Massive Money Injection in an Economy with Broad Liquidity Services: The Japanese Experience 2001-2006

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Abstract

This paper presents a model with broad liquidity services to discuss the consequences of massive money injection in an economy with the zero interest rate bound. We incorporate Goodfriend's (2000) idea of broad liquidity services into the model by allowing the amounts of bonds with various maturities held by a household to enter its utility function. We show that the satiation of money (or the zero marginal utility of money) is *not* a necessary condition for the one-period interest rate to reach the zero lower bound; instead, we present a weaker necessary condition that the marginal liquidity service provided by money coincides with the marginal liquidity service provided by the one-period bonds, both of which are not necessarily equal to zero. This result implies that massive money injection would have some influences on an equilibrium of the economy even if it does not alter the private sector's expectations about future monetary policy. Our empirical results indicate that forward interest rates started to decline relative to the corresponding futures rates just after March 2001, when a quantitative monetary easing policy started by the Bank of Japan, and that the forward and futures spread has never closed until the policy ended in March 2006. We argue that these findings are not easy to explain by a model without broad liquidity services.

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1 Introduction

Recent researches on the optimal monetary policy in an economy with the zero interest rate bound have found the importance of a central bank's commitment about future monetary policy (Woodford (1999), Jung et al. (2005), Eggertsson and Woodford (2003) among others). In a usual environment, a central bank conducts monetary easing by lowering the current overnight interest rate through an additional injection of money to the market. However, this does not work well once the overnight interest rate reaches the zero lower bound. Further monetary easing in such a situation could be implemented only through central bank's announcements about the future path of the overnight interest rate. Specifically, it has been shown that the optimal monetary policy rule is characterized by "history dependence" in the sense that a central bank commits itself to continuing monetary easing even after the economy returns to a normal situation.

Somewhat interestingly, the Bank of Japan's (BOJ's) policymaking since the spring of 1999 has some similarities with the findings provided by these theoretical studies (see Table 1 for recent monetary policy events).[1] One of the keywords frequently used by the BOJ officials and other market participants during this period was *Jikanjiku* effects, or temporal-axis effects.[2] This is an idea that further monetary easing could be implemented by expanding a flat "zero-rate zone" in the yield curve. The length of the zero-rate zone has been regarded as an indicator of monetary easing, which at least partially replaces the role played by the current level of the overnight interest rate in a normal situation.[3]

[1]Iwamura et al. (2005) evaluates the BOJ's monetary policy in 1999-2004 by comparing it with the optimal monetary policy rule.

[2]Temporal-axis effects is a literal translation of *jikanjiku* effects, while the BOJ officially use "policy duration effects" as its English translation. These two translations are both closely related to the zero-rate zone in the yield curve. Interestingly, the word "policy duration effects" implicitly assumes that the zero-rate zone is created and expanded by a central bank's commitment about future monetary policy. For example, as far as the "policy durations effects" are concerned, the zero-rate zone is expanded only when a central bank's commitment about future easing successfully alters the market's expectations. There does not seem to exist any recognition that the zero-rate zone in the yield curve could be expanded by massive money injection.

[3]The idea of "temporal-axis effects" had an important influence upon various aspects of developments in Japanese money markets during this period. First, the Bank of Japan has repeatedly announced that it would continue monetary easing (zero interest rate policy or quantitative monetary easing policy) for longer periods than expected by market participants. For example, Governor Hayami announced in April 1999, just after the introduction of the zero interest rate policy, that the BOJ will continue that policy until "deflationary concerns are dispelled". Also he announced in March 2001, when the BOJ introduced the quantitative monetary easing policy, that it will continue that policy until "the core CPI records a year-on-year increase of zero percent or more on a stable basis". Second, market participants and mass media have been paying a special attention to the length of the zero rate zone in the yield curve as an important source of information about future monetary policy. In some sense, its length has been regarded as an indicator of monetary easing, given that the current level of the overnight interest rate (which is already zero) does not provide any useful information about the degree of monetary easing. Third, the BOJ tended to purchase longer and longer-term bonds over time during this period. Bonds located in the zero-rate zone in the yield curve are already equivalent to money, so financial institutions were not willing to conduct an exchange between these bonds and money. Given that,

The purpose of this paper is to closely look at recent developments in the yield curve, paying a particular attention to developments in the zero-rate zone in the yield curve. One might say this is a simple task, given that the length of the zero-rate zone is determined solely by the market's expectations about future monetary policy, as emphasized by the above mentioned theoretical studies. For example, if one detects a longer zero-rate zone in the yield curve, it should be regarded as an evidence that market participants update their belief about future monetary policy and now believe that monetary easing will be continued longer than it was believed before.

However, there is a serious mismatch between the optimal monetary policy rule described in the theoretical literature and the actual policy decisions made by the BOJ, and it creates a complication. The BOJ switched its operational target in March 2001 to the current account balance (CAB) from the overnight interest rate ("quantitative monetary easing policy"), and has raised the CAB target fairly frequently since then, starting from 5 trillion yen at the beginning of 2001 to 30-35 trillion yen three years later (see Figure 1). According to Eggertsson and Woodford (2003), such an additional injection of money would have no effects upon an equilibrium of the economy when the overnight interest rate is already zero, simply because the marginal utility of money equals to zero as well. However, in a more realistic setting where portfolio rebalancing effects work to some extent, an additional injection of money would lead to an increase in demand for longer-term (or riskier) bonds even when the overnight interest rate is already zero, thereby raising prices for those bonds (Meltzer (2001), Goodfriend (2000), Clouse et al. (2003)). An important point to note here is that an additional money injection, particularly large-scale one, would lead to an expansion in the zero-rate zone in the yield curve through the portfolio rebalancing channel, even if it does not alter at all the market' expectations about future monetary policy (namely, no effects through the expectation channel). This paper seeks to discriminate these two channels by looking at the spread between forward and futures interest rates around the very end of the zero-rate zone in the yield curve, and investigate a mechanism behind the observed expansion of the zero-rate zone during the period of the quantitative monetary easing policy.

The rest of this paper is organized as follows. Section 1 presents a model with broad liquidity services in order to investigate the consequences of massive money injection in an economy with the zero interest rate bound. We incorporate Goodfriend's (2000) idea of broad liquidity services into the

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the BOJ was forced to purchase bonds located outside the zero-rate zone, whose yields were still above zero.

model by allowing the amounts of bonds with various maturities held by a household to enter its utility function. First, we show that the satiation of money (or the zero marginal utility of money) is *not* a necessary condition for the one-period interest rate to reach the zero lower bound; instead, we present a weaker necessary condition that the marginal liquidity service provided by money coincides with the marginal liquidity service provided by the one-period bonds, both of which are not necessarily equal to zero. Second, we show that massive money injection through an open market operation would have some influences on the yield curve even if it does not alter the private sector's expectations about future monetary policy, as long as a central bank purchases bonds located outside the zero-rate zone. Third, we show that the forward and futures spread would become positive (i.e., the forward interest rate is lower than the corresponding futures rate) when a central bank conduct massive money injection, although it would become zero in a limiting case in which the base money is supplied up to the satiation level.[4]

Section 3 turns to empirical analysis. Our empirical results indicate that forward interest rates started to decline relative to the corresponding futures rates just after March 2001, when a quantitative monetary easing policy started by the Bank of Japan, and that the forward and futures spread has never closed until the policy ended in March 2006. We argue that these findings are not easy to explain by a model without broad liquidity services. Section 4 concludes the paper.

2 A Model of Broad Liquidity

Eggertsson and Woodford (2003) shows that an additional provision of the base money would have no effects on an equilibrium of the economy, given that the overnight interest rate has already reached the zero lower bound ("Irrelevance Proposition").[5] In the model of Eggertsson and Woodford (2003), an additional money injection becomes ineffective simply because the marginal utility of money is zero in an economy with the zero overnight interest rate. An additional money injection will never change the marginal utility of money, which is already zero, thereby having no effects on households'

[4]Bernanke (2001) and McGough et al. (2005) argue that a central bank should use a long-term interest rate as its policy instrument in a case when the overnight interest rate reaches the zero lower bound. In responding to this proposal, Woodford (2005) argues that massive money injection might have some impacts on long-term interest rates, but its impacts on the real economic activities would be limited if the operation does not alter the private's sectors expectations about future monetary policy. Our analysis is closely related to this debate.

[5]Given this proposition, an expansion of the zero-rate zone occurs only when the private sector changes their expectations about the future path of overnight interest rates. Based on this understanding, most of recent empirical researches on the Japanese yield curve interpret the observed expansion of the zero-rate zone in the yield curve as coming from changes in the market's expectations (Fujiki and Shiratsuka (2002), Okina and Shiratsuka (2004), Marumo et al. (2003), Nagayasu (2004), Oda and Ueda (2005)).

behaviors, and consequently no effects on an equilibrium of the economy.

Then one may wonder why the marginal utility of money must be zero when the overnight interest rate is zero. Consider a situation that a household carries over its wealth to the next period. The household receives interest as a return if it purchases a one-period bond. On the other hand, if the household carries over through money, it receives no interest but is allowed to enjoy liquidity service provided by money. No-arbitrage condition between money and one-period bonds immediately implies that the marginal utility of money should be equal to the overnight interest rate, and that the marginal utility must be zero if the overnight interest rate is zero. However, this argument depends crucially on the assumption that liquidity services are provided only by money. If one relaxes this assumption by instead assuming that even other financial assets, including overnight bonds, provide liquidity services as money does, then the zero overnight interest rate does not necessarily imply the zero marginal utility of money. If this is the case, an additional money injection into an economy with the zero overnight interest rate could have some impacts on an equilibrium.

Goodfriend (2000) argues that we need to have a model in which liquidity services are provided by other financial assets in order to fully understand the consequences of an open market operation in an economy with the zero overnight interest rate. Specifically, Goodfriend (2000) proposes the idea of "broad liquidity service". The assumption that money is the only financial asset that provides liquidity services might not be a good approximation to actual financial activities, in the sense that financial assets other than money is used as collateral in those activities. According to the definition of Goodfriend (2000), any asset that contributes to reducing external finance premium (i.e., the wedge between internal and external financing costs) must be regarded as providing liquidity services to some extent. Broad liquidity services are defined as those services provided not only by money but also by other financial assets. According to his definition, "all assets provide broadly defined services to one degree or another" (Goodfriend (2000), p.1020).

We introduce broad liquidity services simply by assuming that bonds with various maturities enter the household's utility function as its arguments.[6] This treatment mimics the MIUF (Money in the Utility Function) approach for liquidity services provided by money. Needless to say, this assumes much of what we would like to explain, thus clearly not a satisfactory treatment,[7] but still captures

[6]Our model does not differ from the standard model in other respects, including the assumption of complete capital markets.

[7]See Goodfriend (2004) for an attempt to describe the endogenous emergence of broad liquidity services provided by various financial assets in an economy with imperfect information.

an important aspect of liquidity services provided by various financial assets without sacrificing the tractability of the model. In the rest of this section, we use this approach to investigate how the yield curve is formed, how it is affected by open market operations, and how it is distorted by the zero interest rate bound, in an economy with broad liquidity services.

2.1 Household's optimization

Let us consider a representative household that seeks to maximize a discounted sum of utilities of the form

$$E_0\left[\sum_{t=0}^{\infty}\beta^t u_t\left(z_t^h;\xi_t\right)\right], \tag{2.1}$$

where $u(\cdot)$ is an increasing and concave function, β is the discount factor, and ξ_t represents exogenous stochastic disturbances. The variable z_t^h represents a vector of endogenous variables that is defined by

$$z_t^h \equiv \left[c_t,\ \frac{M_t^h}{P_t},\ \frac{B_{1,t}^h}{P_t},\ \frac{B_{2,t}^h}{P_t},\ldots,\ \frac{B_{n,t}^h}{P_t},\ldots,\frac{B_{\bar{N},t}^h}{P_t}\right] \tag{2.2}$$

where c_t is the private consumption expenditures, M_t^h is the household's nominal end-of-period balances in the base money, and P_t is the price level. We assume that the government issues zero-coupon nominal bonds, each of which pays one yen when it matures, and denote the face value of bonds, with n periods to maturity, held by the household at the end of period t by $B_{n,t}^h$.

The representative household is subject to a flow budget constraint of the form

$$P_t c_t + \left(M_t^d - M_{t-1}^d\right) + \sum_{n=1}^{\bar{N}}(1+i_{n,t})^{-1}\left(B_{n,t}^h - B_{n+1,t-1}^h\right) \leq P_t d_t + B_{1,t-1}^h, \tag{2.3}$$

where d_t is the household's real disposable income, and $B_{1,t-1}^h$ represents the amount of repayment for bonds that mature in period t. The representative household allocates the sum of disposable income and the repayment between consumption expenditures and the purchases of government bonds. The term $B_{n,t}^h - B_{n+1,t-1}^h$ represents the change from the previous period in the face value of bonds that mature in period $t+n$, namely, an amount of net purchase of those bonds in period t. Those new bonds are evaluated by the market price in period t, $(1+i_{n,t})^{-1}$, where $i_{n,t}$ is the yield to maturity for bonds with n periods to maturity.

The household optimization problem defined above implies the following first-order conditions. We differentiate with respect to M_t^h, $B_{n,t}^h$ and c_t, and then substitute a market clearing condition of the form

$$z_t^h = z_t,$$

where z_t represents supply quantities of goods, the base money, and bonds with various maturity, defined by:

$$z_t \equiv \left[y_t,\ \frac{M_t}{P_t},\ \frac{B_{1,t}}{P_t},\ \frac{B_{2,t}}{P_t}, \ldots,\ \frac{B_{n,t}}{P_t}, \ldots,\ \frac{B_{\bar{N},t}}{P_t} \right].$$

The first-order conditions with respect to M_t^h and c_t imply

$$1 = E_t\left(q_{1,t}\right) + \phi_m\left(z_t;\xi_t\right), \tag{2.4}$$

where $\phi_m(z_t;\xi_t)$ represents the marginal utility of money, or equivalently the marginal liquidity service provided in period t, by the base money, which is defined by $\phi_m(z_t;\xi_t) \equiv \partial u(z_t;\xi_t)/\partial(M_t/P_t)$. The variable $q_{j,t}$ is a nominal stochastic discount factor that is defined by

$$q_{j,t} \equiv E_t\left[\beta^j \frac{u_c(z_{t+j};\ \xi_{t+j})}{u_c(z_t;\ \xi_t)} \frac{P_t}{P_{t+j}}\right] \quad \text{for} \quad j \geq 0. \tag{2.5}$$

Similarly, the first-order conditions with respect to $B_{n,t}^h$ and c_t imply

$$\frac{1}{1+i_{n,t}} = E_t\left[\sum_{j=0}^{n-1} q_{j,t}\phi_{n-j}\left(z_{t+j};\xi_{t+j}\right) + q_{n,t}\right], \tag{2.6}$$

where $\phi_{n-j}(z_{t+j};\xi_{t+j})$ represents the marginal liquidity service provided in period $t+j$ by bonds with $n-j$ periods to maturity, which is defined by $\phi_{n-j}(z_{t+j};\xi_{t+j}) \equiv \partial u(z_{t+j};\xi_{t+j})/\partial(B_{n-j,t+j}/P_{t+j})$.

Equation (2.6) implies that the yield for bonds with one period to maturity is given by

$$\frac{1}{1+i_{1,t}} = E_t\left(q_{1,t}\right) + \phi_1\left(z_t;\xi_t\right). \tag{2.7}$$

Together with equation (2.4), we eliminate $E_t\left(q_{1,t}\right)$ to obtain an arbitrage condition between money and these one-period bonds of the form

$$\phi_m\left(z_t;\xi_t\right) = \frac{i_{1,t}}{1+i_{1,t}} + \phi_1\left(z_t;\xi_t\right). \tag{2.8}$$

This equation reduces to a standard money demand relationship in which money demand is a decreasing function in the short-term (namely, one-period) nominal interest rate $i_{1,t}$, if the marginal liquidity service provided by the one-period bonds $\phi_1(\cdot)$ is equal to zero. Otherwise, however, money demand inversely depends on the marginal liquidity service provided by the one-period bonds.

We make the following assumptions about the marginal liquidity services provided by bonds with various periods to maturity. First, we assume that

$$\phi_m(z_t;\xi_t) \geq \phi_1(z_t;\xi_t) \geq \phi_2(z_t;\xi_t) \geq \cdots \geq \phi_{\bar{N}}(z_t;\xi_t) \geq 0. \tag{2.9}$$

This simply states that the marginal liquidity service provided by the base money should be not be less than the one provided by the one-period bonds; the marginal liquidity provided by the one-period bonds should not be less than the one provided by the two-period bonds, and so on. These assumptions must be easy to accept. Second, we assume that the derivative of $\phi_n(z_t;\xi_t)$ with respect to the real money balance $m_t (\equiv M_t/P_t)$, namely $\phi_{nm}(z_t;\xi_t) \equiv \partial\phi_n(z_t;\xi_t)/\partial m_t$ should satisfy

$$\phi_{nm}(z_t;\xi_t) \leq 0 \qquad \text{for} \quad n = 1, \ldots, \bar{N}. \tag{2.10}$$

In other words, the base money and bonds with various periods to maturity are substitutes. The representative household with ample money (and ample liquidity service provided by money) would have little need to obtain additional liquidity services from bonds, which is represented by a lower marginal liquidity service provided by bonds.[8] Third, we assume that

$$|\phi_{mm}(z_t;\xi_t)| \geq |\phi_{1m}(z_t;\xi_t)| \geq |\phi_{2m}(z_t;\xi_t)| \geq \cdots \geq |\phi_{\bar{N}m}(z_t;\xi_t)| \geq 0. \tag{2.11}$$

Figure 2 shows the schedules of $\phi_m(z_t;\xi_t)$, $\phi_1(z_t;\xi_t)$, $\phi_2(z_t;\xi_t)$ and so on, as a function of m_t under the additional assumption (the assumption for simplification) of linearity. As shown in the figure, the marginal liquidity service provided by the base money is greater than the one provided by the one-period bonds if $m_t < \bar{m}_1$ but these two become identical once m_t exceeds $\bar{m}_1$, which is a direct reflection of (2.11). Similarly, $\bar{m}_2$ represents a "junction" for two-period bonds, and $\bar{m}_3$ for three-periods bonds, and so on. These junctions should satisfy

$$\bar{m}_1 > \bar{m}_2 > \cdots > \bar{m}_{\bar{N}} > \bar{\bar{m}}, \tag{2.12}$$

where $\bar{\bar{m}}$ is the level of the base money where the marginal liquidity service provided by the base money equals to zero, namely the satiation level of money.

[8] A similar assumption is adopted by Goodfriend (2000), but in an informal way.

2.2 Necessary conditions for zero interest rates

Consider a necessary condition for the one-period nominal interest rate in period t, $i_{1,t}$, to reach the zero lower-bound. Substituting $i_{1,t} = 0$ into equation (2.8) leads to

$$\phi_1 (z_t; \xi_t) = \phi_m (z_t; \xi_t) . \tag{2.13}$$

An important point to note here is that the satiation of the base money, $\phi_m (z_t; \xi_t) = 0$, is *not* a necessary condition for the one-period interest rate to be zero. Previous studies on the zero bound on nominal interest rates, including Eggertsson and Woodford (2003) and Jung et al. (2005) among others, typically ignore liquidity services provided by bonds, thereby implicitly assuming that $\phi_1 (z_t; \xi_t) = 0$. In their case, equation (2.13) reduces to $\phi_m (z_t; \xi_t) = 0$, implying that the satiation of money is indeed a necessary condition for the one-period interest rate to be zero. The one-period bond provides neither liquidity service nor interest to its holders in their models; thus an arbitrage argument between money and the one-period bonds implies a zero marginal liquidity service (or, zero marginal utility) even for money. However, this kind of arbitrage argument does not necessarily imply the satiation of money in a more general setting in which not only money but also bonds provide liquidity services. In this setting the one-period bonds are still attractive to investors because of its liquidity service, even when its interest rate reaches the zero lower bound. Then an arbitrage argument requires that the marginal liquidity service provided by money should be equal to the marginal liquidity service provided by the one-period bonds (which is positive), as shown in equation (2.13). Note that, as shown in Figure 2, m_t must be greater than $\bar{m}_1$ for the one-period interest rate to be zero, but still could be smaller than $\bar{\bar{m}}$ (namely, the satiation level).

Turning to the two-period bonds, a similar necessary condition for its yield to maturity, $i_{2,t}$, to be zero could be obtained in the following way. Under the assumption of perfect foresight,[9] a simple substitution of equation (2.4) into equation (2.6) leads to

$$\frac{1}{1 + i_{2,t}} = [1 - \phi_m (z_t; \xi_t)] [1 - \phi_m (z_{t+1}; \xi_{t+1}) + \phi_1(z_{t+1}; \xi_{t+1})] + \phi_2 (z_t; \xi_t) .$$

Note that, given the inequality constraint (2.9), the right-hand side of the above equation equals to unity if and only if[10]

$$m_t \geq \bar{m}_2; \;\; m_{t+1} \geq \bar{m}_1,$$

[9] The rest of this section assumes perfect foresight unless otherwise mentioned.

[10] Observe that $\phi_m (z_t; \xi_t) = \phi_2 (z_t; \xi_t)$ if $m_t \geq \bar{m}_2$, and that $\phi_m (z_{t+1}; \xi_{t+1}) = \phi_1(z_{t+1}; \xi_{t+1})$ if $m_{t+1} \geq \bar{m}_1$.

implying that this is a necessary condition for the two-period nominal interest rate, $i_{2,t}$, to reach the zero lower bound.

A similar argument indicates that the yield to maturity for the N-period bonds is given by

$$\frac{1}{1+i_{N,t}} = \prod_{j=0}^{N-1} \left[1-\phi_m(z_{t+j};\xi_{t+j})\right] + \sum_{j=1}^{N-1} \left\{ \left[\prod_{k=0}^{j-1} \left(1-\phi_m(z_{t+k};\xi_{t+k})\right) \right] \phi_{N-j}\left(z_{t+j};\xi_{t+j}\right) \right\} + \phi_N(z_t;\xi_t), \tag{2.14}$$

and, given the inequality condition (2.9), the right-hand side of equation (2.14) could be equal to unity if and only if

$$m_t \geq \bar{m}_N;\ m_{t+1} \geq \bar{m}_{N-1};\ \cdots;\ \ m_{t+N-1} \geq \bar{m}_1. \tag{2.15}$$

This simply states that the yield to maturity for the N period bonds could reach the zero lower bound only when a large-scale money injection is conducted in period t, *and* it is expected by market participants in period t that substantial monetary easing will be continued up until period $t+N-1$. This result sharply differs from those obtained in the previous researches that pay no attention to broad liquidity services. First, a simple substitution of $\phi_n(\cdot)=0$ $(n=1,\ldots,N)$ into equation (2.14) implies that the right-hand side of (2.14) could be equal to unity only when $\phi_m(\cdot)=0$ holds in each period from t to $t+N-1$. In this sense, a necessary condition for $i_{N,t}=0$ in their models is the satiation of money in every period. Second, $i_{1,t},\ldots,i_{1,t+N-1}$ would be zero if $m_t,\ldots,m_{t+N-1}$ are all equal to $\bar{m}_1$. One may think this would be enough to make $i_{N,t}$ to be zero. But this is not true simply because such a path of the one-period nominal interest rate does not satisfy equation (2.15), which requires more monetary easing not only in the current period but also in the future periods. This is in a sharp contrast with the results obtained in the previous studies in that $i_{N,t}$ does indeed become zero if $m_t,\ldots,m_{t+N-1}$ are all equal to $\bar{m}_1$.

2.3 Open market operations

If the condition (2.15) is satisfied, the yield to maturity would equal to zero for all bonds starting from 1-period bonds up to N-period bonds. In other words, the yield curve in period t contains a flat "zero-rate zone." This has been the key feature in the Japanese financial markets since the spring of 2001 until very recently, as we will closely see in the next section. Given such an environment, the Bank of Japan sought to expand the zero-rate zone in the yield curve by conducting a series of large-scale open market operations.

We consider the effects of an additional large-scale open market operation when the yield to maturity equals to zero for all bonds up to N-period bonds, using a theoretical framework prepared in the previous subsections. Specifically, we are interested in the conditions under which an additional operation would expand the zero-rate zone. First, it is important to note that the purchase of bonds with less than N periods to maturity has no consequences on financial prices, as has already been pointed out by many researchers and practitioners. In our model with broad liquidity services, the existence of the zero-rate zone up to the N-period bonds ($i_{1,t} = i_{2,t} = \cdots = i_{N,t} = 0$) implies

$$\phi_m(z_t;\xi_t) = \phi_1(z_t;\xi_t) = \phi_2(z_t;\xi_t) = \cdots = \phi_N(z_t;\xi_t).$$

Note that all bonds up to the N-period bonds are equivalent to money in the sense that they pay no interest to their holders, and that they provide the same marginal liquidity service as money does. Given this, the purchase of bonds with less than N periods to maturity is nothing but an exchange of two identical financial assets, therefore yielding no consequences on an equilibrium in the financial markets. On the contrary, a central bank may not be able even to purchase such bonds if market participants need to pay some transaction costs for each financial transactions including those with a central bank.[11]

Second, consider the purchase of bonds with longer than N periods to maturity. Specifically, suppose that a central bank purchases the $N+1$-period bonds. As explained earlier, all bonds up to the N-period bonds are equivalent to money, so that they can be regarded as "money". The purchase of the $N+1$-period bonds is nothing but an exchange between those bonds and "money". An increase in "money" leads to a decrease in the marginal liquidity service for money and those bonds included in "money" by the same amount. Then the market price of "money" should decline so as to restore an equilibrium in the financial market. However, given that the price of "money" is already equal to unity (namely, zero interest rates), there is no room for a further increase. The only way to restore an equilibrium in this circumstance is to make the interest rate for the $N+1$-period bonds sufficiently lower to create an additional demand for "money", which should coincide with an additional supply of "money" injected though the open market operation. This is the portfolio rebalancing effect of open market operations discussed by Goodfriend (2000).

To evaluate this effect, we investigate the impact of an open market operation in period t upon

[11] The Bank of Japan failed to purchase short-term bonds, and was thus forced to purchase longer-term bonds in 2004-2005.

various interest rates in period t, under the assumption that the market's expectation about future interest rates are not affected by the open market operation in period t. We start by looking at a simple case in which $m_t = \bar{m}_1$ so that $i_{1,t}$ is already zero, but the market expects in period t that $m_{t+1} < \bar{m}_1$ so that $i_{1,t+1}$ will still be positive. We want to see what will happen on $i_{2,t}$ if an additional money is injected through an open market operation in period t, although the market expectation about the relevant future interest rates ($i_{1,t+1}$ in this case) does not change in response to the operation. As we saw in the previous subsection, a necessary condition for $i_{2,t} = 0$ is that $m_t \geq \bar{m}_2$ and $m_{t+1} \geq \bar{m}_1$, suggesting that a further increase in m_t would lower $i_{2,t}$ to a level closer to zero. To see this, we express the price of two-period bond in period t in a slightly different way.[12]

$$\frac{1}{1+i_{2,t}} = \left(\frac{1}{1+i_{1,t}}\right)\left(\frac{1}{1+i_{1,t+1}}\right) + \phi_1\left(z_t;\xi_t\right)\left[\frac{\phi_2\left(z_t;\xi_t\right)}{\phi_1\left(z_t;\xi_t\right)} - \frac{1}{1+i_{1,t+1}}\right]. \tag{2.16}$$

The first term on the right-hand side of this equation does not change in response to the operation, because $i_{1,t}$ is already zero, and the market expectation about $i_{1,t+1}$ is not altered by the operation. Turning to the second term on the right-hand side of the equation, the expectations theory implies that this term should be always equal to zero. This is indeed true if $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are both equal to zero, as assumed in models without broad liquidity services. Then these models imply that an open market operation without changing $i_{1,t+1}$ has no effects on $i_{2,t}$. However, this is not true any more in our setting. For example, when m_t is equal to $\bar{m}_1$, $\phi_2(\cdot)$ is small relative to $\phi_1(\cdot)$, so that the second term on the right-hand side should be negative. If m_t increases through the operation, however, $\phi_2(\cdot)/\phi_1(\cdot)$ monotonically approaches unity, and the second term turns to positive.[13] This implies that an increase in money through the open market operation lowers $i_{2,t}$.[14]

The above argument could be extended to the open market purchase of the $N+1$-period bonds, whose market price is given by

$$\frac{1}{1+i_{N+1,t}} = \left(\frac{1}{1+i_{1,t}}\right)\left(\frac{1}{1+i_{N,t+1}}\right) + \phi_1\left(z_t;\xi_t\right)\left[\frac{\phi_{N+1}\left(z_t;\xi_t\right)}{\phi_1\left(z_t;\xi_t\right)} - \frac{1}{1+i_{N,t+1}}\right].$$

Suppose that $i_{1,t}, i_{2,t}, \ldots, i_{N,t}$ are all equal to zero, and that an open market operation in period t does not alter the market's expectations about the future path of money supply. Then a similar argument

[12] We obtain this equation by substituting (2.4) into (2.6).

[13] If $m_t = \bar{m}_2$, then $\phi_2(\cdot)/\phi_1(\cdot) = 1$, and the second term should become positive. Figure 3 shows the value of the second term as a function of m_t in a case in which the marginal liquidity service schedules for money and various bonds are linear as given in Figure 2.

[14] This could be understood in the following way. The market price of the two-period bonds in period t consists of two parts: the discounted value of the market price of the one-period bonds in period $t+1$ and the marginal liquidity service provided by the two-period bonds in period t ($\phi_2(z_t;\xi_t)$). The latter part increases relative to $\phi_1(z_t;\xi_t)$ in response to an increase in m_t, thereby contributing to a rise in the market price of the two-period bonds in period t.

as before implies that an increase in money though the purchase of the $N+1$-period bonds in period t leads to an increase in the term $\phi_{N+1}(z_t;\xi_t)/\phi_1(z_t;\xi_t)$,[15] thereby raising the market price of the $N+1$-period bonds in period t.

2.4 Forward and futures interest rates

If the second term on the right-hand side of (2.16) is positive, it implies

$$\frac{1+i_{2,t}}{1+i_{1,t}} < 1+i_{1,t+1}.$$

The right-hand side of this equation represents the one-period forward rate in period t that starts in period t+1 and ends in period t+2, while the left-hand side represents the corresponding futures rate. This equation simply states that the forward rate in period t could be lower than the corresponding futures rate if substantial monetary easing is conducted in period t.

The market price for the three-period bonds similarly implies

$$\begin{aligned}\frac{1}{1+i_{3,t}} - \left(\frac{1}{1+i_{2,t}}\right)\left(\frac{1}{1+i_{1,t+2}}\right) &= \phi_2(z_t;\xi_t)\left[\frac{\phi_3(z_t;\xi_t)}{\phi_2(z_t;\xi_t)} - \frac{1}{1+i_{1,t+2}}\right] \\ &+ [1-\phi_m(z_t;\xi_t)]\phi_1(z_{t+1};\xi_{t+1})\left[\frac{\phi_2(z_{t+1};\xi_{t+1})}{\phi_1(z_{t+1};\xi_{t+1})} - \frac{1}{1+i_{1,t+2}}\right]. \end{aligned} \tag{2.17}$$

If the right-hand side of this equation is positive, it implies

$$\frac{1+i_{3,t}}{1+i_{2,t}} < 1+i_{1,t+2}.$$

The left-hand side represents the one-period forward rate that starts in period $t+2$ and ends in period $t+3$, while the right-hand side represents the corresponding futures rate. The right hand-side of (2.17) is likely to be positive when the following conditions are satisfied. First, the base money in period t, m_t, should be close to $\bar{m}_3$ so that $\phi_3(z_t;\xi_t)/\phi_2(z_t;\xi_t)$ becomes as close as its maximum value (namely, unity). Similarly, the market's expectation formed in period t about the base money in period $t+1$, m_{t+1}, should be close to $\bar{m}_2$ so as to make $\phi_2(z_{t+1};\xi_{t+1})/\phi_1(z_{t+1};\xi_{t+1})$ as close as its maximum (namely, unity). These two conditions indicate that substantial monetary easing not only in period t but also in period $t+1$ are necessary conditions for the futures and forward spread to be positive. Second, the base money in period $t+2$ should be less than $\bar{m}_1$ so as to keep $i_{1,t+2}$ at a positive level. If m_{t+2} should exceed $\bar{m}_1$, then the expressions in the two squared brackets on the right-hand side are

[15] Specifically, m_t increases from $\bar{m}_N$ to $m_t = \bar{m}_{N+1}$.

both equal to zero even if both $\phi_3(z_t;\xi_t)/\phi_2(z_t;\xi_t)$ and $\phi_2(z_{t+1};\xi_{t+1})/\phi_1(z_{t+1};\xi_{t+1})$ equal to unity. In fact, $i_{3,t}$ is equal to zero in this case, as we saw in subsection 2.2. In other words, the futures and forward spread becomes zero as soon as the three-period bonds enter the zero-rate zone. This implies that the futures and forward spread is likely to be positive around the end of the zero-rate zone.

The market price for the n-period bonds implies

$$\frac{1}{1+i_{n,t}} - \left(\frac{1}{1+i_{n-1,t}}\right)\left(\frac{1}{1+i_{1,t+n-1}}\right) = \phi_{n-1}(z_t;\xi_t)\left[\frac{\phi_n(z_t;\xi_t)}{\phi_{n-1}(z_t;\xi_t)} - \frac{1}{1+i_{1,t+n-1}}\right]$$
$$+\sum_{j=1}^{n-2}\left\{\prod_{k=0}^{j-1}\left[1-\phi_m(z_{t+k};\xi_{t+k})\right]\right\}\phi_{n-1-j}(z_{t+j};\xi_{t+j})\left[\frac{\phi_{n-j}(z_{t+j};\xi_{t+j})}{\phi_{n-1-j}(z_{t+j};\xi_{t+j})} - \frac{1}{1+i_{1,t+n-1}}\right], \quad (2.18)$$

and

$$\frac{1+i_{n,t}}{1+i_{n-1,t}} < 1+i_{1,t+n-1}$$

holds if the right-hand side of (2.18) is positive. A necessary condition for this is that substantial monetary easing is conducted in period t as well as in periods $t+1$, $t+2$, and so on. For example, a decline in the forward rate relative to the futures rate would be limited if an increase in the base money in period t is perceived by the market to be a temporary one. The future and forward spread would increase over time as market participants gradually update their belief about the future path of the base money.

Figure 4 illustrates how the forward and futures spread depends on the real money balance in period t, m_t, and the expected real balance in period $t+1$, m_{t+1}, for the case of the two-period bonds. First, we note that the forward rate $\frac{1+i_{2,t}}{1+i_{1,t}}$ coincides with the corresponding futures rate $\frac{1}{1+i_{1,t+1}}$, if m_t exceeds $\bar{m}_2$ *and* m_{t+1} exceeds $\bar{m}_1$. In other words, the spread is zero within the zero-rate zone. Second, the spread is again zero if m_t exceeds the satiation level $\bar{\bar{m}}$. This is the situation the previous studies on the optimal monetary policy rule in a liquidity trap, including Eggertsson and Woodford (2003), have been focusing on. Third, the spread would be positive if m_t is close to, or greater than $\bar{m}_2$ *and* m_{t+1} is less than $\bar{m}_1$. In other worlds, the spread could become positive outside the zero-rate zone, if a large-scale open market operation (but not so large to reach the satiation level) is conducted. This is the new possibility our analysis focuses on.

3 Forward and Futures Spreads in 2001-2006

We show that a massive injection of monetary base distorts yield curve in our analysis using the model of broad liquidity in the previous section. More precisely, a massive provision of monetary base reduce forward interest rates, thus possibly inducing forward interest rates lower than futures interest rates. In addition, a spread between forward and futures interest rates, observed at period t and starting from n-period ahead, depends on market expectations regarding the future path of monetary base from period t to $t+n$. This implies that the spread does not necessarily expand at the time of the massive injection of monetary base, but at the time when market expectations regarding the future path of monetary base do change. In this section, we examine these predictions using the Japanese data during the period of quantitative monetary easing policy, from 2001 through 2006.

3.1 Preliminary analysis

We start by comparing the forward and futures yield curves on some typical days. Figure 5 shows these two yield curves on February 5, 1999, April 13, 2000, September 25, 2000, June 27, 2001, June 30, 2005, and June 9, 2006. The forward yield curves represent the three-month rates starting from each day measured on the horizontal axis, which is estimated using the Nelson-Siegel method (see Appendix A for more details). On the other hand, the futures yield curves represent the corresponding futures interest rates, taken from the Euro yen three month futures rate.

A simple comparison between the two yield curves shows the following. First, we do not see any significant discrepancies between the two before the start of a quantitative monetary easing policy in March 19, 2001.[16] Specifically, the two lines are located very close to each other on the three days; February 5, 1999 (just before the start of a zero interest rate policy); April 13, 2000 (the period in which financial markets restored its stability after the end of the fiscal year); September 25, 2000 (just after the termination of the zero interest rate policy). However, the two lines start to deviate from each other after the start of the quantitative monetary easing policy, as shown in the figure for June 27, 2001. The futures yield curve overlaps with the forward one for the maturities shorter than one year, while the futures curve is located well above the forward one for longer maturities. Such a discrepancy between the two yield curve is observed as well on June 30, 2005, when the CAB (current

[16] As we see later, however, there were some phases in which the discrepancies emerged even before the start of the quantitative monetary easing policy, including the end of December 1999 when the spread widened due to the Y2K problem.

account balance) target was 30-35 trillion yen.[17] However, it disappears again on June 9, 2006, when the quantitative monetary easing policy ended and the CAB reduced to about 10 trillion yen.

Figure 6 shows scatter plots with time-to-settlement (relative to the policy duration measured by PD, which is defined in the next subsection) on the horizontal axis, and futures and forward spread on the vertical axis. We show the result for the full sample period (February 2, 1999 to June 30, 2006) as well as the results for five subsamples. We clearly see a hump-shaped curve in each of them. More importantly, these hump shaped curves tend to reach their peaks when period-to-maturity coincides with PD. In other words, the spread is very close to zero for short maturities, but it tends to increase monotonically as the maturity approaches PD. Such a tendency is most clearly seen in the period of QMEP under Governor Hayami, and the period of QMEP under Governor Fukui. These observations are consistent with the theoretical predictions obtained in the previous section.

3.2 Indicators for spreads between futures and forward rates

We propose two indicators to detect spreads between futures and forward rates: horizontal spreads for HSP and vertical spreads for VSP, illustrated in Figure 7. We first estimate the policy duration, PD, as a flattened zone in the short-end of a forward rate curve. We then measure the vertical distance between the futures rate curve and the forward rate curve at PD. We compute a forward rate and a futures rate at PD as $FWPD$ and $FTPD$, respectively, and VSP is defined as a spread between $FTPD$ and $FWPD$. We measure the horizontal distance at $FWPD$. We define PD^* as the time-to-settlement of futures contract that futures rate is equal to $FWPD$. We compute HSP as a difference between PD and PD^*.

We formally define the above mentioned indicators: PD, PD^*, $FWPD$, $FTPD$, HSP, and VSP. First, we follow the definition of PD in Okina and Shiratsuka (2004), based on estimates of the extended Nelson-Siegel model regarding the instantaneous forward rate (IFR) for a settlement at period m, denoted by $r(m)$, below:

$$r(m) = \beta_0 + \beta_1 \exp\big(-\frac{m}{\tau_1}\big) + \beta_2\big(\frac{m}{\tau_1}\big)\exp\big(-\frac{m}{\tau_1}\big) + \beta_3\big(\frac{m}{\tau_2}\big)\exp\big(-\frac{m}{\tau_2}\big), \tag{3.1}$$

where β_0, β_1, β_2, β_3, τ_1 and τ_2 are parameters to be estimated from the data. We expect β_0, τ_1 and τ_2 to be positive. Using the estimates of the extended Nelson-Siegel model, PD is defined as:

$$PD = \tau_2, \tag{3.2}$$

[17]This was the period just before the market's started to expect the termination of the policy.

The estimate of τ_2 corresponds to the duration of the flattened zone of $r(m)$ in the short end, and reflects the market expectations of how long the BOJ will commit to a zero interest rate.

Next, we measure the vertical distance between a forward rate curve and futures rate curve at PD. $FWPD$ and $FTPD$ are defined as 3-month forward rates starting from PD for both euro yen TIBORs/yen swap rates and euro-yen futures rates.

$$FWPD = 4\int_{s=PD}^{PD+1/4} r(s)ds, \tag{3.3}$$

$$FTPD = \frac{t_{n+1} - PD}{t_{n+1} - t_n} FT(t_n) + \frac{PD - t_n}{t_{n+1} - t_n} FT(t_{n+1}), \tag{3.4}$$

where $FT(t)$ is 3-month euro yen futures rates for the time-to-settlement of t, and t_n and t_{n+1} are adjacent time-to-settlements that include PD in their interval, i.e. $t_n \leq PD < t_{n+1}$. Thus, the vertical spread, VSP, is defined as

$$VSP = FTPD - FWPD. \tag{3.5}$$

Finally, we measure the horizontal distance at $FWPD$. We define PD^* as the time-to-settlement of euro yen futures that euro yen futures rate is equal to $FWPD$. More precisely, PD^* satisfies the equation below,

$$\frac{t_{k+1} - PD^*}{t_{k+1} - t_k} FT(t_k) + \frac{PD^* - t_k}{t_{k+1} - t_k} FT(t_{k+1}) = FWPD, \tag{3.6}$$

where t_k and t_{k+1} are adjacent time-to-settlements that include PD^* in their interval, i.e. $t_k \leq PD^* < t_{k+1}$. Thus, PD^* is computed as

$$PD^* = \frac{(t_{k+1} - t_k)FWPD - t_{k+1}FT(t_k) + t_k FT(t_{k+1})}{FT(t_{k+1}) - FT(t_k)}. \tag{3.7}$$

The horizontal spread, HSP, is defined as

$$HSP = PD - PD^*. \tag{3.8}$$

Note that the long-term forward rate, LFR, is also an informative indicator, defined in Okina and Shiratsuka (2004). This indicator is equal to β_0 in equation (3.1) and is considered as a proxy for the summation of expected inflation and expected economic growth, or expected nominal economic growth.

$$LFR = \beta_0, \tag{3.9}$$

3.3 Time-series movements of HSP and VSP

We first look at the day-by-day changes in $FWPD$ and $FTPD$, as well as the changes in PD and PD^*, around the start of the quantitative monetary easing policy in March 19, 2001 (see the two charts on the left-hand side of Figure 8).[18] Note that there was a discrepancy between $FWPD$ and $FTPD$, or between PD and PD^*, even before the start of the policy. This probably reflects the fact that the BOJ's injected money in response to a temporary increase in liquidity demand just before the end of the fiscal year (namely, the end of March). As is consistent with this interpretation, these spreads almost disappeared at the beginning of April, as soon as the new fiscal year started. More importantly, however, the spread started to widen again in April, and continued to increase over the next three months. If we look more closely at the changes in PD and PD^* during this period, we see that PD rose almost consistently during this period while PD^* did not show any significant change. A similar tendency could be seen in $FWPD$ and $FTPD$, although less clearly. These observations suggest that money injection during this period contributed to lowering forward interest rates, and extending the zero-rate zone in the yield curve, while keeping the market's expectations about future monetary policy unchanged.

Turning to the changes in these four variables around the end of the policy in March 9, 2006, we see more clearly that a decrease in the base money lead to a narrower spread, both in terms of the vertical and horizontal ones, until it finally disappeared at the end of May. Note that the current account balance at the BOJ has been declining only gradually even after the decision to terminate the policy was made in March 9.[19] This is consistent with the gradual decline in the spread during this period.

Let us next examine the developments in HSP and VSP from 2001 through 2006 in Figure 9. In this figure we plot PD and PD^* in the top panel, $FWPD$ and $FTPD$ in the second top panel, HSP and VSP in the second bottom panel, and LFR in the bottom panel.

Looking at the top panel, PD jumps upward after the introduction of QMEP and remains high throughout 2001, while PD^* does not respond significantly and rather start declining around the turn of the year from 2001 to 2002. After the sharp decline in PD in mid-2003, PD and PD^* converge to

[18] All figures used in Figure 8 are the moving average over thirteen days. Also note that FWPD and FTPD are depicted using different vertical axes (the left axis for $FWPD$ and the right axis for $FTPD$).

[19] Even at the end of June, more than three months after the decision, the current account balance is above 15 trillion yen, much higher than the level before the start of the quantitative monetary easing policy (about 4 trillion yen).

almost same level at around one year. They then start rising again, but PD increases slightly faster than PD^*. After the second half of 2005, they drop again, as market expectations rise regarding the early termination of QMEP.[20]

In the second top panel, $FWPD$ and $FTPD$ generally show reverse relationship between PD and PD^*. However, most significant movements can be seen in the sharp rise in both $FWPD$ and $FTPD$ after the termination of QMEP, suggesting growing market expectations regarding the early exit from zero interest rates. In the second bottom panel, HSP and VSP generally show similar movements, and especially during the period of QMEP under Governor Hayami, two indicators move very closely.

To sum up, both HSP and VSP steadily expand from 2001 through the summer of 2003 when the series of increase in the target level of the current account balance at the BOJ. HSP and VSP, however, do not necessarily move in response to the increases in the target level. Especially, HSP and VSP are generally insensitive to rapid rises in the target level of the current account balance at the BOJ. Given that LFR follows declining trend from late 2002 through mid-2003, such developments in HSP and VSP suggest that market expectations regarding the future path of the current account balance at the BOJ remain virtually unchanged.

3.4 Granger causality test

We next examine the time-series relation between policy duration indicators and monetary policy actions. To this end, we focus on two variables as indicators for monetary policy actions. The first one, RSV, is the outstanding balance of current accounts at the BOJ, and the second one, OPE, is the maturity of outstanding amounts of outright purchasing operations of commercial bills.

We carry out Granger causality test using the above two variables as well as two sets of policy duration indicators: PD, PD^* and $FWPD$, $FTPD$. Figure 10 and 11 respectively report F-values for Granger causality tests for a relationship among two sets of variables using subsamples of 60 business days ending at each date on the horizontal axis and the four-variable VAR model with five-day lag length.

According to these figures, relationships among these variable are highly sensitive to the choice of sample period. We find almost no evidence regarding the changes in the current account balance at

[20]It should be noted that estimates of PD^* tend to become unstable before significant liquidity events, such as the Y2K problem and the end of fiscal year. This is because short-end of forward rate curve shows a hump-shape reflecting increased concern over liquidity availability, and forward rates at shorter than PD become higher than that at PD, thus making it very difficult to find PD^* within the interval of zero to PD.

the BOJ Granger cause other variables.

4 Conclusion

This paper has presented a model with broad liquidity services to investigate the consequences of massive money injection in an economy with the zero interest rate bound. We incorporate Goodfriend's (2000) idea of broad liquidity services into the model by allowing the amounts of bonds with various maturities held by a household to enter its utility function. Our main findings can be summarized as follows. First, we show that the satiation of money (or the zero marginal utility of money) is *not* a necessary condition for the one-period interest rate to reach the zero lower bound; instead, we present a weaker necessary condition that the marginal liquidity service provided by money coincides with the marginal liquidity service provided by the one-period bonds, both of which are not necessarily equal to zero. Second, we show that massive money injection through an open market operation would have some influences on the yield curve even if it does not alter the private sector's expectations about future monetary policy, as long as a central bank purchases bonds located outside the zero rates zone. Third, we show that the forward and futures spread would become positive (i.e., the forward interest rate is lower than the corresponding futures rate) when a central bank conduct massive money injection, although it would become zero in a limiting case in which the base money is supplied up to the satiation level.

We have also conducted empirical analysis to check the implications of the model. Our empirical results indicate that forward interest rates started to decline relative to the corresponding futures rates just after March 2001, when a quantitative monetary easing policy started by the Bank of Japan, and that the forward and futures spread has never closed until the policy ended in March 2006. We argue that these findings are not easy to explain by a model without broad liquidity services.[21]

[21] Our findings have the following implications. First, the zero-rate zone in the yield curve has been expanded in 2001-2006 mainly through the portfolio rebalancing channel, not through the expectation channel. This is consistent with the fact that the effects of monetary stimulation through quantitative easing have been very limited. Second, the yield curve provides less useful information on the private sector's expectations about future monetary policy when the overnight interest rate has reached the zero lower bound, in the sense that forward interest rates would deviate from the corresponding futures rates.

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Appendix A Indicators for the Policy Duration Effect in Okina and Shiratsuka (2004)

Okina and Shiratsuka (2004) derive indicators for the policy duration effect using the estimates of the extended Nelson and Siegel's (1987) model, proposed by Söderlind and Svensson (1997). They apply the model to euro-yen Tokyo interbank offered rates (TIBOR) and yen swap rates: euro-yen TIBORs for 1-month to 12-month contracts, and yen swap rates for 1-year to 12-year contracts.

As shown in equation (3.1), the instantaneous forward rate (IFR), based on the extended Nelson-Siegel model, includes four terms. For your reference, we show equation (3.1) again below:

$$r(m) = \beta_0 + \beta_1 \exp\big(-\frac{m}{\tau_1}\big) + \beta_2\big(\frac{m}{\tau_1}\big)\exp\big(-\frac{m}{\tau_1}\big) + \beta_3\big(\frac{m}{\tau_2}\big)\exp\big(-\frac{m}{\tau_2}\big), \tag{A.1}$$

where β_0, β_1, β_2, β_3, τ_1 and τ_2 are parameters to be estimated from the data. We expect β_0, τ_1 and τ_2 to be positive.

The first term is a constant β_0. The second term is an exponential function $\beta_1 \exp(-m/\tau_1)$. When β_1 takes a negative (positive) value, this term produces an upward-trending (downward-trending) shape in the short end of the IFR curve. A large (small) value of τ_1 means that this exponential effect decays more slowly (quickly). The third term is $\beta_2(m/\tau_1)\exp(-m/\tau_1)$, producing a U-shape (hump shape) when β_2 takes a negative (positive) value. The fourth term produces a U-shape (hump shape) when β_3 takes a negative (positive) value. τ_2 controls the rate of convergence of the fourth term, as does τ_1 for the second and third terms. The important features of equation (A.1) are that the limits of forward and spot rates when maturity approaches zero and infinity, respectively, are equal to $\beta_0 + \beta_1$ and β_0.[22]

To detect market expectations on the duration of the policy commitment as well as the impacts of the policy, Okina and Shiratsuka (2004) define four indicators for the policy duration effect below, using the estimates of the extended Nelson-Siegel model.

$$PD = \tau_2, \tag{A.2}$$

$$RPD = \frac{1}{PD}\int_{s=0}^{PD} r(s)ds, \tag{A.3}$$

[22]In our estimation, we exploit the first feature to keep the very short end of the IFR curve from becoming negative, by imposing the restriction that the overnight uncollateralized call rate be equal to $\beta_0 + \beta_1$. This restriction is sufficient to prevent the zero-bound of nominal interest rates from influencing the estimates. We also use the second feature to compile an indicator for policy duration effect since it corresponds to the restriction that forward rates for settlements very far into the future be constant.

$$SL = \arctan(r'(2\tau_2)), \tag{A.4}$$

$$LFR = \beta_0, \tag{A.5}$$

The first indicator, PD, corresponds to the duration of the flattened shape of $r(m)$ before its second stage increase reflects the market expectations of how long $r(m)$ will stay close to zero, and thus how long the BOJ will commit to a zero interest rate. The second indicator, RPD, computes spot rates at PD, showing the flatness of $r(m)$ in the short-end and the confidence of market participants regarding the strength of BOJ's commitment to a zero interest rate. The third indicator, SL, shows sharpness of $r(m)$ in medium-term, indicating market expectations as to how rapidly the economy will recover from the zero interest rate conditions. The fourth indicator, LFR, is equal to β_0, or the long-term forward rate, and is considered as a proxy for the summation of expected inflation and expected economic growth, or expected nominal economic growth.[23] This is deemed to reflect market expectations for long-term economic performance.

Figure A-1 updates four indicators up to end-June 2006, which includes the periods of ZIRP and QMEP. We see a significant change in market expectations during the summer of 2003. The forward rate curve shifts upward in the medium to long term, as evidenced by increased SL and LFR, indicating brightening market expectations for the long-term performance of the economy. At the same time, PD shortens, partly because such positive expectations in turn shorten the expected duration of the policy commitment to a zero rate or to quantitative monetary easing.

Looking at the period after the second half of 2005, PD declines steadily and RPD and SL shift upward, reflecting growing expectations of the termination of quantitative monetary easing policy. Shortly before the termination of QMEP, PD stop declining, while SL start increasing rapidly, suggesting that the forward rate curve steepens in the medium-term. After the termination, RPD start rising, reflecting growing market expectations about the exit from zero interest rates. In contrast to the summer of 2003, LFR generally remains unchanged until the termination of QMEP, while it moves up slightly after the termination.

[23] Given the possibility of time-varying risk premiums, we must be cautious in interpreting time-series movements of estimates.

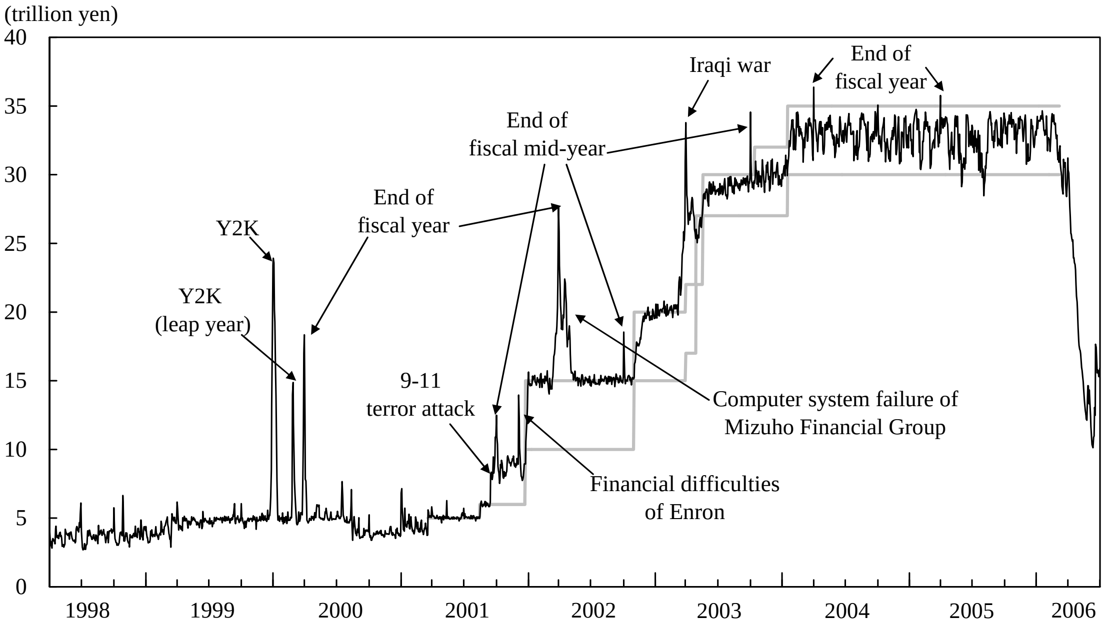
Table 1: Policy events

Date	Changes in Policy Guidelines
September 9, 1998	Reduction of targeted O/N rate (0.5 → 0.25 %)
November 13, 1998	Introduction of new money market operations
February 12, 1999	Introduction of zero interest rate policy
April 13, 1999	Governor's announcement of the commitment to zero interest rate until deflationary concerns are dispelled
October 13, 1999	Expansion of the range of money market operations
August 11, 2000	Termination of zero interest rate policy
February 9, 2001	Reduction of ODR (0.5→0.375%), introduction of new method of liquidity provision
February 28, 2001	Reduction of targeted O/N rate (0.25→0.125%) and ODR (0.375→0.25%)[*1]
March 19, 2001	Decision to introduce quantitative monetary easing policy
August 14, 2001	Increase in the target CAB (5→6 trill. yen) [*1]
September 18, 2001	Increase in the target CAB (6→above 6 trill. Yen)
December 19, 2001	Increase in the target CAB (above 6→10-15 trill. yen)
September 18, 2002	Introduction of stock purchasing plan
October 30, 2002	Increase in the target CAB (10-15→15-20 trill. yen)
(March 20, 2003)	Installation of Governor Fukui
April 30, 2003	Increase in the target CAB (17-22→22-27 trill. yen) [*2]
May 20, 2003	Increase in the target CAB (22-27→27-30 trill. yen)
October 10, 2003	Increase in the target CAB (27-30→27-32 trill. yen)
January 20, 2004	Increase in the target CAB (27-32→30-35 trill. yen)
March 9, 2006	Termination of QMEP[*1]

Note: 1. ODR denotes the official discount rate, CAB denotes the current account balance, and QMEP denotes quantitative monetary easing policy.

2. The target CAB was raised from 15-20 trillion yen to 17-22 trillion yen on April 1, 2003, considering necessary adjustment due to the establishment of the Japan Post.

Figure 1: Current account balance at the BOJ



Note: Shaded lines indicate the target or target range of the current account balance.

Figure 2: Marginal Liquidity Service

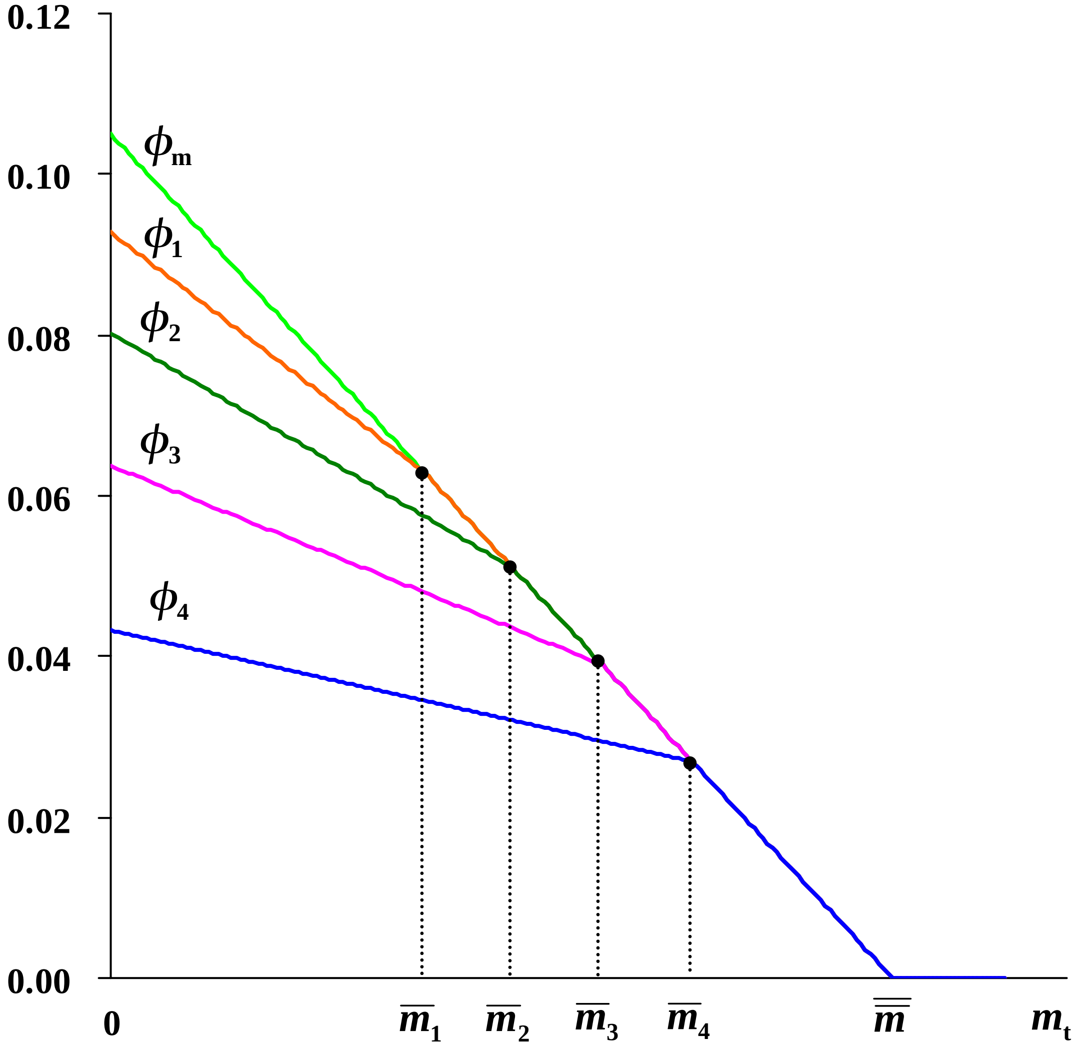


Figure 3: Futures-Forward Spread

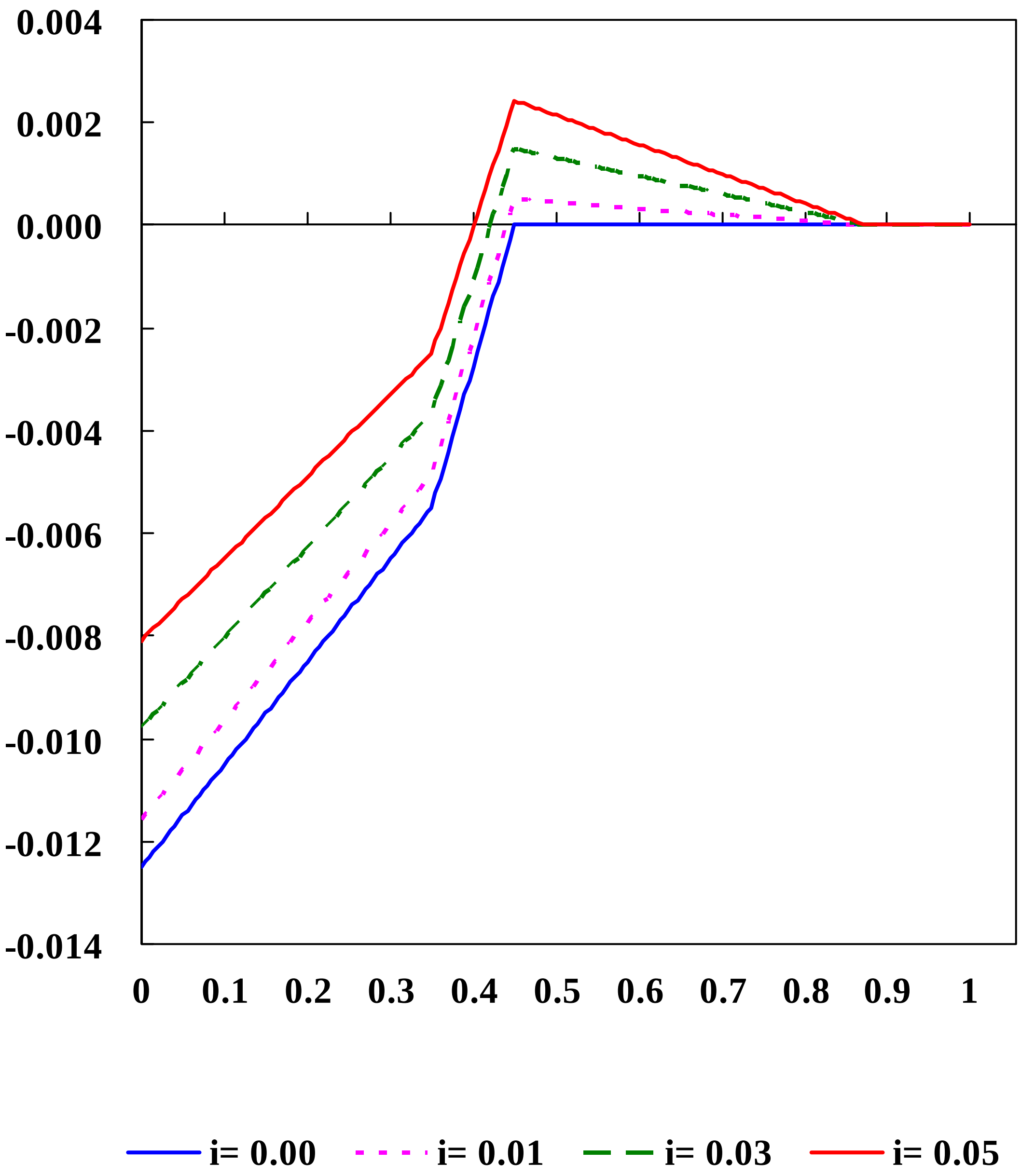


Figure 4: Forward versus Futures Interest Rates

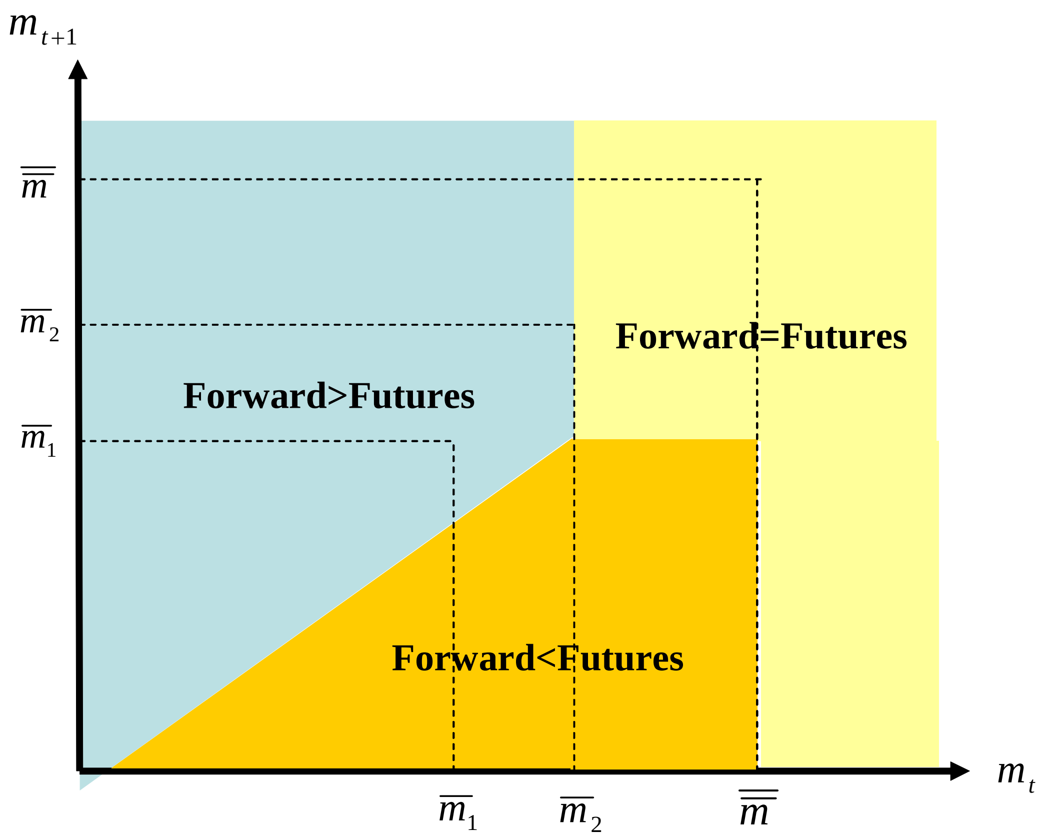


Figure 5: Short-end of yield curves

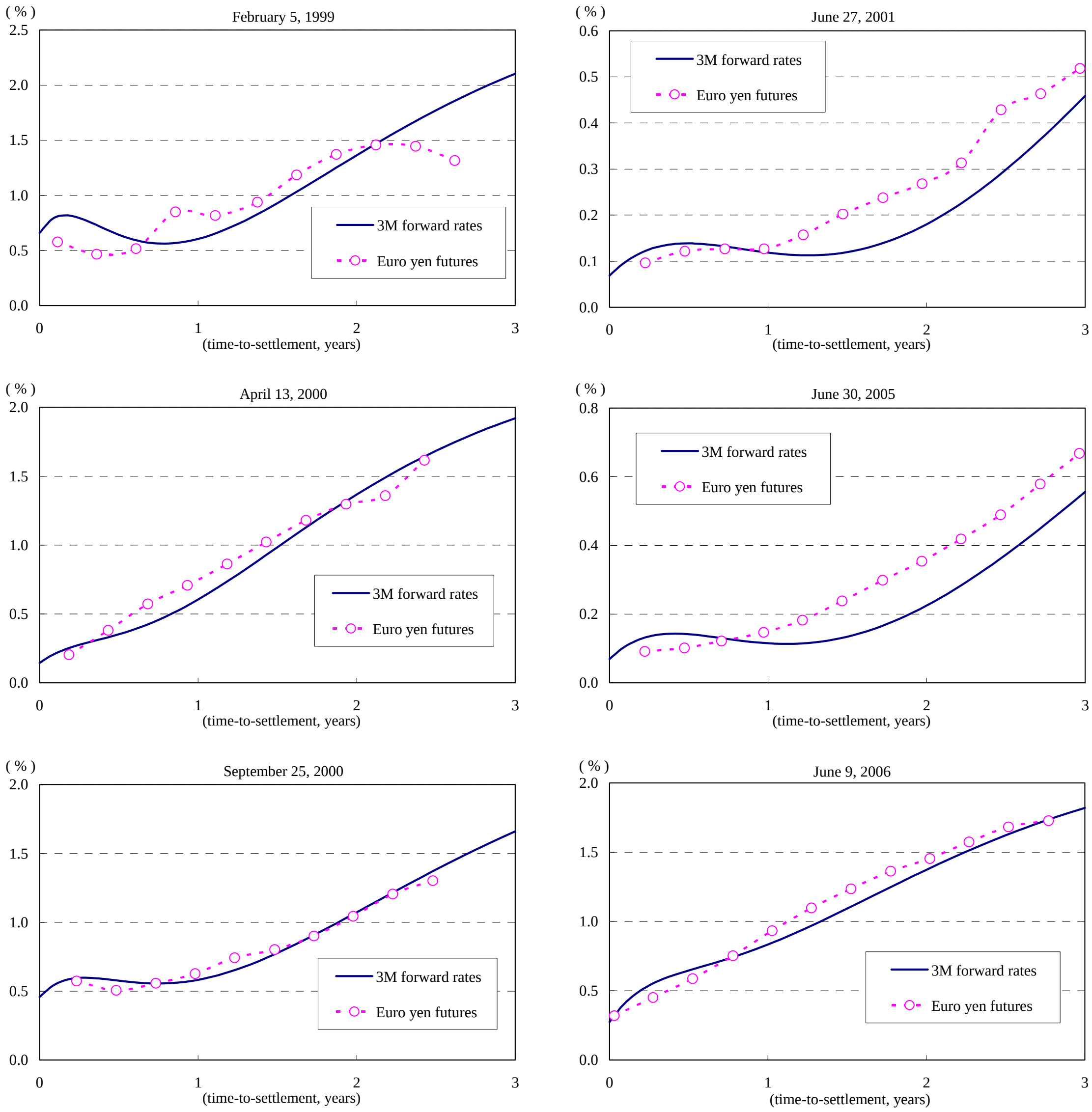


Figure 6: Spreads between futures and forward rates

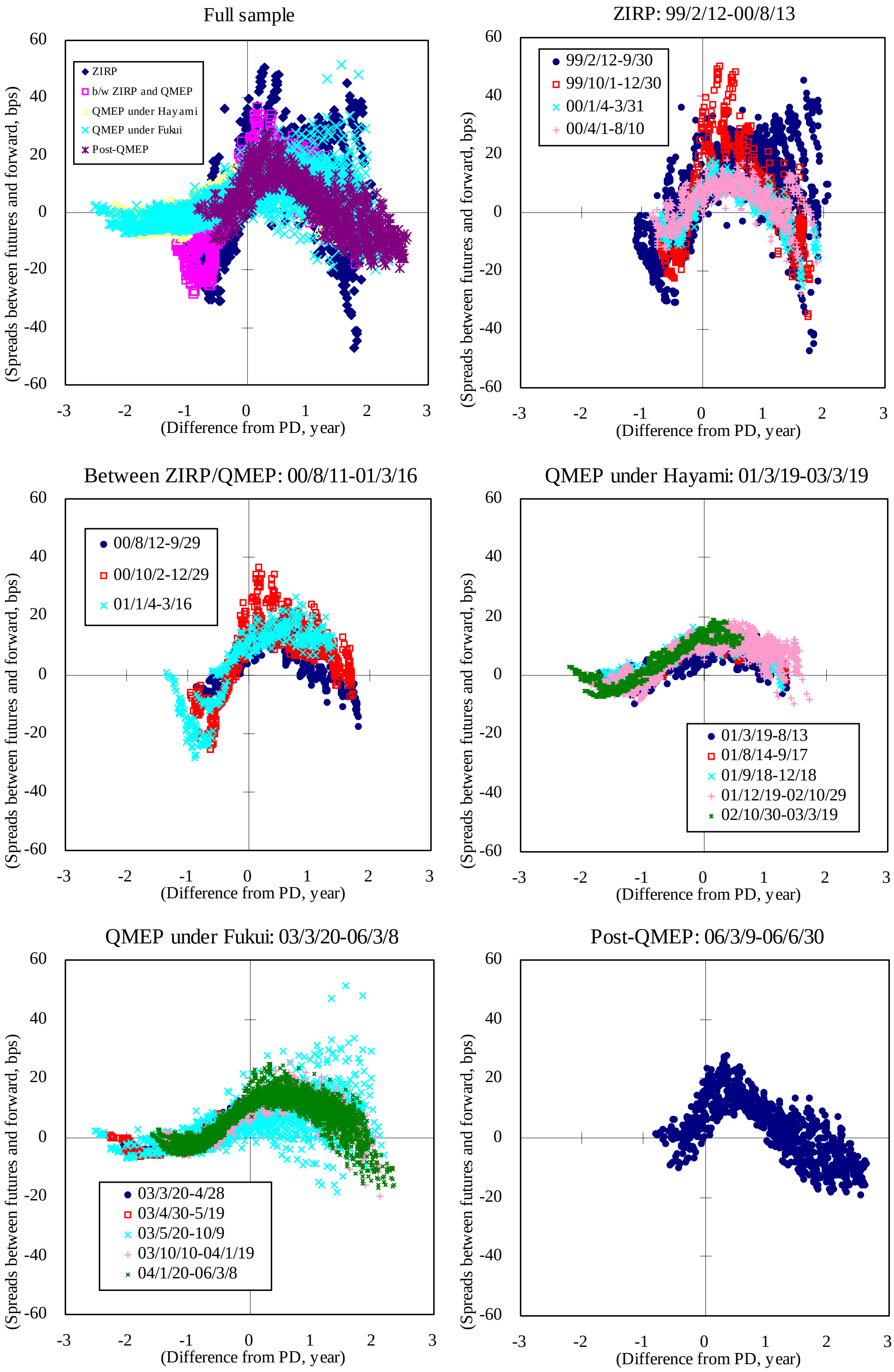


Figure 7: Indicators for spreads between forward and futures interest rates

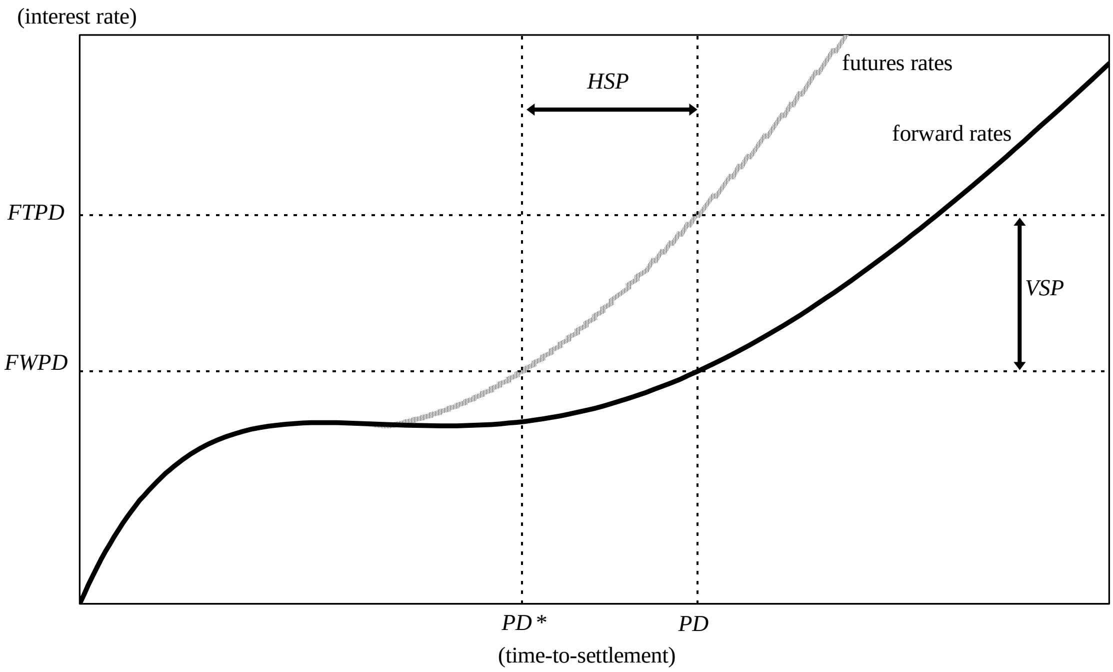


Figure 8: HSP and VSP around the beginning and end of QMEP

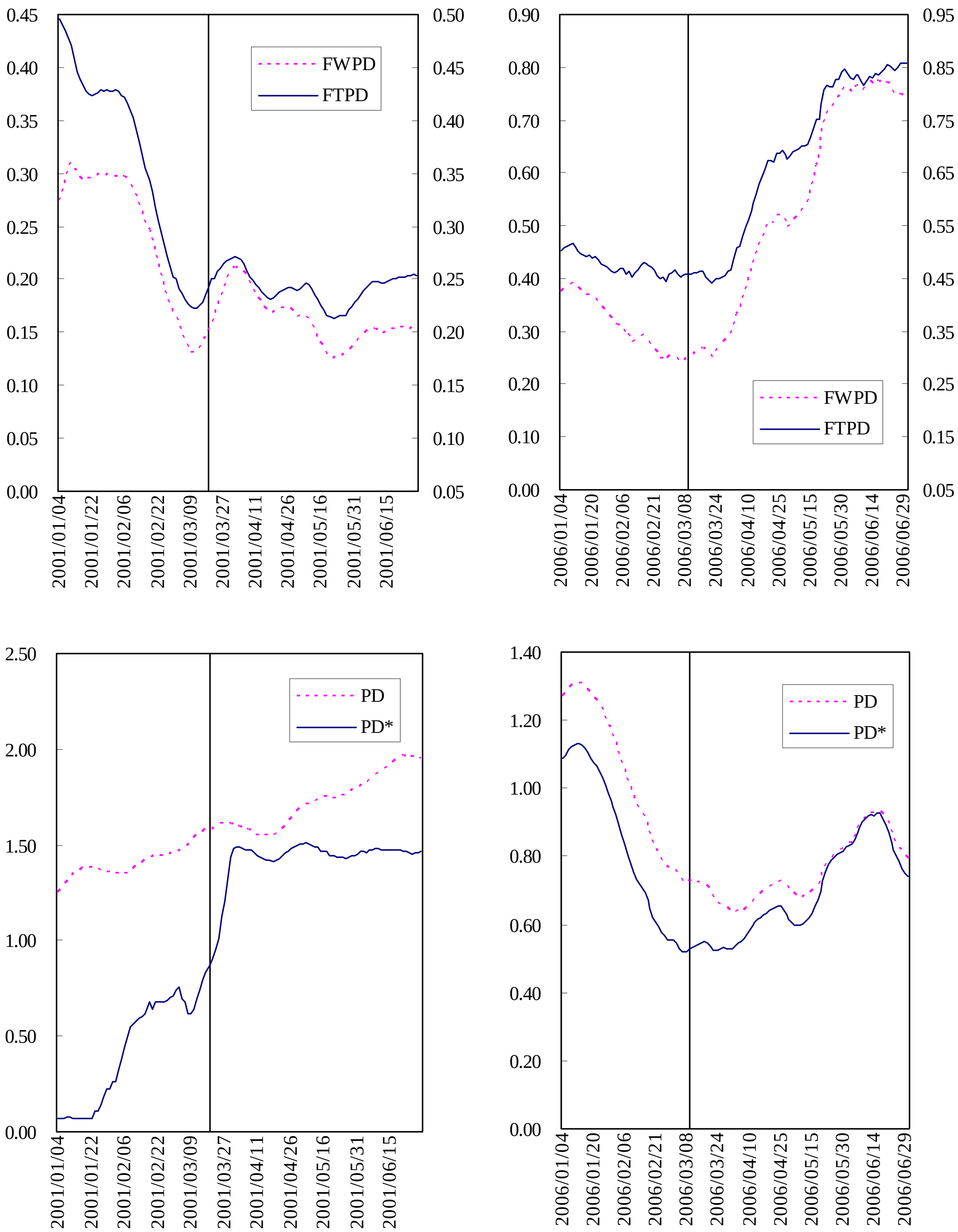
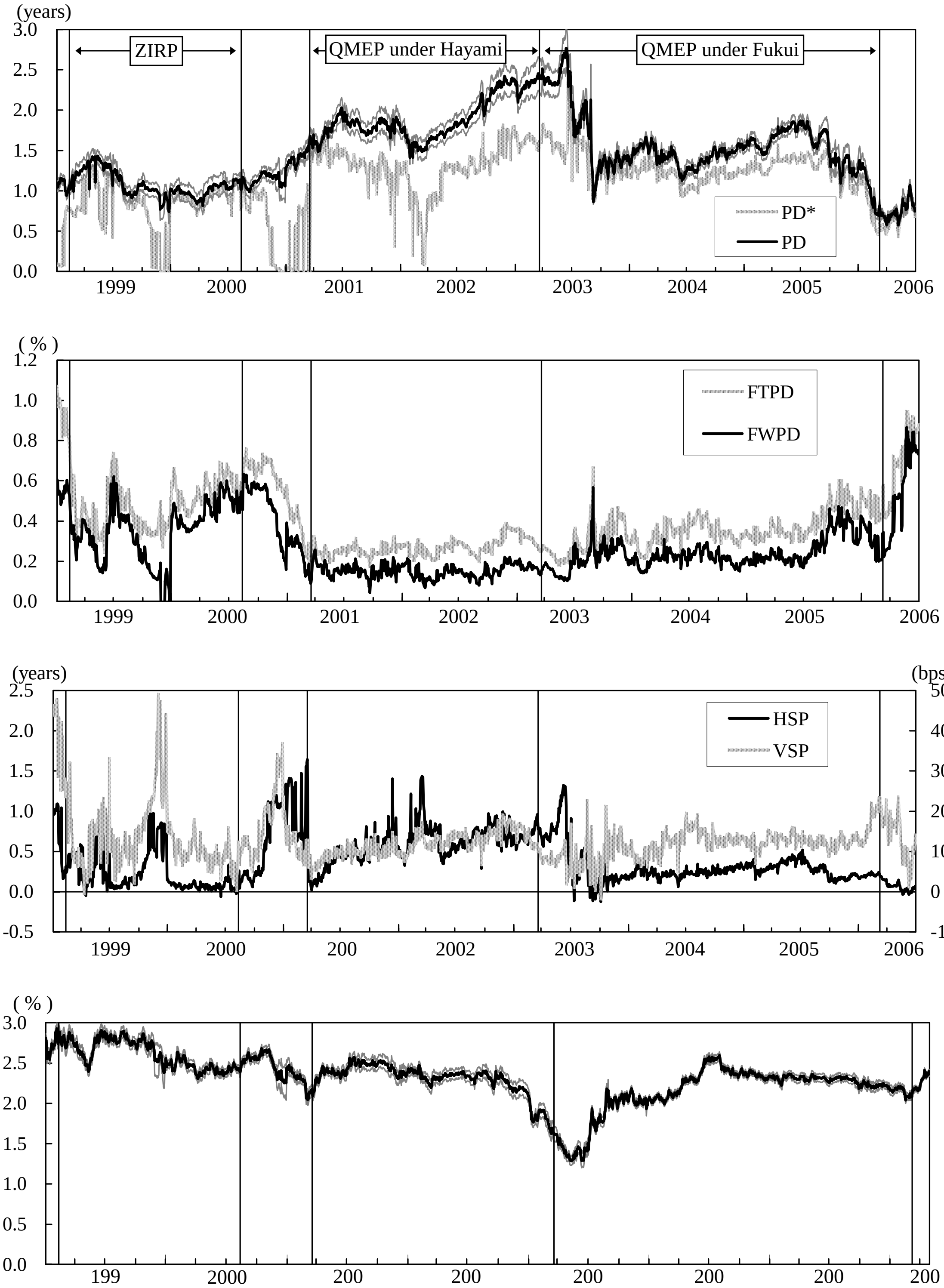
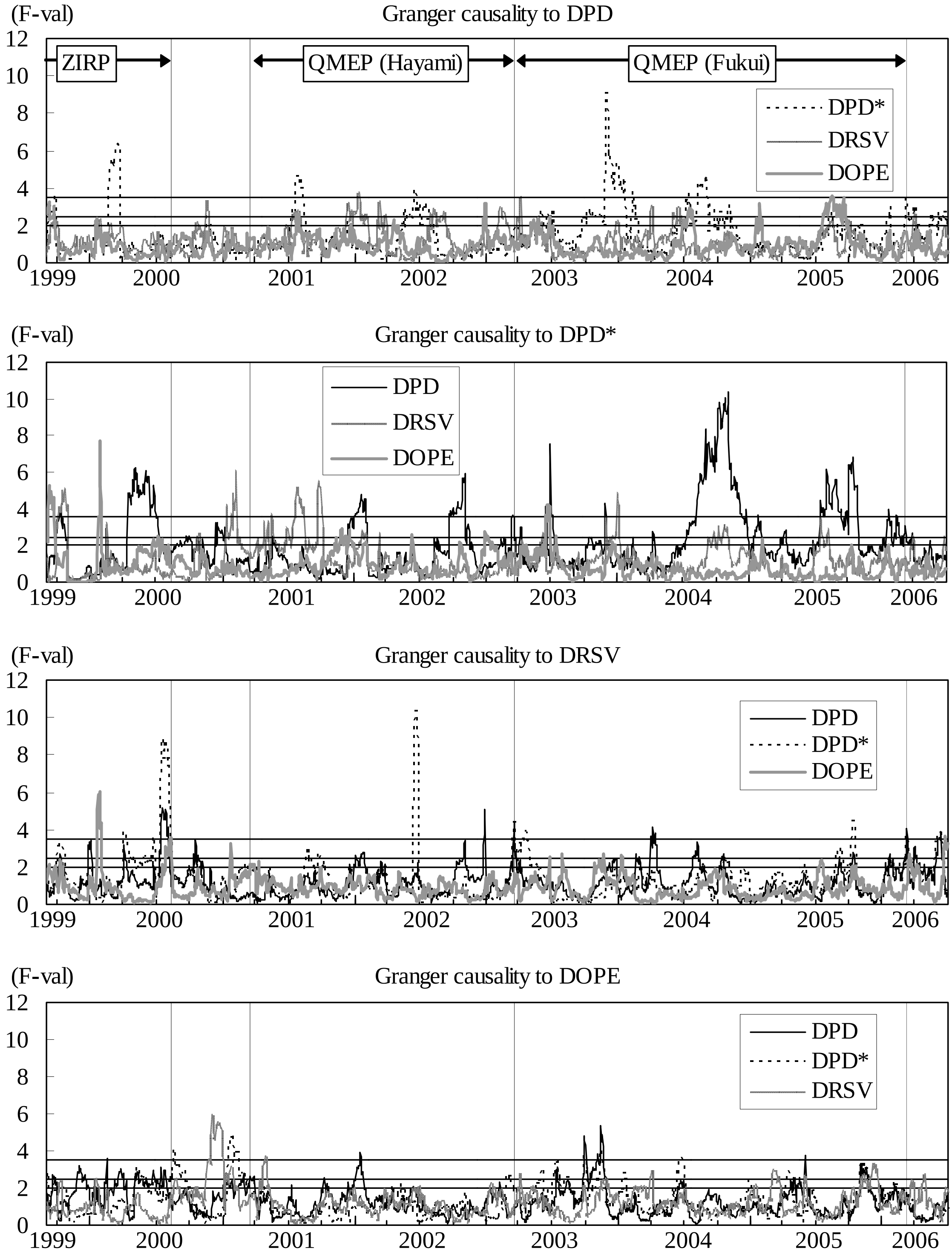


Figure 9: Policy duration indicators



Note: In the top and bottom panels, shaded lines indicate the upper and lower bounds of the confidence interval of the estimate (estimates ± 2*standard errors), respectively.

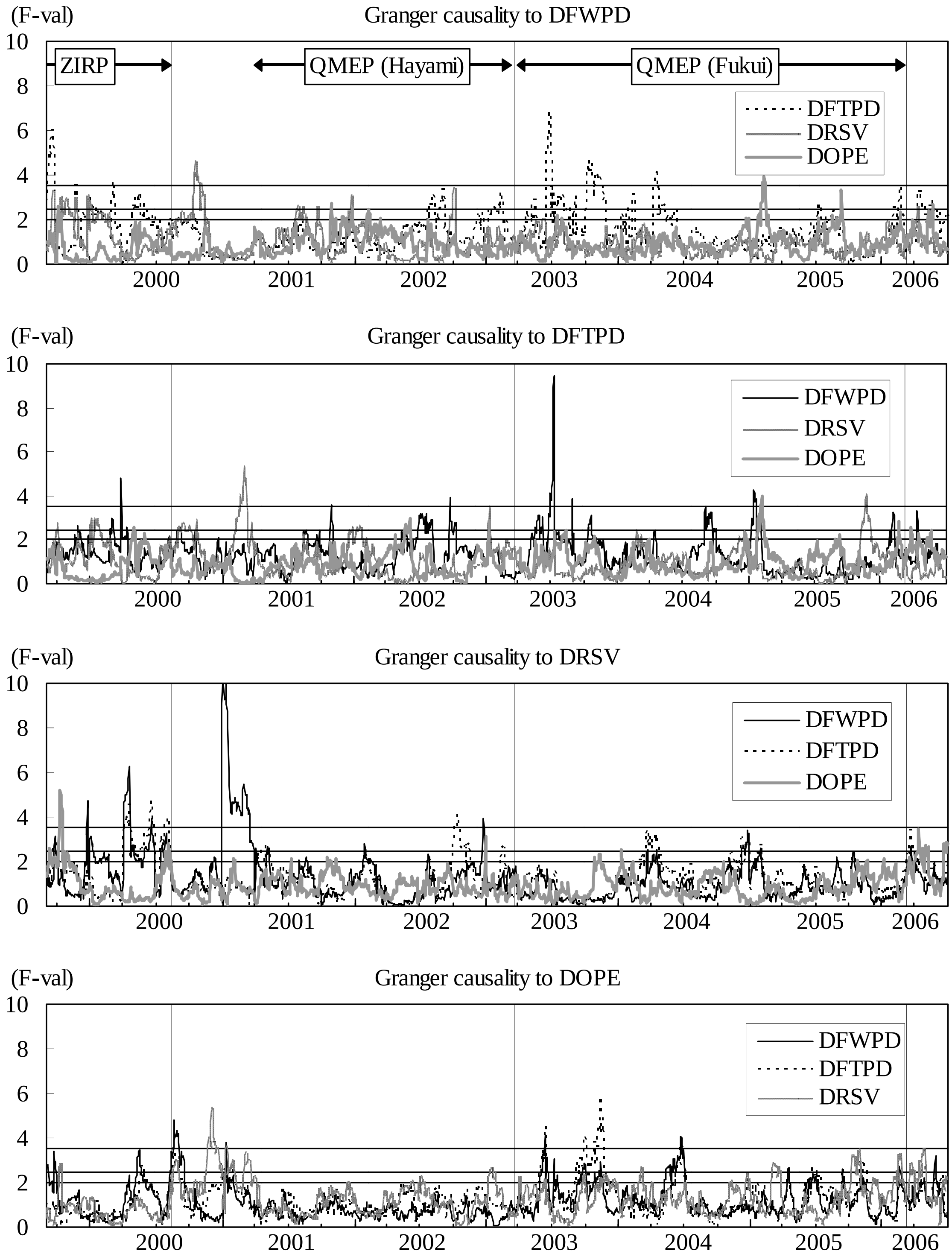
Figure 10: Granger causality tests: horizontal gaps



Notes: 1. Granger causality tests are conducted using subsamples of 60 business days ending at each date on the horizontal axis with 5-day lag length.

2. Horizontal lines shows F-values for 1-percent, 5-percent and 10-percent significance level from top to bottom.

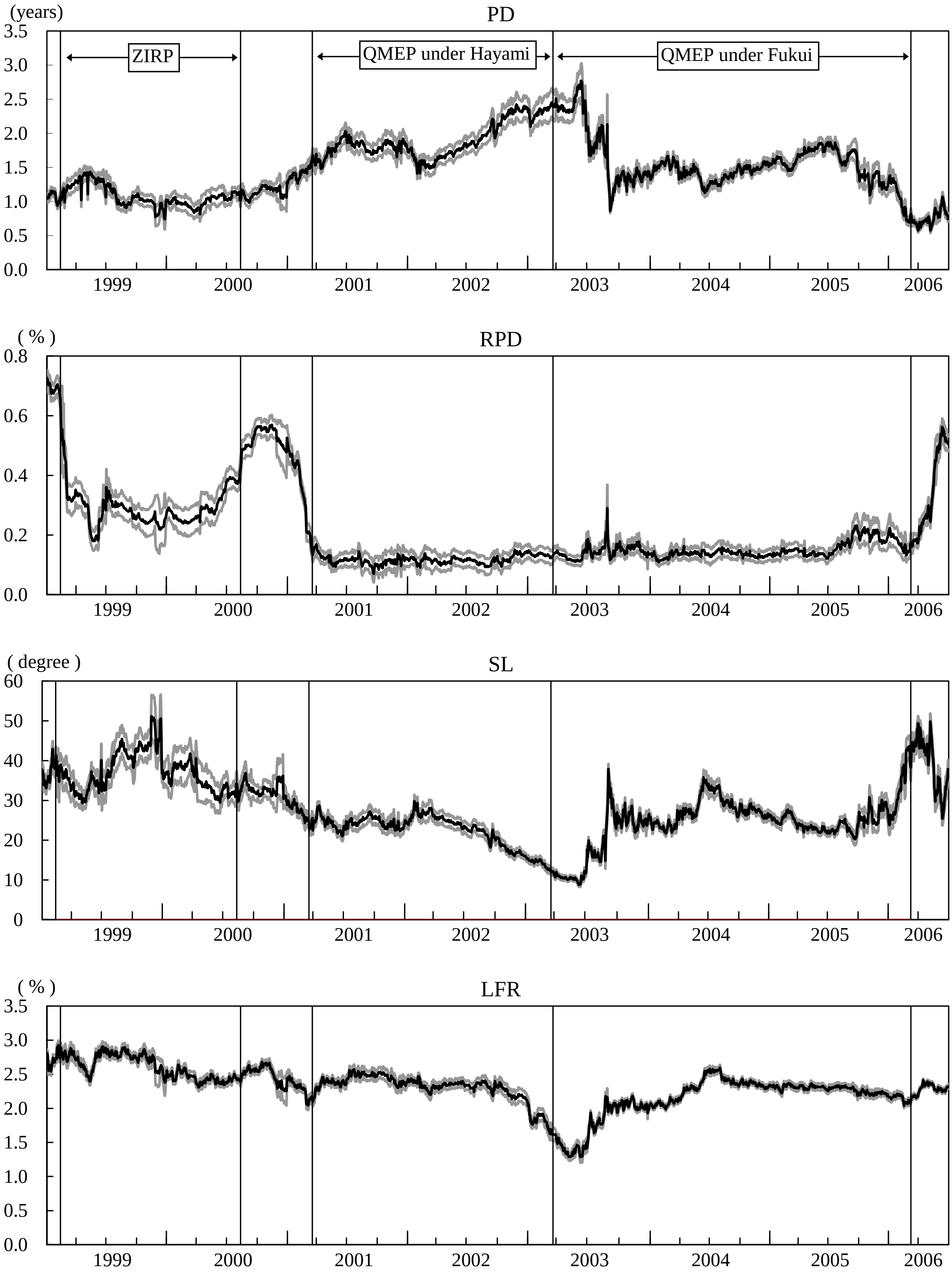
Figure 11: Granger causality tests: vertical gaps



Notes: 1. Granger causality tests are conducted using subsamples of 60 business days ending at each date on the horizontal axis with 5-day lag length.

2. Horizontal lines shows F-values for 1-percent, 5-percent and 10-percent significance level from top to bottom.

Figure A-1: Policy duration indicators



Note: Bold lines are estimates, and shaded lines indicate their upper and lower bounds, respectively, of the confidence interval (estimates ± 2*standard errors).