Oligopolistic and Monopolistic Competition Unified

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Abstract

We study a differentiated product market involving both oligopolistic and monopolistically competitive firms. We show that the size of monopolistically competitive subsector decreases as the number of oligopolistic firms rises. Social welfare level increases as the number of oligopolistic firms rises because the procompetitive effect associated with the entry of a new oligopolistic firm dominates the resulting decrease in product variety.

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1 Introduction

Armchair evidence shows that many industries are characterized by the coexistence of a few large firms, which are able to manipulate the market, as well as of a myriad of very small firms, each of which has a negligible impact on the market. Examples can be found in apparel, catering, publishers and bookstores, retailing, finance and insurances, and IT industries. To the best of our knowledge, such a *mixed market* structure has been overlooked in the literature.[1] This is rather surprising because this situation is fairly common in the real world. Even more strikingly, several countries have passed bills that restrict the entry of large firms or forbid price discounts in order to permit small firms to remain active. For example, in France the 'Lang' Law forbids discounts on books with the aim of preserving a large network of small bookstores, whereas the Net Book Agreement in the United Kingdom between book publishers and retailers, which also prevents price discounting, is argued by the publishers and small book sellers to be justifiable on the same grounds. In France again, the 'Royer-Raffarin' Law imposes severe restrictions on the entry of department stores whose surface exceeds 300 square meters, the purpose being that small shops provide various convenience services. More real-world examples could be cited.

The idea behind such laws and regulations is that small firms allow for a wider array of varieties and, thus, contribute to consumers' welfare. However, such an argument disregards the fact that competition between strategic firms tends to foster lower prices. The purpose of this paper is to provide a unified approach that embodies both large/strategic and small/nonstrategic firms. We then use this framework (i) to study how these two types of firms interact to shape the market and (ii) whether or not it is socially desirable to have large and/or small firms in business. To reach our goal, we blend two standard models, namely the oligopoly model à la Cournot with differentiated products and the monopolistic competition model à la Chamberlin.

The field of industrial organization is dominated by partial equilibrium models of oligopoly in which strategic interactions between firms appear to be the central ingredient. They now serve as the corner-stone of many competition policy studies of real-world markets (Motta, 2004). By contrast, monopolistic competition has been extensively employed as the "main" building-block in the analyses of imperfect competition within general equilibrium models developed in various economic fields. Examples include economic

[1] The main noticeable exception is provided by the dominant firm model in which one large firm and a competitive fringe coexist (Markham, 1951). Another one deals with "big agents" (formally, atoms) whose role in exchange economies have been studied within the context of cooperative game theory (Gabszewicz and Shitovicz, 1992).

policy, growth and innovation, international trade, and economic geography (Matsuyama, 1995). It is fair to say that both approaches are useful to analyze market mechanisms and have their own merits. However, as said in the foregoing, we must also recognize the fact that many industries consist a few large firms and many small firms. In such industries, the large firms behave strategically, whereas small firms maximize their profits on the residual demand in the absence of strategic interactions. This paper may then be viewed as an attempt at providing a reconciliation between such different approaches to market competition.

On the production side, we consider a differentiated product market in which both oligopolistic and monopolistically competitive firms coexist. In measure theoretic-terms, each oligopolistic firm is an atom whereas each monopolistically competitive firm has a zero measure. Because small firms typically exhibit more volatility than large firms in their entry behavior, we assume that the mass of monopolistically competitive firms is adjusted to the number oligopolistic firms until profits in the competitive fringe are zero, as in Chamberlin (1933). On the consumption side, we consider a utility function with a symmetric CES subutility, as in Spence (1976) and Dixit and Stiglitz (1977). However, unlike these contributions, our model involves both *discrete* and *negligible varieties*. By so doing, we are able to embody within the same utility the two specifications of the CES model that have been used in the literature (see, e.g. Anderson *et al.*, 1992; Matsuyama, 1995).[2] Hence, our model may be viewed as a reconciliation of both types of market structure, which may be used for different purposes. In what follows, we restrict ourselves to the positive analysis of a mixed market as well as to its welfare implications regarding the entry of oligopolistic firms. We focus on a symmetric equilibrium in which each type of firms chooses the same output volume.

Our main findings are as follows. The entry of a new oligopolistic firm extends the market share of these firms at the expense of monopolistically competitive firms. Hence, deregulating mixed markets is likely to lead to a progressive disappearance of small firms.[3] More surprisingly, this market expansion is sufficiently strong for the output of each oligopolistic firm to rise when there is one additional oligopolistic firm. Yet, the price at which the large firms sell their product is lower. However, as their profits are higher, large firms are better off when entry arises under the concrete form of a new

[2]Recall that Dixit and Stiglitz assumed a continuum of varieties in their discussion paper reprinted in Brakman and Heijdra (2004).

[3]In the same spirit, there has been in the UK a sharp decline in the number of small groceries after the passage of the Resale Prices Act in 1964 abolishing resale price maintenance (Everton, 1993).

large firm. All these results, established in an otherwise standard model, suggest that *the mixed market structure model obeys different rules than standard oligopoly models*. In terms of welfare, we show the unexpected (at least to us) result that the entry of additional oligopolistic firms is always beneficial to consumers in a mixed market as long as these firms make positive profits, a condition that is quite natural. In other words, the possible loss of welfare that the contraction in the mass of monopolistically competitive firms could generate is always more than compensated by the fact the whole industry becomes more competitive as additional oligopolistic firms operate on the market. This casts some doubt on the welfare foundations of the many laws and regulations that tend to keep active many small businesses. In addition, that result also suggests that the downsizing and rationalization that many firms experienced in the 1990s may not be an unfortunate, anticompetitive outcome but a healthy response to new competitive pressure, which have been especially strong in retail trade, services, and the financial and insurance industries.

To be sure, our results are obtained in the case of a specific model, namely the CES. Being aware of its limits, we want to stress the fact that this model is the workhorse of many contributions dealing with imperfect competition in modern economic theory. So our results cannot be dismissed on that basis only. Although more work is called for, we believe that our analysis provides useful insights about a topic that has been so far neglected. In addition, even though we do not provide a full-fledged general equilibrium analysis, it is worth stressing that our analysis departs from standard partial equilibrium models in that we allow incomes to be endogenous.

The remaining of the paper is organized as follows. The details of the model are provided in Section 2. Section 3 deals with the main properties of a mixed market equilibrium. The welfare analysis is taken up in Section 4, whereas Section 5 concludes.

2 The model

2.1 Consumers

There are two goods, two sectors and one production factor - labor - in the economy. The first good is homogenous and produced under constant returns to scale and perfect competition. It is chosen as the numéraire. Without loss of generality, we may assume that one unit of the homogenous good is produced by using one unit of labor, thus implying that the equilibrium wage is equal to 1. The other good is a horizontally differentiated product.

It is supplied both by oligopolistic firms and by monopolistically competitive firms (in short MC-firms). Variables associated with oligopolistic firms are described by capital letters and those corresponding to MC-firms by lower case letters (this should help the reader to remember that an MC-firm is smaller than an oligopolistic firm). Each firm supplies a single variety, thus implying that oligopolistic firms cannot contribute to product variety by supplying a product line, as in Brander and Eaton (1984). Let $N > 1$ be the *number* of varieties produced by oligopolistic firms and $M > 0$ the *mass* of varieties produced by MC-firms. In other words, the differentiated sector is *mixed* in that it is constituted by *two subsectors governed by distinct forms of competition*, which interact according to rules that will be made precise below.

There exists a representative consumer who describes the aggregated behavior of the whole population of consumers. This agent is endowed with L units of labor, holds the shares of all firms, and has a preference relationship represented by the following utility function (see Anderson *et al.* 1992, for more details):

$$U = \left(\sum_{j=1}^{N} Q_j^{\rho} + \int_0^M [q(i)]^{\rho} di \right)^{(1-\alpha)/\rho} \cdot X^{\alpha} \tag{1}$$

where Q_j is the output level of oligopolistic firm $j = 1, ..., n$, $q(i)$ the output level of MC-firm $i \in [0, M]$, X the aggregate consumption of the homogenous good, whereas α and ρ are two given parameters satisfying the inequalities $0 < \alpha < 1$ and $0 < \rho < 1$. This specification of preferences encapsulates both the oligopolistic and monopolistically competitive modeling strategies of the Spence-Dixit-Stiglitz model.

The novel feature of preferences (1) is that they incorporates both discrete varieties that have each a positive impact on utility as well as a set of negligible varieties in that each of them has a zero impact on U. Clearly, the process of substitution between these two types of varieties is more involved than in standard models. To illustrate how it works, consider the situation in which the quantities of discrete varieties $j = 1, ..., N$ are the same and equal to Q, whereas the quantity density of negligible varieties is uniform and equal to q over $[0, M]$. Let us now assume that there is a $(N+1)$th discrete variety and consider the variation of the total mass of negligible varieties that leaves the utility level unaffected. It is readily verified that M must decrease by a positive amount given by

$$\Delta M = \left(\frac{Q}{q} \right)^{\rho}.$$

Hence, for the utility level to remain the same, the entry of a new discrete variety is to be compensated by a lower decrease in the mass of negligible varieties when the degree of product differentiation (inversely measured by ρ) increases. It is also worth noting that the value of ΔM rises with Q and fall with q.

The representative consumer solves the following maximization problem:

$$\text{Maximize} \qquad \left(\sum_{j=1}^{N} Q_j^{\rho} + \int_0^M [q(i)]^{\rho} di \right)^{(1-\alpha)/\rho} \cdot X^{\alpha}$$

$$\text{subject to} \qquad \sum_{j=1}^{N} P_j Q_j + \int_0^M p(i)q(i)di + X \leq \mathbf{Y}$$

where P_j is the price of variety $j = 1, ..., N$, $p(i)$ the price of variety $i \in [0, M]$, and $\mathbf{Y}$ the total income in the economy.

It appears to be useful to decompose this problem into two steps. In the first one, we solve the following minimization problem:

$$\text{Minimize} \qquad \int_0^M p(i)q(i)di \qquad \text{subject to} \qquad \left[\int_0^M q(i)^{\rho} di \right]^{1/\rho} \equiv Q_0$$

where we interpret Q_0 as the *output index* of the MC-subsector. The first order conditions for an interior maximum are as follows:

$$p(i) = \mu\rho[q(i)]^{-(1-\rho)} \left[\int_0^M q(i)^{\rho} di \right]^{(1-\rho)/\rho}$$

$$\int_0^M q(i)^{\rho} di = Q_0^{\rho}$$

where μ is the Lagrangian multiplier.

Let $R \equiv q(i)/[p(i)]^{-1/(1-\rho)}$ and

$$P_0 \equiv \left[\int_0^M p(i)^{-\rho/(1-\rho)} di \right]^{-(1-\rho)/\rho}. \tag{2}$$

be the *price index* of the monopolistically competitive varieties. We may then rewrite Q_0 as follows:

$$Q_0 = R \cdot \left(\int_0^M p(i)^{-\rho/(1-\rho)} di \right)^{1/\rho} = RP_0^{-1/(1-\rho)} \tag{3}$$

so that $R = Q_0/P_0^{-1/(1-\rho)}$, which in turns implies that

$$q(i) = Rp(i)^{-1/(1-\rho)} = Q_0 \cdot \left[\frac{p(i)}{P_0}\right]^{-1/(1-\rho)} \quad \text{for all } i \in [0, M]. \tag{4}$$

Substituting (3) and (4) into the original maximization problem yields the following reduced maximization problem:

$$\text{Maximize} \quad \left(\sum_{j=0}^{N} Q_j^{\rho}\right)^{(1-\alpha)/\rho} \cdot X^{\alpha} \quad \text{subject to} \quad \sum_{j=0}^{N} P_j Q_j + X \leq \mathbf{Y}.$$

The corresponding first order conditions imply that

$$(1-\alpha)\left(\sum_{j=0}^{N} Q_j^{\rho}\right)^{(1-\alpha-\rho)/\rho} Q_j^{-(1-\rho)} X^{\alpha} = \lambda P_j \qquad j = 0, 1, ..., N \tag{5}$$

$$\alpha \left(\sum_{j=0}^{n} Q_j^{\rho}\right)^{(1-\alpha)/\rho} X^{-(1-\alpha)} = \lambda \tag{6}$$

$$\mathbf{Y} - \sum_{j=0}^{N} P_j Q_j - X = 0 \tag{7}$$

where λ is the Lagrangian multiplier.

Let $\mathbf{P}$ be the price index of *all* the differentiated varieties, which we define as follows:

$$\mathbf{P} \equiv \left(\sum_{j=0}^{N} P_j^{-\rho/(1-\rho)}\right)^{-(1-\rho)/\rho}. \tag{8}$$

so that $\mathbf{P}$ increases with any P_j, $j = 0, 1, ..., N$. It is readily verified that the system (5)-(7) imply that

$$Q_i = (1-\alpha)\mathbf{Y} P_i^{-1/(1-\rho)} \mathbf{P}^{\rho/(1-\rho)} \equiv D(P_i, \mathbf{P}, \mathbf{Y}) \qquad i = 1, ..., N \tag{9}$$

$$X = \alpha \mathbf{Y} \tag{10}$$

$$q(i) = (1-\alpha)\mathbf{Y}[p(i)]^{-1/(1-\rho)} \mathbf{P}^{\rho/(1-\rho)} \equiv d[p(i), \mathbf{P}, \mathbf{Y}] \tag{11}$$

where $D(p_i, \mathbf{P}, \mathbf{Y})$ is to be interpreted as the demand function of the oligopolistic variety $i = 1, ..., N$ and $d[p(i), \mathbf{P}, \mathbf{Y}]$ as that of the monopolistically competitive variety $i \in [0, M]$. The fact that the functional forms D and d are independent of i reflects the symmetry of preferences on the varieties supplied by each subsetor. Both D and d are also decreasing in their own price.

Finally, using (2) and (9), it is easy to show that $\partial D(P_i, \mathbf{P}, \mathbf{Y})/\partial P_j > 0$ for all $i, j = 1, \cdots, N$ and $j \neq i$, implying that the oligopolistically provided varieties are strong gross substitutes. As the same holds for $j = 0$, we may conclude that the output index of the MC-subsector plays the same role in consumption as any variety of the oligopolistic subsector.

2.2 Oligopolistic firms

Each oligopolistic firm selects its output level to maximize its profit. Hence, the solution to the interactive profit-maximizing problem is given by a Nash equilibrium of the following strategic-form game: (i) the players are the N oligopolistic firms; (ii) the strategy of firm $i = 1, ..., N$ is its output level Q_i; and (iii) the payoff for player i is given by its profit function

$$\Pi_i(Q_1, \cdots, Q_N; \mathbf{Y}, Q_0) = \Psi_i(Q_1, \cdots, Q_N; \mathbf{Y}, Q_0)Q_i - CQ_i - F$$

where $\Psi_i(\cdot)$ is the inverse demand function for the product of oligopolistic firm i, $C > 0$ the constant marginal cost and $F > 0$ the fixed cost of an oligopolistic firm, both expressed in terms of labor units.

The demand functions (9) and (11) allow us to describe the market behavior of both types of firms. First, an oligopolistic firm is aware that its output level affects the price index $\mathbf{P}$ and is, therefore,involved in a strategically interdependent environment. It also understands that the price index P is influenced by the aggregate behavior of the MC-firms, as expressed by Q_0. Finally, each oligopolistic firm should account for the income effect that its strategic choice generates through profit distribution. However, for reasons discussed below, we assume that these firms ignore the impact that their output policy has on the total income. By contrast, being negligible to the market, each MC-firm may accurately treats the price index P and the total income Y as parameters when selecting its profit-maximizing output. Hence, unlike the oligopolistic firms, the MC-firms do not behave strategically.

Solving (9) for P_i yields $P_i = [(1-\alpha)\mathbf{Y}]^{1-\rho} Q_i^{-(1-\rho)} \mathbf{P}^{\rho}$. Substituting this expression into (8) yields the price index as function of the consumption levels:

$$\mathbf{P} = (1-\alpha)\mathbf{Y} \left(Q_i^{\rho} + \sum_{j \neq i} Q_j^{\rho} \right)^{-1/\rho}. \tag{12}$$

Plugging (12) into $P_i = [(1-\alpha)\mathbf{Y}]^{-(1-\rho)} Q_i^{1-\rho} \mathbf{P}^{-\rho}$ then yields the inverse demand function for variety $i = 1, ..., N$:

$$\Psi_i(Q_1, \cdots, Q_N; \mathbf{Y}, Q_0) = (1-\alpha)\mathbf{Y} \frac{Q_i^{-(1-\rho)}}{Q_i^{\rho} + \sum_{j \neq i} Q_j^{\rho}}. \tag{13}$$

Consequently, the profit function of firm i may be written as follows:

$$\Pi_i(Q_1, \cdots, Q_N; \mathbf{Y}, Q_0) = (1-\alpha)\mathbf{Y}\frac{Q_i^\rho}{Q_i^\rho + \sum_{j\neq i} Q_j^\rho} - CQ_i - F. \qquad (14)$$

Let $\mathrm{Q}_{-i} \equiv (Q_1, ...[Q_i], ..., Q_N)$ be the vector of all outputs but that of firm i. Then, we have (the proof is given in Appendix A):

Lemma. *For any* $i = 1, ..., N$ *and any given* Q_{-i}, Π_i *is strictly concave with respect to* Q_i *over* $[0, \infty)$.

Hence, the best reply function $Q_i^*(\mathrm{Q}_{-i}; \mathbf{Y}, Q_0)$ of firm i is the unique solution to the following first order condition:

$$[(1-\alpha)\mathbf{Y}]^\rho = [(1-\alpha)\mathbf{Y}]^{2\rho-1}\frac{C}{\rho}\mathbf{P}^{-\rho}\left(Q_i^*\right)^{1-\rho} + \mathbf{P}^\rho\left(Q_i^*\right)^\rho. \qquad (15)$$

Because the oligopolistic varieties are substitutes, one may expect the variables Q_i and Q_j to be strategic complements (Vives, 1999). However, we show below that this need not be the case. Indeed,

$$\frac{\partial \Pi_i}{\partial Q_i} = \frac{\partial \Pi_i(Q_i^*(\mathrm{Q}_{-i}; \mathbf{Y}, Q_0), \mathrm{Q}_{-i}; \mathbf{Y}, Q_0)}{\partial Q_i} = 0.$$

Thus,

$$\frac{\partial Q_i^*}{\partial Q_j} = -\frac{\partial^2 \Pi_i}{\partial Q_i \partial Q_j} / \frac{\partial^2 \Pi_i}{\partial Q_i^2} \qquad \text{for all } i, j = 1, ...N \text{ and } j \neq i.$$

Since $\partial^2\Pi_i/\partial Q_i^2$ is negative by the lemma, the sign of $\partial Q_i^*/\partial Q_j$ is the same as the sign of $\partial^2\Pi_i/\partial Q_i\partial Q_j$. Computing this expression yields

$$\frac{\partial^2 \Pi_i}{\partial Q_i \partial Q_j} = (1-\alpha)\rho^2 Q_i^{-(1-\rho)} Q_j^{-(1-\rho)} \left(Q_i^\rho + \sum_{k\neq i} Q_k^\rho\right)^{-3} \left(Q_i^\rho - \sum_{k\neq i} Q_k^\rho\right).$$

Accordingly, for all $i, j = 1, ...N$ and $j \neq i$, the output levels Q_i and Q_j are strategic complements when $Q_i^\rho > \sum_{k\neq i} Q_k^\rho$ but they become strategic substitutes as long as $Q_i^\rho < \sum_{k\neq i} Q_k^\rho$. Thus, no general characterization arises. Furthermore, even though Q_0 is not chosen by a player per se, the output index of the MC-subsector may be viewed either a strategic substitute or a strategic complement of oligopolistic firms' output because the same inequalities hold for $j = 0$. Among other things, this implies that an expansion of the MC-subsector (through of an increase of M) does not necessarily imply that oligopolistic firms lower their output. However, we will see below that we have a "well-behaved model" in that an increase in the number of firms of each type leads to lower prices for these firms as well as a decrease of the overall price index.

2.3 Monopolistically competitive firms

The profit maximization problem of MC-firm $i \in [0, M]$ is given by

$$\text{Maximize} \quad \pi(i) = p(i)q(i) - cq(i) - f \quad \text{subject to} \quad q(i) = d[p(i), \mathbf{P}, \mathbf{Y}]$$

where $c > 0$ is the constant marginal cost and $f > 0$ the fixed cost of a MC-firm, both expressed in terms of labor units. Note that the resource constraint implies that

$$L > NF + Mf$$

otherwise the economy does not supply the homogenous good.

It follows from (11) that the inverse demand function is given by

$$p(i) = [(1-\alpha)\mathbf{Y}]^{1-\rho} q(i)^{-(1-\rho)} \mathbf{P}^{\rho}.$$

As a result, the profit function of firm i is

$$\pi(i) = \pi[q(i); \mathbf{P}, \mathbf{Y}] = [(1-\alpha)\mathbf{Y}]^{1-\rho} [q(i)]^{\rho} \mathbf{P}^{\rho} - cq(i) - f$$

where each MC-firm accurately treats the price index $\mathbf{P}$ and the total income $\mathbf{Y}$ as parameters. Since $\rho < 1$, $\pi(i)$ *is strictly concave in* $q(i)$. The first order condition for profit maximization leads to

$$q(i) = (1-\alpha)\mathbf{Y} \left(\frac{c}{\rho}\right)^{-1/(1-\rho)} \mathbf{P}^{\rho/(1-\rho)}.$$

Accordingly, we may determine the equilibrium price p^* and output q^* common to all MC-firms as follows:

$$p^* = \frac{c}{\rho} \quad \text{and} \quad q^* = (1-\alpha)\mathbf{Y} \left(\frac{\rho}{c}\right)^{1/(1-\rho)} \mathbf{P}^{\rho/(1-\rho)}. \tag{16}$$

This equilibrium is thus unique and symmetric. Whereas the equilibrium price is constant, the equilibrium output of an MC-firm is a function of the price index $\mathbf{P}$ and, therefore, depends on the quantities chosen by the oligopolistic firms. When the mass of firms is M, the equilibrium profit is then given by

$$\pi^* = \left(\frac{\rho}{c}\right)^{\rho/(1-\rho)} (1-\rho)(1-\alpha)\mathbf{Y}\mathbf{P}^{\rho/(1-\rho)} - f. \tag{17}$$

Finally, the mass M of MC-firms is determined by the zero-profit condition $\pi^* = 0$ in which $P_1, ..., P_N$ and $\mathbf{Y}$ are treated parametrically.

3 Equilibrium

A *mixed market equilibrium* is defined as a state in which the following conditions simultaneously hold: (i) the representative consumer maximizes her utility subject to the budget constraint, (ii) both oligopolistic and MC-firms maximize their own profits, and (iii) the mass of MC-fims is positive and such the profits of these firms are zero. In other words, for any given number N of oligopolistic firms, we assume that the mass M of MC-firms is adjusted until their profits are zero. Even though the output of each oligopolistic firm, the output index of the MC-subsector and the total income in the economy are endogenous, when choosing its own output level, each oligopolistic firm treats the other firms' output as well as the output index of the MC-subsector and the total income as parameters. This implies that these firms behave as *income-takers* in that they neglect the fact that the total income in the economy is positively affected by profits, thus changing their demand level. Handling such an effect is formally very hard and not necessarily empirically meaningful (Bonanno, 1990). Yet, each oligopolistic firm is aware that a higher/lower income influences positively/negatively the level of its demand. Accordingly, even though our model is not a "complete" general equilibrium model, it is a closed, general equilibrium model in which in which oligopolistic firms account for both strategic interactions and endogenous total income. These firms ignore the impact of their policy on the total income in the economy because, perhaps, the industry under consideration represents a small share of the whole economy. Admittedly, such an approach has a partial equilibrium flavor (Hart, 1985). The difference lies in the fact that, in a typical partial equilibrium setting, the total income would be exogenous.

We may characterize our equilibrium concept by means of the following four conditions for some $M > 0$ and $N \geq 1$: (a) the demand functions, (b) the profit-maximization conditions of MC-firms, (c) the profit-maximization conditions of oligopolistic firms, and (d) the zero-profit condition of MC-firms. In this way, we can view 0 as a "pseudo-player" who would choose the mass of MC-firms in order to make zero profits. Let us stress that the oligopolistic firms do not behave here as the leaders of a sequential game in which the MC-firms (or the pseudo-player 0) would be the followers. The size of the MC-subsector is determined simultaneously with the variables of the oligopolistic subsector.

In what follows, we focus on a *symmetric* mixed market equilibrium in which all oligopolistic firms choose the same output Q^* whereas all MC-firms have the same production policy q^* given by (16). Our first proposition is proven in Appendix B.

Proposition 1 *There exists a unique symmetric mixed market equilibrium.*

This result is important because it implies that we stay on the same equilibrium path (if any), when studying the impact of the entry of a new oligopolistic firm.

The equilibrium profit Π^* of an oligopolistic firm at such an equilibrium is then as follows:

$$\Pi^* = [(1-\alpha)\mathbf{Y}]^{1-\rho} (Q^*)^\rho \mathbf{P}^\rho - CQ^* - F. \tag{18}$$

The total income is given by

$$\mathbf{Y} = L + N\Pi^* + M\pi^*$$

which is the unique solution to

$$\begin{aligned} \mathbf{Y} &= L + N\left\{[(1-\alpha)\mathbf{Y}]^{1-\rho} (Q^*)^\rho \mathbf{P}^\rho - CQ^* - F\right\} \\ &\quad + M\left\{\left(\frac{\rho}{c}\right)^{\rho/(1-\rho)} (1-\rho)(1-\alpha)\mathbf{Y}\mathbf{P}^{\rho/(1-\rho)} - f\right\}. \end{aligned} \tag{19}$$

Observe that the equilibrium value of $\mathbf{Y}$ explicitly accounts for the level of fixed costs in each subsector as well as for their respective size.

It follows immediately from (12) that

$$\mathbf{P} = \left[(P_0^*)^{-\rho/(1-\rho)} + N(P^*)^{-\rho/(1-\rho)}\right]^{-(1-\rho)/\rho} = (1-\alpha)\mathbf{Y}\left[(Q_0^*)^\rho + N(Q^*)^\rho\right]^{-1/\rho}. \tag{20}$$

Substituting (16) into (2) and (4), we obtain the equilibrium values of the price and output indices of the MC-firms:

$$P_0^* = \frac{c}{\rho} M^{-(1-\rho)/\rho} \tag{21}$$

and

$$Q_0^* = (1-\alpha)\mathbf{Y}\left(\frac{c}{\rho}\right)^{-1/(1-\rho)} M^{1/\rho}\mathbf{P}^{\rho/(1-\rho)}. \tag{22}$$

The unknown variables $\mathbf{Y}$, $\mathbf{P}$, Q^* and Q_0^* are thus determined in terms of M by using the four equations (15), (19), (20), and (22). This gives us the market outcome when the size M of the MC-subsector is fixed. It is worth studying how this outcome changes with M because this will shed light on the way the two subsectors interact at the market equilibrium. Unfortunately, the fact that the outputs Q_i may be either strategic complements or strategic substitutes does not allow us the determine how Q^* and Q_0^* are affected when M rises. However, we are able to characterize the impact on equilibrium prices. Clearly, (21) implies that the price index of the MC-subsector decreases as the size of this subsector rises. Furthermore, as proven in Appendix C, increasing M has a similar impact upon $\mathbf{P}$ and P^*.

Proposition 2 *Consider a symmetric mixed market equilibrium in which oligopolistic firms earn positive profits. Then, both the price index of the differentiated industry and the price at which oligopolistic firms sell their output decrease when the mass of MC-firms increases.*

To provide a full characterization of the market equilibrium, we still have to determine the size of the MC-subsector. Using (17), the zero-profit condition $\pi^* = 0$ is equivalent to

$$(1-\alpha)\mathbf{Y} = \frac{f}{1-\rho}\left(\frac{c}{\rho}\right)^{\rho/(1-\rho)} \mathbf{P}^{-\rho/(1-\rho)}. \tag{23}$$

Substituting (23) into (22), we obtain

$$Q_0^* = \frac{f}{1-\rho}\left(\frac{c}{\rho}\right)^{-1} (M^*)^{1/\rho}. \tag{24}$$

Hence, both the equilibrium mass and output index of the MC-subsector move in the same direction. The market equilibrium is then described by the five simultaneous equations (15), (19), (20), (23) and (24) whose unknowns are $\mathbf{Y}$, $\mathbf{P}$, Q^*, Q_0^*, and M.

Our objective is now to identify two conditions that will allow us to study the behavior of Q^* and M^*. Let $\kappa \equiv [(1-\rho)/f](c/\rho)^{-\rho/(1-\rho)}$. First, substituting (23) and (24) into (20) leads to

$$\mathbf{P}^{-\rho/(1-\rho)} = M\left(\frac{c}{\rho}\right)^{-\rho/(1-\rho)} + N(\kappa Q^*)^{\rho}. \tag{25}$$

Second, using (20), (23), (24) and the oligopolistic firms' first order condition (15), we have[4]

$$\mathbf{P}^{-\rho/(1-\rho)} = (\kappa Q^*)^{\rho}\left[1 - \frac{C}{\rho}(\kappa Q^*)^{1-\rho}\right]^{-1}. \tag{26}$$

Finally, substituting (23) into (19) yields

$$\mathbf{P}^{-\rho/(1-\rho)} = (1-\alpha)\kappa\left\{L - NF + NQ^*\left[(\kappa Q^*)^{-(1-\rho)} - C\right]\right\}. \tag{27}$$

[4]For this expression to be meaningful, its RHS must be positive. We show below that this amounts to assuming that oligopolistic firms earn positive equilibrium profits.

Equating, respectively, (26) and (27) as well as (25) and (27) give us the equilibrium output of oligopolistic firms and the equilibrium mass of MC-firms:

$$(\kappa Q^*)^\rho = (1-\alpha)\left\{\kappa L + N\left[(\kappa Q^*)^\rho - C\kappa Q^* - \kappa F\right]\right\}\left[1 - \frac{C}{\rho}(\kappa Q^*)^{1-\rho}\right] \quad (28)$$

$$M^* = (1-\alpha)\frac{1-\rho}{f}\left\{L - NF - NQ^*\left[\frac{\alpha}{1-\alpha}(\kappa Q^*)^{-(1-\rho)} + C\right]\right\}. \quad (29)$$

Finally, it follows from (20) and (25) that

$$P^* = (\kappa Q^*)^{-(1-\rho)}. \quad (30)$$

In words, any force inducing oligopolistic firms to expand their output leads to a lower price index for these firms. In particular, a larger number of oligopolistic firms leads to a lower equilibrium price for these firms.

We assume through the rest of the paper that, in equilibrium, oligopolistic firms earn strictly positive profits. Substituting (23) into (18), it is readily verified that this assumption is equivalent to the following inequality:

$$\kappa\Pi^* = (\kappa Q^*)^\rho - C\kappa Q^* - \kappa F > 0. \quad (31)$$

This expression together with $Q^* > 0$ and (28) derived below then implies that

$$1 - (C/\rho)(\kappa Q^*)^{1-\rho} > 0. \quad (32)$$

Proposition 3 *Consider a symmetric mixed market equilibrium in which oligopolistic firms earn positive profits. Then, the equilibrium output of an oligopolistic firm increases when the number of oligopolistic firms rises.*

Proof: Differentiating (28) with respect to N yields

$$\begin{aligned}\frac{dQ^*}{dN} &= \mathbb{A}\left[\kappa^\rho (Q^*)^\rho - C\kappa Q^* - \kappa F\right] \\ &\quad \times \left\{(1-\rho)\frac{C}{\rho}\kappa^{2-\rho}(Q^*)^{-\rho}\mathbb{B} + \rho\kappa^\rho (Q^*)^{-(1-\rho)}\left[(1-\alpha)^{-1} - N\mathbb{A}^2\right]\right\}\end{aligned}$$

where $\mathbb{A} \equiv 1 - (C/\rho)(\kappa Q^*)^{1-\rho}$ and $\mathbb{B} \equiv L + \kappa^{-1}N\left[(\kappa Q^*)^\rho - \kappa C Q^* - \kappa F\right]$. Both $\mathbb{A}$ and $\mathbb{B}$ are positive because of (32) and (31).

The first factor in the expression above is positive by (31). In the second, curly bracketed factor, the first term is positive because $\mathbb{B} > 0$. For the proof to be complete, it remains to show that its second term is positive. Note that (28) may be rewritten as follows:

$$(\kappa Q^*)^\rho = (1-\alpha)\kappa\mathbb{A}\mathbb{B}$$

so that

$$\begin{aligned}\rho\kappa^{\rho}(Q^{*})^{-(1-\rho)}\left[(1-\alpha)^{-1}-N\mathbb{A}^{2}\right] &= \rho(Q^{*})^{-1}\kappa\mathbb{A}\mathbb{B}-\rho\kappa^{\rho}(Q^{*})^{-(1-\rho)}N\mathbb{A}^{2} \\ &= \rho(Q^{*})^{-1}\mathbb{A}\left[\kappa\mathbb{B}-(\kappa Q^{*})^{\rho}N\mathbb{A}\right].\end{aligned}$$

Replacing $\mathbb{A}$ and $\mathbb{B}$ by their respective expression, we get

$$\rho\kappa^{\rho}(Q^{*})^{-(1-\rho)}\left[(1-\alpha)^{-1}-N\mathbb{A}^{2}\right]=\kappa(L-NF)+\frac{1-\rho}{\rho}\kappa CNQ^{*}$$

which is positive because (31) implies that $L > NF$. Q.E.D.

Proposition 4 *Consider a symmetric mixed market equilibrium in which oligopolistic firms earn positive profits. Then, both the equilibrium mass of MC-firms and quantity index of this subsector decrease when the number of oligopolistic firms increases.*

Proof: Differentiating (29) with respect to N yields

$$\begin{aligned}\frac{dM^{*}}{dN} &= -(1-\alpha)\frac{1-\rho}{f}\left\{Q^{*}\left[\frac{\alpha}{1-\alpha}(\kappa Q^{*})^{-(1-\rho)}+C\right]+F\right. \\ &\quad \left.+\left[\frac{\alpha}{1-\alpha}\rho(\kappa Q^{*})^{-(1-\rho)}+C\right]N\frac{dQ^{*}}{dN}\right\}<0.\end{aligned}$$

The second part of the statement follows immediately from (24). Q.E.D.

This proposition has an important implication: *the MC-subsector may disappear when the number N of oligopolistic firms is sufficiently large.* Indeed, using (29), we see that the critical value N_O for which $M^{*}=0$ must be a solution to:

$$N=L\left\{\frac{\alpha}{1-\alpha}\kappa^{-(1-\rho)}[Q^{*}(N)]^{\rho}+CQ^{*}(N)+F\right\}^{-1}.$$

This equation has a single and positive solution because the LHS is increasing in N and equal to zero at $N=0$, whereas the RHS is decreasing by Proposition 3 and always positive. When $N>1$ is an integer such that $N\geq N_O$, we have $M^{*}=0$ so that *the market is entirely oligopolistic.* The equilibrium values of the remaining variables are then given as below:

$$\begin{aligned}Q^{O} &= \frac{\rho(1-\alpha)(N-1)}{CN\left[\alpha N+\rho(1-\alpha)(N-1)\right]}(L-NF) \\ \mathbf{P}^{O} &= P^{O}=\frac{C}{\rho}\frac{N^{(2\rho-1)/\rho}}{N-1} \\ \mathbf{Y}^{O} &= \frac{N}{\alpha N+\rho(1-\alpha)(N-1)}(L-NF). \qquad (33)\end{aligned}$$

That Q^O decreases when the level of fixed costs increases stems from the fact that the total income $\mathbf{Y}$ decreases with F. The values (33) slightly differ from those derived in partial equilibrium models in which the total income is fixed and exogenous because profits are redistributed here (Anderson *et al.*, 1992).

In the foregoing, we have uncovered the existence of a trade-off between the two subsectors: *as the oligopolistic subsector expands, the MC-subsector shrinks and vice-versa.* This in turn allows us to determine the impact of an increase in the number of oligopolistic firms on market prices. Indeed, as N rises, it follows from (30) that P^* decreases. However, by (21), the decrease in the mass of MC-firms leads to an increase of P_0^*. Thus, the total impact on $\mathbf{P}$ is a priori undetermined. Yet, we have:

Proposition 5 *Consider a symmetric mixed market equilibrium in which oligopolistic firms earn positive profits. Then, the price index of the differentiated industry decreases when the number of oligopolistic firms increases.*

Proof: Using (26) leads to

$$\frac{d\mathbf{P}}{dN} = -\frac{1-\rho}{\rho}\kappa^{-\rho}\mathbf{P}^{(1-2\rho)/(1-\rho)}\left(Q^*\right)^{-(1+\rho)}$$
$$\times\left\{\rho\left[1-\frac{C}{\rho}\left(\kappa Q^*\right)^{1-\rho}\right]+(1-\rho)\frac{C}{\rho}(\kappa Q^*)^{1-\rho}\right\}\frac{dQ^*}{dN}<0.$$

which is negative by (32). Q.E.D.

In other words, despite the fact that the entry of a new oligopolistic firm triggers the exit of some MC-firms, *the entry of a new oligopolistic firm makes the global market more competitive.* Thus, even though the market might involve less variety, competition becomes fiercer and prices are lower.

4 Welfare

The social welfare is given by the utility of the representative consumer:

$$W = \left(\sum_{j=1}^{N} Q_j^\rho + \int_0^M [q(i)]^\rho di\right)^{(1-\alpha)/\rho} \cdot X^\alpha$$

Introducing (9)-(11) into W, we the indirect utility:[5]

$$W = (\alpha\mathbf{Y})^\alpha\left[(1-\alpha)\mathbf{Y}\right]^{1-\alpha}\mathbf{P}^{-(1-\alpha)}. \tag{34}$$

[5]Note that it is legitimate to assume the existence of a representative consumer because preferences satisfy the Gorman polar form.

Recall that $\mathbf{Y}$ takes into account both the number and the fixed costs of oligopolistic firms. We may thus consider the impact of increasing N upon both $\mathbf{P}$ and $\mathbf{Y}$ to determine how it affects welfare. We already know from Proposition 5 that $\mathbf{P}$ goes down. It remains to consider how $\mathbf{Y}$ is affected.

Using (23), we see immediately that a lower value $\mathbf{P}$ leads to a higher value of $\mathbf{Y}$. Proposition 5 thus implies:[6]

Proposition 6 *Consider a symmetric mixed market equilibrium in which oligopolistic firms earn positive profits. Then, the total income increases when the number of oligopolistic firms increases.*

We are now ready to show:

Proposition 7 *Consider a symmetric mixed market equilibrium in which oligopolistic firms earn positive profits. Then, the social welfare increases when the number of oligopolistic firms rises.*

Proof: Differentiating (34) leads to

$$\frac{dW}{dN} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\mathbf{P}^{-(1-\alpha)}\frac{d\mathbf{Y}}{dN} - \alpha^{\alpha}(1-\alpha)^{2-\alpha}\mathbf{Y}\mathbf{P}^{-(2-\alpha)}\frac{d\mathbf{P}}{dN}.$$

The result then follows from Propositions 5 and 6. Q.E.D.

In words, this result has the following major implication: *a differentiated market with several large firms and a small number of small firms is more efficient than a market with fewer large firms and a larger number of small firms.*

Given that $\mathbf{Y} = L + N\Pi^*(N)$, the proposition above implies that total profits in the economy rise with the number of oligopolistic firms. However, this does not mean that individual profits increase. To check it, we differentiate (31) and get

$$\frac{d\Pi^*(N)}{dN} = \rho(\kappa Q^*)^{-(1-\rho)}\left[1 - \frac{C}{\rho}(\kappa Q^*)^{1-\rho}\right]\frac{dQ^*}{dN} > 0$$

by (32) and Proposition 3. Unlike what we observe in partial equilibrium models of oligopoly where individual profits decrease with the number of firms (see, e.g. Anderson *et al.*, 1992), such profits increase here as long as there exists an MC-subsector. This is because *the entry of a new oligopolistic firm leads to an expansion of the market supplied by these firms at the expense of the MC-subsector*, the size of which shrinks as shown by Proposition 1. Indeed, as the MC-subsector vanishes, $\Pi^*(N)$ evaluated at the purely

[6] Differentiating (19) with respect to N yields a similar result.

oligopolistic outcome (33) appears to be a decreasing function of N. By contrast, when there is an MC-subsector, the market expansion effect generated by the entry of a new oligopolistic firm dominates the competitive effect associated with the presence of more oligopolistic firms.

5 Concluding remarks

The mixed market model seems to differ significantly from standard oligopoly theory. This is worth noting because we often encountered such markets in the real world and because keeping a competitive fringe in quite a few sectors seems to be a concern in several countries.

To be typed or to be done.

1. The first best outcome

2. Is welfare continuous at N_O ?

3. How is welfare between N_O and L/F?

4. The conditions for N oligopolistic firms to earn positive profits and the condition for $M^*(N) > 0$.

Appendix A

It follows from (14) that

$$\frac{\partial \Pi_i}{\partial Q_i} = (1-\alpha)\rho \mathbf{Y} Q_i^{\rho-1}\left(Q_i^\rho + \sum_{j\neq i} Q_j^\rho\right)^{-2}\left(\sum_{j\neq i} Q_j^\rho\right) - C$$

which, in turn, implies that $\lim_{Q_i\to 0}\partial\Pi_i/\partial Q_i = \infty$. Note that $\Pi_i(0) = -F$. Because we have

$$\begin{aligned}\frac{\partial^2 \Pi_i}{\partial Q_i^2} =&(1-\alpha)\rho \mathbf{Y}\left(\sum_{j\neq i} Q_j^\rho\right) Q_i^{\rho-2}\left(Q_i^\rho + \sum_{j\neq i} Q_j^\rho\right)^{-3}\\ &\times\left[-(1+\rho)Q_i^\rho - (1-\rho)\sum_{j\neq i} Q_j^\rho\right] < 0\end{aligned}$$

Π_i is strongly concave with respect to Q_i.

Appendix B

1. Existence. Because $P_i = [(1-\alpha)\mathbf{Y}]^{1-\rho} Q_i^{-(1-\rho)}\mathbf{P}^\rho$, we have

$$P_i Q_i - CQ_i - F = P_i = [(1-\alpha)\mathbf{Y}]^{1-\rho} Q_i^\rho \mathbf{P}^\rho - CQ_i - F \tag{B.1}$$

whereas (23) leads to

$$\mathbf{P}^\rho = [(1-\alpha)\mathbf{Y}]^{-(1-\rho)}\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{c}{\rho}\right)^\rho. \tag{B.2}$$

Plugging (B.2) into (B.1) and using symmetry yield the equilibrium profit of an oligopolistic firm

$$\Pi^* = \left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{cQ^*}{\rho}\right)^\rho - CQ^* - F$$

so that the equilibrium value of the total income is as follows:

$$Y^* = L + N\left[\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{cQ^*}{\rho}\right)^\rho - CQ^* - F\right].$$

Plugging (B.2) into (15) in which $Q_i^* = Q^*$ and simplifying lead to

$$1 = \frac{C}{\rho}\left(\frac{1-\rho}{f}\right)^{1-\rho}\left(\frac{\rho}{c}\right)^\rho (Q^*)^{1-\rho} + [(1-\alpha)\mathbf{Y}]^{-1}\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{c}{\rho}\right)^\rho (Q^*)^\rho$$

which implies

$$Y^* = \frac{\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{c}{\rho}\right)^{\rho}(Q^*)^{\rho}}{(1-\alpha)\left[1-\frac{C}{\rho}\left(\frac{1-\rho}{f}\right)^{1-\rho}\left(\frac{\rho}{c}\right)^{\rho}(Q^*)^{1-\rho}\right]}.$$

Hence, we have

$$\begin{aligned} & L+N\left[\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{cQ^*}{\rho}\right)^{\rho}-CQ^*-F\right] \\ = & \frac{\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{c}{\rho}\right)^{\rho}(Q^*)^{\rho}}{(1-\alpha)\left[1-\frac{C}{\rho}\left(\frac{1-\rho}{f}\right)^{1-\rho}\left(\frac{\rho}{c}\right)^{\rho}(Q^*)^{1-\rho}\right]}. \end{aligned} \tag{B.3}$$

Because the numerator of the RHS of (B.3) is always positive, its denominator must also be positive for $Y^* > 0$ so that Q must be lower than

$$\overline{Q} \equiv \left[\frac{C}{\rho}\left(\frac{1-\rho}{f}\right)^{1-\rho}\left(\frac{\rho}{c}\right)^{\rho}\right]^{1/(1-\rho)}.$$

Let

$$\begin{aligned} h(Q) \equiv & \left[1-\frac{C}{\rho}\left(\frac{1-\rho}{f}\right)^{1-\rho}\left(\frac{\rho}{c}\right)^{\rho}Q^{1-\rho}\right]\left\{L+N\left[\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{cQ}{\rho}\right)^{\rho}-CQ-F\right]\right\} \\ & -\frac{1}{1-\alpha}\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{c}{\rho}\right)^{\rho}Q^{\rho}. \end{aligned}$$

$$Q_0^* = \left[(1-\alpha)\mathbf{Y}\left(\frac{1-\rho}{f}\right)^{1-\rho}\left(\frac{\rho}{c}\right)^{\rho}-N(Q^*)^{\rho}\right]^{1/\rho}$$

Note that $Q = Q^*$ if and only if Q is a solution of $h(Q) = 0$. Since $h(0) = L - NF > 0$ and since $h(\overline{Q}) < 0$, the intermediate value theorem implies that $Q \in]0, \overline{Q}[$ exists such that $h(Q) = 0$.

2. Uniqueness. Standard algebra shows that

$$\begin{aligned} dh/dQ &= -(1-\rho)\frac{C}{\rho}\left(\frac{1-\rho}{f}\right)^{1-\rho}\left(\frac{\rho}{c}\right)^{\rho}Q^{-\rho}\left\{L+N\left[\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{cQ}{\rho}\right)^{\rho}-CQ-F\right]\right\} \\ &\quad -N\rho\left(\frac{f}{1-\rho}\right)\left(\frac{c}{\rho}\right)^{\rho}Q^{\rho-1}\left[1-\frac{C}{\rho}\left(\frac{1-\rho}{f}\right)^{1-\rho}\left(\frac{\rho}{c}\right)^{\rho}Q^{1-\rho}\right]^{2} \\ &\quad -\frac{\rho}{1-\alpha}\left(\frac{f}{1-\rho}\right)^{1-\rho}\left(\frac{c}{\rho}\right)^{\rho}Q^{\rho-1} \\ &< 0. \end{aligned}$$

Hence, $h(Q)$ intersects the Q-axis only once.

Appendix C

(i) Consider first the impact of a larger M on $\mathbf{P}$. Substituting (22) into (20) and simplifying, we obtain

$$\begin{aligned} Q^{*} &= (1-\alpha)\mathbf{Y}N^{-1/\rho}\mathbf{P}^{-1}\left[1-\left(\frac{c}{\rho}\right)^{-\rho/(1-\rho)}M\mathbf{P}^{\rho/(1-\rho)}\right]^{1/\rho} \\ &= (1-\alpha)\mathbf{Y}N^{-1/\rho}\mathbf{P}^{-1}\mathbb{E}^{1/\rho} \end{aligned} \tag{C.1}$$

where $\mathbb{E} \equiv 1-(\rho/c)^{\rho/(1-\rho)}M\mathbf{P}^{\rho/(1-\rho)}$ is positive provided that $Q^{*}>0$. Substituting (C.1) into (15), we have

$$N\mathbf{P} = \frac{C}{\rho}N^{(2\rho-1)/\rho}\mathbb{E}^{(1-\rho)/\rho}+\mathbf{P}-\left(\frac{c}{\rho}\right)^{-\rho/(1-\rho)}M\mathbf{P}^{1/(1-\rho)}. \tag{C.2}$$

Differentiating (C.2) with respect to M yields

$$\begin{aligned} \frac{d\mathbf{P}}{dM} &= -\left(\frac{c}{\rho}\right)^{-\rho/(1-\rho)}\mathbf{P}^{\rho/(1-\rho)}\left(\mathbf{P}+\frac{1-\rho}{\rho}\frac{C}{\rho}N^{(2\rho-1)/\rho}\mathbb{E}^{(1-2\rho)/\rho}\right) \\ &\quad \times\left[N-1+\left(\frac{c}{\rho}\right)^{-\rho/(1-\rho)}M\mathbf{P}^{(2\rho-1)/(1-\rho)}\left(\frac{1}{1-\rho}\mathbf{P}+\frac{C}{\rho}N^{(2\rho-1)/\rho}\mathbb{E}^{(1-2\rho)/\rho}\right)\right]^{-1} \end{aligned}$$

which is negative.

(ii) Let us now study the variation of P^{*}. Substituting (21) in (20), differentiating the resulting expression with respect to M and simplifying leads to

$$\frac{dP^*}{dM} = N^{-1}(P^*)^{-1/(1-\rho)} \left\{ \left[N(P^*)^{-\rho/(1-\rho)} + M \left(\frac{c}{\rho} \right)^{-\rho/(1-\rho)} \right]^{1/\rho} \frac{d\mathbf{P}}{dM} - \left(\frac{c}{\rho} \right)^{-\rho/(1-\rho)} \right\}$$

which is negative because of $d\mathbf{P}/dM < 0$.

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