Foreign Direct Investment and the Degree of Southern Penetration∗

Chia-Hui Lu†
Institute of Economics, Academia Sinica

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Abstract

I develop a dynamic general equilibrium model of North-South trade to investigate to what extent the South is able to enter international markets through foreign direct investment (FDI) that serves as a channel for technology transfer. In low- and medium-tech industries, the South catches up and competes with the North on the strength of its cost advantage. In response, the North abandons the low-tech industries and instead strengthens its technological lead to escape the South’s competition in medium-tech industries where the North and the South take turns to be the dominant producer in the market. In high-tech industries, the substantial technological gaps deter the South from entering. The measures of the three types of industries are endogenously determined. Countries’ sizes, developmental levels, and openness are linked to the proposed measures of penetration. The effectiveness of FDI and R&D policies is further assessed.

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†Institute of Economics, Academia Sinica, Nankang, Taipei 115, Taiwan. Tel 886-2-2782-2791 ext. 503. E-mail: clu@econ.snica.edu.tw
1 Introduction

Over the last 15 years, the share of foreign direct investment (FDI) being channeled to less developed countries (LDCs) has more than doubled (from 15.3% in 1989 to 30.7% in 2003).\(^1\) Through FDI, industries in developed countries (DCs) migrate to the LDCs to exploit their low production costs, and in the process technologies are also transferred across borders. As a consequence, LDCs have been absorbing new technologies, expanding their product mix, and becoming a far larger presence in international markets. In response to the deepening penetration of world markets by LDCs, industries in DCs have either abandoned certain markets or have intensified their research and development (R&D) in order to further strengthen their technological leads.

This paper develops a dynamic general equilibrium model of North-South trade to analyze the role of international direct investment as a means for the less-developed South to catch up and compete with the North in the world market. By combining the quality-ladder model formalized in Grossman and Helpman (1991a, b) (henceforth, the GH model) with a continuum of industries, each of which has a distinct technological sophistication level, I investigate how far toward the high-tech end of the industry spectrum the South is able to penetrate. It is shown that the North-South technological gaps in high-tech industries are so wide that the South is deterred from entering. However, in low and medium-tech industries, the South is seen to catch up with the North by absorbing production technologies embodied in FDI. As the technological gaps between two countries narrow, the low-wage South is able to enter the international market and compete with the North. This model predicts that in response to competition from the South, the North will abandon the market outright in low-tech industries, but will seek to recapture the market by strengthening its technological lead in medium-tech industries.\(^2\)

This model endogenously determines the ranges of the three types of industries, each

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\(^1\)Source: FDI database, UNCTAD. http://www.unctad.org.

\(^2\)These implications are consistent with the findings of recent empirical studies on North-South FDI and product-cycle trade. For example, Yeaple (2003) shows that the U.S. outward FDI to less-developed countries is concentrated mostly in the low- and medium-skilled industries and least of all in the highly-skilled industries. Schott (2002) examines U.S. product-level trade data during the 1990s and finds that the U.S., facing low-wage competition, has exited the textile goods market in response, but has at the same time engaged in quality-upgrading in the machinery industry.
of which reveals a different degree of penetration. As illustrated in Figure 1, the marginal industry that separates the medium-tech group from the high-tech group denoted by \( \bar{z} \) represents the uppermost industrial boundary that the South has ever been able to penetrate. The marginal industry that separates the low-tech group from the medium-tech group denoted by \( z \) represents the uppermost industrial boundary in which the South’s penetration is permanent—the North has no attempt to recapture the world market after being driven out by the low-cost South. The set of medium-tech industries \( Z_M \equiv (z, \bar{z}) \) measures the industrial range, in which the Southern penetration is temporary. The North and the South take turns to be the dominate producer in the world market, and product cycle trade prevails. A product cycle starts when a Northern firm shifts production to the South via FDI, and it ends when a successful Northern R&D brings production back to the North. From there, FDI intensities \( i_F(z_M) \) determine how quickly, on average, an industry moves to the South. The lower the FDI intensities, the longer it takes for the South to absorb the new technology and replace the North as the dominant producer in the world market. Thus, \( \frac{1}{i_F(z_M)} \) is adopted to reflect the period during which the North dominates the production. Similarly, the R&D intensities \( i_R(z_M) \) determine how long, on average, it will take for the North to upgrade product quality and recapture the market from the South. The lower the Northern R&D intensities, the longer it will take for the North to create a better version that will drive the South out of the market. Therefore, \( \frac{1}{i_R(z_M)} \) is adopted to capture the period during which the South dominates the production in the industry. This paper features four dimensions \( \{\bar{z}, z, \frac{1}{i_F(z_M)}, \frac{1}{i_R(z_M)}\} \) that comprehensively measure the extent of the South’s penetration.

Figure 1: Degrees of Southern Penetration

I relate home and host countries’ sizes, developmental levels, openness as well as the FDI
and R&D subsidy policies to these four measures. It is found that the FDI hosting country’s characteristics and its FDI promoting policies have an inconsistent impact on its ability to penetrate the markets according to measurements based on different perspectives. In other words, improving the South’s ability to penetrate in some respects always comes at the expense of losing in other respects. This implies that the North’s decision regarding which country among the Southern countries with their different developmental levels, sizes, and policies should be chosen to receive FDI plays a critical role in shaping the structure of international production. This study also sheds some light on the impact of FDI on innovation, which serves as the engine of economic growth. Larger country sizes and an improved developmental level, whether in the North or the South, increase effective resource endowments and intensify high-tech innovations. FDI and R&D subsidy policies, that neither augment the labor endowment nor improve labor productivity, alter only investment incentives and may lead to a contraction in high-tech innovation.

In the literature, the chain reaction–FDI affects the home country’s R&D investment, followed by the firms’ investment decisions concerning FDI and R&D determining the extent of the Southern penetration that restructures the international production and trade patterns—is linked for the first time in this paper. Understanding this linkage is essential for economists and policy-makers concerned with the growth and welfare impacts of FDI. FDI liberalization changes the pattern of an industry’s evolution and the rate at which new technologies are developed. As R&D is the engine of economic growth, one can move forward to investigate how the restructuring of the home and host countries’ manufacturing sector affects growth. It is also quite valuable knowing, based on the status quo, whether adjusting their FDI or R&D policies in certain ways would be likely to help governments promote growth. This study also sheds some light on this issue.

The modeling strategies adopted in this paper are to a large extent based on and inspired by existing studies on product-cycle trade. Vernon’s (1966) product cycle hypothesis first links the dynamic patterns of FDI with those of trade. The early wave of the product-cycle literature regards imitation as the only way in which international technology transfer can take place—see Krugman (1979), Jensen and Thursby (1986), Segerstrom, et al. (1990), Grossman, and Helpman (1991), and Glass (1997). The imitation rate is exogenously given
in Krugman (1979) and Segerstrom, et al. (1990), and is chosen by a social planner in Jensen and Thursby (1986). Grossman and Helpman (1991b) endogenize R&D and imitation processes—the driving forces behind the product-cycle trade—and investigate the impacts of country sizes and subsidies on the innovation and imitation rates. In the same spirit as Grossman and Helpman, Glass (1997) incorporates heterogeneous preferences and models a gradual imitation process to determine the extent of Southern penetration from the vertical dimension—that is, she determines how far the South can penetrate the quality ladder via imitation. In contrast to and also to supplement the aforementioned studies, Glass and Saggi (2002), Antrás (2003), Cheng, Qiu, and Tan (2004), and Lu (2004) model endogenous FDI as giving rise to product-cycle trade. However, both Glass and Saggi (2002) and Antrás (2003) consider an economy consisting of a single industry where the discussion is confined to the speed of technology transfer in that industry and is unable to capture the extent of the Southern competition throughout the whole of the manufacturing sector that covers heterogeneous industries. Cheng, Qiu, and Tan (2004) incorporate expatriates, the specific factor for FDI, into a static continuum Ricardian model to identify the industrial range of Southern penetration. My results complement to their. Lu (2004a) serves as a baseline model for this study.

This paper is organized as follows. Section 2 describes the model setup regarding consumers’ preferences, the firms’ production, R&D and FDI technologies, as well as the firms’ pricing strategies under Bertrand competition. Section 3 summarizes the equilibrium market structures. Section 4 describes the steady state equilibrium. In Section 5, the comparative static analyses are looked at in some detail to examine the implications of the countries’ characteristics, as well as the FDI and R&D subsidies. Section 6 concludes.

2 The Model

Built on the GH model, R&D is product-specific, and it moves a product up its quality ladder one step at a time. Inspired by Taylor (1993),3 I assume that the extent of quality improvement with each step up the product ladder is higher in some industries than in others. I consider a continuum of industries, \( z \in [0,1] \). The inventive step \( \lambda(z) \) shows the rate of

3 Taylor (1993) introduces heterogeneous R&D and production technologies across industries.
quality improvement for each innovation with which each industry is associated. \( \lambda(z) \) is exogenously given, \( \lambda'(z) > 0 \), and \( \lambda(0) = 1 \). Within each industry, there is a continuum of varieties, \( y \in [0,1] \). The product space is defined on a unit square, \( (z, y) \in [0,1] \times [0,1] \). Different versions of each product are indexed by \( j = 0, 1, 2, ..., \), which indicates a particular position along the quality ladder (product line). The quality of good \((z, y, j)\) is denoted by \( q(z, y, j) = \lambda(z)^j \). A successful R&D brings a new version of a product to light, and the blueprint is protected by a worldwide infinitely lived patent.

Labor is the only factor that is not mobile across borders. The North is endowed with skilled labor \( L \) that is capable of conducting R&D to bring forth new versions of each product and can engaged in production. The South is endowed with unskilled labor \( L^* \) (hereafter an asterisk is adopted to denote Southern variables). Southern labor is employed in two activities—technology transfer and production. In this paper, FDI refers to the process of technology transfer and is distinguished from production. It takes the form of building up production sites in the South and modifying operating procedures to suit Southern conditions that requires Southern labor input and is characterized by uncertainty. I assume that the cost of transferring the newly invented technology is prohibitively high and only the standardized technologies can be transferred to and implemented in the South. A production technology is said to be standardized if it is leapfrogged by that of a newly-invented version of a product.\(^5\)

After FDI is complete, Southern labor is also used in manufacturing. Northern firms refer to those producing in the North, and multinational entrepreneurs (MNEs) refer to those succeeding in FDI and producing in the South. An MNE hires Southern workers, retains full control over production, and remits all profits back to the North. Let \( J(z, y) \) and \( J^*(z, y) \) denote the versions of product \((z, y)\) produced by the Northern (industry) leader and the MNE leader, respectively, \( J(z, y) - J^*(z, y) \geq 1 \).

Supposing that there is no trade friction, the cheaper labor costs in the South is the only incentive for a firm to undertake costly and risky FDI. This model features endoge-

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\(^4\)For example, there are the textile, office machinery, and road vehicle industries. Within the road vehicle industry, there are different varieties of products, such as sports cars, compact cars, vans, etc.. As I will show later that each product \((z, y)\) follows a stochastic evolution process in the dynamic equilibrium. By assuming a continuum of varieties with homogeneous inventive steps within each industry, I am able to apply the law of large numbers such that the aggregate evolution rates in an industry are non-random and constant.

\(^5\)This assumption keeps the technology of the leading-edge version from being transferred to the South. It is necessary to give rise to a product cycle similar to Vernon’s description.
nous cost-seeking FDI and quality-upgrading R&D process that potentially improve firms competitiveness. The R&D and FDI ventures are financed by Northern Savings.\(^6\)

### 2.1 The Consumer’s Problem

Consumers in the North and South face the same prices and share an identical, time separable, and homothetic utility function. They can save and rent only in the domestic financial market. The representative Northern consumer’s problem is described first. The expected discounted value of the infinite lifetime utility \(U\) is:

\[
U = E_0 \left[ \int_0^\infty e^{-\rho t} \log u(t) \, dt \right],
\]

(1)

where \(E_0\) is the expectation conditional on the information available at time zero, \(\rho\) is the rate of time preference (constant and positive), and \(\log u(t)\) is the utility flow at time \(t\). The instantaneous utility function is defined as:

\[
\log u(t) \equiv \int_0^1 \int_0^1 \log \left[ \sum_{j=0}^{J(z,y,t)} q(z,y,j)d(z,y,j,t) \right] \, dy \, dz,
\]

(2)

where \(q(z,y,j)\) is the quality of good \((z,y,j)\), \(d(z,y,j,t)\) denotes the demand at time \(t\), and \(J(z,y,t)\) represents the leading-edge version of product \((z,y)\) at time \(t\). The representative consumer maximizes utility, (1), subject to an intertemporal budget constraint:

\[
\int_0^\infty e^{-R(t)} E(t) \, dt \leq A(0),
\]

(3)

where \(A(0)\) is the present value of the stream of labor income plus the value of asset holdings at time 0, and \(R(t)\) denotes the cumulative interest factor up to time \(t\). Let \(r(t)\) be the instantaneous interest rate. \(R(t) = \int_0^t r(s)ds\), and \(\dot{R}(t) = r(t)\). The consumer’s expenditure \(E(t)\) can be expressed as:

\[
E(t) = \int_0^1 \int_0^1 \left[ \sum_{j=0}^{J(z,y,t)} p(z,y,j,t)d(z,y,j,t) \right] \, dy \, dz,
\]

(4)

where \(p(z,y,j,t)\) denotes the price of good \((z,y,j)\) at time \(t\).

A utility-maximizing consumer first chooses to spend equal shares of total expenditure on each product, and then, within the same product line, only the version that charges the lowest

\(^6\)Segerstrom et al. (1990). also make this assumption.
quality-adjusted price is purchased. This principle applies to the Southern consumers and the aggregate economy. Let \( j(z, y, t) \) denote the version of product \((z, y)\) with the lowest quality-adjusted price at time \(t\),

\[
j(z, y, t) = \underset{j=1, \ldots, J(z, y, t)}{\text{argmin}} \frac{p(z, y, j, t)}{q(z, y, j)}.\]

The world aggregate demand functions are shown as follows:

\[
d^W(t, z, y, j, t) = \begin{cases} 
E^W(t)/p(z, y, j, t), & \text{if } j = j(z, y, t) \\
0, & \text{otherwise}
\end{cases}, \quad (5)
\]

where \( E^W(t) \) is the world aggregate expenditure. The Northern consumer’s optimal expenditure profile \( E(t) \) is dictated by the following differential equation:

\[
\dot{E}(t)/E(t) = \dot{R}(t) - \rho = r(t) - \rho. \quad (6)
\]

The Southern consumer, however, can not buy shares of either Northern firms or MNEs and spends all wage income on consumption. Let \( w^*(t) \) denote the Southern wage. I normalize \( w^*(t) = 1 \forall t \), and thus \( E^W(t) = L \cdot E(t) + L^* \).

### 2.2 The Firm’s Problem

Notice that \( J(z, y) \) and \( J^*(z, y) \) denote the quality levels of the industry leader and the MNE leader, respectively.\(^7\) By an appropriate choice of units, I set the unit labor requirements of Northern and Southern labor per unit of output to one, i.e. \( a(z, y, j) = 1, \forall j \leq J(z, y) \), and \( a^*(z, y, j) = 1, \forall j \leq J^*(z, y) \) that makes the marginal cost of every good equal to the wage rate of the country where the good is produced.

Firms devote their efforts to R&D. The success of R&D follows a Poisson process, with the arrival rate depending on the current R&D level. A firm engaging in R&D at intensity \( i_R(z, y) \) for a length of time \( dt \) has probability \( i_R(z, y)dt \) of success in developing the next version of product \((z, y)\). Let \( a_R(z, y) = a_R \) denote the unit Northern labor requirement per unit of R&D intensity and \( V_R(z, y) \) denote the present market value of each blueprint. Then, R&D at intensity \( i_R(z, y) \) requires \( a_R \cdot i_R(z, y)dt \) units of Northern labor that creates an expected value of \( V_R(z, y)i_R(z, y)dt \) for the investor. Investors act to maximize expected value, i.e. \( \max_{i_R(z, y) \geq 0} V_R(z, y) - \omega a_R \cdot i_R(z, y)dt \), where \( \omega \) denotes the relative Northern wage. An equilibrium with positive and finite R&D investment implies that \( V^R(z, y) = \omega a_R \), and

\(^7\)Here and henceforth I suppress the time domain when no confusion arises from doing so.
that \( i_R(z, y) = 0 \), if \( V^R(z, y) < \omega a_R \). Since the formation of R&D technology implies constant returns to scale in research efforts together with the assumption of free entry into R&D, investors are prevented from earning excess returns, i.e. \( V^R(z, y) = \omega a_R \), and the individual investor is indifferent to the levels of investment intensity.

Analogous to the R&D technology, the success of FDI also follows a Poisson process. Let \( a^*_F(z, y, j) \) denote the Southern unit labor requirement per unit of FDI intensity. \( a^*_F(z, y, j) = a^*_F \) if \( j < J(z, y) \), and \( a^*_F(z, y, j) = \infty \) if \( j = J(z, y) \). Let \( V^F(z, y, j) \) be the prize for an FDI success of good \((z, y, j)\). An equilibrium with positive and finite FDI investment implies that \( V^F(z, y, j) = a^*_F \). I further assume that the cost of inventing a new blueprint is higher than that of transferring an existing blueprint \( (a^*_R < a^*_F) \) that ensures that, in some industries, followers undertake FDI in an attempt to replace the industry leader.

### 2.3 The Bertrand Pricing Game

Firms owning patents \( j \in \{1, 2, \ldots, J(z, y)\} \) compete as price-setting oligopolists. The one who charges the lowest quality-adjusted price captures the whole market. If all firms are producing in the North, they face the same production cost \( \omega \), and the industry leader charging a price a shade below \( \lambda(z) \omega \) takes the whole market and earns positive profits. The size of the inventive step \( \lambda(z) \) shows the price premium that reflects the consumer’s willingness to pay for a superior quality level. An industry leader’s price markup is positively correlated with \( \lambda(z) \) which suggests that the quality-advantage stemming from R&D is increasing with the industry’s technology sophistication level. In the case where some of the followers have transferred their technologies to the South, faced with the same producing cost \( w^* = 1 \), the MNE leader \( J^*(z, y) \) has the highest ability to undercut the industry leader and sets a price slightly below \( \frac{\omega}{\lambda(z)^{J(z,y)-J^*(z,y)}} \), where \( \lambda(z)^{J(z,y)-J^*(z,y)} \) measures the quality gaps between the industry leader and the MNE leader. The limit price set by \( J^*(z, y) \) to undercut the industry leader cannot be smaller than the production cost and is negatively correlated with \( \lambda(z) \), which suggests that the cost-advantage stemming from FDI is more significant in the relatively low-tech industries than in the high-tech industries. In sum, the relative “importance” of a firm’s quality advantage and a firm’s cost advantage in the price-setting competition is increasing in \( z \).
3 Market Structure

To keep the evolution of each product tractable, I follow Grossman and Helpman (1991a) and Glass (1997)’s assumption that the industry leader has no advantage in conducting R&D and faces the same R&D costs \( (a_R) \) as the followers do. In the case where the industry leader currently dominates the production, the producing leader gains only incremental profits if it succeeds in R&D and becomes a two-step-head industry leader in that its profits are strictly less than the profits gained by a non-producing firm if it succeed in R&D. Since the supply of non-producers willing to invest in R&D at the equilibrium interest rate is perfectly elastic, it follows that the rewards of R&D undertaken by the producing leader do not justify the research cost, and the producing leader does not engage in R&D.\(^8\)

In the absence of FDI moving production to the South, all goods are produced in the North. The evolution of each product is characterized by a process in which the product climbs stochastically up its quality ladder. The industry leader, upon attaining the most superior quality, wins the pricing game and gains temporary monopoly profits until a better version in the product line is invented by non-producing followers.

When the North lifts the ban against outward FDI, followers have one more dimension to improve their competitive edge—the moving-up strategy improves their quality advantages and the moving-out strategy improves their cost advantages.\(^9\) Any investment project is financed in the capital market; only the most profitable projects are funded. Given that the industry leader currently captures the world market, followers choose between two means—R&D and FDI—in an attempt to drive off the industry leader. The choice made by the nearest (second-to-top) follower (henceforth \( NF \)) sufficiently reveals the strategy that would be implemented by other followers. The reason is quiet intuitive. Notice that all followers face the same cost per unit of FDI intensity \( (a^*_F) \). A successful FDI venture, however, brings higher rewards to \( NF \) than it brings to other followers with more obsolete technologies. Since the supply of \( NF \) willing to invest in FDI at the equilibrium interest rate is perfectly elastic, it follows the costs of capital in equilibrium will be such that followers have blueprints older than \( NF \) find that FDI projects to have negative expected present value. Therefore, within a product line,

\(^8\)See Grossman and Helpman (1997, p.93) for the detailed proof.
\(^9\)I assume that there have been at least two versions in each product line prior to time 0, i.e. \( J(z, y, 0) \geq 2 \).
followers apart from NF do not engage in FDI. Moreover, all followers face the same costs per unit of R&D intensity \((a_R \cdot \omega)\). A successful R&D brings the same profits to followers regardless of the vintage of their current technologies. If R&D is more profitable than FDI for NF, it must be true that other followers will also participate in the new round of the patent race. Instead, if R&D is less profitable than FDI from NF’s point of view, NF will implement the moving-out strategy but not the moving-up strategy, and the R&D ventures undertaken by other followers will not be funded in the capital market at the equilibrium interest rate. It is prohibitively expensive for the industry leader to engage in FDI. Therefore, NF’s choice between the moving-out and moving-up strategies determines the evolution direction of each product.

To make a costly and risky investment venture profitable, regardless of whether it involves FDI or R&D, two things must be true. First, a successful investment must be able to outrace the producing firm in the pricing game, and to earn positive profits. Second, the cost of this venture must not be so high as to make it too expensive.

**Assumption 1:** \(\lambda(0) < \omega < \lambda(1).\)

Since \(\lambda(z)\) is continuous and increasing in \(z\), Assumption 1 ensures that \(z’\) exists and is unique, where \(\lambda(z’) = \omega,\) and \(z’ \in (0,1).\) Notice that \(\omega\) reflects the cost advantage accruing to MNEs. In any industry \(z \in [z’, 1], \omega \leq \lambda(z),\) and the improved competitiveness via the costly FDI venture is not sufficient to beat the industry leader in the Bertrand game (the industry leader can set a price equal to \(\lambda(z)\) to undercut the MNE leader whose technology is one step below the leader, and still earn positive profits). Thus NF will implement the moving-up strategy instead of the moving-out strategy. This, however, does not imply that NF in industry \(z \in [0, z’)\) with \(\omega > \lambda(z),\) will necessarily prefer the moving-out strategy to the moving-up strategy. Success in both forms of venture will enable the current NF to drive out the current industry leader from the market. Therefore, NF will then have to compare the rewards and the costs of these two actions and then choose the most profitable one. In industry \(z = 0,\) with \(\lambda(0) = 1,\) the success of the costly R&D does not improve product quality, and NF will undertake the FDI venture for sure. By the property of continuity, there
exists an industry $\bar{z} \in [0, z')$ in which $NF$ is indifferent to the moving-up and moving-out strategies. Since, as shown earlier, the relative importance of a firm’s quality advantage to a firm’s cost advantage in the price-setting competition is increasing in $z$, the moving-up strategy is strictly preferred in industry $z \in (\bar{z}, 1]$, and is strictly dominated by the moving-out strategy in industry $z \in [0, \bar{z})$.

**Proposition 1.** Let $\bar{Z}(z) \equiv \frac{a^*}{a_R} (1 - \frac{1}{\lambda(z)}) + \lambda(z)$ where $\bar{z}$ satisfies $\bar{Z}(\bar{z}) = \omega$. In industry $\bar{z}$, $NF$ is indifferent to the moving-up and the moving out strategies given that the industry leader is the extant producer. (Proof in appendix).

In the situation where $NF$ in industry $z \in [0, \bar{z})$ succeeds in the FDI venture and dominates the production in the South, non-producing firms including the industry leader and other followers have no other choice than to select the moving-up strategy to enhance their competitiveness. However, in industries with $\lambda(z)^2 < \omega$, the inventive steps are relatively small, and the competitiveness gained from R&D accruing to the non-producing firms is not sufficient to drive out the current MNE leader (the MNE leader can set a price equal to $\frac{\omega}{\lambda(z)^2}$ to undercut the new industry leader who has two-step ahead technology, and still earn positive profits). Let $Z(z) \equiv \lambda(z)^2$ and $\bar{Z}(\bar{z}) = \omega$. Non-producing firms have no incentive to undertake R&D to recapture the market in industry $z \in [0, \bar{z})$ but engage in R&D as they seek to recapture the market from the MNE leader in industry $z \in (\bar{z}, \bar{z})$.

**Lemma 1** Suppose Assumption 1 holds. Let $\bar{Z}(z) \equiv \frac{a^*}{a_R} (1 - \frac{1}{\lambda(z)}) + \lambda(z)$, and $Z(z) \equiv \lambda(z)^2$. There exists $0 < \bar{z} < z < 1$, where $\bar{Z}(\bar{z}) = Z(\bar{z}) = \omega$. (Proof in appendix).

Figure 2 shows that two schedules $\bar{Z}(z)$ and $Z(z)$ intersect with $\omega$ at $\bar{z}$ and $\bar{z}$, respectively, that together divide the industry spectrum into three groups—from the low-tech end to the high-tech end, these being in order, $Z_L \equiv [0, \bar{z})$ denoting the set of low-tech industries, $Z_M \equiv (\bar{z}, \bar{z})$ denoting the set of medium-tech industries, and $Z_H \equiv [\bar{z}, 1]$ denoting the set of hi-tech industries. For high-tech industries, $NF$ (as in the cases of other followers) undertakes R&D but not FDI in an attempt to outrace the current leader and recapture the world market.
For any industry below \( \bar{z} \), that is, for the medium-tech and the low-tech industries, FDI is more profitable than R&D; NF undertakes FDI but not R&D in an attempt to outtrace the current leader and recapture the world market. \( \omega \) and \( Z \) intersect at \( \bar{z} \). This denotes the cut-off industry below which, when the product is produced in the South, the R&D ventures undertaken by non-producing firms yield negative profits; outsiders simply exit the market. For any industry above \( \bar{z} \), i.e. for any medium-tech industry, non-producing firms engage in R&D, and race to invent the next generation.

The three industry types reveal different extents of Southern penetration. The Bertrand equilibrium price of each product, and the firms’ optimal R&D and FDI intensities are summarized in the following subsections.

**Hi-tech Industries**

For any *hi-tech* industry \( z_H \in Z_H \), the industry leader undercuts NF by charging price \( \hat{P}(z_H) = \lambda(z_H)\omega \), and realizes sales \( \hat{X}(z_H) = \frac{E(0)}{\lambda(z_H)\omega} \), yielding profits \( \hat{\pi}(z_H) = \left(1 - \frac{1}{\lambda(z_H)}\right)E_W \).

The premium charged, \( \lambda(z_H) \), reflects the consumers’ willingness to pay for a higher-quality version than NF could produce, and reflects the leader’s ability to mark-up price in the Bertrand competition. Since \( \bar{Z}(z_H) > \omega \), all followers, the non-producing firms, initiate a new patent race in an attempt to outtrace the current leader. The free entry in the R&D implies that the expected net profit from R&D equals zero in equilibrium, \( V_R(z_H) = \omega a_R \); the no arbitrage condition in the financial market requires that the stock of any *hi-tech product* yields a certain return equaling the time preference rate, i.e. \( \frac{\pi(z_H)}{V_R(z_H)} - i_R(z_H) = \rho \). The hi-tech R&D intensities are shown in terms of world expenditure, and the North-South relative wage, as follows:

\[ i_R(z_H) = \frac{1 - \frac{1}{\lambda(z_H)}}{\omega a_R} E_W - \rho. \] (7)

Ongoing R&D continually moves each high-tech product up its quality ladder; the *hi-tech industries* never leave the North, and the South never penetrates.
Low-tech Industries

In any low-tech industry, \( z_L \in Z_L \), the inventive steps are very small, and \( \omega > \lambda(z_L) > \lambda(z_L) \). The cost advantage dominates both the one-step quality advantage and the two-step quality advantage. NF undertakes FDI in an attempt to take advantage of the lower production cost in the South, and to outrace the industry leader in the pricing game. When NF succeeds in the FDI venture, it becomes an MNE leader, and undercuts the industry leader by charging price \( \tilde{P}(z_L) = \frac{\omega}{\lambda(z_L)} \), and makes sales \( \tilde{X}(z_L) = \frac{\lambda(z_L)E^W}{\omega} \), yielding positive profits \( \tilde{\pi}(z_L) = \left(1 - \frac{\lambda(z_L)}{\omega}\right)E^W \). The discount charged reflects consumers’ willingness to pay for a lower-quality version than the industry leader could produce. Since the two-step-ahead quality advantage \( \lambda(z_L)^2 \) resulted from further innovation is still dominated by the MNE leader’s cost advantage, the R&D venture aimed to bring the next version to light is not profitable. No R&D ventures are undertaken at the the equilibrium interest rate, and \( i_R(z_L) = 0 \). The MNE leader enjoys permanent monopoly profits. Each low-tech product permanently stagnates in the South once production is moved to the South via FDI.

Medium-tech Industries

In any medium-tech industry, \( z_M \in Z_M \), each product repeatedly experiences a two-stage life cycle. A product cycle starts when a Northern firm shifts production to the South via FDI, and it ends when a successful Northern R&D brings production back to the North. Let \( Z_{M1} \) and \( Z_{M2} \) denote the sets of products shifting in Stage 1, and Stage 2, respectively, where \( Z_{M1} \cup Z_{M2} = Z_M \). In Stage 1, the cost advantage dominates the one-step quality advantage, \( \omega > \lambda(z_{M1}) \), the MNE leader undercuts the industry leader by charging price \( \tilde{P}(z_{M1}) = \frac{\omega}{\lambda(z_{M1})} \), and realizes sales \( \tilde{X}(z_{M1}) = \frac{\lambda(z_{M1})E^W}{\omega} \), yielding positive profits \( \tilde{\pi}(z_{M1}) = \left(1 - \frac{\lambda(z_{M1})}{\omega}\right)E^W \). Since \( \lambda(z_M)^2 > \omega \), the two-step-ahead quality advantage \( \lambda(z_L)^2 \) resulted from further innovation will dominate the MNE leader’s cost advantage. Non-producing firms race to bring a higher version to light in an attempt to recapture the market. The no arbitrage condition, \( \frac{\pi(z_{M1})}{V_F(z_{M1})} - i_R(z_{M1}) = \rho \), together with the free entry in the FDI, i.e. \( V_F(z_{M1}) = a^*_F \) solve the equilibrium medium-tech R&D intensities:

\[
i_R(z_{M1}) = \left(1 - \frac{\lambda(z_{M1})}{\omega}\right)E^W - \rho. \quad (8)
\]
The MNE leader produced in the South gains temporary monopoly profits until a higher-quality version is brought to light by non-producing firms, and the duration of Southern dominance ends. The product cycle then transforms into Stage 2 that is denoted by $Z_{M2}$. In Stage 2, the industry leader charges price $\tilde{P}(z_{M2}) = \lambda(z_{M2})^2$, and realizes sales $\tilde{X}(z_{M2}) = \frac{E^W}{\lambda(z_{M2})^2}$, yielding profits $\tilde{\pi}(z_{M2}) = \left(1 - \frac{\omega}{\lambda(z_{M2})^2}\right)E^W$. The premium charged reflects consumers’ willingness to pay for the superior quality of the product than the MNE leader could produce.

The no arbitrage condition implies that $\pi(z_{M2}) = V_R(z_{M2}) - i_F(z_{M2}) = \rho$, and free entry in the R&D implies that $V_R(z_{M2}) = \omega a_R$. The equilibrium FDI intensities are shown:

$$i_F(z_{M2}) = \left(1 - \frac{\omega}{\lambda(z_{M2})^2}\right)\frac{E^W}{\omega a_R} - \rho. \quad (9)$$

The current industry leader produced in the North captures the world market until $NF$ (the previous industry leader) succeeds in FDI. Once again, the South penetrates this industry and becomes the dominator in the world market. The product shifts to Stage 1, and renews its life cycle.

The North and the South take turns to be the dominate producer in the world market. Two variables $i_F(z_M)$ and $i_R(z_M)$ are used to measure how fast the production of industry $z_M$ is moved to the South and moved back to the North. An FDI venture at intensity $i_F(z_M)$ for a time length of $dt$ has a probability $i_F(z_M)dt$ of succeeding in transferring the technology to the South. The inverse of the probability denotes the expected waiting time for the event (the success of FDI) to occur.$^{10}$ The higher the FDI intensities are, the sooner, on average, the South absorbs the new technology, and replaces the industry leaders. Therefore, FDI intensity $\frac{1}{i_F(z_M)}$ is adopted to capture the duration when the North dominate the production. Similarly, the higher the Northern R&D intensities are, the sooner the North recaptures the market from the South, and the shorter the Southern dominance period is. $\frac{1}{i_R(z_M)}$ is thus adopted to measure the time length during which the South dominates the production in a medium-tech industry.

$^{10}$One of the notable properties of the Poisson process.
4 The Steady State

I consider the balanced growth steady state in which the instantaneous interest rate is constant and equals the time preference rate, \( r(t) = r = \rho \). Allocations of resources to the various activities remain fixed through time, which requires that the rate of each medium-tech industry moving out equals the rate of moving up, and the measures of each industry type are constant.

Definition 1: Let \( n_H, n_L, n_M, n_{M1}, \) and \( n_{M2} \) be the measures of \( Z_H, Z_L, Z_M, Z_{M1}, \) and \( Z_{M2} \), respectively.

By definition 1, \( n_H = 1 - \bar{z}, n_L = \bar{z}, \) and \( n_M = \bar{z} - \bar{z} \) are constant as \( E^W \) and \( \omega \) are constant in the steady state. Within each medium-tech industry, the measures of products produced in the South and the North are also constant if:

\[
i_R(z_{M1}) \cdot n_{M1}(z_M) = i_F(z_{M2})n_{M2}(z_M).
\]

The right-hand side denotes the aggregate products moving to the South; the left-hand side denotes the aggregate products moving back to the North. The net product flow in each medium-tech industry is zero, and \( n_{M1} = \int n_{M1}(z_M)dz_M, \) and \( n_{M2} = \bar{z} - \bar{z} - n_{M1} \) are constant.

The steady-state conditions are simplified into the forms of the Southern and the Northern labor clearing conditions as shown in (11) and (12):

\[
L^* = \int_0^{\bar{z}(\omega)} \frac{E^W}{\lambda(z_L)}dz_L + \int_{\bar{z}(\omega)}^{\bar{z}(\omega)} \frac{E^W}{\lambda(z_M)}n_{M1}(z_M)dz_M + \int_{\bar{z}(\omega)}^{\bar{z}(\omega)} i_F(z_M)a_F^*(1 - n_{M1}(z_M))dz_M.
\]

The left-hand side represents the aggregate Southern labor supply. On the right-hand side, the first brace denotes the aggregate Southern labor employed in production in the low-tech and medium-tech industries; the second brace represents the aggregate Southern labor employed in transferring Northern technology in medium-tech industries.

\[
L = \int_{\bar{z}(\omega)}^{\bar{z}(\omega)} \frac{E^W}{\lambda(z_M)^2}(1 - n_{M1}(z_M))dz_M + \int_{\bar{z}(\omega)}^{1} \frac{E^W}{\lambda(z_H)\omega}dz_H
\]

Labor Employed In Production
\[
\int_{z(\omega)} z(\omega) i_R(z_1) a_R n_{M_1}(z_M) dz_M + \int_{z(\omega)} z(\omega) i_R(z_H) a_R dz_H.
\]

Labor Employed In R&D

The left-hand side denotes the Northern labor supply. The right-hand side represents the aggregate Northern labor employed in manufacturing and in innovation. Let SS and NN schedules depict the combinations of \(E^W\) and \(\omega\) that satisfy (11) and (12) that can be further simplified as:

SS: \[
L^* = \frac{E^W}{\omega} \int_0^{z(\omega)} \lambda(z_L) dz + (E^W - \rho a_P^*) \cdot n_{P_1}(\omega, E^W),
\]

and

NN: \[
L = \left(\frac{E^W}{\omega} - \rho a_R\right) \cdot (1 - z(\omega) - n_{P_1}(\omega, E^W)),
\]

where \(n_{P_1}(\omega, E^W)\) is the aggregate measure of the medium-tech industries produced in the South. The NN schedule is upward sloping. Under Assumption 2,

Assumption 2: \(\frac{1}{2\lambda(z)} < \lambda'(z) < \frac{1}{2}, \forall z \in (0, 1)\)

the SS schedule is downward sloping that ensures the uniqueness of the steady state as illustrated in Figure 3. The intersection of NN and SS that is labeled A represents the steady state world expenditure \(E^W\) and relative wage rate \(\omega\) which together with \(\bar{Z}(z)\) and \(\bar{z}\) pin down the margin industries of product cycle industries, \(z\) and \(\bar{z}\). Iteratively, the hi-tech innovation rate (7) the medium-tech moving-up rate (8) and the medium-tech moving-out rate (9) are solved, which provide us sufficient information to assess the degree of Southern penetration from different perspectives.

5 Comparative Statics Analysis

The paper features four measures \(\{\bar{z}, \bar{z}, \frac{1}{i_P(z_M)}, \frac{1}{i_R(z_M)}\}\) that comprehensively capture the extent of the South’s penetration.

In this section, I analyze how the initial steady state equilibrium, point A, is disturbed by expansion in the Northern labor force \(\hat{L} > 0\), by expansion in the Southern labor force \(\hat{L}^* > 0\), by an R&D subsidy \(s_R > 0\), and by an FDI subsidy \(s_F > 0\). One interpretation of
\( \hat{L} > 0 \) is that the North improves its effective labor force through greater education. I discuss which Northern policy—improving domestic labor productivity in production and innovation or altering the incentive for R&D—is more effective in deterring Southern competition. Similarly, I compare which Southern policy—improving domestic labor productivity in production and absorbing new technology or altering the incentive for FDI—is more effective in strengthening Southern competition. The expansion of the Southern size can also be interpreted as a result of another Southern country which is initially-closed and now integrates into the world economy via trade and FDI liberalization. This exercise provides predictions about how the structure of international production is affected as the less developed Southern countries (for example, China) increase their involvement in the world economy. It also provides predictions about among a group of potential FDI hosting countries, how the degree of Southern penetration varies with their labor endowments or FDI policies. This study alone considers the heterogeneous industry inventive steps and have much riche implications than the single-industry model.

**The Northern Labor Resource Expansion**

The \( NN \) schedule captures the Northern labor market clearing condition, and it shifts to the right with \( \hat{L} > 0 \). The \( SS \) schedule remains in place. Point \( B \) in Figure 3 represents the new steady-state equilibrium in which the relative wage has decreased while the world expenditure has increased.

The Northern expansion weakens the MNEs' cost advantages. In industry \( \bar{z} \), the \textit{moving-up} strategy now becomes more profitable than the \textit{moving-out} strategy. Therefore, the set of hi-tech industries expands, \( \frac{dz}{dL} < 0 \). In industry \( z \), initially, the North exits the market when the South enters, now the North has incentive to undertake the next round of R&D, since the two-step quality advantage dominates the Southern cost advantage. The measure of low-tech industries shrinks, \( \frac{dz}{dx} < 0 \). The range of medium-tech industries shrinks and is pushed toward the low-tech end of the industry spectrum.

In the new steady state, the weakened cost advantage deters competition from the South in terms of the shrinkage in both the temporary and permanent Southern penetration ranges. Within each medium-tech industry, the speed of industry moving out, however, is increased.
The intuition behind this is that the increased world expenditure increases sales, and the decreased relative Northern wage decreases the Northern production cost, and these factors together increase the instantaneous profits of the active industry leader; the profit rate of the active industry leader is higher than the real effective interest rate (the real interest rate plus the risk of losing earnings). The no arbitrage condition implies that an upward adjustment of the leader’s risk of losing its monopoly position is needed in the new steady state, and thus the FDI intensities undertaken by the non-producing firms in an attempt to drive out the industry leader from the market increase as well. The impact on the duration of the Southern dominance further hinges on the magnitudes of the industry inventive steps. Specifically, the period during which the South dominates the production in the market is prolonged if and only if the inventively step is relatively big, i.e. $\lambda(z_M) > \Lambda_L$, where $\Lambda_L \equiv \frac{\omega}{1 - \frac{\omega}{\omega dL dL}}. \text{From (7), it is trivial that the high-tech innovation is intensified by the increased profit rates. Notice that if } \lambda(\bar{z}) < \Lambda_L, \text{ the innovation rate in each medium-tech industry increases. Together with the reinforced high-tech innovation, the long-run growth rate is boosted.}^{11} \text{ Proposition 2 summarizes the impacts of the Northern size expansion:}

**Proposition 2:** The expansion of the North’s size (i) weakens the Southern cost advantage, $\frac{d\omega}{dL} < 0$, (ii) shrinks the set of low-tech industries, $\frac{dz}{dL} < 0$, (iii) expands the set of high-tech industries, $\frac{dz}{dL} < 0$, and (iv) raises high-tech innovation, $\frac{dH(z_M)}{dL} > 0$. (v) The set of medium-tech industries shrinks, $\frac{d(z-\bar{z})}{dL} < 0$, and is pushed toward the low-tech end. For each medium-tech industry $z_P$, (vi) it accelerates industry moving out, $\frac{di_F(z_P)}{dL} > 0$; (vii) it shortens the South’s dominant duration, $\frac{di_R(z_P)}{dL} < 0$, if and only if $\lambda(z_M) < \Lambda_L$. (Proof in Appendix).

**The Southern Labor Resource Expansion**

The SS schedule captures the Southern labor market clearing condition, and it shifts to the right with $\hat{L} > 0$. The NN schedule remains in place. Point C in Figure 4 represents the new steady-state equilibrium in which the Northern relative wage increases, and world expenditure

$^{11}$Similar to G&H(1991a), innovation propels growth, and the growth rate is defined as the rate of increase in the consumption index.
also increases.

The Southern labor resource expansion improves the cost advantages of MNEs. In industry \( \bar{z} \), the \textit{moving-out} strategy now is more profitable than the \textit{moving-up} strategy. The set of the high-tech industries shrinks, \( \frac{dz}{dL^*} > 0 \). In industry \( z + \epsilon \), \( \epsilon > 0 \), in which, initially, the North launches a new round of R&D investment immediately after the South takes over the world market, but now the North exits the market in response, since the two-step quality lead is not sufficient to make the new leader earn a positive profit as it competes with the MNE leader. The set of the low-tech industries expands, \( \frac{dz}{dL^*} > 0 \). The range of the medium-tech industries also expands, and is pushed toward the high-tech end of the industry spectrum.

In the new steady state, the improved Southern cost advantage strengthens the South’s competition in terms of expanding both the temporary and permanent penetration ranges. Within each medium-tech industry, the duration of the South dominance, however, is shortened. By (8), the increased world expenditure and the lowered Southern relative wage together increase the instantaneous profits of the active MNE. The returns of the stock of MNE leader are now higher than the real effective interest rate. By the no arbitrage condition, an adjustment of a higher risk of losing monopoly position faced by the MNE leader is needed, and thus the medium-tech innovation rates increase in response.

The speed of industry moving out increases if only if \( \lambda(z_M) > \Lambda_{L^*} \), where \( \Lambda_{L^*} \equiv \sqrt{\frac{\omega}{1 - \frac{\omega^2}{\bar{z}}}} \). The high-tech innovation is intensified by the expanded market size, even though the Northern relative wage is increased. Proposition 3 summarizes the long-run impacts of Southern size expansion.

\textbf{Proposition 3:} The expansion of the South’s size, \( \hat{L}^* > 0 \), (i) improves the South’s cost advantage, \( \frac{d\omega}{dL^*} > 0 \), (ii) expands the set of low-tech industries, \( \frac{dz}{dL^*} > 0 \), (iii) shrinks the set of high-tech industries, \( \frac{d\bar{z}}{dL^*} > 0 \), and (iv) raises the high-tech innovation rates \( \frac{di_R(zH)}{dL^*} > 0 \). (v) The set of medium-tech industries expands \( \frac{d\bar{z} - z}{dL^*} > 0 \), and is pushed toward the high-tech end. For each medium-tech industry \( z_P \), (vi) it shortens the duration that the South dominates the production, \( \frac{d\left(1i_R(z_P)\right)}{dL^*} < 0 \); (vii) and it slows down the speed of industry moving out, \( \frac{di_F(z_P)}{dL^*} < 0 \), if and only if \( \lambda(z_M) < \Lambda_{L^*} \). (Proof in Appendix).
Although the innovation rates in the high-tech and medium-tech industries are reinforced, the growth rate is not necessarily boosted due to the shrinkage of high-tech industries, and the expansion of low-tech industries. This experiment suggests that the South’s size is positively correlated with range that it is able to penetrate and negatively correlated with the period in which the South maintains its dominant position. Among Southern countries that are identical except for their sizes, the bigger Southern country will penetrate farther, but on average, will dominate for a shorter period of time in each of the medium-tech industries.

**FDI Subsidy**

I discuss the effectiveness of the FDI subsidy in an attempt to promote technology transfer here. Suppose that the Southern government bears a fraction $s_F$ of the cost of FDI uniformly across industries, which is financed by lump-sum taxation; the unit labor requirement per unit of FDI intensity becomes $(1 - s_F)a_F^*$, with $s_F > 0$. A uniform FDI subsidy alters the costs of FDI, and shifts the $\bar{Z}$ schedule downward to $\bar{Z}(s_F > 0)$ as shown in Figure 5.

At the initial steady state wage rates, in industry $z$, the *moving-out* strategy is now more profitable than the *moving-up* strategy. Non-producing firms stop engaging in the R&D ventures, and get involved in the FDI ventures in an attempt to outrace the current leader. The upper bound of the medium-tech industries expands toward the high-tech end, from $\bar{z}$ to $\bar{z}'$ which is the direct effect of the South’s FDI subsidy. The Southern labor market faces excess labor demand, and the Northern labor market encounters excess labor supply which creates a pressure to drive down the Northern relative wage. The $SS$ schedule shifts to the left to $SS'(s_F > 0)$, and the $NN$ schedule shifts to the right to $NN'(s_F > 0)$ in response. The decreased Northern wage weakens the Southern cost advantage, and triggers an indirect effect which offsets part of the FDI incentive created by the subsidy policy. I assume that the direct effect is stronger than the indirect effect. At the new steady state equilibrium, Point $D$, the relative Northern wage decreases $\frac{d\omega}{ds_F} < 0$, and the upper bound of the set of the medium-tech industries expands into the high-tech end, moving from $\bar{z}$ to the right to $\bar{z}_d$; and the set of hi-tech industries shrinks. The upper bound of the set of low-tech industries, however, shrinks due to the weakened Southern cost advantage, moving from $\bar{z}$ to the right to $\bar{z}_d$; in industries $(\bar{z}_d, \bar{z})$, a further widening quality gap is now sufficient for the North to
outrace the MNEs in the pricing game, and the North starts to undertake R&D. The set of medium-tech industries expands and is extended toward both ends.

The Southern FDI subsidy policy lowers the world expenditure level. Equation (8) and (9) show that the impacts of FDI subsidy policy on the industry innovation rate and moving-out rate hinge on the magnitude of $dE_W/ds_F$. Henceforth, I discuss the case in which $(1 - s_F) E_W dE_W/ds_F < -1$. This case occurs if in the initial steady state, the North dominates a great majority of the industries in world markets i.e. $z + nP_1 << 1 - z - nP_1$. I argue that, in this situation, the South is most likely to seek to implement a subsidy policy to strengthen its penetration. Proposition 4 summarizes the long-run impacts of the FDI subsidy policy.

**Proposition 4:** The Southern FDI subsidy ($s_F > 0$) (i) shrinks the set of high-tech industries, $d\bar{z}/ds_F > 0$; however, (ii) this occurs at the expense of shrinking the set of low-tech industries, $d\bar{z}/ds_F < 0$, due to the deteriorating cost advantage. (iii) The set of medium-tech industries is augmented, $d(\bar{z} - z)/ds_F > 0$, and is expanded toward both ends. The FDI subsidy contracts the world expenditure level. If $(1 - s_F) E_W dE_W/ds_F < -1$, (iv) it is effective in prolonging the South’s duration of dominance, $d\left(1iR(zP)\right)/ds_F > 0$, (v) the moving-out speed increases in industries with small inventive steps if and only if $\lambda(z_M) < \Lambda_{s_F}$, where $\Lambda_{s_F} \equiv \sqrt{1 - E_W / \omega dE_W/ds_F}$; however (vi) it contracts innovation investments in the high-tech industries, $dI_R(zH)/ds_F < 0$. (Proof in Appendix).

**R&D Subsidy**

The Northern government might hope to deter competition from the South by subsidizing R&D investment. Suppose that the Northern government bears a fraction $s_R$ of the cost of R&D uniformly across industries, which is financed by lump-sum taxation; the unit labor requirement per unit of R&D intensity becomes $(1 - s_R)a_R$, with $s_R > 0$. The R&D subsidy alters the relative incentive of R&D to FDI, and shifts the $\bar{Z}$ schedule upward to $\bar{Z}(s_R > 0)$, as shown in Figure 6.

At the initial steady state wage rates, in industry $\bar{z}$, the moving-out strategy is now less profitable than the moving-up strategy. Non-producing firms stop engaging in the FDI ventures and start getting involved in the R&D ventures in an attempt to outrace the current
leader in the pricing game. The upper bound of the medium-tech industries is deterred, and is pushed back toward the low-tech end of the industry spectrum, from $\bar{z}$ to $\bar{z}'$ which is the direct effect of the R&D subsidy. The Northern labor market thus faces excess labor demand, and the Southern labor market encounters excess labor supply, all of which creates a pressure to raise the Northern relative wage. The $NN$ schedule shifts to the left to $N'N'(s_R > 0)$, and the $SS$ schedule shifts to the right to $S'S'(s_R > 0)$ in response. The increased Northern wage strengthens the South’s cost advantage and triggers an indirect effect which offsets part of the R&D incentive created by the Northern subsidy policy. I assume that the direct effect is stronger than the indirect effect. At the new steady-state equilibrium, Point $E$, the relative Northern wage goes up. The upper boundary of medium-tech industries is pushed away from the high-tech end in the new equilibrium, moving from $\bar{z}$ to the right to $\bar{z}_e$; and the set of high-tech industries expands. The upper boundary of low-tech industries, however, is extended, moving from $\bar{z}$ to the left to $\bar{z}_e$, due to the improved Southern cost advantage; in industries $(\bar{z}, \bar{z}_e)$, the further widening quality gap is not sufficient for the North to outtrace the MNE leader in the pricing game, and the Northern firms permanently exit these industries. The range of the medium-tech industries shrinks and is contracted from both ends.

The impact of the Northern R&D subsidy on world expenditure is ambiguous. I restrict my discussion by focusing only on the case in which \( \frac{(1-s_R)}{E^W} \frac{dE^W}{ds_R} < -1 \). This situation occurs if in the initial steady state, the South dominates a great majority of industries in world markets, i.e. \( 1 - z - n_{P1} << z + n_{P1} \). This is the situation in which the North is most likely to implement a subsidy policy to deter Southern competition. Proposition 5 summarizes the long-run impacts of the R&D subsidy policy.

**Proposition 5**: The Northern R&D subsidy \((s_R > 0)\) (i) expands the set of high-tech industries, \( \frac{d\bar{z}}{ds_R} < 0 \); however, (ii) it does so at the expense of expanding the set of low-tech industries, \( \frac{dz}{ds_R} > 0 \), due to the improved South’s cost advantage. (iii) The range of medium-tech industries shrinks, \( \frac{d(\bar{z}-\bar{z})}{ds_R} < 0 \), and is contracted from both ends. If \( \frac{(1-s_R)}{E^W} \frac{dE^W}{ds_R} < -1 \), (iv) the North’s R&D subsidy policy is effective in slowing down the speed at which each medium-tech industry moves out from the North to the South, \( \frac{dI_f(z_P)}{ds_R} < 0 \); (v) it shortens the South’s duration of dominance in industries if and only if \( \lambda(z_M) > \Lambda_{s_R} \), where \( \Lambda_{s_R} \equiv \frac{\omega}{1 - \bar{z}_W} \frac{d\bar{z}_R}{\bar{z}_W} \frac{d\bar{z}_R}{ds_R} \).
However, it contracts R&D investments in high-tech industries, \( \frac{d\ln(z_R)}{ds_R} < 0 \). (Proof in Appendix).

It is found that the FDI and R&D subsidy policies have a positive effect on local labor returns. The South, by subsiding FDI, expands its temporary penetration range, but at the expense of contracting its permanent penetration range; the North, by subsiding R&D, deters temporary Southern penetration, but at the expense of permanently losing the market in more industries. I show that the impacts of R&D and FDI subsidies on the speed of industry moving out and the innovation rates hinge on the effects of the policy on the world expenditure level, and the industry characteristic—the size of the inventive steps.

6 Conclusions

Foreign direct investment is considered to be an important channel through which technology transfer takes place across borders. This paper shows that the extent of Southern penetration (competition) via FDI varies across industries with different technological sophistication levels and is dictated by the relative strengths of FDI and R&D intensities on the part of oligopolistic firms competing within a product line. Three types of industries are endogenously grouped. The first is the high-tech group, in which ongoing R&D deters the potential low-cost competition, and the South never enters. In the medium-tech group, the South enters the world market through MNEs that transfer the Northern technologies to the South via FDI and dominate the production until the North further improves its own production technology and recaptures the market. In the low-tech group, FDI moves production to the South, and the North permanently exits the industry. The FDI and R&D intensities serve as two natural measures that capture the degree of Southern penetration in the medium-tech industries. The former reflects the speed at which an industry is moved to (penetrated by) the South and later reflects the period during which the South dominates the production in that industry.

The extent of Southern penetration is linked to the countries’ characteristics and the effectiveness of FDI- and R&D-subsidy policies is also assessed. I show that expanding the size of the South results in the expansion of the low-tech and medium-tech groups, but it also
shortens the period during which the South is in a dominant position in each medium-tech industry. Conversely, expanding the North’s size mitigates the Southern penetration in terms of the shrinkages of the low-tech and medium-tech groups, but it accelerates the speed at which each medium-tech industry moves to the South. The South, by awarding FDI subsidies, effectively expands the medium-tech group, but it is faced with the expense of shrinking the low-tech group due to the weakened Southern cost advantage. The North, however, by subsidizing R&D, effectively expands the high-tech group, but it faces the expense of expanding the low-tech group and having to permanently exit the markets in a growing number of industries. Augmenting the countries’ sizes leads to an increase in high-tech innovation, but R&D and FDI subsidies may cause high-tech innovation to contract.

This study shows that countries’ characteristics and policies have critical and inconsistent impacts on the extent of Southern penetration, which implies that the North’s decision about whom to choose to receive FDI among a group of low-cost Southern countries essentially determines the distribution of R&D investments across industries as well as the structure of international production, both of which together directly determine long-run growth.

In this era of increasingly liberalized international investment, the question as to what are the impacts of FDI on the home and host countries’ growth and welfare is very much a concern of policy-makers and economists. Based on the general-equilibrium framework developed in this paper, we can address and answer this question. Some preliminary results are found in Lu (2004). Moreover, trade policies, such as import tariffs or export subsidies and production subsidies can be regarded as alternative means for promoting or discouraging FDI, or deterring or intensifying Southern penetration. Comparing the effectiveness of different potential policy instruments is also of much practical and theoretical interest.
Appendices

Proof of Proposition 1

In any momentary equilibrium, the values of various types of firms are dictated by the no-arbitrage conditions—the sums of the profit rates and the expected rates of capital gain (or loss) equal to the opportunity cost of funds (the normal rates) in the capital market are shown as follows:

\[
\frac{\pi}{V} + \frac{\dot{V}}{V} - i = r,
\]

where \(\frac{\pi}{V}\) denotes the dividend rates, \(\frac{\dot{V}}{V}\) denotes the capital gains, and \(i\) denotes the probability that the extant producer is replaced by either a new industry leader or an MNE leader and forfeits its future profit stream. Since the R&D and FDI technologies are characterized by constant returns to scale, the supply of non-producing firms willing to invest in an attempt to replace the extant producer at the equilibrium interest rate is perfectly elastic. It follows that in each industry only the most profitable venture is funded in the capital market. Suppose in industry \(\bar{z}\) that the nearest follower \((NF)\) is indifferent to the moving-up and moving-out strategies given that the industry leader is the extant producer. If \(NF\) succeeds in an FDI venture and becomes the MNE leader \(J^*\), it undercuts the industry leader by charging a price equal to \(\frac{\omega}{\lambda(\bar{z})} > 1\) and gains instantaneous profits \(\pi_{J^*}(\bar{z}) = (1 - \frac{\lambda(\bar{z})}{\omega})E^{W}\). The MNE, however, faces a probability of \(i_R\) of displacement from the market by a new industry leader. Instead, if \(NF\) undertakes R&D and becomes the industry’s new leader \(J\), it gains instantaneous profits \(\pi_J(\bar{z}) = (1 - \frac{1}{\lambda(\bar{z})})E^{W}\) and faces a probability of \(i_R\) of displacement from the market by a new industry leader.\(^{12}\) Based on the assumption of free-entry, the stock market value of \(J^*\) and \(J\) are \(V_{J^*}(\bar{z}) = a_F^*\). \(V_J(\bar{z}) = a_R\omega\) that are constant in the steady state \((\dot{V} = 0)\). Together with the no-arbitrage conditions, we have:

\[
\frac{(1 - \frac{\lambda(\bar{z})}{\omega})}{a_F^*} = \frac{(1 - \frac{1}{\lambda(\bar{z})})}{a_R\omega}.
\]

Multiplying both sides of the above equation by \(a_F^*\omega\) and then adding \(\lambda(\bar{z})\), it follows that \(\bar{Z}(\bar{z}) =\omega\).

\(^{12}\)Suppose \(NF\) chooses to undertake R&D venture to outrace the extant producer between these two indifferent strategies.
Proof of Lemma 1

Both \( Z(z) \equiv \lambda(z)^2 \) and \( \tilde{Z}(z) \equiv \frac{a_T}{a_R} (1 - \frac{1}{\lambda(z)^2}) + \lambda(z) \) are increasing in \( z \) given \( \lambda'(z) > 0 \). Since \( \lambda(0) = 1, \lambda(z) > 1, \forall \ z \in (0, 1] \), and \( \frac{a_T}{a_R} < 1 \), it follows that \( \lambda(z) < \tilde{Z}(z) < Z(z) \) \( \forall z \in (0, 1] \), and \( \lambda(0) = \tilde{Z}(0) = Z(0) \). By definition \( \omega = \tilde{Z}(\tilde{z}) = Z(\tilde{z}) \), if Assumption 1 holds, \( \tilde{Z}(0) < \tilde{Z}(\tilde{z}) < \tilde{Z}(1) \), and \( Z(0) < Z(\tilde{z}) < Z(1) \) which implies that \( 0 < \tilde{z} < 1 \) and \( 0 < z < 1 \). Moreover, \( \tilde{Z}(z) > \tilde{Z}(\tilde{z}) \forall z \in (0, 1] \) which implies that \( Z(\tilde{z}) = \tilde{Z}(\tilde{z}) \) if and only if \( \tilde{z} < \tilde{z} \). Therefore, \( 0 < z < \tilde{z} \) \( \forall z \in (0, 1] \).

Proof of Proposition 2–Expanding the Northern Size

I calculate the magnitude of the horizontal shifts in the \( SS \) and \( NN \) schedules in response to an increase in \( L \), and find that

\[
\frac{dE^W}{dL} \bigg|_{SS} = 0, \\
\frac{dE^W}{dL} \bigg|_{NN} = \frac{\omega}{1 - \frac{\tilde{z}(\omega) - n_{P1}(\omega)}{L}} > 0.
\]

The \( SS \) schedule remains in the same position, and the \( NN \) schedule shifts to the right. Therefore, \( \frac{dE^W}{dL} > 0 \), and \( \frac{dE^W}{dL} < 0 \). Moreover,

\[
\frac{d\tilde{z}}{dL} = \frac{d\tilde{z}}{d\omega} \frac{\omega}{d\tilde{L}} = \frac{1}{\frac{a_T}{a_R} (1 - \frac{1}{\lambda(\tilde{z})^2}) + 1} \lambda'(\tilde{z}) \frac{d\omega}{d\tilde{L}} < 0, \\
\frac{d\tilde{z}}{dL} = \frac{d\tilde{z}}{d\omega} \frac{\omega}{d\tilde{L}} = \frac{1}{2\lambda(\tilde{z})^3} \frac{d\omega}{d\tilde{L}} < 0.
\]

The range of temporary South penetration shrinks; so does the range of permanent South penetration. Since \( \frac{d\omega}{d\tilde{L}} < 0 \), and by Assumption 2, \( \left(\frac{a_T}{a_R} \frac{1}{\lambda(\tilde{z})^2} + 1\right) \lambda'(\tilde{z}) < 1 < 2\lambda(\tilde{z})\lambda'(\tilde{z}) \), the range of medium-tech industries also shrinks, \( \frac{d\tilde{z}}{dL} \frac{z(\omega)}{\tilde{L}} < 0 \), and is pushed toward the low-tech end. Recall that \( i_F(z_P) = \frac{1 - \frac{\omega}{\lambda(z_M)^2}}{a_{R\omega}} E^W - \rho \). Taking the total derivative,

\[
\frac{di_F(z_P)}{dL} = \left(1 - \frac{\omega}{\lambda(z_M)^2}\right) \frac{dE^W}{dL} - \frac{E^W}{\omega^2a_R} \frac{d\omega}{dL} > 0,
\]

which implies that the speed of industry moving out is accelerated. Moreover, recall that \( i_R(z_P) = \frac{1 - \frac{\omega}{\lambda(z_M)}}{a_{R\omega}} E^W - \rho \). Taking the total derivative,

\[
\frac{di_R(z_P)}{dL} = \frac{1}{a_{R\omega}} E^W \left[ 1 - \left(1 - \frac{E^W}{\omega} \frac{dL}{dL} \frac{dE^W}{dL} \right) \frac{\lambda(z_M)}{\omega} \right].
\]
An expanded Northern size shortens the duration of Southern dominance, i.e. \( \frac{d}{dL} \left( \frac{1}{\lambda(z_M)} \right) < 0 \), if and only if \( \lambda(z_M) < \frac{\omega}{1 - \frac{E^W}{\omega} \frac{d\omega}{dL^*}} \). On the other hand, the duration of Southern dominance could be prolonged, if industries’ inventive steps were relatively large. Finally, I show that the high-tech innovation rate is reinforced. Recall that \( i_R(z_H) = \frac{(1 - \frac{1}{\lambda(z_H)})E^W}{a_{R\omega}} - \rho \). Taking the total derivative,

\[
\frac{di_R(z_H)}{dL^*} = \frac{(1 - \frac{1}{\lambda(z_H)})E^W}{a_{R\omega}} \left[ \frac{1}{E^W} \frac{dE^W}{dL^*} - \frac{1}{\omega} \frac{d\omega}{dL^*} \right] > 0. \quad Q.E.D.
\]

**Proof of Proposition 3–Expanding the Southern Size**

I calculate the magnitude of the horizontal shifts in the \( SS \) and \( NN \) schedules in response to an increase in \( L^* \), and find that

\[
\frac{dE^W}{dL^*} \bigg|_{SS} = \frac{1}{\int_{0}^{\bar{z}(\omega)} \frac{\lambda(z)}{\omega} dz + n_{P1}(\omega)} > 0,
\]

\[
\frac{dE^W}{dL^*} \bigg|_{NN} = 0.
\]

The \( SS \) schedule shifts to the right, and the \( NN \) schedule remains in the same place. Therefore, \( \frac{dE^W}{dL^*} > 0 \), and \( \frac{dL^*}{d\omega} > 0 \). Moreover,

\[
\frac{d\bar{z}}{dL^*} = \frac{d\bar{z}}{d\omega} \frac{d\omega}{dL^*} = \frac{1}{\left( \frac{a_L^2}{a_R^2} \frac{1}{\lambda(\bar{z})^2} + 1 \right) \lambda'(\bar{z})} > 0,
\]

\[
\frac{d\bar{z}}{dL^*} = \frac{d\bar{z}}{d\omega} \frac{d\omega}{dL^*} = \frac{1}{2\lambda(\bar{z})\lambda'(\bar{z})} > 0.
\]

The range of temporary Southern penetration expands; so does the range of permanent Southern penetration. Since \( \frac{dL^*}{d\omega} > 0 \), and by Assumption 2, \( \left( \frac{a_L^2}{a_R^2} \frac{1}{\lambda(\bar{z})^2} + 1 \right) \lambda'(\bar{z}) < 1 < \lambda(\bar{z})\lambda'(\bar{z}) \), the range of medium-tech industries also expands, \( \frac{d(z_P - \bar{z})}{dL^*} > 0 \), and is pushed toward the high-tech end. Recall that \( i_R(z_P) = \frac{(1 - \frac{\lambda(z_M)}{\omega})E^W}{a_{R\omega}} - \rho \). Taking the total derivative,

\[
\frac{di_R(z_P)}{dL^*} = \frac{(1 - \frac{\lambda(z_M)}{\omega})}{a_{R\omega}^2} \frac{dE^W}{dL^*} + \frac{\lambda(z_M)}{\omega^2 a_{R\omega}^2} \frac{d\omega}{dL^*} > 0,
\]

which implies that the duration of Southern dominance is shortened, i.e. \( \frac{d}{dL^*} \left( \frac{1}{\lambda(z_M)} \right) < 0 \). Recall that \( i_F(z_P) = \frac{(1 - \frac{\lambda(z_M)}{\omega})E^W}{a_{R\omega}} - \rho \). Taking the total derivative,

\[
\frac{di_F(z_P)}{dL^*} = \frac{E^W}{a_{R\omega}^2} \frac{d\omega}{dL^*} \left[ \left( 1 - \frac{\omega}{\lambda(z_M)^2} \right) \frac{dE^W}{dL^*} - \frac{1}{\omega} \frac{d\omega}{dL^*} \right].
\]
The expanded Southern size accelerates the speed of industry moving out, i.e. \( \frac{\text{di}_R(z_H)}{\text{d}L^*} > 0 \), if and only if \( \lambda(z_M)^2 > \frac{1}{1 + \frac{\rho a^*_F}{\rho a^*_{R1}}} \). On the other hand, the speed of industry moving out could be slowed down, if industries’ inventive steps were relatively small. Finally, I show that the high-tech innovation rate is reinforced. Recall that \( \frac{\partial n}{\partial L} \) could be slowed down, if industries’ inventive steps were relatively small. Taking the total derivative,

\[
\frac{\text{di}_R(z_H)}{\text{d}L^*} = \left(1 - \frac{1}{\lambda(z_H)}\right) \frac{\text{d}E^W}{\text{d}L^*} \left[1 - \frac{E^W}{\omega} \frac{\text{d}L^*}{\text{d}E^W} \frac{\text{d}\omega}{\text{d}L^*}\right] > 0,
\]

since \( 0 < \frac{E^W}{\omega} \frac{\text{d}L^*}{\text{d}E^W} \frac{\text{d}\omega}{\text{d}L^*} = \frac{(1-z-n_{p1})}{(1-z-n_{p1})+\omega(\frac{\lambda}{\lambda} + \frac{\partial \omega}{\partial \omega})} < 1 \). Q.E.D.

**Proof of Proposition 4–FDI Subsidy** \( s_F > 0 \)

The simplified forms of the Southern and Northern labor market clearing conditions become:

\[
SS' : L^* = \frac{E^W}{\omega} \int_0 \lambda(z)dz + (E^W - \rho a^*_F)n_{p1}(\omega, s_F) + \frac{s_F}{1-s_F} \frac{E^W}{\omega} \int_0 (\omega - \lambda(z))n_{p1}(z)dz,
\]

\[
NN' : L = \left(\frac{E^W}{\omega} - \rho a_R\right) \left(1 - \frac{1}{\lambda(z)}\right) n_{p1}(\omega, s_F).
\]

I calculate the magnitude of the horizontal shifts in the \( SS \) and \( NN \) schedules in response to an increase in \( s_F \) from 0, and find that

\[
\left. \frac{\text{d}E^W}{\text{d}s_F} \right|_{SS', s_F=0} = - \left(\frac{E^W - \rho a^*_F}{\omega} \frac{\partial n_{p1}(\omega, s_F)}{\partial s_F} + \frac{E^W}{\omega} \int_0 (\omega - \lambda(z))dz \right) \frac{\int_0 \lambda(z)dz}{\int_0 \lambda(z)dz + n_{p1}(\omega)} < 0,
\]

\[
\left. \frac{\text{d}E^W}{\text{d}s_F} \right|_{NN', s_F=0} = \left(\frac{E^W - \rho a_R\omega}{1-s_F} \frac{\partial n_{p1}(\omega, s_F)}{\partial s_F} \right) > 0,
\]

since \( \frac{\partial n_{p1}(\omega, s_F)}{\partial s_F} = \frac{\lambda'(\lambda) - 1}{\lambda(\lambda)} n_{p1}(\tilde{\omega}) > 0 \). The \( SS \) schedule shifts to the left, and the \( NN \) schedule shifts to the right. In the new steady state, \( \frac{\text{d}\omega}{\text{d}s_F} < 0 \). Moreover,

\[
\frac{\text{d}\tilde{\omega}}{\text{d}s_F} = \left(\frac{a^*_F}{a_R} + 1\right) \lambda'(\tilde{\omega}) \left[\frac{a^*_F}{a_R} \left(1 - \frac{1}{\lambda(\tilde{\omega})}\right)\right] + \frac{\text{d}\omega}{\text{d}s_F} > 0,
\]

29
\[
\frac{dz}{ds_F} = \frac{1}{2\lambda(z)\lambda'(z)} \frac{d\omega}{ds_F} < 0.
\]

The range of temporary Southern penetration expands; the range of permanent Southern penetration shrinks. Moreover, \(\frac{d(z-z)}{ds_F} > 0\); the range of medium-tech industries expands toward both ends.

By using similar techniques, I can show the vertical shifts in both schedules:

\[
\left. \frac{d\omega}{ds_F} \right|_{SS', s_F=0} = - \frac{E^W - \rho \alpha_R^*}{\omega} \left[ \frac{\partial n_{P_1(\omega, s_F)}}{\partial s_F} + \int z(\omega - \lambda(z))dz \right] < 0,
\]

\[
\left. \frac{d\omega}{ds_F} \right|_{NN', s_F=0} = - \frac{E^W}{\omega} \left[ \frac{1 - z(\omega - n_{P_1(\omega)})}{(1 - s_F)^2} \alpha_R^* \frac{\partial n_{P_1(\omega, s_F)}}{\partial s_F} \right] < 0
\]

Since the SS schedule shifts down more than the NN schedule does, \(\frac{dE^W}{ds_F} < 0\). I discuss only the case in which \(\frac{(1-s_F)E^W}{ds_F} < -1\). Recall that \(i_R(z_F) = \frac{(1-\lambda(z_F))E^W}{\omega a_R} - \rho\). Taking the total derivative,

\[
\frac{di_R(z_F)}{ds_F} = \frac{1 - \lambda(z_M)}{(1 - s_F)^2} \frac{E^W}{\omega a_R} \left[ 1 + \frac{(1-s_F)E^W}{ds_F} + \frac{(1-s_F)\lambda(z_M)}{(\omega - \lambda(z_M))} \frac{d\omega}{ds_F} \right] < 0.
\]

The FDI subsidy policy is effective in prolonging the South-dominant duration, i.e. \(\frac{d(\frac{i_R(z_F)}{s_F})}{ds_F} > 0\). Recall that \(i_F(z_M) = \frac{(1-\lambda(z_M)^2)}{\omega a_R} - \rho\). Taking the total derivative, and valuing the equation at point \(s_F = 0\), I obtain

\[
\frac{di_F(z_F)}{ds_F} = \frac{1 - \lambda(z_M)^2}{\omega a_R} \frac{E^W}{\omega} \left[ \frac{1}{ds_F} \frac{dE^W}{ds_F} - \frac{1}{1 - \lambda(z_M)^2} \frac{1}{ds_F} \frac{d\omega}{ds_F} \right].
\]

However, whether or not the FDI subsidy is effective in accelerating industry moving out further depends on industry-specific inventive steps. Specifically, \(\frac{di_F(z_F)}{ds_F} > 0\), if and only if

\[
\lambda(z_M) < \sqrt{\frac{\omega}{1 - \frac{E^W \frac{ds_F}{dE^W}}{\omega \frac{d\omega}{ds_F}}}^{13}}.
\]

\[13\text{By assuming that the direct effect dominates the indirect effect, we know that } -1 < \frac{1}{\frac{d\omega}{ds_F}} < 0. \text{ Moreover, since } -1 < -(1-s_F) < \frac{E^W}{\omega} \frac{ds_F}{dE^W} < 0, \text{ we have } 0 < \frac{E^W \frac{ds_F}{dE^W} \frac{d\omega}{ds_F}}{\omega} < 1.\]
Finally, the high-tech innovation rate is shown to decrease in response. Recall that $i_R(z_H) = \left(1 - \frac{1}{\lambda(z_H)}\right)^{E^W} - \rho$. Taking the total derivative,

$$\frac{di_R(z_H)}{ds_F} = \left(1 - \frac{1}{\lambda(z_H)}\right) \frac{dE^W}{ds_F} \left[ 1 - \frac{E^W}{\omega} \frac{ds_F}{d\omega} \frac{d\omega}{ds_R} \right] < 0.$$  

The FDI subsidy policy slows down the hi-tech innovation rates.

**Proof of Proposition 5–R&D Subsidy $s_R > 0$**

The simplified forms of the Southern and the Northern labor market clearing conditions become:

$$SS'' : L^* = \frac{E^W}{\omega} \bar{z}(\omega) \int_0^\omega \frac{\lambda(z)}{\omega} \frac{dz}{\omega} + (E^W - \rho a_F^*) n_{p_1}(\omega, s_R),$$

$$NN'' : L = \left(\frac{E^W}{\omega} - \rho a_R^* \right) \cdot (1 - \bar{z}(\omega) - n_{p_1}(\omega, s_R)) + \frac{s_R}{1 - s_R} \frac{E^W}{\omega} \int_0^{\bar{z}(\omega)} \left(1 - \frac{\omega}{\lambda(z)^2}\right) n_{p_1}(z) \frac{dz}{\omega}.$$ 

I calculate the magnitude of the horizontal shifts in the $SS$ and $NN$ schedules in response to an increase in $s_R$ from 0, and find that

$$\left. \frac{dE^W}{ds_R} \right|_{SS'', s_R=0} = \frac{-(E^W - \rho a_F^*) n_{p_1}(\omega, s_R)}{\int_0^\omega \frac{\lambda(z)}{\omega} \frac{dz}{\omega} + n_{p_1}(\omega)} > 0,$$

$$\left. \frac{dE^W}{ds_R} \right|_{NN'', s_R=0} = \frac{-(E^W - \rho a_R^*) \frac{\partial n_{p_1}(\omega, s_R)}{\partial s_R} + \frac{E^W}{\bar{z}(\omega)} \left(1 - \frac{\omega}{\lambda(z)^2}\right) n_{p_1}(z) \frac{dz}{\omega}}{1 - \bar{z}(\omega) - n_{p_1}(\omega)} < 0,$$

since $\frac{\partial n_{p_1}(\omega, s_R)}{\partial s_R} = -\frac{\partial n_{p_1}(\omega, s_F)}{\partial s_F} < 0$. The $SS''$ schedule shifts to the right, and the $NN''$ schedule shifts to the left. In the new steady state, $\frac{d\omega}{ds_R} > 0$. Moreover,

$$\frac{d\bar{z}}{ds_R} = \frac{1}{\left(\frac{a_F^*}{a_R^*} \lambda(\bar{z}) + 1\right) \lambda(\bar{z}) \left[ -\frac{a_F^*}{a_R^*} \left(1 - \frac{1}{\lambda(\bar{z})}\right) + \frac{d\omega}{ds_R} \right]} < 0,$$

$$\frac{d\bar{z}}{ds_R} = \frac{1}{2\lambda(\bar{z}) \lambda(\bar{z}) \frac{d\omega}{ds_R} > 0}.$$
The range of temporary Southern penetration shrinks; the range of permanent Southern penetration expands. Furthermore, \( \frac{d(z_n)}{d \omega} < 0 \); the range of medium-tech industries shrinks from both ends. The vertical shifts in both schedules are

\[
\frac{d \omega}{d s_R} \bigg|_{SS^n, s_R = 0} = \frac{E^W - \rho a_R}{\partial_R} \frac{\partial n_{P1}(\omega, s_R)}{\partial s_R} > 0,
\]

\[
\frac{d \omega}{d s_R} \bigg|_{NN^n, s_R = 0} = \frac{E^W - \rho a_R}{\partial_R} \frac{\partial n_{P1}(\omega, s_R)}{\partial s_R} + E^W \frac{\partial f_p}{\partial \omega} \left( 1 - \frac{\omega}{\lambda(z)} \right) \frac{n_{P1}(z)}{dZ} > 0.
\]

Both the SS and NN schedules shift upwards, but the sign of \( \frac{dE^W}{ds_R} \) is ambiguous. I restrict my discussion to the case in which \( \frac{(1 - s_R) dE^W}{ds_R} < -1 \).

Recall that \( i_R(z_P) = \frac{(1 - s_R) dE^W}{ds_R} - \rho \). Taking the total derivative,

\[
\frac{di_R(z_P)}{ds_R} = \frac{dE^W}{ds_R} \left[ 1 - \frac{(1 - s_R) dE^W}{ds_R} \right] \frac{\lambda(z_M)}{\omega} > 0.
\]

The effectiveness of the R&D subsidy in shortening the South-dominance duration further depends on the industry-specific inventive step. Specifically, \( \frac{di_R(z_P)}{ds_R} > 0 \) if and only if \( \frac{1 - s_R}{E^W} < \lambda(z_M) \).\(^{14}\) On the other hand, if the inventive steps are relative small, the R&D subsidy might prolong the South-dominance duration! Recall that \( i_F(z_M) = \frac{(1 - s_R)(s_R) - \rho}{\lambda(z_M)} \).

Taking the total derivative,

\[
\frac{di_F(z_P)}{ds_R} = \frac{(1 - s_R)(s_R)}{\omega^2} \frac{dE^W}{ds_R} \left[ 1 + \frac{(1 - s_R) dE^W}{ds_R} \right] \frac{\lambda(z_M)}{\omega} < 0.
\]

The R&D subsidy policy is effective in slowing down the speed of industry moving out \( \frac{di_F(z_P)}{ds_R} < 0 \). Finally, recall that \( i_R(z_H) = \frac{(1 - s_R)(s_R) - \rho}{\lambda(z_H)} \). Taking the total derivative,

\[
\frac{di_R(z_H)}{ds_R} = \frac{(1 - s_R)(s_R)}{\omega^2} \frac{dE^W}{ds_R} \left[ 1 + \frac{(1 - s_R) dE^W}{ds_R} \right] < 0.
\]

The high-tech innovation rate is slowed down even when the R&D is subsidized.

\(^{14}\)Notice that \( \frac{dE^W}{ds_R} < 0 \), and \( \frac{dE^W}{ds_R} > 0 \), thus \( \frac{dE^W}{ds_R} \frac{dE^W}{ds_R} < 0 \), and \( 1 < \frac{dE^W}{ds_R} \frac{dE^W}{ds_R} \).
References


Figure 2: A Continuum of Industries is Endogenously Divided into Low, Medium and Hi-tech Groups
Figure 3: Comparative Statics Analysis—$\hat{L} > 0$

Figure 4: Comparative Statics Analysis—$\hat{L}^* > 0$
Figure 5: Comparative Statics Analysis–FDI Subsidy $s_F > 0$

Figure 6: Comparative Statics Analysis–R&D Subsidy $s_R > 0$