Growth, revenue, and welfare effects of tariff and tax reform: win-win-win strategies

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Abstract
We examine growth, revenue, and welfare effects of tariff and tax reform with a two-good, two-factor endogenous growth model. Learning-by-doing and intersectoral knowledge spillovers contribute to endogenous growth consistent with incomplete specialization. We obtain two main results: First, trade liberalization raises (or lowers) the growth rate if and only if the import sector is more effective-labor-intensive (or capital-intensive). Second, we can attain growth, revenue, and welfare gains by combining consumer-price-neutral tariff and tax reform for growth enhancement with an additional rise in the consumption tax on the less distorted good.

JEL classification: F43; H20; O41
Keywords: Tariff and tax reform; Endogenous growth; Learning-by-doing; Intersectoral knowledge spillovers; Stolper-Samuelson theorem

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1 Introduction

It has become increasingly apparent that governments in developing countries are relying more on consumption taxes such as value-added taxes and less on import tariffs in collecting their revenue. A theoretical rationale for this policy movement is the relative inefficiency of import tariffs: they distort not only consumption but also production decisions, and they have a narrower tax base (i.e., consumption minus production) than consumption taxes. Based on this notion, several authors (e.g., Michael et al., 1993; Hatzipanayotou et al., 1994; Abe, 1995; Keen and Lighthart, 2002) have formulated static general equilibrium trade models to show that tariff and tax reform can bring about a win-win outcome: the reform raises welfare without decreasing, and typically increasing, government revenue.

Although the existing theoretical literature focuses on the welfare and revenue effects of tariff and tax reform, in fact the reform also affects another fundamental policy objective: the growth rate of national income. There is much empirical evidence that changes in the relative price of capital goods to consumption goods, often caused by changes in trade barriers, alter the incentives for investment and hence economic growth (e.g., De Long and Summers, 1991; Lee, 1993; Eaton and Kortum, 2001). Taking account of the growth effect may complicate, or even reverse, the welfare and revenue effects of tariff and tax reform obtained in static models. First, changes in the growth rate mean reallocating the intertemporal consumption stream, which influences welfare and the present value of government revenue in a nontrivial way. Second, when a country imports a capital good, investment demand adds to the tax base of an import tariff (i.e., consumption plus investment minus production), which may now be superior to the consumption tax on the same good in raising revenue. The purpose of this paper is to reconsider how tariff and tax reform affects welfare, government revenue, and growth in a developing country in a dynamic general equilibrium model.

We develop a two-good, two-factor endogenous growth model of a small open economy. A capital good (e.g., machine) is either invested or consumed, whereas a consumption good (e.g., food) is only consumed. Each good is produced from domestically owned capital and labor. The engine of growth is learning-by-doing and economy-wide knowledge spillovers of the Arrow (1962)-Romer (1986) type: the effectiveness of a unit of labor in each sector increases linearly with the aggregate amount of capital stock. As Ohdoi (2003) gave a natural two-sector extension of Barro’s (1990) one-sector endogenous growth model with a flow-type public input, we extend Arrow (1962) and Romer (1986) to a two-sector extension.

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1 According to the World Bank (2002), in low- and middle-income countries, the shares of direct taxes (i.e., taxes on income, profits, and capital gains, plus social security taxes), indirect taxes (i.e., taxes on goods and services), and trade taxes in total current central government revenue were 22%, 26%, and 17%, respectively, in 1999. Those shares became 22%, 36%, and 9% in 1999.

2 Keifer (2002), Franzzen (2002), and Park (2004), among others, provided the empirical evidence of intersectoral knowledge spillovers caused by R&D.
model. Our formulation has two advantages. First, in parallel with static models, our economy is always incompletely specialized. Second, the existence of only one state variable enables us to focus on the steady state, making our problem analytically tractable.

We obtain the following main results. First, the growth effect of tariff and tax reform depends only on factor intensity ranking. Trade liberalization raises (or lowers) the growth rate if and only if the import sector is more effective-labor-intensive (or capital-intensive). This is because the growth rate is, as usual, increasing in the rate of return to capital, which is now governed by the Stolper-Samuelson theorem. Second, we can always design win-win-win (i.e., growth-, revenue-, and welfare-enhancing) tariff and tax reform as long as consumption of either good is more distorted than the other good at the pre-reform equilibrium. When we take consumer-price-neutral tariff and tax reform in the growth-enhancing direction, the present value of government revenue and the consumption index in the initial period may decrease. To compensate for the possible revenue and welfare losses, we should additionally raise the consumption tax on the previously less distorted good as necessary. This is because the tax rise not only increases revenue but also raises welfare by narrowing the consumption distortion.

The rest of this paper is organized as follows. Section 2 formulates the model. Section 3 examines how the economy evolves over time and how the pattern of trade is determined by the world relative price. Section 4 proposes win-win-win tariff and tax reform for each pattern of trade. Section 5 concludes.

2 The model

Suppose that a small open economy has two sectors. Sector 1 produces a capital good (called good 1), which is either invested for capital accumulation or consumed, and sector 2 supplies a pure consumption good (called good 2). We choose good 1 as the numeraire.

2.1 Firms

The representative firm in sector \( j \) \((j = 1, 2)\) maximizes its profit \( \Pi_j = p_j Y_j - rK_j - wL_j \), subject to the production function \( Y_j = F_j(K_j, B_j L_j) \), where \( p_j \) is the producer price of good \( j \); \( Y_j \) is the output; \( r \) is the rental rate; \( K_j \) is the
demand for capital; $w$ is the wage rate; $L_j$ is the demand for labor; and $B_j$ is the effectiveness of a unit of labor. It is assumed that $F_j(\cdot)$ is increasing, concave, linearly homogeneous, and differentiable with respect to capital $K_j$ and the effective amount of labor $B_jL_j$.

The main assumption of this model is that, in each period, the effectiveness of labor is increasing in $K$, the aggregate amount of capital stock, and is common to both sectors:

$$B_1 = B_2 = H(K); H' > 0.$$  

This assumption reflects learning-by-doing and economy-wide knowledge spillovers formulated by Arrow (1962) and Romer (1986). As firms employ new capital to increase their output, at the same time, they learn how to produce more effectively. Moreover, firms’ new knowledge instantaneously spills over across sectors as well as within each sector. Such intersectoral knowledge spillovers are plausible in this model since workers having some technological information can move freely between the two sectors as long as those sectors offer the same wages.\(^6\)

In addition, we assume that the function $H(K)$ is linear in $K$:

$$H = hK; h > 0.$$  

If $H(K)$ were not linear, the total amount of effective labor (with the total amount of raw labor fixed) would grow at a different rate from that of the total capital stock. This means that, from the Rybczynski theorem, the small open economy with constant producer prices should specialize completely in the good that uses the faster-growing factor more intensively within finite time. The linearity assumption is made in order to maintain long-run diversification.\(^7\)

With $B_1 = B_2 = H(K)$ in mind, we rewrite the profit maximization problem in terms of effective labor: maximize $\Pi_j = p_jY_j - rK_j - vH_j$, subject to $Y_j = F_j(K_j, H_j)$, where $v \equiv w/H$ is the wage rate per unit of effective labor; and $H_j \equiv H_L$ is the effective amount of labor.\(^8\) Since the production function is linearly homogeneous, we can define the unit cost function which is independent of $Y_j$. The problem is to minimize the unit cost $c_j = r a_{Kj} + v a_{Hj}$; subject to:

$$1 = F_j(a_{Kj}, a_{Hj}),$$  

where $a_{Kj} \equiv K_j/Y_j$ and $a_{Hj} \equiv H_j/Y_j$ are the demands for capital and effective labor per unit of output, respectively. From the equality between the marginal rate of technical substitution and relative factor price, we have:

$$a_{Kj}/a_{Hj} = k_j(v/r); k'_j > 0.$$  

\(^6\)Drigoen and Venditti (2001), Goenka and Poulsen (2002), Nishimura and Venditti (2002), and Drigoen et al. (2003), among others, dealt with intersectoral knowledge spillovers in their two-sector dynamic models.

\(^7\)Goenka and Poulsen (2002) and Drigoen et al. (2003) also used our Eq. (1) with $h = 1$.

\(^8\)This type of transformation was also made by Ohashi (2003), where the effectiveness of labor was linear in a public input.
From Eqs. (2) and (3), we obtain the solution \((a_{Kj}(r, v), a_{Hj}(r, v))\) and the unit cost function \(c_j(r, v) = r a_{Kj}(r, v) + v a_{Hj}(r, v)\). Throughout this paper, we focus on the case of incomplete specialization. Then, as a result of profit maximization, the producer price must be equal to the unit cost, implying that the maximized profit is zero, for all \(j\):

\[
p_j = c_j(r, v) = r a_{Kj}(r, v) + v a_{Hj}(r, v) \Leftrightarrow p_j Y_j = r K_j + v H_j.
\] (4)

### 2.2 Households

The representative household maximizes its utility \(U = \int_0^\infty \exp(-\rho t)(C(t)^{1-\theta} - 1)/(1 - \theta) dt, C(t) = V(C_1(t), C_2(t))\), where \(\rho\) is the subjective discount rate; \(C\) is the index of consumption; \(\theta\) is the elasticity of marginal instantaneous utility, which is the inverse of the elasticity of intertemporal substitution; \(C_j\) is consumption of good \(j\); and \(t\) denotes time. We omit the time variables where no confusion arises. It is assumed that \(V(\cdot)\) is increasing, concave, linearly homogeneous, and differentiable. Assuming that labor endowment is normalized to unity, the flow budget constraint is given by:

\[
p_1(t)(\dot{K}(t) + \delta K(t)) = r(t)K(t) + v(t)H(t) + T(t) - E(t),
\] (5)

\[
E(t) = q_1(t)C_1(t) + q_2(t)C_2(t),
\] (6)

where \(\delta\) is the depreciation rate; \(T\) is the lump-sum transfer from the government; \(E\) is the value of expenditure; \(q_j\) is the consumer price of good \(j\); and a dot over a variable represents differentiation with respect to time. This problem can be broken down into two stages. In the first stage, the representative household maximizes its consumption index subject to Eq. (6). Since the consumption index function is linearly homogeneous, the optimum requires that relative demand depends solely on relative consumer price:

\[
C_1 / C_2 = c(q_1 / q_2); c' < 0.
\] (7)

From Eqs. (6) and (7), we obtain \(C_2 = [1/(q_1 c + q_2)]E, C_1 = [c/(q_1 c + q_2)]E,\) and \(C = E/c(q_1, q_2)\), where the unit expenditure function \(c(\cdot)\) is increasing, concave, linearly homogeneous, and differentiable. In the second stage, the representative household maximizes \(U\) subject to Eq. (5) and the solution in the first stage. As a result, we obtain the following Euler equation and the transversality condition, respectively:

\[
\gamma_E(t) = (1/\theta)(r(t)/p_1(t) - \delta - \rho),
\] (8)

\[
0 = \lim_{t \to \infty} \exp(-\int_0^t (r(s)/p_1(s) - \delta)ds)K(t),
\] (9)

where \(\gamma_{\bullet, \text{subscript}}\) denotes the growth rate of the subscript. Assume throughout that the growth rate is positive but not high enough to cause unbounded utility: \(\gamma_E(t) > 0, \rho > (1 - \theta)\gamma_E(t)\).
2.3 Government

Let $p_j^*$ denote the constant world price of good $j$, and remember that good 1 is the numéraire: $p_1^* = 1$. When the economy imports the consumption good, domestic prices are given by $p_1 = 1, p_2 = p_2^* + \tau_2, q_1 = 1 + t_1, q_2 = p_2^* + \tau_2 + t_2$, where $\tau_2(\geq 0)$ and $t_2(\geq 0)$ are the permanent specific import tariff rate on the consumption good and the consumption tax rate on good $j$, respectively. Assuming a balanced budget in each period, the government budget constraint is:

$$T = \tau_2(C_2 - Y_2) + t_1C_1 + t_2C_2. \quad (10)$$

As we will see, the balanced budget implies balanced trade in each period, which is standard in the trade literature.

When the economy imports the capital good, we have $p_1 = 1 + \tau_1, p_2 = p_2^*, q_1 = 1 + \tau_1 + t_1, q_2 = p_2^* + t_2$, where $\tau_1(\geq 0)$ is the permanent specific import tariff rate on the capital good, and:

$$T = \tau_1(C_1 + K + \delta K - Y_1) + t_1C_1 + t_2C_2. \quad (11)$$

2.4 Equilibrium

Since we do not allow for international factor movement, we have the market clearing conditions for capital and labor, the latter of which is expressed in effective terms:

$$K = K_1 + K_2 = a_{K1}(r, v)Y_1 + a_{K2}(r, v)Y_2, \quad (12)$$

$$H = H_1 + H_2 = a_{H1}(r, v)Y_1 + a_{H2}(r, v)Y_2. \quad (13)$$

By appropriately summing the zero profit condition (4), the household budget constraint (5) and (6), and the government budget constraint (10) or (11), and imposing the market clearing conditions (12) and (13), we can verify that the following trade balance holds in each period regardless of the patterns of trade:

$$C_1 + K + \delta K - Y_1 + p_2^*(C_2 - Y_2) = 0. \quad (14)$$

3 Free trade

3.1 Equilibrium dynamics

In free trade with $p_1 = 1$ and $p_2 = p_2^*$, Eq. (4) determines the equilibrium values of $r$ and $v$, which in turn determine input coefficients. Eq. (4) also implies that $r$ and $v$ must satisfy:
where $a \equiv a_{K1}a_{H2} - a_{H1}a_{K2}$. The sign of $a$ is positive (or negative) if and only if $a_{K1}/a_{H1} > (or <)a_{K2}/a_{H2}$, that is, good 1 is more capital-intensive (or effective-labor-intensive) than good 2. Note that these equations do not give the reduced-form solution for factor prices. Rather, they simply represent the relationship between factor prices and input coefficients satisfying Eq. (4).

To ensure positive values for factor prices, we assume that $a_{K1}/a_{K2} > (or <)1/p_2^2 > (or <)a_{H1}/a_{H2}$ when $a > (or <)0$.

The equilibrium quantities of output are calculated from Eqs. (12) and (13):

$$Y_1 = (K_a H_2 - a_{K2}H)/a,$$

$$Y_2 = (a_{K1} H - K a_{H1})/a.$$ 

In view of Eqs. (1) and (3), incomplete specialization requires that $k_1 > (or <)1/h > (or <)k_2$ when $a > (or <)0$.

Substituting $C_1 = [c/(q_1 c + q_2)] E$, $C_2 = [(1/q_1 c + q_2)] E$, and the above expressions for outputs into Eq. (14), and using Eq. (1) and the above relationship between factor prices and input coefficients, we obtain:

$$\dot{K} = (A_0 - \delta) K - [(c + p_2^e)/(q_1 c + q_2)]E; 
A_0 \equiv r + vh.$$

This equation and Eq. (8), together with the initial condition and the transversality condition (9), constitute the dynamic system. Our system can be combined into a single linear differential equation with respect to $\kappa \equiv K/E : \dot{\kappa} = (A_0 - \delta - \gamma_E)\kappa - (c + p_2^e)/(q_1 c + q_2)$. Since $A_0 > r$ and $r - \delta - \gamma_E = \rho - (1-\theta)\gamma_E > 0$, this linear differential equation has positive slope $A_0 - \delta - \gamma_E$ and negative vertical intercept $-(c + p_2^e)/(q_1 c + q_2)$ in the $(\kappa, \dot{\kappa})$-plane. Let $\kappa^*$ denote the steady-state value of $\kappa$ such that $\dot{\kappa} = 0$. Since $\kappa < 0$ at $\kappa = 0$ and $\partial \kappa/\partial \kappa > 0$ for all $\kappa \geq 0$, $\kappa^*$ uniquely exists with a positive value, and the dynamics of $\kappa$ are globally unstable. For the economy to grow at a positive rate and to satisfy the transversality condition (9), we must have $\kappa(t) = \kappa^*$ for all $t \in [0, \infty)$. Consequently, $K$ grows at the same constant rate as $E$:

$$\gamma_E = \gamma_K = (1/\theta)(r - \delta - \rho) \equiv \gamma_0.$$

### 3.2 Patterns of trade

Our present task is to examine how the pattern of trade is determined by $p_2^e$. Before doing this, we introduce an indicator of the trade pattern:

**Lemma 1** Let $y \equiv (Y_1 - \dot{K} - \delta K)/Y_2$ denote the relative supply of good 1 to good 2 for consumption, and note that $c \equiv C_1/C_2$ is the relative consumption

$$\gamma_E = \gamma_K = (1/\theta)(r - \delta - \rho) \equiv \gamma_0.$$
demand for good 1 to good 2. The economy exports (or imports) good 1 and
imports (or exports) good 2 if and only if \( y > (or <) c \). No trade occurs if and
only if \( y = c \).

**Proof.** Making use of the trade balance (14) to eliminate \( Y_1 - K - \delta K \) from
\( y \), we obtain 
\[ y - c = \left[(c + p_2^0)/Y_2\right](C_2 - Y_2). \]
This implies that \( y > (or <) c \Leftrightarrow C_2 > (or <) Y_2 \). Remembering Eq. (14) again, we obtain 
\[ C_2 > (or <) Y_2 \Leftrightarrow C_1 + K + \delta K < (or >) Y_1 \]. Needless to say, we have 
\( y = c \Leftrightarrow C_2 = Y_2 \Leftrightarrow C_1 + K + \delta K = Y_1 \).

Since we know from Eq. (7) that \( c \) is positive and monotonically increasing
in \( p_2^0 \), we next see the properties of \( y \). The steps of reasoning are similar to
Jones (1965), except that investment enters \( y \). Totally differentiating Eq. (4) and
applying Shephard’s lemma, we have 
\[ 0 = a_{K1}dr + a_{H2}dv \quad \text{and} \quad dp_2^0 = a_{K2}dr + a_{H2}dv. \]
From these equations, we obtain:

\[ \begin{align*}
dr &= -(a_{H1}/a)dp_2^0, \\
\frac{dv}{dr} &= (a_{K1}/a)dp_2^0.
\end{align*} \]

This indicates the Stolper-Samuelson theorem: a rise in the producer price
of a good raises (or lowers) the price of the factor which is used more (or less)
intensively in the good. Eq. (15) and the Stolper-Samuelson relations imply that:

\[ d\gamma_0 = \gamma_0'(dp^0_2); \gamma_0' \equiv -(1/\theta)a_{H1}/a. \]

The sign of the growth effect of producer price changes depends solely on
factor intensity ranking: \( \gamma_0' > (or <) 0 \Leftrightarrow a < (or >) 0 \). As we will see, this will
open the possibility that trade liberalization does not necessarily boost economic
growth.

From Eqs. (2) and (3), the changes in input coefficients are given by:

\[ \begin{align*}
da_{Hj} &= -(a_{Hj}a_{Kj}\sigma_j/c_j)(r/v)dv - dr, \\
da_{Kj} &= (v/r)(a_{Hj}a_{Kj}\sigma_j/c_j)(r/v)dv - dr,
\end{align*} \]

where \( \sigma_j \equiv \ln k_j/d\ln(v/r)(> 0) \) is the elasticity of substitution between
inputs in sector \( j \). Totally differentiating the factor market clearing conditions
(12) and (13), substituting \( da_{Hj} \) and \( da_{Kj} \) from the above equations into them,
and remembering that \( K \) and hence \( H \) are predetermined in each period, we
obtain:

\[ \begin{align*}
dY_1 &= -[\zeta_{C2}/(ar)](r/v)dv - dr, \quad (16) \\
dY_2 &= [\zeta_{C1}/(ar)](r/v)dv - dr; \quad (17)
\end{align*} \]

\( \zeta \equiv Y_1aH_1aK_1\sigma_1/c_1 + Y_2aH_2aK_2\sigma_2/c_2 > 0. \)
From the Stolper-Samuelson relations, we have \((r/v)dv - dr = |c_1/(av)|dp^*_2\). Substituting this expression into Eqs. (16) and (17), and using Eq. (4), we obtain:

\[
\begin{align*}
  dY_1 &= -\left(\frac{\zeta}{(a^2rv)}\right)dp^*_2, \\
  dY_2 &= \left(\frac{\zeta}{(a^2rv)}\right)dp^*_2.
\end{align*}
\]

Since \(dY_1/dp^*_2 < 0\) and \(dY_2/dp^*_2 > 0\), we can define the range of \(p^*_2\) for incomplete specialization: \(p^*_2 \in (p^*_2, p^*_2)\), where \(p^*_2\) and \(p^*_2\) satisfies \(\lim_{p^*_2 \to p^*_2} Y_1 = 0\) and \(\lim_{p^*_2 \to p^*_2} Y_2 = 0\), respectively.

Finally, let us see the relationship between \(y \equiv (Y_1 - \hat{K} - \Delta K)/Y_2\) and \(p^*_2\). The first derivative is calculated as:

\[
\frac{dy}{dp^*_2} = \left(1/Y_2\right)(dY_1/dp^*_2 - \hat{K}d\gamma_0/dp^*_2 - ydY_2/dp^*_2).
\]

We get the normal sign \(dy/dp^*_2 < 0\) if either good 2 is more capital-intensive or the elasticity of intertemporal substitution is not too large. As for the values of \(y\) at the two boundaries, we have \(\lim_{p^*_2 \to p^*_2} y = \infty\) and \(\lim_{p^*_2 \to p^*_2} y < 0\), since \(\lim_{p^*_2 \to p^*_2} (Y_1 - \hat{K} - \Delta K) > 0\) and \(\lim_{p^*_2 \to p^*_2} (-\gamma_0\hat{K} - \Delta K) < 0\).

The following proposition shows how the pattern of trade is determined by \(p^*_2\):

**Proposition 1** Assume that \(dy/dp^*_2 < 0\). There exists a unique \(p^*_2 \in (p^*_2, p^*_2)\) such that no trade occurs. The economy exports (or imports) good 1 and imports (or exports) good 2 if and only if \(p^*_2 \in (p^*_2, p^*_2)\) or \(p^*_2 \in (p^*_2, p^*_2)\).

**Proof.** We have \(dy/dp^*_2 < 0\), \(\lim_{p^*_2 \to p^*_2} y = \infty\), and \(\lim_{p^*_2 \to p^*_2} y < 0\), whereas \(c\) is positive, finite, and increasing in \(p^*_2\). Since \(d(y - c)/dp^*_2 < 0\), \(\lim_{p^*_2 \to p^*_2} y - c > 0\), and \(\lim_{p^*_2 \to p^*_2} y - c < 0\), there exists a unique \(p^*_2 \in (p^*_2, p^*_2)\) such that \(y - c = 0\). From \(d(y - c)/dp^*_2 < 0\), we have \(y - c > 0\) for \(p^*_2 \in (p^*_2, p^*_2)\), and \(y - c < 0\) for \(p^*_2 \in (p^*_2, p^*_2)\). These results combined with Lemma 1 imply the proposition. ■

### 4 Tariff and tax reform

#### 4.1 The economy importing the consumption good

Suppose that \(p^*_2 \in (p^*_2, p^*_2)\) and hence the economy imports good 2 in free trade, and assume that \(\gamma_2\) is sufficiently low that the pattern of trade should always be preserved. From Eq. (4), we have:

\footnote{Since \(\kappa = \kappa^* > 0\), we have \(E > 0\) and hence \(C_j > 0\) for all \(j\). In view of Eq. (14), as \(p^*_2 \to p^*_2\), this can be possible only if \(\lim_{p^*_2 \to p^*_2} (Y_1 - \hat{K} - \Delta K) > 0\).}
\[
\begin{align*}
r &= [aH_2 - aH_1(p_2^* + \tau_2)]/a, \\
v &= [aK_1(p_2^* + \tau_2) - aK_2]/a.
\end{align*}
\]

Substituting \( C_1 = [c/(q_1 c + q_2)]E \), \( C_2 = [1/(q_1 c + q_2)]E \), and the expressions for outputs that appeared in section 3.1 into Eq. (14), and using Eq. (1) and the above relationship between factor prices and input coefficients, we obtain:

\[
\dot{K} = (A_2 - \delta)K - [(c + p_2^*)/(q_1 c + q_2)]E; A_2 \equiv r + vh - \tau_2(aK_1 h - aH_1)/a.
\]

It can be easily verified that this equation is equivalent to Eqs. (5) and (10). To obtain the standard no-transition property as in section 3.1, we assume that:\footnote{If this assumption were not satisfied, we might have stable dynamics with respect to \( \kappa = K/E \). In that case, transitional dynamics would be indeterminate: the economy would converge to the steady state regardless of the initial value of \( E \). Because our present focus is on tariff and tax reform in a determinist environment, we assume away the possibility of indeterminacy.}

\[
A_2 - r = vh - \tau_2(aK_1 h - aH_1)/a > 0. \tag{18}
\]

Then from Eqs. (8), (9), and the above expression for \( \dot{K} \), we obtain:

\[
\gamma_E = \gamma_K = (1/\theta)(r - \delta - \rho) \equiv \gamma_2(\tau_2), \tag{19}
\]

where the dependence of \( r \) on \( \tau_2 \) through Eq. (4) is taken into account. Substituting Eq. (19) back into the expression for \( \dot{K} \), we have \( C_1 + p_2^*C_2 = (A_2 - \delta - \gamma_2)K \). From this equation and Eq. (7), \( C_2, C_1 \), and \( C \) are solved as \( C_2 = [1/(c + p_2^*)](A_2 - \delta - \gamma_2)K \), \( C_1 = [c/(c + p_2^*)](A_2 - \delta - \gamma_2)K \), and:

\[
C = C_2V(c, 1) = [V(c, 1)]/(c + p_2^*)/(A_2 - \delta - \gamma_2)K. \tag{20}
\]

Since all growing variables grow at the same rate \( \gamma_2 \), we have \( T(t) = T(0) \exp(\gamma_2 t) \). Substituting this into the present value of government revenue \( G_2 \equiv \int_0^\infty T(t) \exp(-(r - \delta)t)dt \), \footnote{This expression appears in the intertemporal budget constraint \( \int_0^\infty E(t) \exp(-(r - \delta)t)dt = K(0) + v \int_0^\infty H(t) \exp(-(r - \delta)t)dt + G_2 \), which is equivalent to the flow budget constraint (5) and the transversality condition (10).} we obtain:

\[
G_2 = T(0)/(r - \delta - \gamma_2) = T(0)/[\rho - (1 - \theta)\gamma_2].
\]

Noting from Eq. (20) that \( C(t) = C(0) \exp(\gamma_2 t) \), the utility of the representative household is rewritten as:

\[
U_2 = [1/(1 - \theta)](C(0)^{1-\theta}/[\rho - (1 - \theta)\gamma_2] - 1/\rho).
\]

We examine how changes in \( \tau_2, t_2 \), and \( t_1 \) affect \( \gamma_2, G_2 \), and \( U_2 \). First, from Eq. (4), we obtain the Stolper-Samuelson relations:

\[\text{10}\]

10If this assumption were not satisfied, we might have stable dynamics with respect to \( \kappa = K/E \). In that case, transitional dynamics would be indeterminate: the economy would converge to the steady state regardless of the initial value of \( E \). Because our present focus is on tariff and tax reform in a determinist environment, we assume away the possibility of indeterminacy.

11This expression appears in the intertemporal budget constraint \( \int_0^\infty E(t) \exp(-(r - \delta)t)dt = K(0) + v \int_0^\infty H(t) \exp(-(r - \delta)t)dt + G_2 \), which is equivalent to the flow budget constraint (5) and the transversality condition (10).
\[
\begin{align*}
  dr &= -(aH_1/a)d\tau_2, \\
  dv &= (aK_1/a)d\tau_2.
\end{align*}
\]

Form these relations and Eq. (19), the growth effect is expressed as:

\[
d\gamma_2 = \gamma'_2 d\tau_2; \quad \gamma'_2 \equiv -(1/\theta)aH_1/a.
\]

Trade liberalization raises (or lowers) the growth rate (i.e., \(d\gamma_2/d\tau_2 < (or > 0)\) if and only if \(a > (or < 0)\), that is, the import sector is more effective-labor-intensive (or capital-intensive), because trade liberalization lowers the price of the factor that is used more intensively in the imported good and raises the price of the other factor. When the import sector is more capital-intensive, the prediction of our model is opposite to the conventional wisdom about the growth-enhancing effect of trade liberalization (e.g., Lee, 1993).

The amount of change in the present value of government revenue is given by (see Appendix A for derivation):

\[
\begin{align*}
dG_2 &= [\chi_2 dt_1 + \phi_2(d\tau_2 + dt_2) + \omega_2 d\tau_2]/[\rho - (1 - \theta)\gamma_2]; \\
\chi_2 &\equiv C_1(0)[1 - \sigma_1(1/(c + p_2^*)](t_1 p_2^* - t_2)/(1 + t_1)], \\
\phi_2 &\equiv C_2(0)[1 - \sigma_2(1/(c + p_2^*)](t_2 + t_2 - t_2 p_2^*/(p_2^* + \tau_2 + t_2)]}, \\
\omega_2 &\equiv -Y_2(0)(\tau_2)/(a^2 r \nu) \\
&- [(t_1 c + \tau_2 + t_2)/(c + p_2^*)]/(\zeta(0)\tau_2/(a^2 r \nu) + K(0)\gamma'_2 + G_2(1 - \theta)\gamma'_2),
\end{align*}
\]

where \(\sigma_1 \equiv -d \ln c/d \ln(q_1/q_2) (> 0)\) is the elasticity of substitution in consumption. In the numerator of the right-hand side of Eq. (21), \(\chi_2\) shows the change in initial government revenue caused by the change in the consumption tax on the exported good. A rise in \(t_1\) directly increases \(T(0)\), and indirectly changes it through the substitution of good 2 for good 1. If \(t_1 > (or <) (\tau_2 + t_2)/p_2^* \iff (1 + t_1)/(p_2^* + \tau_2 + t_2) > (or <) 1/p_2^*,\) that is, consumption of good 1 is more (or less) distorted than good 2 at the pre-reform equilibrium, then the consumption substitution effect works to decrease (or increase) \(T(0)\). Next, \(\phi_2\) represents the effect of the change in \(\tau_2 + t_2\), the total tax burden rate on consumption of the imported good, on initial government revenue. This revenue change also occurs through direct and consumption substitution effects. Finally, \(\omega_2\) indicates the change in the present value of government revenue because of the change in the tariff, with the consumer prices unchanged. In the definition of \(\omega_2\), the first and the second terms express the change in \(-\tau_2 Y_2(0)\), which reflects the difference in tax bases between \(\tau_2\) and \(t_2\). The third term shows the change in initial government revenue caused by the change in the income for consumption \(C_1(0) + p_2^* C_2(0) = Y_1(0) + p_2^* Y_2(0) - K(0) - \delta K(0)\). The latter change occurs because both GDP, evaluated at world prices, and investment change. The fourth term represents the effect of the change in the growth rate on the denominator of \(G_2 = T(0)/[\rho - (1 - \theta)\gamma_2]\). Note that the signs
of $\chi_2$, $\phi_2$, and $\omega_2$ are generally ambiguous. In the absence of the growth effect (i.e., $\gamma_2^* = 0$), $\omega_2$ would be negative as in static models (e.g., Hatzipanayotou et al., 1994; Keen and Lighthart, 2002). In fact, however, $\omega_2$ can be positive if $\gamma_2^* < 0$ and $\theta > 1$.\footnote{There is a lot of empirical evidence (e.g., Hall, 1988; Hahm, 1998; Ogaki and Reinhart, 1998) that the elasticity of intertemporal substitution is significantly below unity, that is, $\theta > 1$.} We can only say that either $\chi_2$ or $\phi_2$ is positive. This is because raising the tax on consumption of the previously less distorted good unambiguously increases initial government revenue.

The welfare effect is calculated as (see Appendix A for derivation):

$$dU_2 = \frac{C(0)^{1-\theta}}{\rho - (1-\theta)\gamma_2} \left[ \sum_2 \left( \frac{d\tau_2 + dt_2}{p_2^* + \tau_2 + t_2} - \frac{dt_1}{1 + t_1} \right) + (Z_2 + \Gamma_2)d\tau_2 \right] ; \quad (22)$$

$$\Sigma_2 \equiv -\sigma_2 \beta_2 \frac{c}{c + p_2^*} \frac{\tau_2 + t_2 - p_2^* t_1}{\tau_2 + t_2} \left\{ \begin{array}{l} > \text{ implies } \frac{\tau_2 + t_2}{p_2^*} \left\{ \begin{array}{l} < \text{ implies } t_1, \end{array} \right. \end{array} \right.$$ 

$$Z_2 \equiv -\frac{\zeta(0)\tau_2}{\partial^2 r_v(A_2 - \delta - \gamma_2) K(0)} < 0,$$

$$\Gamma_2 \equiv \frac{\gamma_2^2 [v - A_2 - aK_1h - ah_1]}{\rho - (1-\theta)\gamma_2} \left\{ \begin{array}{l} > \text{ implies } a \left\{ \begin{array}{l} < \text{ implies } 0, \end{array} \right. \end{array} \right.$$ 

where $\beta_2 = V_2(c, 1)/V(c, 1) \in (0, 1)$ is the elasticity of the consumption index with respect to consumption of good 2. In the brackets of Eq. (22), the first term stands for the change in the initial consumption index coming from the consumption substitution effect. If $(\tau_2 + t_2)/p_2^* > (or <) t_1$, that is, consumption of good 2 is more (or less) distorted than good 1 at the pre-reform equilibrium, then a rise in $\tau_2 + t_2$ relative to $t_1$ enlarges (or narrows) the consumption distortion, bringing about a welfare loss (or gain). Next, $Z_2$ indicates the effect of the change in the production allocation on the initial consumption index. Trade liberalization increases GDP evaluated at world prices, which in turn raises welfare. Finally, $\Gamma_2$ shows the change in welfare caused by the change in the growth rate. A rise in the growth rate raises welfare by increasing future consumption, but it lowers welfare by decreasing present consumption. As long as the regularity condition (18) is assumed, the former always outweighs the latter.

Having examined the properties of $\gamma_2$, $G_2$, and $U_2$ with respect to $\tau_2$, $t_2$, and $t_1$, we characterize tariff and tax reform that aims to improve all three of our policy objectives. Let us first consider the case that the import sector is more effective-labor-intensive so that trade liberalization raises the growth rate, that is, $\gamma_2^* < 0$. We consider the following reform strategies:

(R$_7^*$) \begin{enumerate}
\item $(\tau_2 + t_2)/p_2^* > t_1$ :
  \begin{enumerate}
  \item $\omega_2 < 0 : d\tau_2 < 0, dt_2 + dt_2 > 0, dt_1 \geq 0$;
  \end{enumerate}
\end{enumerate}
(b) $\omega_2 > 0 : dr_2 < 0, dr_2 + dt_2 = 0, dt_1 > -(\omega_2/\chi_2)dr_2$.

2. $(\tau_2 + t_2)/p^*_2 < t_1$ :
   
   (a) $\omega_2 < 0 : dr_2 < 0, dt_1 = 0, dr_2 + dt_2 \geq 0$;
   
   (b) $\omega_2 > 0 : dr_2 < 0, dt_1 = 0, dr_2 + dt_2 > -(\omega_2/\phi_2)dr_2$.

In case 1. (a), consumer-price-neutral tariff and tax reform for trade liberalization (i.e., $dr_2 < 0, dr_2 + dt_2 = 0, dt_1 = 0$) is as effective as in static models (e.g., Hatzipanayotou et al., 1994; Keen and Ligthart, 2002). It increases the present value of government revenue because $\omega_2 < 0$. The reform also raises welfare by raising the growth rate and improving the production allocation. In case 1. (b), the consumer-price-neutral tariff and tax reform would still raise both the growth rate and welfare, but fail to keep the present value of government revenue from falling. Which further tax instruments should we use to increase government revenue? It is appropriate to raise the consumption tax on the less distorted good by more than the revenue-neutral size of change, since this also raises welfare by decreasing the consumption distortion. Case 2. (a) and case 2. (b) are similar to case 1. (a) and case 1. (b), respectively, except that $\tau_2 + t_2$ rather than $t_1$ should be raised.

If the import sector is more capital-intensive so that raising the tariff raises the growth rate, that is, $\gamma_2 > 0$, then $\Gamma_2$ and $Z_2$ have opposite signs. This implies that the welfare effect of consumer-price-neutral tariff and tax reform against trade liberalization (i.e., $dr_2 > 0, dr_2 + dt_2 = 0, dt_1 = 0$) is ambiguous. In this situation, we revise our reform strategy as follows:

$$(\mathbf{R}_2^*)$$

1. $(\tau_2 + t_2)/p^*_2 > t_1$ :
   
   (a) $\omega_2 < 0 : dr_2 > 0, dr_2 + dt_2 = 0, dt_1 > \max\{-(\omega_2/\chi_2)dr_2, (1 + t_1)(Z_2/Z_2)dr_2\}$;
   
   (b) $\omega_2 > 0 : dr_2 > 0, dr_2 + dt_2 = 0, dt_1 \geq (1 + t_1)(Z_2/Z_2)dr_2$.

2. $(\tau_2 + t_2)/p^*_2 < t_1$ :
   
   (a) $\omega_2 < 0 : dr_2 > 0, dt_1 = 0, dr_2 + dt_2 \geq \max\{-(\omega_2/\phi_2)dr_2, -(p^*_2 + \tau_2 + t_2)(Z_2/Z_2)dr_2\}$;
   
   (b) $\omega_2 > 0 : dr_2 > 0, dt_1 = 0, dr_2 + dt_2 \geq -(p^*_2 + \tau_2 + t_2)(Z_2/Z_2)dr_2$.

Let us first look at case 1. (b), in which the consumer-price-neutral tariff and tax reform against trade liberalization would increase the present value of government revenue. To compensate for the welfare loss resulting from the worsened production allocation, the government should raise the consumption tax on the less distorted good by the amount satisfying that $\Sigma_2[-dt_1/(1 + t_1)] + Z_2d\tau_2 \geq 0$. In case 1. (a), raising the consumption tax on the less distorted good has to take another role in increasing government revenue. This can be done by letting $dt_1$ satisfy that $\chi_2dt_1 + \omega_2d\tau_2 > 0$. Case 2. (a) and case 2. (b) are similar to case 1. (a) and case 1. (b), respectively.
4.2 The economy importing the capital good

When the economy imports good 1, the analysis proceeds in a similar way to the case in which the economy imports good 2. From Eqs. (1), (4), and (14), we obtain:

\[ \dot{K} = (A_1 - \delta)K - \frac{(c + p_2^c)}{(q_1 c + q_2)}E; A_1 \equiv \frac{r + v h + \tau_1 p_2^2(a K_1 h - a H_1)}{a} / (1 + \tau_1). \]

Note that \( A_1 - r/(1 + \tau_1) = [v h + \tau_1 p_2^2(a K_1 h - a H_1)]/a \) is positive since \( (a K_1 h - a H_1)/a = Y_2/K \) is positive as long as the economy is incompletely specialized. The steady-state growth rate is given by:

\[ \gamma_E = \gamma_K = \frac{1}{\theta} \frac{r/(1 + \tau_1) - \delta - \rho}{\rho(1 - \theta)} \equiv \gamma_1 \tau_1. \tag{23} \]

The present value of government revenue \( G_1 \equiv \int_0^\infty T(t) \exp(-r/(1 + \tau_1) - \delta t) dt \) and welfare are rewritten as:

\[ G_1 = T(0)/[r/(1 + \tau_1) - \delta - \gamma_1] = T(0)/[\rho/(1 - \theta)\gamma_1], \]
\[ U_1 = [1/(1 - \theta)]\{C(0)^{1-\theta}/[\rho/(1 - \theta)\gamma_1] - 1/\rho]. \]

Making use of Eqs. (4) and (23), the growth effect of trade liberalization is expressed as:

\[ d\gamma_1 = \gamma_1^d \dot{\gamma}_1; \gamma_1^d \equiv (1/\theta)\alpha H_1 p_2^2 / [(1 + \tau_1)^2 \alpha]. \]

It has been confirmed that trade liberalization raises (or lowers) the growth rate if and only if the import sector is more effective-labor-intensive (or capital-intensive): \( \alpha < (\alpha >) 0 \) in the present case.\textsuperscript{13}

In a similar way to Appendix A, the changes in \( G_1 \) and \( U_1 \) are derived as:

\[ dG_1 = \{\chi_1 dt_2 + \phi_1 (dt_1 + dt_1) + \omega_1 d\gamma_1\} / [\rho - (1 - \theta)\gamma_1]; \tag{24} \]
\[ \chi_1 \equiv \bar{C}_2(0)\{1 - \sigma_1[c/(c + p_2^c)](t_2 - (\tau_1 + t_1)p_2^2/(p_2^2 + t_2)), \]
\[ \phi_1 \equiv C_1(0)\{1 - \sigma_1[1/(c + p_2^c)](\tau_1 + t_1)p_2^2 - t_2]/(1 + \gamma_1 + t_1)\}, \]
\[ \omega_1 \equiv \gamma_1 K(0) + \delta K(0) - Y_1(0) + \tau_1[\zeta(0)\gamma_1^2 \alpha^2 r]/(a^2 r v) \]
\[ - \{(1 + \gamma_1)c + t_2]/(c + p_2^c)\}p_2^2 \zeta(0) \gamma_1 / (a^2 r v) + K(0)\gamma_1^3] + G_1(1 - \theta)\gamma_1], \]

\textsuperscript{13}One may wonder if the rate of return to capital \( r/(1 + \tau_1) - \delta \) could rise with trade liberalization even when \( a > 0 \) because not only \( r \) but also \( 1 + \tau_1 \) fall. However, from the Stolper-Samuelson theorem, the rate of return in \( r \) necessarily outweighs that of \( 1 + \tau_1 \), so the rate of return to capital always falls with trade liberalization when \( a > 0 \).
\[ dU_1 = \frac{C(0)^{-\theta}}{\rho - (1 - \theta)\gamma_1} \left[ \Sigma_1 \left( \frac{dr_1 + dt_1}{1 + \gamma_1 + t_1} - \frac{dt_2}{p_2^2 + t_2} \right) + (Z_1 + \Gamma_1)dr_1 \right]; \]  

\[ \Sigma_1 \equiv -\sigma_c\beta_1 \frac{1}{c + p_2^2} \left( \frac{(\tau_1 + t_1)\rho_2^* - t_2}{1 + \gamma_1 + t_1} \right) \begin{cases} > 0 & \tau_1 + t_1 \geq \frac{t_2}{p_2^2} \\ > 0 & \tau_1 + t_1 < \frac{t_2}{p_2^2} \end{cases} \]

\[ Z_1 \equiv -\frac{p_2^2\zeta(0)\gamma_1}{a_r v(A_1 - \delta - \gamma_1)K(0)} < 0, \]

\[ \Gamma_1 \equiv \gamma_1 \left[ \sqrt{\frac{\tau_1 + t_1 \rho_2^*(\rho_2 - \rho_A)}{\rho - (1 - \theta)\gamma_1}} A_1 - \delta - \gamma_1 \right] \begin{cases} > 0 & \frac{\tau_1 + t_1 \rho_2^*(\rho_2 - \rho_A)}{\rho - (1 - \theta)\gamma_1} \geq 0 \\ > 0 & \frac{\tau_1 + t_1 \rho_2^*(\rho_2 - \rho_A)}{\rho - (1 - \theta)\gamma_1} < 0 \end{cases} \]

where \(\beta_1 \equiv V_1(c, 1)c/V(c, 1) \in (0, 1).\) Eqs. (24) and (25) are similar to Eqs. (21) and (22), respectively, except that investment adds to the tax base of the tariff, making the case of positive \(\omega_1\) more likely.

Let \(R_1^-\) (or \(R_1^+\)) denote the reform strategy corresponding to the case where the import sector is more effective-labor-intensive (or capital-intensive). They are designed as follows:

\(R_1^-\)

1. \(\gamma_1 + t_1 > t_2/p_2^* :\)
   a. \(\omega_1 < 0: dr_1 < 0, dr_1 + dt_1 = 0, dt_2 \geq 0;\)
   b. \(\omega_1 > 0: dr_1 < 0, dr_1 + dt_1 = 0, dt_2 > -\omega_1/\chi_1 dr_1.\)

2. \(\gamma_1 + t_1 < t_2/p_2^* :\)
   a. \(\omega_1 < 0: dr_1 < 0, dt_2 = 0, dr_1 + dt_1 \geq 0;\)
   b. \(\omega_1 > 0: dr_1 < 0, dt_2 = 0, dr_1 + dt_1 > -\omega_1/\phi_1 dr_1.\)

\(R_1^+\)

1. \(\gamma_1 + t_1 > t_2/p_2^* :\)
   a. \(\omega_1 < 0: dr_1 > 0, dr_1 + dt_1 = 0, dt_2 > \max\{-\omega_1/\chi_1 dr_1, (p_2^* + t_2)(Z_1/\Sigma_1)dr_1\};\)
   b. \(\omega_1 > 0: dr_1 > 0, dr_1 + dt_1 = 0, dt_2 > (p_2^* + t_2)(Z_1/\Sigma_1)dr_1.\)

2. \(\gamma_1 + t_1 < t_2/p_2^* :\)
   a. \(\omega_1 < 0: dr_1 > 0, dt_2 = 0, dr_1 + dt_1 > \max\{-\omega_1/\phi_1 dr_1, -(1 + \tau_1 + t_1)(Z_1/\Sigma_1)dr_1\};\)
   b. \(\omega_1 > 0: dr_1 > 0, dt_2 = 0, dr_1 + dt_1 \geq -(1 + \tau_1 + t_1)(Z_1/\Sigma_1)dr_1.\)

It is easily verified that \(R_1^-\) and \(R_1^+\) have qualitatively the same effects as \(R_1^-\) and \(R_2^+\), respectively. The following propositions summarize our results:
Proposition 2 Suppose that the economy imports good $j (j = 1 or 2)$, and that the import sector $j$ is more effective-labor-intensive. Then tariff and tax reform for trade liberalization specified as $(R^L_j)$ raises the growth rate, the present value of government revenue, and welfare.

Proposition 3 Suppose that the economy imports good $j (j = 1 or 2)$, and that the import sector $j$ is more capital-intensive. Then tariff and tax reform against trade liberalization specified as $(R^H_j)$ raises the growth rate, the present value of government revenue, and welfare.

As we can see from our reform strategies, it is the consumption tax on the less distorted good that enables us to design growth-, revenue-, and welfare-enhancing tariff and tax reform. This is because raising the tax does not pose a trade-off between revenue and welfare.

5 Concluding remarks

Our model has some policy implications. First, trade liberalization may or may not accelerate economic growth, depending on factor intensity ranking. Rodriguez and Rodrik (2001) found that the hypothesis that trade barriers negatively affect economic growth was not empirically robust, and suggested that future research should identify contingent relationships between trade barriers and economic growth. This paper provides an answer to their research program. Second, whether or not trade liberalization accelerates economic growth, we can recommend tariff and tax reform that simultaneously improves growth, revenue, and welfare. This paper complements the static literature on tariff and tax reform by showing that appropriately designed tariff and tax reform raises welfare and government revenue without sacrificing economic growth.

We can suggest some directions for future research. First, if there are some restrictions on tariff and tax movements, we cannot implement our reform strategy. For example, if for some political reason we cannot change the consumption tax on the less distorted good, then we have to design tariff and tax reform under the restriction, although we may not attain a win-win-win result in this case. Second, we treat learning-by-doing and economy-wide knowledge spillovers only as a mechanism for endogenous growth consistent with incomplete specialization. It will be interesting to see whether our policy recommendation remains valid if we use the revenue gain from tariff and tax reform to correct the distortion from these externalities.

Appendix A. Revenue and welfare effects of tariff and tax changes

When the economy imports good 2, we have $(r/v)dv - dr = [c_1/(aw)]dv_2$ from the Stolper-Samuelson relations. Substituting this expression into Eqs. (16) and (17), and using Eq. (4), we obtain:
\[ dY_1 = -[\zeta(p_2^* + \tau_2)/(a^2rv)]dr_2, \tag{A.1} \]
\[ dY_2 = [\zeta/(a^2rv)]dr_2. \tag{A.2} \]

From Eqs. (7), (14), (A.1), and (A.2), changes in consumption are given by:

\[ dC_2 = \sigma_c[C_1/(c + p_2^*)][dt_1/(1 + t_1) - (dr_2 + dt_2)/(p_2^* + \tau_2 + t_2)] - \{\zeta_2/[c + p_2^*]a^2rv\}dr_2 - [K\gamma_2^2/(c + p_2^*)]dr_2, \tag{A.3} \]
\[ dC_1 = -\sigma_c[C_1p_2^*/(c + p_2^*)][dt_1/(1 + t_1) - (dr_2 + dt_2)/(p_2^* + \tau_2 + t_2)] - \{c_2\tau_2/[c + p_2^*]a^2rv\}dr_2 - [cK\gamma_2^2/(c + p_2^*)]dr_2, \tag{A.4} \]

where \( \sigma_c \equiv -d\ln c/d\ln(q_1/q_2). \) Totally differentiating Eq. (10), and substituting Eqs. (A.2), (A.3), (A.4) into it, we obtain:

\[ dT(0) = \chi_2 dt_1 + \phi_2 (dr_2 + dt_2) + \psi_2 dr_2; \]
\[ \chi_2 \equiv C_1(0)\{1 - \sigma_c[1/(c + p_2^*)](t_1p_2^* - \tau_2 - t_2)/(1 + t_1)\}, \]
\[ \phi_2 \equiv C_2(0)\{1 - \sigma_c[c/(c + p_2^*)](\tau_2 + t_2 - t_1p_2^*/(p_2^* + \tau_2 + t_2))\}, \]

\[ \psi_2 \equiv -Y_2(0) - \tau_2\zeta(0)/(a^2rv) - [(t_1c + \tau_2 + t_2)/(c + p_2^*)]\zeta(0)\gamma_2/(a^2rv) + K(0)\gamma_2^2. \]

Totally differentiating \( G_2 = T(0)/[\rho - (1 - \theta)\gamma_2] \), we have \( dG_2 = [dT(0) + G_2(1 - \theta)\gamma_2 dr_2]/[\rho - (1 - \theta)\gamma_2]. \) Substituting \( dT(0) \) from the above result into this expression, we obtain Eq. (21).

As for the welfare effect, totally differentiating \( U_2 = 1/[(1 - \theta)](C(0))^{1-\theta}/[\rho - (1 - \theta)\gamma_2] - 1/\rho, \) we have \( dU_2 = (C(0))^{1-\theta}/[\rho - (1 - \theta)\gamma_2]d(C(0)/C(0)) + \gamma_2^2/[\rho - (1 - \theta)\gamma_2]d\gamma_2). \) To obtain \( dC(0)/C(0), \) we totally differentiate the consumption index function to have \( dC(0) = V_1(C_1(0), C_2(0))dC_1(0) + V_2(C_1(0), C_2(0))dC_2(0) = V_1(c, 1)dC_1(0) + V_2(c, 1)dC_2(0), \) where \( V_j(C_1(0), C_2(0)) = V_j(c, 1) \) by linear homogeneity of \( V(). \) Substituting Eqs. (A.3) and (A.4) into this expression, using the first-order condition \( V_1(c, 1)/V_2(c, 1) = q_1/q_2 \) and Euler’s formula \( V(c, 1) = V_1(c, 1)c + V_2(c, 1), \) and dividing the resulting equation by Eq. (20), we obtain:

\[ dC(0)/C(0) = -\{\sigma_c\beta_2[c/(c + p_2^*)](\tau_2 + t_2 - p_2^*t_1)/(p_2^* + \tau_2 + t_2)\} \]
\[ \times [(dr_2 + dt_2)/(p_2^* + \tau_2 + t_2) - dt_1/(1 + t_1)] \]
\[ - \{\zeta(0)\gamma_2/[a^2rv(A_2 - \delta - \gamma_2)K(0)]\}dr_2 - [\gamma_2^2/(A_2 - \delta - \gamma_2)]dr_2, \]

where \( \beta_2 \equiv V_2(c, 1)/V(c, 1). \) Substituting this into the expression for \( dU_2, \) and making use of Eq. (18), we obtain Eq. (22).

When the economy imports good 1, we take the same steps of derivation as above to obtain Eqs. (24) and (25).
References


