THE PATTERN OF TRADE, KNOWLEDGE SPILLOVERS AND GLOBAL EFFICIENCY IN R.&D.

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ABSTRACT: This paper aims at discussing how trade patterns and assumptions regarding international transmission of knowledge interact to determine the world allocation of resources to research and innovation. We build an assembly of Dornbush, Fisher and Samuelson's (1977) Ricardian trade model and Grossman & Helpman's (1991) and Aghion's (2003) quality ladder models which differs from that already made by Taylor (1994) in that we do not assume capital mobility for R&D nor licensing. That last feature motivates the analysis of the growth-hindering phenomenon of duplication of R.&D. efforts, which is carried through confronting the former assembled model with trade variations to G.&H.'s basic model where final goods technology is homogenous across countries.

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Introduction

A number of endogenous growth models have been used to analyse the consequences of international trade in a world of global technological competition¹. In this world, new technologies stem from the intentional actions of economic agents responding to market incentives and competing for scarce resources. Countries are typically assumed to differ solely in their factors endowments, so that both the pattern of trade and the rates of innovation tend to be determined by assumed differences in factor intensities between manufacturing and innovating activities. Innovating firms have their monopoly rights to producing new goods protected by some sort of patent system, but a firm's or a country's capacity to improve the quality of a particular product or input is generally regarded as completely independent from the manufacturing of such a product.²

Therefore, the classical Ricardian technological heterogeneity in manufacturing is ruled out by assumption. That leaves unexamined one channel through which heterogeneity in final goods production might influence the innovation activity, even when there is no link between final goods' and innovation technologies, and this last technology is identical across countries: Because in such endogenous growth models R.&D. is guided by market incentives, then the returns to innovation in the same final good may differ across countries solely because of differences in inputs requirements for production. Thence emerges a strong relation between the pattern of trade and the global distribution of R.&D. activities which differs sharply from that existent in Grossman & Helpman's (1991) world where countries differ only in their factors endowments and the innovation activity is arbitrarily supposed to be restricted to some of the final goods.³

In this article an attempt is made to combine a model of Ricardian comparative advantages with a quality ladder model and derive the consequences for the pattern of trade and global efficiency in R&D. That last issue of "efficiency" emerges because, on the contrary of Taylor (1993) who builds an analogous assembled model, we assume that financial capital for R&D is not internationally mobile and there is no "licensing" (when a good is innovated in one country and its production carried over in another). Thus it will not necessarily be the case that a product is innovated only at the cheapest location, what amounts to a problem of simultaneous duplication of R&D efforts at the international level⁴. Indeed, at the national level, we will assume that there exists financial integration, that R&D technology exhibits constant returns, and do away with the traditional assumption in macro models that the arrival of innovations is described by a Poisson process. Working instead with what Dasgupta & Maskin (1987) call "two point distributions" to describe the arrival of innovations pursued by different firms, it will follow that at the national level there will be only one firm targeting each product

¹ See Grossman and Helpman (1991).

 $^{^{2}}$ this happens even when it is assumed, as in Grossman & Helpman's (1991) chapter 3, that the invention of a new good or variety will contribute to the accumulation of a public stock of knowledge, which in turn will reduce uniformly the input requirements for innovation in all goods.

³- this is the world presented in Grossman & Helpman (1991) chapter 7, where the country which specialises in the production of "traditional goods" will present no innovation activity or growth. Indeed, in writing this paper we are to some extent inspired by a criticism to what seems to be an excessive reliance of Grossman and Helpman on asymmetrical structures.

⁴ we speak of **simultaneous** duplication or redundancy as opposed to the case, for example, in which the South (developing countries) spends resources to copy or perform innovations that have already taken place in the North (developed countries), as in Gancia (2002)

simply because constant returns and the possibility of simultaneous success when independent innovators fail to coordinate make that arrangement the only sustainable one. That in turn will require a rational expectations modelling to incorporate the fact that innovative agents are not "atomised" and do take the probability of duplication in account when making their allocation decisions.

As to the relation between the patterns of trade and investment in R&D, Ricardian technological heterogeneity implies that instantaneous profits from innovation are bigger for those goods in which a country has comparative advantage, and indeed the point made in this paper is that duplication of R&D efforts will be smaller between technologically heterogeneous countries or regions than between homogenous ones. However, as Antweiler (1995) points, because innovative firms are maximising not instantaneous profits but the present discounted value of a stream of profits whose duration depends inversely on the rate of innovation, according to a non-arbitrage condition what one expects to observe is a negative relation between innovation intensities and research costs (in our case, relative production costs) across industries⁵. Transposing this prediction to our trade model, we will find that it is possible for a relatively big or high savings' country to "invade" the other country's comparative advantage range of goods to take advantage of smaller obsolescence risks; only that that invasion weakens more and more as one advances through that range in the Ricardian case, while it remains constant in the technologically homogenous case.

The paper is organised as follows: the next section describes the more generalized version of the model, where the above mentioned "invasion" occurs, without showing that duplication will be smaller (and growth faster) in a Ricardian than in a technologically homogenous world. Sections II and III arrive at that result by means of different simplifications of the general model: the first being that countries have the same "size", as in Romer and Rivera-Batiz (1991) integration experiments; the second being actually a discrete-time version of the model where there exists a knowledge spillover such that an innovation becomes common-knowledge and profits are driven to zero after one period, as in Aghion's (2003) simple schumpeterian model. An appendix contains empirical evidence of the relevance of duplication of R&D efforts based on Revealed Technological Advantage (RTA) measures obtained from USPTO patents data.

⁵ Actually, Antweiler's (1995) chief concern is with microeconomic incentives for conducting R&D explaining international differences in growth performance; roughly speaking, following the logic of the "inverse relationship" countries whose economic policies impose high costs on R&D will undergo low rates of innovation

Section I: A Ricardian Model with a Continuum of Rising Quality Products

I.1 – description of the assembled model

Let us briefly review the basic characteristics of the quality ladder model in a closed economy and combine it with a Ricardian trade model with a continuum of goods⁶. The demand side of the economy is determined by agents maximising the following functional:

$$U_{t} = \int_{t}^{\infty} e^{-\rho \cdot (\tau - t)} \cdot \ln D(\tau) \, \mathrm{d}\tau \tag{1}$$

With the instantaneous utility function given by

$$\ln D(\tau) = \int_{0}^{1} \ln \left[\sum_{m} q_{m}(j) \cdot x_{m,t}(j) \right] dj \qquad (2)$$

Where $x_{m,t}(j)$ denotes the consumption of or the demand for the mth quality or generation of good *j* at time *t*, and $q_m(j)$ is an index of quality. It is assumed that $q_o = 1$ for every good *j*. Once the appropriate choices between qualities of the same good and between different goods are made, the instantaneous utility will vary along the equilibrium growth path according to increments in the quality indexes resulting from the innovation activity.

Two important properties of this instantaneous utility function are: 1) it follows from its maximisation that the nominal amount spent on each good will be the same; and 2) once agents choose among qualities or generations of the same good that one which brings the greater quality per unit of money, the elasticity of substitution between any pair of goods will be equal to 1.

At each point in time, income may be broken down into wages and instantaneous profits of monopolist firms and is spent on consumption and acquisition of shares of prospective (innovating) firms. Therefore, aggregate saving is used to hire labour for innovative purposes.

The innovation process and the pattern of firms competition are intimately related: each successful attempt to innovate on good *j* will raise its quality by the exogenously given factor λ , so that $q_m(j) = \lambda^{m-n} \cdot q_{m-n}(j)$, $\lambda > 1^7$. The different qualities of the same good are perfect substitutes of each other. Therefore, each new generation of a good can be charged up to λ^{m-n} times the previous nth generation. If it is charged any infinitesimal amount less than this, the producer of the previous generation will be driven out of the

⁶ This section draws heavily on the Ricardian model presented in Dornbusch et al.(1977) and on the model of rising quality product presented in chapter 4 of Grossman and Helpman (1991).

 $^{^{7}}$ λ may also be determined endogenously, see Grossman and Helpman (1991), page 106

market⁸. Admitting free-disposal, the limit-price for leaving the market is the unit-cost of final good *j*, or a(j). *W*, with *W* representing nominal wages and a(j) the labour input per unit of good j^9 .

Assuming free capital mobility and perfectly non cumulative knowledge¹⁰, then no quality leader will undertake research, and thus goods will be priced by a mark-up that is only one quality index λ over the unit cost:

$$\forall (j,m), p(j,m) = \lambda \cdot a(j) \cdot W = p(j)$$
.¹¹

The model is closed by two market clearing conditions and a free-entry condition in the R&D market: according to this last condition, a positive but limited level of R&D will occur only if the expected value of a new firm or blueprint be equal to the expected cost of performing an innovation. Equilibrium in the labour market requires the sum of demand for labour in manufacturing with that in the R&D sector to be equal to the labour endowment of the economy. Equilibrium in the assets market is expressed in terms of the usual condition that the expect return on any firm's stock be equal to the return on an equal size investment in a riskless bond. This is equivalent to the condition that firms be valued according to the "fundamentals", that is, the present discounted value of their flows of profits.

These conditions determine the dynamics of the two endogenous variables, the aggregate intensity of research and the value of firms at each moment in time. They can be summarised by a differential equation and a contour condition that establishes whether the aggregate value of the firms is rising, falling or is constant. In determining the steady-state of the economy, rational expectations are used to rule out trajectories along which both the aggregate intensity of research and the value of firms tend to zero or the latter grows without bound while the former remains positive.

On the supply side, the rising quality model assumes that labour input is independent of product generation, but Ricardian comparative advantages make it depend upon the particular good being produced and the country which produces, so that:

$$\forall q_m(z), \ a(z,m) = a(z) \text{ and } a^*(z,m) = a^*(z)$$
(3)

where $q_m(z)$ stands for the quality of the mth generation of product z

⁸ The assumption that the quality leader engages in Bertrand price competition and employs a limitpricing strategy is necessary because of Grossman's and Helpman's special utility function and its unitary elasticity of demand. See Barro and Sala-i-Martin (1995), chapter 7, page 245.

 $^{^{9}} a(j)$ is assumed to be equal to 1 for any good *j* in Grossman's & Helpman's version of the model. ¹⁰ "perfectly non cumulative knowledge" is an expression borrowed from Dosi (1984) regarding transmission of product specific knowledge. It means that in spite of property rights or costs which prevent imitation of current state-of-arts products and thus guarantee monopolistic rent to innovators, the current owner of a state-of-arts product has no advantage over other innovators in bringing forth a new vintage of that product. Actually, Dosi himself thinks to be a stylised fact about innovation some degree of cumulativeness. As we shall see at section II below, when transposed to international competition in R.&D. this assumption of perfectly non cumulative knowledge will play a fundamental role in determining the allocation of research efforts.

¹¹ In particular, in Grossman & Helpman's version of the model, with a(j) = 1 for every *j*, and with goods entering the utility function symmetrically, every good will be priced λ . *W* in general equilibrium.

a(z,m) is the labor input per unit produced of the mth generation of product z in the domestic country, with the superscript "*" denoting "the rest of the world".

World-wide consumption expenditure is normalised so that: E = 1. As a result, given the demand function for each good resulting from maximisation of (2), prices and profits may be calculated as:

$$\forall (z,m), \ p(z,m) = p(z) = \lambda \cdots \min(a(z) \cdot W, a^*(z) \cdot W^*)$$
(4) and

$$\pi(z,m) = 1 - \frac{1}{\lambda \cdot \min(a(z) \cdot W, a^*(z) \cdot W^*)} \cdot a(z) \cdot W$$

$$\pi^*(z,m) = 1 - \frac{1}{\lambda \cdot \min(a(z) \cdot W, a^*(z) \cdot W^*)} \cdot a^*(z) \cdot W^*$$
(5),

where $\pi(z,m)$ is the profit earned by the producer of the mth generation of product z in the home country, and the term $[\lambda.min(a(z).W,a^*(z).W^*)]^{-1}$ gives the demanded quantity. Note that, whatever the product or its generation, its price is equal to the quality parameter λ times the internationally minimal unit cost, with W and W* representing nominal wages. Underlying expressions (4) and (5) is a pattern of price competition according to which an innovator will necessarily face a foreign potential competitor able to produce the pre-state-of-arts quality of the same good¹². That is itself a consequence of a well defined **assumption regarding international knowledge spillovers**: Any firm in any country can produce any good z, at period t, with the prestate-of-arts quality, e.g., max $(q_{t-1}(z), q_{t-1}^*(z))$. As a consequence, the maximum that an innovator can charge for the new quality of good z is the mark-up λ times the internationally minimal unit cost of good z. This is a typical "tractability warranting" assumption; without it we would have different prices to the same good, according to where it was produced: $P(z) = \lambda \cdot a(z) \cdot W$ if $q_{t-1}(z) = \max(q_{t-1}(z), q_{t-1}^*(z))$ and $P(z) = \lambda \cdot a^*(z) \cdot W^*$ if $q_{t-1}^* = \max(\bullet)$.¹³

Budget shares are by definition:

$$b(z) \equiv \frac{p(z) \cdot x(z)}{E} \tag{6}$$

¹² - this very pattern is assumed by Yang and Maskus (2001) when they say that "For the leading firm in the Northern market, its closest competitor is the Southern firm that can produce the second-level quality product" (pg. 177) Of course, they also assume there that the South has the lowest wage.

¹³ - In G.&H. (1991, chapter 7, section 2) this problem of two prices to the same good is circumvented by identical technologies and factor price equalisation (with two or more production factors)

where x(z) represents the demanded quantity of good z. Assuming preferences to be identical internationally, that is, $b(z) = b^*(z)$, and given that the choice of the class of utility functions implies b(j) = b(k) for any pair of goods j and k, then we must have

$$\int_{0}^{1} b(z) \cdot dz = 1 \implies b(z) = 1, \ \forall z$$
(7)

that is, Say's law applied on the unity-measure set of goods brings forth an unitary budget share for every good when E is equal to 1.

Technology is assumed to be so smooth that, given a vector of nominal wages (domestic and international), there always exists a good for which domestic and international unitary costs are equal. Formally,

given
$$(W, W^*)$$
, $\exists z \not i a(z) \cdot W = a^*(z) \cdot W^*$ (8)

Reordering the set of goods Z = [0, 1] so that

$$j < k \Longrightarrow \frac{a(j) \cdot W}{a^*(j) \cdot W^*} \le \frac{a(k) \cdot W}{a^*(k) \cdot W^*}$$

that is

$$j < k \Longrightarrow A(j) \equiv a(j)/a * (j) \le A(k) \equiv a(k)/a * (k), \tag{9}$$

we have that

$$\pi(z) = 1 - \frac{1}{\lambda} \equiv \pi \quad \text{for } z \in \left[0, \tilde{z}\right] \text{ and}$$

$$\pi(z) = 1 - \frac{1}{\lambda} \cdot \frac{a(z) \cdot W}{a * (z) \cdot W *} < \pi \quad \text{for } z \in (\tilde{z}, 1] ,$$
(10)

Analogously

$$\pi^*(z) = \pi$$
 for $z \in [\tilde{z}, 1]$ and $\pi^*(z) < \pi$ for $z \in [0, \tilde{z})$

I.2 – the general case without international financial capital mobility

In a model in which financial capital is internationally mobile and there is licensing (innovators can authorise state-of-arts production by subsidiaries abroad), as in Taylor (1993), research and production for each good are carried where they cost less. In the particular case in which research costs are heterogeneous and proportional to production

costs in each country, the ranges of specialisation in production and in R&D will coincide and be given by z defined above, as in Taylor (1994). Here we will assume that there are neither international financial capital mobility nor licensing. We will also follow Grossman & Helpman (1991) homogenous specification of innovation technology, so that in any country it takes *a.t* units of labour for a firm targeting any good to succeed in innovating with probability *t*. Those latter assumptions will blur the clear-cut patterns of specialisation in production and in research found in Taylor (1994). To see why this must be so, consider the standard non-arbitrage condition in the assets market, namely that instantaneous profits plus the change in the value of a firm less the expected value of a total loss due to obsolescence be equal to the instantaneous return to a riskless asset of equal value:

$$\pi(z) + v(z) - \iota \cdot v(z) = r \cdot v(z) \tag{11}$$

where v(z) denotes the discounted value of a firm's profits flow. With $r = \rho$, and in steady-state, (11) gives

$$v(z) = [t(z) + \rho]^{-1} \cdot \pi(z)$$
 (11')

Besides, if there is free-entry in the R&D activity and a finite amount of R&D expenditure, then the expected gain from innovation, namely the value of a firm, must be equal to the research cost. That later being identical for all goods, in equilibrium the values of all firms must be the same. That being so, consider a situation in which the home country is targeting for innovation only $z \in \left[0, \tilde{z}\right]$ while the rest of the world is targeting $z \in \left[\tilde{z}, 1\right]$ and, in particular, the uniform equilibrium innovative efforts are

targeting $z \in [\tilde{z},1]$ and, in particular, the uniform equilibrium innovative efforts are such that $t > t^*$. Call this situation (3). By (9) and (10) we know that the instantaneous profits corresponding to a given good $z' > \tilde{z}$, should the home country hold the patent for its production, will be smaller than those corresponding to a good inside the home country's comparative advantage range. However,

$$v(z') = (\iota^* + \rho^*)^{-1} \cdot \pi(z') = (\iota^* + \rho^*)^{-1} \cdot \left[1 - (A(z'))^{-1} \cdot w \cdot \frac{1}{\lambda}\right]$$
(12)

Because the function A(z) is continuous with A'(z) < 0 and $\left[A\left(\tilde{z}\right)\right]^{-1} \cdot w = 1$ at \tilde{z} ,

then under $\rho = \rho^*$ there must exist some open set Z'' of elements z'' > z such that $v(z'') > v\left(z\right)$. Therefore the situation (3) cannot hold.

This implies that the home country will also target some goods beyond its comparative advantage range. Since there is capital mobility inside the home country, for every good

 $z \in [0, 1]$ targeted, it must be the case that the corresponding firm's value, v(z), is the same. Then the innovation effort on good z', $z' > \tilde{z}$, will be a function t(z'), with t'(z') < 0 since A'(z) < 0. As to the rest of the world, although $\pi^*(z)$ is constant throughout $[\tilde{z}, 1]$, equalisation of firms' values in face of the home country's behaviour t(z') implies an analogous $t^*(z')$ function, but with $t^{*'}(z') > 0$ in the range $[\tilde{z}, \tilde{z}]$ where \hat{z} is such that $t(\hat{z}) = 0$. For $z > \hat{z}$ there exists an homogenous t^* , as there is a homogeneous t for $z < \tilde{z}$.

there is a homogeneous t for $z \le z$. Bearing in mind the above notation, one can establish the following non-arbitrage and free-entry conditions on firms' values:

$$v(z) = (\rho + \iota)^{-1} \cdot \left(1 - \frac{1}{\lambda}\right) = v(z') = \left[\iota^*(z') + \iota(z') - \iota^*(z') \cdot \iota(z') + \rho\right]^{-1} \cdot \left[1 - (A(z'))^{-1} \cdot \frac{w}{\lambda}\right] ,$$

with $v(z) = W \cdot a_1(z) = W \cdot a$ (13)

and analogously for the rest of the world:

$$v^{*}(z) = (\rho^{*} + \iota^{*})^{-1} \cdot \left(1 - \frac{1}{\lambda}\right) = v(z') = [\iota^{*}(z') + \iota(z') - \iota^{*}(z') \cdot \iota(z') + \rho^{*}]^{-1} \cdot \left[1 - \frac{1}{\lambda}\right] ,$$

with $v^{*}(z) = W^{*} \cdot a_{\iota}^{*}(z) = W^{*} \cdot a$ (13')

Notice that in conditions (13) and (13') it is assumed that in a rational expectations equilibrium in which agents are not atomistic, they make their research allocation decisions fully taking in account other agents' innovative efforts as well as the probability of simultaneous discoveries. Recall also that (13) and (13') hold for $z' \in \begin{bmatrix} \tilde{z} & \hat{z} \end{bmatrix}$. Accordingly, it must be the case that

$$v(z) = \left[\iota^*\left(\overset{\circ}{z}\right) + \rho\right]^{-1} \cdot \left[1 - (A(z'))^{-1} \cdot \frac{w}{\lambda}\right] \quad \text{, so that } \iota^*\left(\overset{\circ}{z}\right) \text{ is already the homogenous } \iota^*.$$

Now the labour market clearing conditions become:

$$L = a \cdot \left[\iota \cdot z + \int_{z}^{z} \iota(z') \cdot dz' \right] + z \cdot \frac{1}{\lambda \cdot W} + \int_{z}^{z} \frac{\iota(z') - \iota(z') \cdot \iota^{*}(z')}{\iota(z') + \iota^{*}(z') - 2 \cdot \iota(z') \cdot \iota^{*}(z')} \cdot \frac{1}{\lambda \cdot W^{*}} \cdot [A(z')]^{-1} \cdot dz'$$
(14)

 $L^{*} = a \cdot \left[\iota^{*} \cdot (1 - z) + \int_{z}^{z} \iota^{*}(z') \cdot dz' \right] + (1 - z) \cdot \frac{1}{\lambda \cdot W^{*}} + \int_{z}^{z} \frac{\iota^{*}(z') - \iota(z') \cdot \iota^{*}(z')}{\iota(z') + \iota^{*}(z') - 2 \cdot \iota(z') \cdot \iota^{*}(z')} \cdot \frac{1}{\lambda \cdot W^{*}} \cdot dz'$ (14')

where the first term in the second integral of each expression is a country's proportion of non simultaneous innovations, which will be its share in state-of-arts production.

This picture of the world is to be compared now with that emerging from a completely homogeneous one, where production and innovation costs are the same for both countries. With the linear (constant returns) research technology adopted by Grossman&Helpman (1991), if agents are risk averse and disperse investment uniformly across all goods in [0, 1], the resulting amount of duplication of R&D efforts will be maximal. The comparison with the amount of duplication resulting from the Ricardian world's equilibrium can be made in absolute (and not relative) terms because, due to the special functional forms chosen by G.&H., the two models display the same intensities of innovation both in autarky and under international capital mobility and licensing.¹⁴

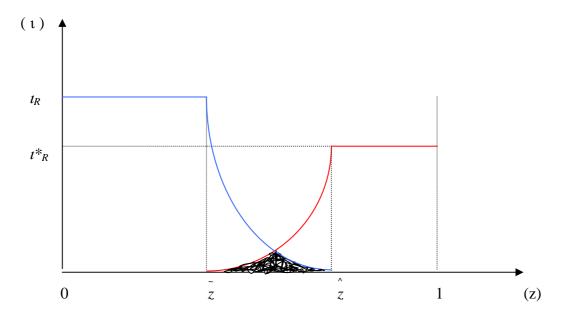


Figure 1: the pattern of R&D investment in a Ricardian World (the home country's investment function is drawn in blue, the rest of the world's in red; duplication of R&D efforts corresponded to the shaded area; the subscript "R" stands for Ricardian)

and

¹⁴ the demonstration of this is relegated to an appendix, available at the reader's request

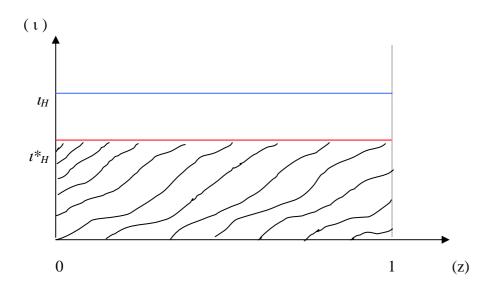


Figure 2: the pattern of R&D investment in a technologically homogenous world (subscript H)

Finding solutions for the research intensities in (13), (13'), (14), (14') requires imposing A(z) some functional form, and still that proves to be analytically cumbersome. Instead we propose here two simplifications to calculate the amounts of innovation (and duplication of R&D efforts) in the Ricardian world.

section II - innovation and duplication for countries of "same size"

II.1 – the Ricardian model

The first, obvious simplification, consists in assuming that A(z) decays so fast at z > z

that $z \cong z$, so that we can ignore duplication in the Ricardian model. Alternatively, one could think of countries of "same size", as Romer and Rivera-Batiz (1991) do in their "integration experiments", so that the intensities of innovation are the same and one doesn't invade the comparative advantage range of the other, with what maximisation of firm's values implies simply maximisation of instantaneous profits.¹⁵ Each country will devote its research efforts only to those goods in which it has comparative advantage

The domestic country shall produce in

¹⁵ as we shall see below, this alternative of assuming equal aggregate research intensities implies equal nominal wages exactly as in the case where there exists international mobility of financial capital and research activity takes place in both countries.

$$Z_{\tilde{z}} = \left\{ z \in [0,1]; \ 0 \le z \le \tilde{z} \right\}$$

With this, and $b(z)=1 \quad \forall z$, the proportion of income spent anywhere on those goods in which the domestic country has comparative advantage will be

$$\mathscr{G}\left(\tilde{z}\right) \equiv \int_{0}^{z} b\left(z\right) \cdot dz = \tilde{z} \qquad (15)$$

If innovation technology is such that it takes a.t units of labour for a firm targeting any good to succeed in innovating with probability t (constant returns) and agents in possession of savings are risk averse, then in lending resources to prospective innovative firms they will distribute resources uniformly across goods. With a unit measure continuum of goods, and by the law of large numbers, t will also be the measure of the set of goods being upgraded at each period, or the aggregate intensity of innovation. So the condition of equilibrium in the labour market may be established as:

$$\tilde{z} \cdot a \cdot \iota + \int_{0}^{z} \frac{1}{\lambda \cdot a(z) \cdot W} \cdot a(z) \cdot dz = L$$

Where $\tilde{z} \cdot a \cdot \iota$ is the demand for labour in research and $\int_{0}^{z} \frac{1}{\tau \cdot a(z) \cdot w} \cdot a(z) \cdot dz$ the demand for labour in manufacturing. Simplifying the term on the integral comes

$$\tilde{z} \cdot a \cdot \iota + \frac{1}{\lambda . W} \cdot \tilde{z} = L$$
 (16)

and analogously for the rest of the world

$$\left(1 - \tilde{z}\right) \cdot a \cdot \iota^* + \frac{1}{\lambda \cdot W^*} \cdot \left(1 - \tilde{z}\right) = L^*$$
(17)

Now, solving (16) for W and (17) for W*, comes

$$W = \frac{1}{\lambda} \cdot \frac{z}{\left(L - \tilde{z} \cdot a \cdot \iota\right)}$$
(18)

$$W^* = \frac{1}{\lambda} \cdot \left[\frac{1 - \tilde{z}}{L^* - (1 - \tilde{z}) \cdot a \cdot \iota^*} \right]$$

and

$$\omega = \frac{W}{W*} = \frac{\overbrace{\left(L-\tilde{z}\cdot a\cdot \iota\right)}^{\tilde{z}}}{\overbrace{\left[L*-\left(1-\tilde{z}\right)\cdot a\cdot \iota^{*}\right]}^{\tilde{z}}} = \frac{\tilde{z}}{1-\tilde{z}} \cdot \frac{\left[L*-\left(1-\tilde{z}\right)\cdot a\cdot \iota^{*}\right]}{\left(L-\tilde{z}\cdot a\cdot \iota\right)}$$
(19)¹⁶

with $\frac{d\omega}{d\bar{z}} > 0$ everywhere.

Given l and l^* , equation (19) and the specialisation condition

 $a * \left(\tilde{z}\right) / a \left(\tilde{z}\right) = \omega$ determine $\bar{\omega}$ and \bar{z} , the trade and research equilibrium

values of ω and z.

In this simplified version of the model, from the non-arbitrage condition in the assets market (11), with $r = \rho$ and $\pi = 1 - \frac{1}{\lambda}$, comes

$$\frac{v}{v} = \iota + \rho - \frac{1 - \frac{1}{\lambda}}{v}$$

The free-entry condition is, as usual, $v \le W \cdot a$. When l = 0, we have $v \le W \cdot a$ and $W = \frac{\tilde{z}}{\lambda \cdot L}$, by (16) above. Therefore, when l = 0, $v \le \frac{a \cdot \tilde{z}}{\lambda \cdot L}$. When l > 0, $v = W \cdot a$. In order for the employment in R&D to be non-negative, we have, by (16), that $L - \frac{\tilde{z}}{\lambda W} \ge 0$, e.g., $W \ge \frac{\tilde{z}}{\lambda \cdot L}$. In short, $l > 0 \Leftrightarrow v > \frac{a \cdot \tilde{z}}{\lambda \cdot L}$. Again, by (16), $l = \frac{L}{a \cdot \tilde{z}} - \frac{1}{\lambda \cdot W \cdot a}$, or, with $v = W \cdot a$, $l = \frac{L}{a \cdot \tilde{z}} - \frac{1}{\lambda \cdot v}$

¹⁶ This function is analogous to the one derived in the static model of Dornbusch et al. (1977).

Defining $V = \frac{1}{v}$

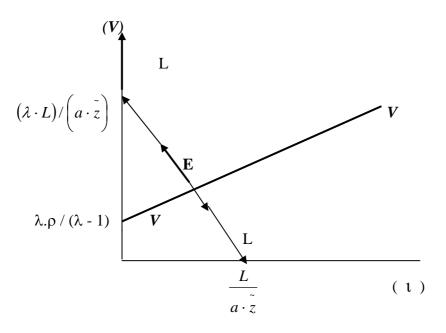
$$\frac{\mathbf{\dot{V}}}{V} = \left(1 - \frac{1}{\lambda}\right) \cdot V - \iota - \rho \tag{20}$$

$$\iota = \begin{cases} 0 & \text{for } V \ge \frac{\lambda \cdot L}{\tilde{a \cdot z}} \\ \frac{L}{a \cdot \tilde{z}} - \frac{V}{\lambda} & \text{for } V < \frac{\lambda \cdot L}{a \cdot \tilde{z}} \end{cases}$$
(21)

setting $\dot{V}/V = 0$ comes

$$\iota = \left(1 - \frac{1}{\lambda}\right) \cdot V - \rho \tag{22}$$

So the phase diagram discribing the dynamics of the two endogenous variables V and t is



Immediate we can see that if $\frac{\lambda \cdot L}{a} > \frac{\rho}{1 - \frac{1}{\lambda}}$, which is the condition for growth in

G.&H.'s basic autarky model, then, with more reason $\frac{\lambda \cdot L}{a \cdot \tilde{z}} > \frac{\rho}{1 - \frac{1}{\lambda}}$, since $0 < \tilde{z}$

As to the solution l:

$$\bar{\iota} = \frac{L}{a \cdot \bar{z}} - \frac{\bar{V}}{\lambda} = \left(1 - \frac{1}{\lambda}\right) \cdot \bar{V} - \rho \implies \left(1 - \frac{1}{\lambda} + \frac{1}{\lambda}\right) \cdot \bar{V} = \frac{L}{a \cdot \bar{z}} + \rho \qquad (23)$$

then, solving (23) for \overline{V} and replacing it in any of the equations for l:

$$\bar{i} = \left(1 - \frac{1}{\lambda}\right) \cdot \frac{L}{a \cdot \bar{z}} - \frac{\rho}{\lambda}$$
(24)

Analogously, for the rest of the world,

$$\bar{\iota}^* = \left(1 - \frac{1}{\lambda}\right) \cdot \frac{L^*}{a \cdot \left(1 - \bar{z}\right)} - \frac{\rho^*}{\lambda}$$
(25).

The rates of innovation in both countries are greater than their rates in autarky, which are given by the expression $\left(1-\frac{1}{\lambda}\right)\cdot\frac{L}{a}-\frac{\rho}{\lambda}$, regardless from the presence of an homogeneous final goods technology production or Ricardian comparative advantages.¹⁷ As expected, a Ricardian specialisation in trade releases work from inefficient employment, to be allocated here in more final goods production or more research, according to the agents' intertemporal preferences.

It should also be noted that in both countries nominal wages are smaller in free trade than in autarky, but real wages are greater in free trade. As the rates of innovation are higher with free trade, consumers in both countries will be better off, because they will be able to purchase a larger quantity of higher quality goods.

¹⁷ Indeed, it can easily be shown that the autarky solution to the "quality ladder" model is independent from both the chosen normalisation of the aggregate expenditure and the final goods' production technology. This, in turn, certainly depends crucially on the rather special functional form adopted for the utility function.

In equilibrium, and assuming intertemporal preferences to be equal internationally, e.g., $\rho = \rho^*$, the country where nominal wage is smaller will present both the rate of profit and the rate of innovation larger than in the rest of the world¹⁸. If manufacturing technology improves across the whole set of goods in the lower wage country, real wage, relative wage and the share of goods domestically produced will rise, but the rate of innovation and the rate of profit will fall domestically. Wage equalisation (nominal and real) will take place when the set of domestically produced goods becomes equal to relative size of the country.

Next, in order to examine the question of the existence and uniqueness of an equilibrium to our Ricardian model, it would be convenient to review the equations determining the endogenous variables t, t^*, W, W^* and z, in steady-state: the specialisation condition (8), (16), (17), (24) and (25). Notice that, instead of (24) and (25), we might as well have used only (22), together with the free-entry condition $V = \frac{1}{W \cdot a}$

Substituting (24) in (16) and solving for W, substituting (25) in (17) and solving for W^* , and finally substituting the resulting expressions for W and W^* in (8), comes

$$a\left(\tilde{z}\right)\cdot\frac{z}{L+a\cdot z\cdot\rho} = a*\left(\tilde{z}\right)\cdot\frac{(1-z)}{L*+a\left(1-\tilde{z}\right)\cdot\rho*}$$

what can be rewritten as

$$\frac{a^{*}\left(\tilde{z}\right)}{a\left(\tilde{z}\right)} \equiv A\left(\tilde{z}\right) = \frac{\frac{z}{L+a\cdot z\cdot \rho}}{\left(1-\tilde{z}\right)}$$

$$(26)$$

$$\frac{\tilde{z}}{L^{*}+a\cdot\left(1-\tilde{z}\right)\cdot \rho^{*}}$$

Now, $A(\tilde{z})$ is a continuous function in [0,1] taking values $[A(1), A(0)] \ge 0 < A(1)$ and $A'(z) \le 0$

¹⁸ This will happen to the country in which the set of domestically produced goods is smaller than the size of the country relatively to the world.

On the other side,
$$F(\tilde{z}) \equiv \frac{\frac{z}{L + a \cdot \tilde{z} \cdot \rho}}{\frac{1 - \tilde{z}}{L + a \cdot (1 - \tilde{z}) \cdot \rho^*}}$$
 is a continuous function of \tilde{z} in [0,1],

with

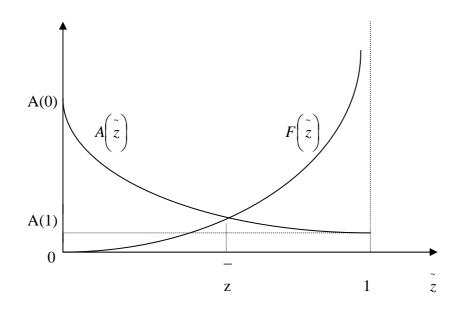
$$F(0) = 0 \quad \text{and} \quad F'\left(\tilde{z}\right) > 0, \quad \text{since}$$
$$\frac{\partial \left[\left(1 - \tilde{z}\right) / \left(L^* + a \cdot \left(1 - \tilde{z}\right) \cdot \rho^*\right)\right]}{\partial \tilde{z}} < 0$$

$$\frac{\partial \left(\frac{\tilde{z}}{L+a\cdot z\cdot \rho}\right)}{\tilde{z}} > 0$$

and

Besides,
$$\lim_{z \to 1} F(z) = \infty$$

Thus $A([0,1]) \cap F([0,1]) \neq \emptyset$ and $\#(A([0,1]) \cap F([0,1])) = 1$



II.2: Innovation in a World with no Comparative Advantages

Let us now examine the effect of trade on the rates of innovation, assuming that both countries have identical technologies for manufacturing goods, so that trade will solely reflect the innovative power or position of each country. We will again assume that any firm in any country can produce any good z, at period t, with the pre-state-of-arts quality, e.g., max $(q_{t-1}(z), q_{t-1}^*(z))$, what we called at section I.1 above a "tractability warranting" assumption regarding international knowledge spillovers.

Let us also assume that any firm in any country, with the same labour input may become with the same probability leader in quality in a market z at period t + 1, even if the leader in quality at period t lay in the other country.¹⁹ Therefore, whatever the pattern of specialisation in production at period t, investment in R&D will befall on the whole set of final goods $\overline{z} = [0,1]$.

In order to simplify the model, labour input per unit of product is taken to be unitary, so that a(z) = a * (z) = 1, $\forall z$. Under free trade, it follows that $p(z) = \lambda \cdot \min(W, W^*)$, and to preserve an incentive to innovate in good z in the highest wage country, it is required that $\lambda \cdot \min(W, W^*) = \mu \cdot \max(W, W^*)$ and $\mu > 1$. Together these conditions imply $\lambda > \max\left(\frac{W^*}{W}, \frac{W}{W^*}\right)$. Recalling the definition $\omega = \frac{W}{W^*}$, this amounts to the following "wages harmonisation range":

$$\omega \in \left(\frac{1}{\lambda}, \lambda\right)$$
 (27)

In a steady-state equilibrium with stable shares of the world production reflecting the sets of goods in which countries detain quality leadership, and this quality leadership being captured at each time by stable innovation efforts of intensity t and t^* , both uniformly aimed at the whole set Z = [0, 1] of final goods, the fractions of Z where each country has quality leadership must be proportional to the respective intensities in R&D.²⁰ Therefore, labour market equilibrium may be written as:

$$a \cdot \iota + \frac{\iota}{\iota + \iota^*} \cdot \frac{1}{\lambda \cdot \min(W, W^*)} = L$$
(28)

$$a \cdot \iota^* + \frac{\iota^*}{\iota + \iota^*} \cdot \frac{1}{\lambda \cdot \min(W, W^*)} = L^*$$
(29)

¹⁹ this forward looking assumption regarding transmission of knowledge amounts to saying, as Grossman & Helpman (1991, chapter 4) do, that innovation is a memoryless process; only that now we take it as holding internationally and not only at a country's level. Dosi (1984) would dub this assumption "perfectly internationally non cumulative knowledge".

²⁰ This can be seen by slightly adapting 7.18 in Grossman & Helpman (1991), pg. 194

adding (28) to (29), it follows that

$$\iota + \iota^* = \left[\left(L + L^* \right) - \frac{1}{\lambda \cdot \min(W, W^*)} \right] \cdot \frac{1}{a}$$
(30)

solving (28) for t and using (30):

$$\Rightarrow l = \frac{L}{a} - \frac{L}{L+L^*} \cdot \frac{1}{a \cdot \min(W, W^*) \cdot \lambda}$$

Analogously

$$t^* = \frac{L^*}{a} - \frac{L^*}{L + L^*} \cdot \frac{1}{a \cdot \min(W, W^*) \cdot \lambda}$$

$$\therefore \quad \frac{\iota}{\iota + \iota^*} = \frac{L}{L + L^*} \tag{32}$$

Before jointing (31) and the non-arbitrage conditions in the assets markets, let us notice that, when t > 0 and $v = W \cdot a$, and $t^* > 0$ and $v^* = W^* \cdot a$, we have

(31)

$$\begin{cases} t = \frac{L}{a} - \frac{L}{L + L^*} \cdot \frac{1}{v \cdot \lambda} \\ \text{and} \\ t^* = \frac{L^*}{a} - \frac{L^*}{L + L^*} \cdot \frac{1}{v \cdot \lambda} \end{cases} \quad \text{if } W = \min(W, W^*) \tag{33}$$

or

$$\begin{cases} t = \frac{L}{a} - \frac{L}{L+L^*} \cdot \frac{1}{v^* \cdot \lambda} \\ \text{and} \\ t = \frac{L}{a} - \frac{L^*}{L+L^*} \cdot \frac{1}{v^* \cdot \lambda} \end{cases} \quad \text{if } W^* = \min(W, W^*) \quad (34)$$

in any case, $v^* = W^* \cdot a = \omega^{-1} \cdot W \cdot a = \omega^{-1} \cdot v$

Under those general conditions ($W \neq W^*$), notice that, defining

$$\pi \equiv 1 - \frac{1}{\lambda \cdot \min(W, W^*)} \cdot W \text{ and}$$

$$\pi^* \equiv 1 - \frac{1}{\lambda \cdot \min(W, W^*)} \cdot W^* \text{ , what is possible in the absence of capital mobility,}$$

given the $\min(W, W^*)$ and adopting the normalisation worldwide expenditure =1, there follows that both (28) and (29) adequately express equilibrium in the trade balance:

$$WL + \frac{\iota}{\iota + \iota^*} \cdot \pi = \frac{\iota}{\iota + \iota^*} \cdot \left[WL + \frac{\iota}{\iota + \iota^*} \cdot \pi + W^* \cdot L^* + \frac{\iota^*}{\iota + \iota^*} \cdot \pi^* - a \cdot \iota \cdot W - a^* \cdot \iota^* \cdot W^* \right] + a \cdot \iota \cdot W$$

$$WL + \frac{\iota}{\iota + \iota^*} \cdot \left(1 - \frac{1}{\lambda}\right) = \frac{\iota}{\iota + \iota^*} \cdot \left(1\right) + a \cdot \iota \cdot W$$
, which is simply equation (28) times W.

And analogously for the rest of the world

$$W * L^* = \frac{\iota^*}{\iota + \iota^*} \cdot \frac{W^*}{\lambda \cdot W} + a \cdot \iota^* \cdot W^* \quad \text{is simply equation (29) times W*.}$$

Therefore, we do not yet have another equation to determine W/W^* inside what we called the "wages harmonisation range". So, let us suppose for the moment that $W < W^*$, and bring forth the conditions of non-arbitrage in the assets market (N-A) and free-entry (F-E):

F.-E.
$$\begin{cases} t > 0 \Longrightarrow v = W \cdot a \\ t^* > 0 \Longrightarrow v^* = W^* \cdot a \end{cases}$$
N-A
$$\begin{cases} \pi^* + v^* - (t^* + t - t^* \cdot t) \cdot v^* = r^* \cdot v^* \\ \cdot \\ \pi + v - (t + t^* - t \cdot t^*) \cdot v = r \cdot v \end{cases}$$
(35)

Notice that in the above Non-Arbitrage condition the term $(t + t^* - t \cdot t^*)$ represents the probability of a total capital loss for a monopolist firm when home and foreign prospective innovators are **independently** targeting a typical final good.²¹

²¹ this differs sharply from Jones & William's (1999) assumption that "atomistic" agents perceive constant returns to the R&D activity while, from the social point of view, there are diminishing returns.

With $r = \rho$ and $r^* = \rho^*$ and $\pi = 1 - \frac{1}{\lambda}$ and $\pi^* = 1 - \frac{1}{\lambda \cdot \omega}$, and defining $V = \frac{1}{\nu}$ and $V^* = \frac{1}{\nu^*}$, comes

N-A
$$\begin{cases} \frac{\mathbf{v}}{V} = \left(1 - \frac{1}{\lambda}\right) \cdot V - (\iota + \iota^* - \iota \cdot \iota^*) - \rho \\ \frac{\mathbf{v}}{V^*} = \left(1 - \frac{1}{\lambda \cdot \omega}\right) \cdot V^* - (\iota + \iota^* - \iota \cdot \iota^*) - \rho^* \end{cases}$$
(36)

Setting $\frac{v}{v} = \frac{v^*}{v^*} = 0$, comes

$$\left(1 - \frac{1}{\lambda}\right) \cdot V - \rho = \left(1 - \frac{1}{\lambda \cdot \omega}\right) \cdot V^* - \rho^*$$
(37)

From (37) we have

$$\left(1-\frac{1}{\lambda}\right)\cdot\frac{1}{W\cdot a}-\rho=\left(1-\frac{1}{\lambda\cdot\omega}\right)\cdot\frac{1}{W^*\cdot a}-\rho^* \Rightarrow (1-\omega)=W\cdot a\cdot(\rho-\rho^*)$$

Because $\omega \leq 1$, that is, $(1-\omega) \geq 0$, we must have $(\rho \geq \rho^*)$, and the greater is the difference $\rho - \rho^*$, the smaller is ω . In particular, when $\rho = \rho^*$, $\omega = 1$ or $W = W^*$. Notice this very same relation between the intertemporal preference rates and relative wages also obtains from the comparative statics of equations (8), (16), (17), (24) and (25) in the Ricardian model.

Solving the first equation in (36) for V, with $\frac{V}{V} = 0$ and condition (32), and substituting in the expression for ι in (33), gives the equation whose sole unknown is the steady-state solution for ι , $\overline{\iota}$:

$$\bar{i} = \frac{L}{a} - \frac{L}{L+L*} \cdot \frac{1}{\lambda - 1} \cdot \left(\bar{i} + \bar{i} \cdot \frac{L*}{L} - \bar{i}^2 + \rho\right)$$
(38)

Because private agents do not "internalise" duplication of R&D efforts, ultimately there will be overinvestment.

which unfortunately does not yield a clean, unique analytical solution. This, nevertheless, will not prevent us from comparing the two models we have just developed (the Ricardian world in II.1 and the technologically homogeneous world in II.2) as to their innovation achievements.

Section II.3: Global Efficiency in R&D

In comparing the two trade and innovation models we have just developed as to their innovation achievements, it must be born in mind that because in the Ricardian model comparative advantages establish an international division of labour in R.&D., with each country targeting a partition of the set Z = [0, 1] of final goods, then the object of comparison cannot be i, called by Grossman & Helpman the "aggregate intensity" of research efforts. Originally, t was the hazard rate of a common Poisson process guiding innovation in every good and, as shown by Feller (1968, pg. 159), when a large number N of such identical processes are carried over during a time interval of length dt, then N.t.dt will be the number of events (quality improvements, in our case) observed. In Grossman & Helpman's quality ladder model, with a unity measure continuum of goods, the "number" or measure of quality improvements observed in an interval of length dt was simply 1. t. dt = t dt. This must be adapted now in order to account for: the varying measures of sets targeted by each country's innovative activity; the possibility of simultaneous discoveries, what was ruled out in G.&H. (1991) by the adoption of a Poisson process. So, let us proceed to compare the global measures of innovation of the Ricardian model and the "homogeneous technology" model:

Using the subscript "G" to denote global or total world innovation, the measure of the set of goods that will undergo a quality raise under the Ricardian model is

$$m(Q_{GR}) = \bar{i}_{R} \cdot \bar{z} + \bar{i}_{R} \cdot (1 - \bar{z}) = \text{under } \rho = \rho^{*} = \left[\left(1 - \frac{1}{\lambda}\right) \cdot \frac{L + L^{*}}{a} - \frac{\rho}{\lambda} \right]$$
(39)

As to the analogous for the homogeneous technology or unitary labour input model, we begin by noticing that solving (36) for *V*, with $\frac{\dot{V}}{V} = 0$, yields *v* as a negative function of the risk of obsolescence. And by (33) *t* is a positive function of *v*, as expected. Therefore the unitary labour input model must present a smaller rate of innovation than that hypothetically generated by (33) and the following non-arbitrage condition:

$$\begin{cases} \pi^{*} + v^{*} - \iota^{*} \cdot v^{*} = r^{*} \cdot v^{*} \\ \bullet \\ \pi + v - \iota \cdot v = r \cdot v \end{cases}$$
(36')

where current state-of-arts owners would face a risk of obsolescence only from their countryfellow's research efforts.²² The rates of innovation emerging from this hypothetical model are easily calculable as

$$\bar{\iota}_{1} = \frac{1 - \frac{1}{\lambda}}{\left[1 - \frac{1}{\lambda} \cdot \left(1 - \frac{L}{L + L^{*}}\right)\right]} \cdot \left(\frac{L}{a} + \rho\right) - \rho$$

and

٢

(40)

$$\bar{\iota^{*}}_{1} = \frac{L^{*}}{L} \cdot \left\{ \frac{1 - \frac{1}{\lambda}}{\left[1 - \frac{1}{\lambda} \cdot \left(1 - \frac{L}{L + L^{*}}\right)\right]} \cdot \left(\frac{L}{a} + \rho\right) - \rho \right\}$$

The global measure of innovation corresponding to this hypothetical model is

$$m(Q_{G1}) = \overline{i \cdot 1} + \overline{i^* \cdot 1} - \overline{i \cdot i^*}$$
(41)
since each country's probability of simultaneous innovation
research efforts befall on
the whole unitary measure
set of final goods

$$= \bar{\iota} \cdot \left[1 + \frac{L^*}{L} \cdot \left(1 - \bar{\iota} \right) \right], \text{ since } \bar{\iota}^* = \frac{L^*}{L} \cdot \bar{\iota}$$

In a first comparison, we will neglect the second order big term $-\overline{i}\cdot\overline{i}^*$ and take the approximation

$$m(Q_{G1A}) \equiv \overline{i} + \overline{i} * = \frac{L + L^*}{L} \cdot \left[\frac{1 - \frac{1}{\lambda}}{1 - \frac{1}{\lambda} \cdot \left(1 - \frac{L}{L + L^*}\right)} \cdot \left(\frac{L}{a} + \rho\right) - \rho \right]$$
(42)

 $^{^{22}}$ - one might wonder if the rates of innovation generated by the unitary labour input model, t and t*, could not be so smaller than those generated by (30) and (33') that the corresponding risk of obsolescence, despite of incorporating the world joint probability of innovation, would actually be also smaller. But this would lead to an absurd; for, if the risk of obsolescence in the unitary labour input model were actually smaller, then necessarily the innovation rates would be bigger.

Then:

$$\mathbf{m}(\mathbf{Q}_{GR}) - \mathbf{m}(\mathbf{Q}_{GIA}) =$$

$$= (L+L^*) \cdot \left\{ \frac{1-\frac{1}{\lambda}}{a} \cdot \left(1-\frac{1}{1-\frac{1}{\lambda}(1-l)}\right) + \rho \cdot \frac{1}{L} - \rho \cdot \frac{1}{L} \cdot \left[\frac{1-\frac{1}{\lambda}}{1-\frac{1}{\lambda}(1-l)}\right] - \frac{\rho}{\lambda \cdot (L+L^*)} \right\}$$

 $m(Q_{GR}) > m(Q_{G1A}) \Longrightarrow \{\bullet\} > 0 \Longrightarrow$

$$\Rightarrow \frac{L}{a} \cdot \left\{ \left(1 - \frac{1}{\lambda}\right) \cdot \left(\frac{l}{\lambda} - \frac{1}{\lambda}\right) \right\} > \rho \cdot \left\{ \frac{l}{\lambda} - 1 + \left[\bullet\right] \right\}$$
$$\Rightarrow \frac{L}{a} \cdot \left\{ \left(1 - \frac{1}{\lambda}\right) \cdot \left(l - 1\right) \right\} > \rho \cdot \left\{ l \cdot \left(1 - \frac{1}{1 - \frac{1}{\lambda}(1 - l)}\right) \right\}$$
$$\underbrace{-1 - \frac{1}{\lambda}(1 - l)}_{<0} \right\}$$

Since the condition for growth in the unitary labour input quality ladder model under trade is $\frac{(\lambda - 1)}{l} \cdot \frac{L}{a} > \rho$, which must always hold, then a sufficient condition to $m(Q_{GR}) > m(Q_{G1A})$ is

$$\left(1-\frac{1}{\lambda}\right)\cdot (l-1) \ge \frac{(\lambda-1)}{l}\cdot l\cdot \left(1-\frac{1}{1-\frac{1}{\lambda}(1-l)}\right) \Longrightarrow$$

$$\Rightarrow \frac{l-1}{\lambda} \ge \frac{l \cdot \frac{1}{\lambda} \cdot (1-l)}{1 - \frac{1}{\lambda} \cdot (1-l)} \Rightarrow$$

 $\Rightarrow \lambda - 1 + l \leq \lambda \Rightarrow l \leq 1$, which is always true. Therefore, $m(Q_{GR}) > m(Q_{GIA})$ and, this latter measure being bigger than that of the unitary labour input model, we may conclude that the measure of the set of goods that will undergo a quality raise under the Ricardian model is safely bigger²³.

section III - discrete time version of the models for arbitrary countries' sizes

The result just obtained in section II.3 might arguably seem to obtain only under the over restrictive assumption, adopted in II.1, that countries be of same "size" or have identical research intensities. Thus next we develop a discrete time version of our previous models allowing for $t \neq t^*$ while still preserving the identity between a firm's value and instantaneous profits. This is done assembling Grossman&Helpman's (1991) quality ladder trade model with Aghion's (2003) simple schumpeterian model, where it is further assumed that due to a knowledge spillover any firm becomes able to produce a state of arts good after one period elapses.

Research arbitrage condition under G.&H.' s technology²⁴ when profits last only one period:

 $\{a_R \cdot \iota \text{ units of labor} \rightarrow \text{probability } \iota \text{ of innovating}\} \Rightarrow \\ \Rightarrow \left\{ N \text{ units of labor} \rightarrow \text{probability } \frac{N}{a_R} \text{ of innovating} \right\}$

So equating the marginal expected benefit to the marginal cost of investment in R&D,

$$\frac{1}{a_R} \cdot \pi = w \implies W = \frac{1}{a_R} \cdot \left(\frac{\lambda - 1}{\lambda}\right) \text{ and } W^* = \frac{1}{a_R^*} \cdot \left(\frac{\lambda - 1}{\lambda}\right)$$
(43)

But then z (the critical good for research specialization) will be given simply by

 $^{^{23}}$ - in establishing this section's comparisons between worlds among other things technologically different, it is important to verify that the chosen normalisation to the world nominal consumption expenditure (E_W) is neutral with relation to real variables both in the assembled Ricardian model and in the unitary labour input model, just as E was neutral under autarky. Thanks to this neutrality property we were authorised in using E_W = 1 in all calculations of sections II.1 and II.2. Here, a prerequisite of making E_W in the assembled Ricardian model somehow compatible with E_W under the unitary labour input model would be something quizzical because in those variations we cannot determine the global income or E_W even in terms of an arbitrary numeraire, since the precise relative wage remains undetermined inside what we labelled "harmonisation range". For the Ricardian assembled model case,

which could seem a bit more complicated because of the problem of determining z, on which the

solution t in turn depends, the neutrality property can be verified by substituting "E_W" for "1" in equations (16) and (17) above, what would all the same leave the relative wage (W/W*) and therefore condition (26) unaltered.

²⁴ Notice that if we preserved Aghion's (2003) original decreasing returns research technology, then the "bigger" country would still target some goods beyond its comparative advantage range, since for those goods there would be a bigger marginal probability of innovation multiplied by an infinitesimally smaller instantaneous profit.

$$\frac{a^{*}\left(\tilde{z}\right)}{a\left(\tilde{z}\right)} \equiv A\left(\tilde{z}\right) = \frac{W}{W^{*}} = \frac{a_{R}^{*}}{a_{R}}$$
(44)

While equilibrium in the home country's labor market requires

$$\tilde{z} \cdot a_R \cdot \iota + \frac{1}{\lambda \cdot W} \cdot \tilde{z} = L \implies \text{by } (43) \implies \iota = \frac{L}{a_R \cdot \tilde{z}} - \frac{1}{\lambda - 1} \text{ and } \iota^* = \frac{L^*}{a_R^* \cdot (1 - \tilde{z})} - \frac{1}{\lambda - 1}$$

(45)

So the global measure (quantity) of innovation is

$$m_{GR} \equiv i \cdot \tilde{z} + i \cdot \left(1 - \tilde{z}\right) = \frac{L}{a_R} + \frac{L^*}{a_R^*} - \frac{1}{\lambda - 1}$$
(46)

Now consider the technologically homogenous world with $a(z) = a^*(z) = 1 \quad \forall z$, where thus duplication is bound to happen.

N units of labor to R&D \rightarrow probability $\frac{N}{a_R} \cdot \left(1 - \frac{N^*}{a_R^*}\right)$ of becoming a leading-edge producer, implying the following research arbitrage conditions:

$$\begin{cases} \frac{1}{a_R} \cdot \left(1 - \frac{N^*}{a_R^*}\right) \cdot \pi = W \\ \frac{1}{a_R^*} \cdot \left(1 - \frac{N}{a_R}\right) \cdot \pi^* = W^* \end{cases}$$
(47)

where

$$\begin{cases} \pi = 1 - \frac{W}{\lambda \cdot \min(W, W^*)} \\ \pi^* = 1 - \frac{W^*}{\lambda \cdot \min(W, W^*)} \end{cases}$$
(48)

And we may write the equilibrium conditions in the labor markets as

$$\left(N + \frac{N/a_R}{N/a_R + N * a_R^*} \cdot \frac{1}{\lambda \cdot \min(W, W *)} = L$$

$$\left(N * + \frac{N * a_R^*}{N/a_R + N * a_R^*} \cdot \frac{1}{\lambda \cdot \min(W, W *)} = L *\right)$$
(49)

which implies

 $\lambda \cdots \min(W, W^*) = (L + L^* - N - N^*)^{-1}$ (50)

Now substituting (50) in any of the equations in (49) comes

$$N \cdot a_R^* \cdot (L^* - N^*) = N^* \cdot a_R \cdot (L - N)$$
(51)

which in particular under $a_R^* = a_R = a$ amounts to

$$N^* = \frac{L^*}{L} \cdot N \tag{51'}$$

Assume now, without loss of generality, that $W = \min(W, W^*)$, and then substitute (51') and (48) in (47) with $a_R^* = a_R = a$:

$$\begin{cases} \frac{1}{a} \cdot \left(1 - \frac{(L^*/L) \cdot N}{a}\right) \cdot \frac{\lambda - 1}{\lambda} = W \\ \frac{1}{a} \cdot \left(1 - \frac{N}{a}\right) \cdot \frac{\lambda \cdot \omega - 1}{\lambda \cdot w} = W^* \end{cases}$$
(52)

where $w \equiv W/W^{*25}$

Besides, from the first equation in (49) and (51') comes

$$W = \frac{1}{\lambda \cdot (L - N)} \cdot \frac{L}{L + L^*}$$
(53)

Substituting (53) in the first equation in (52) we have

$$\frac{L^*}{L} \cdot N^2 - (L^* + a) \cdot N + a \cdot L \cdot \left(1 - \frac{a}{L + L^*} \cdot \frac{1}{\lambda - 1}\right) = 0$$
(54)

²⁵ notice that w<w* implies, then, L*>L

whose sole unknown is *N*, the amount of labor allocated to R&D at the home country. At N = 0 the derivative of the left side of (54) with relation to *N* is < 0. If $\left(1 - \frac{a}{L+L^*} \cdot \frac{1}{\lambda-1}\right) < 0$, due, for example, to too small quality jumps λ , then at N = 0 the left side expression will be <0 and there will be only one positive root/solution for *N*. In any case, for comparative purposes (comparison with the Ricardian world), we will take the greatest root

$$\bar{N} = \frac{(L^* + a) + \sqrt{(L^* + a)^2 - 4 \cdot \frac{L^*}{L} \cdot a \cdot L \cdot \left(1 - \frac{a}{L + L^*} \cdot \frac{1}{\lambda - 1}\right)}{2 \cdot L^* / L}$$
(55)

and, because of (51'),

$$\bar{N}^* = \frac{(L^* + a) + \sqrt{\bullet}}{2}$$

Then for this equilibrium with maximal investment in R&D, the global quantity of innovation will be

$$m_{GD} = \frac{\bar{N}}{a} + \frac{\bar{N}^*}{a} - \frac{\bar{N}}{a} \cdot \frac{\bar{N}^*}{a} =$$

$$= \frac{L^*}{a} \cdot \frac{\left[(L^* + a) + \sqrt{\bullet} \right]}{2 \cdot L^*} - \frac{\left(L^{*2} + 2 \cdot L^* \cdot a + a^2 \right) \cdot L}{2 \cdot L^* \cdot a^2} + \frac{L}{a} - \frac{L}{L + L^*} \cdot \frac{1}{\lambda - 1}$$
(56)

Next recall that under $a_R = a_R^* = a$, the global quantity of innovation for the Ricardian model will be

$$m_{GR} = \frac{L+L^*}{a} - \frac{1}{\lambda - 1} \tag{57}$$

So

$$m_{GR} - m_{GD} = \frac{L}{a} + \frac{L^*}{a} \cdot \left(1 - \frac{\left[L^* + a + \sqrt{\bullet}\right]}{2 \cdot L^*} + \frac{L}{2 \cdot a}\right) + \frac{L}{2 \cdot L^*} + \left(\frac{L}{L + L^*} - 1\right) \cdot \frac{1}{\lambda - 1}$$
(58)

Since $\frac{L}{a} - \frac{1}{\lambda - 1} > 0$ is the condition for growth in autarky, then a sufficient condition

for $m_{GR} > m_{GD}$ is

$$\frac{L^*}{a} \cdot \left(1 - \frac{\left[L^* + a + \sqrt{\bullet}\right]}{2 \cdot L^*} + \frac{L}{2 \cdot a}\right) + \frac{L}{2 \cdot L^*} + \left(\frac{L}{L + L^*}\right) \cdot \frac{1}{\lambda - 1} > 0$$

This in turn will hold as long as

$$\frac{L^{*2} \cdot L^2}{a^2} + 2 \cdot L \cdot L^* \cdot \left(\frac{L^*}{a} - 1\right) + \frac{L \cdot a}{2 \cdot L^{*2}} + \frac{a}{L^*} \cdot \frac{L}{L + L^*} \frac{1}{\lambda - 1} > 4 \cdot \frac{L^*}{L + L^*} \cdot a^2 \cdot \frac{1}{\lambda - 1}$$

But as long as $\frac{L^*}{a} > 1$ and the condition for growth in autarky holds for the rest of the world, namely, $\frac{L^*}{a} > \frac{1}{\lambda - 1}$, the above inequality amounts only to

$$L^* \cdot (L+L^*) \ge 4 \cdot a^4 \tag{59}$$

Again, as in Grossman & Helpman's conditions for perpetual growth, countries mustn't be too small or unproductive in research.

Conclusions

The analysis in this article has shown that in a non-Ricardian world, where comparative advantage does not play any role, momentary quality leadership cannot prevent R&D investment to disperse through the whole set of goods in each country. This implies duplication of research efforts, causing the global measure of innovation to fall short of what it could be with the same aggregate research effort. In such world, it is only by means of a further restriction on knowledge transmission that an efficient global allocation of research effort will obtain²⁶: In APPENDIX 1 we derive the global measure of innovation for the unitary labour input model when innovative knowledge is perfectly cumulative in the sense that once a country takes up the quality leadership in a final good this leadership will be kept forever. In a steady-state with all final goods already so captured, both a country's production and innovation will be restricted to the set of goods in which it has this well established quality leadership, a set whose size or measure is determined by the past history of R.&D. in the world, and that we will here assume to be exogenously given. Meaningfully, when this set's relative size is equal to the country's participation in world's population, this variation to the unitary labour input model will yield the same global measure of innovation of the Ricardian model.

Under Ricardian conditions, on the contrary, global efficiency in the allocation of research efforts will naturally obtain, even when product-specific prospective knowledge is internationally public. If restraining knowledge transmission is bound to be costly, then a Ricardian world will present a net measure of global innovation greater than that of a technologically uniform world. On the normative side, this suggests that although from the point of view of imitation of R&D results the enforcement of Intellectual Property Rights and legislation harmonisation are chief issues for promoting growth in a "North-South" context²⁷, from the point of view of the phenomenon of duplication of R&D efforts those issues are relevant on the contrary in a North-North context or, more generally, for trade between similar regions.

On the empirical side, our analysis purports the same predictions that may be expected by assuming: 1) that owing to capital market imperfections final goods monopolist firms have to finance their own R&D investment; or 2) that there are externalities which stem from comparative advantage in final goods production to comparative advantage in innovation at sectors' level, what is certainly the case for what Patel and Pavitt (1995, pgs. 20, 21) call "production-based" classes of technology (mechanical, for example) though not clearly the case for science-based classes (chemical, electronic). Those predictions are basically that: 1) on the whole, in spite of phenomena like licensing and multinational companies, there is a strong correlation between countries' shares in final

 $^{^{26}}$ - duplication of research efforts might also be prevented in a world with perfect mobility of capital allocated to R&D, causing the world's total savings to be distributed uniformly across goods and so that there is no more than one firm targeting a given final good to be innovated. This last feature of an unified capital market in Grossman & Helpman's quality ladder model, namely that the number of firms is no bigger than the number of goods, is a simple consequence from the fact that the expected return to the total investment of two firms independently targeting a given good to be innovated is smaller than the return to an equal size investment of a single firm.

²⁷ - in Gancia's (2003) model, growth is hindered more and more as the North becomes richer and through Ricardian specialisation a wider range of final goods production is transferred to the South, where IPRs are weaker, thus reducing the incentive to innovation in a world where financial capital mobility is assumed to exist "prior" to trade.

goods or types of final goods production (or exports) and their shares in innovation in those same classes of final goods; 2) that duplication of efforts/achievements or absence of specialisation in R&D is more frequent between countries/regions with similar comparative advantages in final goods production.

Although our analysis focused on efficiency aspects of R.&D. under two different patterns of production and trade, we have already made at the INTRODUCTION a brief comment on asymmetric structures and uneven trade²⁸, a subject upon which we would like to enlarge here: Nowadays, it is well known how trade may lead to uneven growth in the presence of asymmetric structures: In Krugman (1981), with agriculture exhibiting constant returns to scale and manufacture increasing returns, the country which specialise in this latter sector will undergo a sustained rise in per capita income, while the other economy will stagnate. Likewise, and perhaps more obviously, in Grossman & Helpman (1991, chapter 6), with an intermediate goods sector pushing the growth process through innovation, and two competitive final goods sectors, the "hightech" one employing qualified labour and intermediate goods in production, and the other employing non-qualified labour and intermediate goods, trade specialisation in the high-tech final good sector will hinder the growth process when innovation is a R&D activity employing qualified labour.²⁹ In our Ricardian model, with only one production factor and a continuous function A(z) describing comparative advantage in final goods all equally prone to be innovated, there is not such associations between Heckscher-Ohlin specialisation and asymmetric structures by which opening to trade may reduce research activity and growth in some country. Indeed, it can be seen by (18) above that wages in both countries will be shorter under trade than under autarky, while profits remain constant, thus stimulating research.

 $^{^{28}}$ - see note 3

²⁹ - this is a simple consequence of the conjunction of Heckscher-Ohnlin and Stolper-Samuelson theorems: trade specialisation will rise the price of high-tech good and with that the cost of the production factor qualified labour, rendering the R&D activity less profitable. This result may be itself counterfactual in that countries which produce high-tech goods are also outstanding in research, what could in turn be accounted for by the introduction of some positive externality of the level of activity in high-tech's production on the productivity of qualified labour in research, this latter effect outmatching the first, so that the cost of qualified labour measured in efficiency units will be falling over time.

Ishikawa (1992) has conceived an entirely analogous model in order to explain the famous Akamatsu's catching-up product cycle hypothesis, only that a producer services sector takes the place of Grossman & Helpman's intermediate goods sector, and growth steems from an Arrow-like learning by doing process raising the producer services sector productivity.

APPENDIX 1 - the unitary labour input model under restricted international knowledge spillovers

Let the measure of the set $\bar{Z} = \left\{ \bar{z} \in [0,1]; q(\bar{z}) > q^*(\bar{z}) \right\}$ be *s*, exogenously given.

And suppose that the assumption of free international imitation of pre-state-of-arts technology keeps holding. Then the condition of equilibrium in labour markets is

$$a \cdot \iota \cdot s + \frac{1}{\lambda \cdot \min(W, W^*)} \cdot s = L \tag{1}$$

$$a \cdot \iota^* \cdot (1 - s) + \frac{1}{\lambda \cdot \min(W, W^*)} \cdot (1 - s) = L^*$$
(2)

adding (1) and (2) and solving for t^* :

$$\iota^* = \frac{\left(L + L^* - \frac{1}{\lambda \cdot \min(W, W^*)}\right) \cdot \frac{1}{a} - \iota \cdot s}{1 - s}$$
(3)

Without loss of generality, suppose that $W = \min(W, W')$ and that the assumption of free international imitation of pre-state-of-arts technology keeps holding:

Then (3)
$$\Rightarrow a \cdot \iota \cdot s + \frac{1}{\lambda \cdot W} \cdot s = L \Rightarrow by$$
 the usual free-entry
condition $\Rightarrow a \cdot \iota \cdot s + \frac{V \cdot a}{\lambda} \cdot s = L \Rightarrow \quad V = \lambda \cdot \left(\frac{L}{a \cdot s} - \iota\right)$ (4)

On the other side, the non-arbitrage condition is, under $\frac{V}{V} = 0$,

$$\iota = \left(1 - \frac{1}{\lambda}\right) \cdot V - \rho \Longrightarrow (44) \implies \iota = \left(1 - \frac{1}{\lambda}\right) \cdot \lambda \cdot \left(\frac{L}{a \cdot s} - \iota\right) - \rho \Longrightarrow$$

$$\Rightarrow \bar{i}_2 = \frac{\lambda - 1}{\lambda} \cdot \frac{L}{a \cdot s} - \frac{\rho}{\lambda}$$
(5)

The non negativity of $\overline{\iota}_2$ is guaranteed by the condition for growth under autarky. On the other hand, with $W = \min(W, W^*)$, (3) and (5) imply:

$$\bar{\iota}_2^* = \frac{L^*}{a \cdot (1-s)} - \frac{1}{\lambda} \cdot \frac{L}{a \cdot s} - \frac{\rho}{\lambda} \quad \text{, which must be} \ge 0 \tag{6}$$

Notice that this imposes a restriction on *s*, in order to trade keep on taking place – namely, $s_{\min}\left(\bar{L^*}, \bar{L}, \bar{\rho}\right)$.

Resuming (2), with s given, comes

$$L^* - \frac{1}{\lambda \cdot W} \cdot (1 - s) \ge 0 \quad \text{or} \quad W \ge \frac{(1 - s)}{\lambda \cdot L^*}$$
(7)

Analogously, from (1) comes

$$W \ge \frac{s}{\lambda \cdot L} \tag{8}$$

Taking in (7) and (8) the restriction which is binding, that is

$$\max\left(\frac{1-s}{\lambda\cdot L^*},\frac{s}{\lambda\cdot L}\right)$$

If this max is $s/(\lambda L)$, then $s > l = L/(L+L^*)$. That is, when the country with minimal wage has a more than proportional to 1 advantage in quality, this salary cannot be so small that the demand for final goods absorbs more than the whole labour endowment of this country. Anyway, when trade takes place, an excess demand for work in this country will make wages increase.

As to the global measure of innovation, it the will be:

$$m(Q_{G2}) = \overline{i}_{2} * \cdot (1-s) + \overline{i}_{2} \cdot s = by (5) and (6) =$$
$$= \frac{L^{*}}{a} + \frac{L}{\cdot a \cdot \lambda} \cdot \left(\lambda - 1 - \frac{(1-s)}{s}\right) - \frac{\rho}{\lambda}$$
(9)

Comparing $m(Q_{GR}) = m(Q_{G2})$: Ricardian 2^{nd} variation

$$m(Q_{GR}) - m(Q_{G2}) = \left(\frac{\lambda - 1}{\lambda} - 1\right) \cdot \frac{L^*}{a} + \left(\frac{1 - s}{s \cdot \lambda}\right) \cdot \frac{L}{a}$$

 $m(Q_{GR}) > m(Q_{G2}) \Leftrightarrow \frac{L^*}{L} < \frac{1-s}{s} \Leftrightarrow s < l \equiv \frac{L}{L+L^*}$

Therefore

$$s = l \equiv \frac{L}{L + L^{*}} \Leftrightarrow m(Q_{GR}) = m(Q_{G2})$$
$$s > l \Leftrightarrow m(Q_{GR}) < m(Q_{G2})$$
(10)

and we expect this last relation (10) to be reversed when $W^* = \min(W, W^*)$.

These relations are all the more easy to see when one notices that while $m(Q_{GR})$ does not depend on the endogenous shares z and (1-z), $m(Q_{G2})$ is such that $\frac{dm(Q_{G2})}{ds} > 0$ when $W < W^*$. This latter signal is explained by the fact that, when W $< W^*$, then

$$\pi \equiv 1 - \frac{1}{\lambda \cdot \min(W, W^*)} \cdot W > \pi^* \equiv 1 - \frac{1}{\lambda \cdot \min(W, W^*)} \cdot W^* .$$

Setting $m(Q_{G2})=0$, one obtains

$$s_{\min} = \frac{L}{L + L^* \cdot \lambda + (\lambda - 1) \cdot L - \rho \cdot a} \quad \text{or} \quad s_{\min} = f\left(\stackrel{+}{L}, \stackrel{-}{L^*}, \stackrel{+}{\lambda}, \stackrel{+}{\rho}\right) \quad (11)$$

Inspecting (11), it is immediate to verify that if the condition for growth under autarky is satisfied, then $s_{min} < l$. As to $m(Q_{G2})$, it is an increasing function of s, intersecting the horizontal axis at s_{min} , and being equal to $m(Q_{GR})$ when s = l.

So, s > l means that the proportion of final goods targeted for innovation where it is more lucrative is greater than this country's participation in world population, what amounts in a global positive incentive to allocate labour in research. Conversely, $W < W^*$ and s < l, a situation that might very well represent "an innovating North and a Copying South", would constitute a global negative incentive to allocate labour in research. This result lies at the root of such propositions as Yang & Maskus (2001)'s that, when licensing is a channel of technology diffusion from North to South, an increase in Southern Intellectual Property Rights, by reducing the costs to licensing, would foster the northern firms' taking advantage of lower wages in South and thereby incentive them to innovate.³⁰

In comparing $m(Q_{G1})$ and $m(Q_{G2})$, a situation with $s = (L/L+L^*)$ in the second variation would differ from the first variation only in that there would be no duplication of research efforts; accordingly, as can be seen by the above calculations, the global measure of innovation would be bigger. But since s is exogenous, $s = (L/L+L^*)$ is only a benchmark value, and one might conjecture whether such a situation could arise in which, owing to the concentration of a great share of final goods targeted for innovation in the higher wages' country, the global measure of innovation of the second variation

³⁰ - see Yang and Maskus (2001), pg. 171

would be smaller than that of the first, in spite of the effect of duplication of research efforts.

APPENDIX 2 – testing for "duplication" of R&D efforts

Based on RTA (revealed technological advantage) measures for the over 300 hundred USPTO patent classes, we performed chi-square tests of independence to verify whether the 3 blocks of countries (US, Japan and Europe) are significantly specialised in R&D or if, on the contrary, they conduct research on the same technological classes and therefore there is room for duplication of R&D efforts. RTA measures are obtained by dividing a country's share in a given technological class by its share in overall patents granted; a contingency table in which RTAs are close to 1 means that there is not much specialisation in R&D activity.

CHI-SQUARE TESTS

NULL HIPOTHESIS: NON SPECIALISATION IN R&D

Table 1: based on number of patents

SQ. DEV. EURO	SQ.DEV.JAPAN	SQ.DEV.USA	CHI-SQ. STAT	critical value at 5%
184,35	194,93	32,05	411,33	635

Data source: U.S. Patent and Trademark Office

Table 2: based on patent citations (number of citations received)

SQ. DEV. EURO	SQ.DEV.JAPAN	SQ.DEV.USA	CHI-SQ. STAT	critical value at 5%	critical value at 1%
637,72	198	32,15	868,3	864,8	893,9

Data source: NBER, compiled from the USPTO raw data

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