Analysis of Fiscal Shocks in a Small-Open Economy with Home Production*

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Abstract

In view of the large scale of nonmarket home sector in both developed and developing economies, in this paper we construct an open economy model with home production. We discuss both long-run as well as short-run impacts of fiscal policies, such as income tax, consumption tax and government spending, on factor allocation, capital formation and current account. The main purpose of the paper is to elucidate the role of home production in an open economy environment. We show that in our framework both fiscal policy and the rate of substitution between market and home goods may affect the dynamic behaviors of the economy. As a result, introducing home production may alter the effects of fiscal shocks on the key variables in the economy. We confirm this fact by examining various policy experiments both in the short run and in the long run.

JEL classification: H31, D13, O41

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1 Introduction

Production activities within the households are substantial. Time and resources devoted to home production share considerable portions even in advanced countries. According to Eisner (1988), in the United States an estimate of home-produced output relative to measured gross national production is in the range of from 20 to 50 percent. Wrase (2001) reports that a married couple in the U.S., on average, devotes 25 percent of discretionary time to unpaid home production and 33 percent of discretionary time to work in the market place for pay. Because of the nonmarket property of the home sector, an immediate question is whether the inclusion of this home sector affects the usual predictions on public policy.

The idea that home production may play a relevant role in macroeconomics has generated a bulk of the recent studies focusing on how households' production activities affect business cycles, macroeconomic policy performances and long-term economic growth[1]. Most of this literature has tried to reveal that introducing a home production sector into the otherwise standard macroeconomic models improves the models' ability in explaining observed data. For example, Benhabib et al. (1991) and Greenwood and Hercowitz (1991) show that the introduction of home production into the standard real business cycle theory significantly improves the performances of the calibrated models. The intuition behind such a good fitness is that the incorporation of a home sector in the standard one-sector real business cycle model brings about possibility of substitution between market and nonmarket production over time. Therefore, relative productivity differentials between the two sectors may enhance volatility in market activity. Furthermore, the substitution between home and market commodities at a given date, not just at different dates, affects the size of fluctuations induced by productivity shocks.[2] As for explanation of the observed economic development facts, Parente et al. (2000) illustrate that, by adding a home production sector to the neoclassical growth model, international income differences can be accounted well

[1] The idea that home sector acts parallel to market sector is originated from Becker (1965).

[2] The empirical work of McGattan, Rogerson and Wright (1997) claims that the elasticity of substitution between home and market goods is considerably high.

under relatively small differences in policies. This is because, in the presence of household production, fiscal policy affects not only capital accumulation but also the shares between market and nonmarket activities.

Inspired by the work of Parente et al. (2000), the first objective of this paper is to explain economic development facts. Rather than numberical analysis in Parennte et al. (2000), however, we aim to analysis this problem theoretically. Furthermore, this paper aims to explain diverse fiscal policy effects illustrated in existing empirical studies. We show that, depending on the factor intensity ranking between the market and home sector, right opposite policy effects can be obtained.

There is a huge body of public policy literature. In a framework of small-open economy, public policy especially those of fiscal policy and government spending, have been explored intensively[3]. Among others, we are especially interested in the way of resource allocation between the market and home sectors. Though there are existing works that consider labor-leisure choice, rigorous public policy investigation with non-market home sector is rare.[4] It has been shown in the field of macroeconomic that the non-market household production sector is a meaningful addition to the business sector in explaining the observed economic fluctuations and development facts[5]. Though recently there are studies in the field of international real business cycle (e.g., Raffo, 2006) incorporating home production into the models, comprehensive study of the home production activity in an open-economy framework is still rare. The present work contributes to the literature in this direction.

The main findings of the paper include the following. When income taxation does not change the factor-intensity ranking between market and home sectors, then the model economy exhibits standard results. However, in the case that taxation on capital and labor incomes changes the factor intensity ranking between the market and home sectors, then

[3] Among others, see Brock (1996) and Turnovsky (1997) and the references there, for example.

[4] A recent work by Hu and Mino (2005) presents analytical policy investigation in an endogenous growth model with home production.

[5] Among others, see Benhabib, Rogerson and Wright (1991), and Parente, Rogerson and Wright (2000), for example.

different policy predictions from the standard models occur. On the steady-state analysis, similar to the existing literature, for given initial condition, there is one and only one steady state in the model economy which is locally saddle-point stable. However, different from the existing small-open economy literature, fiscal policy and the scale of the rate of substitution between market and home consumptions affect the stability conditions of the steady state. Long-run policy effects are investigated analytically. We show, in general, that it depends on the ranking of factor intensity between the home and market sectors that an expansion in the government spending or in the tax rate how endogenous variables to change.

Analytical method the present study adopted is in line with that of the dependent economy models[6] in international macroeconomics[7]. While "nontraded" goods are the focus of those existing contributions, our interests lie in pinning down the implication of the "nonmarket" sector in a small-open economy. Since a nonmarket sector is isolated from taxation, the incorporation of a home production sector leads to asymmetry between sectors which will play an important role in determining the policy effects.

The rest of the paper is arranged as follows. Section 2 lays out the model. The dynamic system and stability analysis of the model are reported in Section 3. Section 4 is devoted to long-run policy analysis, and Section 5 reports transitional dynamic results when shocks occur. Finally Section 6 concludes the paper.

2 The model

Considering a small-open economy which faces an integrated capital market. There are three kinds of agents, firms, households and a government. Firms produce a consumable-capital good with capital and labor. Households, as factor owners, supply capital and labor either to the factor markets for earning rental and wage, or to home sector for

[6] According to Salter (1959), dependent economy describes an economy that is a price taker on world markets but also produces nontraded goods for domestic use.

[7] In this respect, see Turnovsky (1997, ch. 4) for a detail discussion.

producing non-market home goods, which are utility promoting. It is worth noting that it is the assumption on home production distinguishes this study from most of the existing contributions in the literature. The central government levies flat-rate of taxes on incomes in order to finance its spending. To isolate the taxation effects, we assume the government repays the income after its spending to households in a lump-sum form (tax or transfer depending on the relative size of government income and spending).

We assume that market good and capital are tradable internationally, while home goods can be consumed at home solely. Labor cannot move across borders, however agents can choose to work in the market place or to stay at home engaging in nonmarket production. Both market and home goods need capital and labor as inputs. Both market and home-made goods are preference promoting, while only market goods can be invested in domestic capital stock or to the world credit market. Furthermore, all markets are competitive.

2.1 Production

Specify the production functions as the Cobb-Douglas form

$$Y_m = A_m K_m^{\alpha_m} L_m^{1-\alpha_m}, \quad Y_h = A_h K_h^{\alpha_h} L_h^{1-\alpha_h},$$

where variables with subscripts "m", "h" represent the market sector and home sector respectively. A_j represents the total factor productivity, and Y_j, K_j and L_j are output, capital and labor in sector j $(j = m, h)$ respectively. $x_j \equiv K_j/L_j$ represents the capital/labor ratio in sector j $(j = m, h)$.

Market competition implies equalization between rental rates and marginal production in market sector. That is,

$$R = \frac{\partial Y_m}{\partial K_m} = A_m \alpha_m x_m^{\alpha_m - 1}, \quad w = \frac{\partial Y_m}{\partial L_m} = A_m (1-\alpha_m) x_m^{\alpha_m}, \tag{1}$$

where R and w are (gross) rental rate and wage rate respectively.

2.2 Households

Assuming away population growth and normalizing the number of households to unity. For given factor prices and world interest rate, the representative household maximizes its life-time utility

$$\int_0^\infty u(c_m, c_h, n)e^{-\rho t}dt,$$

where c_m and c_h are consumptions of the market and home goods respectively, n is the pure leisure time. The market good consumption c_m could be domestic produced or imported from the foreign countries. In order to concentrate to the fiscal policy, we omit tariff and assume the domestic produced market good is the same as the imported good. Following the convention, we assume households own capital and labor. Suppose the each household owns one unit of labor at each moment of time, and denote the aggregate capital as K. Then households allocate capital between the market and home sectors: $K_m + K_h = L_m x_m + L_h x_h = K$, while allocate time between market work, L_m, home work, L_h, and leisure $n = 1 - L_m - L_h$.

Following Benhabib, Rogerson and Wright (1991), we specify the momentary utility as

$$u(c_m, c_h, n) = log[\mu c_m^\varepsilon + (1-\mu)c_h^\varepsilon]^{1/\varepsilon} + \gamma log n,$$

where $-\infty < \varepsilon < 1$ is the parameter expressing the rate of substitution between market and home-made goods, with $\mu > 0$, $\gamma > 0$ and , where c_h represents the part of home products of which close market substitutions exist. Since we do not consider trade policy in this paper, so there is no need to distinguish the domestic-made with the imported market goods. Recognizing that some home activities (e.g. sleep) have less market substitutions, we specify leisure and consumption in a log-additive form.

Facing a unified international capital market, the representative agent allocates its total income to goods consumption, physical capital investment, and foreign assets investment. Denote B as the value of the economy's net claims on the rest of the world. Therefore the flow budget constraint of the representative household is

$$\dot{B} = (1-\tau_k)RK_m + (1-\tau_l)wL_m + r^*B - (1+\tau_c)c_m - \phi(I) + T, \tag{2}$$

where τ_k, τ_l and τ_c are tax rates on market capital income, labor income and consumption, and T is a lump-sum transfer (or tax) from the government. Since the model economy considered here is small and faces a perfect international credit market, then it takes the world interest rate r^* as exogenous given.

To retain nondegenarate dynamics, we introduce a capital adjustment cost in capital accumulation. This is reflected in the function, ϕ, which satisfies $\phi' > 0$ and $\phi'' > 0$. That is, to accomplish a unit increase in physical capital stock, a more than one unit input is needed, and larger the investment is, more input per unit of investment is needed.

On the other hand,

$$\dot{K} = I \tag{3}$$

The current value Hamiltonian of the representative household is

$$\begin{aligned} H &\equiv u(c_m, c_h, n) + p\dot{B} + q\dot{K} + \lambda(Y_h - c_h) \\ &= log[\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}]^{1/\varepsilon} + \gamma log(1 - L_m - L_h) + p\Big[(1-\tau_k)RK_m \\ &\quad +(1-\tau_l)wL_m + (1-\tau_b)rB - (1+\tau_c)c_m - (1-\tau_i)\phi(I) + T\Big] \\ &\quad +qI + \lambda[A_h(K-K_m)^{\alpha_h} L_h^{1-\alpha_h} - c_h], \end{aligned}$$

The representative household maximizes it life-time utility by choosing c_m, c_h, L_m, L_h,

K_m and I. At the interior solution, the first-order conditions at each point of time are

$$\frac{\mu c_m^{\varepsilon-1}}{\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}} = p(1+\tau_c), \tag{4}$$

$$\frac{(1-\mu)c_h^{\varepsilon-1}}{\mu c_m^{\varepsilon} + (1-\mu)c_h^{\varepsilon}} = \lambda, \tag{5}$$

$$\frac{\gamma}{1-L_m-L_h} = p(1-\tau_l)w, \tag{6}$$

$$\frac{\gamma}{1-L_m-L_h} = \lambda(1-\alpha_h)A_h x_h^{\alpha_h}, \tag{7}$$

$$p(1-\tau_k)R = \lambda A_h \alpha_h x_h^{\alpha_h-1}, \tag{8}$$

$$p\phi'(I) = q, \tag{9}$$

and the intertemporal conditions are

$$\dot{p} = p(\rho - r^*), \tag{10}$$

$$\dot{q} = q\rho - p(1-\tau_k)R, \tag{11}$$

while the transversility conditions are

$$\lim_{t\to\infty} pBe^{-\rho t} = 0 = \lim_{t\to\infty} qKe^{-\rho t}. \tag{12}$$

Notice that, (10) and (11) can be rearranged to express arbitrage conditions between the foreign asset and capital investments.

$$\frac{\dot{p}}{p} + r^* = \rho \text{ and } \frac{\dot{q}}{q} + \frac{p}{q}(1-\tau_k)R = \rho.$$

2.3 The government

For the time being, we assume the government keeps its budget balance in each point of time by transferring the gap of its income and expenditure to households in a lump sum form. That is, for given government spending G we have

$$T = \tau_k R K_m + \tau_l w L_m + \tau_c c_m - G.$$

2.4 Market equilibrium

In equilibrium, households use up all the home-made goods, that is

$$Y_h = c_h. \tag{13}$$

And completely employment in factor markets implies

$$n = 1 - L_m - L_h,\ L_m x_m + L_h x_h = K. \tag{14}$$

The economy as a whole, the current account is

$$\dot{B} = R K_m + w L_m + r^* B - c_m - \phi(I) - G. \tag{15}$$

When the economy produces more output than its domestic demands, it exports goods to gain ownership of the foreign capital, which improves its current account. On the other hand, the trade deficit leads to financial deficit and worsens its current account.

3 Equilibrium analysis

3.1 The dynamic system

Note firstly, it must be assumed that $\rho = r^*$ in order that the economy has a steady-state. Thus (10) shows that p stays constant over time. That is, for a small country, when facing constant world interest rate, has this rate as its time preference rate as well.

From (4) and (5), we have $c_m = c_m(\lambda, p; \tau_c)$, $c_h = c_h(\lambda, p; \tau_c)$ and

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} dc_m \\ dc_h \end{pmatrix} = \begin{pmatrix} d[p(1+\tau_c)] \\ d\lambda \end{pmatrix}$$

where

$$\begin{aligned} a_{11} &= \frac{-\mu c_m^{\varepsilon-2}\left[\mu c_m^\varepsilon + (1-\mu)(1-\varepsilon)c_h^\varepsilon\right]}{\left[\mu c_m^\varepsilon + (1-\mu)c_h^\varepsilon\right]^2} \\ a_{12} &= a_{21} = \frac{-\mu(1-\mu)\varepsilon c_m^{\varepsilon-1}c_h^{\varepsilon-1}}{\left[\mu c_m^\varepsilon + (1-\mu)c_h^\varepsilon\right]^2} \\ a_{22} &= \frac{-(1-\mu)c_h^{\varepsilon-2}\left[\mu(1-\varepsilon)c_m^\varepsilon + (1-\mu)c_h^\varepsilon\right]}{\left[\mu c_m^\varepsilon + (1-\mu)c_h^\varepsilon\right]^2} \end{aligned}$$

and the determinant of the coefficient matrix is

$$D_1 = a_{11}a_{22} - a_{12}a_{21} = \frac{\mu(1-\mu)(1-\varepsilon)c_m^{\varepsilon-2}c_h^{\varepsilon-2}}{\left[\mu c_m^\varepsilon + (1-\mu)c_h^\varepsilon\right]^2} > 0$$

Then

$$\begin{aligned} dc_m &= \frac{a_{22}}{D_1}d[p(1+\tau_c)] - \frac{a_{12}}{D_1}d\lambda \\ dc_h &= \frac{a_{11}}{D_1}d\lambda - \frac{a_{12}}{D_1}d[p(1+\tau_c)] \end{aligned}$$

and

$$\begin{aligned} \frac{\partial c_h}{\partial \lambda} &= \frac{a_{11}}{D_1} = -\frac{\mu c_m^\varepsilon + (1-\mu)(1-\varepsilon)c_h^\varepsilon}{(1-\mu)(1-\varepsilon)c_h^{\varepsilon-2}} \\ \frac{\partial c_m}{\partial \lambda} &= -\frac{a_{12}}{D_1} = \frac{\varepsilon c_m c_h}{(1-\varepsilon)} \end{aligned}$$

Lemma 1. (Market and home consumptions)

$$\begin{aligned} \operatorname{sign}\left[\frac{\partial c_m}{\partial \lambda}\right] &= \operatorname{sign}\left[\varepsilon\right], \ \frac{\partial c_m}{\partial p} < 0, \ \frac{\partial c_m}{\partial \tau_c} < 0 \qquad (16) \\ \frac{\partial c_h}{\partial \lambda} &< 0, \ \operatorname{sign}\left[\frac{\partial c_h}{\partial p}\right] = \operatorname{sign}\left[\varepsilon\right], \ \frac{\partial c_h}{\partial \tau_c} > 0 \qquad (17) \end{aligned}$$

That is, an increase in one consumption's price lowers the consumption of this goods. While the effect of this price change on the consumption of the other good depends on the rate of substitution of these two goods. For example, when $\varepsilon < 0$, that is, the market good consumption is complementary to the home good consumption, then the price increase in the home good will lower the consumption of the market good as well.

Similarly, from (6)-(8), we have $x_m = x_m(\lambda, p; \tau_k, \tau_l)$, $x_h = x_h(\lambda, p; \tau_k, \tau_l)$ which satisfy

$$\frac{x_m}{x_h} = \left(\frac{1-\alpha_h}{\alpha_h}\right)\left(\frac{\alpha_m}{1-\alpha_m}\right)\left(\frac{1-\tau_k}{1-\tau_l}\right) \qquad (18)$$

and

$$\begin{aligned} &\begin{pmatrix} p(1-\tau_l)A_m\alpha_m(1-\alpha_m)x_m^{\alpha_m-1} & -\lambda A_h\alpha_h(1-\alpha_h)x_h^{\alpha_h-1} \\ p(1-\tau_k)A_m\alpha_m(\alpha_m-1)x_m^{\alpha_m-2} & -\lambda A_h\alpha_h(\alpha_h-1)x_h^{\alpha_h-2} \end{pmatrix}\begin{pmatrix} dx_m \\ dx_h \end{pmatrix} \\ = &\begin{pmatrix} A_h(1-\alpha_h)x_h^{\alpha_h}d\lambda - A_m(1-\alpha_m)x_m^{\alpha_m}d[p(1-\tau_l)] \\ A_h\alpha_h x_h^{\alpha_h-1}d\lambda - A_m\alpha_m x_m^{\alpha_m-1}d[p(1-\tau_k)] \end{pmatrix} \end{aligned}$$

where the determinant of the coefficient matrix is

$$\begin{aligned} D_2 &\equiv pA_m\alpha_m(1-\alpha_m)x_m^{\alpha_m-2}\lambda A_h\alpha_h(1-\alpha_h)x_h^{\alpha_h-2}\begin{vmatrix} (1-\tau_l)x_m & -x_h \\ -(1-\tau_k) & 1 \end{vmatrix} \\ &= pA_m\alpha_m(1-\alpha_m)x_m^{\alpha_m-2}\lambda A_h\alpha_h(1-\alpha_h)x_h^{\alpha_h-2}\left[(1-\tau_l)x_m - (1-\tau_k)x_h\right] \end{aligned}$$

That is, sign$[D_2]$ =sign$[(1-\tau_l)x_m-(1-\tau_k)x_h]$. Thus

$$dx_m=\frac{\lambda A_h\alpha_h(1-\alpha_h)x_h^{\alpha_h-2}}{D_2}\left\{\begin{array}{c}A_h x_h^{\alpha_h}d\lambda\\ -A_m x_m^{\alpha_m-1}[(1-\alpha_m)(1-\tau_l)x_m+\alpha_m x_h(1-\tau_k)]dp\\ +pA_m(1-\alpha_m)x_m^{\alpha_m}d\tau_l+pA_m\alpha_m x_m^{\alpha_m-1}x_h d\tau_k\end{array}\right\}$$

and

$$dx_h=\frac{pA_m\alpha_m(1-\alpha_m)x_m^{\alpha_m-2}}{D_2}\left\{\begin{array}{c}\left[(1-\tau_l)A_h\alpha_h x_m x_h^{\alpha_h-1}+(1-\tau_k)A_h(1-\alpha_h)x_h^{\alpha_h}\right]d\lambda\\ -(1-\tau_l)(1-\tau_k)A_m x_m^{\alpha_m}dp+p(1-\tau_l)A_m\alpha_m x_m^{\alpha_m}d\tau_k\\ +p(1-\tau_k)A_m(1-\alpha_m)x_m^{\alpha_m}d\tau_l\end{array}\right\}$$

Notice that

$$\begin{aligned}\text{sign}\left[\left(\frac{1-\tau_l}{1-\tau_k}\right)x_m-x_h\right] &= \text{sign}\left[\frac{\alpha_m}{1-\alpha_m}-\frac{\alpha_h}{1-\alpha_h}\right]=\text{sign}\left[\alpha_m-\alpha_h\right]\\ \text{sign}\left[x_m-x_h\right] &= \text{sign}\left[\left(\frac{\alpha_m}{1-\alpha_m}\right)\left(\frac{1-\tau_k}{1-\tau_l}\right)-\frac{\alpha_h}{1-\alpha_h}\right]\end{aligned}$$

that is, sign$[(1-\tau_l)x_m-(1-\tau_k)x_h]$ represents the pre-tax capital/labor ratio rank between the market and home sector, while sign$[x_m-x_h]$ represents the market and home sector capital/labor ratio after the taxation. The above calculations give out the effects of the consumption price change upon the capital/labor ratios in the market and home sectors.

Define ϕ_m, $\tilde{\phi}_m$ as the pre-tax and after-tax capital/labor ratios in the market sector, while ϕ_h as the capital/labor ratio in the home sector. That is,

$$\phi_m\equiv\frac{\alpha_m}{1-\alpha_m},\ \tilde{\phi}_m\equiv\left(\frac{\alpha_m}{1-\alpha_m}\right)\left(\frac{1-\tau_k}{1-\tau_l}\right),\ \phi_h\equiv\frac{\alpha_h}{1-\alpha_h}.$$

Lemma 2. (Capital-labor ratios) For $j = m, h$

$$\begin{aligned} \operatorname{sign}\left[\frac{\partial x_j}{\partial \lambda}\right] &= -\operatorname{sign}\left[\frac{\partial x_j}{\partial p}\right] = \operatorname{sign}\left[\frac{\partial x_j}{\partial \tau_k}\right] = \operatorname{sign}\left[\frac{\partial x_j}{\partial \tau_l}\right] = \operatorname{sign}\left[\phi_m - \phi_h\right] \\ \frac{\partial x_m}{\partial p} &= \left(-\frac{x_m}{p}\right)\left(\frac{1}{\alpha_m - \alpha_h}\right) \end{aligned}$$

Proof See Appendix.

That is, it depends on the capital/labor ratio ranking between the market and home sectors that how these ratios response to a consumption price change. For example, if the market good is relatively capital intensive, then a price increase in the home good raises the capital/labor ratios in the two sectors, while a price change in the market good sector will decrease these ratios.

Since $R = A_m \alpha_m x_m^{\alpha_m - 1}$, $w = A_m(1-\alpha_m)x_m^{\alpha_m}$, we have $\operatorname{sign}\left[\frac{\partial R}{\partial *}\right] = -\operatorname{sign}\left[\frac{\partial x_m}{\partial *}\right]$, $\operatorname{sign}\left[\frac{\partial w}{\partial *}\right] = \operatorname{sign}\left[\frac{\partial x_m}{\partial *}\right]$, $* = \lambda, p, \tau_k, \tau_l$. Using Lemma 2, the following factor price results can can be derived consequently.

Lemma 3. (Rental rate, wage rate)

$$\begin{aligned} -\operatorname{sign}\left[\frac{\partial R}{\partial \lambda}\right] &= \operatorname{sign}\left[\frac{\partial R}{\partial p}\right] = \operatorname{sign}\left[\phi_m - \phi_h\right] \\ \operatorname{sign}\left[\frac{\partial w}{\partial \lambda}\right] &= -\operatorname{sign}\left[\frac{\partial w}{\partial p}\right] = \operatorname{sign}\left[\phi_m - \phi_h\right] \end{aligned}$$

and

$$R + P\frac{\partial R}{\partial p} = R\left(\frac{1-\alpha_h}{\alpha_m - \alpha_h}\right)$$

In words, when the market sector is relatively capital intensive ($\alpha_m > \alpha_h$), an increase in the price of the market good (p) will raise the return rate to capital (R) and lower the wage rate (w); while an increase in the implicit price of the home good (λ) has right opposite effects on them.

From (6) and the factor market equilibrium conditions (14), we have $L_i = L_i(K, \lambda, p; \tau_k, \tau_l)$, that is

$$L_m = \frac{(1-n)x_h - K}{x_h - x_m}, \tag{19}$$

$$L_h = \frac{K - (1-n)x_m}{x_h - x_m}. \tag{20}$$

and $n = n(\lambda, p; \tau_k, \tau_l)$

$$n = 1 - L_m - L_h = \frac{\gamma}{p(1-\tau_l)A_m(1-\alpha_m)x_m^{\alpha_m}}. \tag{21}$$

Lemma 4. (Labor and leisure time)

$$\begin{aligned}
\text{sign}\left[\frac{\partial n}{\partial \lambda}\right] &= -\text{sign}\left[\phi_m - \phi_h\right], \ \text{sign}\left[\frac{\partial n}{\partial p}\right] = \text{sign}\left[\phi_m - \phi_h\right] \\
\text{sign}\left[\frac{\partial L_h}{\partial \lambda}\right] &= \text{sign}\left[\tilde{\phi}_m - \phi_h\right] \text{sign}\left[\phi_m - \phi_h\right] \\
-\text{sign}\left[\frac{\partial L_h}{\partial K}\right] &= \text{sign}\left[\tilde{\phi}_m - \phi_h\right] = \text{sign}\left[\frac{\partial L_m}{\partial K}\right]
\end{aligned}$$

Proof. See Appendix.

It is worth noting that, capital and labor income taxation can affect the factor intensity ranking between the market and home sector. That is, $\text{sign}\left[\tilde{\phi}_m - \phi_h\right]$ may be different from $\text{sign}[\phi_m - \phi_h]$. While this factor-intensity-reverse force does not disturb households's choice of market and home good consumption and the factor prices, it can affect the labor time allocation between sectors. For example, in the standard models without home production, $\text{sign}\left[\tilde{\phi}_m - \phi_h\right] = \text{sign}[\phi_m - \phi_h]$ always. Therefore, an increase in the home goods price will lead to corresponding increase in home work time. However, when the income taxation is distorted sufficiently that $\text{sign}\left[\tilde{\phi}_m - \phi_h\right] = -\text{sign}[\phi_m - \phi_h]$, then an increase in the home goods price will lead to less home work time.

Lemma 5. (Outputs)

$$
\begin{aligned}
\text{sign}\left[\frac{\partial Y_h}{\partial \lambda}\right] &= \text{sign}\left[\tilde{\phi}_m - \phi_h\right]\text{sign}\left[\phi_m - \phi_h\right] = -\text{sign}\left[\frac{\partial Y_m}{\partial \lambda}\right] \\
\text{sign}\left[\frac{\partial Y_h}{\partial p}\right] &= -\text{sign}\left[\tilde{\phi}_m - \phi_h\right]\text{sign}\left[\phi_m - \phi_h\right] \\
\text{sign}\left[\frac{\partial Y_h}{\partial K}\right] &= -\text{sign}\left[\tilde{\phi}_m - \phi_h\right] = -\text{sign}\left[\frac{\partial Y_m}{\partial K}\right]
\end{aligned}
$$

Proof. See Appendix.

Similar to the previous result, it depends on the before-tax and after-tax factor intensity ranking that whether an increase in the good price can raise the output of this good or not.

Lemma 6. (Implicit price of the home good) If sign$\left[\tilde{\phi}_m - \phi_h\right]$=sign$[\phi_m - \phi_h]$, then

$$\text{sign}\left[\frac{\partial \lambda}{\partial K}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right];$$

If $sign\left[\tilde{\phi}_m - \phi_h\right] = -sign\left[\phi_m - \phi_h\right]$ and $\tilde{\phi}_m - \phi_h$ is close to 0, then

$$\text{sign}\left[\frac{\partial \lambda}{\partial K}\right] = -\text{sign}\left[\tilde{\phi}_m - \phi_h\right].$$

Proof. See Appendix.

In summary, from (11), (9) and (3)

$$\dot{q} = q\rho - p(1-\tau_k)A_m\alpha_m {x_m}^{\alpha_m - 1}. \tag{22}$$

$$\dot{K} = (\phi'^{-1})(\frac{q}{p}), \tag{23}$$

together with the equilibrium condition in the home sector

$$0 = Y_h - c_h, \tag{24}$$

we obtain the dynamic system with respect to K, q, λ of the model economy.

3.2 The steady state

Notice that p is constant. From $\dot{K} = 0$ and $\dot{q} = 0$,

$$\bar{q}/p = \phi'(0), \tag{25}$$

$$\bar{q}\rho = p(1-\tau_k)A_m\alpha_m x_m(\lambda, p)^{\alpha_m - 1}. \tag{26}$$

From the above relations and $Y_h(K, \lambda, p) = c_h(\lambda, p)$, we can derive the steady state values of K, q and λ, which are denoted with barred notations.

Let $\dot{B} = 0$ in (15) and substituting $\bar{K}$, $\bar{q}$ into it, we obtain the steady-state value of B, $\bar{B}$. Similarly we can get the steady-state values of other variables.

Proposition 1 *For any given p, a unique steady state $(\bar{K}, \bar{B}, \bar{c_m}, \bar{c_h}, \bar{L_m}, \bar{L_h}, \bar{q})$ exists.*

Proof. From (25), $\bar{q} = p\phi'(0)$. Then, combining the above result with (26), we obtain

$$\bar{x}_m = \left[\frac{A_m\alpha_m(1-\tau_k)}{\rho\phi'(0)}\right]^{\frac{1}{1-\alpha_m}}. \tag{27}$$

Hence

$$\bar{n} = \frac{\gamma}{pA_m(1-\alpha_m)(1-\tau_l)}\left[\frac{\rho\phi'(0)}{A_m\alpha_m(1-\tau_k)}\right]^{\frac{\alpha_m}{1-\alpha_m}}$$

Substituting $\bar{x}_m$ into (18) to obtain

$$\bar{x}_h = \frac{\phi_h}{\tilde{\phi}_m}\left[\frac{A_m\alpha_m(1-\tau_k)}{\rho\phi'(0)}\right]^{\frac{1}{1-\alpha_m}}. \tag{28}$$

Therefore, from (8), the relative price of the home good is

$$\bar{\lambda}/p = \frac{[A_m \alpha_m]^{\frac{1-\alpha_h}{1-\alpha_m}}}{A_h \alpha_h [\rho \phi'(0)]^{\frac{\alpha_m - \alpha_h}{1-\alpha_m}}} \left(\frac{\phi_h}{\phi_m} \right)^{1-\alpha_h} (1-\tau_l)^{1-\alpha_h} (1-\tau_k)^{\frac{\alpha_m(1-\alpha_h)}{1-\alpha_m}} .$$

Substitute $\bar{\lambda}/p \equiv \bar{v}$ into (4) and (5), then

$$\bar{c}_m = \frac{1}{p} \Big[\frac{(\frac{1-\mu}{\mu})^{1/(\varepsilon-1)} (1+\tau_c)^{1/(\varepsilon-1)} \bar{v}^{\varepsilon/(1-\varepsilon)}}{(\frac{1-\mu}{\mu})^{1/(\varepsilon-1)} (1+\tau_c)^{\varepsilon/(\varepsilon-1)} \bar{v}^{\varepsilon/(1-\varepsilon)} + 1} \Big] \tag{29}$$

$$\bar{c}_h = \frac{1}{p\bar{v}} \Big[\frac{1}{(\frac{1-\mu}{\mu})^{1/(\varepsilon-1)} (1+\tau_c)^{\varepsilon/(\varepsilon-1)} \bar{v}^{\varepsilon/(1-\varepsilon)} + 1} \Big] \tag{30}$$

Notice that for a standard model (with no home production considered), that is $\mu = 1$, we have $\bar{c}_h = 0$, $\bar{c}_m = p^{-1}$.

From

$$A_h \left(\frac{K - (1-n) x_m}{x_h - x_m} \right) x_h^{\alpha_h} = c_h(\lambda, p)$$

and (15), we obtain

$$\bar{K} = \bar{x}_m \Big[1 + \Big(\frac{x_h/x_m - 1}{A_h} \Big) \bar{x}_h^{-\alpha_h} \bar{c}_h + \Big(\frac{\gamma}{A_m(1-\alpha_m)} \Big) \frac{\bar{x}_m^{-\alpha_m}}{p(1-\tau_l)} \Big], \tag{31}$$

$$\bar{B} = \frac{1}{r^*} \Big[\bar{c}_m + \phi(\psi(\bar{q}/p)) - A_m \bar{L}_m \bar{x}_m^{\alpha_m} + G \Big]. \tag{32}$$

where ψ is the inverse function of ϕ'.

3.3 Stability analysis

To investigate the local stability of the steady state, let us linearize the dynamic system (22)-(24) in the neighborhood of the steady state. This yields

$$\begin{pmatrix} \dot{K} \\ \dot{q} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & \partial\dot{K}/\partial q & 0 \\ 0 & \rho & -p(1-\tau_k)\partial R/\partial\lambda \\ \partial Y_h/\partial K & 0 & \partial Y_h/\partial\lambda - \partial c_h/\partial\lambda \end{pmatrix} \begin{pmatrix} K-\bar{K} \\ q-\bar{q} \\ \lambda-\bar{\lambda} \end{pmatrix} \tag{33}$$

where

$$\frac{\partial\dot{K}}{\partial q} = \frac{1}{p}\psi'(\bar{q}/p) > 0 \tag{34}$$

in view of $\psi' = 1/(\phi'') > 0$. The eigen equation of the coefficient matrix in the above linear system is

$$\chi^2 - \rho\chi + D = 0$$

where

$$D = -p(1-\tau_k)\frac{\partial R}{\partial\lambda}\frac{\partial Y_h}{\partial K}\frac{\partial\dot{K}}{\partial q}/\left(\frac{\partial Y_h}{\partial\lambda} - \frac{\partial c_h}{\partial\lambda}\right)$$

Proposition 2 (i) *If sign*$\left[\tilde{\phi}_m - \phi_h\right]$=*sign*$[\phi_m - \phi_h]$, *or (ii) sign*$\left[\tilde{\phi}_m - \phi_h\right]$=$-$*sign*$[\phi_m - \phi_h]$ *and* $\tilde{\phi}_m - \phi_h$ *is close to* 0, *then the steady state is locally saddle-point stable.*

Proof. Note first that $\partial\dot{K}/\partial q > 0$ always. From Lemmas 3 and 5, we know sign$[\partial R/\partial\lambda]$=$-$sign$[\phi_m-\phi_h]$ and sign$[\partial Y_h/\partial K]$=$-$sign$[\tilde{\phi}_m-\phi_h]$. On the other hand, $\partial c_h/\partial\lambda < 0$ and sign$[\partial Y_h/\partial\lambda]$ =sign$[\phi_m-\phi_h]$sign$[\tilde{\phi}_m - \phi_h]$ from Lemmas 1 and 5, then if sign$[\phi_m - \phi_h]$ =sign$[\tilde{\phi}_m - \phi_h]$, $\partial Y_h/\partial\lambda - \partial c_h/\partial\lambda > 0$, and $(\partial R/\partial\lambda)(\partial Y_h/\partial K) > 0$, therefore $D < 0$. If sign$[\phi_m - \phi_h]$ $=$ $-$sign$[\tilde{\phi}_m - \phi_h]$, since $(\partial R/\partial\lambda)(\partial Y_h/\partial K) < 0$ now, in order for $D < 0$, $\partial Y_h/\partial\lambda - \partial c_h/\partial\lambda$ must be negative, which can be ensured if $\partial c_h/\partial\lambda$ is dominated by $\partial Y_h/\partial\lambda$.This is case when $\tilde{\phi}_m - \phi_h$ is sufficient close to 0. □

It is worth noting that, unlike the standard model without home production, the stability of the steady state depends on the ranking of capital/labor ratio between market and home sector. In the case that the taxation does not affect the ranking of the two sectors, the saddle-point stability of the steady state can be assured. However, when the post-tax capital/labor ratio ranking is reversed additional condition is needed.

3.4 Current account and the determinant of p

Although p is constant over time, it is endogenously determined. In the following, through the dynamic analysis of the current account, the condition p must satisfy can be derived.

Recall the national budget constraint in (15) and take (12) into consideration, then we have

$$B_0 + \int_0^\infty [RK_m + wL_m - c_m - \phi(I) - G]e^{-rt}dt = 0$$

This means that a net creditor country can not run trade surplus permanently: at some point it must run a trade deficit in order for the above relation to be satisfied. Under a given government spending G, this relation does not necessarily holds, because both production and consumption are determined by market force. Therefore, in addition to the optimal conditions, the above relation gives an extra constraint of the economy. This additional constraint determines what value p should take.

To determine this endogenously determined constant, we rely on the linearized dynamic system (33). Having indicated the saddle point stability of the steady state,[8] there must be a stable eigenvalue since K is predetermined. The stable eigenvalue of system (33) is

$$\chi = \frac{1}{2}\left[\rho - \sqrt{\rho^2 - 4D}\right] < 0.$$

The stable saddle path on the $K - q$ plane can be expressed as

$$K - \bar{K} = a_1(K_0 - \bar{K})e^{\chi t},$$
$$q - \bar{q} = a_2(q_0 - \bar{q})e^{\chi t},$$

[8] Our discussion below is confined to this case only, which we think is economically meaningful.

where $(a_1, a_2)^T$ is a eigenvector of χ. Let $a_1 = 1$, we get $a_2 = \chi/\frac{1}{p}\psi'$. Since q_0 can be chose freely, we can express the stable saddle-path as

$$K - \bar{K} = (K_0 - \bar{K})e^{\chi t}, \tag{35}$$

$$q - \bar{q} = \frac{\chi}{\psi'/p}(K - \bar{K}). \tag{36}$$

From (15),

$$\dot{B} = Y_m(K, \lambda, p) + rB - c_m(\lambda, p) - \phi(\psi(\frac{q}{p})).$$

Thus, in the neighborhood of $(\bar{K}, \bar{q}, \bar{B})$, the above relation can be approximated with

$$\dot{B} = r(B - \bar{B}) + \Lambda(K - \bar{K}), \tag{37}$$

where

$$\Lambda \equiv \frac{\partial Y_m}{\partial K} + \left(\frac{\partial Y_m}{\partial \lambda} - \frac{\partial c_m}{\partial \lambda}\right)\frac{\partial \lambda}{\partial K} - \chi\phi'(\bar{q}/p).$$

Notice that, given the saddle-point property of the system, the 1st and 3rd terms on the right hand side of the above expression are positive, while the 2nd term has the same sign with $[-v'(\bar{K})]$ since $c_m'(v(\bar{K})) > 0$. It looks $\Lambda > 0$ is likely the case at least when sign$[x_m - x_h]$=sign$[\alpha_m - \alpha_h] < 0$.

Lemma 7. *(i) If* $\tilde{\phi}_m - \phi_h > 0$, $\phi_m - \phi_h > 0$ and $\varepsilon < 0$ or $\varepsilon \simeq 0$, $\Lambda > 0$*; (ii) If* $\tilde{\phi}_m < \phi_h$, $\phi_m < \phi_h$ *and* $-\varepsilon$ *is sufficient large*, $\Lambda < 0$; *(iii) If* $\tilde{\phi}_m > \phi_h$, $\phi_m < \phi_h$, $\tilde{\phi}_m - \phi_h \simeq 0$ and $\varepsilon > 0$ or $\varepsilon \simeq 0$, $\Lambda > 0$. (iv) $\tilde{\phi}_m < \phi_h$, $\phi_m > \phi_h$ and $\tilde{\phi}_m - \phi_h \simeq 0$, $\Lambda < 0$.

Proof. Notice that $-\chi\phi'(\bar{q}/p) > 0$ always.

Case (i) $\tilde{\phi}_m > \phi_h$ and $\phi_m > \phi_h$, then $\partial Y_m/\partial K > 0$, $\partial Y_m/\partial \lambda > 0$, and $\partial \lambda/\partial K > 0$. Since sign$[\partial c_m/\partial \lambda]$=sign$[\varepsilon]$ $(-\infty < \varepsilon < 1)$, as long as $\varepsilon < 0$ or $\varepsilon \simeq 0$, we have $\Lambda > 0$.

Case (ii) $\tilde{\phi}_m < \phi_h$ and $\phi_m < \phi_h$, then $\partial Y_m/\partial K < 0$, $\partial Y_m/\partial \lambda > 0$ and $\partial \lambda/\partial K < 0$.For sufficient large $-\varepsilon$, we have $\Lambda < 0$.

Case (iii) $\tilde{\phi}_m > \phi_h$ and $\phi_m < \phi_h$, then $\partial Y_m/\partial K > 0$, $\partial Y_m/\partial \lambda < 0$, and if in addition

$\tilde{\phi}_m - \phi_h \simeq 0$, $\partial\lambda/\partial K < 0$. As long as $\varepsilon > 0$ or $\varepsilon \simeq 0$, we have $\Lambda > 0$.

Case (iv) $\tilde{\phi}_m < \phi_h$ and $\phi_m > \phi_h$, then $\partial Y_m/\partial K < 0$, $\partial Y_m/\partial\lambda < 0$ and if in addition $\tilde{\phi}_m - \phi_h \simeq 0$, $\partial\lambda/\partial K > 0$. Hence, $\Lambda < 0$. □

The solution of (37) is

$$(B_t - \bar{B})e^{-r^* t} = C + \Lambda(K_0 - \bar{K})\Big(\frac{1}{\chi - r^*}\Big)e^{(\chi - r^*)t}, \tag{38}$$

where C is a constant, given by

$$C = (B_0 - \bar{B}) - \Lambda(K_0 - \bar{K})\Big(\frac{1}{\chi - r^*}\Big).$$

On the other hand, the transversility condition in (12) means $C = 0$ in (38), so that

$$B_0 - \bar{B} = \Lambda(K_0 - \bar{K})\Big(\frac{1}{\chi - r^*}\Big) \tag{39}$$

From Subsection 3.2, we have known that $\bar{B} = \bar{B}(p)$ and $\bar{K} = \bar{K}(p)$. For given K_0, B_0, substitute these expressions into the above relation, which is a unitary equation of p, from which the value of $p = p(K_0, B_0)$ can be determined.

We emphasize that the steady state to which the economy converges depends upon the initial conditions. The transition pattern is determined by the adjustment of the implicit price of home good in terms of market goods p. Different from the corresponding closed economy, there is no room for a small economy to adjust its interest rate for achieving a unique steady state. Instead, the initial conditions affect the destination of the economy in the long run.

4 Long-run policy effect

To obtain the long-run effect of policy changes, by differentiating (15), (22)-(24) and (39) around the steady state $(\bar{K}, \bar{q}, \bar{\lambda}, \bar{B}, \bar{p})$, we have

$$\begin{bmatrix} 0 & 1/p\phi''(0) & 0 & 0 & -\phi'(0)/p\phi''(0) \\ 0 & \rho & -p(1-\tau_k)\partial R/\partial\lambda & 0 & -(1-\tau_k)\left(R+p\partial R/\partial p\right) \\ \partial Y_h/\partial K & 0 & \partial Y_h/\partial\lambda - \partial c_h/\partial\lambda & 0 & \partial Y_h/\partial p - \partial c_h/\partial p \\ \frac{\partial Y_m}{\partial K} & -\frac{\phi'(0)}{p\phi''(0)} & \frac{\partial Y_m}{\partial\lambda} - \frac{\partial c_m}{\partial\lambda} & r^* & \frac{\partial Y_m}{\partial p} - \frac{\partial c_m}{\partial p} + \frac{\phi'(0)^2}{p\phi''(0)} \\ \Lambda/(\chi - r^*) & 0 & 0 & -1 & 0 \end{bmatrix} \begin{pmatrix} d\bar{K} \\ d\bar{q} \\ d\bar{\lambda} \\ d\bar{B} \\ d\bar{p} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ p\left[(1-\tau_k)\frac{\partial R}{\partial\tau_k} - R\right]d\tau_k + p(1-\tau_k)\frac{\partial R}{\partial\tau_l}d\tau_l \\ -\frac{\partial Y_h}{\partial\tau_k}d\tau_k - \frac{\partial Y_h}{\partial\tau_l}d\tau_l + \frac{\partial c_h}{\partial\tau_c}d\tau_c \\ dG - \frac{\partial Y_m}{\partial\tau_k}d\tau_k - \frac{\partial Y_m}{\partial\tau_l}d\tau_l + \frac{\partial c_m}{\partial\tau_c}d\tau_c \\ 0 \end{pmatrix} \tag{40}$$

from which

$$\frac{\partial\bar{p}}{\partial G} = \frac{1}{\Delta\rho\phi''(0)}\frac{\partial Y_h}{\partial K}\frac{\partial R}{\partial\lambda}p(1-\tau_k)$$

can be derived. Since $\mathrm{sign}\left[\frac{\partial Y_h}{\partial K}\right] = -\mathrm{sign}\left[\tilde{\phi}_m - \phi_h\right]$ and $\mathrm{sign}\left[\frac{\partial R}{\partial\lambda}\right] = -\mathrm{sign}[\phi_m - \phi_h]$, and the determinant of the coefficient matrix of the above system $\Delta < 0$ from the stable condition, then

$$\mathrm{sign}\left[\frac{\partial\bar{p}}{\partial G}\right] = -\mathrm{sign}\left[\tilde{\phi}_m - \phi_h\right]\mathrm{sign}\left[\phi_m - \phi_h\right].$$

Here

$$\Delta = \frac{-1}{\rho\phi''(0)}\begin{vmatrix} 0 & -p(1-\tau_k)\frac{\partial R}{\partial\lambda} & \rho\phi'(0) - (1-\tau_k)\left(R + p\frac{\partial R}{\partial p}\right) \\ \frac{\partial Y_h}{\partial K} & \frac{\partial Y_h}{\partial\lambda} - \frac{\partial c_h}{\partial\lambda} & \frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} \\ \frac{\partial Y_m}{\partial K} + \frac{r^*\Lambda}{\chi - r^*} & \frac{\partial Y_m}{\partial\lambda} - \frac{\partial c_m}{\partial\lambda} & \frac{\partial Y_m}{\partial p} - \frac{\partial c_m}{\partial p} \end{vmatrix}$$

4.1 Effects of government spending expansion

From (40) and notice that $\rho\phi'(0)/(1-\tau_k)=R$, we obtain

$$\begin{aligned}\frac{\partial\bar{q}}{\partial G} &= \phi'(0)\frac{\partial\bar{p}}{\partial G}\\ \frac{\partial\bar{\lambda}}{\partial G} &= \frac{\rho\phi'(0)-(1-\tau_k)\left(R+p\partial R/\partial p\right)}{p(1-\tau_k)\partial R/\partial\lambda}\frac{\partial\bar{p}}{\partial G}=-\frac{\partial R/\partial p}{\partial R/\partial\lambda}\frac{\partial\bar{p}}{\partial G}\end{aligned}$$

and

$$\begin{aligned}-\frac{\partial Y_h}{\partial K}\frac{\partial\bar{K}}{\partial G} &= \left[\left(\frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p}\right)+\left(\frac{\partial Y_h}{\partial\lambda}-\frac{\partial c_h}{\partial\lambda}\right)\frac{\partial\bar{\lambda}}{\partial G}\right]\frac{\partial\bar{p}}{\partial G}\\ \frac{\partial\bar{B}}{\partial G} &= \left(\frac{\Lambda}{\chi-r^*}\right)\frac{\partial\bar{K}}{\partial G}\end{aligned}$$

Capital level and net foreign asset. *If* sign$\left[\tilde{\phi}_m-\phi_h\right]$ =sign$[\phi_m-\phi_h]$, $\varepsilon>0$ *and* $\tilde{\phi}_m\approx\phi_h$, *then* $\frac{\partial\bar{K}}{\partial G}>0$. *Only in the case that* $\tilde{\phi}_m-\phi_h>0$, $\phi_m-\phi_h>0$ *and* $\varepsilon\simeq 0$, *an unambiguous result,* $\frac{\partial\bar{B}}{\partial G}<0$ *can be derived.*

Proof. Notice that under the stable condition in Proposition 2

$$\operatorname{sign}\left[\frac{\partial Y_h}{\partial\lambda}-\frac{\partial c_h}{\partial\lambda}\right]=\operatorname{sign}\left[\tilde{\phi}_m-\phi_h\right]\operatorname{sign}\left[\phi_m-\phi_h\right]$$

On the other hand,

$$\begin{aligned}\frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p} &< 0,\ \text{if sign}\left[\tilde{\phi}_m-\phi_h\right]=\text{sign}\left[\phi_m-\phi_h\right]\ \text{and}\ \varepsilon>0\\ \frac{\partial Y_h}{\partial p}-\frac{\partial c_h}{\partial p} &> 0,\ \text{if sign}\left[\tilde{\phi}_m-\phi_h\right]=-\text{sign}\left[\phi_m-\phi_h\right]\ \text{and}\ \varepsilon<0\end{aligned}$$

and

$$\operatorname{sign}\left[-\frac{\partial Y_h}{\partial K}\right]=\operatorname{sign}\left[\tilde{\phi}_m-\phi_h\right],\ \operatorname{sign}\left[\frac{\partial\bar{p}}{\partial G}\right]=-\operatorname{sign}\left[\tilde{\phi}_m-\phi_h\right]\operatorname{sign}\left[\phi_m-\phi_h\right]$$

Hence,

(i) $\tilde{\phi}_m > \phi_h$ and $\phi_m > \phi_h$. Then $-\frac{\partial Y_h}{\partial K} > 0$ and $\frac{\partial \bar{p}}{\partial G} < 0$. If in addition $\varepsilon > 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} < 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} > 0$. Hence $\frac{\partial \bar{K}}{\partial G} > 0$.

(ii) $\tilde{\phi}_m > \phi_h$ and $\phi_m < \phi_h$. Then $-\frac{\partial Y_h}{\partial K} > 0$ and $\frac{\partial \bar{p}}{\partial G} > 0$. If in addition $\varepsilon < 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} > 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} < 0$. Hence $\text{sign}\left[\frac{\partial \bar{K}}{\partial G}\right]$ =?.

(iii) $\tilde{\phi}_m < \phi_h$ and $\phi_m < \phi_h$. Then $-\frac{\partial Y_h}{\partial K} < 0$ and $\frac{\partial \bar{p}}{\partial G} < 0$. If in addition $\varepsilon > 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} < 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} > 0$. Hence $\frac{\partial \bar{K}}{\partial G} > 0$.

(iv) $\tilde{\phi}_m < \phi_h$ and $\phi_m > \phi_h$. Then $-\frac{\partial Y_h}{\partial K} < 0$ and $\frac{\partial \bar{p}}{\partial G} > 0$. If in addition $\varepsilon < 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} > 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} < 0$. Hence $\text{sign}\left[\frac{\partial \bar{K}}{\partial G}\right]$ =?.

From Lemma 7, we know $\Lambda > 0$ if $\tilde{\phi}_m - \phi_h > 0$, $\phi_m - \phi_h > 0$ and $\varepsilon \simeq 0$. That is, we have now $\frac{\partial \bar{B}}{\partial G} < 0$. This is the only case that the sign of $\frac{\partial \bar{B}}{\partial G}$ can be determined. $\square$

Market and home consumptions. *(i) If* $\text{sign}\left[\tilde{\phi}_m - \phi_h\right]$*=sign*$[\phi_m - \phi_h]$ *and* $\varepsilon < 0$, *then an expansion in the government spending lowers households' market and home goods consumption. (ii) If* $\text{sign}\left[\tilde{\phi}_m - \phi_h\right]$*=-sign*$[\phi_m - \phi_h]$ *and* $\varepsilon < 0$, *then an expansion in the government spending lowers households' market and home goods consumption.*

The results here can be understood by inspecting the following calculations ($i = m, h$).

$$\begin{aligned} \frac{\partial \bar{c}_i}{\partial G} &= \frac{\partial c_i}{\partial \lambda}\frac{\partial \bar{\lambda}}{\partial G} + \frac{\partial c_i}{\partial p}\frac{\partial \bar{p}}{\partial G} \\ &= \left\{\frac{\partial c_i}{\partial \lambda}\left(-\frac{\partial R/\partial p}{\partial R/\partial \lambda}\right) + \frac{\partial c_i}{\partial p}\right\}\frac{\partial \bar{p}}{\partial G} \end{aligned}$$

The statement in case (i) describe the usual results, while that of case (ii) are new to the home production model. If the income taxation distorts the economy sufficiently that the factor intensity ranking between the market and home sectors is reversed, then when the substitution rate between the home good and market good is small ($\varepsilon < 0$), in the long run government spending expansion leads to market good price to increase. Therefore the demand on home consumption should increase. In the case that $\varepsilon < 0$, even facing a higher p, households' market good consumption still can increase.

Factor prices. *An expansion in government spending does not affect the factor intensity in market sector, hence leaves the factor prices unaffected.*

$$\begin{aligned}\frac{\partial \bar{x}_m}{\partial G} &= \frac{\partial x_m}{\partial \lambda}\frac{\partial \bar{\lambda}}{\partial G} + \frac{\partial x_m}{\partial p}\frac{\partial \bar{p}}{\partial G} = \left[\frac{\partial x_m}{\partial \lambda}\left(-\frac{\partial R/\partial p}{\partial R/\partial \lambda}\right) + \frac{\partial x_m}{\partial p}\right]\frac{\partial \bar{p}}{\partial G} \\ &= \left[\frac{\partial x_m}{\partial \lambda}\left(-\frac{\partial x_m/\partial p}{\partial x_m/\partial \lambda}\right) + \frac{\partial x_m}{\partial p}\right]\frac{\partial \bar{p}}{\partial G}\end{aligned}$$

Labor time allocation. *An increase in the government spending G leads to the leisure time n to increase if sign$\left[\tilde{\phi}_m - \phi_h\right]$ =sign$[\phi_m - \phi_h]$, and the leisure time to decrease if sign$\left[\tilde{\phi}_m - \phi_h\right]$ $\neq$sign$[\phi_m - \phi_h]$. The effects on the work time allocation between market and home work of this expansion in G are generally ambiguous. In the case that sign$\left[\tilde{\phi}_m - \phi_h\right]$ =sign$[\phi_m - \phi_h]$, $\varepsilon > 0$ and $\tilde{\phi}_m \approx \phi_h$, sign$\left[\partial \bar{L}_m/\partial G\right]$=$-$sign$\left[\partial \bar{L}_m/\partial G\right]$=sign$[\phi_m - \phi_h]$.*

Notice that $\frac{\partial \bar{x}_m}{\partial G} = 0$ and $\frac{\partial n(x_m,p)}{\partial p} < 0$, then

$$\text{sign}\left[\frac{\partial \bar{n}}{\partial G}\right] = -\text{sign}\left[\frac{\partial \bar{p}}{\partial G}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right]\text{sign}\left[\phi_m - \phi_h\right]$$

From the expressions of L_m, L_h in (20) and (19)

$$\begin{aligned}\frac{\partial \bar{L}_m}{\partial G} &= \frac{\partial x_h/\partial x_m}{\partial x_h/\partial x_m - 1}\left(-\frac{\partial \bar{n}}{\partial G}\right) - \frac{1/x_m}{\partial x_h/\partial x_m - 1}\frac{\partial \bar{K}}{\partial G} \\ \frac{\partial \bar{L}_h}{\partial G} &= \frac{1/x_m}{\partial x_h/\partial x_m - 1}\frac{\partial \bar{K}}{\partial G} - \frac{1}{\partial x_h/\partial x_m - 1}\left(-\frac{\partial \bar{n}}{\partial G}\right)\end{aligned}$$

4.2 Effects of taxation shocks

Similar to the above analysis, we can investigate the effects of taxation shocks. For example, taking the change of consumption rate as an example. From (40), we have

$$\frac{\partial \bar{p}}{\partial \tau_c} = \frac{p(1-\tau_k)}{\Delta\rho\phi''(0)}\frac{\partial R}{\partial \lambda}\left[\frac{\partial Y_h}{\partial K}\frac{\partial c_m}{\partial \tau_c} - \frac{\partial c_h}{\partial \tau_c}\left(\frac{\partial Y_m}{\partial K} + \frac{r^*\Lambda}{\chi - r^*}\right)\right]$$

$$\begin{aligned}\frac{\partial \bar{q}}{\partial \tau_c} &= \phi'(0)\frac{\partial \bar{p}}{\partial \tau_c}\\ \frac{\partial \bar{\lambda}}{\partial \tau_c} &= \left(-\frac{\partial R/\partial p}{\partial R/\partial \lambda}\right)\frac{\partial \bar{p}}{\partial \tau_c}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial Y_h}{\partial K}\frac{\partial \bar{K}}{\partial \tau_c} &= \frac{\partial c_h}{\partial \tau_c} - \left[\left(\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p}\right) + \left(\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda}\right)\frac{\partial \bar{\lambda}}{\partial \tau_c}\right]\frac{\partial \bar{p}}{\partial \tau_c}\\ \frac{\partial \bar{B}}{\partial \tau_c} &= \left(\frac{\Lambda}{\chi - r^*}\right)\frac{\partial \bar{K}}{\partial \tau_c}\end{aligned}$$

As long as the effect of τ_c's change on p is clear, we can derive τ_c's change effects on other variables. However, from Lemma 7, we know $\partial Y_m/\partial K$ and $r^*\Lambda/(\chi - r^*)$ have opposite signs always. Except numerical calculations, it is generally impossible to determine the sign of $\partial\bar{p}/\partial\tau_c$. In the following, we will assume that $r^*\Lambda/(\chi - r^*)$ is dominated by $\partial Y_m/\partial K$ (this is at least the case that $x_m \approx x_h$), then we have the following results.

Shadow price of wealth p. *An increase in the rate of consumption tax raises* p *if* *sign*$\left[\tilde{\phi}_m - \phi_h\right]$ =*sign*$[\phi_m - \phi_h]$, *and lowers* p *if sign*$\left[\tilde{\phi}_m - \phi_h\right] = -$*sign*$[\phi_m - \phi_h]$.

Notice that $\frac{\partial c_m}{\partial \tau_c} < 0$, $\frac{\partial c_h}{\partial \tau_c} < 0$, sign$\left[\frac{\partial Y_h}{\partial K}\right]$=$-$sign$\left[\frac{\partial Y_m}{\partial K}\right]$=-sign$\left[\tilde{\phi}_m - \phi_h\right]$ and sign$\left[\frac{\partial R}{\partial \lambda}\right] = -$sign$[\phi_m - \phi_h]$. When $r^*\Lambda/(\chi - r^*)$ is dominated by $\partial Y_m/\partial K$, we have

$$\text{sign}\left[\frac{\partial \bar{p}}{\partial \tau_c}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right]\text{sign}\left[\phi_m - \phi_h\right].$$

Market and home consumptions. *(i) If sign*$\left[\tilde{\phi}_m - \phi_h\right]$*=sign*$[\phi_m - \phi_h]$ *and* $\varepsilon < 0$*, then an increase in the rate of the consumption tax lowers market. (ii) If sign*$\left[\tilde{\phi}_m - \phi_h\right]$*=-sign*$[\phi_m - \phi_h]$ and $\varepsilon < 0$ *then an increase in the rate of the consumption tax raises home goods consumption.*

From $(i = m, h)$.

$$
\begin{aligned}
\frac{\partial \bar{c}_i}{\partial \tau_c} &= \frac{\partial c_i}{\partial \lambda}\frac{\partial \bar{\lambda}}{\partial \tau_c} + \frac{\partial c_i}{\partial p}\frac{\partial \bar{p}}{\partial \tau_c} + \frac{\partial c_i}{\partial \tau_c} \\
&= \left\{\frac{\partial c_i}{\partial \lambda}\left(-\frac{\partial R/\partial p}{\partial R/\partial \lambda}\right) + \frac{\partial c_i}{\partial p}\right\}\frac{\partial \bar{p}}{\partial G} + \frac{\partial c_i}{\partial \tau_c}
\end{aligned}
$$

the above results can be obtained.

Factor prices: *An expansion in the rate of consumption tax does not affect the factor intensity in market sector, hence leaves the factor prices unaffected.*

$$
\begin{aligned}
\frac{\partial \bar{x}_m}{\partial \tau_c} &= \frac{\partial x_m}{\partial \lambda}\frac{\partial \bar{\lambda}}{\partial \tau_c} + \frac{\partial x_m}{\partial p}\frac{\partial \bar{p}}{\partial \tau_c} = \left[\frac{\partial x_m}{\partial \lambda}\left(-\frac{\partial R/\partial p}{\partial R/\partial \lambda}\right) + \frac{\partial x_m}{\partial p}\right]\frac{\partial \bar{p}}{\partial \tau_c} \\
&= \left[\frac{\partial x_m}{\partial \lambda}\left(-\frac{\partial x_m/\partial p}{\partial x_m/\partial \lambda}\right) + \frac{\partial x_m}{\partial p}\right]\frac{\partial \bar{p}}{\partial \tau_c}
\end{aligned}
$$

Capital level and net foreign asset. *If* sign$\left[\tilde{\phi}_m - \phi_h\right] = -$sign$[\phi_m - \phi_h]$, $\varepsilon < 0$, $\varepsilon \approx 0$ *and* $\tilde{\phi}_m \approx \phi_h$*, then* $\frac{\partial \bar{K}}{\partial \tau_c} < 0$*. In the case that* $\tilde{\phi}_m - \phi_h < 0$, $\phi_m - \phi_h > 0$*, we have* $\frac{\partial \bar{B}}{\partial \tau_c} < 0$.

Notice that under the stable condition in Proposition 2

$$
\text{sign}\left[\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right]\text{sign}\left[\phi_m - \phi_h\right]
$$

On the other hand,

$$
\begin{aligned}
\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} &< 0, \text{ if sign}\left[\tilde{\phi}_m - \phi_h\right] = \text{sign}\left[\phi_m - \phi_h\right] \text{ and } \varepsilon > 0 \\
\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} &> 0, \text{ if sign}\left[\tilde{\phi}_m - \phi_h\right] = -\text{sign}\left[\phi_m - \phi_h\right] \text{ and } \varepsilon < 0
\end{aligned}
$$

and

$$\text{sign}\left[-\frac{\partial Y_h}{\partial K}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right], \ \text{sign}\left[\frac{\partial \bar{p}}{\partial \tau_c}\right] = \text{sign}\left[\tilde{\phi}_m - \phi_h\right] \text{sign}\left[\phi_m - \phi_h\right]$$

Hence,

(i) $\tilde{\phi}_m > \phi_h$ and $\phi_m > \phi_h$. Then $\frac{\partial Y_h}{\partial K} < 0$ and $\frac{\partial \bar{p}}{\partial \tau_c} > 0$. If in addition $\varepsilon > 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} < 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} > 0$. Hence $\frac{\partial \bar{K}}{\partial \tau_c}$?.

(ii) $\tilde{\phi}_m > \phi_h$ and $\phi_m < \phi_h$. Then $\frac{\partial Y_h}{\partial K} < 0$ and $\frac{\partial \bar{p}}{\partial \tau_c} < 0$. If in addition $\varepsilon < 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} > 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} < 0$. If additionally, $\varepsilon \approx 0$ so that $\frac{\partial c_h}{\partial \tau_c}$ is close to 0. Then, $\frac{\partial \bar{K}}{\partial \tau_c} < 0$.

(iii) $\tilde{\phi}_m < \phi_h$ and $\phi_m < \phi_h$. Then $\frac{\partial Y_h}{\partial K} > 0$ and $\frac{\partial \bar{p}}{\partial \tau_c} > 0$. If in addition $\varepsilon > 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} < 0$. From the sufficient conditions of a stable steady state in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} > 0$. Hence $\frac{\partial \bar{K}}{\partial \tau_c}$?0.

(iv) $\tilde{\phi}_m < \phi_h$ and $\phi_m > \phi_h$. Then $\frac{\partial Y_h}{\partial K} > 0$ and $\frac{\partial \bar{p}}{\partial \tau_c} < 0$. If in addition $\varepsilon < 0$, we have $\frac{\partial Y_h}{\partial p} - \frac{\partial c_h}{\partial p} > 0$. From the sufficient stable condition in Proposition 2, we know $\frac{\partial Y_h}{\partial \lambda} - \frac{\partial c_h}{\partial \lambda} < 0$. And if additionally, $\varepsilon \approx 0$ so that $\frac{\partial c_h}{\partial \tau_c}$ is close to 0. Then, $\frac{\partial \bar{K}}{\partial \tau_c} < 0$.

Examining the above cases for Λ, we find from Lemma 7 that, only in case (iv), we have a clear sign for $\Lambda : \Lambda < 0$. Hence, $\frac{\partial \bar{B}}{\partial \tau_c} < 0$. This is the only case that the sign of $\frac{\partial \bar{B}}{\partial G}$ can be determined. $\square$

5 Transitional dynamics

For showing the transitional dynamics of the economy, we take an expansion of government spending as an example. The analysis on other shocks can be accomplished in a similar way. Based on the linearized dynamic system (33), we know that around the steady state the stable equilibrium path takes a shape like SS' in Figure 1.

(CASE I): $\partial p/\partial G < 0$.

When a sudden increase in G occurs, p jumps up at once. Since $\bar{q}$ must move in a same way as p in view of (25), then $\bar{q}$ will go down. On the other hand, Proposition 3 shows $\bar{K}$ will move to a higher level. Then this shock results in a right shift of the stable arm on the $K-q$ plane, to S_1S_1' for example. Since the $K-q$ dynamic system is complete, moving patterns of other variables can be derived therefrom, except those of B, which we will examine below.

In view of (38), the relation between B and K around the steady state can be illustrated as XX' in Figure 2, which is negative sloped on the $K-B$ plane. An increase in G lowers $\bar{B}$ and raises $\bar{K}$ in the long run. This corresponds to a right down moving along locus XX'.

(CASE II): $\partial p/\partial G > 0$.

As the above case, similar analysis can be made.

6 Concluding remarks

Including home sector into a standard small-open neoclassical growth model, in this paper we explored the equilibrium dynamics and the policy effects of the model economy analytically. We found that nonmarket home sector and fiscal policy asymmetry together play important role in determining the equilibrium property and policy effects in the model economy. We shown also that the rate of substitution between home and market consumptions can affect the above analysis.

We found also that sole theoretical study has limitation in understanding the whole policy effects. For some parameter ranges, we have to rely on numerical calculation to determine the exact effects. This numerical study is left for the future.

Appendix

A.1 Proof of Lemma 2

From the expression of dx_m in Section 3.1, the definition of D_2 and the relation

$$x_m(1-\tau_l)(1-\alpha_m) = \left(\frac{1-\alpha_h}{\alpha_h}\right)\alpha_m(1-\tau_k)x_h$$

we have

$$\begin{aligned}\frac{\partial x_m}{\partial p} &= \frac{-1}{D_2}A_m x_m^{\alpha_m-1}\lambda A_h\alpha_h(1-\alpha_h)x_h^{\alpha_h-2}[(1-\alpha_m)(1-\tau_l)x_m+\alpha_m x_h(1-\tau_k)] \\ &= \left(-\frac{x_m}{p}\right)\left(\frac{1}{\alpha_m-\alpha_h}\right)\end{aligned}$$

A.2 Proof of Lemma 4

From Lemma 2

$$\frac{1}{\alpha_m}+\frac{p}{x_m}\frac{\partial x_m}{\partial p} = \frac{-\alpha_h}{\alpha_m(\alpha_m-\alpha_h)}.$$

On the other hand, from $n = n(\lambda, p; \tau_k, \tau_l)$ in (21)

$$\begin{aligned}\frac{\partial n}{\partial p} &= \frac{-\gamma\alpha_m}{A_m(1-\alpha_m)(1-\tau_l)}\frac{(1/\alpha_m+p/x_m\cdot\partial x_m/\partial p)}{p^2x_m^{\alpha_m}} \\ &= \frac{\gamma\alpha_h/(\alpha_m-\alpha_h)}{A_m(1-\alpha_m)(1-\tau_l)p^2x_m^{\alpha_m}}\end{aligned}$$

A.3 Proof of Lemma 5

From $Y_j = A_jL_j(K,\lambda,p)x_j^{\alpha_j}(\lambda,p), j = m, h,$

$$\begin{aligned}Y_h &= \frac{A_h}{1-x_m/x_h}\left[\frac{K}{x_h}-(1-n)\frac{x_m}{x_h}\right]x_h^{\alpha_h} \\ Y_m &= \frac{A_m}{x_h/x_m-1}\left[(1-n)\frac{x_h}{x_m}-\frac{K}{x_m}\right]x_m^{\alpha_m}\end{aligned}$$

then

$$\begin{aligned}
\frac{\partial Y_h}{\partial \lambda} &= -\left(\frac{A_h}{1-x_m/x_h}\right)\left[\begin{array}{c} \frac{{x_h}^{\alpha_h} x_m}{x_h}\left(-\frac{\partial n}{\partial \lambda}\right) \\ +(1-\alpha_h)\frac{K{x_h}^{\alpha_h-1}}{x_h}\frac{\partial x_h}{\partial \lambda}+\frac{(1-n)\alpha_h x_h{x_h}^{\alpha_h-1}x_m}{x_h}\frac{\partial x_h}{\partial \lambda}\end{array}\right] \\
\frac{\partial Y_h}{\partial p} &= \left(\frac{A_h}{1-x_m/x_h}\right)\left\{x_m x_h^{\alpha_h-1}\frac{\partial n}{\partial p}-\left[(1-\alpha_h)K+\alpha_h(1-n)x_m\right]x_h^{\alpha_h-2}\frac{\partial x_h}{\partial p}\right\}
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial Y_m}{\partial \lambda} &= \left(\frac{A_m}{x_h/x_m-1}\right)\left[\begin{array}{c} \frac{x_h{x_m}^{\alpha_m}}{x_m}\left(-\frac{\partial n}{\partial \lambda}\right) \\ +(1-\alpha_m)\frac{K{x_m}^{\alpha_m-1}}{x_m}\frac{\partial x_m}{\partial \lambda}+\frac{(1-n)\alpha_m x_h{x_m}^{\alpha_m-1}}{x_m}\frac{\partial x_m}{\partial \lambda}\end{array}\right] \\
\frac{\partial Y_h}{\partial K} &= \frac{A_h{x_h}^{\alpha_h}}{x_h-x_m},\ \frac{\partial Y_m}{\partial K}=-\frac{A_m{x_m}^{\alpha_m}}{x_h-x_m}
\end{aligned}$$

A.4 Proof of Lemma 6

Totally differentiate the two sides of $Y_h(K,\lambda,p)=c_h(\lambda,p)$, we obtain

$$\frac{\partial \lambda}{\partial K}=-\frac{\partial Y_h}{\partial K}/\left(\frac{\partial Y_h}{\partial \lambda}-\frac{\partial c_h}{\partial \lambda}\right).$$

If $\text{sign}\left[\tilde{\phi}_m-\phi_h\right]$ $=\text{sign}[\phi_m-\phi_h]$, then $\partial Y_h/\partial\lambda-\partial c_h/\partial\lambda>0$, hence $\text{sign}[\partial\lambda/\partial K]=-\text{sign}[\partial Y_h/\partial K]=\text{sign}\left[\tilde{\phi}_m-\phi_h\right]$. If $\text{sign}\left[\tilde{\phi}_m-\phi_h\right]=-\text{sign}[\phi_m-\phi_h]$, as long as $\tilde{\phi}_m-\phi_h$ is sufficient close to 0, $\partial c_h/\partial\lambda$ will be dominated by $\partial Y_h/\partial\lambda$, that is $\partial Y_h/\partial\lambda-\partial c_h/\partial\lambda<0$. Then in this case, $\text{sign}[\partial\lambda/\partial K]=\text{sign}[\partial Y_h/\partial K]=-\text{sign}\left[\tilde{\phi}_m-\phi_h\right]$.

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