta}$	$\hat{\gamma}$
		AR(1) Error Term					
DOLS	0	0.944 (0.054)	-0.090 (0.015)	0.889 (0.057)	-0.308 (0.058)	0.850 (0.085)	0.906 (0.280)
	1	0.958 (0.048)	-0.096 (0.014)	0.884 (0.046)	-0.313 (0.045)	0.843 (0.066)	0.915 (0.216)
	2	0.970 (0.051)	-0.101 (0.014)	0.879 (0.044)	-0.320 (0.043)	0.837 (0.072)	0.941 (0.233)
	3	0.975 (0.055)	-0.104 (0.015)	0.871 (0.036)	-0.328 (0.035)	0.832 (0.062)	0.975 (0.205)
	4	0.967 (0.054)	-0.108 (0.015)	0.855 (0.029)	-0.334 (0.028)	0.824 (0.065)	0.995 (0.215)
	BIC [lag]	[3]		[5]		[5]	
GLS-corrected	0	0.407 (0.081)	-0.014 (0.004)	0.419 (0.079)	-0.086 (0.022)	0.388 (0.078)	0.300 (0.082)
	1	0.654 (0.119)	-0.025 (0.010)	0.685 (0.115)	-0.177 (0.046)	0.643 (0.115)	0.506 (0.148)
	2	0.837 (0.134)	-0.050 (0.013)	0.848 (0.130)	-0.248 (0.053)	0.787 (0.133)	0.620 (0.161)
	3	0.856 (0.145)	-0.067 (0.017)	0.884 (0.140)	-0.289 (0.061)	0.816 (0.146)	0.725 (0.185)
	4	0.962 (0.161)	-0.086 (0.022)	0.898 (0.151)	-0.283 (0.067)	0.811 (0.153)	0.654 (0.195)
	BIC [lag]	[2]		[2]		[5]	
FGLS-corrected AR(1)	0	0.942 (0.052)	-0.083 (0.023)	0.893 (0.049)	-0.290 (0.079)	0.858 (0.071)	0.850 (0.435)
	1	0.888 (0.040)	-0.065 (0.009)	0.872 (0.035)	-0.278 (0.030)	0.815 (0.045)	0.744 (0.115)
	2	0.940 (0.045)	-0.081 (0.010)	0.901 (0.036)	-0.309 (0.031)	0.840 (0.054)	0.797 (0.128)
	3	0.980 (0.050)	-0.096 (0.011)	0.905 (0.029)	-0.330 (0.026)	0.851 (0.046)	0.912 (0.124)
	4	1.010 (0.045)	-0.108 (0.011)	0.886 (0.025)	-0.333 (0.023)	0.833 (0.051)	0.895 (0.133)
	BIC [lag]	[4]		[5]		[5]	
FGLS-corrected AR(2)	0	0.942 (0.052)	-0.083 (0.023)	0.893 (0.049)	-0.290 (0.079)	0.858 (0.071)	0.850 (0.435)
	1	0.900 (0.039)	-0.069 (0.009)	0.872 (0.038)	-0.276 (0.031)	0.809 (0.049)	0.722 (0.118)
	2	0.948 (0.042)	-0.086 (0.010)	0.894 (0.033)	-0.312 (0.029)	0.839 (0.049)	0.830 (0.131)
	3	0.991 (0.044)	-0.100 (0.011)	0.907 (0.029)	-0.332 (0.026)	0.853 (0.050)	0.903 (0.128)
	4	1.012 (0.042)	-0.109 (0.010)	0.889 (0.026)	-0.335 (0.023)	0.827 (0.061)	0.856 (0.142)
	BIC [lag]	[3]		[5]		[5]	
HAUSMAN-TEST	0	289.892‡		113.485‡		79.188‡	
	1	54.427‡		10.203‡		9.946‡	
	2	15.059‡		1.867		3.978	
	3	4.690†		0.460		1.852	
	4	1.112		0.820		3.102	

Note:

$$
\begin{aligned}
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma i_t + u_t, \quad \text{(equation 1)} \\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln(i_t) + u_t, \quad \text{(equation 2)} \\
\ln\left(\frac{M}{P}\right)_t &= \alpha + \beta \ln(y_t) + \gamma \ln\left[\frac{1+i_t}{i_t}\right] + u_t. \quad \text{(equation 3)}
\end{aligned}
$$

'GLS-corrected (FGLS-corrected)' denotes the GLS (FGLS) corrected dynamic regression estimator. Figures in the parenthesis represent standard errors. 'k' denotes the maximum length of leads and lags. In FGLS corrected estimation, the serial correlation coefficient in error term is estimated before being applied to the Cochrane-Orcutt transformation whereas it is assumed to be unity in GLS corrected estimation which is analogous to regressing the first difference of variables without constant term. AR(1) error term in the FGLS corrected estimation is structured as $u_t = \rho u_{t-1} + \epsilon_t$ while AR(2) error term is $u_t = \delta_1 u_{t-1} + \delta_2 u_{t-2} + \epsilon_t$. Hausman test represents the Hausman-type cointegration test as stipulated in section 2.4. The test statistic is constructed as $(\hat{\Gamma}_{OLS} - \tilde{\Gamma}_{GLS})\Sigma(\hat{\Gamma}_{OLS} - \tilde{\Gamma}_{GLS})' \rightarrow \chi^2(2)$ where $\Gamma = [\beta, \gamma]$ and $\Sigma = \begin{bmatrix} var(\tilde{\beta}_{GLS} & cov(\tilde{\beta}_{GLS}, \tilde{\gamma}_{GLS}) \\ cov(\tilde{\beta}_{GLS}, \tilde{\gamma}_{GLS}) & var(\tilde{\gamma}_{GLS}) \end{bmatrix}$. The critical values of $\chi^2(2)$ are 4.61, 5.99 and 9.21 for 10%, 5%, and 1% significance levels. ‡(†) represents that the null hypothesis of $\hat{\beta}_{OLS} = \tilde{\beta}_{GLS}$ can be rejected at 5% (10%).

Table 2: Application to Preference Parameter (β) Estimation

Estimator	k	ND	NDS
DOLS	0	1.865 (0.218)	1.102 (0.052)
	1	1.865 (0.192)	1.103 (0.052)
	2	1.870 (0.181)	1.102 (0.042)
	3	1.873 (0.193)	1.100 (0.041)
	4	1.877 (0.204)	1.099 (0.036)
	5	1.880 (0.196)	1.095 (0.033)
	BIC [lag]	[0]	[0]
GLS-corrected	0	0.222 (0.061)	0.480 (0.066)
	1	0.628 (0.091)	0.796 (0.080)
	2	0.720 (0.102)	0.855 (0.084)
	3	0.850 (0.110)	0.924 (0.085)
	4	0.963 (0.117)	0.967 (0.087)
	5	1.041 (0.123)	0.995 (0.088)
	BIC [lag]	[3]	[3]
FGLS-corrected AR(1)	0	1.874 (0.151)	1.106 (0.095)
	1	0.983 (0.087)	0.912 (0.041)
	2	1.160 (0.089)	0.952 (0.034)
	3	1.199 (0.097)	0.978 (0.032)
	4	1.207 (0.107)	0.998 (0.030)
	5	1.391 (0.104)	1.018 (0.028)
	BIC [lag]	[4]	[3]
FGLS-corrected AR(2)	0	1.874 (0.151)	1.106 (0.095)
	1	0.928 (0.086)	0.932 (0.035)
	2	1.101 (0.092)	0.911 (0.038)
	3	1.155 (0.099)	0.975 (0.031)
	4	1.280 (0.107)	1.000 (0.029)
	5	1.346 (0.110)	1.009 (0.029)
	BIC [lag]	[4]	[3]
HAUSMAN-TEST	0	737.38‡	87.81‡
	1	185.37‡	14.56‡
	2	125.88‡	8.56‡
	3	86.79‡	4.30†
	4	60.53‡	2.31
	5	46.22‡	1.30

Note: Results for $W(t) = \frac{v'(l(t))}{C(t)^{-\beta}}$. 'GLS-corrected (FGLS-corrected)' denotes the GLS (FGLS) corrected dynamic regression estimator. Figures in the parenthesis represent standard errors. 'k' denotes the maximum length of leads and lags. In FGLS corrected estimation, the serial correlation coefficient in error term is estimated before being applied to the Cochrane-Orcutt transformation whereas it is assumed to be unity in GLS corrected estimation which is analogous to regressing the first difference of variables without constant term. AR(1) error term in the FGLS is structured as $u_t = \rho u_{t-1} + \epsilon_t$ while AR(2) error term is $u_t = \delta_1 u_{t-1} + \delta_2 u_{t-2} + \epsilon_t$. Hausman test represents the Hausman-type cointegration test as stipulated in section 2.4. The test statistic is constructed as $\frac{(\hat{\beta}_{OLS} - \tilde{\beta}_{GLS})^2)}{Var(\tilde{\beta}_{GLS})} \rightarrow \chi^2(1)$. The critical values of $\chi^2(1)$ are 2.71, 3.84 and 6.63 for ten, five, and one percent significance level. ‡(†) asterisk represent that the null hypothesis of $\hat{\beta}_{OLS} = \tilde{\beta}_{GLS}$ can be rejected at 5% (10%) significance level.

Table 3-1: Application to Output Convergence (Regressand: developing countries; Regressor: industrial countries)

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
COL	GER	0	0.680 (0.156)	0.464 (0.091)	0.635 (0.030)	5.678‡
		1	0.755 (0.179)	0.501 (0.111)	0.963 (0.109)	5.221‡
		2	0.876 (0.245)	0.490 (0.130)	1.087 (0.121)	8.850‡
		3	0.963 (0.235)	0.409 (0.154)	0.979 (0.058)	12.984‡
		4	1.105 (0.267)	0.607 (0.169)	1.029 (0.067)	8.635‡
		BIC	[0]	[0]	[3]	
	LUX	0	0.923 (0.144)	0.322 (0.097)	0.915 (0.009)	38.115‡
		1	0.953 (0.208)	0.642 (0.151)	0.669 (0.143)	4.276‡
		2	0.993 (0.136)	0.633 (0.167)	0.892 (0.103)	4.633‡
		3	1.035 (0.118)	0.657 (0.193)	1.030 (0.095)	3.841‡
		4	1.087 (0.093)	0.739 (0.223)	1.119 (0.075)	2.443
		BIC	[1]	[1]	[1]	
	NZL	0	1.218 (0.454)	0.363 (0.117)	1.219 (0.053)	53.309‡
		1	1.213 (0.368)	0.650 (0.199)	0.931 (0.270)	7.969‡
		2	1.203 (0.309)	0.600 (0.241)	1.425 (0.221)	6.245‡
		3	1.218 (0.330)	0.748 (0.261)	1.608 (0.235)	3.232†
		4	1.178 (0.328)	0.788 (0.294)	1.801 (0.191)	1.760
		BIC	[0]	[0]	[1]	
	SWI	0	0.972 (0.349)	0.493 (0.121)	0.969 (0.021)	15.639‡
		1	0.967 (0.343)	0.663 (0.162)	0.875 (0.284)	3.498‡
		2	0.948 (0.327)	0.662 (0.189)	1.350 (0.248)	2.289
		3	0.901 (0.291)	0.567 (0.212)	1.435 (0.175)	2.496
		4	0.926 (0.437)	0.576 (0.210)	1.491 (0.212)	2.769‡
		BIC	[0]	[0]	[1]	
ECU	GER	0	0.784 (0.311)	0.344 (0.159)	0.779 (0.009)	7.639‡
		1	0.816 (0.400)	0.472 (0.191)	0.742 (0.413)	3.228†
		2	0.873 (0.583)	0.472 (0.231)	0.956 (0.503)	3.008†
		3	0.913 (0.596)	0.357 (0.273)	1.259 (0.289)	4.155‡
		4	1.041 (0.743)	0.501 (0.322)	1.245 (0.252)	2.815†
		BIC	[0]	[0]	[1]	
	LUX	0	1.078 (0.697)	0.129 (0.157)	1.067 (0.034)	36.651‡
		1	1.158 (0.661)	0.393 (0.263)	0.085 (0.293)	8.442‡
		2	1.261 (0.410)	0.397 (0.291)	0.631 (0.246)	8.806‡
		3	1.347 (0.696)	0.734 (0.301)	0.573 (0.407)	4.153‡
		4	1.438 (0.518)	0.848 (0.343)	1.076 (0.362)	2.963†
		BIC	[0]	[0]	[1]	
	NZL	0	1.496 (0.418)	0.299 (0.182)	1.505 (0.091)	43.377‡
		1	1.499 (0.446)	0.608 (0.331)	0.988 (0.366)	7.230‡
		2	1.487 (0.536)	0.662 (0.387)	1.087 (0.436)	4.538‡
		3	1.520 (0.573)	0.795 (0.409)	1.367 (0.465)	3.148†
		4	1.535 (0.631)	0.928 (0.462)	1.934 (0.486)	1.725
		BIC	[0]	[0]	[1]	
	SWI	0	1.155 (0.439)	0.287 (0.202)	1.176 (0.052)	18.378‡
		1	1.139 (0.405)	0.368 (0.294)	0.819 (0.321)	6.864‡
		2	1.086 (0.400)	0.418 (0.326)	1.139 (0.294)	4.208‡
		3	1.022 (0.340)	0.470 (0.352)	1.354 (0.235)	2.455
		4	1.061 (0.378)	0.563 (0.352)	1.332 (0.246)	2.010
		BIC	[0]	[0]	[1]	

Note: See the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). Regression equation is $ln(y_{DEV}) = \alpha + \beta ln(y_{IND}) + \varepsilon$.

Table 3-1: Continued–

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
EGT	GER	0	0.916 (0.108)	0.372 (0.133)	0.851 (0.009)	16.815‡
		1	1.011 (0.145)	0.550 (0.143)	1.224 (0.094)	10.387‡
		2	1.125 (0.256)	0.663 (0.165)	1.278 (0.140)	7.792‡
		3	1.160 (0.342)	0.691 (0.192)	1.263 (0.143)	5.965‡
		4	1.233 (0.449)	0.767 (0.223)	1.227 (0.151)	4.369‡
		BIC	[1]	[1]	[1]	
	LUX	0	1.228 (0.270)	0.059 (0.136)	1.203 (0.056)	73.478‡
		1	1.313 (0.362)	0.568 (0.198)	0.596 (0.199)	14.176‡
		2	1.390 (0.141)	0.611 (0.229)	1.239 (0.107)	11.574‡
		3	1.436 (0.144)	0.831 (0.264)	1.280 (0.122)	5.253‡
		4	1.476 (0.100)	1.025 (0.287)	1.396 (0.091)	2.469
		BIC	[5]	[1]	[1]	
	NZL	0	1.674 (0.317)	0.299 (0.155)	1.658 (0.115)	78.339‡
		1	1.737 (0.301)	0.829 (0.257)	1.666 (0.260)	12.516‡
		2	1.794 (0.388)	1.090 (0.298)	1.653 (0.331)	5.570‡
		3	1.837 (0.456)	1.355 (0.324)	1.940 (0.412)	2.210
		4	1.852 (0.594)	1.473 (0.359)	2.719 (0.497)	1.109
		BIC	[3]	[1]	[4]	
	SWI	0	1.344 (0.243)	0.388 (0.169)	1.311 (0.075)	32.144‡
		1	1.422 (0.266)	0.666 (0.229)	1.464 (0.246)	10.889‡
		2	1.498 (0.387)	0.991 (0.235)	1.571 (0.372)	4.627‡
		3	1.537 (0.483)	1.080 (0.268)	1.636 (0.462)	2.909†
		4	1.756 (0.453)	1.176 (0.297)	2.005 (0.254)	3.820†
		BIC	[2]	[2]	[2]	
PAK	GER	0	0.746 (0.112)	0.328 (0.155)	0.696 (0.007)	7.223‡
		1	0.858 (0.144)	0.454 (0.177)	0.930 (0.096)	5.223†
		2	0.981 (0.193)	0.526 (0.213)	0.994 (0.103)	4.550†
		3	1.020 (0.269)	0.727 (0.241)	1.066 (0.131)	1.479
		4	0.999 (0.339)	0.771 (0.283)	0.985 (0.157)	0.649
		BIC	[0]	[0]	[1]	
	LUX	0	0.980 (0.233)	0.267 (0.148)	0.972 (0.031)	23.103‡
		1	1.014 (0.257)	0.315 (0.245)	0.649 (0.188)	8.119‡
		2	1.055 (0.167)	0.330 (0.282)	0.935 (0.127)	6.606‡
		3	1.059 (0.196)	0.617 (0.321)	0.932 (0.164)	1.898
		4	1.038 (0.169)	0.809 (0.355)	0.937 (0.140)	0.417
		BIC	[0]	[0]	[1]	
	NZL	0	1.354 (0.318)	0.294 (0.177)	1.347 (0.079)	35.760‡
		1	1.377 (0.268)	0.675 (0.314)	1.392 (0.258)	5.007‡
		2	1.382 (0.273)	0.598 (0.381)	1.453 (0.251)	4.244‡
		3	1.447 (0.311)	0.936 (0.409)	1.454 (0.303)	1.564
		4	1.538 (0.582)	1.409 (0.384)	1.701 (0.633)	0.113
		BIC	[4]	[0]	[4]	
	SWI	0	1.121 (0.185)	0.557 (0.182)	1.088 (0.048)	9.580‡
		1	1.177 (0.176)	0.655 (0.260)	1.320 (0.172)	4.037‡
		2	1.254 (0.169)	0.722 (0.304)	1.413 (0.143)	3.064
		3	1.333 (0.202)	1.036 (0.312)	1.579 (0.170)	0.906
		4	1.542 (0.170)	1.208 (0.331)	1.692 (0.133)	1.020
		BIC	[0]	[0]	[3]	

Note: Refer to the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). Regression equation is $ln(y_{DEV}) = \alpha + \beta ln(y_{IND}) + \varepsilon$.

Table 3-2: Application to Output Convergence (Regressand: industrial countries; Regressor: industrial countries)

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
		0	1.291 (0.246)	0.623 (0.112)	1.330 (0.044)	35.849‡
		1	1.302 (0.207)	1.018 (0.162)	0.719 (0.079)	3.095†
	LUX	2	1.312 (0.174)	1.094 (0.174)	0.849 (0.077)	1.572
		3	1.294 (0.222)	1.040 (0.184)	0.706 (0.101)	1.910
		4	1.293 (0.262)	1.074 (0.178)	0.694 (0.119)	1.508
		BIC	[4]	[4]	[1]	
		0	1.829 (0.125)	0.494 (0.159)	1.889 (0.078)	70.817‡
		1	1.802 (0.165)	1.089 (0.240)	1.465 (0.165)	8.852‡
GER	NZL	2	1.775 (0.105)	1.268 (0.285)	1.658 (0.096)	3.165†
		3	1.760 (0.060)	1.480 (0.301)	1.643 (0.056)	0.869
		4	1.692 (0.076)	1.671 (0.174)	1.746 (0.079)	0.014
		BIC	[5]	[5]	[5]	
		0	1.494 (0.087)	0.811 (0.147)	1.531 (0.064)	21.513‡
		1	1.482 (0.127)	1.260 (0.150)	1.305 (0.121)	2.178
	SWI	2	1.452 (0.127)	1.326 (0.169)	1.379 (0.107)	0.557
		3	1.413 (0.118)	1.339 (0.171)	1.331 (0.090)	0.185
		4	1.369 (0.184)	1.347 (0.148)	1.421 (0.134)	0.022
		BIC	[4]	[4]	[4]	
		0	0.726 (0.239)	0.702 (0.126)	0.678 (0.045)	0.037
		1	0.800 (0.168)	0.581 (0.136)	0.909 (0.092)	2.586
	GER	2	0.925 (0.191)	0.536 (0.163)	0.938 (0.088)	5.687‡
		3	1.030 (0.265)	0.723 (0.185)	1.039 (0.111)	2.743
		4	1.104 (0.328)	0.838 (0.209)	1.000 (0.133)	1.618
		BIC	[0]	[1]	[1]	
		0	1.289 (0.633)	0.412 (0.176)	1.290 (0.040)	24.806‡
		1	1.256 (0.852)	0.796 (0.294)	0.445 (0.447)	2.434
LUX	NZL	2	1.227 (0.555)	0.797 (0.354)	1.231 (0.503)	1.472
		3	1.255 (0.886)	1.098 (0.362)	0.464 (0.814)	0.189
		4	1.188 (0.676)	1.277 (0.383)	2.311 (0.608)	0.054
		BIC	[3]	[1]	[1]	
		0	1.040 (0.499)	0.603 (0.184)	1.035 (0.007)	5.623‡
		1	1.016 (0.642)	0.898 (0.240)	0.932 (0.480)	0.238
	SWI	2	1.006 (0.461)	0.771 (0.273)	1.443 (0.469)	0.747
		3	0.984 (0.616)	0.875 (0.309)	1.578 (0.626)	0.124
		4	0.932 (0.900)	0.967 (0.336)	1.766 (0.845)	0.011
		BIC	[1]	[1]	[1]	

Note: Refer to the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). Regression equation is $ln(y_{IND}) = \alpha + \beta ln(y_{IND}) + \varepsilon$.

Table 3-2: Continued–

Regressand	Regressor	k	DOLS	GLS-corrected	FGLS-corrected	Hausman Test
NZL	GER	0	0.509 (0.044)	0.395 (0.127)	0.501 (0.046)	0.811
		1	0.498 (0.060)	0.449 (0.149)	0.507 (0.047)	0.110
		2	0.498 (0.075)	0.459 (0.175)	0.488 (0.055)	0.052
		3	0.520 (0.096)	0.285 (0.177)	0.511 (0.060)	1.758
		4	0.517 (0.142)	0.315 (0.170)	0.452 (0.081)	1.406
		BIC	[5]	[5]	[5]	
	LUX	0	0.676 (0.232)	0.292 (0.125)	0.671 (0.044)	9.446‡
		1	0.703 (0.233)	0.552 (0.202)	0.314 (0.161)	0.558
		2	0.731 (0.133)	0.576 (0.224)	0.485 (0.105)	0.484
		3	0.734 (0.209)	0.483 (0.206)	0.496 (0.164)	1.487
		4	0.745 (0.184)	0.628 (0.228)	0.597 (0.144)	0.264
		BIC	[5]	[5]	[5]	
	SWI	0	0.797 (0.043)	0.613 (0.145)	0.783 (0.013)	1.599
		1	0.801 (0.050)	0.742 (0.201)	0.791 (0.050)	0.087
		2	0.778 (0.047)	0.817 (0.224)	0.769 (0.046)	0.031
		3	0.763 (0.057)	0.681 (0.227)	0.789 (0.055)	0.131
		4	0.844 (0.080)	0.812 (0.238)	0.832 (0.080)	0.019
		BIC	[0]	[0]	[3]	
SWI	GER	0	0.634 (0.045)	0.531 (0.097)	0.639 (0.045)	1.133
		1	0.649 (0.079)	0.623 (0.100)	0.653 (0.063)	0.070
		2	0.671 (0.119)	0.650 (0.114)	0.665 (0.078)	0.034
		3	0.676 (0.179)	0.572 (0.130)	0.672 (0.091)	0.638
		4	0.709 (0.278)	0.672 (0.149)	0.696 (0.122)	0.063
		BIC	[1]	[1]	[1]	
	LUX	0	0.831 (0.340)	0.350 (0.107)	0.848 (0.022)	20.185‡
		1	0.850 (0.244)	0.724 (0.162)	0.421 (0.142)	0.610
		2	0.857 (0.214)	0.746 (0.183)	0.469 (0.137)	0.371
		3	0.845 (0.250)	0.702 (0.214)	0.423 (0.169)	0.446
		4	0.824 (0.268)	0.611 (0.238)	0.442 (0.182)	0.795
		BIC	[0]	[1]	[1]	
	NZL	0	1.206 (0.073)	0.503 (0.119)	1.233 (0.014)	34.878‡
		1	1.219 (0.098)	0.991 (0.187)	1.148 (0.106)	1.488
		2	1.224 (0.095)	1.005 (0.232)	1.171 (0.095)	0.890
		3	1.233 (0.088)	1.056 (0.260)	1.180 (0.091)	0.465
		4	1.234 (0.116)	1.194 (0.272)	1.263 (0.124)	0.022
		BIC	[1]	[1]	[1]	

Note: Refer to the notes in Table 2. Annual data covering 1950-1992 are used for four developing countries (COL: Columbia; ECU: Ecuador; EGT: Egypt; PAK: Pakistan) and four industrial countries (GER: Germany; LUX: Luxemburg; NZL: New Zealand; SWI: Switzerland). Regression equation is $ln(y_{IND}) = \alpha + \beta ln(y_{IND}) + \varepsilon$.

Table 4: Application to PPP for traded and non-traded goods

Estimator	k	Traded Goods					Non-traded Goods				
		FRA	ITA	JPN	U.K.	U.S.	FRA	ITA	JPN	U.K.	U.S.
DOLS	0	1.149	1.379	1.558	1.306	1.053	1.872	2.142	2.357	2.059	1.711
		(0.312)	(0.165)	(0.326)	(0.201)	(0.198)	(0.165)	(0.241)	(0.299)	(0.278)	(0.443)
	1	1.179	1.456	1.485	1.439	1.078	1.887	2.175	2.376	2.066	1.728
		(0.424)	(0.217)	(0.459)	(0.262)	(0.276)	(0.151)	(0.222)	(0.299)	(0.290)	(0.441)
	2	1.195	1.511	1.442	1.533	1.092	1.898	2.216	2.399	2.067	1.748
		(0.515)	(0.277)	(0.646)	(0.342)	(0.328)	(0.161)	(0.237)	(0.331)	(0.272)	(0.473)
	3	1.186	1.531	1.390	1.571	1.102	1.888	2.250	2.397	2.054	1.762
		(0.524)	(0.308)	(0.561)	(0.392)	(0.381)	(0.165)	(0.247)	(0.346)	(0.261)	(0.511)
	4	1.195	1.553	1.388	1.613	1.109	1.871	2.287	2.402	2.042	1.763
		(0.502)	(0.353)	(0.471)	(0.412)	(0.401)	(0.159)	(0.252)	(0.343)	(0.233)	(0.484)
	BIC	[0]	[0]	[0]	[2]	[0]	[0]	[0]	[5]	[1]	[0]
GLS-corrected	0	0.833	1.114	1.086	1.030	0.919	0.375	0.448	0.372	0.351	0.159
		(0.393)	(0.381)	(0.411)	(0.365)	(0.140)	(0.178)	(0.176)	(0.198)	(0.171)	(0.080)
	1	0.984	1.086	1.221	1.324	1.027	0.864	0.900	0.888	0.988	0.397
		(0.454)	(0.436)	(0.477)	(0.419)	(0.161)	(0.315)	(0.309)	(0.352)	(0.295)	(0.142)
	2	1.259	1.333	1.415	1.516	1.103	1.127	1.104	1.046	1.001	0.569
		(0.469)	(0.454)	(0.477)	(0.430)	(0.167)	(0.369)	(0.368)	(0.408)	(0.355)	(0.166)
	3	1.374	1.391	1.246	1.560	1.158	1.255	1.171	1.192	1.088	0.751
		(0.501)	(0.485)	(0.504)	(0.460)	(0.176)	(0.422)	(0.422)	(0.456)	(0.407)	(0.185)
	4	1.549	1.661	1.248	1.600	1.184	1.577	1.411	1.379	1.166	0.785
		(0.520)	(0.511)	(0.535)	(0.492)	(0.188)	(0.456)	(0.468)	(0.503)	(0.457)	(0.208)
	BIC	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[0]	[1]	[0]
FGLS-corrected	0	1.156	1.358	1.605	1.269	1.049	1.868	2.129	2.354	2.063	1.712
		(0.259)	(1.163)	(0.783)	(0.125)	(0.054)	(0.232)	(1.223)	(0.760)	(0.156)	(0.073)
	1	1.229	1.456	1.607	1.248	0.766	1.909	2.005	1.947	1.671	0.291
		(0.329)	(0.149)	(0.339)	(0.162)	(0.242)	(0.141)	(0.201)	(0.257)	(0.226)	(0.159)
	2	1.178	1.487	1.717	1.323	0.802	1.932	2.102	2.095	1.864	0.357
		(0.355)	(0.181)	(0.396)	(0.217)	(0.266)	(0.152)	(0.218)	(0.286)	(0.217)	(0.191)
	3	1.152	1.463	1.625	1.376	0.822	1.920	2.158	2.173	1.983	0.392
		(0.339)	(0.204)	(0.318)	(0.250)	(0.301)	(0.157)	(0.233)	(0.311)	(0.214)	(0.216)
	4	1.252	1.603	1.506	1.423	0.826	1.922	2.285	2.202	2.052	0.631
		(0.333)	(0.245)	(0.277)	(0.272)	(0.284)	(0.153)	(0.240)	(0.310)	(0.189)	(0.234)
	BIC	[1]	[1]	[2]	[1]	[1]	[2]	[1]	[5]	[1]	[3]
Hausman Test	0	0.644	0.482	1.319	0.573	0.924	70.35‡	93.02‡	100.88‡	99.87‡	377.01‡
	1	0.184	0.720	0.308	0.075	0.102	10.57‡	17.01‡	17.86‡	13.32‡	87.77‡
	2	0.019	0.153	0.003	0.001	0.004	4.36‡	9.14‡	11.00‡	8.99‡	50.20‡
	3	0.140	0.084	0.081	0.001	0.101	2.25	6.52‡	6.97‡	5.63‡	29.93‡
	4	0.464	0.045	0.069	0.001	0.158	0.42	3.52†	4.15‡	3.67†	22.21‡

Note: Results are for $f_t^T = \alpha + \beta p_t^T + \epsilon_t$ and $f_t^N = \alpha + \beta p_t^N + \epsilon_t$ using Canada as a base country. Figures in the parenthesis represent standard errors. 'k' denotes the maximum length of leads and lags. For the FGLS corrected estimator, AR(1) error term is structured as $u_t = \rho u_{t-1} + \epsilon_t$. Hausman test represents the Hausman-type cointegration test as stipulated in section 2.4. The test statistic is constructed as $\frac{(\hat{\beta}_{OLS} - \tilde{\beta}_{GLS})^2)}{Var(\tilde{\beta}_{GLS})} \to \chi^2(1)$. The critical values of $\chi^2(1)$ are 2.71, 3.84 and 6.63 for ten, five, and one percent significance level. ‡(†) represent that the null hypothesis of $\hat{\beta}_{OLS} = \tilde{\beta}_{GLS}$ can be rejected at 5% (10%) significance level.

References

[1] Billingsley, P. (1986), *Probability and Measure*, *Second Edition*, New York: Wiley.

[2] Blough, S.R. (1992), "Spurious Regressions, with AR(1) Correction and Unit Root Pretest," *Mimeo*, Johns Hopkins University.

[3] Cooley, T.F. and M. Ogaki (1996), "A Time Series Analysis of Real Wages, Consumption, and Asset Returns," *Journal of Applied Econometrics*, 11, 119-134.

[4] de Jong, R. and J. Davidson (2000), "The functional central limit theorem and weak convergence to stochastic integrals I: weakly dependent processes," *Econometric Theory*, 16, 621-642.

[5] Durlauf, S.N. and P.C.B. Phillips (1988), "Trends versus Random Walks in Time Series Analysis," *Econometrica*, 56, 1333-1354.

[6] Eichenbaum, M.S., L.P. Hansen, and K.J. Singleton (1988), "A Time-series Analysis of Representative Agent Models of Consumption and Leisure Choice under Uncertainty," *Quarterly Journal of Economics*, 103, 51-78.

[7] Engel, C. (1999), "Accounting for US Real Exchange Rate Changes," *Journal of Political Economy*, 107, 507-538.

[8] Granger, C.W.J. and P. Newbold (1974), "Spurious Regressions in Econometrics," *Journal of Econometrics*, 74, 111-120.

[9] Judge, G.G., W.E. Griffiths, R.C. Hill, H. Lutkepohl, and T.-C. Lee. 1985. *The Theory And Practice of Econometrics*, Wiley.

[10] Kakkar, V., and M. Ogaki (1999), "Real Exchange Rates and Nontradables: A Relative Price Approach," *Journal of Empirical Finance*, 6, 193-215.

[11] Kim, J. (2004), "Convergence Rates to PPP for Traded and Non-traded Goods: A Structural Error Correction Model Approach," *Journal of Business and Economic Statistics*, *forthcoming.*

[12] Nelson, C.R. and H. Kang (1981), "Spurious Periodicity in Inappropriately Detrended Time Series," *Econometrica*, 49, 741-751.

[13] ______________ (1983), "Pitfalls in the Use of Time as an Explanatory Variable in Regression," *Journal of Business and Economic Statistics,* 2, 73-82.

[14] Ogaki, M. and C.Y. Choi (2001), "The Gauss-Marcov Theorem and Spurious Regressions," WP 01-13, Department of Economics, The Ohio State University.

[15] Ogaki, M. and J.Y. Park (1998), "A Cointegration Approach to Estimating Preference Parameters," *Journal of Econometrics,* 82, 107-134.

[16] Park, J.Y. and P.C.B. Phillips (1988), "Asymptotic Equivalence of Ordinary Least Squares and Generalized least Squares in Regressions with Integrated Regressors," *Journal of the American Statistical Association*, 83, 111-115.

[17] Phillips, P.C.B. (1986), "Understanding Spurious Regressions in Econometrics," *Journal of Econometrics,* 33, 311-340.

[18] ______________ (1989), "Partially Identified Econometric Models," *Econometric Theory*, 5, 95-132.

[19] ______________ (1998), "New Tools for Understanding Spurious Regression," *Econometrica,* 66, 1299-1325.

[20] Phillips, P.C.B. and D.J. Hodgson (1994), "Spurious Regression and Generalized Least Squares," *Econometric Theory*, 10, 957-958.

[21] Phillips, P.C.B. and M. Loretan (1991), "Estimating Long-run Economic Equilibria," *Review of Economic Studies*, 58, 407-436.

[22] Phillips, P.C.B. and S. Ouliaris (1990), "Asymptotic Properties of Residual Based Tests for Cointegration," *Econometrica*, 58, 165-193.

[23] Stock, J.H. and M.W. Watson (1993), "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems," *Econometrica,* 61, 783-820.

[24] Stockman, A.C., and L. Tesar (1995), "Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements," *American Economic Review*, 85, 168-185.

[25] Summers, R. and A. Heston (1991), "The Penn World Table (Mark 5): An Expanded Set of International Comparison, 1950-1988," *Quarterly Journal of Economics*, 106, 327–368.