

Import Tariffs and Growth in a Model with Habits

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1. Introduction

This paper studies the relationship between tariffs and economic growth in an endogenous growth framework. While most existing documents find a negative relationship between import tariffs and the rate of economic growth, there are works finding an ambiguous, or even positive, relationship.

In existing theoretical works, most studies maintained that tariffs slow down economic growth (e.g., Jones and Manuelli, 1990; Easterly and Rebelo, 1993; Osang and Pereira, 1996; and Ben-David and Loewy, 1998). However, others held different points of view. In a R&D model, for example, Grossman and Helpman (1990) argued that the relationship between tariffs and economic growth is ambiguous, depending on how the fraction of resources to the R&D sector is affected. Similar argument has been made in a R&D model by Rivera-Batiz and Romer (1991).¹

In empirical works, while most papers documented a negative relationship between tariff rates and economic growth (e.g., Lee, 1993; Harrison, 1996; and Edwards, 1998), Rodriguez and Rodrik (2001) and Sala-i-Martin (1997) contended that the relationship is not robust. Indeed, Yanikkaya (2003) has found a positive relationship using cross-country data over the period of 1970–1997. Moreover, O’Rourke (2000) and Irwin (2002) reported a positive relationship between tariffs and economic growth in the late 19th and early 20th century, as opposed to the otherwise consensus of a negative relationship in the Post-War era. Recently, Clemens and Williamson (2004) have confirmed that high tariffs were associated with fast growth before World War II, but with slow growth thereafter.

We build a dynamic trade model in a two-country (Home and Foreign), two-output (pure consumption and investment goods) Ricardian framework with capital accumulation as the source of economic growth, as opposed to R&D as the source of economic growth in the Grossman and Helpman (1990) and Rivera-Batiz and Romer (1991) model. In this paper, we use Ak technologies in production. The choice is made in order to differentiate the role of physical

¹ Rivera-Batiz and Romer (1991) show that the growth rate is a non-monotone function of the tariff rate: it first declines as the tariff rate rises from zero, and then rises after a positive critical tariff rate, albeit the growth rate in the free trade regime is never reached.

capital accumulation from the human capital accumulation as a source of economic growth.² A special feature of our model is to incorporate time non-separable preferences; we introduce the endogenous rate of time preference and the average living standard in the foreign as benchmark consumption.

A growing body of empirical evidence using country or cross-country data has confirmed the importance of time non-separable preferences.³ In particular, an extensive literature on asset pricing and real business cycles lends further credit to the level of benchmark consumption being a significant determinant of consumption behavior.⁴ Habit-forming consumers dislike large and rapid cuts in consumption. While the endogenous rate of time preference equalizes the marginal products of capital to the subjective rate of discount across countries in order to avoid sudden capital flight following a shock, the average living standard results in a large premium to hold risky assets that might force a rapid cut in consumption.

Despite this evidence supporting the relevance of benchmark consumption levels for current consumption decisions, no attempts have been made to introduce time non-separable preferences into endogenous growth literature in an open economy framework. Recently, this approach has been pursued by Carroll et al. (2000), Fisher and Hof (2000), Alonso-Carrera et al. (2005) and Chen (2006) in an endogenous growth, closed-economy framework.⁵ Our model may be considered as an extension of these works to an open-economy framework. In particular, we investigate the effects of tariffs on the trade patterns and the sustainable rate of economic growth that have never been studied in these existing works.

Our main findings may be summarized as follows. First, the Ricardian theorem of comparative advantage holds in the two-country world even if the preferences are different across countries. Second, in the case of incomplete specialization, local indeterminacy emerges under milder conditions, as compared to Dugeon (1998) and Nishimura and Shimomura (2006). Third, in the case of complete specialization, the relationship between tariffs and economic growth is

² The introduction of human capital accumulation needs a third sector that is non-tradable, pure investment goods. This makes the analysis complicated without adding the value.

³ See Van de Stadt et al. (1985) using data in the Netherlands, Osborne (1988) using data in the U.K., and Further and Klein (1998) using data in the G-7 countries.

⁴ See Abel (1990), Constantinides (1990), Campbell and Cochrane (1999) on asset pricing and Ljungqvist and Uhlig (2000) and Boldrin et al. (2001) on real business cycles.

⁵ In contrast, there is a number of works in a bounded growth framework. Examples include Dupor and Liu (2003), Alonso-Carrera et al. (2004, 2006), Liu and Turnovsky (2005), among others.

ambiguous. When the Home country specializes in the investment (*resp.* consumption) goods, a sufficiently higher rate of tariffs on the consumption (*resp.* investment) goods reverses the trade pattern in the long run and decreases economic growth when the productivity coefficient of the investment goods in the Home (*resp.* Foreign) country is larger than the threshold. However, economic growth is increased when the productivity coefficient of the investment goods in the Home (*resp.* Foreign) country is smaller and in the Foreign (*resp.* Home) country is larger than the threshold. Finally, tariffs increase (*resp.* decrease) the long-run welfare in the Home country when it specializes in the investment (*resp.* consumption) goods.

Intuitively, the effects of a tariff on economic growth work through its effect on the difference between the interest rate and the rate of time preference. The import tariff raises the price of the imported goods in the Home country, attracting more resources into this sector. As a result of a higher demand for capital, the interest rate is increased in the Home country. However, the international price of the imported goods is reduced by augmented output supply, thereby decreasing the demand for capital and the interest rate in the Foreign. Higher domestic and lower foreign interest rates indicate that the ratio of Home to Foreign consumption increases (*resp.* decreases) when the Home country is specialized in the investment (*resp.* consumption) goods, resulting in a higher (*resp.* lower) rate of time preference in the Home country. Moreover, the change in the interest rates in the Home and Foreign countries depends on the productivity level in the investment goods sector. When the productivity level of the investment goods in the Home (*resp.* Foreign) country is higher than the threshold, the increase in the interest rate is smaller than the increase in the rate of time preference. As a result, economic growth is lower. When the productivity level of the investment goods in the Home (*resp.* Foreign) country is smaller than the threshold and in the Foreign (*resp.* Home) country is larger than the threshold, the former makes the interest rate increase more, whereas the latter makes the rate of time preference increase less. As a result, the rate of economic growth is increasing in tariffs.

Finally, the reason for the effect on long-run welfare is simple. The long-run welfare in the Home country is increasing in the ratio of consumption between the Home and Foreign country. When the Home country specializes in the investment (*resp.* consumption) goods, tariffs on the imported consumption (*resp.* investment) goods increase (*resp.* decrease) the ratio of consumption between the Home and Foreign country in the long run, thereby increasing (*resp.* decreasing) long-run welfare in the Home country.

The structure of the paper is as follows. In section 2, we set up the basic model. In Section

3, we analyze the balanced growth path and the transitional dynamics. In Section 4, we study the relationship between the import tariff policy and economic growth. In Section 5, we investigate the welfare effect of tariffs. Finally, some concluding remarks are made in Section 6.

2. The Basic Model

2.1. Firms

There are two sectors in each country: a pure consumption good and a pure investment good sector; both use only capital, k_i .⁶ The capital may be thought of as a composite of various types of physical and human capital as outlined in Rebelo (1991). Following the Oniki and Uzawa tradition, we assume that while the two goods are tradable, capital stock is not internationally mobile. The production function of good i is denoted as

$$y_i = a_i k_i, \quad i=1, 2. \quad (1)$$

where $i=1$ (resp. 2) corresponds to the consumption (resp. investment) good. The full employment conditions are

$$k = k_1 + k_2, \quad (2)$$

where k is the total capital stock.

The first-order conditions for Home's representative competitive firm are

$$p a_1 = r, \quad (3a)$$

$$a_2 = r, \quad (3b)$$

where p is the price of consumption good in terms of the investment good; r is the interest rate, and thus the net rental rate of capital if we assume that there is no depreciation of capital

There are two countries in the world, Home and Foreign, which may have different production technologies and preferences. Suppose the Foreign has a similar production structure to those in (1) and (2). Denote the variables with an asterisk as in the Foreign. Then this is a Ricardian model, which means that the trade pattern is determined by comparative advantage. Thus, we classify into three types of the trade pattern.

Case 1: $a_1/a_2 = a_1^*/a_2^*$

This is the case of incomplete specialization with each country producing both goods. Conditions (3) imply that both p and r are constant; i.e. $p = a_2/a_1$ and $r = a_2$.

Case 2: $a_1/a_2 < a_1^*/a_2^*$

⁶ With only one consumption good, the quadratic utility form used in existing studies can be relaxed.

Complete specialization arises in this case. The Home country only produces and exports the investment goods and imports the consumption goods that is produced solely by the Foreign; hence, $k_1=0, k_2=k, k_1^*=k^*$ and $k_2^*=0$. Therefore, Conditions (3) indicates that $r=a_2$ and $r^*=a_1^*p$.

Case 3: $a_1/a_2 > a_1^*/a_2^*$

In this case, the trade pattern is reversed and thereby $k_1=k, k_2=0, k_1^*=0$ and $k_2^*=k^*$. Thus, Conditions (3) indicates that $r=a_1p$ and $r^*=a_2^*$.

In sum, combining (2) and (3), we obtain

$$\left\{ \begin{array}{l} k = k_1, k_2 = 0; \quad k^* = k_2^*, k_1^* = 0; \quad r = a_1p; \quad r^* = a_2^*; \quad a_2/a_1 < p < a_2^*/a_1^*; \quad a_1/a_2 > a_1^*/a_2^* \\ k = k_1 + k_2; \quad k^* = k_1^* + k_2^*; \quad r = a_2; \quad r^* = a_2^*; \quad p = a_2/a_1; \quad \text{if } a_1/a_2 = a_1^*/a_2^*. \\ k = k_2, k_1 = 0; \quad k^* = k_1^*, k_2^* = 0; \quad r = a_2; \quad r^* = a_1^*p; \quad a_2/a_1 > p > a_2^*/a_1^*; \quad a_1/a_2 < a_1^*/a_2^* \end{array} \right. \quad (4)$$

2.2 Households

The representative household earns factor income with $k(0)$ units of capital endowed initially. The household's budget constraint is

$$\dot{k} = rk - pc, \quad (5)$$

where c is consumption.

The felicity of the representative agent is $u(c, c^*)$, where c^* is the average consumption level in the foreign country, a consumption externality. The felicity allows for the idea that the representative household cares about his living standard relative to the average living standard in the Foreign. We assume that $u(c, c^*)$ is increasing and concave in c . Moreover, we assume that $u(c, c^*)$ is decreasing in c^* . The assumption is similar the one made in Abel (1990) and Dupor and Liu (2003) in which an individual is *keeping up with the Joneses*.

The representative agent maximizes the discounted sum of the lifetime felicity, with discount factor X . The discount factor changes in the following way;

$$\dot{X} = -r(u)X. \quad (6)$$

Following Uzawa (1968), we call $r(u)$ the time preference function and consider

Assumption 1 $0 < r(0) < a_2$ and $r(u) > 0, r'(u) > 0, r''(u) < 0, r(u) - ur'(u) > 0$ for all $u > 0$.

We use the following form for the felicity,

$$u(c, c^*) = u\left(\frac{c}{c^*}\right). \quad (7)$$

While the above form is consistent with the assumption of a positive and decreasing marginal

utility in c and a decreasing utility in c^* , let us make some remarks. In Abel (1990), Gali (1994), Chen (2006) and others, the form $u[\frac{c}{(c^*)^\beta}]$ is used, where $0 < \beta = 1$. Our form is a special case of this class of felicity that takes the extreme case at $\beta = 1$. The reason for choosing $\beta = 1$ is due to the endogenous rate of time preference in an open economy with perpetual growth as follows.

In a two-country dynamic Heckscher-Ohlin model by Baxter (1992), the responses to taxes and other shocks are very dramatic and not continuous. Chen, et al. (2005) finds that introducing an endogenous rate of time preference avoids such a problem and assures continuous responses. In a recent paper with endogenous rates of time preference Palivos et al. (1997) showed that the rate of time preference must be constant along a *Balanced Growth Path (BGP)*. Given that the rate of time preference is monotone in the felicity, the only way to guarantee a constant rate of time preference along a BGP is $\beta = 1$. Moreover, our form (7) is consistent with the recent paper on endogenous rates of time preference by Druegon (1998) that shows that the homogeneity of degree zero with respect to an individual consumption and the living standard is a necessary condition for the existence of a BGP.⁷ Thus, although our function form is restrictive, it is necessary in order to assure a BGP. Nevertheless, when we introduce the distortionary tariff policy, the consumption ratio in a BGP could deviate from unity.

Let $z = c/c^*$. We make the following standard assumptions for the felicity in (7).

Assumption 2 $u(0) = 0$, $u'(z) > 0$, $u''(z) < 0$, $[u'(z)z]/u(z) < 1$.

If we denote \mathbf{I} and \mathbf{m} as the co-state variables of k and X , the Hamiltonian of the household optimization problem is

$$\mathfrak{H} = u\left(\frac{c}{c^*}\right)X + \mathbf{I}\{rk - pc\} + \mathbf{m}\{-r(u)X\},$$

The necessary conditions for optimality are

$$u'\left(\frac{c}{c^*}\right)[1 - \mathbf{m}r'(u\left(\frac{c}{c^*}\right))] = \frac{\mathbf{I}pc^*}{X}, \quad (8a)$$

$$\frac{\dot{\mathbf{I}}}{\mathbf{I}} = -r, \quad (8b)$$

$$\frac{\dot{\mathbf{m}}}{\mathbf{m}} = r(u\left(\frac{c}{c^*}\right)) - \frac{1}{\mathbf{m}}u\left(\frac{c}{c^*}\right), \quad (8c)$$

⁷ In a recent paper Druegon (1998) introduces endogenous discounting that is increasing in consumption and decreasing in the living standard in a society, measured by the average consumption in the society. Our formulation is consistent with Druegon (1998) in that the discounting is decreasing in the average foreign living standards.

with the two transversality conditions $\lim_{t \rightarrow \infty} \mathbf{I}k = 0$, and $\lim_{t \rightarrow \infty} \mathbf{m}X = 0$.

While condition (8a) equates the marginal utility of consumption to the marginal cost of foregone savings, conditions (8b-c) are Euler equations for capital and the discount, respectively.

3. Two-country World Market Equilibrium

The world commodity market-clearing condition for the consumption goods is

$$c + c^* = a_1 k_1 + a_1^* k_1^*. \quad (9)$$

Once (9) is satisfied, the world market for the investment goods is automatically cleared.

3.1. Transformation of the Economic System

In order to analyze the equilibrium, it is necessary to transform the equilibrium conditions with perpetual growth into a system with stationary variables. Denote $m = pc/k$, $m^* = pc^*/k^*$, $v_1 = k_1/k$, $v_2 = k_2/k$, $v_1^* = k_1^*/k^*$ and $v_2^* = k_2^*/k^*$. In what follows we briefly explain the transformation.

First, (8c) in terms of the transformed variables and its counterpart for the Foreign are

$$\frac{\dot{m}}{m} = \mathbf{r}(u(z)) - \frac{1}{m}u(z), \quad (10a)$$

$$\frac{\dot{m}^*}{m^*} = \mathbf{r}^*(u(\frac{1}{z})) - \frac{1}{m^*}u^*(\frac{1}{z}). \quad (10b)$$

Next, differentiating (8a) and its foreign counterpart, with (8b) (10a) and (10b), yields

$$\frac{\dot{c}}{c} = \frac{1}{c(z)} \left\{ r - \mathbf{r}(u(z)) - [1 - \mathbf{c}(z)] \frac{\dot{c}^*}{c^*} - \frac{\dot{p}}{p} - \frac{\mathbf{r}'(u(z))[\mathbf{m}\mathbf{r}(u(z)) - u(z)]}{1 - \mathbf{m}\mathbf{r}'(u(z))} \right\}, \quad (11a)$$

$$\frac{\dot{c}^*}{c^*} = \frac{1}{c^*(z)} \left\{ r^* - \mathbf{r}^*(u^*(\frac{1}{z})) - [1 - \mathbf{c}^*(z)] \frac{\dot{c}}{c} - \frac{\dot{p}}{p} - \frac{\mathbf{r}^{*\prime}(u^*(\frac{1}{z}))[\mathbf{m}^*\mathbf{r}^*(u^*(\frac{1}{z})) - u^*(\frac{1}{z})]}{1 - \mathbf{m}^*\mathbf{r}^{*\prime}(u^*(\frac{1}{z}))} \right\}, \quad (11b)$$

where $\mathbf{c}(z) \equiv \mathbf{s}(z) + \frac{\mathbf{m}\mathbf{r}'(u(z))}{1 - \mathbf{m}\mathbf{r}'(u(z))}\mathbf{y}(z) > 0$, in which $\mathbf{s}(z) \equiv \frac{-u''(z)z}{u'(z)} > 0$ denotes the reciprocal of the intertemporal elasticity of substitution in the Home country and $\mathbf{y}(z) \equiv \frac{\mathbf{r}'(u(z))u'(z)z}{\mathbf{r}''(u(z))} > 0$, the elasticity of the marginal time preference rate with respect to z in the Home country. Similarly, $\mathbf{c}^*(z) \equiv \mathbf{s}^*(z) + \frac{\mathbf{m}^*\mathbf{r}^{*\prime}(\frac{1}{z})}{1 - \mathbf{m}^*\mathbf{r}^{*\prime}(\frac{1}{z})}\mathbf{y}^*(z) > 0$, $\mathbf{s}^*(z) \equiv \frac{-u^{*\prime}(\frac{1}{z})}{u^{*\prime}(\frac{1}{z})z}$ and $\mathbf{y}^*(z) \equiv \frac{\mathbf{r}^{*\prime}(\frac{1}{z})u^{*\prime}(\frac{1}{z})}{\mathbf{r}^{*\prime}(\frac{1}{z})z}$ are the corresponding counterparts in the Foreign country.

Then, if we substitute $\frac{\dot{p}}{p}$ in (11a) into (11b), we obtain

$$\frac{\dot{z}}{z} = \frac{1}{\mathbf{c}(z) + \mathbf{c}^*(z) - 1} \left\{ r - \mathbf{r}(u(z)) - r^* + \mathbf{r}^*(u^*(\frac{1}{z})) - \frac{\mathbf{r}'(u(z))[\mathbf{m}\mathbf{r}(u(z)) - u(z)]}{1 - \mathbf{m}\mathbf{r}'(u(z))} + \frac{\mathbf{r}'(u^*(\frac{1}{z}))[\mathbf{m}^*\mathbf{r}^*(u^*(\frac{1}{z})) - u^*(\frac{1}{z})]}{1 - \mathbf{m}^*\mathbf{r}'^*(u^*(\frac{1}{z}))} \right\}. \quad (12)$$

Moreover, (5) is rewritten as

$$\frac{\dot{k}}{k} = r - m. \quad (13)$$

Using (11a)-(13), we obtain

$$\frac{\dot{m}}{m} = \mathbf{c}^*(z) \frac{\dot{z}}{z} + r^* - \mathbf{r}^*(u^*(\frac{1}{z})) - \frac{\mathbf{r}'^*(u^*(\frac{1}{z}))[\mathbf{m}^*\mathbf{r}^*(u^*(\frac{1}{z})) - u^*(\frac{1}{z})]}{1 - \mathbf{m}^*\mathbf{r}'^*(u^*(\frac{1}{z}))} - r + m, \quad (14a)$$

$$\frac{\dot{m}^*}{m^*} = -\mathbf{c}(z) \frac{\dot{z}}{z} + r - \mathbf{r}(u(z)) - \frac{\mathbf{r}'(u(z))[\mathbf{m}\mathbf{r}(u(z)) - u(z)]}{1 - \mathbf{m}\mathbf{r}'(u(z))} - r^* + m^*. \quad (14b)$$

Further, dividing (9) by c^* , the world market-clearing condition may be rewritten as

$$z + 1 = p \left(\frac{a_2 v_1}{m} + \frac{a_1 v_1^*}{m^*} \right). \quad (15)$$

Finally, the full employment conditions and the first-order conditions for the firm summarized in (4) can now be rewritten as follows.

$$\begin{cases} v_1 = v_2^* = 1; & v_2 = v_1^* = 0; & r = a_1 p; & r^* = a_2^*; & a_2/a_1 < p < a_2^*/a_1^*; & a_1/a_2 > q^*/a_2^* \\ v_1 + v_2 = 1; & v_1^* + v_2^* = 1; & r = a_2; & r^* = a_2^*; & p = a_2/a_1; & \text{if } a_1/a_2 = q^*/a_2^*. \\ v_2 = v_1^* = 1; & v_1 = v_2^* = 0; & r = a_2; & r^* = a_1^* p & a_2/a_1 > p > a_2^*/a_1^*; & a_1/a_2 < q^*/a_2^* \end{cases} \quad (16)$$

Equations (10a), (10b), (12) and (14a)-(16) are the transformed equilibrium system. The system determines the equilibrium paths of the twelve control variables, z , m , m^* , \mathbf{m} , \mathbf{m}^* , p , v_1 , v_2 , v_1^* , v_2^* , r and r^* .

3.2. Balanced Growth Path

We now analyze the equilibrium in a steady state. A steady state is a perfect foresight equilibrium with a *balanced growth path* (BGP) under which z , m , m^* , \mathbf{m} , \mathbf{m}^* , p , v_1 , v_2 , v_1^* , v_2^* , r and r^* are constant, and thus $\dot{z}/z = \dot{m}/m = \dot{m}^*/m^* = \dot{p}/p = \dot{\mathbf{m}}/\mathbf{m} = \dot{\mathbf{m}}^*/\mathbf{m}^* = 0$. Denote $\tilde{z}, \tilde{m}, \tilde{m}^*, \tilde{\mathbf{m}}, \tilde{\mathbf{m}}^*, \tilde{p}, \tilde{v}_1, \tilde{v}_2, \tilde{v}_1^*, \tilde{v}_2^*, \tilde{r}$ and \tilde{r}^* as the values in a BGP. Then, they are determined by the following relationships.

$$\mathbf{r}[u(\tilde{z})]\tilde{\mathbf{m}} = u(\tilde{z}), \quad (17a)$$

$$\mathbf{r}^*[u^*(\frac{1}{\tilde{z}})]\tilde{\mathbf{m}}^* = u^*(\frac{1}{\tilde{z}}), \quad (17b)$$

$$\tilde{r} - \mathbf{r}[u(\tilde{z})] = \tilde{r}^* - \mathbf{r}^*[u^*(\frac{1}{\tilde{z}})], \quad (17c)$$

$$\tilde{r}^* - \mathbf{r}^*[u^*(\frac{1}{\tilde{z}})] = \tilde{r} - \tilde{m}, \quad (17d)$$

$$\tilde{r} - \mathbf{r}[u(\tilde{z})] = \tilde{r}^* - \tilde{m}^*, \quad (17e)$$

$$\tilde{z} + 1 = \tilde{p} \left(\frac{a_2 \tilde{v}_1}{\tilde{m}} + \frac{a_1^* \tilde{v}_1^*}{\tilde{m}^*} \right), \quad (17f)$$

$$1 = \tilde{v}_1 + \tilde{v}_2, \quad 1 = \tilde{v}_1^* + \tilde{v}_2^*, \quad (17g)$$

$$\begin{cases} \tilde{v}_2 = \tilde{v}_1^* = 0, & \text{if } a_1/a_2 > a_1^*/a_2^*, \\ \tilde{p} = \frac{a_2}{a_1} = \frac{a_2^*}{a_1^*}, & \text{if } a_1/a_2 = a_1^*/a_2^*, \\ \tilde{v}_1 = \tilde{v}_2^* = 0, & \text{if } a_1/a_2 < a_1^*/a_2^*, \end{cases} \quad (17h)$$

$$\tilde{r} = \begin{cases} a_1 \tilde{p} \geq a_2, & \text{if } a_1/a_2 \geq a_1^*/a_2^*, \\ a_2 \geq a_1 \tilde{p}, & \text{if } a_1/a_2 \leq a_1^*/a_2^*, \end{cases} \quad \tilde{r}^* = \begin{cases} a_2^* \geq a_1^* \tilde{p}, & \text{if } a_1/a_2 \geq a_1^*/a_2^*, \\ a_1^* \tilde{p} \geq a_2^*, & \text{if } a_1/a_2 \leq a_1^*/a_2^*. \end{cases} \quad (17i)$$

Three cases are in order to analyze the existence of BGP.

Case 1: $a_1/a_2 = a_1^*/a_2^*$ (Incomplete specialization.)

First, (17h) and (17i) indicate that $\tilde{p} = a_2/a_1$, $\tilde{r} = a_2$ and $\tilde{r}^* = a_2^*$ and are all constant. Substituting them into (17c) yields

$$a_2 - \mathbf{r}[u(\tilde{z})] = a_2^* - \mathbf{r}^*[u^*(\frac{1}{\tilde{z}})] \quad (18a)$$

that determines a unique \tilde{z} under Assumptions 1 and 2.

Next, if we substitute \tilde{z} into (17a), (17b), (17d) and (17e), $\tilde{\mathbf{m}}$, $\tilde{\mathbf{m}}^*$, \tilde{m} , and \tilde{m}^* are determined uniquely, respectively. Finally, if we substitute in \tilde{z} , \tilde{m} and \tilde{m}^* , the three relationships in (17f) and (17g) need to determine the four endogenous variables: $\tilde{v}_1, \tilde{v}_2, \tilde{v}_1^*$ and \tilde{v}_2^* . Multiplicity of BGPs thus emerges. Given a set of values for \tilde{v}_1 and \tilde{v}_1^* , then \tilde{v}_2 and \tilde{v}_2^* can be uniquely determined by the two relationships in (17g).

Using (17f), the set of values for \tilde{v}_1 and \tilde{v}_1^* is determined by

$$(\tilde{v}_1, \tilde{v}_1^*) = \left\{ (\tilde{v}_1, \tilde{v}_1^*) \mid \tilde{z} + 1 = \tilde{p} \left(\frac{a_2 \tilde{v}_1}{\tilde{m}} + \frac{a_1^* \tilde{v}_1^*}{\tilde{m}^*} \right) \right\}.$$

It is obvious that there is a continuum of BGPs, indexed by \tilde{v}_1 . For each \tilde{v}_1 , there exists a

unique \tilde{v}_1^* that satisfies the equilibrium condition in the above expression. As the choice of the value of \tilde{v}_1 in $(0,1)$ is free, there is a continuum of BGPs. The results indicate that any combination of incomplete specialization that satisfies the market-clearing conditions in the two countries is a long-run equilibrium. As both $\tilde{v}_1 > \tilde{v}_1^*$ and $\tilde{v}_1 < \tilde{v}_1^*$ could emerge, the long term trade patterns are thus indeterminate for these two countries.

Case 2: $a_1/a_2 < a_1^*/a_2^*$. (Complete specialization.)

In this case, the Home country completely specializes in the production of and exports the investment goods, while the Foreign country completely specializes in and exports the consumption goods.

Substituting (17h) and (17g) into (17f), we obtain $a_1\tilde{p}/(\tilde{z}+1) = \tilde{m}^*$. This relationship, together (17c), (17e) and (17i), yields

$$a_2 - \mathbf{r}(u(\tilde{z})) = \mathbf{r}^* \left(u^* \left(\frac{1}{\tilde{z}} \right) \right) \tilde{z}. \quad (18b)$$

The left hand side of (22) is decreasing in \tilde{z} and the right hand side of (18b) is increasing in \tilde{z} under Assumptions 1 and 2.⁸ Thus, (18b) determines a unique \tilde{z} . Then, variables $\tilde{r}, \tilde{r}^*, \tilde{\mathbf{m}}, \tilde{\mathbf{m}}^*, \tilde{m}$ and \tilde{m}^* are uniquely determined by the other equations. In particular, $\tilde{v}_1 = \tilde{v}_2^* = 0$ and $\tilde{v}_2 = \tilde{v}_1^* = 1$. Using (17i), the equilibrium relative price of the consumption goods in the long run is therefore uniquely determined as $\tilde{p} = \tilde{r}^* / a_1^*$.

Case 3: $a_1/a_2 > a_1^*/a_2^*$. (Complete specialization.)

In this case, the Home country completely specializes in the production of and exports the consumption goods, while the Foreign completely specializes in and exports the investment goods.

First, we use (17c), (17d), (17f), (17h) and (17i) to obtain

$$a_2^* - \mathbf{r}^* \left[u^* \left(\frac{1}{\tilde{z}} \right) \right] = \frac{\mathbf{r}[u(\tilde{z})]}{\tilde{z}}. \quad (18c)$$

Assumptions 1 and 2 assure the existence of a unique \tilde{z} . Then, other variables are determined in the fashion similar to the way used in Case 2. The equilibrium relative price of the consumption goods is $\tilde{p} = \tilde{r} / a_1$.

We should mention that in a BGP, the rates of economic growth for consumption and income

⁸ $\frac{\partial}{\partial \tilde{z}} \left(\mathbf{r}^* \left(u^* \left(\frac{1}{\tilde{z}} \right) \right) \tilde{z} \right) = \mathbf{r}^* - u^* \mathbf{r}^{*'} \left(\frac{u^*}{\tilde{z}} \right) > 0$, and $\frac{\partial}{\partial \tilde{z}} \left(\frac{\mathbf{r}(u(\tilde{z}))}{\tilde{z}} \right) = \frac{-[\mathbf{r} - u \mathbf{r}'(u/\tilde{z})]}{\tilde{z}^2} < 0$, under Assumption 1 and 2.

are equal. The long-run rate of economic growth is determined as follows.

In a BGP, $\frac{\dot{p}}{p} = 0$, and moreover, (17a) and (17b) indicates $\tilde{m} = u / r$ and $\tilde{m}^* = u^* / r^*$.

Substituting these relationships into (11a) and (11b), the economic growth rates in the two countries are respectively

$$\frac{\dot{c}}{c} = \frac{1}{c(z)} \{r - r(u(z)) - [1 - c(z)] \frac{\dot{c}^*}{c^*}\}, \quad (19a)$$

$$\frac{\dot{c}^*}{c^*} = \frac{1}{c^*(z)} \{r - r(u(z)) - [1 - c(z)] \frac{\dot{c}}{c}\}. \quad (19b)$$

Then, if we substitute $\frac{\dot{c}^*}{c^*}$ in (19b) into (19a), we obtain $\frac{\dot{c}}{c} = \frac{c^*}{[c+c^*-1]} \{r - r - \frac{1-c}{c^*} (r^* - r^*)\}$.

Using (17c), we then obtain

$$\frac{\dot{c}}{c} = r - r[u(z)]. \quad (20a)$$

In a similar fashion, we obtain

$$\frac{\dot{c}^*}{c^*} = r^* - r^*[u^*(\frac{1}{z})]. \quad (20b)$$

To summarize the results analyzed above, we obtain

Proposition 1 *Under AK technologies and Assumptions 1 and 2, the Ricardo theorem of comparative advantage holds in the two-country world even if the preferences are different across countries.*

Remark 1 The model is a Ricardo two-country endogenous growth model. The price of consumption is determined by the production technology. Thus, the trade pattern may be changed by the distortion of the firm's behavior but not of the household's behavior. For examples, output taxes or import tariffs may affect the trade pattern.

Remark 2 When the two countries have the same technologies, the *factor price equalization theorem* holds in the case of incomplete specialization, in which $r = a_2 = a_2^* = r^*$, but fails in the case of complete specialization with exception only when the following conditions are met.

$$\begin{cases} r \equiv [a_2^* - r^*(u^*(\frac{1}{z}))](1 + \tilde{z}) = a_2^* \equiv r^*, & \text{if } a_1/a_2 > a_1^*/a_2^*. \\ r \equiv a_2 = [a_2 - r(u(\tilde{z}))](\frac{1+\tilde{z}}{\tilde{z}}) \equiv r^*, & \text{if } a_1/a_2 < a_1^*/a_2^*. \end{cases}$$

3.3. Local Dynamics

In this subsection, we analyze the local dynamics. We only study the situation when both countries are incomplete specialization; i.e., under Case 1 where $a_1/a_2 = a_1^*/a_2^*$.

It is obvious that the dynamic system is recursive: While (10a), (10b), (12), (14a) and (14b) simultaneously govern the dynamics of z , m , m^* , \mathbf{m} and \mathbf{m}^* , the dynamics of other variables are easily determined by substituting these variables into other equations. Note that although there is a continuum of equilibrium values for $\tilde{v}_1, \tilde{v}_2, \tilde{v}_1^*$ and \tilde{v}_2^* , the equilibrium values for z, m, m^*, \mathbf{m} and \mathbf{m}^* , together \tilde{p}, \tilde{r} and \tilde{r}^* , are unique along a BGP. We consider

$$\text{Condition D: } \mathbf{s} + \frac{\mathbf{m}\mathbf{r}'}{1 - \mathbf{m}\mathbf{r}'}\mathbf{y} > 1 \quad \text{and} \quad \mathbf{s}^* + \frac{\mathbf{m}^*\mathbf{r}^{*\prime}}{1 - \mathbf{m}^*\mathbf{r}^{*\prime}}\mathbf{y}^* > 1.$$

Condition D requires a sufficiently small intertemporal elasticity of substitution; namely, a sufficiently large value of s .⁹

If we apply linear Taylor expansion of (10a), (10b), (12), (14a) and (14b) near the unique $(\tilde{\mathbf{m}}, \tilde{\mathbf{m}}^*, \tilde{z}, \tilde{m}, \tilde{m}^*)$, we obtain

$$\begin{pmatrix} \dot{\mathbf{m}} \\ \dot{\mathbf{m}}^* \\ \dot{z} \\ \dot{m} \\ \dot{m}^* \end{pmatrix} = \begin{pmatrix} \mathbf{r} & 0 & b_{13} & 0 & 0 \\ 0 & \mathbf{r}^* & b_{23} & 0 & 0 \\ b_{31} & b_{32} & 0 & 0 & 0 \\ b_{41} & b_{42} & 0 & \tilde{m} & 0 \\ b_{51} & b_{52} & 0 & 0 & \tilde{m}^* \end{pmatrix} \begin{pmatrix} \mathbf{m} - \tilde{\mathbf{m}} \\ \mathbf{m}^* - \tilde{\mathbf{m}}^* \\ z - \tilde{z} \\ m - \tilde{m} \\ m^* - \tilde{m}^* \end{pmatrix}, \quad (21)$$

where $b_{13} = -u'(1 - \mathbf{m}\mathbf{r}') < 0$,

$$b_{23} = \frac{u^{*\prime}}{z^2}(1 - \mathbf{m}^*\mathbf{r}^{*\prime}) > 0,$$

$$b_{31} = \frac{-\mathbf{r}'\mathbf{r}z}{(\mathbf{c} + \mathbf{c}^* - 1)(1 - \mathbf{m}\mathbf{r}')} < 0,$$

$$b_{32} = \frac{\mathbf{r}^{*\prime}\mathbf{r}^*z}{(\mathbf{c} + \mathbf{c}^* - 1)(1 - \mathbf{m}^*\mathbf{r}^{*\prime})} > 0,$$

$$b_{41} = \frac{-\mathbf{c}^*m\mathbf{r}'\mathbf{r}}{(\mathbf{c} + \mathbf{c}^* - 1)(1 - \mathbf{m}\mathbf{r}')} < 0,$$

$$b_{42} = \frac{-m(\mathbf{c} - 1)\mathbf{r}^{*\prime}\mathbf{r}^*}{(\mathbf{c} + \mathbf{c}^* - 1)(1 - \mathbf{m}^*\mathbf{r}^{*\prime})} < 0,$$

⁹ It is in general documented that s is larger than 1, so Condition D is easily met. Ogaki and Reinhart (1998) estimated the value using the US data and obtained $\sigma \in [2.22, 3.125]$. Yogo (2004) estimated data using cross-country data and uncovered $\sigma > 2$ across eleven developed countries which was smaller than $\sigma > 5$ he estimated for the U.S. economy.

$$b_{51} = \frac{-\mathbf{r}'\mathbf{r}m^*(\mathbf{c}^* - 1)}{(\mathbf{c} + \mathbf{c}^* - 1)(1 - \mathbf{m}\mathbf{r}')} < 0,$$

$$b_{52} = \frac{-m^*\mathbf{c}\mathbf{r}'\mathbf{r}^*}{(\mathbf{c} + \mathbf{c}^* - 1)(1 - \mathbf{m}^*\mathbf{r}'\mathbf{r}^*)} < 0.$$

Under Condition D, we obtain $b_{13} < 0$, $b_{23} > 0$, $b_{31} < 0$, $b_{32} > 0$, $b_{41} < 0$, $b_{42} < 0$, $b_{51} < 0$, and $b_{52} < 0$.

Notice that the system in (21) only includes variables that may jump instantaneously and does not involve any state variables whose values are predetermined. As a result, there exists a unique equilibrium path toward the BGP if the number of eigenvalues with negative real parts for the Jacobean matrix on the right hand side of (21) is zero. On the other hand, if the number of eigenvalues with negative real parts is larger than or equal to one, then there exists a continuum of equilibrium paths toward the BGP.

Denote by J the Jacobean matrix in (21), by \mathbf{q} the corresponding eigenvalues and by I an identity matrix of order 5. The eigenvalues are then determined by

$$\Omega(\mathbf{q}) \equiv |J - \mathbf{q}I| = (\tilde{m} - \mathbf{q})(\tilde{m}^* - \mathbf{q})\Gamma(\mathbf{q}) = 0,$$

where $\Gamma(\mathbf{q}) = \{-\mathbf{q}^3 + (\mathbf{r} + \mathbf{r}^*)\mathbf{q}^2 - (\mathbf{r}\mathbf{r}^* - b_{13}b_{31} - b_{23}b_{32})\mathbf{q} - (b_{13}b_{31}\mathbf{r}^* + b_{23}b_{32}\mathbf{r})\}$.

It is clear to see that two of the five eigenvalues are $\mathbf{q}_1 = \tilde{m} > 0$ and $\mathbf{q}_2 = \tilde{m}^* > 0$. The remaining three roots, denoted by \mathbf{q}_3 , \mathbf{q}_4 and \mathbf{q}_5 , are determined by $\Gamma(\mathbf{q})$. Using the result that

$$\Gamma(0) \equiv \mathbf{q}_3\mathbf{q}_4\mathbf{q}_5 = -(b_{13}b_{31}\mathbf{r}^* + b_{23}b_{32}\mathbf{r}) < 0,$$

there are either one or three roots with negative real parts. As a consequence, the dynamic equilibrium path toward the BGP is local indeterminacy. To summarize,

Proposition 2. *Under Condition D, suppose that the two countries in the world have technologies with identical comparative advantage, i.e. $a_1/a_2 = a_1^*/a_2^*$. Then, the BGP is locally indeterminate.*

Our result may be compared with that of Nishimura and Shimomura (2006), which finds the emergence of local indeterminacy in the neighborhood of a steady state in a similar two-country trade model with bounded growth.¹⁰ The local indeterminacy in our model with unbounded growth is established with weaker requirements. Our model shares with that of Nishimura and

¹⁰ In Nishimura and Shimomura (2006), the emergence of local indeterminacy requires that (i) the technology be of Leontief-type and the explicit utility form be quadratic, (ii) the consumption good be more labor intensive than the investment good and (iii) parameter values for technology and preference are restricted.

Shimomura (2006) in the use of an AK technology that is similar to a Leontief technology. Different from these authors, we use an implicit functional form for the felicity by assuming only one kind of consumption goods. As a result, the emergence of local indeterminacy in our model requires neither a particular factor intensity ranking nor specific restrictions on parameters, except for a sufficiently small intertemporal elasticity of substitution. However, there is an external effect in the felicity in our model emerging from the consumption standard in the Foreign that is absent in Nishimura and Shimomura (2006). Because of the external effect, restrictive conditions necessary to obtain indeterminacy in Nishimura and Shimomura (2006) are not required in our model.

The external effect is reminiscent of the consumption externalities in a closed-economy, unbounded growth model by Drugeon (1998). By assuming an agent's rate of time preference that is decreasing in the consumption standard measured by average consumption, Drugeon (1998) finds the emergence of local indeterminacy. Like Drugeon (1998) the rate of time preference is decreasing in the consumption externalities in our model. Yet, different from Drugeon (1998), the felicity is decreasing in the consumption externalities in our model. As a result, the consumption externalities may be likely to offset the possibilities of the emergence of local indeterminacy via the negative effect on the felicity. Because of the open economies in our model, the emergence of local indeterminacy does not require the former effect to dominate the latter effect.

4. Import Tariff and Growth

In this section, we examine the long-run relationship between import tariffs and growth. To simplify the analysis, we assume two countries identical in every aspect, except for the technologies. As a result, $\mathbf{r}^*(u^*(\frac{1}{Z})) = \mathbf{r}(u(\frac{1}{Z}))$. As the case of incomplete specialization (Case 1) involves a continuum of BGPs, our comparative-static analysis is conducted under the case of complete specialization (Cases 2 and 3) that has a unique BGP.

4.1 Home country specializes in the investment goods

This is under Case 2 in Section 3.2, in which $a_1/a_2 < a_1^*/a_2^*$, and thereby, the Home country exports the investment goods and imports the consumption goods.

Suppose that the government in the Home country imposes an ad-valorem import tariff on the consumption goods with the amount of tariff revenue transferring to the households in a lump-sum

fashion.¹¹ The government budget constraint is

$$T = sp_T(c - y_1), \quad (22a)$$

where s is the tariff rate of the imports, p_T is the international price of the consumption goods under the tariff policy, and T is the transfer payment. In what follows, a variable with subscript T is used to denote the variable under the tariff regime, while a variable with subscript F is used to denote the variable under the free trade regime.

As the Home country completely specializes in investment goods, $y_1=0$ and (20a) becomes $T=sp_Tc$. Now, $p_T(1+s)$ is the Home's domestic prices of the pure consumption good. Thus, the flow budget constraint of the household is

$$\dot{k} = rk - p_T(1+s)c + T. \quad (22b)$$

Note that $m = p(1+s)c/k$ when $(1+s)a_1/a_2 > a_1^*/a_2^*$. In a BGP, equilibrium system (17) remains the same, except for (17f), (17h) and (17i) that now become

$$\begin{cases} \tilde{z} + 1 = \tilde{p}_T \left(\frac{(1+s)a_1\tilde{z}\tilde{v}_1}{\tilde{m}} + \frac{a_1^*\tilde{v}_1^*}{\tilde{m}^*} \right), & \text{if } (1+s)a_1/a_2 \geq a_1^*/a_2^*, \\ \tilde{z} + 1 = \tilde{p}_T \left(\frac{a_1\tilde{z}\tilde{v}_1}{\tilde{m}} + \frac{a_1^*\tilde{v}_1^*}{\tilde{m}^*} \right), & \text{if } (1+s)a_1/a_2 \leq a_1^*/a_2^*, \end{cases} \quad (17f)'$$

$$\begin{cases} \tilde{v}_2 = \tilde{v}_1^* = 0, & \text{if } (1+s)a_1/a_2 > a_1^*/a_2^*, \\ \tilde{p}_T = \frac{a_2}{(1+s)a_1} = \frac{a_2^*}{a_1^*}, & \text{if } (1+s)a_1/a_2 = a_1^*/a_2^*, \\ \tilde{v}_1 = \tilde{v}_2^* = 0, & \text{if } (1+s)a_1/a_2 < a_1^*/a_2^*, \end{cases} \quad (17h)'$$

$$\tilde{r} = \begin{cases} a_1\tilde{p}_T(1+s) \geq a_2, & \text{if } (1+s)a_1/a_2 \geq a_1^*/a_2^*, \\ a_2 \geq a_1\tilde{p}_T(1+s), & \text{if } (1+s)a_1/a_2 \leq a_1^*/a_2^*, \end{cases} \quad \tilde{r}^* = \begin{cases} a_2^* \geq a_1^*\tilde{p}_T, & \text{if } (1+s)a_1/a_2 \geq a_1^*/a_2^*, \\ a_1^*\tilde{p}_T \geq a_2^*, & \text{if } (1+s)a_1/a_2 \leq a_1^*/a_2^*. \end{cases} \quad (17i)'$$

To analyze the impacts of import tariffs on economic growth in a BGP, we compare the economic growth rate under tariff with the rate under free trade. Denote $\hat{s} \equiv \frac{a_1^*/a_2^*}{a_1/a_2} - 1$. Apparently, $\hat{s} > 0$ in the case under study.

First, if $s < \hat{s}$, then $(1+s)a_1/a_2 < a_1^*/a_2^*$, and thereby, the trade pattern is not changed. The rate of economic growth depends upon the value of \tilde{z} along the BGP. As \tilde{z} is not affected, the rate of economic growth is study.

Second, if $s > \hat{s}$, then $(1+s)a_1/a_2 > a_1^*/a_2^*$. As a result, the trade pattern is reversed.

¹¹ The results under a specific import tariff are qualitatively similarly to those of an ad-valorem import tariff, but the analysis under an ad-valorem import tariff is simpler. We thus focus on the analysis of the effects of an ad-valorem import tariff.

The economy changes from the importer to the exporter of the consumption and imports the investment goods. The tariff is on the imports of consumption goods, but the Home country imports the investment goods after the tariff policy is enforced. As a consequence, the Home country does not have any tariff revenues, and thus, the transfer payment is zero in the long-run equilibrium.

To investigate the effect of the tariff policy on the interest rate, we note from (17i) that under the free trade regime $\tilde{r}_F = a_2$ and $\tilde{r}_F^* = a_1^* \tilde{p}_F$. Because of the tariff policy, there is an excess of supply of the consumption goods in the world market at \tilde{p}_F , making the world-market clearing price lower, $\tilde{p}_T < \tilde{p}_F$. Thus, after the policy is enforced, (17i)' indicates that $\tilde{r}_T = a_1 \tilde{p}_T (1+s) > a_2$ and $a_1^* \tilde{p}_F > \tilde{r}_T^* = a_2^* > a_1^* \tilde{p}_T$. Therefore, while the interest rate in the Home country is raised under the tariff regime, i.e., $\tilde{r}_T > \tilde{r}_F$, the interest rate in the Foreign is lowered under the tariff regime, i.e., $\tilde{r}_T^* < \tilde{r}_F^*$. As a result, (17c) suggests $\tilde{z}_T > \tilde{z}_F$. Moreover, (18b) indicates that

$$\tilde{z}_F \begin{cases} \geq 1, & \text{if } a_2 \geq 2\mathbf{r}(u(1)), \\ < 1, & \text{if } a_2 < 2\mathbf{r}(u(1)), \end{cases} \quad (23a)$$

while (18c) indicates that

$$\tilde{z}_T \begin{cases} \geq 1, & \text{if } a_2^* \leq 2\mathbf{r}(u(1)), \\ < 1, & \text{if } a_2^* > 2\mathbf{r}(u(1)). \end{cases} \quad (23b)$$

We are ready to examine the effect of the tariff policy on economic growth. Under free trade, from (17i) and (20a), the economic growth rate is equal to $g_F(z) = a_2 - \mathbf{r}(u(z))$, which using (18b) is rewritten as

$$g_F(\tilde{z}_F) \equiv \mathbf{r}(u(\frac{1}{\tilde{z}_F})) \tilde{z}_F. \quad (24a)$$

Under the tariff regime, on the other hand, the economic growth rate in the Home country is $g_T = a_1 \tilde{p}_T (1+s) - \mathbf{r}(u(z))$, which using (17c), (17d) and (17f)' is rewritten as

$$g_T(\tilde{z}_T) \equiv \mathbf{r}(u(\tilde{z}_T)) / \tilde{z}_T. \quad (24b)$$

Under Assumptions 1 and 2, g_F is increasing in \tilde{z}_F , while g_T is decreasing in \tilde{z}_T . Moreover, Loci g_F and g_T intersect at $\tilde{z}_F = \tilde{z}_T = 1$. (See Figure 1.)

[Insert Figure 1 here]

1. $a_2 \geq 2\mathbf{r}(u(1))$.

In this case, (23a) indicates that the ratio of Home to Foreign consumption is $\tilde{z}_F \geq 1$ under free trade (see point F_1 in Figure 1). Recall that the import tariff policy leads to a higher ratio of

Home to Foreign consumption; i.e., $\tilde{z}_T > \tilde{z}_F$. (Say at Point T_1 in Figure 1.) As a result, $g_T(\tilde{z}_T) < g_F(\tilde{z}_F)$. The rate of economic growth is lower under the tariff regime.

2. $a_2 < 2\mathbf{r}(u(1))$.

In this case, $\tilde{z}_F < 1$ under free trade according to (23a). Under the tariff regime, $\tilde{z}_T > \tilde{z}_F$. It is possible that economic growth is higher under the tariff regime, depending upon the relative magnitude between a_2^* and $2\mathbf{r}(u(1))$.

2.1. If $a_2^* \geq 2\mathbf{r}(u(1))$, (23b) indicates that $\tilde{z}_T \leq 1$. As $\tilde{z}_T > \tilde{z}_F$, it indicates $\tilde{z}_F < \tilde{z}_T \leq 1$. (See F_2 for free trade and T_2 for the tariff regime in Figure 1.) As a result, growth rate is increased under the tariff regime.

2.2. If $a_2^* < 2\mathbf{r}(u(1))$, (23b) suggests $\tilde{z}_T > 1$, say at T_1 in Figure 1. As a result, it must be $\tilde{z}_F < 1 < \tilde{z}_T$. In this situation, the economic growth is ambiguous after the tariff. Therefore, the growth effect is ambiguous in this case.

We summarize the results in these different cases in Table 1.

[Insert Table 1 here]

The effects on economic growth can be understood by the effect of the tariff policy on the interest rate relative to the rate of time preference in the Home country *a la* (17c). The import tariff makes the price of consumption goods higher in the Home country, thereby attracting more resources into this sector. As a result of higher demand for capital, the interest rate is increased in the Home country. However, the international price of the consumption goods is reduced by augmented output supply, thereby decreasing the demand for capital and the interest rate in the Foreign. As the result of higher domestic interest rates and lower foreign interest rates, (17c) indicates that the ratio of Home to Foreign consumption is increased and the rate of time preference in the Home country is increasing, $\mathbf{r}'(u(\tilde{z}_T))u'(\tilde{z}_T) > 0$. So, the economic growth rate, $\tilde{r}_T - \mathbf{r}(u(z))$, is ambiguous.

In particular, the change in the interest rates is $\Delta\tilde{r}_T = a_1\tilde{p}_T(1+s) - a_2 > 0$ and $\Delta\tilde{r}_T^* = a_2^* - a_1^*\tilde{p}_F < 0$. If a_2 is sufficiently large so that $a_2 \geq 2\mathbf{r}(u(1))$, then the increase in $\Delta\tilde{r}_T > 0$ is smaller than the increase in $\mathbf{r}'(u(\tilde{z}_T))u'(\tilde{z}_T) > 0$; hence, the economic growth rate $\tilde{r} - \mathbf{r}(u(z))$ is reduced. Alternately, if a_2 is small, such that $a_2 < 2\mathbf{r}(u(1))$, then the increase in $\Delta\tilde{r}_T > 0$ is large. Moreover, from (17c), the increment of z depends on the magnitude of $\Delta\tilde{r}_T^*$. If a_2^* is sufficiently large, such that $a_2^* \geq 2\mathbf{r}(u(1))$, the foreign interest rate decreases less and so

does the increment of z . This causes the domestic time preference rate to increase less. Thus, $\tilde{r}_T - \mathbf{r}(u(z))$ increases in equilibrium, making economic growth higher. However, if a_2^* is sufficiently small, such that $a_2^* < 2\mathbf{r}(u(1))$, the foreign interest rate decreases more and so does the increment of z . This higher domestic time preference rate leads to an ambiguous sign of $\tilde{r}_T - \mathbf{r}(u(z))$.

To summarize the results,

Proposition 3. *Suppose that the felicity and the rate of time preference be identical initially across countries. Let $\hat{s} \equiv \frac{a_1^*/a_2^*}{a_1/a_2} - 1 > 0$, so the Home country imports the consumption goods. Then, the effect of imposing the tariff rate s on the imports of the consumption goods is as follows.*

- (i) *If $s < \hat{s}$, nothing is affected;*
- (ii) *If $s > \hat{s}$, the trade pattern is reversed in the long run, and the Home country has a higher ratio of Home to Foreign consumption and*
 - a. lower economic growth when $a_2 \geq 2\mathbf{r}(u(1))$;*
 - b. higher economic growth when $a_2 < 2\mathbf{r}(u(1)) \leq a_2^*$;*
 - c. ambiguous economic growth when $a_2 < 2\mathbf{r}(u(1))$ and $a_2^* < 2\mathbf{r}(u(1))$.*

4.2 Home country specializes in the consumption goods

This is Case 3 in Section 3.2, in which $a_1/a_2 > a_1^*/a_2^*$, and thereby, the Home country exports the consumption goods and imports the investment goods.

Suppose now that the government in the Home country imposes an import tariff on the investment goods. Then, the government budget constraint is

$$T = \mathbf{t}(\dot{k} - y_2), \quad (25a)$$

where \mathbf{t} is the rate of tariff.

As the Home country completely specializes in the production of consumption goods, $y_2=0$ and (25a) becomes $T = \mathbf{t}\dot{k}$. Now $(1+\mathbf{t})$ is the domestic prices of the pure investment goods in the Home country. Thus, the flow budget constraint of the household in (5) now turns into

$$\dot{k} = \frac{1}{1+\mathbf{t}}(rk - p_T c + T). \quad (25b)$$

Similarly, let $m = pC/[(1+\mathbf{t})k]$ when $a_1/[a_2(1+\mathbf{t})] < a_1^*/a_2^*$. As a result, the equilibrium conditions (17) in a BGP remain the same except for (17c), (17e), (17g), (17h) and (17i) that now

become, respectively,¹²

$$\frac{\tilde{r}}{1+\mathbf{t}} - \mathbf{r}(u(\tilde{z})) = \tilde{r}^* - \mathbf{r}^*(u^*(\frac{1}{z})), \quad (17c)''$$

$$\frac{\tilde{r}}{1+\mathbf{t}} - \mathbf{r}(u(\tilde{z})) = \tilde{r}^* - \tilde{m}^*, \quad (17e)''$$

$$\begin{cases} \tilde{z} + 1 = \tilde{p}_T \left(\frac{a_1 \tilde{v}_1}{\tilde{m}} + \frac{a_1^* \tilde{v}_1^*}{\tilde{m}^*} \right), & \text{if } a_1/[a_2(1+\mathbf{t})] \geq a_1^*/a_2^*, \\ \tilde{z} + 1 = \tilde{p}_T \left(\frac{a_1 \tilde{v}_1}{(1+\mathbf{t})\tilde{m}} + \frac{a_1^* \tilde{v}_1^*}{\tilde{m}^*} \right), & \text{if } a_1/[a_2(1+\mathbf{t})] \leq a_1^*/a_2^*, \end{cases} \quad (17f)''$$

$$\begin{cases} \tilde{v}_2 = \tilde{v}_1^* = 0, & \text{if } a_1/[a_2(1+\mathbf{t})] > a_1^*/a_2^*, \\ \tilde{p}_T = \frac{(1+\mathbf{t})a_2}{a_1} = \frac{a_2^*}{a_1^*}, & \text{if } a_1/[a_2(1+\mathbf{t})] = a_1^*/a_2^*, \\ \tilde{v}_1 = \tilde{v}_2^* = 0, & \text{if } a_1/[a_2(1+\mathbf{t})] < a_1^*/a_2^*, \end{cases} \quad (17h)''$$

$$\tilde{r} = \begin{cases} a_1 \tilde{p}_T \geq (1+\mathbf{t})a_2, & \text{if } a_1/[a_2(1+\mathbf{t})] \geq a_1^*/a_2^*, \\ (1+\mathbf{t})a_2 \geq a_1 \tilde{p}_T, & \text{if } a_1/[a_2(1+\mathbf{t})] \leq a_1^*/a_2^*, \end{cases} \quad \tilde{r}^* = \begin{cases} a_2^* \geq a_1^* \tilde{p}_T, & \text{if } a_1/[a_2(1+\mathbf{t})] \geq a_1^*/a_2^*, \\ a_1^* \tilde{p}_T \geq a_2^*, & \text{if } a_1/[a_2(1+\mathbf{t})] \leq a_1^*/a_2^*. \end{cases} \quad (17i)''$$

The effects of the tariff policy depends on the rate of the tariff relative to the threshold $\mathbf{t} \equiv \frac{a_1/a_2}{a_1^*/a_2^*} - 1 > 0$. As in the former subsection, if it is smaller than the threshold, the comparative advantage is not changed and as a result, the trade patterns and economic growth both are not affected. Alternatively, if the rate of tariff is larger than the threshold, the comparative advantage is distorted sufficiently, so the comparative advantage is changed. Then, the Home country produces the investment goods and imports the consumption goods in the long run. In equilibrium, there is no tariff revenue. Moreover, the domestic interest rate is reduced but the foreign interest rate is raised. As a result, z decreases. As in the former subsection, the effect on economic growth depends upon the magnitude of the productivity coefficients relative to the time preferences rate in both countries evaluated at an identical level of consumption. The results are summarized in Figure 2 and Table 2.

[Insert Figure 2 and Table 2 here]

To summarize the results in this subsection,

Proposition 4. *Suppose that the felicity and the rate of time preference be identical initially*

¹² The necessary conditions in (8a) and (8b) now becomes $u'(\frac{c}{c^*})[1 - \mathbf{m}'(u(\frac{c}{c^*}))] = \frac{1 p_r c^*}{X(1+\mathbf{t})}$ and $\frac{j}{I} = -\frac{r}{1+\mathbf{t}}$.

across countries. Let $\hat{\mathbf{t}} \equiv \frac{a_1/a_2}{a_1^*/a_2^*} - 1 > 0$, so the Home country imports the investment goods.

Then, the effects of imposing the tariff rate t on the imports of the investment goods are as follows.

- (i) If $t < \hat{\mathbf{t}}$, nothing is affected;
- (i) If $t > \hat{\mathbf{t}}$, the trade pattern is reversed in the long run, and the Home country has a smaller ratio of Home to Foreign consumption and
 - a. lower economic growth when $a_2^* \geq 2\mathbf{r}(u(1))$;
 - b. higher economic growth when $a_2^* < 2\mathbf{r}(u(1)) \leq a_2$;
 - c. ambiguous economic growth when $a_2 < 2\mathbf{r}(u(1))$ and $a_2^* < 2\mathbf{r}(u(1))$.

5. Import Tariff and Welfare

Finally, we turn to examining the relationship between tariffs and welfare in the long run. In a BGP, the representative agent's lifetime utility in the Home country is

$$U = \int_0^{\infty} u(\tilde{z}) e^{-r(u(\tilde{z}))t} dt = \frac{u(\tilde{z})}{\mathbf{r}(u(\tilde{z}))}.$$

This indicates that the long-run welfare is increasing in \tilde{z} .¹³ Changes in the welfare depend on the goods the Home country imports.

Under Case 2, in which the Home country specializes in the investment goods, the consumption goods are imported. Tariffs s on the imports of the consumption goods lead to a higher \tilde{z} in the long run if $s > \hat{s}$. As a result, the tariffs increase the long-run welfare in the Home country if $s > \hat{s}$.

In contrast, under Case 3, in which the Home country specializes in the consumption goods, the investment goods are imported. Tariffs on the imports of the investment goods result in a lower \tilde{z} in the long run if $t > \hat{\mathbf{t}}$. It follows that the tariffs decrease the long-run welfare in the Home country if $t > \hat{\mathbf{t}}$,

To summarize the relationship between tariffs and welfare in the long run, we obtain

Proposition 5. *Let $a_1/a_2 < (\text{resp. } >) a_1^*/a_2^*$. Then, a tariff on the imports of the consumption (resp. investment) goods raises (resp. reduces) the long-run welfare in the Home country if the*

¹³ $\frac{dU}{d\tilde{z}} = \frac{u'[\mathbf{r}-\mathbf{r}'u]}{\mathbf{r}^2} > 0$ based on Assumption 1.

tariff reverses the trade pattern.

6. Conclusion

While most existing documents find a negative relationship between the import tariffs and the rate of economic growth, there are works finding an ambiguous, or even positive, relationship. This paper builds a two-country, two-output trade model and studies the relationship between tariffs and economic growth in an endogenous growth framework. The Ak technologies are used in order to differentiate the role of physical capital accumulation from the human capital accumulation as a source of economic growth. The choice leads to Ricardian technologies; yet, the Ricardian theorem of comparative advantage holds in the two-country world even if the preferences are different across countries.

A special feature of our model is to incorporate time non-separable preferences. A growing body of empirical evidence using country or cross-country data has confirmed the importance of time non-separable preferences. We introduce the endogenous rate of time preference. An extensive literature on asset pricing and real business cycles lends further credit to the level of benchmark consumption being a significant determinant of consumption behavior. We incorporate the average living standard in the foreign as benchmark consumption. We find that in the case of incomplete specialization, local indeterminacy arises in the neighborhood of a BGP.

We examine the relationship between tariffs and economic growth in the case of complete specialization. We find an ambiguous relationship. In particular, when the Home country specializes in the investment (*resp.* consumption) goods, a sufficient higher rate of tariffs on the consumption (*resp.* investment) goods reverses the trade pattern in the long run and decreases economic growth when the productivity coefficient of the investment goods in the Home (*resp.* Foreign) country is larger than the threshold. However, economic growth is increased when the productivity coefficient of the investment goods in the Home (*resp.* Foreign) country is smaller and in the Foreign (*resp.* Home) country is larger than the threshold. Finally, tariffs increase (*resp.* decrease) the long-run welfare in the Home country when it specializes in the investment (*resp.* consumption) goods.

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Table 1: Import Tariff and Growth Rate ($a_1/a_2 < q^*/a_2^*$)?

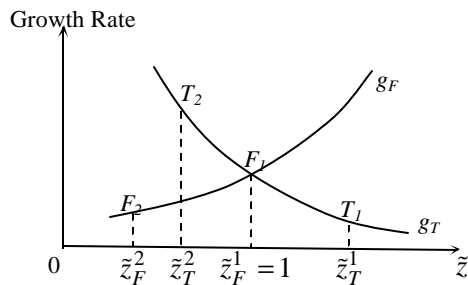
Import Tariffs	Conditions	Variation of Growth Rate
$s < \hat{s} \equiv \frac{a_1^* a_2}{a_1 a_2^*} - 1$	–	unchanged
$s > \hat{s} \equiv \frac{a_1^* a_2}{a_1 a_2^*} - 1$	$a_2 \geq 2r[u(1)]$	lower
	$a_2 < 2r[u(1)] \leq a_2^*$	higher
	a_2 and $a_2^* < 2r[u(1)]$	ambiguous

?The preferences and time preference rates are assumed to be the same between two countries.

Table 2: Import Tariff and Growth Rate ($a_1/a_2 > q^*/a_2^*$)?

Import Tariffs	Conditions	Variation of Growth Rate
$t < \hat{t} \equiv \frac{a_1 a_2^*}{a_1^* a_2} - 1$	–	unchanged
$t > \hat{t} \equiv \frac{a_1 a_2^*}{a_1^* a_2} - 1$	$a_2^* \geq 2r[u(1)]$	lower
	$a_2^* < 2r[u(1)] \leq a_2$	higher
	a_2 and $a_2^* < 2r[u(1)]$	ambiguous

?The preferences and time preference rates are assumed to be the same between two countries.

Figure 1: Growth Rates ($a_1/a_2 < q^*/a_2^*$)Figure 2: Growth Rates ($a_1/a_2 > q^*/a_2^*$)