Repeated Duopolistic Competition with Endogenous Timing*

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Abstract

An infinitely repeated duopolistic competition with endogenous timing is considered in this paper. The underlying market structure covers various duopolistic competition such as quantity and price competition. The necessary and sufficient condition for the credible security level penal code is derived. A sufficient condition for an outcome to be an subgame perfect equilibrium (SPE) outcome is derived when the credible security level penal code is available. The conditions imply that the set of SPE outcomes is enlarged by introducing the endogenous timing. In other words, firms may use timing strategically to sustain more profitable collusion if the discount factor is not sufficiently large.

Keywords: Collusion; Endogenous timing; Repeated games

JEL classification: D43; L13

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1 Introduction

An infinitely repeated duopolistic competition with endogenous timing is considered in this paper. The underlying market structure covers various duopolistic competition such as quantity and price competition. In the stage game, there are two rounds. In the first round, firms simultaneously choose either to commit some action in the stage game or to postpone its decision until the second round. If a firm postpones its decision in the first round, it can determine its action after observing the opponent's action in the first round. Timing *per se* is payoff-irrelevant. At the end of the second round, each firm collects its payoff according to the actions.

The aim of this paper is to investigate whether or not this small deviation from simultaneous moves yields some new strategic effects: Does introducing timing problem changes the structure of collusion? If it does, how does the structural change occurs? The main results are as follows. First, the necessary and sufficient condition for the credible security level penal code (that gives each firm its minimax profit as an average payoff on its punishment path) is derived. Second, a sufficient condition for an outcome to be an subgame perfect equilibrium (SPE) outcome is derived when the credible security level penal code is available. The condition, together with the first result, implies that the set of SPE outcomes is enlarged by introducing the endogenous timing. In other words, firms may use timing strategically to sustain more profitable collusion if the discount factor is not sufficiently large.

Recently, some literature analyzes repeated games in which players do not move simultaneously in a stage game. For example, Lagunoff and Matsui (1997) consider repeated coordination games in which players move asynchronously. Rubinstein and Wolinsky (1995) and Wen (2002) consider repeated games in which a stage game is an extensive form game. There are two differences between their papers and the present paper. The first is on the structure of games. In the present paper, players strategically choose their timing of moves in each stage game. In the papers mentioned above, the timing structure is given exogenously and players cannot use their decision timings strategically. The second is on the motivations. The above papers consider whether or not the Folk Theorem holds when a stage game is not a standard normal form game. In this paper, the Folk Theorem holds obviously because players can choose not to use the timing structure strategically.[1] Our concern is to investigate how the collusive structure changes when firms can use timing as an additional tool in the long-term strategic interaction.

As the title of this paper implies, this paper is also related to the literature on collusive behaviors in the industrial organization. From the Folk Theorem, it is obvious that firms can collude efficiently in most competition if the discount factor is close to 1. However, even if the discount factor is not too large, firms may use various means strategically to sustain more profitable collusion. For example, it is known that firms may hold inventories for this purpose. Rotemberg and Saloner (1989) present a model in which firms can hold inventories. They show that there are cases in which firms can sustain the monopoly outcome if and only if they use inventories strategically. Matsumura (1999) also shows that holding inventories encourages firms to take collusive actions even in finitely repeated competition. Another example is collusive price leadership. Ishibashi (2004) analyzes repeated price competition by n firms with capacity constraints. Assuming the existence of the Nash equilibrium in a one-shot competition with simultaneous moves, I show that the leadership structure allows firms to achieve more profitable collusion compared to collusion without leadership structure. In the context of collusion, this paper shows that the timing structure is important not only in capacity-constrained price competition but also in various kinds of competition.

The remaining of the paper proceeds as follows. In section 2, the model is described. The underlying market structure captures various kinds of duopolistic competition. Based on the market structure, the action commitment game by Hamilton and Slutsky (1990) is used as the stage game. Then, the simple strategy profile by Abreu (1988) is defined with a little modification so that it can apply to the action commitment stage game. In section 3, the main results are derived. The necessary and sufficient condition for the credible security level penal code is derived. Then, we show that, if the credible security level penal code is available, an outcome is an SPE outcome if at least one firm's discounted profit exceeds its profit from the optimal deviation (followed by the minimax punishment). In section 4, a simple quantity competition model is investigated to see the results more clearly. A possible extension to n firms case is also argued in the model. Concluding remarks are in the last section.

[1] That is, all players move in the first round after any history.

2 Model

There are two firms, firm 1 and firm 2. The underlying duopoly game is as follows. Let $z_i \in Z_i$ be firm i's action.[2] When we use i and j at the same time, $i \neq j$ is assumed unless we explicitly mention (otherwise). For $z = (z_1, z_2) \in Z = Z_1 \times Z_2$, firm i's payoff is given by $u_i(z)$. We assume a reasonable property on payoff functions.

Assumption 1
$\max_{z_i \in Z_i} u_i(z)$ is well defined for all z_j.

The assumption seems inappropriate for description of the standard price competition. However, if we use a sufficiently fine grid as the price space, the assumption holds in the modified price competition. We introduce some notations for later analysis.

Define firm i's minimax payoff m^i by

$$m^i = \min_{z_j \in Z_j} \max_{z_i \in Z_i} u_i(z). \tag{1}$$

Let $\underline{z}^i = (\underline{z}_1^i, \underline{z}_2^i)$ be the profile that gives (1).[3]

Define M^i by

$$\begin{aligned} M^i &= \max_{z \in Z} u_i(z) \qquad (2) \\ \text{s.t. } u_j(z) &= m^j \end{aligned}$$

M^i is firm i's maximal payoff subject to the constraint that firm j's payoff is equal to m^j. Let $\bar{z}^i = (\bar{z}_1^i, \bar{z}_2^i)$ be the maximizer of (2).

Given z_j, define $\hat{u}_i(z_j)$ by

$$\hat{u}_i(z_j) = \max_{z_i \in Z_i} u_i(z) \tag{3}$$

$$\text{s.t. } u_j(z) \leq m^j. \tag{4}$$

Given z_j, $\hat{u}_i(z_j)$ is firm i's maximal payoff subject to the constraint that firm j's payoff is less than or equal to m^j. Let $\hat{z}_i(z_j)$ be the maximizer of (3).

[2]We describe general duopolistic competition, which covers various kinds of price competition and quantity competition. So, z_i is interpreted as firm i's price or output, dependent on the context.

[3]If there are multiple solutions, we choose one that maximizes firm j's payoff.

Note that (4) holds for any z_j if z_i is chosen properly. Otherwise, there exists z_j such that firm j can earn more than m^j by choosing z_j regardless of z_i. This contradicts the definition of m^j.

The following properties on the payoff functions are assumed.

Assumption 2
If $u_i(z) > u_i(\bar{z}^k)$, $u_j(z) < u_j(\bar{z}^k)$ for $k = 1, 2$.

Assumption 3
For all $z_j \in Z_j$,

$$
\begin{aligned}
u_i^*(z_j) - \hat{u}_i(z_j) &\leq u_i^*(\underline{z}_j^j) - u_i(\underline{z}^j) \quad \text{and} \qquad (5)\\
u_i^*(\bar{z}_j^i) - M^i &\leq u_i^*(\underline{z}_j^j) - u_i(\underline{z}^j), \qquad (6)
\end{aligned}
$$

where $u_i^*(z_j)$ is firm i's maximal payoff given z_j.

Assumption 2 says that (M^1, m^2) and (m^1, M^2) are Pareto efficient in terms of firms' payoffs.

To see what (5) implies, let us consider the following situation. Firm i is ordered to give firm j no more than m^j as a punishment. However, firm i may deviate from the order. (5) says that firm i's incentive to deviate is larger when firms move simultaneously than when firm i moves after firm j. (5) holds in both price competition and quantity competition. In either case, punishment is costly and the cost is reduced if the punishing firm moves after the punished firm.

(6) implies that, given the constraint that firm j's payoff must be equal to m^j, firm i's incentive to deviate is weakened if its own payoff is maximized.

Now, we construct the stage game based on the above market structure.

The stage game consists of two rounds. In the first round, firms simultaneously choose actions. Firm i chooses an action a_i from $A_i = \{W\} \cup Z_i$, where W is interpreted as waiting and $a_i \in Z_i$ is interpreted as committing to a_i. In the second round, firms that chose W in the first round observe their opponents' first round choices and then decide their actions.[4] At the end of the second round, firms collect their payoff depending on the realization of z.

[4]Thus, our stage game is an action commitment game formulated by Hamilton and Slutsky (1990). Note that the choice of timing is payoff-irrelevant.

Formally, firm i's stage game strategy is expressed as $x_i = (a_i, b_i) \in X_i = A_i \times B_i$, where $B_i = \{b_i | b_i : A_j \to Z_i\}$.[5] The profile of the stage game strategies is denoted by $x = (x_1, x_2)$.

Let $y_i(x) : X_1 \times X_2 \to Z_i$ be a function such that

$$y_i(x) = \begin{cases} a_i & \text{if } a_i \in Z_i \text{ and} \\ b_i(a_j) & \text{if } a_i = W. \end{cases}$$

Firm i's payoff in the stage game is denoted by $u_i(y_1(x), y_2(x))$. From now on, we abuse the notation and use $u_i(x)$ instead of $u_i(y_1(x), y_2(x))$.

Time is discrete and indexed by $t \in T = \{1, 2, \cdots\}$. $\delta \in (0, 1)$ is the common discount factor.

Denote x_i in period t by $x_i(t)$. A path Q is a sequence $\{x(t)\}_{t \in T}$, where $x(t) = (x_1(t), x_2(t))$. Denote x_i in the tth period of Q by $x_i(t|Q)$. $a_i(t|Q)$ and $b_i(t|Q)$ are defined analogously. Let $Q(t)$ be a path that begins from the tth period of Q.

We adopt the subgame perfect equilibrium (SPE) and restrict our attention to simple strategy profiles (Q^0, Q^1, Q^2) developed by Abreu (1988). Q^i $(i = 1, 2)$ is called firm i's punishment path and (Q^1, Q^2) is called a (simple) penal code. First, firms follow the initial path Q^0. If firm i deviates unilaterally from any ongoing prescribed path, Q^i is played from the next period.

In this paper, a penal code is credible if the following inequalities are satisfied for all t and $h, i, j = 1, 2$.

$$\begin{aligned} V_i(Q^h(t)) &\geq u_i^*(x_j(t|Q^h)) + \delta V_i(Q^i) \text{ and} \qquad (7) \\ u_i(x_i(t|Q^h), (a_j, b_j(t|Q^h)) + \delta V_i(Q^j) &\geq u_i^*((a_j, b_j(t|Q^h))) + \delta V_i(Q^i) \qquad (8) \\ &\forall \quad a_j \in A_j - \{a_j(t|Q^h)\} \text{ if } a_i(t|Q^h) = W, \end{aligned}$$

where $V_i(Q)$ is firm i's discounted profit on a path Q.

(7) is the familiar condition that any unilateral deviation is not profitable. (8) says that the second round mover does not deviate after any first round mover's deviation . (8) is needed in our model in order to check the subgame perfection in the stage game.

We say that a penal code is an optimal penal code if it minimizes $V_i(Q^i)$ for $i = 1, 2$ over the set of credible penal codes. Because firm i can earn

[5] For simplicity, we restrict our attention to pure strategies. Note that firms which choose $a_i \in Z_i$ also choose b_i. This assumption is made for the purely technical reason to check the subgame perfection for deviations in the first round.

at least m^i by choosing W in every first round on any ongoing path, any credible penal code must satisfy $V_i(Q^i) \geq \frac{m^i}{1-\delta}$. Following Lambson (1987), a penal code is called a security level penal code if $V_i(Q^i) = \frac{m^i}{1-\delta}$ for $i = 1, 2$.

3 Analysis

In this section, the main results are derived. First of all, an important property on the set of SPE outcomes is described as follows. Suppose an SPE such that all firms commit to actions in the first round after any history. Then, any deviation such that a firm chooses its action in the second round is strategically the same as committing to the action in the first round. That is, strategic interaction in the SPE is the same as that in the corresponding SPE of the game with simultaneous moves. Therefore, given any discount factor, the set of SPE outcomes in the game with endogenous timing includes that in the corresponding game with simultaneous moves. This immediately implies that the Folk theorem also holds in the repeated game with endogenous timing.

Given this argument, the most important issue becomes the characterization of the (enlarged) set of SPE outcomes for relatively small discount factors. In general, to check whether a path is sustainable or not depends on the available credible penal codes. However, as in the repeated game with simultaneous moves, the characterization of credible penal code is difficult especially for low discount factors. Therefore, we focus on the credible security level penal code. Restricting this class of credible penal codes, we can see clearly the essential effect of the endogenous timing in the context of the long-term strategic interactions. First, we derive the condition for the existence of the credible security level penal code. Then, we specify the set of SPE outcomes when the credible security level penal codes are available.

Proposition 1 states the necessary and sufficient condition for the existence of credible security level penal codes in repeated duopolistic competition with endogenous timing.

Proposition 1
Credible security level penal code exists if and only if, for all $i = 1, 2$,

$$u_i(\underline{z}^j) + \frac{\delta M^i}{1-\delta} \geq u_i^*(\underline{z}_j^j) + \frac{\delta m^i}{1-\delta} \tag{9}$$

Proof

"if" part

First, we construct firm 1's punishment path $Q^1 = (x^1, x^1, \cdots)$, where $x_1^1 = (\bar{z}_1^2, b_1^1)$ and $x_2^1 = (W, b_2^1)$. b_1^1 is an arbitrary function in B_1 such that $b_1^1(W) = \underline{z}_1^1$. b_2^1 is the function in B_2 such that

$$b_2^1(a_1) = \begin{cases} \bar{z}_2^2 & \text{if } a_1 = \bar{z}_1^2, \\ \hat{z}_2(a_1) & \text{if } a_1 \in Z_1 - \{\bar{z}_1^2\} \text{ and} \\ \underline{z}_2^1 & \text{if } a_1 = W. \end{cases}$$

Similarly, we construct firm 2's punishment path $Q^2 = (x^2, x^2, \cdots)$, where $x_1^2 = (W, b_1^2)$ and $x_2^2 = (\bar{z}_2^1, b_2^2)$. b_2^2 is an arbitrary function in B_2 such that $b_2^2(W) = \underline{z}_2^2$. b_1^2 is the function in B_1 such that

$$b_1^2(a_2) = \begin{cases} \bar{z}_1^1 & \text{if } a_2 = \bar{z}_2^1, \\ \hat{z}_1(a_2) & \text{if } a_2 \in Z_2 - \{\bar{z}_2^1\} \text{ and} \\ \underline{z}_2^2 & \text{if } a_2 = W. \end{cases}$$

Note that $V_i(Q^i) = \frac{m_i}{1-\delta}$ and $V_j(Q^i) = \frac{M^j}{1-\delta}$.

Now, we show that (Q^1, Q^2) is a credible penal code. For firm i on Q^i, (7) is satisfied because $u_i^*(x_j(t|Q^i)) \leq m^i$ for all t. For firm i on Q^j, (7) is satisfied if

$$\begin{aligned} \frac{M^i}{1-\delta} &\geq u_i^*(\bar{z}_j^i) + \frac{\delta m^i}{1-\delta} \\ \Longleftrightarrow \frac{\delta(M^i - m^i)}{1-\delta} &\geq u_i^*(\bar{z}_j^i) - M^i. \end{aligned} \tag{10}$$

Since rewriting (9) yields

$$\frac{\delta(M^i - m^i)}{1-\delta} \geq u_i^*(\underline{z}_j^j) - u_i(\underline{z}^j),$$

(10) holds from (6).

Finally, we show that the second round mover credibly punishes the first round mover's deviation in the first round. From (5), it is enough to check the credibility when the first round mover waits. So, (8) is satisfied if

$$u_i(\underline{z}^j) + \frac{\delta M^i}{1-\delta} \geq u_i^*(\underline{z}_j^j) + \frac{\delta m^i}{1-\delta}.$$

This is exactly the same as (9).

Thus, (Q^1, Q^2) is a credible penal code and we obtain the desired result.

"only if" part

Without loss of generality, let firm i be the punishing firm. If credible security level penal code exists, firm i must be willing to choose $\underline{z}_i^j$ at least once. Otherwise, firm j can obtain more than m^j by repeatedly waiting in the first round and choosing some z_j in the second round. Firm i accepts $\underline{z}_i^j$ at least once if its future payoff is sufficiently large. Because the maximal individually rational payoff is M^i from Assumption 2, (9) must hold.

Q.E.D.

The rough sketch of the proof of Proposition 1 is as follows.

In general, in order to set a punished firm's profit to its minimax level, a punishing firm often needs to suffer loss so that a punished firm cannot deviate profitably. The punishing firm accepts the loss if the future rewards after the punishing phase is sufficiently large. From (5) and (6), the credibility of the security level penal code is the most vulnerable to the simultaneous move.

For a moment, suppose that firm i is willing to choose $\underline{z}^j$ in every period on Q^j. Then, firm i (j) is made to choose $\bar{z}_i^i$ ($\bar{z}_j^i$) in every second (first) round, respectively, by using $\underline{z}^i$ as a credible threat when firm j deviates and waits in the first round. Therefore, firm i does not have to choose $\underline{z}^j$ actually. Moreover, firm i can obtain M^i, which is the maximal payoff in the individually rational payoff set, in every period with keeping firm j's payoff to m^j. This is the effect of the immediate punishment.

Now we turn back to the question whether or not firm i is willing to choose $\underline{z}^j$ in every period on Q^j. This problem is similar to that in stick-and-carrot strategies where firm i accepts the costly punishment for firm j in order to obtain sufficiently large future payoff. The same is true in our problem. Firm i obeys $\underline{z}^j$ in the current period if its future payoff $\frac{\delta M^i}{1-\delta}$ is enough to compensate the costly punishment. (9) is the formal expression of this condition.

The effect of endogenous timing mentioned above can occur not only in penal codes but also in the initial path. It is interesting to see the overall structural change of collusion brought about by endogenous timing. The following proposition shows a sufficient condition for an outcome to be an

SPE outcome.

Proposition 2
Suppose that (9) holds. For $z^0 = (z_1^0, z_2^0)$ such that $u_i(z^0) \geq m^i$ $(i = 1, 2)$, there exists a sustainable initial path Q^0 such that $V_i(Q^0) = \frac{u_i(z^0)}{1-\delta}$ $(i = 1, 2)$ if the following inequality holds for some i.

$$\frac{u_i(z^0)}{1-\delta} \geq u_i^*(z_j^0) + \frac{\delta m^i}{1-\delta} \tag{11}$$

Proof
Since the security level penal code is available, we only check the credibility on the equilibrium path.
Without loss of generality, we assume that (11) holds for firm 1. Let $Q^0 = (x^0, x^0, x^0, \cdots)$ be the initial path, where $x_1^0 = (W, b_1^0)$ and $x_2^0 = (z_2^0, b_2^0)$. b_2^0 is a function in B_2 such that $b_2^0(W) = \underline{z}_2^2$. b_1^0 is a function in B_1 such that

$$b_1^0(a_2) = \begin{cases} z_1^0 & \text{if } a_2 = z_2^0, \\ \hat{z}_1(a_2) & \text{if } a_2 \in Z_2 - \{z_2^0\} \text{ and} \\ \underline{z}_1^2 & \text{if } a_2 = W. \end{cases}$$

Given x_1^0, firm 2 does not deviate because it cannot earn more than $u_2(z^0)$. Given x_2^0, firm 1 does not deviate if the following two inequalities hold.

$$\begin{aligned} \frac{u_1(z^0)}{1-\delta} &\geq u_1^*(x_2^0) + \frac{\delta m^1}{1-\delta} \\ u_1(x_1^0, (a_2, b_2^0)) + \frac{\delta M^1}{1-\delta} &\geq u_1^*((a_2, b_2^0)) + \frac{\delta m^1}{1-\delta} \quad \forall a_2 \in A_2 - \{z_2^0\} \end{aligned}$$

The first inequality, which is exactly the same as (11), means that firm 1 does not deviate unilaterally. The second inequality, which holds from (5) and (9), means that firm 1 does not deviate after observing any firm 2's deviation. So, we obtain the desired result.
Q.E.D.

It should be mentioned that (11) is required to both firms in the corresponding repeated game with simultaneous moves while (11) is required to only one of the firms in Proposition 2. This is the result of introducing endogenous timing into the initial path given the availability of the credible security level penal code. Also remember that (9) does not guarantee the existence of the credible security level penal code in the repeated game with simultaneous moves. Therefore, it can be said that the set of SPE outcomes is enlarged by the endogenous timing.

4 Example: Duopolistic Quantity Competition

In this section, simple duopolistic quantity competition is investigated to see the results in the previous section more clearly.[6]

Let $P(Q) = 1 - Q$ be the inverse demand function, where Q is the sum of firms' outputs. ($P(Q) = 0$ if $Q \geq 1$.) Firm i's ($i = 1, 2$) output is denoted by $z_i \in [0, 1]$. Marginal cost is assumed to be constant and zero after normalization. Fixed cost is assumed to be zero.

Simple calculations yield

$$\begin{aligned}
m^1 = m^2 &= 0 \\
(\underline{z}_1^1, \underline{z}_2^1) &= (0, 1) \\
(\underline{z}_1^2, \underline{z}_2^2) &= (1, 0) \\
M^1 = M^2 &= \frac{1}{4} \\
(\bar{z}_1^1, \bar{z}_2^1) &= (0, \frac{1}{2}) \\
(\bar{z}_1^2, \bar{z}_2^2) &= (\frac{1}{2}, 0) \\
u_i^*(z_j) &= \frac{(1 - z_j)^2}{4}.
\end{aligned}$$

Because a firm's incentive to deviate is the largest when the opponent's output is zero, (5) is satisfied. Similarly, (6) is satisfied. Also, Assumption 2 is satisfied because $M^i = \max_z u_i(z)$ for $i = 1, 2$.

From Proposition 1, security level penal code exists if and only if $\frac{\delta}{1-\delta} \geq 1$, that is, $\delta \geq \frac{1}{2}$. Furthermore, if $\delta \geq \frac{1}{2}$, any Pareto efficient outcome ($z_1 + z_2 = \frac{1}{2}$) is sustainable. This result, which is derived as follows, comes from Proposition 2.

[6] A similar analysis is presented in section 4.1 of Ishibashi (2004), which considers duopolistic price competition with capacity constraints.

Because it is enough to show that the result holds for the smallest discount factor, suppose that $\delta = \frac{1}{2}$. In this example, (11) is rewritten as

$$2z_i(1 - Q) \geq \frac{(1 - z_j)^2}{4}. \tag{12}$$

Here, without loss of generality, it is assumed $z_1 \geq z_2$. If an outcome is Pareto efficient, Q must be $\frac{1}{2}$. Note that these imply $z_1 \geq \frac{1}{4}$ and $z_2 = \frac{1}{2} - z_1$.

Using these properties, (12) is rewritten as

$$z_1 \geq \frac{(\frac{1}{2} + z_1)^2}{4}.$$

Solving this inequality yields

$$\frac{3 - 2\sqrt{2}}{2} \leq z_1 \leq \frac{3 + 2\sqrt{2}}{2}.$$

Because $\frac{3-2\sqrt{2}}{2} < \frac{1}{4}$, any Pareto efficient outcome is sustainable if $\delta = \frac{1}{2}$.

Furthermore, this example implies a possibility for the extension to n firms case. One of the possible constructions is as follows. First, construct firm i's punishment path Q^i $(i = 1, 2, \cdots, n)$ applying the method in Proposition 1. For Q^i $(i = 2, 3, \cdots, n)$, let firm 1 be a punishing firm and other firms be punished firms.[7] Note that other $n - 2$ firms than firm i are punished together. The essential structure is the same as that in the proof of Proposition 1. $n - 1$ punished firms commit to 0 in every first round on Q^i. Firm 1 produces the monopoly output $(\frac{1}{2})$ in the second round and obtain all of the monopoly profit. These behaviors are credible as follows. By the same logic as that in the above duopoly example, firm 1 is willing to punish any other firm's deviation in the first round if $\delta \geq \frac{1}{2}$. Given this credible immediate punishment, no firm wants to deviate in the first round. Firm 1 does not deviate unilaterally as Proposition 1 shows. Therefore, security level penal code in the n firms case is available if and only if $\delta \geq \frac{1}{2}$. Moreover, as in the duopoly case, any Pareto efficient outcome such that one firm produces more than $\frac{1}{4}$ is sustainable if the security level penal code is available. This extension is based on the fact that minimax punishment is possible by one punishing firm in this quantity competition. If minimax punishment requires multiple punishing firms, the above arguments do not work.

[7] If $i = 1$, firm 2 is the punishing firm.

5 Concluding Remarks

This paper demonstrates that the endogenous timing enlarges the set of SPE outcomes compared to the corresponding repeated game with simultaneous moves. The key factor is the immediate punishment. It allows the second round mover to obtain a large payoff on the opponent's punishment path. The similar effect also occurs on the initial path. The threat of the immediate punishment weakens the first round mover's incentive to deviate. Therefore, the condition to sustain an outcome becomes easy to be satisfied.

The analysis in the paper focus on the credible security level penal code. It is conjectured that the endogenous timing enlarges the set of SPE outcomes even if the credible security level penal code is not available. Unfortunately, it is difficult to characterize optimal penal codes for quite small discount factors. Without characterization of optimal penal codes, the set of SPE outcomes cannot be determined exactly. This line of extension is left for future research.

Two topics should be mentioned. The first topic is on the roles of firms. Throughout this paper, firms' roles of the first (second) round movers on a path are fixed. However, this is not necessary to obtain the essence of the results. The initial path on which a firm sometimes plays the first round mover and at other times plays the second round mover can be sustainable under certain conditions. The conditions would be such that: (i) the first round mover's average payoff exceeds its minimax profit for all t and (ii) the second round mover's discounted payoff is sufficiently large not to deviate for all t.

The second topic is on the repeated competition by n firms with endogenous timing. This extension is briefly analyzed in section 4 using a very special example. If a game has the same property as the example, that is, only one punishing firm is enough to punish a firm at its minimax level, the logic shown in the example would apply to the game. If multiple firms are necessary for the minimax punishment, the effect of the immediate punishment would depend on the details of the games. On the one hand, if the number of the second round movers is large, the immediate punishment prevents the first round movers' deviations more easily. On the other hand, however, the second round mover has a larger incentive to deviate compared to when it is the first round mover because deviation in the second round cannot be punished immediately. Therefore, it becomes difficult to allocate profits as the number of the second mover increases. Therefore, strengthening the effect of the immediate punishment is not necessarily preferable. It would be also interesting to consider the stage game with n rounds. In the game, the set of SPE outcomes would be enlarged further by the optimally determined order of moves. These extensions are left for future research.

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