

Trade Openness and International Conflict

by

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Abstract

Unlike scholars of international relations and political science, economists have paid little attention to the possible connections between trade openness, inter-state disputes over resources and appropriative competition. This paper addresses this issue in the context of a neoclassical trade model in which two “small” countries that have ownership claims over a productive resource resolve their dispute non-violently in a military contest. We first clarify how the initial distribution of factor endowments and technology condition arming incentives, and how that determines the division of the contested resource and welfare under alternative trade regimes (“autarky” vs “free trade”). We then show that, under conditions of factor price equalization, free trade in goods may “level the playing field” in appropriative competition. Through its impact on product and then factor prices, trade brings about gains from exchange and specialization in production. However, trade may also intensify appropriative competition – so much so that the waste of resources in the contest may outweigh the traditional gains from trade leaving one or both countries worse off. The paper also explores the dependence of trade patterns on fundamentals and elaborates on the broader implications of trade policies for national security.

1. Introduction

In the decades that preceded World War I, the proportion of world trade to world GDP had reached unprecedented heights (O'Rourke and Williamson, 1999). Yet, international conflict ensued with much ferocity and despite expectations to the contrary.¹ More recently, the expansion of trade during the latest (and ongoing) wave of globalization has been spectacular. Still, insecurity and contention have not only subsided but are flaring in many parts of the world.² In Europe, however, members of the European Economic Community (now the European Union) have enjoyed diminished tensions and increased security as their internal trade expanded over time. Regardless of whether countries are involved in actual or potential conflict though, often they expend non-trivial portions of their national incomes on arming to safeguard their perceived interests. This raises several important questions. Does trade openness amplify or mitigate arming incentives? How? And through what channels? Are there circumstances under which free trade hurts the countries that embrace it? Can trade policy moderate arming incentives and contain opportunistic conduct?

This paper attempts to theoretically address selected aspects of the above issues. To be sure, philosophers, political scientists, and scholars of international relations have debated variants of these ideas for decades. Two prominent schools of thought stand out: the classical liberal school, which posits that freer trade alleviates conflict because of its costs – conflict preempts the possibility of realizing the classical gains from exchange and specialization in production (Polachek, 1980);³ and the realist/neorealist schools, which view relations between states pri-

¹The prediction before World War I, for example, that war was impossible or unthinkable – because Britain and Germany had become so economically interdependent that conflict would be “commercial suicide” (Angell, 1933) – was flatly contradicted by experience.

²From the Mediterranean and Africa to the Middle East, Caucasus, Central Asia, Kashmir, and the Spratly islands in the South China sea, international conflict continues to devastate societies with death, destruction, poverty, and despair.

³This view also emphasizes the possibility that free trade may create constituencies that favor peace by ex-

marily as adversarial with trade being an instrument that may in fact hurt one or all parties. Realists argue that the benefits from freer trade may fuel frictions and conflict because some states might perceive that they (or their rivals) will develop a military edge in security competition (Waltz, 1979; Grieco, 1988). Neorealists point out that freer trade between adversaries creates an adverse “security externality” that may in fact outweigh the gains from trade (Gowa and Mansfield, 1993; Gowa, 1994).

Unfortunately, the theoretical underpinnings of both schools are vague and weak (1) because the models they employ which purport to capture the nuanced relationship between trade openness and international conflict are ill-suited to address this task, and (2) because, until only recently, economists have not given these issues the attention they deserve.⁴ As a consequence, the extant literature has not managed to clearly and unambiguously establish the channels through which trade openness affects national arming incentives and conflict. It has also not explored how the distinctive characteristics of adversaries (e.g., technology, resources, consumer preferences, and national objectives) or the international environment within which they interact matter.

Generally, the costs and benefits of interstate conflict depend on the causes of conflict and the form it takes, its impact on product prices and national incomes, the institutional environment within which governments operate, the objectives of policy designers and the nature

panding global contacts and increasing commercial interests that oppose conflict (Schneider et al., 2003). Related perspectives also propound the notion that, by fostering understanding of cultures, mutually advantageous exchange discourages violence and diminishes the likelihood of overt conflict. The British policy of appeasement towards Germany during the 1930s was based on classical liberal arguments about the use of economic carrots to avoid war (Kagan, 1995).

⁴Recent research that has considered aspects of the relationship between security and trade include Anderson and Marcouiller (1997) and Anderton et al. (1999) who have analyzed Ricardian models in which traded commodities are insecure either because of the presence of pirates and bandits or because the two sides influence the terms of trade through arming. Both approaches emphasize the basic point that much international trade can be hampered by the anarchy that characterizes international relations. Models like that of Hirshleifer (1988), suitably adapted, could also make the same point. Skaperdas and Syropoulos (1996, 2001, 2002) have begun to address some of the implications of insecure property in simple models of exchange. Barbieri and Schneider (1999) have surveyed much of the theoretical and empirical literatures on the subject.

of their strategic interactions in international relations. A key aspect of international conflict (actual or potential) is that it diverts real resources away from productive tasks and thus reduces consumption opportunities. Of course, conflict may also benefit one or more parties by enabling them to appropriate a larger share of, say, a contested territory, or by preempting costly frictions in the future. Thus, to rationalize national arming incentives, formal models of interstate conflict ought to identify and then connect the costs and benefits associated with such conflict. In particular, to explore how trade openness affects arming, it is crucial to recognize that the introduction of trade affects not just real incomes but also factor prices and, through its impact on these prices, the opportunity costs and marginal benefits of arming. In other words, theoretical work is bound to be incomplete if it does capture the factor price effects of trade. For that we need general equilibrium models that are capable of capturing the price effects of trade together with the strategic interactions in security and possibly trade policies.⁵

In the spirit of neoclassical trade theory, this paper focuses on the prices of traded goods as the vehicle through which trade openness affects arming incentives and welfare. We build a canonical model of trade and non-violent conflict in which two “small” sovereign states dispute the control of a productive resource and sort out their ownership claims militarily. As in the Heckscher-Ohlin trade model, there are two intersectorally mobile (but internationally immobile) resources, and two tradable consumption goods. An important departure from the model is our inclusion of a third sector that produces arms (“guns” for short). Guns enable adversaries to enhance their power. We model the interaction between states as an appropriative contest between nationalistic but welfare-oriented governments over shares of the contested resource (land) and focus on the effects of two trade regimes: autarky (no trade) and global free trade.

⁵That trade and security policies should be related appears obvious. In fact, many embargoes, sanctions, and various other forms of trade restrictions that have been used throughout history and continue to be used today can be considered extensions of security policies (Hirschman, 1945).

We first aim to identify how these trade regimes condition countries' valuations of resources and, consequently, the interacting states' marginal benefits and marginal costs of arming. Our ultimate goal is to characterize non-cooperative interactions in security policies and then explore what these policies mean for national welfare when free trade is introduced.

After settling the issue of existence and uniqueness of (Nash) equilibrium in security policies under autarky, we identify endowment distributions which ensure that the contestants produce identical quantities of arms. We then show how technology and asymmetries in secure endowments affect relative investments in arms and the prices of consumption goods. This sets the stage for our analysis of free trade. We establish the existence of conditions under which the introduction of free trade in goods leads to the equalization of factor prices, which in turn ensures the equalization of marginal benefits and marginal costs of arming across countries and, ultimately, national arming incentives. An intriguing finding is that, under certain circumstances (which we make precise), free trade in consumption goods "level the playing field" in appropriative competition and induces adversaries to be equally powerful.

Turning to welfare, we find that, depending on prevailing product prices, the introduction of free trade may amplify both countries' incentives to arm. Our analysis reveals that the waste of resources that may accompany this effect may be so strong that it may overshadow the traditional gains from free trade, thereby causing free trade to become a Pareto-inferior regime. In other words, we formally establish the sense in which (and some of the circumstances under which) autarky may dominate free trade.

In this setting, activist commercial and/or industrial policies can have important effects on appropriative competition and welfare even when countries are unable to affect world prices. Our analysis reveals that this possibility arises only when governments can commit to use these

policies before they design their security policies. As in perfectly competitive environments, and for any given arms, the aforementioned policies create deadweight losses. However, with competition in security policies, these policies can have important effects on the strategic incentives of adversaries to build guns and, ultimately, on welfare. We explore these possibilities and demonstrate how trade policy may turn into “power policy” through its impact on appropriative competition.

Lastly, our analysis discusses the possible dependence of trade patterns on inter-country differences in secure relative endowments and technology. In contrast to the Heckscher-Ohlin theorem – which links autarky prices to relative endowments and trade patterns – we find that, in the presence of insecure property, neither autarky prices nor intercountry differences in relative endowments are necessarily accurate predictors of trade patterns.

In the next section, we present the formal model and some preliminary analysis that proves useful in subsequent sections. Then, in Section 3, we investigate optimal security policies and conflict under autarky and free trade. In Section 4, we contrast and compare these regimes in terms of their implications for arming and welfare. In Section 5, we briefly examine the effects of trade policies. Lastly, in Section 6, we offer several concluding comments. Most proofs have been placed in the Appendix.

2. Model and Preliminary Analysis

We consider a world that consists of two countries, labeled 1 and 2, and the rest of the world (ROW) which for simplicity we treat as a single entity. Every country may produce two consumption goods, also labeled 1 and 2, using two resources, labor and land. The production function for each good is concave and linear homogeneous. For concreteness, we suppose good 1 (2) is labor (land) intensive but, when appropriate, we indicate how the analysis depends on the

ranking on factor intensities. In the spirit of the Heckscher-Ohlin trade model, we rule out factor intensity reversals and allow all countries to have access to the same technology. Moreover, we suppose consumers have identical and homothetic preferences, and goods 1 and 2 are essential in consumption. All markets are perfectly competitive.

Each country $i = 1, 2$ possesses L^i units of *secure* labor (or human capital) and K^i units of *secure* land. (Land could be interpreted as a natural resource, e.g., oil or water, or just physical capital.) In addition, there are K_0 units of disputed land which the two countries contest militarily. Importantly, the international environment we consider is anarchic, so binding contracts on the proliferation of arms and the partition of K_0 are impossible. We abstract from special interest politics and focus on benevolent policymakers who aim to maximize their respective national welfare levels.

Denote with G^i country i 's military strength. For brevity, we shall refer to G^i as “guns” or “arms” but perhaps it is more useful to think of this variable as a producible composite commodity that encapsulates country i 's military power which manpower and weaponry. Let ϕ^i be country i 's share of K_0 , and identify it with the contest success function (CSF)

$$\phi^i(G^i, G^j) = \begin{cases} \frac{f(G^i)}{f(G^i)+f(G^j)} & \text{if } G^i + G^j > 0 \text{ for } i \neq j = 1, 2 \\ \frac{1}{2} & \text{if } G^1 = G^2 = 0 \end{cases} \quad (1)$$

We suppose $f(\cdot) \geq 0$ with $f(0) = 0$, $f'(\cdot) > 0$, $\lim_{G^i \rightarrow 0} f'(G^i) = \infty$, and $f''(\cdot) \leq 0$. It follows that a country's share of the contested resource is increasing in its own guns (i.e., $\phi_{G^i}^i = \partial \phi^i / \partial G^i > 0$) and decreasing in the guns of its adversary ($\phi_{G^j}^i = \partial \phi^i / \partial G^j < 0$ for $i \neq j$).⁶ This dependence

⁶As we will see, $\lim_{G^i \rightarrow 0} f'(G^i) = \infty$ implies that a country's marginal benefit of investing in arms is infinitely large when $G^i \rightarrow 0$. This assumption could be relaxed without much loss of generality. In the Appendix we describe several additional properties of ϕ^i that prove useful in our analysis. A functional form for $f(\cdot)$ that has been widely used in the rent-seeking literature, as well as the literatures on tournaments and conflict, is the “Tullock form”, $f(G^i) = (G^i)^\eta$ where $\eta \in (0, 1]$ (Tullock, 1980). See Hirshleifer (1989) for a comparison of the

of ϕ^i on guns can be taken literally or could be interpreted as the reduced form of a bargained outcome in which relative arming figures prominently in the division of the contested resource.

Generally, a country may build its own military constellation but it may also import some weapons and hire mercenaries and foreign security experts. Here we focus on the benchmark case in which arms are domestically produced and, for reasons that will become clear later, view trade in consumption goods as a substitute for trade in arms. The important point is that there is both a marginal benefit to producing arms and an opportunity cost. By producing more guns, a country can raise its income through the appropriation of a larger share of the contested resource. However, a larger production of guns also requires a country to divert some of its resources away from income generating activities which diminishes its consumption possibilities.

To give the model a neoclassical texture, we suppose arms require the employment of land and labor under a convex technology that exhibits constant returns to scale. Denote with w^i and r^i the competitive wage and rental rates paid to workers and landowners, respectively. Now let $\psi^i \equiv \psi(w^i, r^i)$ and $c_j^i \equiv c_j(w^i, r^i)$ respectively represent the unit cost function of guns and good $j = 1, 2$. Under our assumptions on technologies, these functions are strictly concave and linear homogeneous in factor prices. By Shephard's lemma, $\psi_w^i \equiv \partial\psi^i/\partial w^i$ and $\psi_r^i \equiv \partial\psi^i/\partial r^i$ capture the unit labor and land requirements in arms, respectively. We can similarly define the unit land and labor requirements in good j as $a_{Kj}^i \equiv \partial c_j^i/\partial r^i$ and $a_{Lj}^i \equiv \partial c_j^i/\partial w^i$. It follows that the land/labor ratio in the arms sector is $k_G^i \equiv \psi_r^i/\psi_w^i$ and the corresponding ratio in industry j is $k_j^i \equiv a_{Kj}^i/a_{Lj}^i$. Industry 2 is land (labor) intensive in the absence of factor intensity reversals if $k_2^i > k_1^i$ ($k_2^i < k_1^i$) at all factor prices.

Now let good 1 be the numeraire and denote with p^i the relative price of good 2 in country i .

properties of this and another prominent functional form.

In the absence of specialization in production, perfect competition in product markets requires

$$c_1(w^i, r^i) = a_{L1}^i w^i + a_{K1}^i r^i = 1 \quad (2)$$

$$c_2(w^i, r^i) = a_{L2}^i w^i + a_{K2}^i r^i = p^i \quad (3)$$

Furthermore, diversification in production and factor market clearing require

$$a_{K1}^i X_1^i + a_{K2}^i X_2^i = K_X^i \quad (\equiv K^i + \phi^i K_0 - \psi_r^i G^i) \quad (4)$$

$$a_{L1}^i X_1^i + a_{L2}^i X_2^i = L_X^i \quad (\equiv L^i - \psi_w^i G^i) \quad (5)$$

where X_j^i is the output of good j , and K_X^i and L_X^i are the quantities of land and labor that are available for the production of consumption goods in country i .

The cost of producing arms in each country is financed with non-distortionary income taxes. This fact together with the existence of perfect competition imply that net national income, R^i , coincides with the (maximized) value of the country's domestic production of consumption goods, so $R^i = w^i L_X^i + r^i K_X^i = X_1^i + p^i X_2^i$. From equations (2)-(5), it can be verified that R^i is a function of the relative price, p^i , security policies, G^1 and G^2 , and factor supplies.⁷ From (4) and (5) it can also be confirmed that

$$k_X^i \equiv \frac{K^i + \phi^i K_0 - \psi_r^i G^i}{L^i - \psi_w^i G^i}. \quad (6)$$

Lemmas 1 and 2 use the above ideas to prepare the ground for the analysis that follows.

⁷ R^i should be identified with the familiar gross domestic product (*GDP*) or revenue function (Dixit and Norman, 1980), excluding arms expenditures. R^i is increasing and convex in p^i , and increasing and concave in factor inputs K_X^i and L_X^i . It can be verified that $X_2^i = R_p^i$ ($\equiv \partial R^i / \partial p^i$) and $\partial X_2^i / \partial p^i = R_{pp}^i \geq 0$. Furthermore, factor prices satisfy $w^i = R_L^i$ ($\equiv \partial R^i / \partial L_X^i$) and $r^i = R_K^i$ ($\equiv \partial R^i / \partial K_X^i$).

Lemma 1. *Suppose $k_2^i > k_1^i$ and $X_j^i > 0$ for every $j = 1, 2$. Then*

- a) $\frac{p^i w_p^i}{w^i} < 0$, $\frac{p^i r_p^i}{r^i} > 1$, and $\frac{p^i \omega_p^i}{\omega^i} < -1$ where $\omega^i \equiv w^i/r^i$;
- b) $\frac{\partial X_1^i/\partial L_X^i}{X_1^i/L_X^i} > 1$, $\frac{\partial X_2^i/\partial L_X^i}{X_2^i/L_X^i} < 0$, $\frac{\partial X_1^i/\partial K_X^i}{X_1^i/K_X^i} < 0$, and $\frac{\partial X_2^i/\partial K_X^i}{X_2^i/K_X^i} > 1$;
- c) $\frac{\partial(X_2^i/X_1^i)/\partial k_X^i}{(X_2^i/X_1^i)/k_X^i} > 1$;
- d) $\frac{\partial(X_2^i/X_1^i)/\partial p^i}{(X_2^i/X_1^i)/p^i} > 0$.

Part (a) of Lemma 1 describes the dependence of factor prices on product prices (Stolper-Samuelson (1941) theorem). In words, an increase in the relative price of good 2 raises the real rewards paid to owners of the resource employed intensively in this industry (i.e., landowners) and reduces the real rewards paid to owners of the other factor (i.e., labor). Part (b) unveils the dependence of outputs on the available factor supplies (Rybczynski (1955) theorem). Part (c) is a variant of the Rybczynski theorem – it clarifies that an increase in the relative supply of land, k_X^i , causes the relative supply of the land-intensive good 2 to rise more than proportionately. Part (d) is a reflection of the fact that the opportunity cost of good 2 is increasing. (It should be fairly obvious to infer how parts (a)-(d) would change if $k_2^i < k_1^i$ instead.)

In Lemma 1, k_X^i was treated as exogenous. Inspection of (6) reveals, however, that k_X^i is increasing in K^i and K_0 and decreasing in L^i . Lemma 2 below unveils the dependence of k_X^i on price and guns.

Lemma 2. *Define $k^i \equiv \frac{K^i + \phi^i K_0}{L^i}$ and suppose $X_j^i > 0$ for every $j = 1, 2$. Then*

- a) $\frac{\partial k_X^i}{\partial p^i} \geq 0$ as $k_2^i \geq k_1^i$;
- b) $\frac{\partial k_X^i}{\partial G^i} > 0$ for all G^i that satisfy $K_0 \phi_{G^i}^i - \psi^i/r^i \geq 0$;
- c) $\frac{\partial k_X^i}{\partial G^j} < 0$ for $i \neq j = 1, 2$;

- d) $\frac{\partial k_X^i}{\partial G^i} + \frac{\partial k_X^i}{\partial G^j} \geq 0$ as $k^i \geq k_G^i$ whenever $G^i = G^j$ for $i \neq j = 1, 2$;
- e) $\frac{\partial k_X^i}{\partial K_0} > 0$, $\frac{\partial k_X^i}{\partial K^i} > 0$, and $\frac{\partial k_X^i}{\partial L^i} < 0$.

Suppose $k_2^i > k_1^i$. Part (a) of Lemma 2 establishes that, when both consumption goods are produced, the available land/labor ratio, k_X^i , is increasing in the relative price, p^i . By Lemma 1(a), an increase in p^i forces the wage/rental ratio, ω^i , to fall thereby reducing the demand for land in the arms sector and raising the amount left (i.e., K_X^i) for the production of consumption goods. At the same time, the fall in ω^i boosts demand for labor in the arms sector thus causing the quantity left (i.e., L_X^i) for the production of consumption goods to rise.

From (6), it can be seen that k_X^i is ambiguously affected by an increase in country i 's arms. Part (b) points out that k_X^i will rise unambiguously if an increase in the country's guns does not reduce its net national income (see below).⁸ Part (c) clarifies that an arms increase by country i 's adversary forces k_X^i to fall – this is so because the quantity of country i 's appropriated land is reduced. Part (d) notes that, under conditions of symmetry on guns, a simultaneous increase in both countries' arms will raise or reduce k_X^i depending on how the country's overall land/labor ratio, k^i , compares with the land/labor ratio, k_G^i . Part (e) is obvious. Lemmas 1 and 2 will help us characterize conflict in the presence and absence of trade.

Turning to preferences, for simplicity, we suppose consumers are risk neutral. Thus, country i 's indirect utility (aggregate welfare) function may be written as

$$V^i \equiv V^i(p^i, G^i, G^j) = v(p^i)R^i(p^i, K^i + \phi^i K_0 - \psi_r^i G^i, L^i - \psi_w^i G^i) \quad (7)$$

where $v^i \equiv v(p^i)$ is the marginal utility of income. From Roy's identity, country i 's Marshallian demand function for good 2 is $D_2^i = -\frac{v_p(p^i)}{v(p^i)}R^i$ or, equivalently, $D_2^i = \alpha_D^i R^i/p^i$ where $\alpha_D^i =$

⁸The proof in the Appendix clarifies that $\partial k_X^i/\partial G^i \geq 0$ for more G^i s than the ones defined in part (b).

$\alpha_D(p^i) = -\frac{p^i v_p^i(p^i)}{v^i(p^i)}$ is the expenditure share on good 2. Total differentiation of (7) and utilization of these ideas yields the welfare decomposition

$$dV^i = v(p^i) \left[-M_2^i dp^i + r^i (K_0 \phi_{G^i}^i - \psi^i / r^i) dG^i + r^i K_0 \phi_{G^j}^i dG^j \right] \quad \text{for } j \neq i \quad (8)$$

where $M_2^i \equiv D_2^i - X_2^i$ is the excess demand function for good 2. The first term inside the square brackets captures the effect of a price change on country i 's net income. If country i is an importer of good 2 (i.e., $M_2^i > 0$), an increase in p^i will reduce its welfare because imports become locally more expensive. Exactly the opposite is true if country i is an exporter of good 2 (i.e., $M_2^i < 0$). Obviously, if country i does not engage in trade (i.e., $M_2^i = 0$) the welfare effect of a price change vanishes. The second term inside the brackets captures the net income effect of a change in country i 's guns. *Ceteris paribus*, an increase in G^i is welfare-improving because it expands the country's share of the contested land (the first term in the parentheses) and therefore its national income. At the same time, however, the G^i increase reduces the country's welfare because it absorbs resources away from the production of consumption goods which reduces its national income (the second term in the parentheses). *Ceteris paribus*, country i will be able to raise its income, and thus improve its welfare, through an increase in the production of arms as long as the expression in the parentheses is positive. The third term in the brackets captures the income effect of an arms change by country i 's adversary. For an increase in G^j this effect is negative because country i 's share of the contested resource is reduced. On the other hand, if $G^i = G^j$ and both quantities are increased proportionately, welfare in both countries will necessarily fall because there is no benefit to raising guns – only costs.

We will model bilateral conflict as a contest over K_0 in which the adversaries use their security policies (national arming) for nationalist reasons. More specifically, we consider an

anarchic environment in which the possibility of signing binding arms treaties is ruled out and conflictual equilibria are identified with noncooperative (Nash) equilibria in security policies. A distinguishing feature of this problem is that, at an interior solution, country i 's first-order condition (FOC) for welfare maximization satisfies

$$V_{G^i}^i \equiv \frac{\partial V^i}{\partial G^i} = v^i r^i (K_0 \phi_{G^i}^i - \psi^i / r^i) = 0 \quad (9)$$

no matter whether the trade regime we consider is *autarky* or *free trade*. This is so for the following reasons. Under autarky, price p^i adjusts until product markets clear or, more precisely, until $M_2^i = 0$. This causes the first term in the brackets in (8) to vanish, thus leaving (9) as the relevant optimality condition. As mentioned earlier, in the case of free trade, the focus is going to be on adversaries that are “small” (i.e., price takers) in world markets. For this reason, the first term in (8) will vanish once again yielding (9) as country i 's optimality condition. Notice that in both cases welfare maximization coincides with *GDP* maximization. Also notice this importance difference between trade regimes: under autarky p^i is endogenously determined; under free trade p^i will be exogenous.⁹

From (9) it can be deduced that a country's marginal benefit function, $MB^i = K_0 \phi_{G^i}^i$, measured in land units, is independent of price and the trade regime considered. However, its marginal cost function, $MC^i = \psi^i / r^i$, differs across trade regimes because of their differing implications for the determination of product prices. Lemmas 3 and 4 unveil several properties of the indirect utility V^i which help explain the shapes of MB^i and MC^i and some of their consequences.

⁹Our discussion here and later in the paper is based on the assumptions that, under free trade, the world price and/or factor endowments are such that production is diversified. Similarly, under autarky, the distribution of secure factor endowments between the adversaries is such that their production of arms is not constrained by their initial land holdings

Lemma 3. *A country's indirect utility function, V^i , has the following properties:*

- a) $V_{G^i G^i}^i < 0$;
- b) $V_{G^i G^j}^i \geq 0$ as $G^i \geq G^j$;
- c) $V_{G^i p^i}^i \geq 0$ as $k_2^i \geq k_1^i$ at the G^i that solves $V_{G^i}^i = 0$;
- d) V^i is strictly quasi-convex in p^i and is minimized at the p^i that solves $M_2^i = 0$.

Part (a) of Lemma 3 establishes that V^i is strictly concave in country i 's guns, thereby explaining the downward sloping shape of MB^i , portrayed as the solid-line curve in Fig. 1.¹⁰ Part (b) clarifies the idea that a country's marginal payoff of investing in arms rises or falls with its rival's arms (i.e., MB^i shifts up or down) depending on which of the two countries supports a larger army to start with. As shown in the Appendix, both parts (a) and (b) are direct consequences of the properties of the CSF in (1).

Part (c) captures the idea that at the optimum the net marginal welfare effect of country i 's arming rises or falls with p^i depending on the ranking of factor intensities between industries 1 and 2. This effect is a consequence of the Stolper-Samuelson theorem and is driven by the impact of a price change on the country's opportunity cost, MC^i . The linear homogeneity of ψ^i in factor prices implies $\psi^i/r^i = \psi(\omega^i, 1)$, which is increasing in the wage/rental ratio, ω^i . But, by Lemma 1(a), ω^i is decreasing or increasing in p^i depending on whether $k_2^i > k_1^i$ or $k_2^i < k_1^i$. Since, under global free trade, MC^i is pinned down by the world price – and is, therefore, invariant to changes in G^i – it must be the case that MC^i is a constant function under this regime (the dotted-line curve in Fig. 1). In the next section, we show that MC^i is increasing in G^i under autarky (the dashed-line curve in Fig. 1).¹¹

¹⁰From the definition of MB^i and the properties of the CSF it can also be affirmed that $\lim_{G^i \rightarrow 0} MB^i = \infty$.

¹¹Strictly speaking, $\partial MC^i / \partial G^i > 0$ for values of G^i in the neighborhood of its intersection with MB^i .

Part (d) is a well-known property of indirect utility functions (Dixit and Norman, 1980) that highlights the notion that, for given guns, a country is not worse off when product prices in the world market differ from autarky prices.

Now consider the optimal security policies implied by the FOCs in (9). Lemma 4 below explains how price differences across countries are related to possible differences in security policies, and conversely.

Lemma 4. *For each country i , optimal investments in arms and relative prices are related as follows:*

- a) *If $k_2^i > k_1^i$ then $p^1 \gtrless p^2 \Leftrightarrow G^1 \gtrless G^2$;*
- b) *If $k_2^i < k_1^i$ then $p^1 \gtrless p^2 \Leftrightarrow G^1 \lesseqgtr G^2$.*

Proof: The properties of the CSF (see (A.1) and (A.2) in the Appendix) and the fact that $\psi^i/r^i = \psi(\omega^i, 1)$ can be used in (9) to obtain

$$\frac{MB^1}{MB^2} = \frac{f'(G^1)/f(G^1)}{f'(G^2)/f(G^2)} = \frac{\psi(\omega(p^1), 1)}{\psi(\omega(p^2), 1)} = \frac{MC^1}{MC^2}.$$

Focusing on part (a), suppose $k_2^i > k_1^i$ and, for concreteness, assume that $p^1 \geq p^2$. By Lemma 1(a), $\psi^i/r^i = \psi(\omega(p^i), 1)$ is decreasing in p^i ; therefore, $\psi^1/r^1 \leq \psi^2/r^2$ which implies $MC^1/MC^2 \leq 1$ and requires $MB^1/MB^2 \leq 1$. The concavity of $f(\cdot)$ implies that the latter inequality will be satisfied if $G^1 \geq G^2$. We can prove the *only if* portion of part (a) using similar reasoning. Part (b) can be established following a similar procedure. $\quad ||$

3. Trade Regimes and Security Policies

In this section, we explore the implications of autarky and free trade for arming and welfare. Under either trade regime, the sequencing of decisions is going to be as follows. The adversaries

first determine their (irreversible) investments in arms in anticipation of the fact that private agents everywhere will complete their production and consumption plans. When arming decisions are made, the contested resource is divided in proportion to relative power, and then markets clear under the conditions dictated by the prevailing trade regime.

Henceforth, we use subscripts A and F to associate variables with *autarky* and *free trade*, respectively. We also use a star “*” to identify the values of variables under conflict.

3.1 Autarky

The equations in (2)-(5) define w^i , r^i , X_1^i and X_2^i as functions of prices and guns. Utilization of these relationships in (9) makes it possible to write guns as functions of product prices; thus, to close the model we need two additional equations that help determine product prices. By Walras’ law, requiring prices to clear the domestic markets for good 2 (i.e., $M_2^i = 0$ for $i = 1, 2$) accomplishes this task. An equivalent and analytically more convenient procedure is to search for the prices that ensure $RD^i \equiv D_2^i/D_1^i = X_2^i/X_1^i \equiv RS^i$.

Homotheticity of preferences implies RD^i is uniquely determined by p^i .¹² By parts (c) and (d) of Lemma 1, RS^i is a function of p^i and k_X^i ; and, furthermore, by Lemma 2, k_X^i depends on p^i , G^i , G^j , and factor endowments. In short, we can view a country’s autarky price, p_A^i , as the solution to

$$RD(p^i) = RS(p^i, k_X^i(p^i, G^i, G^j)). \quad (10)$$

We may now state

Lemma 5. *A country’s autarky price, p_A^i , depends on the land/labor supply, k_X^i , that is*

¹²Since $D_2^i = \alpha_D^i R^i / p^i$, where α_D^i is the expenditure share on good 2, we will have $D_1^i = (1 - \alpha_D^i) R^i$; therefore, $RD^i = \frac{1}{p^i} \frac{\alpha_D(p^i)}{1 - \alpha_D(p^i)}$ is a function of p^i (but not of income R^i).

available for the production of consumption goods as follows: $\frac{\partial p_A^i}{\partial k_X^i} \leq 0$ as $k_2^i \geq k_1^i$. Furthermore,

- a) $\frac{\partial p_A^i}{\partial G^i} \leq 0$ as $k_2^i \geq k_1^i$ for all G^i that satisfy $V_{G^i}^i \geq 0$;
- b) $\frac{\partial p_A^i}{\partial G^j} \geq 0$ as $k_2^i \geq k_1^i$ for $i \neq j = 1, 2$;
- c) $\frac{\partial p_A^i}{\partial G^i} + \frac{\partial p_A^i}{\partial G^j} \leq 0$ as $k_2^i \geq k_1^i$ and $k^i \leq k_G^i$ whenever $G^i = G^j$ for $i \neq j = 1, 2$;
- d) $\frac{\partial p_A^i}{\partial K_0} \leq 0$, $\frac{\partial p_A^i}{\partial K^i} \leq 0$ and $\frac{\partial p_A^i}{\partial L^i} \geq 0$ as $k_2^i \geq k_1^i$.

By Lemma 1(c), an increase in k_X^i raises (reduces) RS^i when $k_2^i > k_1^i$ ($k_2^i < k_1^i$). Market stability then requires the equilibrium price, p_A^i , to fall (rise) as stated in Lemma 5. Parts (a)-(d) of Lemma 5 now follow readily parts (b)-(e) of Lemma 2.¹³

We may now begin our formal analysis of equilibrium. We first prove that a pure-strategy equilibrium in security policies exists and is unique. Then, in the remainder of this section, we investigate the nature of this equilibrium.

Theorem 1. (*Existence and Uniqueness of Nash Equilibrium*) *An interior Nash equilibrium in pure strategies (security policies) exists. Moreover, if arms inputs are sufficiently close substitutes for each other or if the technology for arms is sufficiently labor-intensive, the equilibrium is unique.*

As pointed out in the proof of Theorem 1, uniqueness of equilibrium can be assured under fairly general circumstances. The conditions mentioned above are only sufficient. For more insight on the nature of this equilibrium let us now refer to Fig. 2. The solid-line curves labeled B_A^1 and B_A^2 depict the two countries' best-response functions under autarky.¹⁴ (Ignore the other curves for now.) The following points should be underscored here. *First*, country i 's instrument

¹³As noted earlier in connection with Lemma 2(b), part (a) of Lemma 5 is valid for a larger set of G^i values, but this is not essential for our argument. This relationship, together with the dependence of ψ^i/r^i on p^i (described in the proof to Lemma 3(c)), establish that MC^i is increasing in G^i as claimed earlier.

¹⁴In the proof of Theorem 1, we explore the shapes of these functions in some detail. Country i 's best-response function $B_A^i(G^j)$ is the (unique) solution to (9) with $p^i = p_A^i$, captured by the intersection of the appropriate MB^i and MC^i schedules at point A in Fig. 1.

is a strategic complement for its rival j 's guns at all quantities that satisfy $G^j \leq B_A^i(G^j)$. For country 1, this is captured by the fact that B_A^1 is upward-sloping up to (and including) the point of intersection with the 45° line. However, at some point beyond this intersection, B_A^1 becomes negatively sloped, capturing the idea that G^1 eventually becomes a strategic substitute for G^2 . *Second*, in the neighborhood of the Nash equilibrium point A (where B_A^1 and B_A^2 intersect), both countries' instruments are necessarily strategic complements. *Third*, point $(0, 0)$ cannot be a Nash equilibrium because a country always has an incentive to produce some arms when its rival produces no arms at all.

Fig. 2 depicts a situation in which country 1 builds more guns and therefore obtains a larger share of the contested resource than its adversary. This raises several important questions. What are the determinants of arming incentives and military superiority (power) in this setting? Does the distribution of secure factor endowments between the two adversaries matter? If so, how? Is it necessarily the case that the relatively more affluent economy will produce more arms? What can be said about autarky prices? Will the country that is relatively abundant in, say, land always enjoy a lower relative price for the good that employs land intensively?

Fig. 3 helps address these questions. By construction, the sides of the outer rectangle depict the aggregate supplies of labor and land (including K_0) that are available to the two contestants. Point O^i is country i 's origin. The inner box, $A^1B^1A^2B^2$, identifies the aggregate quantities of the secure endowments with point A^i being country i 's origin.

Let us now focus on the benchmark case in which countries 1 and 2 have identical secure endowments, as indicated by point D along the diagonal A^1A^2 , and let a tilde “ \sim ” over variables identify the resulting equilibrium. Vector O^iC^i depicts the quantity of resources country i devotes to the production of arms.¹⁵ Due to symmetry, both countries will face identical arming

¹⁵It can be shown that the quantity of capital $K_G^i = \frac{1}{1-\alpha} p_r^i G^i$ employed in the production of arms in country i

incentives; therefore, $G_A^{i*} = \tilde{G}_A^*$ for every $i = 1, 2$ with each country receiving one half of the contested resource, as indicated in Fig. 3 by the facts that $O^1C^1 = O^2C^2$ and $O^1A^1 = O^2A^2 = \frac{1}{2}K_0$. Vectors C^1D and C^2D respectively depict the quantities of land and labor countries 1 and 2 devote to the production of consumption goods. Once again, by symmetry, $C^1D = C^2D$. By Lemma 4, it follows that $p_A^{i*} = \tilde{p}_A^*$, $\omega_A^{i*} = \tilde{\omega}_A^*$, and $k_X^{i*} = \tilde{k}_X^*$ for $i = 1, 2$. (Fig. 3 is based on the assumption that $k_G^i < k_1^i < k_2^i$.)

To see how arms and autarky prices compare across countries as functions of the distribution of secure endowments, consider the following sets in Fig. 3: $S^0 \equiv F^1F^2$, $S^1 \equiv F^1F^2A^2B^1$, and $S^2 \equiv F^2F^1A^1B^2$. We may now state

Proposition 1. *Under autarky, for each country $i \neq j = 1, 2$, we will have*

- a) $G_A^{i*} = \tilde{G}_A^*$ and $p_A^{i*} = \tilde{p}_A^*$ for endowment allocations in S^0 ;
- b) $G_A^{i*} > G_A^{j*}$ and $p_A^{i*} \geq p_A^{j*}$ as $k_2^i \geq k_1^i$ for endowment allocations in S^i .

Part (a) of Proposition 1 reveals that interstate arming incentives and product prices do not differ when the distribution of factor endowments between the competing states is such that $k_X^1 = k_X^2 = \tilde{k}_X^*$. For some intuition, consider the following endowment redistribution in S^0 . Temporarily suppose the contestants do not adjust their security policies. Starting at the point of symmetry D in Fig. 3, transfer resources from country 2 to country 1 along F^1F^2 and in the direction of point E so that $k_X^1 = k_X^2 = \tilde{k}_X^*$. Since, by Lemma 5, this redistribution will leave intact both countries' relative supply (as well as relative demand) functions there will be no reason for either country's autarky price to change. But, if product prices do not change then neither will arming incentives as was initially hypothesized. It follows that symmetry is a sufficient (but not necessary) condition for two competing states to end being equally powerful.

does not exceed $\frac{1}{2}K_0$.

Part (b) unveils circumstances under which a country arms more heavily than its adversary and clarifies how autarky prices compare across countries as functions of technology and the distribution of secure endowments. To see the logic, suppose $k_2^i > k_1^i$ and, once again, consider an allocation on S^0 , say point E in Fig. 3.¹⁶ By Proposition 1(a) and Theorem 1, $B_A^{1'}$ and $B_A^{2'}$ will cross the 45° line in Fig. 2 only once and intersect each other at, say, point A' . Now arbitrarily transfer some labor from country 2 to country 1 in the direction of point H in S^1 . By Lemma 5(d), a direct effect of this transfer will be to raise (reduce) country 1's (2's) autarky price. By Lemma 3(c), this price adjustment will reduce (raise) country 1's (2's) opportunity cost of producing guns; in turn, this will induce country 1 (2) to expand (contract) its optimal production of arms, as indicated by the clockwise rotation of the best-response functions that become B_A^1 and B_A^2 and intersect at point A below the 45° line. Clearly, country 1 arms more heavily than its adversary and receives a bigger share of the contested resource. Using this observation in Lemma 4 necessarily implies $p_A^{1*} > p_A^{2*}$.

Proposition 1 highlights two important ideas. *First*, under autarky, the relationship between relative factor abundance, arming, and military superiority is complex. For one thing, it is not always the case that the relatively more affluent economy will necessarily arm more heavily. For example, even though country 2's secure factor endowments at point J in S^1 of Fig. 3 are larger than country 1's, the former country *will not* build more arms or obtain a bigger share of the contested resource than the latter when $k_2^i > k_1^i$. It may be tempting to conclude that country 1 will arm more heavily because it is relatively less well endowed in the contested resource (land). However, this reasoning is incorrect because country 1 also produces more arms at point H where it is relatively more abundant in land. An interesting special case arises when we consider

¹⁶The dotted-line parallelograms illustrate how the vector endowments at E can be decomposed into sectoral input demand vectors. For example, C^1Q^1 captures (K_1^1, L_1^1) and C^1Q^2 captures (K_2^1, L_2^1) .

endowment configurations along the diagonal A^1A^2 where only scale effects matter. It can be verified that, in this case, the relationship between arming incentives and relative size is indeed monotonic. What part (b) clearly reveals is that a necessary condition for military superiority is that a country's secure endowments exceed the threshold levels implied by allocations in S^0 .

Second, Proposition 1 clarifies that, in the presence of insecure property, the relationship between a country's autarky price and its secure land/labor endowment ratio need not be monotonic. Alternatively, and in contrast to the world of the Heckscher-Ohlin trade model, where property rights are well-defined and costless to enforce, the country with the largest land/labor ratio will not necessarily enjoy the lowest relative price for the good that employs land intensively – compare, for example, relative endowments at different points along S^0 , keeping in mind that $p_A^{i*} = \tilde{p}_A^*$ at all such points. This makes sense. What matters for autarky prices are the *ex-post* endowment ratios that are available for the production of consumption goods which, as we have seen, are endogenously determined. As we will see later, this plays a key role in the determination of the direction of international trade flows.

Proposition 1 and the above discussion suggest that endowment redistributions have important implications for arming and national welfare. While it is possible to explore the comparative static effects of factor growth and factor reallocations in fine detail here, to save on space we only highlight the effects of small endowment redistributions in the neighborhood of S^0 . Starting with an allocation in S^0 , consider a small transfer of, say, labor from country j to country i . Extending the welfare decomposition in (8) to include the direct (income) effect of a change in L^i on country i 's welfare, utilizing the envelope theorem and the fact that $M_2^i = 0$ under

autarky yields

$$\frac{dV_A^{i*}}{dL^i} = v(p^i) \left[w^i + r^i K_0 \phi_{G^j}^i \frac{dG^j}{dL^i} \right] \quad \text{for } j \neq i = 1, 2. \quad (11)$$

There are two income (and, therefore, welfare) effects associated with a labor transfer. A direct effect which involves the wage adjustment of a country's income (the first term in the brackets); and an indirect (strategic) effect associated with the accompanying change in rentals on the appropriated land. The welfare effect of land transfers can be similarly derived. Proposition 2 details the exact findings.

Proposition 2. *Consider an allocation of secure endowments in S^0 . Then, a small transfer of labor (land) from country j to country $i \neq j$, so that $-dL^j = dL^i > 0$ ($-dK^j = dK^i > 0$), has the following implications for security policies and welfare:*

- a) $dG_A^{i*}/dL^i = -dG_A^{j*}/dL^i > 0$ and $dG_A^{i*}/dK^i = -dG_A^{j*}/dK^i < 0$;
- b) $dV_A^{i*}/dL^i = -dV_A^{j*}/dL^i > \nu(\tilde{p}_A^*)w(\tilde{p}_A^*)$ and $\nu(\tilde{p}_A^*)r(\tilde{p}_A^*) > dV_A^{i*}/dK^i = -dV_A^{j*}/dK^i$.

Part (a) of Proposition 2 points out that the transferee (transferor) of labor builds more (less) guns, but the aggregate production of arms does not change.¹⁷ Additionally, part (a) reveals that the direction of change in the equilibrium quantity of guns is reversed when land transfers are considered instead. As shown in the Appendix, these results are driven by the dependence of the autarky price, p_A^i , on secure endowments noted in Lemma 5(d), and rely on the fact we start at a symmetric equilibrium. The welfare effects described in part (b) are natural consequences of the strategic effect noted in (11) and part (a) which signs this effect. Proposition 2 will prove helpful when we address the question of how asymmetries in secure

¹⁷It can be shown that when the initial endowment allocation is in the interior of S^i , labor transfers into country i typically induce a reduction in the aggregate production of guns. Perhaps more interestingly though when the initial allocation is sufficiently asymmetric, both countries produce less guns. We consider the implications of asymmetries in Section 4.2.

endowments affect the welfare comparison of conflict under free trade and autarky.

3.2 Free Trade

Let π be the relative price of good 2 in the world market and abstract from transport costs and the possibility that ROW may adopt discriminatory trade policies vis-a-vis countries 1 and 2. Our assumption that countries 1 and 2 are “price takers” in world markets implies $p^i = \pi$ for $i = 1, 2$; therefore, p^i is independent of national security policies and the country’s payoff function can be identified with its indirect utility function described in Section 2. For given π , the intersection of MB^i and MC^i (the dotted-line schedule) at point A in Fig. 1 determines country i ’s best-response function, $B_F^i(G^j)$, as predicted by the country’s optimality condition in (9).

Depending on the value of the world price and the distribution of secure endowments, a country’s production of arms may be constrained by its secure land endowment and/or a country may specialize completely in the production of a consumption good. To keep the analysis simple and direct, in what follows we abstract from these possibilities, so we may use parts (a) and (b) of Lemma 3 to characterize the contestants’ best-response functions. It can be verified that $\partial B_F^i / \partial G^j = -V_{G^i G^j}^i / V_{G^i G^i}^i = -\phi_{G^i G^j}^i / \phi_{G^i G^i}^i$; therefore, the shape of B_F^i is determined by the properties of the CSF. The dashed-line curves in Fig. 2 portray B_F^1 and B_F^2 . Three features of these functions stand out. *First*, they are positively sloped (strategic complementarity) up to their point of their intersection with the 45° line, and negatively sloped (strategic substitutability) thereafter. *Second*, B_F^1 and B_F^2 meet at point F on the 45° line, the Nash equilibrium in security policies. *Third*, as before, point $(0, 0)$ is not a Nash equilibrium. Theorem 2 and Corollaries 1 and 2 elaborate on these points further.

Theorem 2. (*Existence and Uniqueness of Nash Equilibrium*). Suppose the world price,

the size of the contested resource, and the distribution of secure endowments are such that free trade in consumption goods leads to the equalization of factor prices but with arms production in either country being unconstrained by the initial land endowment. Then, an interior Nash equilibrium in pure strategies (security policies) will exist, and will be unique and symmetric.

Corollary 1. (*Arms Equalization*). Under the conditions of Theorem 2, the introduction of free trade in consumption goods levels the playing field in arms competition, so that $\phi^1 = \phi^2 = \frac{1}{2}$.

Corollary 2. Under the conditions of Theorem 2, the security policies of competing states are independent of their factor endowments. Thus, if some labor (land) is transferred from country j to country $i \neq j$, so that $-dL^j = dL^i > 0$ ($-dK^j = dK^i > 0$), then

- a) $dG_F^{i*}/dL^i = dG_F^{j*}/dL^i = 0$ and $dG_F^{i*}/dK^i = dG_F^{j*}/dK^i = 0$;
- b) $dV_F^{i*}/dL^i = -dV_F^{j*}/dL^i = v(\pi)w(\pi)$ and $dV_F^{i*}/dK^i = -dV_F^{j*}/dK^i = v(\pi)r(\pi)$.

We illustrate the key ideas behind Theorem 2 and Corollaries 1 and 2 with the help of Fig.

4. This figure assumes $\pi = \widehat{p}_A^*$ and is similar to Fig. 3. (The analysis goes through for other π values.) Under conditions of factor price equalization, the world price will pin down factor prices in each country and will equalize the opportunity costs of arming, provided initial land holdings do not altogether get absorbed into the production of guns.¹⁸ Under these circumstances, we will have $G_F^{1*} = G_F^{2*} = \widetilde{G}_A^*$ with vectors $O^i C^i = A^i N^i$ in Fig. 4 capturing the resources absorbed into country i 's military, and $O^i A^i = C^i N^i = \frac{1}{2}K_0$ capturing the appropriated land; therefore, the rectangle going through points C^1 and C^2 depicts the aggregate quantities of inputs left for the production of consumption goods, and parallelogram $C^1 J^1 C^2 J^2$ portrays the region of factor price equalization (FPE). This region has the property that, for given guns, competitive equilibria with free trade in consumption goods are replicated by equilibria of the integrated economy involving countries 1 and 2. (An integrated economy is one in which both goods

¹⁸The general conditions for factor prices equalization include constant returns to scale in production, the absence of factor intensity reversals, identical technologies, diversification in production, absence of market failures or distortions, no trade barriers, and the existence of at least as many productive factors in the tradable goods sectors as there are traded goods.

and factors are freely traded (Dixit and Norman, 1985).) The sides of $C^1J^1C^2J^2$ depict the integrated economy's sectoral employment in goods 1 and 2. The sectoral employment in each country can be obtained by decomposing the resource vector associated from an allocation in $C^1J^1C^2J^2$ along the sides of this parallelogram.¹⁹

The shaded subset of the FPE region is the set of secure factor allocations that conform to Theorem 2; that is, allocations in this set, which we may christen the “arms equalization” (AE) region, ensure that free trade in goods leads to arms equalization. It can be verified, that for endowment allocations outside the AE region, either factor prices are not equalized, or at least one country's arms production is constrained by its secure land endowment.

Existence and uniqueness of equilibrium are guaranteed under more general conditions than the ones stated in Theorem 2. However, explicit consideration of these conditions requires that we examine the possibility of specialization in the production of tradables. This would complicate the analysis but is not crucial and for this reason we do not pursue it formally here.²⁰ Theorem 2, and Corollaries 1 and 2 should be viewed as benchmarks that flesh out several noteworthy ideas. First and foremost, they unveil an important channel through which international trade affects arming incentives: the price of traded consumption goods. The literature on the subject did not recognize this channel and, as a consequence, has failed to demonstrate that the implications of free trade for arms competition can be strong, indeed. A central contribution of the analysis is that it identifies salient circumstances under which the introduction of free trade levels the

¹⁹If the contestants did not obtain a portion of the contested land, parallelogram $N^1I^1N^2I^2$ (instead of $C^1J^1C^2J^2$) would describe the joint sectoral employment in goods 1 and 2. It is the infusion of K_0 units of capital into the integrated economy that changes $N^1I^1N^2I^2$ into $C^1J^1C^2J^2$. This also causes the aggregate production of the land-intensive good 2 to expand and the production of the labor-intensive good 1 to contract (Rybczynski Theorem).

²⁰If the world price is significantly different from \tilde{p}_A^* , or the size of K_0 is sufficiently large, or the international distribution of secure endowments is considerably unequal, then one and possibly both countries may specialize in the production of a single consumption good. In the presence of such specialization, a country's opportunity cost would be increasing in its arms and would no longer be determined by the world price.

playing field in arms competition, *even when arms themselves are not traded internationally*. Another way to see this is to note that trade in arms is neither a necessary nor a sufficient condition for trade openness to influence arming incentives. Second, the analysis unveils which factors matter for the actual tradability of arms. Lastly, the analysis reveals that, at least when arms do not depend on the distribution of factor endowments, relatively larger countries will spend a relatively a smaller fraction of their incomes on arms.

Under free trade, two key determinants of investments in arms are: the size of the contested resource, K_0 , and the world price, π . The importance of K_0 is fairly obvious here: the larger K_0 , the larger the marginal benefit of arming and, consequently, the larger each country's incentive to arm. The role of π is a bit more involved.

Proposition 3. *Suppose the arms equalization region is not empty for certain prices and allocations of secure endowments. Then, there will exist a range of world prices*

- a) $(\underline{\pi}, \bar{\pi})$ such that $\partial G_F^{i*}/\partial \pi \geq 0$ as $k_2^i \geq k_1^i, \forall \pi \in (\underline{\pi}, \bar{\pi})$; and
- b) $(\underline{\pi}', \bar{\pi}')$ such that $\partial G_F^{i*}/\partial \pi = 0 \forall \pi \notin (\underline{\pi}', \bar{\pi}')$, where $\underline{\pi}' \leq \underline{\pi}$ and $\bar{\pi}' \geq \bar{\pi}$.

To see the logic of part (a), suppose $k_2^i > k_1^i$ and consider an increase in π . By Lemma 1(a), this price increase will force both contestants' wage/rental ratio, ω^i , to fall; provided the initial endowment allocation remains within the arms equalization set, this will depress both contestants' opportunity costs of arming and will induce them to arm more heavily. Fig. 1 captures this point for an individual country with the downward shift in its MC^i curve. In the context of Fig. 2, this point would be portrayed with an outward shift in best-response functions B_F^1 and B_F^2 (not shown) that would lead to the new equilibrium point F' along the 45° line.

The increase in π also affects the sectoral employment and production of tradables through two distinct channels. First, the aforementioned fall in ω^i causes the land/labor ratio, k_j^i , demanded in each industry $j = 1, 2$ to fall and bids resources into (away from) industry 2 (1).

At the same time, however, the price increase affects the supplies of K_X^i and L_X^i and, therefore, outputs.²¹ Typically, the overall effect of a π increase entails an expansion (contraction) in the output of good 2 (1), and for sufficiently large price adjustments specialization may arise. Depending on the distribution of secure endowments, it is possible that, for a certain range of prices, only one country specializes completely. Part (b) shows how equilibrium arms respond to price changes when *both* adversaries specialize completely either in the production of good 1 (which arises when $\pi \leq \underline{\pi}'$) or in the production of good 2 (which arises when $\pi \geq \bar{\pi}'$). The independence of security policies from prices arises because, in the presence of such specialization in production, factor prices (and, therefore, opportunity costs) are determined locally in domestic markets. Proposition 3 has important implications for welfare.

Turning to the relationship between welfare and price, the decomposition in (8) together with the envelope theorem give

$$\frac{\partial V_F^{i*}}{\partial \pi} = \nu(\pi) \left[-M_2^i + r^i K_0 \phi_{G^j}^i \frac{\partial G_F^{j*}}{\partial \pi} \right], \text{ for } i \neq j. \quad (12)$$

As pointed out in Section 2, the first term inside the brackets captures the terms of trade effect (TOT) of a price change and its sign is determined by the country's trade pattern. The second term is the strategic effect of a price change. Owing to the negative security externality ($\phi_{G^j}^i < 0$), this effect will necessarily be negative if country i 's adversary alters its arms production in the same direction as the change in the world price (Proposition 3(a)). However, as pointed out in Proposition 3(b), the strategic effect vanishes if the world price is sufficiently high or

²¹Suppose the CSF takes the Tullock form; that is, $f(G) = G^\eta$ for some $\eta \in (0, 1]$. Then, it is fairly easy to show that $G_F^* = \frac{\eta\psi}{4r} K_0$, $K_X^i = K^i + \frac{1}{2}K_0 - \frac{\eta}{4}\theta_{KG}K_0$, and $L_X^i = L^i - \frac{\eta}{4\omega}\theta_{LG}K_0$, where $\theta_{KG} \equiv \frac{r\psi_r}{\psi}$ and $\theta_{LG} \equiv \frac{w\psi_w}{\psi}$. Now suppose π rises. It is easy to show that $dL_X^i/d\pi < 0$ but $dK_X^i/d\pi \gtrless 0$ depending on whether $\sigma_G \gtrless 1$, where σ_G is the elasticity of substitution between factor inputs in guns.

sufficiently low. The following ideas can now be confirmed. First, there exists a world price that ensures $M_2^i = 0$, even when arms are endogenously determined.²² Second, and unlike part (d) of Lemma 3, welfare is not necessarily minimized when $M_2^i = 0$. As can be verified from (12), if $k_2^i > k_1^i$ ($k_2^i < k_1^i$), welfare is minimized when $M_2^i < 0$ ($M_2^i > 0$). This leads to our third point: there exists a range of prices under which an improvement in country i 's terms of trade is “immiserizing” (because its adversary produces more arms).

4. Autarky vs Free Trade in the Presence of Conflict

Two important insights of neoclassical trade theory are: (1) in the absence of market failures and/or distortions, the introduction of free trade does not leave a country worse off, as compared to autarky; and (2) a country's trade pattern can be predicted by comparing its free trade and autarky prices or from information on the interplay between intersectoral differences in technology and international differences in relative endowments. In this section, we examine the robustness of these ideas on the introduction of insecurity and costly enforcement of property rights. We do this in two subsections: in the first, we deal with symmetric adversaries; in the second, we consider asymmetries in factor ownership.

4.1 Symmetric Adversaries

Here the symmetric allocation at point D in Figs. 3 and 4 describes the competing states' initial factor ownership. In the spirit of these figures, temporarily suppose $k_2^i > k_1^i > k_G^i$ and rule out the possibility of factor intensity reversals. We establish the key elements of the analysis with the help of Fig. 5. The shaded regions in this figure capture the previously mentioned idea that, for any symmetric configuration of guns, $G = G^1 = G^2$, there exists a sufficiently low

²²For example, for allocations in S^0 , we have $M_2^i \gtrless 0$ as $\pi \gtrless \tilde{p}_A^*$.

(high) price of good 2 in country i that brings about specialization in the production of good 1 (2).²³

The positively sloped solid-line curve going through point A describes how the simultaneous solution to $V_G^i = 0$ for $i = 1, 2$ depends on price $p = p^i$. (This relationship follows readily from parts (a)-(c) of Lemma 3.) As explained in Proposition 3(b), for sufficiently high or low product prices, factor prices (and, therefore, opportunity costs) are determined by the clearing of domestic factor markets. The dotted-line extensions of the curve in the regions of specialization portray the independence of equilibrium guns from price in these regions.

The downward sloping curve that goes through point A describes the dependence of country i 's autarky price, $p_A = p_A^i$, on G . This relationship is a direct consequence of Lemma 5(c) and the assumption that $k_2^i > k_1^i > k_G^i$. (It can be shown that p_A would depend positively on G (not shown in Fig. 5) if $k_G^i > k_2^i > k_1^i$ instead.) The non-monotonic dashed-line curve portrays yet another possible relationship between p_A and G , when $k_2^i > k_G^i > k^i > k_1^i$ for small quantities of guns but $k_2^i > k^i > k_G^i > k_1^i$ for larger quantities.²⁴

For concreteness, let us now turn back to the case $k_2^i > k_1^i > k_G^i$. The intersection of the aforementioned curves at point A captures the equilibrium quantity of guns, \tilde{G}_A^* , and price, \tilde{p}_A^* , discussed in Subsection 3.1. Turning to free trade, initially suppose $\pi = \tilde{p}_A^*$ so that $M_2^i = 0$, $G_F^{i*} = \tilde{G}_A^*$, and $V_F^{i*} = V_A^{i*}$. From Proposition 3(a), we know that $\partial G_F^*/\partial \pi > 0$; therefore, $G_F^* > \tilde{G}_A^*$ for all $\pi > \tilde{p}_A^*$, and $G_F^* < \tilde{G}_A^*$ for $\pi < \tilde{p}_A^*$. We now explore the desirability of free trade relative to autarky.

First note that, for any given price, every country's welfare falls when all countries expand their (equal) production of arms proportionately, regardless of whether they engage in free trade

²³In the context of Proposition, the assumption of symmetry implies $\underline{\pi}' = \underline{\pi}$ and $\overline{\pi}' = \overline{\pi}$.

²⁴The latter case emerges when the increase in p_A^i , brought about by the rise in G , forces k_G^i to fall below k^i through the incipient fall in the country's wage/rental ratio, ω^i (Lemma 1(a)).

or remain in a state of autarky. Furthermore, by Lemma 3(d), for any given guns, a country's welfare increases with the distance of the world price from curve p_A . (This is the familiar argument that a country's welfare increases with improvements in its TOT.) As a consequence, welfare contours in the $G - p$ space will be orthogonal to the p_A curve, as indicated by the ones in Fig. 5.

Now suppose π rises marginally above \tilde{p}_A^* . As can be seen from (12), under free trade, a country's welfare will necessarily fall below the autarky level because the TOT effect will vanish and the adversary will produce more guns. We thus have

Proposition 4. (*Arming and Welfare*) *Suppose free trade in consumption goods leads to arms equalization when $\pi = \tilde{p}_A^*$. Then,*

- a) *trade openness induces more arming $\forall \pi > \tilde{p}_A^*$ if $k_2^i > k_1^i$ ($\forall \pi < \tilde{p}_A^*$ if $k_2^i < k_1^i$);*
- b) *there exists a set of prices with infimum (supremum) at \tilde{p}_A^* when $k_2^i > k_1^i$ ($k_2^i < k_1^i$), and $V_F^{i*} \leq V_A^{i*}$ for all π in this set.*

It can be seen from Fig. 5, that autarky is Pareto-superior to free trade for all world prices that span interval AB . Indeed, the free trade welfare level is minimized at point C , where the positive TOT effect in (12) just offsets the strategic effect of a world price change. It should be noted that this result arises even with eventual specialization in the production of good 2. (This is so because the classical benefits of free trade outweigh the waste of resources diverted into the arms sector for world prices beyond the one associated with point C .) In short, Proposition 4 clarifies that, in the presence of insecure property, trade openness may intensify the arms buildup and, through that, it may raise the appeal of autarky. Perhaps more importantly, Proposition 4 unveils a fundamental conduit through which the possibly adverse effects of free trade may be transmitted: the world price.

We may now address the question of how the representative country's trade pattern with

conflict compares to the trade pattern that would arise in the absence of arming. If neither country arms, each will receive one half of the contested land and the autarky price in Fig. 5 will coincide with p_A^0 ; therefore, in the absence of conflict, the representative country will export (import) good 2 (1) if $\pi > p_A^0$, and conversely if $\pi < p_A^0$. More generally, for any positive symmetric configuration of guns, the representative country will be an exporter (importer) of good 2 for all world price and gun configurations north (south) of the p_A curve. But, for any world price, the conflictual equilibrium with free trade is along the curve going through points A and B . We thus have

Proposition 5. *(Trade Patterns) Under conditions of symmetry and in the presence of conflict, a country's trade pattern may differ from the one that would arise under complete property rights, but can be determined by comparing π with \tilde{p}_A^* .*

Inspection of Fig. 5 reveals that, with conflict, the representative country's trade pattern gets reversed for all $\pi \in (\tilde{p}_A^*, p_A^0)$. However, exactly how the country's pattern of trade may change depends on the ranking of factor intensities. If $k_2^i > k_1^i > k_G^i$, country i will export good 2 in the absence of conflict but will import it under conflict, for all $\pi \in (\tilde{p}_A^*, p_A^0)$. In contrast, if $k_G^i > k_2^i > k_1^i$, country i will import good 2 in the absence of conflict but will import it in its presence, for all $\pi \in (p_A^0, \tilde{p}_A^*)$.

Let us now consider some implications of the degree of land insecurity. Let $K \equiv K^1 + K^2 + K_0$ be the (fixed) aggregate supply of land. We say that land becomes more insecure if K_0 rises and $K^1 + K^2$ falls. When adversaries are symmetric, $K^2 = K^1$; therefore, $dK_0 + 2dK^i = 0$. Suppose K_0 rises. Since this does not affect country i 's land endowment, $\frac{1}{2}K_0 + K^i$, when $G = G^1 = G^2$, the relationship between the autarky price p_A and G depicted in Fig. 5 will not be affected. However, the increase in T_0 will cause every country's marginal benefit of arming to rise (MB^i shifts up in Fig. 1), as indicated by the rightward shift in the positively sloped schedule in

Fig. 5, which moves the new equilibrium under autarky at point E . If $\pi = \widehat{p}_A^*$ initially and π remained at that level, the new equilibrium under free trade would be at point F . It follows that, in this case, increased land insecurity induces both contestants to arm more heavily under either trade regime. The arms adjustment however is relatively less pronounced under autarky because of the endogeneity of p^i ; therefore, in this case, trade openness reduces both countries' welfare relative to autarky.²⁵

What does increased land insecurity imply for the range of world prices under which (1) autarky is Pareto superior to free trade, and (2) trade patterns are reversed under conflict? At this point, there appears to be no general analytical answer to the first question. However, numerical analysis of a model in which consumer preferences and production functions are Cobb-Douglas and the CSF takes the Tullock form indicates that the price range mentioned in the first question is enlarged. The answer to the second question follows from the above analysis and Fig. 5; that is, the aforementioned price range is also enlarged, provided that either $k^i > k_G^i$ or $k^i < k_G^i$ for all factor prices.

4.2 Asymmetric Adversaries

We first discuss the implications for welfare and trade patterns of asymmetric allocations in S^0 . This turns out to be a valuable exercise in its own right but also because it is a useful benchmark. We then go on to explore some of the effects of secure factor allocations in S^i .

Much of the analysis in the previous subsection goes through for general allocations in S^0 .²⁶ For example, after a minor adjustment in the argument to take into account the possibility that, for some prices, only one of the adversaries may specialize completely under free trade, we find

²⁵The argument implicitly assumes that the secure land constraint does not become binding as $K_0 \uparrow$.

²⁶Relative to the symmetric allocation, the analysis of asymmetric allocations in S^0 differs in that the price ranges under which production becomes specialized across countries do not coincide.

that Propositions 4 and 5 remain intact.

Let us now examine asymmetric endowments in S^i . Assuming $k_2^i > k_1^i$, suppose $\pi = \tilde{p}_A^* + \varepsilon$ for some $\varepsilon > 0$, so that $V_F^{1*}/V_A^{1*} = V_F^{2*}/V_A^{2*} < 1$ for any asymmetric allocation in S^0 . By continuity, there will exist subsets of endowment configurations in S^1 and S^2 on the two sides of S^0 (but sufficiently close to S^0), under which autarky dominates free trade. This raises the question of whether there exists circumstances under which trade openness countries have divergent preferences over trade regimes. Proposition 6 provides the answer.

Proposition 6. *Suppose $\pi = \tilde{p}_A^*$. Then, will exist a subset $D^i \subseteq S^i$ of factor allocations in the neighborhood of S^0 such that $V_F^{i*} \leq V_A^{i*}$ and $V_F^{j*} \geq V_A^{j*}$ as $k_2^i \geq k_1^i$ for $i \neq j = 1, 2$.*

Proof: For concreteness, suppose $k_2^i > k_1^i$ and $\pi = \tilde{p}_A^*$. Now, starting at an arbitrary allocation in S^0 , transfer a small quantity of labor from country 2 to country 1 (i.e., $-dL^2 = dL^1$) so that the final allocation is in S^1 , as indicated by the move from point E to point H in Figs 3 and 4. Together, Proposition 2(b) and Corollary 2(b) imply $dV_F^{1*}/dL^1 < dV_A^{1*}/dL^1$ and $dV_F^{2*}/dL^1 > dV_A^{2*}/dL^1$. Since $V_F^{i*} = V_A^{i*}$ initially, we will have $V_F^{1*} < V_A^{1*}$ and $V_F^{2*} > V_A^{2*}$, which completes the proof. ||

Together with the preceding analysis, Proposition 6 clarifies how the world price, technology and asymmetries in endowments determine this just-described divergence in national preferences over trade regimes. It should now be clear how the above reasoning could be extended to consider the broader implications of asymmetric allocations for the welfare ranking of trade regimes when $\pi \neq \tilde{p}_A^*$.

In the world of neoclassical trade theory, to predict a country's trade pattern it is sufficient to compare its price under autarky to the world price. In the world of insecure property and conflict, however, things are not the same.

- Proposition 7.** *For factor allocations in S^i ($i = 1, 2$), insecure property and conflict imply*
- a) *it may be impossible to predict trade patterns by comparing world and autarky prices;*
 - b) *a land (labor) abundant country may export the labor (land) intensive commodity.*

We establish the validity of Proposition 7 informally and with the help of Figs 2, 3 and 4.

For specificity, suppose $k_2^i > k_1^i$ and consider an allocation in S^1 . By Proposition 1(a), we will have $p_A^{1*} > p_A^{2*}$ and $G_A^{1*} > G_A^{2*}$, as shown at point A in Fig. 2. Now suppose $\pi = p_A^{2*}$ and allow both countries to trade freely in the world market. Since the new Nash equilibrium will be at point F , country 1 will produce less arms but country 2 will produce more. Now focus on country 2. By parts (b) and (c) of Lemma 2, country 2's land/labor ratio, k_X^2 , will rise. In turn, by Lemma 1(c), this will cause the relative supply of good 2 to increase and thereby create an excess supply (demand) for good 2 (1). In other words, country 2 will become an exporter (importer) of good 2 (1) even though the world price does not differ from its autarky price. The reason for this is simple: what matters for country i 's trade pattern is **not** how π differs from p_A^{i*} , but how π differs from the autarky price that would arise if guns were set at their free trade Nash equilibrium levels.

For part (b), consider the allocation H in Fig. 4. Now suppose $\pi = \tilde{p}_A^*$ and keep in mind that country 1 (2) will export good 1 (2). In addition, note that at point H , country 1 (2) is relatively abundant in land (labor) both in terms of its initial secure endowments and its final endowments in the conflictual equilibrium under free trade. This confirms part (b). Moreover, since $p_A^{1*} > p_A^{2*}$ at point H , the land (labor) intensive commodity does not necessarily command the lowest (largest) autarky price in the land (labor) abundant country.

5. Security Aspects of Trade Policies

So far, our analysis focused on the extreme regimes of complete autarky and global free trade, so it would be of interest to examine the implications of less restrictive policies. Suppose

country i intervenes in trade with an ad valorem trade tax, τ^i , on good 2. (This requires $\tau^i > 0$ when $M_2^i > 0$, and $\tau^i < 0$ when $M_2^i < 0$.) Under these circumstances, $p^i = (1 + \tau^i)\pi > 0$ for $i = 1, 2$. Assuming that in each country i tariff revenues, $\tau^i \pi M_2^i = (p^i - \pi)M_2^i$ are redistributed to consumers in lump-sum fashion, country i 's welfare changes may be decomposed as follows:

$$dV^i = \nu^i \left[-M_2^i d\pi + \tau^i \pi dM_2^i + (r^i K_0 \phi_{G^i}^i - \psi^i) dG^i + r^i K_0 \phi_{G^j}^i dG^j \right], \quad i \neq j = 1, 2. \quad (13)$$

The above equation clarifies the channels through which the effects of security and trade policies will travel now. The first and second terms inside the brackets capture the familiar terms of trade and volume of trade effects of trade policies, respectively. By our assumption that countries 1 and 2 are “small” in world markets, the first term will vanish when we consider the effects of trade and security policies. If country i participates in world trade, the second term will not vanish; it will depend on country i 's trade and security policies, and on its adversary j 's security (but not trade) policy. The third and fourth terms in (13) capture the direct effects of security policies that we discussed earlier.

Now suppose country i 's trade and security policies are simultaneously determined. It can be easily inferred from (13) that its optimal trade policy will be free trade (i.e., $\tau^{i*} = 0$). Since this implies that the second term will also vanish, country i 's optimal security policy will coincide with the one we described earlier in the context of global free trade. It is straightforward to verify that if security policies are determined prior to trade policies the analysis is similar.

The above reasoning raises the question of whether trade policy commitments before the implementation of security policies alter the analysis in a substantive way. To explore this possibility, consider a two-stage game in which countries determine their trade policies in stage 1 and their security policies in stage 2. In the presence of trade taxes, country i 's optimal

security policy would have to include its possible effect on the volume of trade. Starting with the last stage, at an interior solution, country i 's ($= 1, 2$) FOC for welfare maximization will be

$$\frac{\partial V^i}{\partial G^i} = \nu^i \left[\tau^i \pi \frac{\partial M_2^i}{\partial G^i} + r^i K_0 \phi_{G^i}^i - \psi^i \right] = 0. \quad (14)$$

The effects of trade policy on arming and power can be identified with standard comparative statics exercises performed on (14). The important point for our purposes is that precommitments on trade policy can strategically affect the security policies of even small countries.

Now, identify with a star (*) the solution to the above system of equations and let subscript T reflect the presence of tariffs. For simplicity, suppose country i imports good 2. Going back to stage 1, we may summarize the welfare effect of a change in country i 's trade policy as follows:

$$\frac{\partial V_T^i}{\partial \tau^i} = \nu^i \left[\tau^i \pi \frac{\partial M_2^i}{\partial \tau^i} + \left(\tau^i \pi \frac{\partial M_2^i}{\partial G^j} + r^i K_0 \phi_{G^j}^i \right) \frac{\partial G^{j*}}{\partial \tau^i} \right]. \quad (15)$$

In (15), the direct effect of trade policy on country i 's optimal security strategy vanished, by the envelope theorem. An increase in country i 's tariff will have a negative welfare effect due to its distortionary impact on the country's volume of trade (i.e., $\partial M_2^i / \partial \tau^i < 0$), as indicated by the first term in (15). But there also exists a strategic effect – one associated with the possible impact of the country's trade policy on the rival country's security policy, captured by the second term inside the brackets. This latter effect has two components. Suppose a restrictive trade policy induces country i 's adversary to behave less aggressively in security competition (i.e., $\partial G^{j*} / \partial \tau^i < 0$). Then, if $k_2^i > k_1^i$, the first component of the strategic effect will be negative because $\partial M_2^i / \partial G^j > 0$, but the second will be negative because $\phi_{G^j}^i < 0$. The former effect is due to the volume effect of the rival country's security policy and the latter effect is due to the

income effect of this policy. Numerical analysis of a model with Cobb-Douglas production and utility functions, and a Tullock CSF indicates that restrictive unilateral trade policies do not raise welfare.

6. Concluding Remarks

In this paper, we have constructed a simple but sufficiently general model of trade and appropriative conflict that enable us to provide a systematic assessment of the implications of different trade regimes for arming and welfare. By design, our focus was on “small” countries, so that we could abstract from the possible terms of trade effects of security and trade policies. This is appropriate for some countries but not necessarily for others that have monopoly/monopsony power in world trade, so it would be interesting and worthwhile to extend the formal analysis in this direction. One important difference from the current setting is that countries’ incentives to produce guns and intervene in trade become more complex, with trade no longer inducing the equalization of arming incentives even under conditions of factor price equalization. Still, the qualitative welfare effects are broadly consistent with the ones obtained here.²⁷ Another difference is that trade and security policies can be used simultaneously, the former to balance terms of trade with volume of trade effects, and the latter security considerations. This is a rich and promising environment within which the implications of policy interactions could be explored, including the economics of incentive-compatible free trade agreements and their possible spillover effects on national security. An additional extension in a different direction is the possible examination of overt conflict – as that may be captured, for example, in a “winner-

²⁷Skaperdas and Syropoulos (2002) explored a framework similar to the one considered here but with the terms of trade determined by bargaining. Welfare under unrestricted trade in that article is shown to be higher than welfare under autarky only when the countries have sufficiently different secure endowments – so that their gains from trade are large enough. Skaperdas and Syropoulos (1996) have analyzed a more conventional interaction of large countries. They showed that even going to war, an alternative that we have not examined here, can yield (in a limited set of occasions) higher welfare than unrestricted trade.

take-all” contest – and the identification of circumstances under which conflict with no trade may arise as an equilibrium outcome. Pursuing this alternative in the environment of this paper and exploring in finer detail the interactions of trade and security policies are clearly goals worthy of further exploration.

Ultimately, solving the problem of insecurity entails the development of commitment devices that aim to reduce, and possibly eliminate, the need to arm. Such commitment devices, however, are not easy to come by and, judging from particular historical instances, they take a long time to develop. Europe is a good example of this. After the experience of the two world wars, the original six members of the European Community slowly began to develop mechanisms of economic integration that were in large part institutions of conflict management. That twin process of economic integration and conflict resolution through bureaucratic and political struggle, instead of conflict in the battlefield, is ongoing and far from complete after a century of tribulations. Trade openness and, more generally, economic interdependence may ameliorate conflict, but it would be naive to think that they could achieve this by themselves.

7. Appendix

We first unveil several useful properties of the CSF in (1). For simplicity, define $f_i \equiv f(G^i)$. Recalling that $f'_i > 0$ and $f''_i \leq 0$, differentiate $\phi^i(G_i, G_j)$ with respect to its arguments to obtain

$$\phi_{G^i}^i = \frac{f'_i f_j}{(f_1 + f_2)^2} > 0 \tag{A.1}$$

$$\phi_{G^j}^i = -\frac{f'_j f_i}{(f_1 + f_2)^2} < 0 \tag{A.2}$$

$$\phi_{G^i G^i}^i = \frac{f_j}{(f_1 + f_2)^3} [f''_i (f_1 + f_2) - 2(f'_i)^2] < 0 \tag{A.3}$$

$$\phi_{G^i G^j}^i = \frac{(f_i - f_j) f'_i f'_j}{(f_1 + f_2)^3} \geq 0 \text{ as } G^i \geq G^j \text{ for } i \neq j. \tag{A.4}$$

Proof of Lemma 1: Following Jones (1965), denote with $\theta_{hj}^i \equiv r^i a_{hj}^i / c_j^i$ the shares of factor $h = K, L$ in the cost of producing good $j = 1, 2$, and with $\theta_{KG}^i \equiv r^i \psi_r^i / \psi^i$ and $\theta_{LG}^i \equiv w^i \psi_w^i / \psi^i$ the corresponding cost shares in guns. Furthermore, let $\lambda_{Kj}^i \equiv K_j^i / L_X^i$ and $\lambda_{Lj}^i \equiv L_j^i / L_X^i$ respectively capture the proportion of capital and labor employed in industry $j = 1, 2$ and let a hat ($\widehat{}$) over a variable denote percentage change (e.g., $\widehat{x} = \frac{dx}{x}$).

Part (a): Differentiating (2) and (3) totally, using the above definitions and solving the resulting system of equations for the p^i -induced changes in factor prices yields

$$\frac{p^i w_p^i}{w^i} = -\frac{\theta_{K1}^i}{|\theta^i|} < 0 \quad \text{and} \quad \frac{p^i r_p^i}{r^i} = \frac{\theta_{L1}^i}{|\theta^i|} > 1 \quad \text{if} \quad k_2^i > k_1^i \quad (\text{A.5})$$

where

$$|\theta^i| \equiv \theta_{K2}^i - \theta_{K1}^i = \theta_{L1}^i - \theta_{L2}^i = \frac{\omega^i (k_2^i - k_1^i)}{(\omega^i + k_1^i)(\omega^i + k_2^i)} \geq 0 \quad \text{as} \quad k_2^i \geq k_1^i.$$

It can be confirmed that $|\theta^i|$ is the determinant of the coefficient matrix obtained from the differentiation of (2) and (3). It can also be verified from (A.5) that $\frac{p^i \omega_p^i}{\omega^i} = \frac{p^i w_p^i}{w^i} - \frac{p^i r_p^i}{r^i} = -\frac{1}{|\theta^i|}$, thus completing the proof to this part.

Parts (b) and (c): First note that we can combine (4) and (5) to obtain $\lambda_{L1}^i k_1^i + \lambda_{L2}^i k_2^i = k_X^i$.

Differentiating (4) and (5) totally and solving the resulting system of equations gives

$$\widehat{X}_1^i = \frac{1}{|\lambda^i|} \left(-\lambda_{L2}^i \widehat{K}_X^i + \lambda_{K2}^i \widehat{L}_X^i \right) - \frac{1}{|\lambda^i| |\theta^i|} \left(\lambda_{L2}^i \delta_K^i + \lambda_{K2}^i \delta_L^i \right) \widehat{p}^i \quad (\text{A.6a})$$

$$\widehat{X}_2^i = \frac{1}{|\lambda^i|} \left(+\lambda_{L1}^i \widehat{K}_X^i - \lambda_{K1}^i \widehat{L}_X^i \right) + \frac{1}{|\lambda^i| |\theta^i|} \left(\lambda_{L1}^i \delta_K^i + \lambda_{K1}^i \delta_L^i \right) \widehat{p}^i \quad (\text{A.6b})$$

where $\delta_K^i \equiv \lambda_{K1}^i \theta_{L1}^i \sigma_1^i + \lambda_{K2}^i \theta_{L2}^i \sigma_2^i$, $\delta_L^i \equiv \lambda_{L1}^i \theta_{K1}^i \sigma_1^i + \lambda_{L2}^i \theta_{K2}^i \sigma_2^i$, with σ_j^i being the elasticity of

substitution between land and labor in industry j and

$$|\lambda^i| \equiv \lambda_{K2}^i - \lambda_{L2}^i = \lambda_{L1}^i - \lambda_{K1}^i = \frac{(k_2^i - k_X^i)(k_X^i - k_1^i)}{k_X^i(k_2^i - k_1^i)} \geq 0 \text{ as } k_2^i \geq k_1^i.$$

The proofs to these parts now follow from (A.6a) and (A.6b).

Part (d): Noting that $\widehat{k}_X^i = \widehat{K}_X^i - \widehat{L}_X^i$, it can be seen from (A.6a) and (A.6b) that

$$\frac{d(X_2^i/X_1^i)}{X_2^i/X_1^i} = \widehat{X}_2^i - \widehat{X}_1^i = \frac{1}{|\lambda^i|} \widehat{k}_X^i + \frac{\delta_K^i + \delta_L^i}{|\lambda^i| |\theta^i|} \widehat{p}^i \quad (\text{A.7})$$

thus completing the proof to this part and, therefore, Lemma 1. $\quad ||$

Proof of Lemma 2: Denote with $s_K^i \equiv \frac{r^i K_X^i}{R^i}$ and $s_L^i \equiv \frac{w^i L_X^i}{R^i}$ country i 's land and labor shares in total net income R^i , and let $\sigma_G^i = \frac{\psi^i \psi_w^i \psi_r^i}{\psi^i \psi_w^i \psi_r^i}$ be the elasticity of substitution between land and labor in the military sector. Total differentiation of (6) yields

$$\begin{aligned} \widehat{k}_X^i &= \left(\frac{\psi^i \theta_{LG}^i \theta_{KG}^i \sigma_G^i G^i}{|\theta^i| R^i s_K^i s_L^i} \right) \widehat{p}^i + \\ &\left(\frac{K_0 \phi_{G^i}^i}{K_X^i} - \frac{\psi_r^i}{K_X^i} + \frac{\psi_w^i}{L_X^i} \right) dG^i + \left(\frac{K_0 \phi_{G^j}^i}{K_X^i} \right) dG^j + \\ &\frac{1}{K_X^i} dK^i + \frac{\phi^i}{K_X^i} dK_0 - \frac{1}{L_X^i} dL^i \end{aligned} \quad (\text{A.8})$$

or, equivalently,

$$\begin{aligned} \widehat{k}_X^i &= \left(\frac{\psi^i \theta_{LG}^i \theta_{KG}^i \sigma_G^i G^i}{|\theta^i| R^i s_K^i s_L^i} \right) \widehat{p}^i + \\ &\frac{\psi^i}{R^i s_K^i s_L^i} \left[\frac{r^i s_L^i}{\psi^i} \left(K_0 \phi_{G^i}^i - \frac{\psi^i}{r^i} \right) + \theta_{LG}^i \right] dG^i + \frac{r^i}{R^i s_K^i} (K_0 \phi_{G^j}^i) dG^j + \\ &\frac{\phi^i}{K_X^i} dK_0 + \frac{1}{K_X^i} dK^i - \frac{1}{L_X^i} dL^i. \end{aligned} \quad (\text{A.8}') \quad 39$$

Parts (a)-(c) & (e): The proofs follow from (A.8), (A.8') and the linear homogeneity of ψ^i .

Part (d): Suppose $G^i = G^j$ so that $\phi_{G^i}^i = -\phi_{G^j}^i$. Now use $dG^i = dG^j$ in (A.8) to obtain $\frac{\partial k_X^i / \partial G^i}{k_X^i} = -\frac{\psi_r^i}{K_X^i} + \frac{\psi_w^i}{L_X^i}$. Utilizing the definitions of K_X^i and L_X^i in (4) and (5) we may transform this relationship into

$$\frac{\partial k_X^i / \partial G^i}{k_X^i} = \frac{\psi_w^i}{K_X^i} (k_X^i - k_G^i) = \frac{\psi_w^i L^i}{K_X^i L_X^i} (k^i - k_G^i) = \frac{\psi^i}{R^i s_K^i s_L^i} (\theta_{LG}^i - s_L^i) \quad (\text{A.9})$$

which proves part (d). $\quad ||$

Proof of Lemma 3. Part (a): Differentiating (9) with respect to G^i and using (A.3) gives

$$V_{G^i G^i}^i = v^i r^i K_0 \phi_{G^i G^i}^i < 0, \quad (\text{A.10})$$

thereby establishing part (a). This proves that country i 's indirect utility is concave in its security policy.

Part (b): To prove this part, differentiate (8) with respect to G^j and utilize (A.4) in the resulting expression to obtain

$$V_{G^i G^j}^i = v^i r^i K_0 \phi_{G^i G^j}^i \geq 0 \quad \text{as} \quad G^i \geq G^j. \quad (\text{A.11})$$

Part (c): Differentiating (8) with respect to price and evaluating at the optimum gives (by Lemma 1(a))

$$\begin{aligned} V_{G^i p^i}^i &= -v^i r^i \frac{\partial(\psi^i / r^i)}{\partial p^i} = -v^i r^i \left(\frac{\psi^i / r^i}{p^i} \right) \left(\frac{w^i \psi_w^i}{\psi^i} \right) \left(\frac{p^i \omega_p^i}{\omega^i} \right) \\ &= v^i r^i \frac{\psi^i / r^i}{p^i} \frac{\theta_{LG}^i}{|\theta^i|} \geq 0 \quad \text{as} \quad k_2^i \geq k_1^i. \end{aligned} \quad (\text{A.12})$$

Part (d): This is a standard property of indirect (trade) utility functions. ||

Proof of Lemma 5. Let σ_D^i be the elasticity of substitution in consumption. Focusing on percentage changes, note that $\widehat{RD}^i = -\sigma_D^i \widehat{p}^i$ and that the expression for \widehat{RS}^i is given in (A.7). Totally differentiating (10) and rearranging terms gives

$$\widehat{RD}^i = \widehat{RS}^i \implies \left(\sigma_D^i + \frac{\delta_K^i + \delta_L^i}{|\lambda^i| |\theta^i|} \right) \widehat{p}^i + \frac{1}{|\lambda^i|} \widehat{k}_X^i = 0.$$

The above relation reveals that p^i is negatively (resp., positively) related to k_X^i if $k_2^i > k_1^i$ (resp., $k_2^i < k_1^i$). Taking into account (A.8) gives

$$\widehat{p}_A^i = -\frac{1}{\Delta^i |\lambda^i|} \left[\frac{\partial k_X^i / \partial G^i}{k_X^i} dG^i + \frac{\partial k_X^i / \partial G^j}{k_X^i} dG^j + \frac{\phi^i}{K_X^i} dK_0 + \frac{1}{K_X^i} dK^i - \frac{1}{L_X^i} dL^i \right] \quad (\text{A.13})$$

where

$$\Delta^i \equiv \sigma_D^i + \frac{\delta_K^i + \delta_L^i}{|\lambda^i| |\theta^i|} + \frac{\psi^i \theta_{LG}^i \theta_{KG}^i \sigma_G^i}{|\lambda^i| |\theta^i| R^i s_K^i s_L^i} G^i > 0. \quad (\text{A.14})$$

The proofs to parts (a)-(c) now follow from (A.13), (A.8) and (A.8'). ||

Proof of Theorem 1. (Existence) We establish existence of equilibrium in pure strategies, by showing that every country i 's payoff function V_A^i is strictly quasi-concave in its strategy G^i . To establish strict quasi-concavity of V_A^i in G^i it is sufficient to show either that V_A^i is strictly monotonic in G^i or that V_A^i is first strictly increasing and then strictly decreasing over the agent's strategy space.

Let $F(K_G^i, L_G^i)$ be the production function for guns that is dual to the unit cost function $\psi(w^i, r^i)$ and define $\overline{G}^i \equiv F(K^i, L^i)$. Country i 's strategy space is $[0, \overline{G}^i]$. For any $G^j \in [0, \overline{G}^j]$,

country i ($\neq j$) will be unable to produce any of the consumption goods if $G^i = \bar{G}^i$; therefore, $V_A^i(\bar{G}^i, G^j) < V_A^i(G^i, G^j)$ for any $G^i \in [0, \bar{G}^i)$ which implies that, under autarky, no country will use all its resources to produce arms. Furthermore, since $\lim_{G^i \rightarrow 0} f'(G^i) = \infty$, we must have $\partial V_A^i / \partial G^i > 0$ as $G^i \rightarrow 0$. By the continuity of V_A^i in G^i , there will exist a $B_A^i(G^j) \equiv \min\{G^i \in (0, \bar{G}^i) \mid \partial V_A^i / \partial G^i = 0\}$ with the property that $\partial V_A^i / \partial G^i > 0 \forall G^i < B_A^i(G^j)$. Thus, to establish strict quasi-concavity of V_A^i in G^i it remains to prove that $\partial V_A^i / \partial G^i < 0 \forall G^i > B_A^i(G^j)$.

Suppose $\partial V_A^i / \partial G^i \geq 0$. Since V_A^i must eventually fall to $V_A^i(\bar{G}^i, G^j)$, the function must attain a local minimum at some $G^i > B_A^i(G^j)$ which would imply that $\partial^2 V_A^i / \partial (G^i)^2 > 0$. We now prove that this is impossible. Recalling that $p_A^i = p_A^i(G^i, G^j)$ under autarky and that factor prices are functions of p^i , we may differentiate (9) with respect to G^i and apply (9) on the resulting expression to obtain

$$\frac{\partial^2 V_A^i}{\partial (G^i)^2} = [V_{G^i G^i}^i]_{p^i=p_A^i}^{(-)} + [V_{G^i p^i}^i]_{p^i=p_A^i}^{(\pm)} \left(\frac{\partial p_A^i}{\partial G^i} \right)^{(\mp)} < 0. \quad (\text{A.15})$$

By Lemma 3(a), the first term in the right-hand side (RHS) of the above expression is negative regardless of the ranking of factor intensities. Furthermore, by Lemmas 3(c) and 4(a), the second term will also be negative.²⁸ It follows that $\partial^2 V_A^i / \partial (G^i)^2 < 0$ at any G^i point at which $\partial V_A^i / \partial G^i = 0$ regardless of the ranking of factor intensities. This proves $B_A^i(G^j)$ is unique. In addition, it establishes the existence of a pure-strategy equilibrium.

(Uniqueness) To establish uniqueness of equilibrium we prove that at any equilibrium point the determinant of the Jacobian of the marginal payoffs in (9) is positive (i.e., $|J| = \frac{\partial^2 V_A^1}{\partial (G^1)^2} \frac{\partial^2 V_A^2}{\partial (G^2)^2} - \frac{\partial^2 V_A^1}{\partial G^1 \partial G^2} \frac{\partial^2 V_A^2}{\partial G^2 \partial G^1} > 0$) and some boundary conditions are satisfied (Kolstad and Mathiesen, 1987).

²⁸In (A.15) and elsewhere, the top signs in “ \pm ” and “ \mp ” apply when $k_2^i > k_1^i$ and the bottom signs apply when $k_2^i < k_1^i$.

First, note that

$$\frac{\partial^2 V_A^i}{\partial G^i \partial G^j} = [V_{G^i G^j}^i]_{p^i=p_A^i} + \left[V_{G^i p^i}^i \right]_{p^i=p_A^i} \left(\frac{\partial p_A^i}{\partial G^j} \right)^{(\pm)}. \quad (\text{A.16})$$

By Lemma 3(b) (see also (A.11)), the first term in the RHS of the above expression is positive or negative depending on whether $B_A^i(G^j) > G^j$ or $B_A^i(G^j) < G^j$, respectively. By Lemmas 3(c) and 4(b), the second term is always positive.

Utilizing (A.15) and (A.16), we may write the slope of country i 's best-response function as

$$\frac{\partial B_A^i}{\partial G^j} = -\frac{\partial^2 V_A^i / \partial G^i \partial G^j}{\partial^2 V_A^i / \partial (G^i)^2} = -\frac{[V_{G^i G^j}^i]_{p^i=p_A^i} + [V_{G^i p^i}^i]_{p^i=p_A^i} \left(\frac{\partial p_A^i}{\partial G^j} \right)}{[V_{G^i G^i}^i]_{p^i=p_A^i} + [V_{G^i p^i}^i]_{p^i=p_A^i} \left(\frac{\partial p_A^i}{\partial G^i} \right)}. \quad (\text{A.17})$$

From (A.15), (A.16) and (A.17) it can be seen that $|J| > 0$ if $(\partial B_A^1 / \partial G^2) (\partial B_A^2 / \partial G^1) < 1$ at an equilibrium point. Since $\partial^2 V_A^i / \partial (G^i)^2 < 0$, the sign of $\partial B_A^i / \partial G^j$ is determined by the sign of $\partial^2 V_A^i / \partial G^i \partial G^j$. As can be affirmed from the above, $\partial^2 V_A^i / \partial G^i \partial G^j > 0$ when $B_A^i(G^j) \geq G^j$, so G^i is a strategic complement for G^j in this case. Inspection of (A.16) reveals that G^i can become a strategic substitute for G^j when $B_A^i(G^j)$ is sufficiently smaller than G^j . Furthermore, from (A.11) it follows that $\text{sign} [V_{G^1 G^2}^1] = -\text{sign} [V_{G^2 G^1}^2]$ since $\phi_{G^1 G^2}^1 = -\phi_{G^2 G^1}^2$; therefore, there are two possibilities with regards to the signs of best-response functions at an equilibrium point. Either (i) $\partial B_A^i / \partial G^j > 0$ and $\partial B_A^j / \partial G^i \leq 0$ for $i \neq j = 1, 2$, or (ii) $\partial B_A^i / \partial G^j > 0 \quad \forall i \neq j = 1, 2$. It is easy to check that, in case (i), $|J| > 0$. Turning to case (ii), we now establish the existence of (sufficient) conditions that ensure $(\partial B_A^1 / \partial G^2) (\partial B_A^2 / \partial G^1) < 1$ and, therefore, $|J| > 0$.²⁹

²⁹Note that, in case (ii), $|J| > 0$ is also the condition for local stability of equilibrium.

First, apply (9) onto (A.8') and then onto (A.13) to obtain

$$\frac{\partial p_A^i}{\partial G^i} = -\frac{p_A^i \psi^i}{\Delta^i |\lambda^i| R^i s_K^i s_L^i} \theta_{LG}^i$$

$$\frac{\partial p_A^i}{\partial G^j} = \frac{p_A^i \psi^i}{\Delta^i |\lambda^i| R^i s_K^i s_L^i} \left(-\frac{\phi_{G^j}^i}{\phi_{G^i}^i} \right) s_L^i.$$

The above expressions together with (A.10), (A.11), and (A.12) can be substituted into (A.17)

to obtain $\partial B_A^i / \partial G^j = -(\phi_{G^j}^i / \phi_{G^i}^i) \Gamma_A^i$, where

$$\Gamma_A^i = \frac{-\frac{\phi_{G^i G^j}^i}{\phi_{G^j}^i} \Delta^i + \frac{\psi^i}{|\lambda^i| |\theta^i| R^i s_K^i s_L^i} (\theta_{LG}^i s_L^i)}{-\frac{\phi_{G^i G^i}^i}{\phi_{G^i}^i} \Delta^i + \frac{\psi^i}{|\lambda^i| |\theta^i| R^i s_K^i s_L^i} (\theta_{LG}^i)^2}.$$

From (A.1) and (A.2), we have $(\phi_{G^2}^1 / \phi_{G^1}^1)(\phi_{G^1}^2 / \phi_{G^2}^2) = 1$ which implies $(\partial B_A^1 / \partial G^2) (\partial B_A^2 / \partial G^1) = \Gamma_A^1 \Gamma_A^2$; therefore, if $\Gamma_A^i \in (0, 1) \forall i = 1, 2$, then $|J| > 0$. But, in case (ii), both the numerator and the denominator of Γ_A^i are positive, so $\Gamma_A^i > 0$. Now subtract the numerator of Γ_A^i from its denominator to obtain

$$\frac{\eta^i}{G^i} \left(\sigma_D^i + \frac{\delta_K^i + \delta_L^i}{|\lambda^i| |\theta^i|} \right) + \frac{\psi^i \theta_{LG}^i}{|\lambda^i| |\theta^i| R^i s_K^i s_L^i} (\theta_{LG}^i + \theta_{KG}^i \sigma_G^i \eta^i - s_L^i) \quad (\text{A.18})$$

where $\eta^i \equiv G^i f_i' / f_i - G^i f_i'' / f_i' (> 0)$. (To derive (A.18) we used the definition of Δ^i in (A.14) and (A.1)-(A.4) which imply $-\phi_{G^i G^i}^i / \phi_{G^i}^i + \phi_{G^i G^j}^i / \phi_{G^j}^i = \eta^i / G^i$.) Clearly, a sufficient condition for $\Gamma_A^i < 1$ is that (A.18) is positive. Inspection of (A.18) reveals that this is almost always true. A somewhat restrictive (but hardly necessary) condition for $\Gamma_A^i < 1$ is $\theta_{LG}^i + \theta_{KG}^i \sigma_G^i \eta^i - s_L^i \geq 0$ which is satisfied under a wide range of circumstances including the following two: (i) $\sigma_G^i \eta^i \geq s_L^i$ which requires arms inputs to be sufficiently close substitutes;³⁰ (ii) $\theta_{LG}^i \geq s_L^i$ (or, by (A.9),

³⁰This condition is always satisfied when the production function for guns is Cobb-Douglas and the CSF

$k^i > k_G^i$) which requires the guns sector to be sufficiently labor-intensive, regardless of the degree of substitutability between inputs in arms. The above conditions and $B_A^i(G^j) \in (0, \bar{G}^i)$ $\forall i \neq j = 1, 2$ (boundary conditions) establish uniqueness of equilibrium. $\quad ||$

Proof of Proposition 1. Since the logic behind part (a) was outlined in the main text, here we prove part (b). A small reallocation of labor from one country to another expands (reduces) the “recipient” (“donor”) country’s labor endowment. Differentiating country i ’s FOC condition in (9) appropriately gives

$$\frac{\partial^2 V_A^i}{\partial (G^i)^2} dB_A^i + \frac{\partial^2 V_A^i}{\partial G^i \partial L^i} dL^i = 0 \implies \frac{dB_A^i}{dL^i} = -\frac{\partial^2 V_A^i / \partial G^i \partial L^i}{\partial^2 V_A^i / \partial (G^i)^2}.$$

But, as can be ascertained from (9), Lemma 3(c), and Lemma 5(d),

$$\frac{\partial^2 V_A^i}{\partial G^i \partial L^i} = [V_{G^i L^i}^i]_{p^i=p_A^i} = [V_{G^i p^i}^i]_{p^i=p_A^i}^{(\pm)} \left(\frac{\partial p_A^i}{\partial L^i} \right)^{(\pm)} > 0. \quad (\text{A.19})$$

Since $\partial^2 V_A^i / \partial (G^i)^2 < 0$, by (A.15), it follows that $\frac{dB_A^i}{dL^i} > 0$; therefore, for any arms choice by its rival, the recipient (donor) country’s best-response will be to produce more (less) arms than before. Thus, if we start with an arbitrary endowment configuration on S^0 and then keep transferring labor from country j to country $i \neq j$ (so that we end up somewhere in S^i) we will necessarily have $G_A^{i*} > G_A^{j*}$. Applying this observation in Lemma 5 readily implies that $p_A^{i*} \geq p_A^{j*}$ as $k_2^i \geq k_1^i$, thus completing the proof. $\quad ||$

Proof of Proposition 2. To identify the effects of (some) endowment changes on equilibrium security policies we differentiate the FOCs in (9) solve the resulting system of equations

assumes the Tullock form (i.e., $f(G^i) = (G^i)^\gamma$, $\forall \gamma \in (0, 1]$). This is so because $\sigma_G^i = 1$ and $\eta^i = 1$, hence, $\theta_{LG}^i + \theta_{KG}^i \sigma_G^i \eta^i - s_L^i = 1 - s_L^i \geq 0$

to obtain

$$\begin{pmatrix} dG_A^{1*} \\ dG_A^{2*} \end{pmatrix} = \frac{1}{|J|} \begin{pmatrix} \frac{\partial^2 V_A^2}{\partial(G^2)^2} & -\frac{\partial^2 V_A^1}{\partial G^1 \partial G^2} \\ -\frac{\partial^2 V_A^1}{\partial G^2 \partial G^1} & \frac{\partial^2 V_A^1}{\partial(G^1)^2} \end{pmatrix} \begin{pmatrix} -\frac{\partial^2 V_A^1}{\partial G^1 \partial L^1} dL^1 - \frac{\partial^2 V_A^1}{\partial G^1 \partial K^1} dK^1 \\ -\frac{\partial^2 V_A^2}{\partial G^2 \partial L^2} dL^2 - \frac{\partial^2 V_A^2}{\partial G^2 \partial K^2} dK^2 \end{pmatrix} \quad (\text{A.20})$$

where, of course, all expressions are evaluated at the equilibrium. Start with an endowment allocation on S^0 , so that $G_A^{1*} = G_A^{2*} = \tilde{G}_A^*$ and $|J| > 0$.

Part (a): The fact that we evaluate expressions at a symmetric equilibrium implies: (i) $\frac{\partial^2 V_A^1}{\partial G^1 \partial L^1} = \frac{\partial^2 V_A^2}{\partial G^2 \partial L^2} > 0$ by (A.19) and, using similar logic, $\frac{\partial^2 V_A^1}{\partial G^1 \partial K^1} = \frac{\partial^2 V_A^2}{\partial G^2 \partial K^2} < 0$; (ii) $\frac{\partial^2 V_A^1}{\partial G^1 \partial G^2} = \frac{\partial^2 V_A^2}{\partial G^2 \partial G^1} > 0$ by (A.16) and because $V_{G^1 G^2}^1 = V_{G^2 G^1}^2 = 0$; (iii) $\frac{\partial^2 V_A^1}{\partial(G^1)^2} = \frac{\partial^2 V_A^2}{\partial(G^2)^2} < 0$ by (A.15); and (iv) $\frac{\partial B_A^1}{\partial G^2} = -\frac{\partial^2 V_A^1 / \partial G^1 \partial G^2}{\partial^2 V_A^1 / \partial(G^1)^2} = -(\phi_{G^2}^1 / \phi_{G^1}^1) \Gamma_A^1 \in (0, 1)$ by (A.17) and the analysis on the proof of Theorem 1. For concreteness, focus on a small transfer of labor from country 2 to country 1, so that $-dL^2 = dL^1 > 0$. Using the above observations in (A.20) yields

$$\frac{dG_A^{1*}}{dL^1} = -\frac{dG_A^{2*}}{dL^1} = \frac{1}{|J|} \left[-\frac{\partial^2 V_A^1}{\partial(G^1)^2} \right] \left(1 - \frac{\partial B_A^1}{\partial G^2} \right) \left(\frac{\partial^2 V_A^1}{\partial G^1 \partial L^1} \right) > 0.$$

The result on land transfers can be similarly derived.

Part (b): The proof is established by invoking symmetry and applying part (a) to (11). \parallel

Proof of Theorem 2. The proof is similar to that of Theorem 1. \parallel

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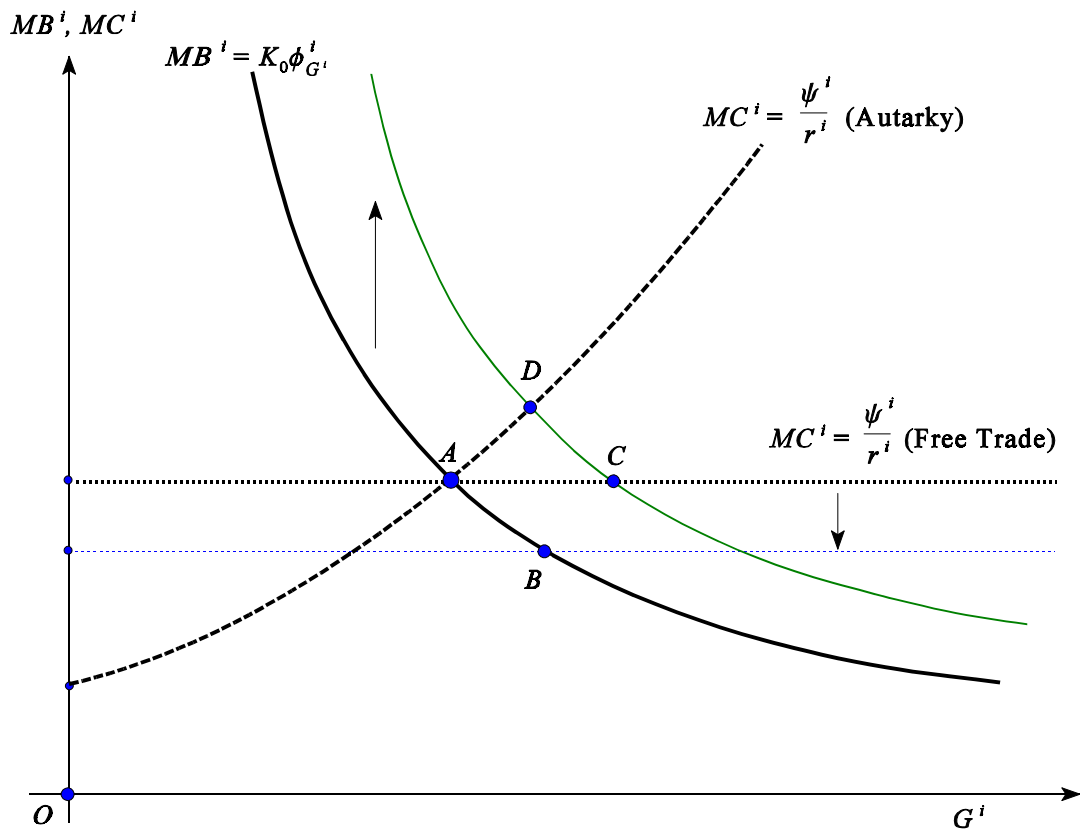


Figure 1

Individually Optimal Security Policies

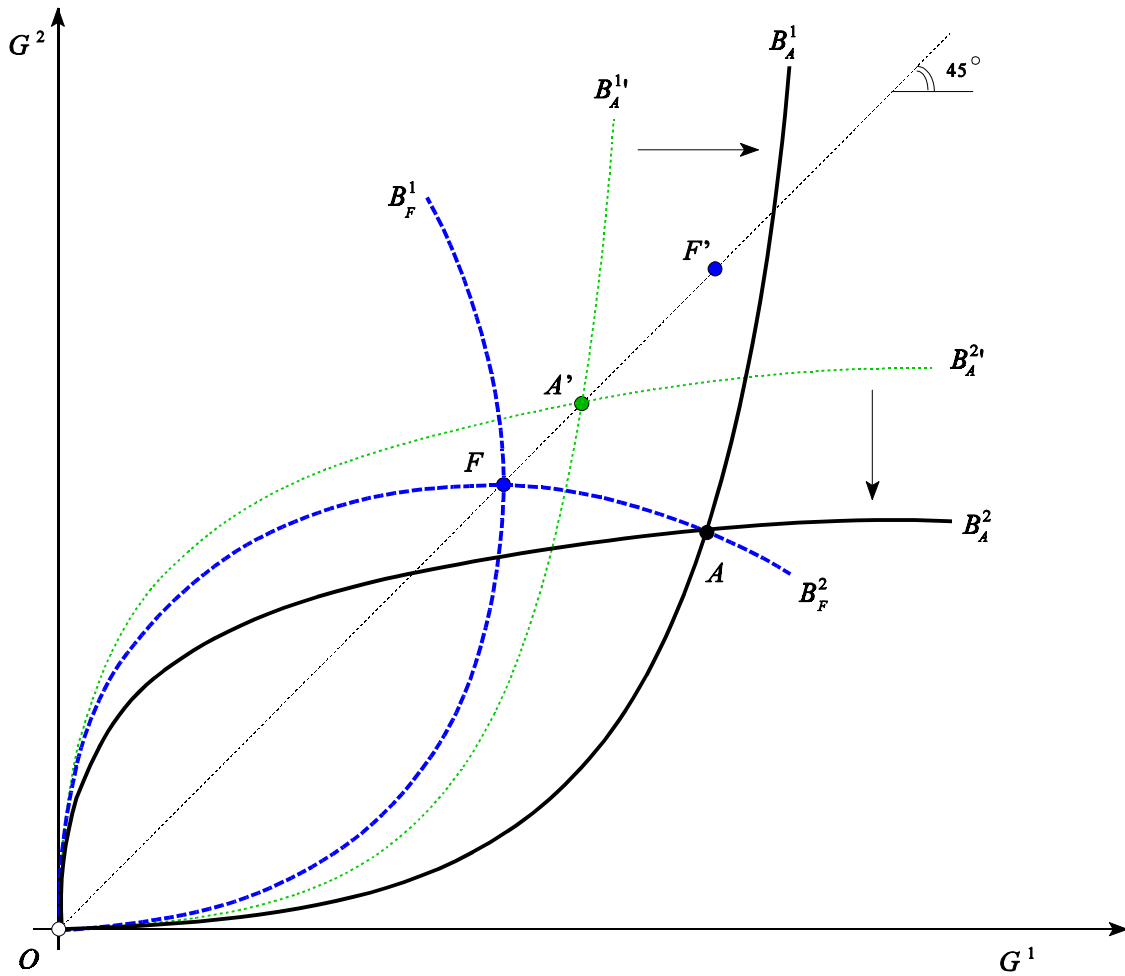


Figure 2

Best-Response Functions in Security Policies

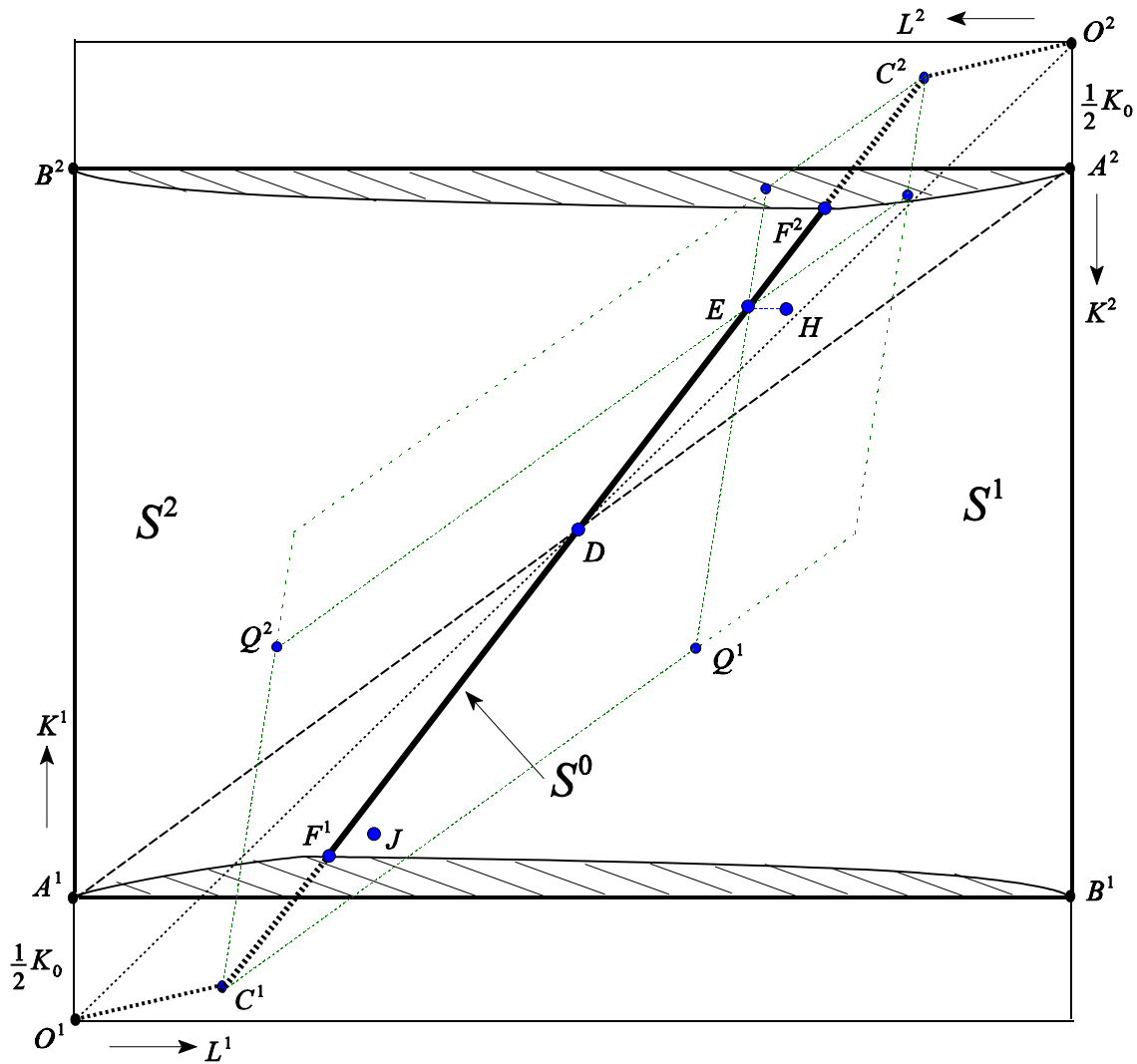


Figure 3

The Distribution of Factor Endowments, Sectoral Decomposition of Production, and Arming Incentives under Autarky

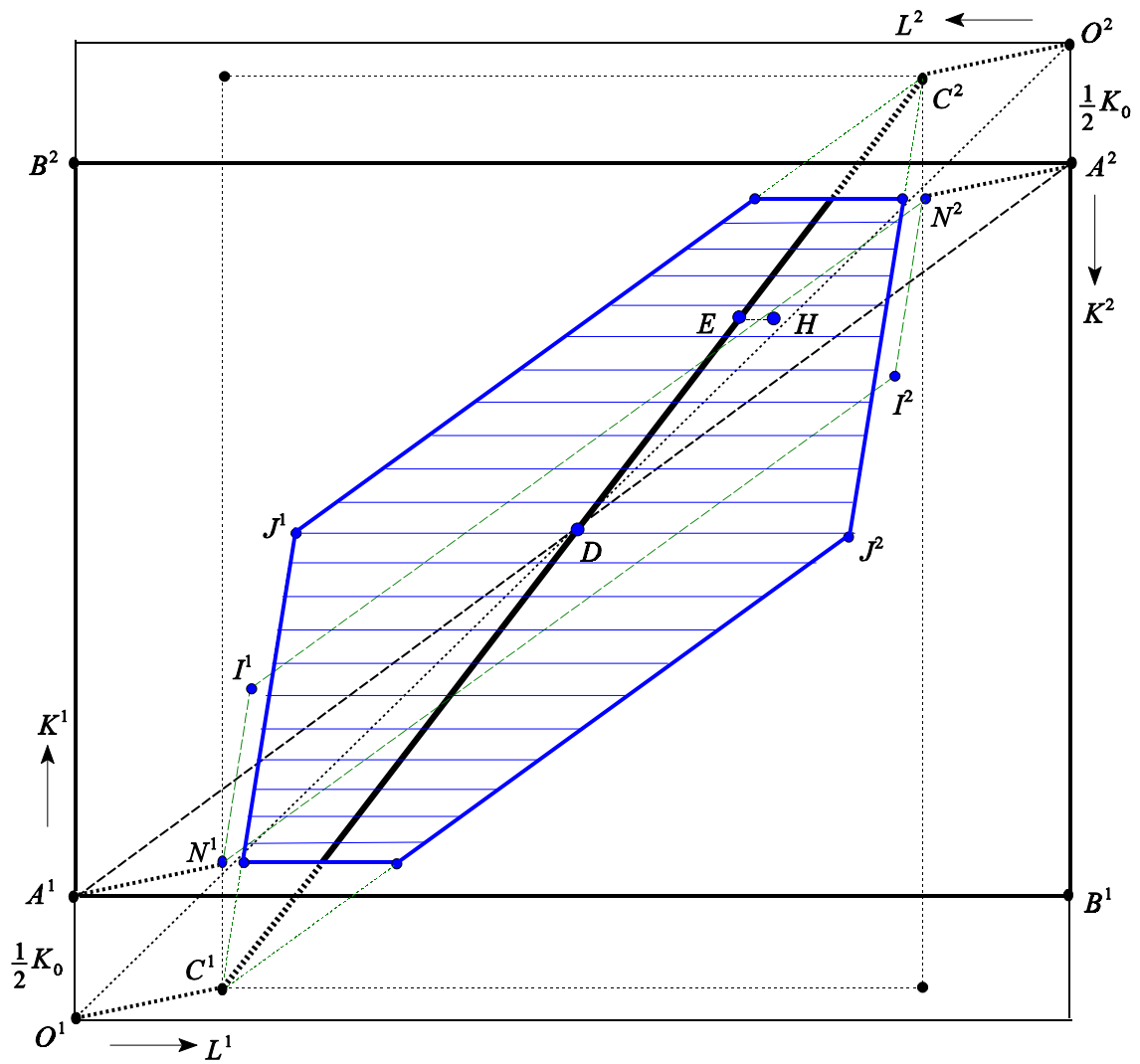


Figure 4

Free Trade and Arms Equalization Region

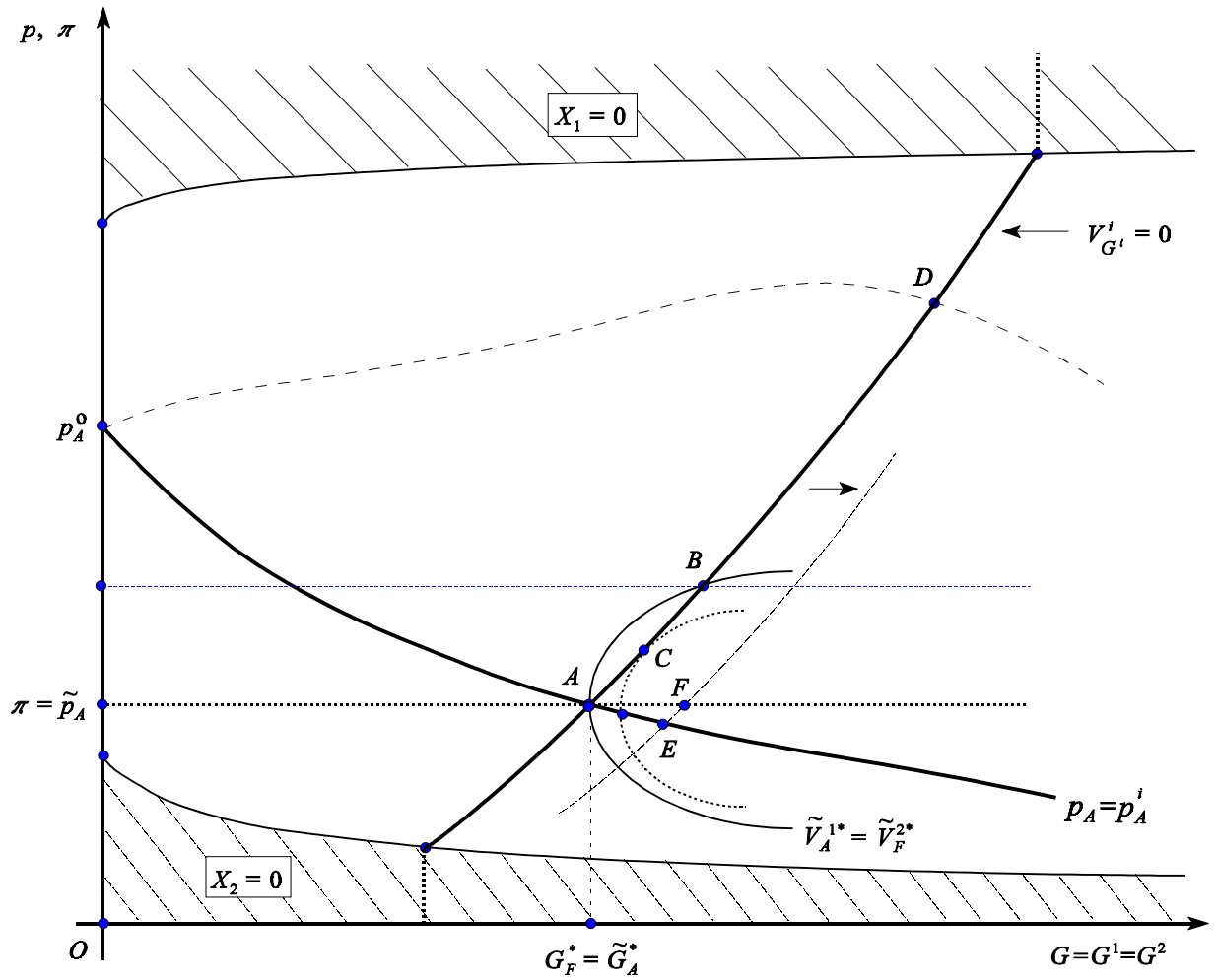


Figure 5

Welfare, Patterns of Trade, and Equilibrium Security Policies
with Identical Adversaries