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Intermediate goods and the spatial structure of an economy

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Abstract

We develop a monopolistic competition model of spatial economy in which manufacturing requires a large variety of intermediate goods. The economy yields two types of monocentric configurations: an *integrated city equilibrium* (*I-specialized city equilibrium*) when transaction costs of intermediate goods are high (low). In the former, both manufacturing and intermediate sectors agglomerate in a single city. In the latter, the city is specialized in the provision of intermediate goods. When the economy is in an integrated city equilibrium, it is in a *primacy trap* such that population growth alone never leads to the formation of new cities.

Keywords: City formation; Intermediate goods; Primacy trap; Monopolistic competition

JEL classification: R12; F12; 014

1. Introduction

In this paper, we develop a monopolistic competition model of a spatial economy in which manufacturing requires a large variety of intermediate goods. In our model, cities are formed in a continuous location space due the agglomeration forces that arise from the vertical linkages between the manufacturing and

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intermediate-good sectors. The dispersion forces, in contrast, arise from the demand for the manufactured goods by the agricultural workers who are spatially dispersed due to the necessity of land input. In this context, we examine the role of the variety of intermediate goods and their transport costs in shaping the spatial structure of the economy.

From a methodological viewpoint, our model is not completely new. In fact, our model is closely related to the spatial economy model of Fujita and Krugman (1995), called F–K model hereafter. In the F–K model, the agglomeration forces for city formation arise from the *love for variety* on the consumer side. In contrast, in the present model, the agglomeration forces arise from the *product variety in intermediate goods*. Therefore, the present model is essentially dual to the F–K model. For several reasons, however, we claim relevance of the present work in explaining the formation of cities in the real world.

First, on the empirical side, although the love for variety in consumer goods may play an important role in city formation, casual observations suggest that the accessibility to a large variety of intermediate goods (such as producer services and specific inputs) seems to play an even more important role in the formation of both specialized cities and mega cities. In particular, many metropolises in developed countries have been experiencing a new cycle of growth since the early 1980s (The Economist, 1995). The resurgence of these cities seems to be largely due to the growth of intermediate good sectors. In trying to concentrate in their core-competence and to save overhead costs, firms in most industries are increasingly utilizing the externally-provided goods and services.[1] In particular, in many developed countries, the demand for specialized producer services has grown rapidly, which includes both non-professional and professional services (financial services, legal services, information system management, advertising, accounting, insurance, personnel training, management consultancy, etc.). For example, between 1970 and 1990, employment in the producer service industries in the United States, Japan, and Germany expanded annually 4.77, 4.29, and 2.55%, respectively, while total employment grew at much lower rates (Sassen, 1994).

Intermediate good and service firms create a tacit knowledge in cooperation with final good manufactures. Since such a knowledge is communicated most commonly on the face-to-face basis, interactions between suppliers and users are quite sensitive to distance, while manufactured goods are much easier to transport. Thus, intermediate good firms tend to anchor their users. However, in developed countries, intermediate good and service firms in large cities are able to cater to

[1]For the increasing use of externally-provided intermediate goods and services, there exist other reasons such as to avoid dealing with unions and for flexibility in hiring and firing. Our model in this paper is limited in the sense that the separation of the intermediate sector from the manufacturing sector is assumed a priori, and we focus on the spatial implications of such an industrial structure. The simultaneous determination of both the industrial structure and spatial structure is left for future.

their customers over the long distance owing to the well-developed transportation/communication infrastructure. Hence, their users can be dispersed. For example, based on 1991 data, 47.2% of the total sales of producer services in Japan originated in Tokyo (Statistic Bureau, 1992). Furthermore, as is well-known, London and New York City are international centers of finance, insurance, real estate and other producer services.

On the other hand, in developing countries, all kinds of non-agricultural production tend to be heavily concentrated in primate cities. For example, in Thailand, 1990 data shows that 40% of manufacturing jobs is in Bangkok and the additional 32% is in the five provinces surrounding Bangkok (Labour Studies and Planning Division, 1990). Meanwhile, the financial sector and various producer services account for about 10% of the total employment in Bangkok, which represents the 75% of the total employment of this sector in Thailand. Although a firm in Bangkok could achieve cost savings by moving to a periphery location, the loss of accessibility to the producer services provided in Bangkok tends to make such a relocation unprofitable. Consequently, the sprawl of urbanization tends to occur only within a limited distance from Bangkok.

Next, on the theoretical side, the present model (based on the product variety in intermediate goods) yields a set of outcomes richer than the F–K model (based on the product variety in consumer goods). In particular, the present model yields two types of monocentric configurations involving very different patterns of trade. In one type of monocentric configuration, called an *integrated city equilibrium*, both the manufacturing sector (producing a homogeneous consumer good) and the intermediate sector (supplying a large variety of intermediate goods to the manufacturing sector) are agglomerated together in a single city that exports the manufactured goods to the agricultural hinterland. Such a city resembles primate cities in developing countries. In the other type of monocentric configuration called an *I-specialized city equilibrium*, the intermediate good sector is concentrated in a single city; as for the manufacturing sector, it is partially concentrated in the city, while the rest is mixed with the agricultural sector in such a way that each area produces the manufactured goods for its own needs. The city now exports the intermediate goods only, a pattern which resembles that of several cities in developed countries. Not surprisingly, the integrated city equilibrium (respectively, I-specialized city) tends to arise when the transport costs of intermediate goods are relatively high (respectively, relatively low).

We will also show that once the economy is in an integrated city equilibrium, it is in a *primacy trap* such that the growth of the economy's population alone can never lead to the formation of new major cities. In order to escape from such a primacy trap, it is necessary to sufficiently lower the transport cost of intermediate goods so that the utilization of such goods becomes possible in remote areas. In contrast, in the case of an I-specialized city equilibrium, the population growth of the economy eventually leads to the formation of new cities.

Although our model can potentially yield many different patterns of spatial equilibria (involving multiple cities), in this paper we focus on the two types of monocentric configurations mentioned above. This limitation of scope turns out to be helpful in illuminating the essential role of intermediate goods in shaping the spatial structure of an economy.

The plan of the paper is as follows. In the next section, we present the model and establish equilibrium conditions. In Section 3, we examine the integrated city equilibrium, followed by Section 4 in which the I-specialized city equilibrium is examined. Finally, Section 5 concludes the paper.

2. The model

We consider a boundless, one-dimensional location space of the economy, X, along which lies land of homogeneous quality, with one unit of land per unit of distance. The economy has two final-good sectors, the agricultural sector (A-sector) and the manufacturing sector (M-sector), and a single intermediate-good sector (I-sector). The A-sector produces a homogeneous agricultural good (A-good) under constant returns using labor and land. The M-sector also produces a homogeneous consumer-good, called M-good, under constant returns, using labor and a continuum of intermediate goods (I-goods). The differentiated I-goods are produced in the I-sector under an increasing returns technology, using labor only. For the transportation of each good, we assume Samuelson's 'iceberg' form of transport technology. That is, if a unit of the A-good [the M-good, or any variety of I-good] is shipped from a location $x \in X$ to another location $y \in X$, only a fraction, $e^{-\tau^{A}|x-y|}$[$e^{-\tau^{M}|x-y|}$, or $e^{-\tau^{I}|x-y|}$], of the original unit actually arrives, while the rest melts away en route, where each τ^{A}, τ^{M}, and τ^{I} is a positive constant. Finally, the economy has a continuum of homogeneous workers with a given size, N. Each worker is endowed with a unit of labor, and is free to work in any location and sector.

Although each consumption and production takes place at a specific location, first we describe each type of activity without explicitly referring to the location.

The consumers of the economy consist of N workers plus a class of landlords, who for simplicity are assumed to live on their own land holdings so that land rents are consumed where they are accrued. Every consumer shares the same Cobb–Douglas utility tastes,

$$U = (A)^{1-\alpha}(M)^{\alpha} \tag{1}$$

where A and M, respectively, represent the consumption of the A-good and that of the M-good and α is a constant ($0<\alpha<1$) representing the expenditure share on the M-good. Given an income Y and a pair of prices, p^{A} for the A-good and p^{M} for the M-good, the consumer's utility maximization yields the following demand functions:

$$A = (1-\alpha)Y/p^{\mathrm{A}}, \quad M = \alpha Y/p^{\mathrm{M}}, \tag{2}$$

which in turn yield the following indirect utility function:

$$U = \alpha^{\alpha}(1-\alpha)^{1-\alpha}Y(p^{\mathrm{A}})^{-(1-\alpha)}(p^{\mathrm{M}})^{-\alpha}. \tag{3}$$

Next, the A-good is produced under an input–output technology such that each unit of A-good requires a unit of land and a^{A} units of labor. In contrast, the M-good is produced with a Cobb–Douglas production function,

$$M = (L^{\mathrm{M}})^{1-\mu}I^{\mu}, \quad 0 < \mu < 1 \tag{4}$$

where M is the amount of M-good produced, L^{M} is the associated labor input, and I represents a composite index of I-goods given by

$$I = \left\{ \int_0^n q(i)^{\rho}\mathrm{d}i \right\}^{1/\rho}, 0 < \rho < 1. \tag{5}$$

Here, n represents the range of the I-good varieties supplied by the I-sector, $q(i)$ is the input of each available variety $i \in [0, n]$, and ρ is the substitution parameter. A smaller ρ means that I-goods are more highly differentiated. The variable n is to be determined endogenously in equilibrium.

Given a wage rate, w, and the price of each I-good, $p^{\mathrm{I}}(i)$ for each $i \in [0, n]$, the unit production cost of the M-good associated with the production function (4) is given by

$$c^{\mathrm{M}} = \mu^{-\mu}(1-\mu)^{-(1-\mu)}w^{1-\mu}G^{\mu}, \tag{6}$$

while the requirement for each input associated with an output level M is given by

$$L^{\mathrm{M}} = (1-\mu)c^{\mathrm{M}}M/w, \tag{7}$$

$$q(i) = \mu c^{\mathrm{M}}Mp^{\mathrm{I}}(i)^{-\sigma}G^{\sigma-1} \quad \text{for} \quad i \in [0, n], \tag{8}$$

where $\sigma \equiv 1/(1-\rho)$, and G is the price index of I-goods defined by

$$G = \left\{ \int_0^n p^{\mathrm{I}}(j)^{-(\sigma-1)}\mathrm{d}j \right\}^{-1/(\sigma-1)}. \tag{9}$$

Notice that since $\sigma > 1$, an increase in n *ceteris paribus* reduces the price index G in Eq. (9), which in turn reduces the unit production cost c^{M} in Eq. (6).

Turning to the production of I-goods, each I-good is produced under an increasing-return technology, using labor only. All I-goods have the same production technology such that the production of quantity $q(i)$ of any variety i at any given location requires labor input $l(i)$, given by

$$l(i) = F + a^{\mathrm{I}}q(i), \tag{10}$$

where F and a^{I} are the fixed and marginal labor requirements, respectively. Due to scale economies in the specialized production of I-goods, each variety of I-goods is assumed to be produced by a single firm that chooses its location and f.o.b. (mill) price in a non-strategic manner a la Chamberlin. Hence, the number of active firms equals the number of varieties being produced in the I-sector.

Given the general framework of the model above, next we turn to the equilibrium conditions of the spatial economy. First, we describe them informally. Given a spatial distribution of workers and production activities, the associated spatial economy is an equilibrium if and only if the following five conditions are satisfied:

(i) Equilibrium of workers: regardless of their location and job, all workers achieve the same equilibrium utility level.
(ii) Zero-profit of existing production activities: every active production activity earns zero profit at its present location.
(iii) Market clearing and no arbitrage in trade: markets for all goods (i.e. the A-, M-, and I-goods) are cleared at every location, and no arbitrage is possible in the trade of any good.
(iv) Clearing in the economy-wide labor market.
(v) Location equilibrium of production activity: no production activity can earn a positive profit at any possible location.

Given that there exist many variations of possible equilibrium configurations, the formal representation of the equilibrium conditions above for the generic case requires an introduction of heavy notations. In this paper, however, it is not worth doing so, for we focus our analysis only on two possible equilibrium configurations. Therefore, instead of dealing with a generic case, we consider below two special cases of the monocentric economy.

In the model, increasing returns are involved only in the production of I-goods. Therefore, considering the agglomeration economies caused by the linkages between the M-sector and the I-sector, we assume that the entire production of I-goods takes place at the unique city of the economy located at $x=0$. A part of M-production is assumed to take place in the city, and the rest (if any exists) is dispersed over the agricultural hinterland X^{A}, a subset of X in which land is used for agriculture.

In the context above, let n be the range of I-goods produced in the city, L^{I} be the number of workers in the I-sector (in the city), M_0 be the amount of the M-good produced in the city, and L_0^{M} be the number of workers in the M-sector in the city. Let $M(x)$ be the density of the M-good produced (per unit of distance) at each $x \neq 0$, and $L^{\mathrm{M}}(x)$ be the density of associated workers at each $x \neq 0$. By assumption, the density of agricultural workers is a^{A} at every $x \in X^{\mathrm{A}}$. And, let $p^{\mathrm{A}}(x)$ and $p^{\mathrm{M}}(x)$ be the A-good price and M-good price, respectively, at each

$x \in X$, and $w(x)$ be the wage rate at each x. Finally, let p_0^{I} be the f.o.b. price of each I-variety produced in the city, and U^* be the equilibrium utility level common for all workers in the economy.

In this context, first using Eq. (3), the equilibrium of workers (condition (i)) is achieved by specifying the equilibrium wage rate at each $x \in X$ as follows:

$$w(x) = U^* \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} p^{\mathrm{M}}(x)^{\alpha} p^{\mathrm{A}}(x)^{1-\alpha}, \tag{11}$$

under which all workers achieve the same utility, U^*.

Next, in condition (ii), the zero profit of A-production is assured by specifying the land rent at each location as follows:

$$R(x) = p^{\mathrm{A}}(x) - a^{\mathrm{A}} w(x) \geq 0 \quad \text{for} \quad x \in X^{\mathrm{A}}, \tag{12}$$

$$R(x) = 0 \geq p^{\mathrm{A}}(x) - a^{\mathrm{A}} w(x) \quad \text{for} \quad x \notin X^{\mathrm{A}}, \tag{13}$$

The zero-profit condition in M-production means that

$$M_0 > 0 \Rightarrow p^{\mathrm{M}}(0) = c^{\mathrm{M}}(0), \tag{14}$$

$$M(x) > 0 \Rightarrow p^{\mathrm{M}}(x) = c^{\mathrm{M}}(x) \quad \text{for} \quad x \neq 0, \tag{15}$$

where the unit production cost, $c^{\mathrm{M}}(x)$, at each $x \in X$ is given by Eq. (6) as follows:

$$c^{\mathrm{M}}(x) = \mu^{-\mu}(1-\mu)^{-(1-\mu)} w(x)^{1-\mu} G(x)^{\mu}. \tag{16}$$

Here, the price index of I-goods, $G(x)$, at each x can be obtained as follows. If p_0^{I} is the common f.o.b. price of every variety of I-goods produced at the city, then, due to the assumption of iceberg transport technology, its delivered price, $p^{\mathrm{I}}(0, x)$, at each x is

$$p^{\mathrm{I}}(0, x) = p_0^{\mathrm{I}} \mathrm{e}^{\tau^{\mathrm{I}}|x|}. \tag{17}$$

Thus, using Eq. (9), the price index of I-goods at each x can be obtained as

$$\begin{aligned} G(x) &= \{n[p_0^{\mathrm{I}} \mathrm{e}^{\tau^{\mathrm{I}}|x|}]^{-(\sigma-1)}\}^{-1/(\sigma-1)} \\ &= n^{-1/(\sigma-1)} p_0^{\mathrm{I}} \mathrm{e}^{\tau^{\mathrm{I}}|x|}, \end{aligned} \tag{18}$$

which decreases in n while increases in the distance from the city, $|x|$.

To specify the zero profit condition in I-production, suppose in general that a firm producing a variety of I-good locates at x and chooses an f.o.b. price, $p^{\mathrm{I}}(x)$, for its product. Then since its delivered price at each location $y \in X$ equals $p^{\mathrm{I}}(x)\mathrm{e}^{\tau^{\mathrm{I}}|x-y|}$, the firm's total sales can be obtained by using Eq. (8) as follows:

$$q(x,\, p^{\mathrm{I}}(x)) = \mu c^{\mathrm{M}}(0)M_0\Big[p^{\mathrm{I}}(x)\mathrm{e}^{\tau^{\mathrm{I}}|x|}\Big]^{-\sigma}G(0)^{\sigma-1}\mathrm{e}^{\tau^{\mathrm{I}}|x|} + \int_{X^{\mathrm{A}}} \mu c^{\mathrm{M}}(y)M(y)[p^{\mathrm{I}}(x)\mathrm{e}^{\tau^{\mathrm{I}}|x-y|}]^{-\sigma}G(y)^{\sigma-1}\mathrm{e}^{\tau^{\mathrm{I}}|x-y|}\mathrm{d}y, \tag{19}$$

where the first term on the right-hand side represents the demand originated at the city, and the second term is the demand originated form the dispersed M-production in the active area of the economy, X^{A}. Notice that on each term above, the multiplication at the end by $\exp(\tau^{\mathrm{I}}|x|)$ or $\exp(\tau^{\mathrm{I}}|x-y|)$ reflects the consumption of the firm's product in transportation. Rewriting Eq. (19), we have

$$q(x,\, p^{\mathrm{I}}(x)) = (p^{\mathrm{I}}(x))^{-\sigma}\varphi(x), \tag{20}$$

where

$$\varphi(x) = \mu c^{\mathrm{M}}(0)M_0\mathrm{e}^{-(\sigma-1)\tau^{\mathrm{I}}|x|}G(0)^{\sigma-1} + \int_{X^{\mathrm{A}}} \mu c^{\mathrm{M}}(y)M(y)\mathrm{e}^{-(\sigma-1)\tau^{\mathrm{I}}|x-y|}G(y)^{\sigma-1}\mathrm{d}y. \tag{21}$$

Since the firm is assumed to take all components in Eq. (21) as given, Eq. (20) implies that the price elasticity of the aggregate demand of a firm in the I-sector equals σ. Thus, the equality of its marginal revenue and marginal cost leads to the equilibrium price given by

$$p^{\mathrm{I}}(x) = a^{\mathrm{I}}w(x)/(1-1/\sigma) = a^{\mathrm{I}}w(x)/\rho. \tag{22}$$

Under this pricing rule, the firm's profit equals

$$\pi(x) = a^{\mathrm{I}}\rho^{-1}w(x)q(x, a^{\mathrm{I}}\rho^{-1}w(x)) - w(x)[F + a^{\mathrm{I}}q(x, a^{\mathrm{I}}\rho^{-1}w(x))],$$

or

$$\pi(x) = a^{\mathrm{I}}(\sigma-1)^{-1}w(x)[q(x, a^{\mathrm{I}}\rho^{-1}w(x)) - (\sigma-1)F/a^{\mathrm{I}}]. \tag{23}$$

Hence, in general, if the firm actually operates at x, then by the zero-profit condition under free entry and exit, it must hold that

$$q(x, a^{\mathrm{I}}\rho^{-1}w(x)) = (\sigma-1)F/a^{\mathrm{I}} \equiv q^{*}, \tag{24}$$

while the firm's labor requirement equals $F + (\sigma-1)F = \sigma F \equiv l^{\mathrm{I}*}$. Therefore, for each firm in the I-sector, both the zero-profit output level, q^*, and zero-profit labor input, $l^{\mathrm{I}*}$, are constants which are independent of location.

In the present context of the monocentric economy, since all firms in the I-sector is assumed to be in the city, all I-goods produced (in the city) have the same equilibrium price,

$$p_0^{\mathrm{I}} \equiv p^{\mathrm{I}}(0) = a^{\mathrm{I}}w(0)/\rho, \tag{25}$$

using Eq. (22), and the zero-profit condition in the I-sector means that

$$q(x, a^{\mathrm{I}}\rho^{-1}w(0)) = (\sigma - 1)F/a^{\mathrm{I}} \equiv q^*, \tag{26}$$

using Eq. (24), and the size of the I-good varieties actually produced is given by

$$n = L^{\mathrm{I}}/l^{\mathrm{I}*} = L^{\mathrm{I}}/(\sigma F). \tag{27}$$

Substituting Eqs. (25) and (27) into Eq. (18) yields

$$G(x) = (\sigma F/L^{\mathrm{I}})^{1/(\sigma-1)} a^{\mathrm{I}}\rho^{-1}w(0)\mathrm{e}^{\tau^{\mathrm{I}}|x|}. \tag{28}$$

Finally, setting $x=0$ in Eqs. (20) and (21), and using Eqs. (26) through (28), we obtain the following relation,

$$w(0)L^{\mathrm{I}} = \mu \left\{ c^{\mathrm{M}}(0)M_0 + \int_{X^{\mathrm{A}}} c^{\mathrm{M}}(x)M(x)\mathrm{d}x \right\},$$

which simply means that the total (labor) cost of the I-sector accounts for the $\mu \times 100\%$ of the M-sector's total cost. Using Eq. (7), we have $c^{\mathrm{M}}(0)M_0 = w(0)L_0^{\mathrm{M}}/(1-\mu)$ and $c^{\mathrm{M}}(x)M(x) = w(x)L^{\mathrm{M}}(x)/(1-\mu)$. Hence the equation above can be rewritten as:

$$L^{\mathrm{I}} = \frac{\mu}{1-\mu} \left\{ L_0^{\mathrm{M}} + \int_{X^{\mathrm{A}}} w(x)L^{\mathrm{M}}(x)/w(0)\mathrm{d}x \right\}. \tag{29}$$

For normalization, we set

$$w(0) = 1. \tag{30}$$

Furthermore, through the appropriate normalization of the unit of I-goods, we can assume without the loss of generality that

$$a^{\mathrm{I}} = \rho, \tag{31}$$

which implies that $q^* = (\sigma - 1)F/a^{\mathrm{I}} = \sigma F$.

Using Eqs. (30) and (31), the equilibrium conditions, (22), (11), (16), (28) and (29) can be rewritten, respectively, as:

$$p^{\mathrm{I}}(x) = w(x), \tag{32}$$

$$w(x) = (p^{\mathrm{M}}(x)/p^{\mathrm{M}}(0))^{\alpha}(p^{\mathrm{A}}(x)/p^{\mathrm{A}}(0))^{1-\alpha}. \tag{33}$$

$$c^{\mathrm{M}}(x) = \mu^{-\mu}(1-\mu)^{-(1-\mu)}w(x)^{1-\mu}G(x)^{\mu}, \tag{34}$$

$$G(x) = (\sigma F/L^{\mathrm{I}})^{1/(\sigma-1)}\mathrm{e}^{\tau^{\mathrm{I}}|x|}, \tag{35}$$

$$L^{I} = \frac{\mu}{1-\mu}\left\{L_0^{M} + \int_{X^{A}} w(x)L^{M}(x)\mathrm{d}x\right\}, \tag{36}$$

and the equilibrium utility level is

$$U* = \alpha^{\alpha}(1-\alpha)^{1-\alpha}p^{M}(0)^{-\alpha}p^{A}(0)^{-(1-\alpha)}, \tag{37}$$

using Eq. (11) at $x=0$.

Thus far, we have obtained Eqs. (12) through (15) and Eqs. (32) through (37) defining the equilibrium conditions in (i) and (ii). Turning to condition (iii), the income at the city equals $w(0)(L_0^{M} + L^{I}) = L_0^{M} + L^{I}$ using Eq. (30), and the income (per unit of distance) at each $x \neq 0$ is $p^{A}(x) + w(x)L^{M}(x)$. Hence, using Eq. (2), the consumer demand for the A-good at the city and that for the M-good are, respectively

$$D_0^{A} = (1-\alpha)(L^{M} + L^{I})/p^{A}(0), \tag{38}$$

$$D_0^{M} = \alpha(L_0^{M} + L^{I})/p^{M}(0), \tag{39}$$

while those at each $x \neq 0$ are, respectively

$$D^{A}(x) = (1-\alpha)(p^{A}(x) + w(x)L^{M}(x))/p^{A}(x), \tag{40}$$

$$D^{M}(x) = \alpha(p^{A}(x) + w(x)L^{M}(x))/p^{M}(x). \tag{41}$$

The production of the A-good per unit distance at each $x \in X^{A}$ equals 1 by assumption. Using Eq. (7) and (30), the production of the M-good at the city and that at each $x \neq 0$ can be obtained, respectively, as

$$M_0 = L_0^{M}/(1-\mu)c^{M}(0), \tag{42}$$

$$M(x) = w(x)L^{M}(x)/(1-\mu)c^{M}(x). \tag{43}$$

The matching of the demand and supply of each good at each location, however, cannot be done without specifying the trade pattern of each good. Hence, we postpone the remaining analysis of condition (iii) to the following sections where the trade pattern of each good is explicitly specified in association with each possible equilibrium configuration.

Concerning condition (iv), the clearing at the economy-wide labor market means that

$$a^{A}|X^{A}| + \left\{L_0^{M} + \int_{X^{A}} L^{M}(x)\mathrm{d}x\right\} + L^{I} = N,$$

where $|X^{\mathrm{A}}|$ represents the size of the agricultural area. Using Eq. (36), this equation can be rewritten as follows:

$$a^{\mathrm{A}}|X^{\mathrm{A}}| + L_0^{\mathrm{M}}/(1-\mu) + \int_{X^{\mathrm{A}}} \left(1 + \frac{\mu}{1-\mu} w(x)\right) L^{\mathrm{M}}(x)\mathrm{d}x = N. \tag{44}$$

Next, concerning condition (v), the location equilibrium of agriculture has already been assured by Eqs. (12) and (13). The location equilibrium of manufacturing means that

$$p^{\mathrm{M}}(x) \leq c^{\mathrm{M}}(x) \text{ for all } x \in X. \tag{45}$$

By Eqs. (23) and (31), the location equilibrium of I-industry means that

$$q(x, w(x)) \leq \sigma F \equiv q^* \text{ for all } x \in X. \tag{46}$$

For convenience, let us define the *(market) potential function*, $\Omega(x)$, of I-industry at each location $x \in X$ by

$$\Omega(x) \equiv \frac{q(x, w(x))}{q^*}, \tag{47}$$

which is the ratio of the sales of a potential firm in I-industry at x to the zero-profit output level, $q^* = \sigma F$. Then, the location equilibrium of I-industry can be simply expressed as:

$$\Omega(x) \leq 1 \text{ for all } x \in X, \tag{48}$$

saying that the market potential of I-industry never exceeds 1.

Notice by Eqs. (26) and (31) that the market potential Eq. (47) always equals unity at the city:

$$\Omega(0) = 1. \tag{49}$$

Hence, the market potential, $\Omega(x)$, measures the relative profitability of each location $x \in X$ (for I-production) in comparison with the city location. Substituting Eqs. (32), (42) and (43) into Eqs. (20) and (21), and using Eq. (31), the market potential of I-industry can be rewritten as follows:

$$\Omega(x) = \frac{\mu}{(1-\mu)F\sigma} w(x)^{-\sigma} \Bigg\{ L_0^{\mathrm{M}} \mathrm{e}^{-(\sigma-1)\tau^{\mathrm{I}}|x|} G(0)^{\sigma-1} + \int_{X^{\mathrm{A}}} L^{\mathrm{M}}(y) w(y) \mathrm{e}^{-(\sigma-1)\tau^{\mathrm{I}}|x-y|} G(y)\mathrm{d}y \Bigg\}. \tag{50}$$

Finally, concerning the welfare analysis of the economy, notice that our economy has two groups of consumers, workers and landlords. The welfare of each worker, of course, is represented by the equilibrium utility, U^*, given by Eq.

(37). Since the landlords at x (per unit of distance) receive the land rent $R(x)$, their (aggregate) utility, $U^{L}(x)$, can be obtained as:

$$U^{L}(x) = \alpha^{\alpha}(1-\alpha)^{1-\alpha}R(x)p^{A}(x)^{-(1-\alpha)}p^{M}(x)^{-\alpha}, \tag{51}$$

where $R(x) = p^{A}(x) - a^{A}w(x)$ at each $x \in X^{A}$. Hence, using Eq. (11), we obtain $U^{L}(x)/U^{*} = (p^{A}(x)/w(x)) - a^{A}$, or

$$U^{L}(x) = U^{*}\left(\frac{p^{A}(x)}{w(x)} - a^{A}\right)\text{for } x \in X^{A}. \tag{52}$$

Of course, $U^{L}(x) = 0$ for $x \notin X^{A}$.

In summary, we have obtained a set of equilibrium conditions, (12)–(15), (32)–(37), and (44)–(52), which are common for any monocentric spatial configuration. In each section below, first we introduce additional specifications on the spatial configuration, and obtain the remaining equilibrium conditions specified in page 6 using Eqs. (38) through (43). Then, by using the entire set of equilibrium conditions, the parameter range of each equilibrium configuration can be determined.

3. The integrated city equilibrium

Given the model in the previous section, we consider two specific spatial configurations in turn. First, in this section we consider the integrated city configuration depicted in Fig. 1.

3.1. The spatial structure of the integrated city economy

In this configuration, the entire production activity of both the M- and I-sectors is assumed to take place in the city located at $x=0$. The agricultural area surrounds the city symmetrically from $-f$ to f. The city exports the M-good to each location in the agricultural area, and imports the A-good.

Let $p^{A}(0)$ and $p^{M}(0)$ be the price of the A-good and that of the M-good, respectively, at the city. Then, in order to support the trade of the A-good and M-good described, the price of each good at each location $x \in X$ must be such that

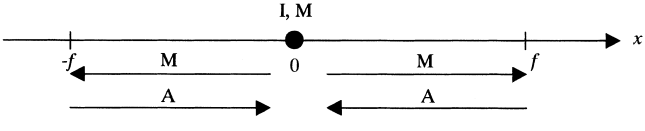

Fig. 1. The spatial configuration of the integrated city economy.

$$p^{\mathrm{A}}(x) = p^{\mathrm{A}}(0)\mathrm{e}^{-\tau^{\mathrm{A}}|x|}, \tag{53}$$

$$p^{\mathrm{M}}(x) = p^{\mathrm{M}}(0)\mathrm{e}^{\tau^{\mathrm{M}}|x|}. \tag{54}$$

Since the land rent conditions, (12) and (13), together imply that the land rent at the agricultural boundary equals zero, we have $R(f) = p^{\mathrm{A}}(f) - a^{\mathrm{A}}w(f) = 0$, or

$$p^{\mathrm{A}}(f) = a^{\mathrm{A}}w(f). \tag{55}$$

Furthermore, we have by assumption

$$M(x) = 0 = L^{\mathrm{M}}(x) \text{ for all } x \neq 0. \tag{56}$$

Hence, the market clearing of the M-good means that

$$M_0 = D_0^{\mathrm{M}} + \int_{-f}^{f} D^{\mathrm{M}}(x)\mathrm{e}^{\tau^{\mathrm{M}}|x|}\mathrm{d}x,$$

where the term, $\exp(\tau^{\mathrm{M}}|x|)$, reflects the transport consumption of the M-good. Using Eqs. (39), (41) and (56), this equation can be rewritten as

$$M_0 = \alpha(L_0^{\mathrm{M}} + L^{\mathrm{I}})/p^{\mathrm{M}}(0) + \int_{-f}^{f} \alpha p^{\mathrm{A}}(x)\mathrm{e}^{\tau^{\mathrm{M}}|x|}/p^{\mathrm{M}}(x)\mathrm{d}x. \tag{57}$$

Since $M_0 > 0$, by Eq. (14) we have

$$p^{\mathrm{M}}(0) = c^{\mathrm{M}}(0). \tag{58}$$

Now, using the equilibrium conditions explained in Section 2 (specifically, Eqs. (32)–(37)) together with Eqs. (53)–(58), it is not difficult to solve for each variable as a function of a single unknown, f. In particular, we have:

$$L^{\mathrm{I}} = \mu(N - 2a^{\mathrm{A}}f), \tag{59}$$

$$L_0^{\mathrm{M}} = (1-\mu)(N - 2a^{\mathrm{A}}f), \tag{60}$$

$$p^{\mathrm{A}}(x) = a^{\mathrm{A}}\mathrm{e}^{\alpha(\tau^{\mathrm{A}}+\tau^{\mathrm{M}})f}\mathrm{e}^{-\tau^{\mathrm{A}}|x|}, \tag{61}$$

$$p^{\mathrm{M}}(x) = \mu^{-\mu/\rho}(1-\mu)^{-(1-\mu)}(\sigma F)^{\mu/(\sigma-1)}(N - 2a^{\mathrm{A}}f)^{-\mu/(\sigma-1)}\mathrm{e}^{\tau^{\mathrm{M}}|x|}, \tag{62}$$

$$w(x) = \mathrm{e}^{[\alpha\tau^{\mathrm{M}} - (1-\alpha)\tau^{\mathrm{A}}]|x|}, \tag{63}$$

$$G(x) = (\sigma F/\mu)^{1/(\sigma-1)}(N - 2a^{\mathrm{A}}f)^{-1/(\sigma-1)}\mathrm{e}^{\tau^{\mathrm{I}}|x|}. \tag{64}$$

In order to determine the remaining unknown, f, we substitute Eqs. (42), (59) and (60) into the M-good market clearing condition, (57). Then, using Eqs. (53), (54) and (58), we can obtain the following relation;

$$N - 2a^{\mathrm{A}} f = \frac{2\alpha a^{\mathrm{A}}}{1-\alpha} \frac{1 - \mathrm{e}^{-\tau^{\mathrm{A}} f}}{\tau^{\mathrm{A}}} \mathrm{e}^{\alpha(\tau^{\mathrm{A}} + \tau^{\mathrm{M}}) f}, \tag{65}$$

which defines the size of the urban population. Since the left-hand side decreases in f while the right-hand side increases in f (from 0 towards ∞), we can readily conclude that there exists a unique equilibrium value of f, which in turn determines all other variables uniquely. We can also readily see by Eq. (65) that the equilibrium f increases from 0 to ∞ as N increases from 0 to ∞. Hence, in the following discussion, we can treat the two parameters, N and f, interchangeably.

3.2. Sustainability of the integrated city economy

So far, we have assumed a priori that the entire production activity of the M- and I-sectors takes place in the city. To claim, however, that this configuration is really an equilibrium, we must check the location equilibrium conditions, (45) and (48).

First, to examine the location equilibrium of M-production, we substitute Eq. (62) into the left-hand side of Eq. (45), while the right-hand side of Eq. (45) is obtained by substituting Eqs. (63) and (64) into Eq. (34). Then condition (45) is reduced to

$$\mathrm{e}^{\tau^{\mathrm{M}}|x|} \le \mathrm{e}^{(1-\mu)[\alpha\tau^{\mathrm{M}} - (1-\alpha)\tau^{\mathrm{A}}]|x|} \mathrm{e}^{\mu\tau^{\mathrm{I}}|x|},$$

that is, $(1-\alpha)(1-\mu)\tau^{\mathrm{A}} + (1 - \alpha + \alpha\mu)\tau^{\mathrm{M}} \le \mu\tau^{\mathrm{I}}$, or

$$\frac{(1-\alpha)(1-\mu)}{\mu} + \frac{1 - \alpha + \alpha\mu}{\mu} \frac{\tau^{\mathrm{M}}}{\tau^{\mathrm{A}}} \le \frac{\tau^{\mathrm{I}}}{\tau^{\mathrm{A}}}. \tag{66}$$

Next, turning to the location equilibrium of I-production, we can obtain the potential function $\Omega(x)$ by substituting Eqs. (56), (60), (63) and (64) into Eq. (50) and using Eq. (65) as follows:

$$\Omega(x) = \mathrm{e}^{-\sigma[\alpha\tau^{\mathrm{M}} - (1-\alpha)\tau^{\mathrm{A}} + \rho\tau^{\mathrm{I}}]|x|}. \tag{67}$$

Hence, the location equilibrium condition for the I-sector, (48), holds if and only if $\alpha\tau^{\mathrm{M}} - (1-\alpha)\tau^{\mathrm{A}} + \rho\tau^{\mathrm{I}} \ge 0$, or

$$\frac{1-\alpha}{\rho} - \frac{\alpha}{\rho} \frac{\tau^{\mathrm{M}}}{\tau^{\mathrm{A}}} \le \frac{\tau^{\mathrm{I}}}{\tau^{\mathrm{A}}}. \tag{68}$$

In summary, we can conclude as follows:

Theorem 1. *The integrated city configuration is an equilibrium if and only if parameters satisfy the two conditions*, (66) *and* (68).

The parameter range in which both conditions are satisfied is marked by the shaded area in Fig. 2 for the case in which $(1-\mu)/\mu > 1/\rho$.[2]

This figure indicates that when the transport cost of I-good is sufficiently high in comparison with the transport costs of both A-good and M-good, the integrated city configuration is an equilibrium. In this situation, the vertically-linked M-sector and I-sector cluster together at a single location.[3] This is due to the strong agglomeration forces created through the backward and forward linkages between the M- and I-sectors. As Eq. (63) indicates, if $\alpha\tau^{M} < (1-\alpha)\tau^{A}$, there is labor cost advantage in moving away from the city. Yet, given high transport costs in the I-sector, labor cost savings are outweighed by an increase in the procurement costs of I-good for the M-production, and by the loss of accessibility of I-firms to the large M-good demand at the city.

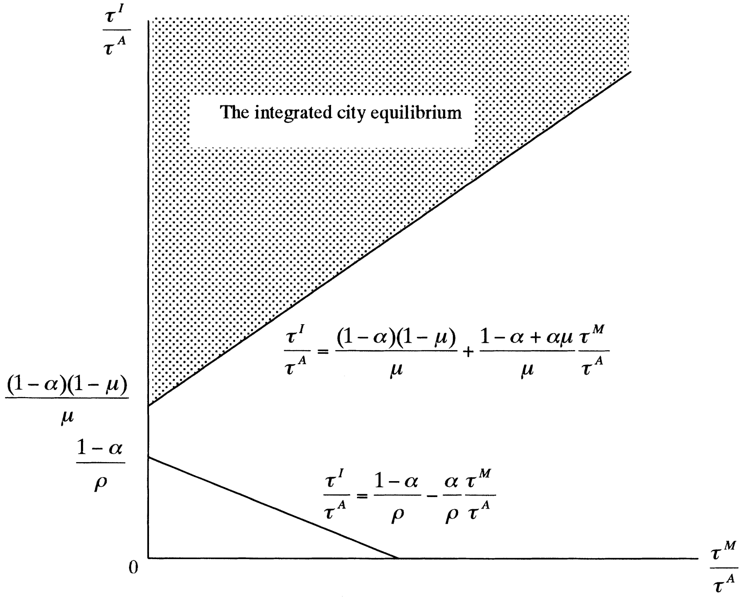

Fig. 2. The parameter range of the intergrated city equilibrium.

[2] When the opposite inequality holds, the order of the two intercepts on the vertical axis is reversed. This difference, however, is not important for our analysis of the equilibrium.

[3] Actually, most existing models of urban agglomeration based on intermediate-good variety assume a priori that intermediate goods are not tradable, i.e. $\tau^{I} = \infty$. See, for example, Rivera-Batiz (1988) and Abdel-Rahman and Fujita (1990). Hence, these models can be considered as a special case of our model.

Notice also in Fig. 2 that the sustainability of the integrated city configuration is independent of the economy's population size, N. Therefore, once the integrated city is formed, the growth of the economy's population alone can never generate new cities. Hence, with the increase of the economy's population, the city population $(N - 2a^{\mathrm{A}} f)$ keeps growing by attracting non-agricultural activities.[4]

3.3. Welfare analysis

To examine the impact of population growth on the economy's welfare, we substitute Eqs. (61) and (62) into Eq. (37), and use Eq. (65) and the identity, $(1-\alpha+\alpha\mu)-(1-\alpha)\sigma = \sigma[\alpha\mu-(1-\alpha+\alpha\mu)\rho]$. Then, we obtain

$$U^* = k(1-\mathrm{e}^{-\tau^{\mathrm{A}} f})^{\alpha\mu/(\sigma-1)} \mathrm{e}^{\{[\alpha\mu-(1-\alpha+\alpha\mu)\rho]/\rho\}\alpha(\tau^{\mathrm{A}}+\tau^{\mathrm{M}})f}, \tag{69}$$

where

$$k \equiv \alpha^{\alpha}(1-\alpha)^{1-\alpha}\mu^{\alpha\mu}(1-\mu)^{\alpha(1-\mu)}(a^{\mathrm{A}})^{[\alpha\mu-(1-\alpha+\alpha\mu)\rho]/\rho} \times \left[\frac{2\alpha\mu}{(1-\alpha)\sigma F\tau^{\mathrm{A}}}\right]^{\alpha\mu/(\sigma-1)}.$$

Likewise, substituting Eqs. (61) and (63) into Eq. (52), we have

$$U^{\mathrm{L}}(x) = U^* a^{\mathrm{A}}[\mathrm{e}^{\alpha(\tau^{\mathrm{A}}+\tau^{\mathrm{M}})(f-|x|)} - 1] \text{ for } |x| \le f. \tag{70}$$

Differentiating Eq. (69) with respect to f, we get

$$\frac{\mathrm{d}U^*}{\mathrm{d}f} = \frac{U^*\alpha}{\sigma-1}\left\{\sigma[\alpha\mu-(1-\alpha+\alpha\mu)\rho](\tau^{\mathrm{A}}+\tau^{\mathrm{M}}) + \frac{\mu\tau^{\mathrm{A}}}{\mathrm{e}^{\tau^{\mathrm{A}} f}-1}\right\}. \tag{71}$$

Now, if I-goods are highly differentiated from each other so that $\alpha\mu - (1-\alpha + \alpha\mu)\rho \ge 0$, or

$$\rho \le \frac{\alpha\mu}{1-\alpha+\alpha\mu}, \tag{72}$$

then U^* always increases with f (and hence, with N). Furthermore, in the right-hand side of Eq. (70), the term inside the brackets always increases with f. Therefore, as worker population, N, increases, the welfare of both workers and landlords increases. This is the case where the scale economies of the population through the increasing I-good varieties always dominates the scale diseconomies of the population caused by the increasing trade costs of the A- and M-goods in the agricultural hinterland.

Conversely, if it holds that

[4] As noted earlier, the equilibrium value of f increases as N increases. Hence, since the right-hand side of Eq. (65) increases in f, the urban population, $N - 2a^{\mathrm{A}} f$, also increases with N.

$$\rho > \frac{\alpha\mu}{1-\alpha+\alpha\mu}, \tag{73}$$

then the first term inside the braces of Eq. (71) is negative, while the second term decreases continuously from infinity to zero as N (and f) increases from zero to infinity. Thus, U^* is inverse-U shaped, implying that the utility of workers achieves the maximum at a certain population, $\hat{N}$, which is defined by the associated $\hat{f}$ satisfying the following relation:

$$(1-\alpha+\alpha\mu)\rho - \alpha\mu = \frac{\tau^{\mathrm{A}}}{\sigma(\tau^{\mathrm{A}}+\tau^{\mathrm{M}})}\frac{\mu}{e^{\tau^{\mathrm{A}}\hat{f}}-1}. \tag{74}$$

Therefore, the equilibrium utility level of workers starts declining when the population grows beyond $\hat{N}$. This is the case where I-goods are not sufficiently differentiated from each other, so that the weak scale economies of the population through increasing I-good varieties will eventually be overwhelmed by the scale diseconomies of increasing trade costs of the A- and M-goods. Since $\hat{N}$ is the optimal for workers, we call $\hat{N}$ the *w-optimal* population.

Fig. 3 illustrates the impact of population increase on the equilibrium utility,

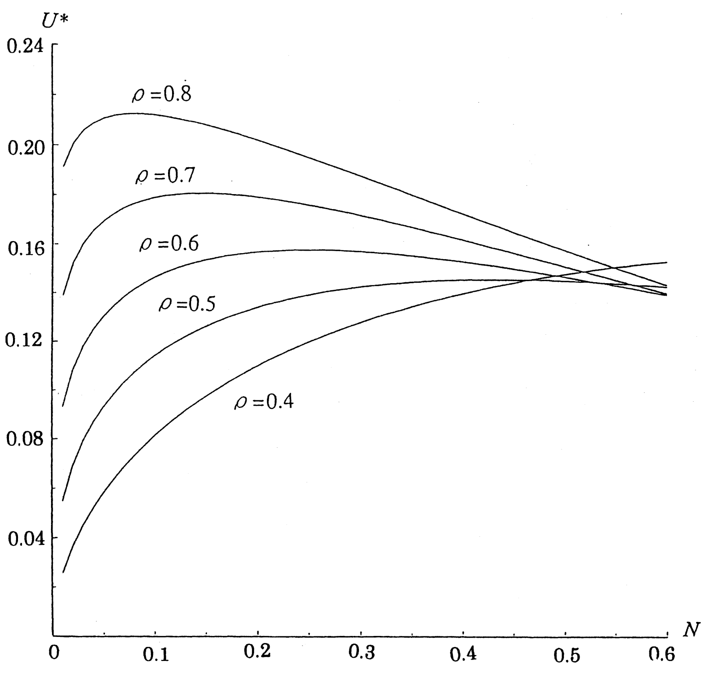

Fig. 3. Impact of population increase on the equilibrium utility in the integrated city economy.

U^*, where each curve corresponds to a specific value of substitution parameter ρ.[5] We can see by the figure that for each $\rho < 0.538 = \bar{\rho}$ (where $\bar{\rho}$ is defined by the value of ρ satisfying Eq. (72) with an equality), the utility curve achieves the maximum at a certain population. Since f increases with N and since $\hat{f}$ (identified by Eq. (74) above) increases with $\sigma = 1/(1-\rho)$, the w-optimal population, $\hat{N}$, decreases with ρ, i.e.,

$$\frac{\mathrm{d}\hat{N}}{\mathrm{d}\rho} < 0, \tag{75}$$

meaning that the w-optimal population is smaller when I-goods are less differentiated.

Although the equilibrium utility level of workers starts declining when the population grows beyond $\hat{N}$, the welfare of landlords can continue to increase, for their income (i.e. the land rent) keeps increasing as the frontier distance f becomes larger. Indeed, using Eqs. (70) and (71), we obtain

$$\begin{aligned}\frac{\mathrm{d}U^{\mathrm{L}}(x)/\mathrm{d}f}{U^{\mathrm{L}}(x)} &= \frac{\alpha}{\sigma-1}\left\{\sigma[\alpha\mu-(1-\alpha+\alpha\mu)\rho](\tau^{\mathrm{A}}+\tau^{\mathrm{M}})+\frac{\mu\tau^{\mathrm{A}}}{\mathrm{e}^{\tau^{\mathrm{A}}f}-1}\right\}\\ &\quad+\frac{\alpha(\tau^{\mathrm{A}}+\tau^{\mathrm{M}})\mathrm{e}^{\alpha(\tau^{\mathrm{A}}+\tau^{\mathrm{M}})(f-|x|)}}{\mathrm{e}^{\alpha(\tau^{\mathrm{A}}+\tau^{\mathrm{M}})(f-|x|)}-1}\\ &> \frac{\alpha}{\sigma-1}\{\sigma[\alpha\mu-(1-\alpha+\alpha\mu)\rho](\tau^{\mathrm{A}}+\tau^{\mathrm{M}})\}+\alpha(\tau^{\mathrm{A}}+\tau^{\mathrm{M}})\\ &= \frac{\alpha^2}{\sigma-1}(\sigma+\mu-1)(\tau^{\mathrm{A}}+\tau^{\mathrm{M}}) > 0,\end{aligned} \tag{76}$$

implying that the welfare of landlords always increases with population growth.

Summarizing the results above, we can conclude as follows:

Theorem 2. *Suppose that the economy is in the integrated city equilibrium. Then, if the population size of workers increases,*

(*i*) *the welfare of landlords always increases within the agricultural area*;

(*ii*) *when condition* (72) *holds, i.e.*

$$\rho \le \alpha\mu/(1-\alpha+\alpha\mu),$$

workers' welfare always increases;

---

[5]Fig. 3 is based on the parameters, $\alpha = 0.7, \mu = 0.5, a^{\mathrm{A}} = a^{\mathrm{I}} = F = 1, \tau^{\mathrm{A}} = 0.2$, $\tau^{\mathrm{M}} = 1.5$. To be precise, since we are concerned on the impact of ρ-change in U^*, we do not use the normalization in Eq. (31). When this normalization is not used, the right-hand side of Eq. (69) should be multiplied by $(\rho/a^{\mathrm{I}})^{\alpha\mu}$. Notice, however, that this multiplication does not change the relation (74), which is independent of parameter a^{I}. Hence, the w-optimal population, $\hat{N}$, is independent of a^{I}.

(*iii*) *when condition* (73) *holds, i.e.*

$$\rho > \alpha\mu/(1 - \alpha + \alpha\mu),$$

workers' welfare rises until a certain population level is reached and then declines beyond.

Recall the previous assertion through Theorem 1 that once the integrated city is formed, the growth of the economy's population alone can never generate new cities, and hence the integrated city keeps growing by attracting non-agricultural activities of the economy. In this situation, if I-goods are not sufficiently differentiated so that the relation (73) holds, then the population growth of the economy eventually leads to the continual decline of workers' welfare. This means that the economy is in the 'primacy trap' in which the agglomeration economies of the primate city continues to attract industrial activities even though the welfare of workers keeps declining.

4. The I-specialized city equilibrium

Next, we consider the economy with an I-specialized city, of which the spatial configuration is depicted in Fig. 4.

4.1. The spatial structure of the I-specialized city economy

In this configuration, the production of I-goods is concentrated exclusively in the city located at $x=0$. In contrast, the M-good is produced at every location $x \in [-f, f]$ so as to satisfy only the demand in the same location. That is, every location in the economy is self-sufficient in the M-good, implying that

$$M_o = D_o^M \quad \text{at the city,} \tag{77}$$

$$M(x) = D^M(x) \quad \text{at each } x \in X^A. \tag{78}$$

The city exports I-goods to every location $x \in X^A$(which are used for the M-good

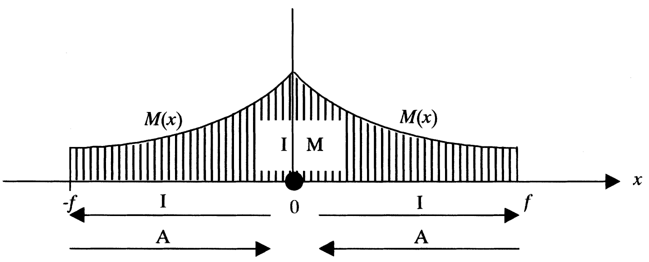

Fig. 4. The spatial configuration of the I-specialized city economy.

production there) and imports the A-good in return, while no trade of M-good occurs. The price configuration of the A-good is the same as before:

$$p^{\mathrm{A}}(x) = p^{\mathrm{A}}(0)\mathrm{e}^{-\tau^{\mathrm{A}}|x|}. \tag{79}$$

The price configuration of the M-good is implicitly determined by the zero-profit conditions in Eqs. (14) and (15) as follows:

$$p^{\mathrm{M}}(x) = c^{\mathrm{M}}(x) \quad \text{for all} \quad x \in [-f, f]. \tag{80}$$

Since $R(f) = 0$ at the agricultural boundary, f, it holds that

$$p^{\mathrm{A}}(f) = a^{\mathrm{A}} w(f). \tag{81}$$

Now, using the equilibrium conditions in Section 2 (specifically, (32) to (37), and (38) to (43)) together with (77) to (81), we can solve for all variables as functions of a single unknown, f. Among others, we have:

$$L^{\mathrm{I}} = \frac{2\alpha\mu a^{\mathrm{A}}}{1-\alpha} \frac{(1-\mathrm{e}^{-\tau^{\mathrm{A}} f})}{\tau^{\mathrm{A}}} \mathrm{e}^{[\alpha\mu(\tau^{\mathrm{A}}+\tau^{\mathrm{I}})/(1-\alpha+\alpha\mu)]f}, \tag{82}$$

$$L_0^{\mathrm{M}} = \frac{\alpha(1-\mu)}{1-\alpha+\alpha\mu} L^{\mathrm{I}}, \tag{83}$$

$$L^{\mathrm{M}}(x) = \frac{\alpha(1-\mu)a^{\mathrm{A}}}{1-\alpha+\alpha\mu} \mathrm{e}^{[\alpha\mu(\tau^{\mathrm{A}}+\tau^{\mathrm{I}})/(1-\alpha+\alpha\mu)](f-|x|)} \quad \text{for} \quad x \in X^{\mathrm{A}}, \tag{84}$$

$$p^{\mathrm{A}}(x) = a^{\mathrm{A}} \mathrm{e}^{[\alpha\mu(\tau^{\mathrm{A}}+\tau^{\mathrm{I}})/(1-\alpha+\alpha\mu)]f} \mathrm{e}^{-\tau^{\mathrm{A}}|x|} \quad \text{for} \quad x \in X, \tag{85}$$

$$p^{\mathrm{M}}(x) = \mu^{-\mu}(1-\mu)^{-(1-\mu)}[\sigma F/L^{\mathrm{I}}]^{\mu/(\sigma-1)} \mathrm{e}^{\{[\mu\tau^{\mathrm{I}}-(1-\alpha)(1-\mu)\tau^{\mathrm{A}}]/(1-\alpha+\alpha\mu)\}|x|} \quad \text{for} \quad x \in X^{\mathrm{A}} \tag{86}$$

$$w(x) = \mathrm{e}^{\{[\alpha\mu\tau^{\mathrm{I}}-(1-\alpha)\tau^{\mathrm{A}}]/(1-\alpha+\alpha\mu)\}|x|} \quad \text{for } x \in X, \tag{87}$$

$$G(x) = [\sigma F/L^{\mathrm{I}}]^{1/(\sigma-1)} \mathrm{e}^{\tau^{\mathrm{I}}|x|} \quad \text{for } x \in X. \tag{88}$$

In order to determine f, we substitute Eqs. (83), (84) and (87) and the relation, $|X^{\mathrm{A}}| = 2a^{\mathrm{A}} f$, into the labor-market clearing Eq. (44), and obtain

$$N - 2a^{\mathrm{A}} f = \frac{2\alpha a^{\mathrm{A}}}{1-\alpha+\alpha\mu} \mathrm{e}^{\lambda f} \left\{ \frac{\mu}{1-\alpha} \frac{1-\mathrm{e}^{-\tau^{\mathrm{A}} f}}{\tau^{\mathrm{A}}} + \frac{(1-\mu)(1-\mathrm{e}^{-\lambda f})}{\lambda} \right\}. \tag{89}$$

where $\lambda \equiv \alpha\mu(\tau^{\mathrm{A}} + \tau^{\mathrm{I}})/(1-\alpha+\alpha\mu)$. Since the left-hand side decreases in f while the right-hand side increases in f, we can readily see the unique existence of the equilibrium value of f. We can also easily verify that the equilibrium f increases as N increases.

4.2. Sustainability of the I-specialized city economy

For the I-specialized city configuration obtained above to be really in equilibrium, two additional conditions need to be satisfied. First, although the configuration is based on the assumption that no trade of M-good occurs in the economy, this condition is actually satisfied if and only if the rate of the spatial variation in the M-good price never exceed the transport rate, τ^{M}, in the economy:

$$\left|\mathrm{d}p^{\mathrm{M}}(x)/\mathrm{d}x\right|/p^{\mathrm{M}}(x) \le \tau^{\mathrm{M}} \text{ for all } x \in X.$$

Using Eq. (86), this condition is satisfied if and only if

$$\left|\mu\tau^{\mathrm{I}} - (1-\alpha)(1-\mu)\tau^{\mathrm{A}}\right|/(1-\alpha+\alpha\mu) \le \tau^{\mathrm{M}},$$

or,

$$\begin{aligned} &\frac{(1-\alpha)(1-\mu)}{\mu} - \frac{1-\alpha+\alpha\mu}{\mu}\frac{\tau^{\mathrm{M}}}{\tau^{\mathrm{A}}} \le \frac{\tau^{\mathrm{I}}}{\tau^{\mathrm{A}}} \\ &\le \frac{(1-\alpha)(1-\mu)}{\mu} + \frac{1-\alpha+\alpha\mu}{\mu}\frac{\tau^{\mathrm{M}}}{\tau^{\mathrm{A}}}. \end{aligned} \tag{90}$$

Next, the location equilibrium condition of I-industry, (48), must be satisfied.[6] Substituting Eqs. (83), (84), (87) and (88) into Eq. (50) yields the market potential of I-industry as follows:

$$\begin{aligned} \Omega(x) = {} & \frac{\alpha\mu}{1-\alpha+\alpha\mu} \mathrm{e}^{-\{\sigma[\alpha\mu\tau^{\mathrm{I}} - (1-\alpha)\tau^{\mathrm{A}}]/(1-\alpha+\alpha\mu)\}|x|} \\ & \times \left\{ \mathrm{e}^{-(\sigma-1)\tau^{\mathrm{I}}|x|} + \frac{(1-\alpha)\tau^{\mathrm{A}}}{2\alpha\mu(1-\mathrm{e}^{-\tau^{\mathrm{A}}f})} \int_{-f}^{f} \mathrm{e}^{-\tau^{\mathrm{A}}|y| - (\sigma-1)\tau^{\mathrm{I}}(|x-y|-|y|)} \mathrm{d}y \right\}, \end{aligned}$$

or,

$$\begin{aligned} \Omega(x) = {} & \frac{\alpha\mu}{1-\alpha+\alpha\mu} \mathrm{e}^{-\eta|x|} \\ & \times \left\{ 1 + \frac{(1-\alpha)\tau^{\mathrm{A}}\mathrm{e}^{(\sigma-1)\tau^{\mathrm{I}}|x|}}{2\alpha\mu(1-\mathrm{e}^{-\tau^{\mathrm{A}}f})} \int_{-f}^{f} \mathrm{e}^{-\tau^{\mathrm{A}}|y| - (\sigma-1)\tau^{\mathrm{I}}(|x-y|-|y|)} \mathrm{d}y \right\}, \end{aligned} \tag{91}$$

where

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[6] Under Eq. (79), we have $p^{\mathrm{M}}(x) = c^{\mathrm{M}}(x)$ for all $x \in X$, and hence the location equilibrium of M-industry is automatically satisfied.

$$\eta \equiv \frac{\sigma\{[\alpha\mu + (1-\alpha+\alpha\mu)\rho]\tau^{\mathrm{I}} - (1-\alpha)\tau^{\mathrm{A}}\}}{1-\alpha+\alpha\mu} \tag{92}$$

Eq. (91) represents a rather complex function. Since it is symmetric with respect to $x=0$, hereafter we focus on the right-hand side. Then, we can rewrite Eq. (91) in a simpler form for $x \geq 0$ as follows (refer to Appendix A for the derivation):

$$\Omega(x) = \mathrm{e}^{-\eta x}\left\{1 + \frac{(1-\alpha)(\sigma-1)\tau^{\mathrm{I}}}{1-\alpha+\alpha\mu}\int_0^x \mathrm{e}^{2(\sigma-1)\tau^{\mathrm{I}}y}\left(1 - \frac{1-\mathrm{e}^{-\tau^{\mathrm{A}}y}}{1-\mathrm{e}^{-\tau^{\mathrm{A}}f}}\right)\mathrm{d}y\right\}. \tag{93}$$

The potential function is illustrated in Fig. 5 in which parameters are fixed as follows:

$$\alpha = 0.6,\ \mu = 0.7,\ F = a^{\mathrm{I}} = 1,\ \rho = 0.8,\ \tau^{\mathrm{A}} = 1,\ \tau^{\mathrm{M}} = 1.5,\ \tau^{\mathrm{I}} = 0.7. \tag{94}$$

Notice that for each fixed f, Eq. (93) defines a curve (as a function of x) which we call a *potential curve*. Since the equilibrium value of f is uniquely determined by the economy's population, N, each potential curve is associated with a specific value of N. Hence, by changing N, we obtain a set of potential curves as illustrated in Fig. 5. For the I-specialized city configuration under a given value of N to be an equilibrium, the associated potential curve should not exceed 1 anywhere in the

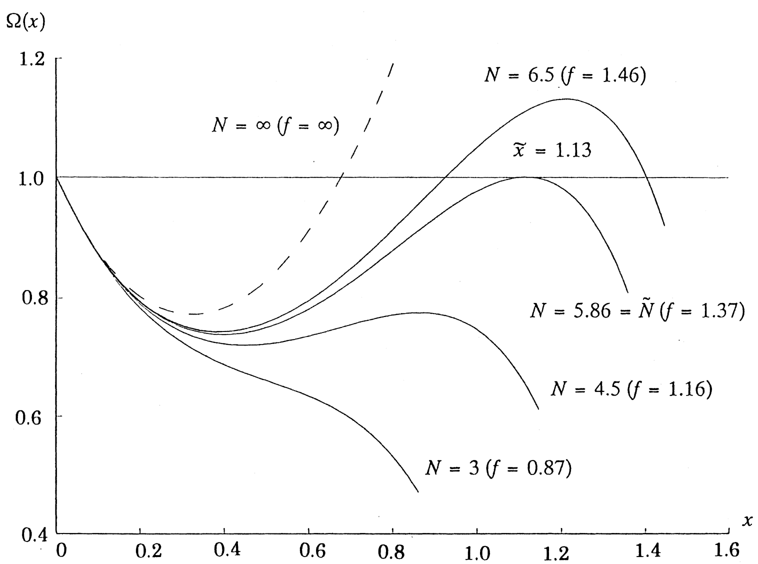

Fig. 5. Potential curves for the I-specialized city configuration.

economy. In order to investigate when this condition holds, we need to know the general properties of function (93).

First, it is obvious that $\Omega(0)=1$. Hence, for the market potential not to exceed 1 in the neighborhood of the city, its slope at the city,

$$\Omega'(0_+)=\sigma\{(1-\alpha)\tau^{\mathrm{A}}-\alpha\mu(1+\rho)\tau^{\mathrm{I}}\}/(1-\alpha+\alpha\mu), \tag{95}$$

should not be positive, or

$$\frac{1-\alpha}{\alpha\mu(1+\rho)}\le\frac{\tau^{\mathrm{I}}}{\tau^{\mathrm{A}}}. \tag{96}$$

If this condition holds, the potential curve starts declining from the city, and hence firms in the I-sector will not find it profitable to move a short distance away from the city. Notice that the slope at the origin, Eq. (95), is common for all potential curves.

Next, as we can readily see by Eq. (93), the value of $\Omega(x)$ increases with f at every $x\neq 0$. Since the fringe distance f increases with N, this implies that an increase in N shifts the potential curve upward. This happens because a larger N implies a larger area of M-good production on each side of the city. Hence, given all other I-firms staying in the city, the I-firm that defects into a periphery location can capture a larger demand for M-goods there. It is even possible that, with a sufficiently large population, the potential curve eventually exceeds 1 somewhere. In order to investigate this possibility, we introduce the *limiting potential curve*, $\bar{\Omega}(x)$, which is associated with $f=\infty$ (and hence, $N=\infty$), representing the upper limit of all potential curves. Setting $f=\infty$ in Eq. (93), and rewriting this equation, we obtain the following expression (see Appendix B):

$$\bar{\Omega}(x)=(1-K)\mathrm{e}^{-\eta x}+K\mathrm{e}^{\nu x}, \tag{97}$$

where η, ν and K are constants such that[7]

$$\eta=\frac{(1-\alpha)(\sigma-1)\tau^{\mathrm{I}}}{1-\alpha+\alpha\mu}-\Omega'(0_+),$$

$$\nu\equiv\frac{\sigma[(1-\alpha+\alpha\mu)\rho-\alpha\mu](\tau^{\mathrm{A}}+\tau^{\mathrm{I}})}{1-\alpha+\alpha\mu},$$

$$K\equiv\frac{(1-\alpha)\rho\tau^{\mathrm{I}}}{(1-\alpha)\rho\tau^{\mathrm{I}}+[(1-\alpha+\alpha\mu)\rho-\alpha\mu](\tau^{\mathrm{A}}+\tau^{\mathrm{I}})-(1-\alpha+\alpha\mu)\Omega'(0_+)/\sigma}.$$

In the equation of η and K above, $\Omega'(0_+)(=\bar{\Omega}'(0_+))$ is given by Eq. (95), representing the slope of potential curves at the origin (which is common for all f).

[7] The constant, η, below represents the rewriting of the same one defined in Eq. (92).

Since we consider only the case where $\Omega'(0_+) \leq 0$, the limiting potential curve slopes downward at the origin. Furthermore, since Eq. (97) is a sum of two exponential functions, it has at most one turning point (at which $\Omega'(x) = 0$ for $x > 0$). Hence, $\Omega(x)$ exceeds unity at some x if and only if $\Omega(\infty) > 1$. Here, since $\Omega'(0_+) \leq 0$ implies $\eta > 0$, we have by Eq. (97) that $\Omega(\infty) = 0 + Ke^{\nu\infty} = Ke^{\nu\infty}$. Therefore, $\Omega(x)$ exceeds 1 at some x if and only if $Ke^{\nu\infty} > 1$.

Suppose that $(1 - \alpha + \alpha\mu)\rho - \alpha\mu < 0$. Then, $\nu < 0$, and hence $Ke^{\nu\infty} = 0$. If $(1 - \alpha + \alpha\mu)\rho - \alpha\mu = 0$, then $\nu = 0$ and $K \leq 1$, and hence $Ke^{\nu\infty} = K \leq 1$. Therefore, if $(1 - \alpha + \alpha\mu)\rho - \alpha\mu \leq 0$, that is, if condition (72) holds, then $\Omega(x) \leq 1$ for all x, implying that under any finite N, $\Omega(x) < 1$ for all $x \neq 0$. Condition (72) means that I-goods are highly differentiated from each other, and hence their price elasticity is very low. In this case, I-firms locating at the city do not lose much demand even in the peripheral market, making the city still the most profitable location for I-good production.

Conversely, if $(1 - \alpha + \alpha\mu)\rho - \alpha\mu > 0$, that is, if condition (73) holds, then $\nu > 0$ and $0 < K < 1$, and hence $Ke^{\nu\infty} = \infty$, implying that $\Omega(x)$ exceeds 1 at some x. Therefore, in this case, as N keeps increasing, the potential $\Omega(x)$ eventually exceeds unity at some $x \neq 0$. Since condition (73) means that I-goods are highly substitutable for each other and hence their price elasticity is very high, I-firms locating at the city lose much demand for their products in the periphery. Thus, when N becomes sufficiently large, a large local demand at the outskirts will eventually make firm location there more profitable than at the city.

The last case is illustrated by Fig. 5 in which the potential curve reaches unity at a *critical distance* $\tilde{x} = 1.13$ when N becomes a critical population, $\tilde{N} = 5.86$. Any further increase in the population size above $\tilde{N}$ makes the location $\tilde{x}$ more profitable than the city for any single firm, thus suggesting that a new city might well emerge there. In fact, using the same adjustment dynamics of city formation as in Fujita and Mori (1997), we can show that a pair of new cities emerge, respectively, at $x = \tilde{x}$ and $x = -\tilde{x}$ when the population reaches $\tilde{N}$. However, we postpone the analysis of the formation of new cities for future research.

Pulling together the results, we can conclude as follows:

Theorem 3. *For the I-specialized city configuration to be a spatial equilibrium, conditions* (90) *and* (96) *must hold such that*

$$\frac{(1-\alpha)(1-\mu)}{\mu} - \frac{1-\alpha+\alpha\mu}{\mu}\frac{\tau^{\mathrm{M}}}{\tau^{\mathrm{A}}} \leq \frac{\tau^{\mathrm{I}}}{\tau^{\mathrm{A}}}$$
$$\leq \frac{(1-\alpha)(1-\mu)}{\mu} + \frac{1-\alpha+\alpha\mu}{\mu}\frac{\tau^{\mathrm{M}}}{\tau^{\mathrm{A}}},$$

and

$$\frac{1-\alpha}{\alpha\mu(1+\rho)} \leq \frac{\tau^{\mathrm{I}}}{\tau^{\mathrm{A}}}.$$

(i) *Provided that the two conditions above hold, and if condition* (72) *holds further, i.e.*

$$\rho \leq \alpha\mu/(1-\alpha+\alpha\mu),$$

then the ICE configuration is an equilibrium regardless of the population size;

(ii) *Provided that the two conditions above hold, and condition* (73) *holds further, i.e.*

$$\rho > \alpha\mu/(1-\alpha+\alpha\mu),$$

then there exists a critical population, $\tilde{N}$, *such that the I-specialized city configuration is an equilibrium for any population* $N \leq \tilde{N}$, *but ceases to be an equilibrium as soon as* $N > \tilde{N}$.

In Fig. 2, we add the parameter range of the I-specialized city equilibrium in which conditions (90) and (96) hold, and obtain Fig. 6.[8] We can see in the figure that the I-specialized city configuration is an equilibrium when the relative transport cost of I-goods, τ^{I}/τ^{A}, is sufficiently low. (However, it should not be too

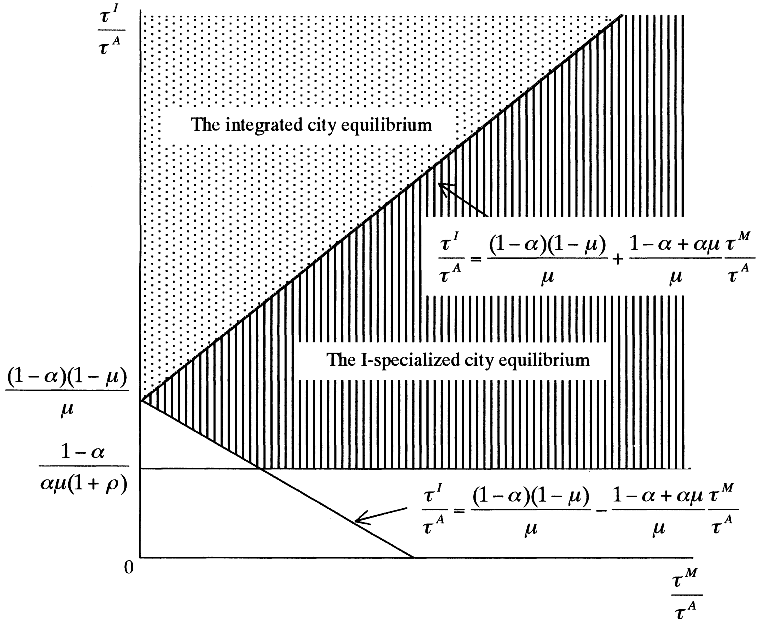

Fig. 6. Parameter ranges of the two monocentric equilibria.

[8] In Fig. 6, the order of the two intercepts on the vertical axis changes as $1-\mu$ is larger or smaller than $1/\alpha(1+\rho)$. This fact, however, does not essentially affect the following discussion.

low, otherwise I-firms would move to the periphery to enjoy the lower wage prevailing there).

4.3. Welfare analysis

Next, to examine the welfare implications of population growth on the I-specialized city equilibrium, we substitute Eqs. (85) and (86) (after setting $x=0$) into Eq. (37), and obtain

$$U^* = k(1 - e^{-\tau^A f})^{\alpha\mu/(\sigma-1)} e^{\{[\alpha\mu - (1-\alpha+\alpha\mu)\rho]/\rho\}[\alpha\mu(\tau^A+\tau^I)/(1-\alpha+\alpha\mu)]f}, \tag{98}$$

where k in the same constant as in Eq. (69). Furthermore, substituting Eqs. (85) and (87) into Eq. (52), we obtain

$$U^L(x) = U^* a^A [e^{[\alpha\mu(\tau^A+\tau^I)/(1-\alpha+\alpha\mu)](f-|x|)} - 1] \text{ for } |x| \le f. \tag{99}$$

Differentiation of Eq. (98) by f yields

$$\frac{dU^*}{df} = \frac{U^*\alpha}{\sigma-1}\left\{\sigma[\alpha\mu - (1-\alpha+\alpha\mu)\rho]\frac{\mu(\tau^A+\tau^I)}{1-\alpha+\alpha\mu} + \frac{\mu\tau^A}{e^{\tau^A f}-1}\right\}. \tag{100}$$

Further, differentiating Eq. (99) by f and using the same approach as in Eq. (76), we obtain

$$\begin{aligned}\frac{dU^L(x)/df}{U^L(x)} &> \frac{\alpha}{\sigma-1}\left\{\sigma[\alpha\mu-(1-\alpha+\alpha\mu)\rho]\frac{\mu(\tau^A+\tau^I)}{1-\alpha+\alpha\mu}\right\} + \frac{\alpha\mu(\tau^A+\tau^I)}{1-\alpha+\alpha\mu} \\ &= \frac{\alpha^2}{\sigma-1}\frac{\mu}{1-\alpha+\alpha\mu}(\sigma+\mu-1)(\tau^A+\tau^I) > 0.\end{aligned} \tag{101}$$

Notice that Eqs. (98)–(101) are very similar to Eqs. (69)–(71) and (76), respectively, except that τ^M is replaced by τ^I in the latter set of equations. In particular, in both Eqs. (69) and (98), the equilibrium worker utility U^* contains the same term, $[\alpha\mu - (1-\alpha+\alpha\mu)\rho]/\rho$, in each exponential function. Therefore, recalling the derivation in Theorem 2, we can conclude as follows:

Theorem 4. *When the economy is in the I-specialized city equilibrium, the impact of population growth on the economy's welfare is qualitatively the same as in* (*i*), (*ii*) *and* (*iii*) *of Theorem* 2, *respectively.*

Although the impact of population growth on the economy's welfare appears to be the same in the two types of equilibria, there is actually a major difference in the overall impact of population growth in the two cases. That is, as noted previously, if $\rho \ge \alpha\mu/(1-\alpha+\alpha\mu)$ and the economy is an integrated city equilibrium, then it is in the primacy trap, implying that the welfare of workers keeps declining forever as the population grows beyond a critical size. In contrast, if the economy is in the I-specialized city equilibrium, the primacy trap does not arise even when $\rho \geq \alpha\mu/(1-\alpha+\alpha\mu)$. In fact, in the latter case, growing population eventually destroys the I-specialized city equilibrium, leading to the emergence of new cities and thus boosting the welfare of workers.[9]

5. Conclusion

In this paper, we have presented a monopolistic competition model of city formation, in which the agglomeration forces arise from the product variety in intermediate goods. The present model is essentially dual to the F–K model of city formation in which the agglomeration forces arise from the love for variety on the consumer side. However, the present model have yielded new results of empirical significance. In particular, in the context of the F–K model, there exists, not surprisingly, only one type of monocentric spatial equilibrium in which the entire manufacturing sector is agglomerated in a single city. However, in the present model, there exist two types of monocentric configuration, i.e. the integrated city equilibrium and the I-specialized city equilibrium.

Although the spatial configuration of the integrated city equilibrium looks similar to that of the monocentric equilibrium of the F–K model, the impact of population growth is very different in the two settings. In the present setting, as we have seen in Section 3, raising the population size of the economy alone never destroys the integrated city equilibrium; furthermore, when intermediate goods are not sufficiently differentiated to each other, population growth beyond a threshold level makes the workers' welfare decline. That is, the economy is in a situation of primacy trap in which workers' welfare declines while the metropolis keeps growing by attracting non-agricultural activities. Today, some urban giants in developing countries (think of Bangkok and Jakarta) might be examples of such primate cities.

The primacy trap never arises, however, in the setting of the F–K model. On one hand, as has been shown in their paper (Fujita and Krugman, 1995), when the manufactured consumption-goods are highly differentiated from each other, the monocentric configuration remains an equilibrium however large the population size becomes, but the welfare of workers also keeps rising with the population growth. On the other hand, when the manufactured consumption-goods are not sufficiently differentiated from each other, the welfare of workers starts declining beyond a certain population size; however, the monocentric configuration ceases to be an equilibrium when the population becomes sufficiently large, implying that new cities emerge, and hence workers' welfare starts growing again.

[9]Using the framework of the F–K model, Fujita and Mori (1997) shows that the formation of new cities boosts the welfare of workers.

Such a difference in results finds its origin in the fact that in the F–K model based on the love for variety in consumption goods, it is implicitly assumed that consumers themselves ‘put together’ the differentiated goods for their consumption (implied by the CES sub-utility function for differentiated goods). In contrast, in the present model, the manufacturing sector ‘puts together’ the differentiated goods for making a homogeneous product for consumers. In the former case, the producers of differentiated goods sell their output directly to all consumers who are dispersed in the agricultural hinterland and partly located in the city; whereas, in the latter, the producers of differentiated inputs sell their output to the manufacturing sector that is entirely concentrated within the city. Hence, not surprisingly, *the lock-in effect of an integrated city with intermediate commodities is much stronger than that of a monocentric economy based on love for variety of consumer goods*. This suggests the usefulness of the present model in explaining the continued dominance of primate cities in many developing countries today.

Fig. 6 is also useful in investigating a way to escape from such a trap. That is, suppose that the economy is in the integrated city equilibrium and in the primacy trap, implying that transport cost parameters are in the dotted area in Fig. 6. A possible strategy for the economy to escape from such a trap is to reduce the trade costs of intermediate goods so that transport cost parameters move down to the area of the I-specialized city equilibrium in Fig. 6. In this parameter region, then, growing population will eventually lead to the formation of new cities, thus escaping from the primacy trap. The development of modern telecommunication infrastructure might be an important measure for such a purpose. However, given that many kinds of intermediate goods such as producer services are often transacted on a face-to-face basis, the development of high-speed passenger transport systems also seems to be essential. In order to make such an idea practical, however, we must explicitly consider also the financial side of infrastructure development. This is an interesting topic left for the future.

In certain aspects, the study of this paper is rather preliminary. First, in order to fulfill the entire parameter region of Fig. 6, we must investigate other types of equilibrium configurations (including those with multiple cities).[10] Second, we must study the process of the emergence of new cities more explicitly. Third, an important direction in extending our model is to introduce multiple groups of intermediate goods, with each group having different characteristics in terms of transport costs, degree of substitutability, and labor intensity. Such an extended model will generate a hierarchical urban system based on intermediate-good varieties. Fourth, in order to make our study more useful for empirical purposes, we must consider the variety in both consumer goods and intermediate goods. This requires us to combine the results of urban models in Fujita et al. (1999) with

[10] It is shown in Hamaguchi (1995) that the bottom part of the parameter region in Fig. 6 belongs to the dispersed equilibrium in which both the manufacturing and intermediate sectors are entirely mixed in the agricultural area.

those in the present line of modeling. Finally, like most existing works in the so called New Geographical Economics, our model is based on the usage of particular functional forms and transport technology. To examine the robustness of the results, we need to conduct more general analyses based on less restrictive specifications.

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Appendix A. Derivation of Eq. (93)

By Eq. (91), we have for $x \geq 0$ that

$$\Omega(x) = \frac{\alpha\mu}{1-\alpha+\alpha\mu}\, \mathrm{e}^{-\eta x} A(x), \tag{A.1}$$

where

$$A(x) \equiv 1 + \frac{(1-\alpha)\tau^{\mathrm{A}} \mathrm{e}^{(\sigma-1)\tau^{\mathrm{I}} x}}{2\alpha\mu(1-\mathrm{e}^{-\tau^{\mathrm{A}} f})} \int_{-f}^{f} \mathrm{e}^{-\tau^{\mathrm{A}}|y| - (\sigma-1)\tau^{\mathrm{I}}(|x-y|+|y|)} \mathrm{d}y$$

$$= 1 + \frac{(1-\alpha)\tau^{\mathrm{A}}}{2\alpha\mu(1-\mathrm{e}^{-\tau^{\mathrm{A}} f})} \left\{ \int_{-f}^{x} \mathrm{e}^{-\tau^{\mathrm{A}}|y| + (\sigma-1)\tau^{\mathrm{I}}(y+|y|)} \mathrm{d}y + \mathrm{e}^{2(\sigma-1)\tau^{\mathrm{I}} x} \int_{x}^{f} \mathrm{e}^{-\tau^{\mathrm{A}} y} \mathrm{d}y \right\}.$$

Taking the logarithm of both sides of Eq. (A.1), and then differentiating each side by x, we have

$$\Omega'(x)/\Omega(x) = -\eta + A'(x)/A(x),$$

or,

$$\Omega'(x) = -\eta\Omega(x) + \frac{\alpha\mu}{1-\alpha+\alpha\mu}\, \mathrm{e}^{-\eta x} A'(x), \tag{A.2}$$

where

$$A'(x) = \frac{(1-\alpha)\tau^{A}(\sigma-1)\tau^{I}}{\alpha\mu(1-e^{-\tau^{A}f})} e^{2(\sigma-1)\tau^{I}x} \int_{x}^{f} e^{-\tau^{A}y} dy.$$

Solving the differential Eq. (A.2) under the initial condition, $\Omega(0) = 1$, we have

$$\Omega(x) = e^{-\eta x}\left\{1 + \int_{0}^{x} e^{\eta y}\left[\frac{\alpha\mu}{1-\alpha+\alpha\mu} e^{-\eta y}A'(y)\right]dy\right\},$$

which leads to Eq. (93).

Appendix B. Derivation of Eq. (97)

Setting $f = \infty$ in Eq. (93) yields

$$\begin{aligned}\overline{\Omega}(x) &= e^{-\eta x}\left\{1 + \frac{(1-\alpha)(\sigma-1)\tau^{I}}{1-\alpha+\alpha\mu}\int_{0}^{x} e^{[2(\sigma-1)\tau^{I}-\tau^{A}]y}dy\right\} \\ &= e^{-\eta x}\left\{1+\frac{(1-\alpha)(\sigma-1)\tau^{I}}{1-\alpha+\alpha\mu}\frac{e^{[2(\sigma-1)\tau^{I}-\tau^{A}]x}-1}{2(\sigma-1)\tau^{I}-\tau^{A}}\right\} \\ &= e^{-\eta x}\{(1-K)+Ke^{[2(\sigma-1)\tau^{I}-\tau^{A}]x}\} \\ &= (1-K)e^{-\eta x}+Ke^{[2(\sigma-1)\tau^{I}-\tau^{A}-\eta]x},\end{aligned} \tag{B.1}$$

whereη is give by Eq. (92), while K is defined as

$$K \equiv \frac{(1-\alpha)(\sigma-1)\tau^{I}}{(1-\alpha+\alpha\mu)[2(\sigma-1)\tau^{I}-\tau^{A}]}. \tag{B.2}$$

Using the identity, $(1-\alpha)\sigma - (1-\alpha+\alpha\mu) = \sigma[(1-\alpha+\alpha\mu)\rho - \alpha\mu]$, we can obtain that

$$2(\sigma-1)\tau^{I} - \tau^{A} - \eta = \nu \tag{B.3}$$

where ν is defined just below Eq. (97). Substituting Eq. (B.3) into Eq. (B.1) yields $\overline{\Omega}(x) = (1-K)e^{-\eta x} + Ke^{\nu x}$.

Finally, using Eq. (95), we can rewrite η and K as those given just below Eq. (97).

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