Imitation as a Stepping Stone to Innovation

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Abstract

By creating a knowledge base that reduces innovation costs, imitation can serve as a stepping stone enabling firms from lagging countries to undertake innovation. This paper constructs a product cycle model with this feature to determine what forces could cause a lagging country to shift its R&D from imitation toward innovation. Increasing resources or subsidizing R&D in the lagging South increases Southern innovation, while increasing resources or subsidizing R&D in the North decreases Southern innovation. Effects on Northern innovation, Southern imitation, aggregate expenditure, and the relative wage are compared with the case where innovation does not occur in the South.

Keywords: Innovation, Imitation, Appropriate technology

JEL classification: F1; F4; O3

1.   Introduction

Learning through imitation may enable firms to improve existing technologies. South Korea, Taiwan, and increasingly China, Malaysia, Indonesia and Thailand, have begun to follow Japan's path by imitating technologies from abroad (see Carolan et al 1998). Firms from these countries are now beginning to innovate.

The World Competitiveness Yearbook (2002) indicates that, as of 2001, 29.34% of Korea's manufactured exports are high-tech, a value similar to the United States (29.70%) and Japan (26.60%). Korea's expenditure on R&D in 2001 was $12,489 million, which ranks eighth and is similar in magnitude to Canada ($13,517 million) but well below the United States and Japan at $282,293 million and $142,014 million, respectively. Mainland China's R&D in 2001 was $12,595 million (rank 7), and Taiwan's was $6,063 (rank 10). Korea's R&D is 2.921% of GDP, which ranks second, close behind Japan at 2.981% and just above the United States at 2.800%. Taiwan's R&D is 2.156% of GDP (rank 6, above Canada and the United Kingdom), and China's R&D is 1.087% of GDP (rank 11, above Italy and Spain). An average of 34,052 patents were granted to Korean residents per year over the period 1998-2000, which ranks third, well behind Japan and

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the United States at 123,978 and 83,090, respectively. Taiwan ranks fourth with 20,094 patents a year, and China ranks tenth with 3,742. In 2000, Korean residents secured 7,032 patents abroad, ranking eighth and similar to Canada at 7,473 (rank 7). Taiwan secured 2,486 patents abroad (rank 10), and China 308 (rank 15).

What determines the degree that firms in a lagging country innovate? For lagging economies, innovation may initially be prohibitively difficult. By bringing practical knowledge of the current state-of-the-art into the country, imitation can provide the vital knowledge base needed to make innovation attractive. Knowledge assimilation stemming from imitation is endogenous due to imitation being endogenous. Therefore, changes in the incentives for (or capacity) for imitation are linked to changes in the incentives for firms in lagging countries to innovate.

What are the effects on the United States and other advanced countries of the increasing integration of developing countries into the world trading system? How do these effects depend on the level of development of the lagging countries and, in particular, on whether they engage in imitation or innovation? As countries enter into freer trade with the rest of the world, they seek to absorb technology from abroad through imitation and, ultimately, to be able to achieve state-of-the-art technology through innovation. One can think of the entry and further integration of developing countries into the world trading system as an expansion in the size (or resources) of the developing world that is open to trade. One can also think of subsidies to R&D as capturing policies countries pursue in order to try to promote R&D.

This paper builds a product cycle model to determine the effects of forces such as resource accumulation or government R&D incentives on the mix of a lagging country's R&D between innovation and imitation. The model has two countries, the North (an advanced country) and the South (a lagging country). The results indicate that increasing the supply of Southern resources can provide the added resources needed to increase Southern innovation. Government incentives to R&D could also encourage Southern firms to increase innovation. Such incentives need not be able to distinguish innovation from imitation. Increasing Northern resources or a Northern R&D subsidy shifts Southern R&D out of innovation and into imitation.

Most of the product cycle literature, such as Grossman and Helpman (1991a) and Segerstrom et al (1990), assumes firms from the lagging country only imitate and thus do not address what determines the degree that the South innovates. One exception is Currie et al (1999), where spillovers that lower the cost of Southern innovation can occur directly from the knowledge base built by Northern innovation. That model is based on innovations that are new varieties (rather than new qualities) and does not consider a general subsidy to Southern R&D (both innovation and imitation). Considering a general Southern R&D subsidy is important because distinguishing between innovation and imitation when subsidizing R&D is nearly impossible in practice.

Van Elkan (1996) also provides a model where the South shifts from imitation to innovation. There, productivity in imitation depends on the difference between the body of world knowledge and the stock of human capital (the knowledge gap), while productivity in innovation depends on past behavior through learning-by-doing. Imitation is easier the larger the knowledge gap due to the larger pool of potential imitations (larger world knowledge stock), so as the knowledge gap closes with successive imitation, imitation becomes relatively more difficult and innovation relatively more attractive. The van Elkan model has the unrealistic trait that a technological advantage in imitation assures higher per capital output than a technological advantage in innovation.

The paper is organized as follows. After establishing the behavior of consumers and firms (Section 2), we find the steady-state equilibrium when Southern innovation awaits the knowledge base generated by Southern imitation (Section 3). Then we examine what forces promote Southern innovation relative to imitation and compare those effects to the case where the South only imitates (Section 4). Proofs of results appear in the Appendix.

2.   The Economy

The economy is composed of two countries, each containing a representative consumer and many firms. Firms differ in their R&D abilities so that Northern firms innovate while Southern firms imitate and only innovate if imitation has lowered the costs of innovation. The Southern wage falls below the Northern wage, so imitation is profitable for Southern firms but not

for Northern firms. A fixed supply of one factor, labor, is used for R&D and production in each country. In equilibrium, expected profits from the product market compensate firms for their R&D costs, and resources are fully employed in each country.

2.1. Consumers

The specification of the consumer's problem follows Grossman and Helpman (1991a). Consumers choose from a continuum of products $j \in [0, 1]$. Quality level m of product j provides quality $q_m(j) \equiv \lambda^m$. By the definition of quality improvement, new generations are better than the old: $q_m(j) > q_{m-1}(j) \rightarrow \lambda^m > \lambda^{m-1} \rightarrow \lambda > 1$. All products start at time $t = 0$ at quality level $m = 0$, so the base quality is $q_0(j) = \lambda^0 = 1$.

A consumer from country $i \in \{N, S\}$ has additively separable intertemporal preferences given by lifetime utility

$$U_i = \int_0^\infty e^{-\rho t} \log u_i(t) dt, \tag{1}$$

where ρ is the common subjective discount factor. Instantaneous utility is

$$\log u_i(t) = \int_0^1 \log \sum_m (\lambda)^m x_{im}(j, t) dj, \tag{2}$$

where $x_{im}(j, t)$ is consumption by consumers from country i of quality level m of product j at time t.

Consumers maximize lifetime utility subject to an intertemporal budget constraint. Since preferences are homothetic, aggregate demand is found by maximizing lifetime utility subject to the aggregate intertemporal budget constraint

$$\int_0^\infty e^{-R(t)} E_i(t) dt \leq A_i(0) + \int_0^\infty e^{-R(t)} Y_i(t) dt, \tag{3}$$

where $R(t) = \int_0^t r(s) ds$ is the cumulative interest rate up to time t and $A_i(0)$ is the aggregate value of initial asset holdings by consumers from country i. Individuals hold assets in the form of ownership in firms, but with a diversified portfolio, any capital losses appear as capital gains elsewhere so only initial asset holdings remain. Aggregate labor income of all consumers from country i is $Y_i(t) = L_i w_i(t)$, where $w_i(t)$ is the wage in country i at

time t and L_i is the labor supply there, so $L_i w_i(t)$ is total labor income in country i at time t. Aggregate expenditure of all consumers in country i is

$$E_i(t) = \int_0^1 \left[\sum_m p_m(j,t) x_{im}(j,t) \right] dj, \tag{4}$$

where $p_m(j,t)$ is the price of quality level m of product j at time t, and $E_i(t)$ is aggregate expenditure of consumers in country i, where aggregate expenditure is $E(t) = E_N(t) + E_S(t)$. Due to assumed free trade, price levels do not vary across countries.

A consumer's maximization problem can be broken into three stages: the allocation of lifetime wealth across time, the allocation of expenditure at each instant across products, and the allocation of expenditure at each instant for each product across available quality levels. In the final stage, consumers allocate expenditure for each product at each instant to the quality level $\widetilde{m}(j,t)$ offering the lowest quality-adjusted price, $p_m(j,t)/\lambda^m$. Consumers are indifferent between quality level m and quality level $m-1$ if the relative price equals the quality difference $p_m(j,t)/p_{m-1}(j,t) = \lambda$. Settle indifference in favor of the higher quality level so the quality level selected is unique. Only the highest quality level available of each product will sell in equilibrium.

In the second stage, consumers then evenly spread expenditure across the unit measure of all products, $E_i(j,t) = E_i(t)$, as the elasticity of substitution between any two products is constant at unity. Consumers demand $x_{i\widetilde{m}}(j,t) = E_i(t)/p_{\widetilde{m}}(j,t)$ units of quality level $\widetilde{m}(j,t)$ of product j and no units of other quality levels of that product. In the first stage, consumers evenly spread lifetime expenditure across time, $E_i(t) = E_i$, as the utility function for each consumer is time separable and the aggregate price level will not vary across time $\log p_{\widetilde{m}}(j,t) = \log p_{\widetilde{m}}(j)$. Since aggregate expenditure is constant across time, the interest rate at each point in time reflects the discount rate $r(t) = \rho$, so $R(t) = \rho t$ in the intertemporal budget constraint.

In summary, consumers evenly spread expenditure across time and products. For each product, they are willing to pay a premium λ for a quality increment and will purchase only the highest quality level available.

2.2. Production

The firm's problem can be broken down into two stages: R&D and then production. In the production stage (once successful in R&D), each firm then chooses the price of its product to maximize its value, given prices and R&D intensities of other firms. Assume that firms have access to discarded technology (technologies that no longer yield profits from production, as described later). This assumption assures that Southern firms have the ability to produce lower quality levels.

Normalize the labor supplies to make the labor requirement in production be one in each country. Normalize prices to make the Southern wage be one $w_S = 1$. Define $w \equiv w_N/w_S$ as the relative wage. Consequently, the marginal cost of production is $c_S = c_M = 1$ for Southern firms (innovators or imitators) and $c_N = w$ for Northern firms. Profits are price minus cost times sales $\pi = (p - c)x$.

Consider first markets where a Northern firm has just succeeded in innovation. The Northern firm engages in limit pricing against the Southern firms with access to discarded technology one quality level below, charging a premium reflecting consumer's valuation of the quality improvement $p_N = \lambda$ and making sales $x_N = E/\lambda$. This price is just low enough to keep Southern firms from being able to sell the lower quality level at a profit, since consumers would be willing to pay $p_N/\lambda = 1$ for the lower quality level, which is the production cost for a Southern firm. Instantaneous profits for the Northern firm are

$$\pi_N = E\left(1 - \frac{w}{\lambda}\right). \tag{5}$$

Call these markets *Northern-led* since only Northern firms have the ability to produce the state-of-the-art technology.

Now consider markets where a Southern firm has just imitated. The Southern firm engages in limit pricing against the Northern firm at the same quality level by charging a price equal to the Northern firm's cost $p_M = w$ and makes sales $x_M = E/w$. Instantaneous profits for the Southern firm are

$$\pi_M = E\left(1 - \frac{1}{w}\right). \tag{6}$$

Call these markets *imitated* since both Northern and Southern firms have the ability to produce the state-of-the-art technology.

Finally consider markets where a Southern firm has just succeeded in innovation. The Southern firm engages in limit pricing against the other Southern firms with access to discarded technology one quality level below, charging a premium reflecting consumer's valuation of the quality improvement $p_S = \lambda$ and making sales $x_S = E/\lambda$. Instantaneous profits for the Southern firm are

$$\pi_S = E \left(1 - \frac{1}{\lambda} \right). \tag{7}$$

Call these markets *Southern-led* since only Southern firms have the ability to produce the state-of-the-art technology.

Northern firms target all markets for innovation. Southern firms target only Northern-led markets for imitation; they do not target imitated or Southern-led markets since they have no cost advantage against other Southern firms. Southern firms target imitated and Southern-led markets for innovation; they do not target Northern-led markets since lack of a sufficient knowledge base makes innovation targeting Northern-led markets prohibitively difficult (to be shown).

A successful Northern innovator gains the value from producing in a Northern-led market v_N, a successful Southern imitator gains the value from producing in an imitated market v_M, and a successful Southern innovator gains the value from producing in a Southern-led market v_S. To find the value of producing firms, discount the flow of instantaneous profits to account for the chance that the profit stream will be terminated due to innovation or imitation.

Northern-led markets are targeted by both imitation and Northern innovation, so the value from producing in a Northern-led market is

$$v_N = \frac{\pi_N}{\rho + \iota_N + \mu_S}. \tag{8}$$

The effective discount rate (the denominator) is the subjective discount rate ρ plus the sum of all the chances that the profit stream will be terminated at an instant in time. Imitated markets are targeted by both Northern and Southern innovation, so the value from producing in an imitated market is

$$v_M = \frac{\pi_M}{\rho + \iota_N + \iota_S}. \tag{9}$$

Southern-led markets are also targeted by both Northern and Southern innovation, so the value from producing in a Southern-led market is

$$v_S = \frac{\pi_S}{\rho + \iota_N + \iota_S}. \tag{10}$$

These values indicate the reward to successful Northern innovation, imitation and Southern innovation. These rewards must offset the costs of innovation and imitation for innovation and imitation to occur in equilibrium.

2.3. Innovation and Imitation

In the R&D stage, each firm chooses its intensity of R&D to maximize its expected value, given the R&D intensities of other firms. R&D races occur simultaneously for all products. R&D races are like lotteries in that certain costs lead to uncertain outcomes. A Northern firm undertaking innovation intensity ι_N for a time interval dt requires $a_N \iota_N dt$ units of Northern labor at cost $w a_N \iota_N dt$ and leads to success with probability $\iota_N dt$. Southern innovation and imitation operate similarly but with different labor requirements (and Southern workers are paid the Southern wage of one). A Southern firm undertaking imitation intensity μ_S for a time interval dt requires $a_M \mu_S dt$ units of Southern labor at cost $a_M \mu_S dt$ and leads to success with probability $\mu_S dt$. A Southern firm undertaking innovation intensity ι_S for a time interval dt requires $a_S \iota_S dt$ units of Southern labor at cost $a_S \iota_S dt$ and leads to success with probability $\iota_S dt$. Imitation is easier than Southern innovation, and thus requires relatively fewer resources: $\gamma \equiv a_M/a_S < 1$. The quality increment λ is fixed.

Assume each country is at worst one step below the world technology frontier for each product. Such a situation occurs if knowledge of the design of quality levels no longer produced (discarded technology) spreads globally. A technology becomes discarded when the firm that invented that quality level no longer earns any profits from producing it; therefore, no firm would have any reason to protect its design. Thus, there are the three possible situations. Either a Southern firm has a one quality level lead, a Northern firm has a one quality level lead, or a Northern firm and a Southern firm are at the same quality level.

Southern firms innovate only *appropriate technologies*, quality levels that have already been imitated by a Southern firm. The underlying assumption is that the resource requirement a_S applies to innovation targeting an imitated or Southern-led market but a substantially higher resource requirement $A > a_S$ applies to innovation targeting a Northern-led market, so that Southern innovation costs exceed the expected reward to innovation targeting Northern-led markets in equilibrium: $A > v_S$. Successful prior imitation of a quality level of a product generates a knowledge base that makes subsequent Southern innovation for that product easier. The North might well enjoy lower innovation costs following successful imitation of products in Southern-led markets, but Northern imitation does not occur in equilibrium because the North does not have a production cost advantage relative to the South.

The assumption that Southern innovation proceeds only after successful imitation resembles Glass (1997), which assumes (in a setting where two quality levels sell) that imitation of the low quality level provides a knowledge base for imitation of the high quality level of each product. Aghion et al (2001) and Aghion et al (1997) also make a similar assumption in a one-country model of innovation: innovation in their models must proceed in a step-by-step fashion.

Firms engage in positive rates of innovation or imitation whenever the expected gains are no less than their costs. As usual, only firms that are not currently producing engage in R&D. To generate finite rates of innovation and imitation, the expected gains must not exceed their cost. Northern firms conduct innovation at intensity ι_N, earning the reward v_N if successful.

$$v_N \leq wa_N, \ \iota_N > 0 \Leftrightarrow v_N = wa_N \tag{11}$$

Southern firms conduct imitation at intensity μ_S, earning the reward v_M if successful.

$$v_M \leq a_M, \ \mu_S > 0 \Leftrightarrow v_M = a_M \tag{12}$$

Southern firms also conduct innovation at intensity ι_S, earning the reward v_S if successful.

$$v_S \leq a_S, \ \iota_S > 0 \Leftrightarrow v_S = a_S \tag{13}$$

Resource constraints and conditions for the measure of markets to remain constant complete the model.

3. Steady-State Equilibrium

The purchasing decisions of consumers and the R&D and pricing decisions of firms have been established. Now R&D and production must be constrained according to resource availability in each country. Further, the measures of Northern-led, imitated and Southern-led markets must remain constant. We also establish important relationships between variables of interest in a steady-state equilibrium, both with and without Southern innovation.

Focus mainly on the case where all three forms of R&D occur in equilibrium, so that these three R&D conditions all hold with equality. Section 4.5 will confirm that parameters leading to such an equilibrium do indeed exist. Section 4.6 will consider the case where innovation is prohibitively costly in the South even following imitation.

3.1. Resource Constraints

Let n_N be the measure of Northern-led markets, n_M be the measure of imitated markets and n_S be the measure of Southern-led markets, which together sum to one. The measure of Northern-led markets is the fraction of products that have a Northern firm serving the market. Similarly, the measure of Southern-led markets is the fraction of products that have a Southern innovator serving the market (and likewise for imitated markets).

Labor is used for R&D and production. Define $\delta \equiv 1/\lambda$. In the North, labor demand for innovation is $a_N \iota_N$, while labor demand for production is $n_N E \delta$ (sales of $x_N = E\delta$ for the fraction n_N of markets). In the South, labor demand for imitation is $a_M \mu_S n_N$, labor demand for innovation is $a_S \iota_S (n_M + n_S)$, while labor demand for production is $n_M E/w + n_S E \delta$. For equilibrium in the labor markets, the demand for labor must equal the fixed supply of labor in each country.

$$a_N \iota_N + n_N E \delta = L_N \tag{14}$$

$$a_M \mu_S n_N + a_S \iota_S (n_M + n_S) + n_M \left(\frac{E}{w} \right) + n_S E \delta = L_S \tag{15}$$

The resource constraints limit the amount of R&D and production performed in each country.

3.2. Constant Measures

Product markets change between Northern-led, imitated, and Southern-led over time as innovation or imitation occurs; however, each market type must have flows in match flows out for the measure of each market type to remain constant at the aggregate level in a steady-state equilibrium. For Northern-led markets, the measure of products newly innovated by Northern firms must match the measure of products newly imitated

$$\iota_N \left(n_M + n_S\right) = \mu_S n_N. \tag{16}$$

For Southern-led markets, the measure of products newly innovated by Southern firms must match the measure of Southern products innovated over by Northern firms

$$\iota_S n_M = \iota_N n_S. \tag{17}$$

Finally, the measures of market types must sum to one

$$n_N + n_M + n_S = 1. \tag{18}$$

These conditions can be solved to find an expression for each market measure in terms of the intensities of Northern innovation, imitation and Southern innovation.

Since Northern innovation targets all markets, the aggregate rate of Northern innovation is the intensity of innovation ι_N. Since imitation targets only Northern-led markets, the aggregate rate of imitation is the intensity of imitation times the measure of Northern-led markets $\mu \equiv \mu_S n_N$. Since Southern innovation targets imitated and Southern-led markets, the aggregate rate of Southern innovation is the intensity of Southern innovation times the measure of these markets: $I_S \equiv \iota_S \left(n_M + n_S\right)$. The rate of Southern innovation relative to imitation is

$$\psi \equiv \frac{I_S}{\mu} = \frac{\iota_S \left(n_M + n_S\right)}{\mu_S n_N} = \frac{\iota_S \left(n_M + n_S\right)}{\iota_N \left(n_M + n_S\right)} = \frac{\iota_S}{\iota_N}. \tag{19}$$

By (16), the rate of Southern innovation relative to imitation reflects the intensity of Southern innovation relative to the intensity of Northern innovation. The rate of Southern innovation can be recovered from the rate of imitation μ and the ratio ψ using $I_S \equiv \psi\mu$. Overall, the aggregate rate

of innovation is the sum of the Northern and Southern rates of innovation $\iota \equiv \iota_N + \iota_S \left(n_M + n_S\right) \equiv \iota_N + I_S$. The aggregate rate of innovation is the fraction of all products that experience quality improvement at each point in time.

3.3. With Southern Innovation

The equations reduce to a system of five equations in five unknowns. Assuming Northern innovation $\iota_N > 0$, Southern innovation $\iota_S > 0$, and imitation $\mu_S > 0$ all occur, three conditions are required to ensure that costs equal benefits for each form of R&D. Plugging profits (5) and producing firm value (8) into the return to R&D condition (11) yields the Northern innovation condition.

$$E\left(1 - w\delta\right) = wa_N\left(\rho + \iota_N + \mu_S\right) \tag{20}$$

Similarly (6), (9), and (12) yield the imitation condition

$$E\left(1 - \frac{1}{w}\right) = a_M\left(\rho + \iota_N + \iota_S\right) \tag{21}$$

and (7), (10), and (13) yield the Southern innovation condition.

$$E\left(1 - \delta\right) = a_S\left(\rho + \iota_N + \iota_S\right) \tag{22}$$

The resource constraints (14, 15), with the market measures substituted out using the constant measure conditions (16, 17, 18), form the remaining two equations.

As shown in the Appendix, the definitions of the rate of imitation μ and rate of Southern innovation relative to imitation ψ can be applied to translate the system into rates rather than intensities, which proves convenient for directly determining the effects of the parameters on the *aggregate rates* of R&D. Also, the system can be reduced to four equations, with the relative wage determined separately.

The relative wage is exclusively determined to ensure that the returns to Southern innovation and imitation are equal in equilibrium. If both imitation and Southern innovation occur in equilibrium ($\mu_S > 0$, $\iota_S > 0$), the R&D conditions (21, 22) must hold with equality. The relative wage must equal

$$w = \frac{1}{1 - \gamma\left(1 - \delta\right)} \tag{23}$$

so that the profits from imitation are less than profits from Southern inno-
vation in proportion to the relative ease of imitation.

$$\frac{\pi_M}{\pi_S} = \frac{a_M}{a_S} \equiv \gamma < 1 \tag{24}$$

Southern firms all have the same unit costs of production, but imitators sell
at price w due to competition from Northern firms capable of producing the
same quality level whereas innovators sell at price λ reflecting their quality
lead. The relative wage is assured of being above one $w > 1$ (wages higher
in North than South) provided innovations are quality improvements $\lambda > 1$
and imitation is easier than innovation in the South $\gamma \equiv a_M/a_S < 1$. The
relative wage is assured of being smaller than the quality increment $w < \lambda$
(imitators charge a lower price than innovators) provided imitation is easier
than innovation.

3.4. Without Southern Innovation

Before considering parameter changes, the key equations when Southern
innovation does not occur should be described so that the situations with
and without Southern innovation can later be compared. If the resource
requirement in Southern innovation (following imitation) is large enough
$a_S > \underline{a_S}$, no Southern firms will innovate $\iota_S = 0$ and thus there would
be no markets led by Southern firms $n_S = 0$. The Southern innovation
condition would be an inequality.

$$E\left(1 - \delta\right) < a_S\left(\rho + \iota_N\right) \tag{25}$$

The Northern resource constraint (14) is unaffected, and the Southern re-
source constraint (15) reduces to

$$a_M \mu_S n_N + n_M \left(\frac{E}{w}\right) = L_S. \tag{26}$$

The Northern innovation condition (20) is unaffected, and the imitation
condition reduces to

$$E\left(1 - \frac{1}{w}\right) = a_M\left(\rho + \iota_N\right). \tag{27}$$

The steady-state conditions for the market measures to be constant are $\mu_S n_N = \iota_N n_M$ and $n_N + n_M = 1$, which lead to the substitutions $n_N = 1 - n_M$ and $\mu_S = \iota_N n_M/(1 - n_M)$.

The two R&D conditions and the two resource constraints then determine Northern innovation ι_N, the measure of imitated markets n_M, aggregate expenditure E, and the relative wage w. The rate of imitation can be determined from Northern innovation and the measure of imitated markets $\mu \equiv \mu_S n_N = \iota_N n_M$. This scenario closely resembles the case of inefficient Northern followers in Grossman and Helpman (1991a). The key difference is that with access to discarded technologies, Northern innovators target all markets here, not just markets in which imitation has occurred.

Now the effects of parameters such as resource availability on the endogenous variables can be determined to find what forces can cause Southern innovation to increase.

4.   Innovation Through Imitation

In this section we explore how imitation, Northern innovation, Southern innovation, Southern innovation relative to imitation, aggregate spending and the relative wage depend on the parameters of the model. Southern countries such as the East Asian Tigers are striving to push forward the technology frontier. They want to increase the mix of innovation in their R&D. What parameters increase Southern innovation relative to imitation? Can any of these forces increase Southern innovation without reducing aggregate innovation? The propositions below address these questions for the case where Southern innovation does occur, but only following imitation.

The Southern government might have means of influencing the Southern labor supply or the cost of conducting R&D in the South. The Southern labor supply represents all factors of production and is measured in efficiency units (recall one unit of labor produces one unit of output). So increasing the supply of Southern labor represents increasing the supply of any resource useful for R&D and production. It could stem from an increase in human capital due to educational programs, for example. An expansion in Southern resources could stem from entry of additional developing countries into the world trading system. The Southern government could also offer subsidies to lower the cost of R&D. Since imitation is hard to distinguish

from innovation, we examine a general R&D subsidy (to both innovation and imitation) in the South. Changes in resources and R&D subsidies in the North could also affect the amount of innovation done in the South.

To determine their effects, introduce the parameter ϕ_N to represent an increase in Northern resources, ϕ_S to represent an increase in Southern resources, σ_N to represent a subsidy to Northern R&D, and σ_S to represent a subsidy to Southern R&D. As a result, $(1 + \phi_N) L_N$ is the Northern resource supply, and $(1 + \phi_S) L_S$ is the Southern resource supply. Additionally, the term $(1 - \sigma_N)$ is added to the cost side of the Northern innovation valuation condition (20), as the Northern government bears some of Northern firms' innovation costs; the term $(1 - \sigma_S)$ is added to the cost side of the imitation and Southern innovation conditions (21) and (22), as the Southern government bears some of the costs of Southern R&D.

4.1.   Northern Resources

Larger Northern resources support a faster rate of Northern innovation and more Northern production. Faster Northern innovation erodes the South's knowledge base, so the rate of Southern innovation relative to imitation falls. Aggregate expenditure rises ,which increases the resources used in Northern production by increasing the sales of each Northern producer. The labor used in Northern production also rises due to the increase in Northern-led markets.

Proposition 1 *The rate of imitation, the rate of Northern innovation, and the aggregate rate of innovation increase while the rate of Southern innovation relative to imitation and the rate of Southern innovation decrease with larger Northern resources ϕ_N. Aggregate expenditure rises, while the relative wage is unaffected. More markets become led by Northern firms, and fewer by Southern firms, with fewer imitated products.*

4.2.   Southern Resources

Similarly, larger Southern resources support more Southern R&D and more Southern production. An expansion in Southern resources causes both Southern innovation and imitation to rise. Since imitation is needed to provide the knowledge base for innovation, Southern innovation relative

to imitation grows. Aggregate expenditure rises, which increases resource demand in Southern production by increasing the sales of each Southern producer. The labor used in Southern production also rises due to the decrease in Northern-led markets (so more production occurs in the South).

Proposition 2 *The rate of imitation, the rate of Southern innovation relative to imitation, the rate of Southern innovation, and the aggregate rate of innovation all increase with larger Southern resources ϕ_S. Aggregate expenditure rises, while the relative wage and the rate of Northern innovation are unaffected. More markets become led by Southern firms, and fewer by Northern firms, with more imitated products.*

4.3.   Northern R&D Subsidy

A subsidy to Northern innovation reduces the cost of Northern innovation, which encourages Northern innovation. Aggregate expenditure falls to reduce the reward to innovation to match the lower cost of innovation born by Northern firms. A subsidy to Northern innovation encourages imitation but reduces the mix of Southern innovation in Southern R&D.

Proposition 3 *The rate of imitation, the rate of Northern innovation, and the aggregate rate of innovation increase while the rate of Southern innovation relative to imitation and the rate of Southern innovation decrease with a larger Northern innovation subsidy σ_N. Aggregate expenditure falls, while the relative wage is unaffected. Fewer markets become led by Northern and Southern firms, so more products have been imitated.*

4.4.   Southern R&D Subsidy

Imitation may be difficult to distinguish from innovation when allocating R&D subsidies. The results below suggest that R&D incentives can encourage innovation, even if innovation cannot be distinguished from imitation when making subsidy payments.

A subsidy to Southern R&D causes Southern innovation to rise but imitation to fall. Although the subsidy reduces the costs of imitation, the increase in Southern innovation reduces the reward to imitation by making the profits of imitators shorter in duration. The reward to imitation is

further reduced by a reduction in aggregate expenditure, so profits are smaller as well as shorter in duration.

Proposition 4 *The rate of Southern innovation relative to imitation, the rate of Southern innovation and the aggregate rate of innovation increase while the rate of imitation decreases with a larger Southern R&D subsidy σ_S. Aggregate expenditure falls, while the relative wage and the rate of Northern innovation are unaffected. More markets become led by Northern and Southern firms, so fewer products have been imitated.*

Table 1 summarizes the sign of the effects on the endogenous variables.

Table 1. Analytical Results

	ι_N	ψ	μ	I_S	ι	E	w	n_N	n_M	n_S
L_N	+	-	+	-	+	+	0	+	-	−
L_S	0	+	+	+	+	+	0	-	+	+
σ_N	+	-	+	-	+	-	0	-	+	−
σ_S	0	+	-	+	+	-	0	+	-	+

To sign the derivatives, the analytical results are done in the limit as the discount rate approaches zero $\rho \to 0$. The discount rate is presumably close to zero and thus this practice should not much alter the results. However, the effects on Northern innovation that are zero above could be nonzero for positive discounting. The effects on the relative wage are necessarily zero since they were determined prior to taking the limit. To explore the robustness of the results for $\rho > 0$, we turn to numerical solutions.

4.5.   Numerical Examples

When the discount rate approached zero, larger profits with a shorter duration yielded the same reward to innovation when Southern resources expand. For a positive discount rate, the larger size of the profits would then dominate their shorter length (as the termination of profits occurs at a lag), causing the reward to innovation to rise. So intuitively, for an increase in Southern resources, Northern innovation should rise slightly.

On the other hand, for a Southern R&D subsidy, the smaller but longer profits should yield a somewhat smaller reward to innovation with discounting, so Northern innovation should fall slightly. Since the decrease in

Northern innovation should be slight, the increase in Southern innovation should still ensure that aggregate innovation increases.

Indeed, numerical solutions confirm this intuition regarding the effects on Northern innovation for a positive discount rate. The following results are calculated for discount rate $\rho = 1/25 = 4\%$, a quality increment $\lambda = 3$, Northern resources $L_N = 4$, Southern resources $L_S = 2$, resource requirement in Northern innovation $a_N = 2$, resource requirement in Southern innovation $a_S = 3$, and resource requirement in imitation $a_M = 1$.

Starting from the base case, the parameter shifts considered are a 50 percent increase in Northern resources $\phi_N = 1/2$, a 50 percent increase in Southern resources $\phi_S = 1/2$, a Northern R&D subsidy that pays one-quarter of the R&D costs of Northern firms $\sigma_N = 1/4$, or a Southern R&D subsidy that pays half of the R&D costs of Southern firms $\sigma_S = 1/2$. Table 2 reports the equilibrium values and the implied percentage changes in the endogenous variables relative to the base values (the relative wage remains fixed at $w = 1.3$).

Table 2A. Numerical Results

	ι_N	ψ	μ	I_S	ι	E	n_N	n_M	n_S
$base$	1.130	0.39	0.32	0.12	1.25	7.2	0.72	0.20	0.08
L_N	1.701	0.26	0.35	0.09	1.79	9.8	0.79	0.16	0.04
L_S	1.132	0.58	0.42	0.24	1.37	8.2	0.63	0.23	0.14
σ_N	1.270	0.09	0.40	0.03	1.31	6.4	0.68	0.29	0.03
σ_S	1.129	1.55	0.23	0.36	1.49	6.6	0.79	0.08	0.13

Table 2B. Percentage Changes

	ι_N	ψ	μ	I_S	ι	E	n_N	n_M	n_S
L_N	50%	-33%	11%	-26%	43%	36%	10%	-19%	-46%
L_S	**0.13%**	50%	32%	98%	9%	14%	-12%	16%	74%
σ_N	12%	-77%	27%	-71%	4%	-12%	-5%	44%	-68%
σ_S	**-0.11%**	302%	-26%	196%	19%	-9%	10%	-60%	61%

The effects on Northern innovation, which are zero when $\rho \to 0$, are all quite small when $\rho > 0$. Northern innovation rises slightly with Southern resources and falls slightly with Southern R&D subsidies. Even though subsidizing Southern R&D does reduce Northern innovation, Southern innovation expands by more so that aggregate innovation rises.

4.6.   No Southern Innovation

We now briefly consider how the model behaves if Southern innovation is too difficult to occur even after imitation. Aggregate innovation is simply Northern innovation when Southern innovation is absent. The results here are identical to the case of inefficient Northern followers in Grossman and Helpman (1991a): an increase in either Southern resources or a subsidy to imitation increases both innovation and imitation.

- The rate of innovation and the rate of imitation increase with larger Northern resources ϕ_N. Aggregate expenditure and the relative wage both increase. More markets become led by Northern firms, so fewer products have been imitated.

- The rate of innovation and the rate of imitation increase with larger Southern resources ϕ_S. Aggregate expenditure increases, while the relative wage decreases. Fewer markets become led by Northern firms, so more products have been imitated.

- The rate of innovation and the rate of imitation increase with a larger Northern imitation subsidy σ_N. Aggregate expenditure decreases, while the relative wage increases. Fewer markets become led by Northern firms, so more products have been imitated.

- The rate of innovation and the rate of imitation increase with a larger Southern imitation subsidy σ_S. Aggregate expenditure and the relative wage both decrease. More markets become led by Northern firms, so fewer products have been imitated.

Table 3 summarizes the signs of these effects for the case when Southern innovation does not occur.

Table 3. Results When No Southern Innovation

	ι	μ	E	w	n_N	n_M
L_N	+	+	+	$+^*$	+	-
L_S	+	+	+	$-^*$	-	+
σ_N	+	+	-	$+^*$	-	+
σ_S	+	$+^*$	-	$-^*$	+	-

The sign of cells marked by an asterisk * in Table 3 depends on whether there is Southern innovation. The effects on the relative wage, which were zero with Southern innovation, are not in its absence. Increasing Northern resources or a Northern R&D subsidy increase the relative wage, while increasing Southern resources or a Southern R&D subsidy decreases the relative wage. When both Southern innovation and imitation occur, the relative wage adjusts to equalize their returns. Any policy that affects Southern R&D symmetrically (the same consequences for innovation as for imitation) will therefore not affect the relative wage.

In the absence of Southern innovation, a subsidy to imitation lowers the cost of imitation, and so the benefit of imitation must fall to restore equality between the costs and benefits. A lower relative wage is one means of lowering the reward to imitation: imitators charge a price equal to the costs of the Northern firm whose product they imitated, which is the relative wage, so the profits of an imitator are lower when the relative wage falls.

When Southern resources increase, a lower relative wage leads to a lower price charged by imitators and thus larger sales, and thus more resources absorbed into production in the South. A lower relative wage, by increasing the demand for Southern resources, helps restore equality in the Southern resource constraint when the supply of Southern resources expands.

The effects on aggregate innovation are the same regardless of whether innovation occurs in the South. Increased Southern resources or a subsidy to Southern R&D increase aggregate innovation. However, the effects on imitation do depend on whether some innovation occurs in the South. A subsidy to Southern R&D clearly expands imitation if there is no Southern innovation, but decreases imitation if Southern innovation then targets products after an imitation has occurred. A subsidy, even though it lowers the cost of imitation, may increase the onslaught of innovation to such a degree that less imitation occurs in equilibrium.

The equilibrium without Southern innovation can be solved for the same parameter values as for Table 2 but for $a_S > \underline{a_S} = 3.79$ as needed for the cost of innovation to exceed its expected benefit in the South. Table 4 reports the equilibrium values and their corresponding percentage changes relative to the base values.

Table 4A. Numerical Results When No Southern Innovation

	μ	ι	E	w	n_N	n_M
$base$	0.34	1.18	6.9	1.21	0.71	0.29
L_N	0.37	1.75	9.5	1.23	0.83	0.17
L_S	0.46	1.20	7.8	1.19	0.61	0.39
σ_N	0.41	1.28	6.3	1.27	0.68	0.32
σ_S	0.36	1.25	6.3	1.11	0.71	0.29

Table 4B. Percentage Changes When No Southern Innovation

	μ	ι	E	w	n_N	n_M
L_N	8.5%	49%	37%	1.6%	16%	-40%
L_S	35%	1.6%	13%	-2.1%	-13%	33%
σ_N	20%	8.8%	-9.0%	4.3%	-4.0%	10%
σ_S	-5.0%	5.8%	-8.5%	-8.3%	0.3%	-0.7%

These numerical values confirm the analytical results reported in Table 3 for the case without Southern innovation. Table 4 can also be used to contrast the magnitude of effects with and without Southern innovation when the effects share the same sign.

5. Conclusion

This paper develops a model of Southern firms switching R&D from imitation towards innovation. Imitation may be necessary for the South to build a sufficient knowledge base to make innovation attractive to Southern firms. Absent the knowledge base from imitation, the expected reward to Southern innovation would not cover the innovation cost.

In this setting, how does a Southern country, such South Korea or Taiwan, shift its R&D toward innovation? One option is to encourage resource accumulation, which releases the Southern resource constraint and permits all Southern R&D activities to grow. Subsidizing Southern R&D also promotes both innovation and imitation in the South. Thus, Southern innovation can be promoted even if it is not possible to distinguish innovation from imitation when allocating subsidies. However, when innovation and imitation both occur in the South, a general subsidy to Southern

R&D decreases Southern imitation due to the surge in Southern innovation (whereas subsidizing Southern imitation in the absence of Southern innovation increases imitation). Expanding Northern resources or Northern R&D subsidies discourage Southern innovation.

Further research should explore other specifications of how imitation can help to promote innovation in developing countries. Here, it was assumed that every Northern innovation had to be imitated by a Southern firm before innovation would occur for that product in the South. An alternative specification could assume that imitation of a product by a Southern firm permanently lowers the cost of innovation in the South, not just for the current generation, but also for all further improvement, even if a Northern innovation intercedes. Southern imitation would thus fall over time as fewer products remain that have not yet been imitated. The interaction between imitation and innovation in the South could differ in such a setup.

A   Appendix

Substitute for the quality increment $\lambda = 1/\delta$, the intensity of imitation $\mu_S = \mu/n_N$, the intensity of Southern innovation $\iota_S = \psi\mu/\left(n_M + n_S\right)$, the measure of Northern-led markets $n_N = 1 - \mu/\iota_N$, the measure of imitated markets $n_M = \mu/\left[\iota_N\left(1 + \psi\right)\right]$, the measure of Southern-led markets $n_S = \psi\mu/\left[\iota_N\left(1 + \psi\right)\right]$, and the relative wage (23). The remaining system of four equations is

$$a_N\iota_N^2 + E\delta\left(\iota_N - \mu\right) - \left(1 + \phi_N\right)\iota_N L_N = 0 \tag{28}$$

$$\begin{aligned} \mu\iota_N a_S\left(1 + \psi\right)\left(a_M + \psi a_S\right) - \left(1 + \psi\right)\left(1 + \phi_S\right)a_S\iota_N L_S \\ + \mu E\left[a_S\left(1 + \psi\delta\right) - a_M\left(1 - \delta\right)\right] = 0 \end{aligned} \tag{29}$$

$$E\left(1 - \delta\right)\left[a_S + a_M\right]\left(\iota_N - \mu\right) - \left(1 - \sigma_N\right)a_N a_S\left[\rho\left(\iota_N - \mu\right) + \iota_N^2\right] = 0 \tag{30}$$

$$E\left(1 - \delta\right) - a_S\left(1 - \sigma_S\right)\left[\rho + \iota_N\left(1 + \psi\right)\right] = 0 \tag{31}$$

The four remaining variables are: aggregate spending E, the rate of Northern innovation ι_N, the rate of imitation μ, and the rate of Southern inno-

vation relative to imitation ψ. Derivatives are generated by the system

$$\begin{bmatrix} b_{11} & 0 & -\delta E & \delta\left(\iota_N - \mu\right) \\ b_{21} & b_{22} & b_{23} & b_{24} \\ b_{31} & 0 & b_{33} & b_{34} \\ b_{41} & b_{42} & 0 & 1-\delta \end{bmatrix} \begin{bmatrix} d\iota_N \\ d\psi \\ d\mu \\ dE \end{bmatrix} = \begin{bmatrix} 0 \\ c_2 \\ 0 \\ c_4 \end{bmatrix} \tag{32}$$

where

$$b_{11} = 2a_N\iota_N + \delta E - \left(1+\phi_N\right)L_N$$

$$b_{21} = -a_S\left(1+\psi\right)\left[\left(1+\phi_S\right)L_S - \mu\left(\psi a_S + a_M\right)\right]$$

$$b_{22} = -a_S\left[\left(1+\phi_S\right)\iota_N L_S - \mu\iota_N\left(a_M + \left(1+2\psi\right)a_S\right) - \mu\delta E\right]$$

$$b_{23} = \iota_N a_S\left(1+\psi\right)\left(a_M + \psi a_S\right) + E\left[a_S\left(1+\psi\right) + \left(a_S - a_M\right)\left(1-\delta\right)\right]$$

$$b_{24} = \mu\left[a_S\left(1+\psi\right) + \left(a_S - a_M\right)\left(1-\delta\right)\right]$$

$$b_{31} = E\left(1-\delta\right)\left[a_S + a_M\right] - \left(1-\sigma_N\right)a_N a_S\left(\rho + 2\iota_N\right)$$

$$b_{33} = -E\left(1-\delta\right)\left[a_S + a_M\right] - \left(1-\sigma_N\right)a_N a_S\rho$$

$$b_{34} = \left(1-\delta\right)\left[a_S + a_M\right]\left(\iota_N - \mu\right)$$

$$b_{41} = -a_S\left(1+\psi\right)\left(1-\sigma_S\right)$$

$$b_{42} = -a_S\iota_N\left(1-\sigma_S\right)$$

$$c_1 = \iota_N L_N\partial\phi_N$$

$$c_2 = \iota_N\left(1+\psi\right)L_S\partial\phi_S$$

$$c_3 = -a_N a_S\left[\rho\left(\iota_N - \mu\right) + \iota_N^2\right]\partial\sigma_N$$

$$c_4 = -\left[\rho + \iota_N\left(1+\psi\right)\right]a_S\partial\sigma_S$$

The following derivatives are evaluated at $\sigma_N = \phi_N = \sigma_S = \phi_S = 0$ and in the limit as $\rho \to 0$ for expositional ease since the discount rate should be near zero.

$$|B| = \frac{E^5\left(1-\gamma\right)\left(1-\delta\right)^4\left(\iota_N - \mu\right)\left[1 - \gamma\left(1-\delta\right)\right]}{\iota_N^2\left(1+\psi\right)^2} > 0 \tag{33}$$

A1. Proof of Proposition 1

An increase in Northern resources increases the rate of Northern innovation

$$\frac{\partial \iota_N}{\partial \phi_N} = \iota_N^2 > 0 \tag{34}$$

has no effect the relative wage $\partial w / \partial \phi_N = 0$, decreases the rate of Southern innovation relative to imitation

$$\frac{\partial \psi}{\partial \phi_N} = -\frac{\mu \left(1 + \psi\right)}{\iota_N} < 0 \tag{35}$$

increases the rate of imitation

$$\frac{\partial \mu}{\partial \phi_N} = \frac{\mu^2}{\iota_N} > 0 \tag{36}$$

increases aggregate spending

$$\frac{\partial E}{\partial \phi_N} = \frac{E \left(\iota_N - \mu\right)}{\iota_N} > 0 \tag{37}$$

as $\iota_N - \mu > 0$ for $n_N > 0$, decreases the rate of Southern innovation

$$\frac{\partial I_S}{\partial \phi_N} = -\frac{\mu^2}{\iota_N} < 0 \tag{38}$$

increases the rate of innovation

$$\frac{\partial \iota}{\partial \phi_N} = \frac{\left(\iota_N + \mu\right)\left(\iota_N - \mu\right)}{\iota_N} > 0 \tag{39}$$

increases the measure of Northern-led markets

$$\frac{\partial n_N}{\partial \phi_N} = \frac{\mu}{\iota_N} n_N > 0 \tag{40}$$

decreases the measure of imitated markets

$$\frac{\partial n_M}{\partial \phi_N} = -\frac{\mu}{\iota_N \left(1 + \psi\right)} < 0 \tag{41}$$

and decreases the measure of Southern-led markets.

$$\frac{\partial n_S}{\partial \phi_N} = \tag{42}$$

A2. Proof of Proposition 2

An increase in Southern resources has no effect on the rate of Northern innovation or the relative wage $\partial\iota_N/\partial\phi_S = \partial w/\partial\phi_S = 0$, increases the rate of Southern innovation relative to imitation

$$\frac{\partial\psi}{\partial\phi_S} = \frac{1+\psi}{E} > 0 \tag{43}$$

increases the rate of imitation

$$\frac{\partial\mu}{\partial\phi_S} = \frac{\iota_N - \mu}{E} > 0 \tag{44}$$

increases the rate of Southern innovation and the aggregate rate of innovation

$$\frac{\partial I_S}{\partial\phi_S} = \frac{\partial\iota}{\partial\phi_S} = \frac{\mu + \psi\iota_N}{E} > 0 \tag{45}$$

increases aggregate spending

$$\frac{\partial E}{\partial\phi_S} = 1 > 0 \tag{46}$$

decreases the measure of Northern-led markets

$$\frac{\partial n_N}{\partial\phi_S} = -\left(\frac{1}{\iota_N}\right)\frac{\partial\mu}{\partial\phi_S} < 0 \tag{47}$$

increases the measure of imitated markets

$$\frac{\partial n_M}{\partial\phi_S} = \frac{\frac{\partial\mu}{\partial\phi_S} - \left(\frac{\mu}{1+\psi}\right)\frac{\partial\psi}{\partial\phi_S}}{\iota_N(1+\psi)} = -\frac{1-2n_N}{E(1+\psi)} > 0 \longleftrightarrow n_N > \frac{1}{2} \tag{48}$$

and increases the measure of Southern-led markets.

$$\frac{\partial n_S}{\partial\phi_S} = \frac{\psi\frac{\partial\mu}{\partial\phi_S} - \left(\frac{\mu}{1+\psi}\right)\frac{\partial\psi}{\partial\phi_S}}{\iota_N(1+\psi)} > 0 \tag{49}$$

A3. Proof of Proposition 3

An increase in the subsidy to Northern innovation increases the rate of Northern innovation

$$\frac{\partial \iota_N}{\partial \sigma_N} = \frac{\iota_N \delta}{1 - \gamma (1 - \delta)} > 0 \tag{50}$$

has no effect on the relative wage $\partial w / \partial \sigma_N = 0$, decreases the rate of Northern innovation relative to imitation

$$\frac{\partial \psi}{\partial \sigma_N} = -\frac{(1 + \psi) [(\iota_N - \mu) [1 - \gamma (1 - \delta)] + \mu \delta]}{\iota_N [1 - \gamma (1 - \delta)]} < 0 \tag{51}$$

increases the rate of imitation

$$\frac{\partial \mu}{\partial \sigma_N} = \frac{\mu [(\iota_N - \mu) [1 - \gamma (1 - \delta)] + \mu \delta]}{\iota_N [1 - \gamma (1 - \delta)]} > 0 \tag{52}$$

decreases aggregate spending

$$\frac{\partial E}{\partial \sigma_N} = -\frac{E (1 - \delta) (\iota_N - \mu) (1 - \gamma)}{\iota_N [1 - \gamma (1 - \delta)]} < 0 \tag{53}$$

decreases the rate of Southern innovation

$$\frac{\partial I_S}{\partial \sigma_N} = -\frac{\mu [(\iota_N - \mu) [1 - \gamma (1 - \delta)] + \mu \delta]}{\iota_N [1 - \gamma (1 - \delta)]} < 0 \tag{54}$$

and increases the rate of innovation (if λ is not too large)

$$\frac{\partial \iota}{\partial \sigma_N} = \frac{(\iota_N - \mu) [\delta \iota_N - \mu (1 - \gamma) (1 - \delta)]}{\iota_N [1 - \gamma (1 - \delta)]} > 0 \tag{55}$$

decreases the measure of Northern-led markets

$$\frac{\partial n_N}{\partial \sigma_N} = -\frac{\mu (1 - \delta) (1 - \gamma) n_N}{\iota_N [1 - \gamma (1 - \delta)]} < 0 \tag{56}$$

increases the measure of imitated markets

$$\frac{\partial n_M}{\partial \sigma_N} = \tag{57}$$

and decreases the measure of Southern-led markets.

$$\frac{\partial n_S}{\partial \sigma_N} = \tag{58}$$

A4. Proof of Proposition 4

An increase in the subsidy to Southern R&D has no effect on the rate of Northern innovation or the relative wage $\partial \iota_N / \partial \sigma_S = \partial w / \partial \sigma_S = 0$, increases the rate of Southern innovation relative to imitation

$$\frac{\partial \psi}{\partial \sigma_S} = 1 + \psi - (1 - \delta)(1 - n_N) > 0 \tag{59}$$

decreases the rate of imitation

$$\frac{\partial \mu}{\partial \sigma_S} = -\frac{\mu n_N (1 - \delta)(\gamma + \psi)}{1 + \psi} < 0 \tag{60}$$

increases the rate of Southern innovation and the aggregate rate of innovation

$$\frac{\partial I_S}{\partial \sigma_S} = \frac{\partial \iota}{\partial \sigma_S} = \mu (1 + \psi) \left[1 - (1 - \delta) \left(\frac{\mu / \iota_N + \psi}{1 + \psi} \right) \left(\frac{a_M / a_S + \psi}{1 + \psi} \right) \right] > 0 \tag{61}$$

decreases aggregate spending

$$\frac{\partial E}{\partial \sigma_S} = -\mu (a_M + a_S \psi) < 0 \tag{62}$$

increases the measure of Northern-led markets

$$\frac{\partial n_N}{\partial \sigma_S} = \left(\frac{1}{\iota_N} \right) \frac{\partial \mu}{\partial \sigma_S} > 0 \tag{63}$$

decreases the measure of imitated markets

$$\frac{\partial n_M}{\partial \sigma_S} = \frac{\frac{\partial \mu}{\partial \sigma_S} - \left(\frac{\mu}{1 + \psi} \right) \frac{\partial \psi}{\partial \sigma_S}}{\iota_N (1 + \psi)} < 0 \tag{64}$$

and increases the measure of Southern-led markets, with $C \equiv \gamma + \psi$.

$$\frac{\partial n_S}{\partial \sigma_S} = \frac{\psi \frac{\partial \mu}{\partial \sigma_S} - \left(\frac{\mu}{1 + \psi} \right) \frac{\partial \psi}{\partial \sigma_S}}{\iota_N (1 + \psi)} = \left(\frac{1 - n_N}{1 + \psi} \right) [1 - (1 - \delta)(1 - n_N) C] > 0 \tag{65}$$

A5.   Northern Resources When No Southern Innovation

Define $D \equiv 1 - w\delta + (w - 1)\left[1 - n_M\left(1 - w\delta\right)\right] > 0$. An increase in Northern resources increases the relative wage

$$\frac{\partial w}{\partial \phi_N} = \frac{\mu w\left(w - 1\right)\left(1 - w\delta\right)}{D} > 0 \tag{66}$$

increases the rate of imitation

$$\frac{\partial \mu}{\partial \phi_N} = \frac{\mu^2\left(1 - w\delta\right)}{D} > 0 \tag{67}$$

increases the aggregate rate of innovation

$$\frac{\partial \iota}{\partial \phi_N} = \frac{\iota\left[\left(\iota - \mu\right)\left(w - 1\right) + \iota\left(1 - w\delta\right)\right]}{D} > 0 \tag{68}$$

increases aggregate spending

$$\frac{\partial E}{\partial \phi_S} = \frac{Ew\left(1 - \delta\right)\left(\iota - \mu\right)}{D} > 0 \tag{69}$$

increases the measure of Northern-led markets

$$\frac{\partial n_N}{\partial \phi_N} = \frac{n_N\left[\left(1 - w\delta\right) + \mu\left(w - 1\right)\right]}{D} > 0 \tag{70}$$

and therefore decreases the measure of imitated markets $\partial n_M / \partial \phi_N = -\partial n_N / \partial \phi_N < 0$.

A6.   Southern Resources When No Southern Innovation

An increase in Southern resources decreases the relative wage

$$\frac{\partial w}{\partial \phi_S} = -\frac{wn_M\left(w - 1\right)\left(1 - w\delta\right)}{D} < 0 \tag{71}$$

increases the rate of imitation

$$\frac{\partial \mu}{\partial \phi_S} = \frac{\iota wn_M\left[\left(1 - n_M\right)\left(1 - \delta\right) + n_M\delta\left(w - 1\right)\right]}{D} > 0 \tag{72}$$

increases the aggregate rate of innovation

$$\frac{\partial \iota}{\partial \phi_S} = \frac{\iota wn_M\delta\left(w - 1\right)}{D} > 0 \tag{73}$$

increases aggregate spending

$$\frac{\partial E}{\partial \phi_S} = \frac{E n_M \left[1 - w\delta + w\delta \left(w - 1\right)\right]}{D} > 0 \tag{74}$$

increases the measure of imitated markets

$$\frac{\partial n_M}{\partial \phi_S} = \frac{w n_M \left(1 - \delta\right) \left(1 - n_M\right)}{D} > 0 \tag{75}$$

and therefore decreases the measure of Northern-led markets $\partial n_N / \partial \phi_S = -\partial n_M / \partial \phi_S < 0$.

A7. Subsidy to Northern R&D When No Southern Innovation

An increase in the subsidy to Northern R&D increases the relative wage

$$\frac{\partial w}{\partial \sigma_N} = \frac{w \left(w - 1\right) \left(1 - w\delta\right) \left[\iota - \mu \left(1 - w\delta\right)\right]}{D} > 0 \tag{76}$$

increases the rate of imitation

$$\frac{\partial \mu}{\partial \sigma_N} = \frac{\mu \left(1 - w\delta\right) \left[\iota - \mu \left(1 - w\delta\right)\right]}{D} > 0 \tag{77}$$

increases the aggregate rate of innovation

$$\frac{\partial \iota}{\partial \sigma_N} = \frac{w \iota^2 \delta \left(1 - w\delta\right)}{D} > 0 \tag{78}$$

decreases aggregate spending

$$\frac{\partial E}{\partial \sigma_N} = -\frac{E \left(\iota - \mu\right) \left(1 - w\delta\right)^2}{D} < 0 \tag{79}$$

decreases the measure of Northern-led markets

$$\frac{\partial n_N}{\partial \sigma_N} = -\frac{\mu n_N \left(1 - w\delta\right)^2}{D} < 0 \tag{80}$$

and therefore increases the measure of imitated markets $\partial n_M / \partial \sigma_N = -\partial n_N / \partial \sigma_N > 0$.

A8.   Subsidy to Southern R&D When No Southern Innovation

An increase in the subsidy to Southern R&D decreases the relative wage

$$\frac{\partial w}{\partial \sigma_S} = -\frac{(w-1)(1-w\delta)\left[w(1-n_M)+n_M\right]}{D} < 0 \qquad (81)$$

increases the rate of imitation

$$\frac{\partial \mu}{\partial \sigma_S} = \frac{\iota n_M \delta (w-1)}{D} > 0 \qquad (82)$$

increases the aggregate rate of innovation

$$\frac{\partial \iota}{\partial \sigma_S} = \frac{\iota \delta (w-1)\left[w(1-n_M)+n_M\right]}{D} > 0 \qquad (83)$$

decreases aggregate spending, with $N \equiv w(1-n_M) + n_M$

$$\frac{\partial E}{\partial \sigma_S} = -E\left(1-\frac{1}{w}\right)\left(\frac{wn_M\delta(w-1)+(1-w\delta)N}{D}\right) < 0 \qquad (84)$$

decreases the measure of imitated markets

$$\frac{\partial n_M}{\partial \sigma_S} = -\frac{n_M \delta (1-n_M)(w-1)^2}{D} < 0 \qquad (85)$$

and therefore increases the measure of Northern-led markets $\partial n_N/\partial \sigma_S = -\partial n_M/\partial \sigma_S > 0$.

A9.   Innovation Independent of Southern Resources and R&D Subsidy

Propositions 2 and 4 showed that an increase in Southern resources or in the Southern R&D subsidy had no effect on the rate of Northern innovation in the limit as the discount rate approaches zero $\rho \to 0$. To see why, solve the Northern innovation valuation condition (20) for aggregate expenditure

$$E = \frac{wa_N(\rho + \iota_N + \mu_S)}{1-w\delta}. \qquad (86)$$

Insert this expression into the Northern labor constraint (14)

$$a_N\iota_N + \left(\frac{\iota_N}{\iota_N + \iota_S}\right)\left[\frac{wa_N(\rho + \iota_N + \mu_S)}{1-w\delta}\right]\delta = L_N, \qquad (87)$$

where the conditions for constant market measures require $n_N = \iota_N / (\iota_N + \iota_S)$, and simplify

$$a_N \iota_N \left[1 + \left(\frac{\rho + \iota_N + \mu_S}{\iota_N + \iota_S} \right) \left(\frac{w\delta}{1 - w\delta} \right) \right] = L_N \tag{88}$$

$$a_N \iota_N \left[1 + \left(1 + \frac{\rho}{\iota_N + \iota_S} \right) \left(\frac{w\delta}{1 - w\delta} \right) \right] = L_N. \tag{89}$$

As $\rho \to 0$, the Northern labor constraint approaches

$$a_N \iota_N \left[1 + \frac{w\delta}{1 - w\delta} \right] = L_N \tag{90}$$

$$\frac{a_N \iota_N}{1 - w\delta} = L_N, \tag{91}$$

which solves for a Northern rate of innovation of

$$\iota_N = \frac{L_N}{a_N} (1 - w\delta). \tag{92}$$

This solution is independent of any endogenous variables yet to be determined other than the relative wage (23), which depends only on the size of the quality increment and the difficulty of imitation relative to Southern innovation. This solution is also independent of Southern resources and the Southern R&D subsidy. The solution for the rate of Northern innovation does depend on Northern resources and the Northern R&D subsidy (the latter through the solution for aggregate expenditure).

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