On the formation of Pareto-improving trading club without income transfer

Koji Shimomura
Kobe University

Koichi Hamada
Yale University

Masahiro Endoh
Keio University/Yale University

March 1, 2005

Abstract
Constructing a multi-country general equilibrium model, we show that a Pareto-improving coordinated tariff reforms by a subset of countries (a trading club) is possible without intra-club income transfer, if for each good traded between club member countries there are two groups of members such that one group adjusts a tariff/subsidy on its net import while the other adjusts it on its net export.

1 Introduction

Consider a trading world that consists of an arbitrary number of countries, say $n + 1$, such that there is a tariff-ridden world equilibrium for given tariff vectors imposed by those countries. Suppose that a part of the countries, say $n$ countries, form a trading club by adjusting their tariff rates. This paper studies under what conditions the formation of the trading club be Pareto-improving in the sense that, as a result of the adjustments of their tariffs, (1) at least one member country is better-off and (2) no country, whether it is a member or non-member, is worse-off. We show that it is possible to form a Pareto-improving trading club without any international income transfer if for each good traded between club member countries there are two groups of members such that one group adjusts a tariff/subsidy on its net import while the other adjusts it on its net export.

[We need to review the literature. One recent paper which is closely related to this paper would be]
Section 2 sets up the model. Section 3 shows the main theorem in a general setting. Section 4 applies the theorem to a case such that the Armington Assumption holds in a three-good and three-country framework. Section 5 provides concluding remarks.

2 The Model

We consider a multi-country tariff-ridden general equilibrium model which consists of \( n + 1 \) countries and \( m + 1 \) tradable goods. The countries and goods are indexed as Country 0, Country 1, ..., Country \( n \), and Good 0, Good 1, ..., Good \( m \), respectively. Country 1, ..., and Country \( n \) form a trading club. Good 0 is the numeraire and Country 0 represents "the rest of the world". Using revenue and expenditure functions, we can describe the multi-country model as follows.

\[
E^0(P, u^0) = F^0(P) \tag{1}
\]

\[
E^i(P + \Lambda^i, u^i) = F^i(P + \Lambda^i) + \Lambda^i[E^i_p(P + \Lambda^i, u^i) - F^i_p(P + \Lambda^i)], \quad i = 1, ..., n \tag{2}
\]

\[
-[E^0(P, u^0) - F^0(P)] = \sum_{i=1}^{n} [E^i_p(P + \Lambda^*, u^*) - F^i_p(P + \Lambda^*)], \tag{3}
\]

where \( P \equiv (p_1, ..., p_m)^T \) and \( \Lambda^i \equiv (\tau^1_i, ..., \tau^m_i)^T \) are the international price vector and the import tariff/export tax vector imposed by country \( i \), respectively. \( P^i = P + \Lambda^i \), where \( P^i \equiv (p^i_1, ..., p^i_m)^T \) is the domestic price vector in country \( i \). \( u^i, i = 0, 1, ..., n \), is the community utility level of Country \( i \). The above system determines the international price of each good and the community utility level for given tariff rates, \( \tau^j_i, i = 1, ..., n, j = 1, ..., m \), are given. \( E^i_p(P + \Lambda^i, u^i) \equiv (E^i_{p_1}, ..., E^i_{p_m})^T \) and \( F^i_p(P + \Lambda^i) \equiv (F^i_{p_1}, ..., F^i_{p_m})^T \), where \( E^i_{p_j} \equiv \frac{\partial}{\partial p^i_j} E^i \) and \( F^i_{p_j} \equiv \frac{\partial}{\partial p^i_j} F^i, j = 1, ..., m. \)

The superscript \( T \) attached to vectors denotes the transpose of them. We assume that vectors without the super-script are column vectors.
3 The Main Theorem

First, let us list the main assumptions.

**Assumption 1:** All revenue and expenditure functions satisfy the standard textbook properties. Income effects are always normal in the sense that

\[ E_i^uP \equiv (E_i^{u_{P_1}}, \ldots, E_i^{u_{P_m}})^T > 0 \]

Moreover, for any \( i = 1, \ldots, n \), the second derivatives

\[ E_{P_iP}^i(P + \Lambda^i, u^i) - F_{P_iP}^i(P + \Lambda^i) \]

are non-singular, where

\[ E_i^{P_{p_jp_h}} \equiv \frac{\partial^2 E_i}{\partial p_j \partial p_h} \]

and

\[ F_{P_iP}^i(P + \Lambda^i, u^i) \equiv \left[ \begin{array}{ccc} F_{1P1}^i & \cdots & F_{1Pm}^i \\ \vdots & \ddots & \vdots \\ F_{nP1}^i & \cdots & F_{nPm}^i \end{array} \right], \quad E_i^{P_{p_jp_h}} \equiv \frac{\partial^2 E_i}{\partial p_j \partial p_h} \]

**Assumption 2:** There exists a unique pre-club equilibrium, \((\bar{P}, \bar{u}^j, j = 0, 1, \ldots, n)\) for given tariff rates, \( i = 1, \ldots, n \), where

\[ \bar{P} \equiv (\bar{p}_1, \ldots, \bar{p}_m)^T > -\Lambda^i, \]

\( i = 1, \ldots, n \). Moreover, a pre-club equilibrium uniquely exists for any tariff rates in a neighborhood of the given tariff rates.

**Assumption 3:** In the pre-club equilibrium, for any good \( j, j = 1, \ldots, m \), there are two types of club countries such that the first type, say Country \( i(j) \), is to impose a positive tariff \( \tau_{i}^{i(j)} > 0 \) on the net import of Good \( j \) and the second type, say Country \( i^*(j) \), is to impose a non-negative tariff, \( \tau_{i^*}^{i*(j)} \leq 0 \) on the net export of it\(^2\).

In what follows, we denote the sets of the first type countries and the second type countries by \( \Delta \) and \( \Delta^* \), respectively.

Let us state the main theorem.

**Theorem 1** Under Assumptions 1-3, if the negative tariff rates in the pre-club equilibrium are not very large in their absolute values, \( |\tau_{j}^{i*(j)}|, j = 1, \ldots, m \), then it is possible for \( n \) countries to form a trading club that undertakes a differential and non-discriminatory reform of tariffs in such a way that at least some club countries are better off without hurting all other club countries and the rest of the world.

\(^2\)Thus, if the net import is negative, a positive (resp. negative), \( \tau_{i}^{i(j)} > 0 \) (resp. \( \tau_{i^*}^{i*(j)} < 0 \)), means export (import) subsidy.
Proof. Let us consider the following tariff policies.

\[ \Lambda^i(\varepsilon^i) \equiv \Lambda^i - \{ E_{PP}^i(\bar{P} + \Lambda^i, \bar{u}^i) - F_{PP}^i(\bar{P} + \Lambda^i) \}^{-1} \times \{ E_u^i(\bar{P} + \Lambda^i, \bar{u}^i) - (\Lambda^i)^T E_{pu}^i(\bar{P} + \Lambda^i, \bar{u}^i) \} \varepsilon^i, \quad (4) \]

where \( \varepsilon^i = (\varepsilon_1^i, ..., \varepsilon_m^i)^T \). Totally differentiating (2) and (3) with respect to \( \Lambda^i \) and \( u^i \), \( i = 1, ..., n \), around the pre-club equilibrium in such a way that both \( P \) and \( u^0 \) are left unchanged, and considering (4), we have.

\[ du^i = \frac{(\Lambda^i)^T [E_{PP}^i(\bar{P} + \Lambda^i, \bar{u}^i) - F_{PP}^i(\bar{P} + \Lambda^i)]d\Lambda^i}{E_u^i(\bar{P} + \Lambda^i, \bar{u}^i) - (\Lambda^i)^T E_{pu}^i(\bar{P} + \Lambda^i, \bar{u}^i)}, \quad i = 1, ..., n, \quad (5a) \]

\[ 0_m = \sum_{s=1}^{n} \{ [E_{PP}^s(\bar{P} + \Lambda^s, \bar{u}^s) - F_{PP}^s(\bar{P} + \Lambda^s)]d\Lambda^s + E_{uP}^s(\bar{P} + \Lambda^s, \bar{u}^s)du^s \} \]

\[ + \frac{E_{uP}^s(\bar{P} + \Lambda^s, \bar{u}^s)(\Lambda^i)^T [E_{PP}^i(\bar{P} + \Lambda^i, \bar{u}^i) - F_{PP}^i(\bar{P} + \Lambda^i)]}{E_u^i(\bar{P} + \Lambda^i, \bar{u}^i) - (\Lambda^i)^T E_{pu}^i(\bar{P} + \Lambda^i, \bar{u}^i)}d\Lambda^s(\varepsilon^s)^{(6)} \]

where \( 0_m \equiv (0, ..., 0)^T \), an \( m \)-dimensional zero vector, and, from (4),

\[ d\Lambda^i(\varepsilon^i) = \frac{[E_{PP}^i(\bar{P} + \Lambda^i, \bar{u}^i) - F_{PP}^i(\bar{P} + \Lambda^i)]^{-1} \times \{ E_u^i(\bar{P} + \Lambda^i, \bar{u}^i) - (\Lambda^i)^T E_{pu}^i(\bar{P} + \Lambda^i, \bar{u}^i) \} d\varepsilon^i}{E_u^i(\bar{P} + \Lambda^i, \bar{u}^i) - (\Lambda^i)^T E_{pu}^i(\bar{P} + \Lambda^i, \bar{u}^i)} \]

The substitution of (7) into (5a) and (6) yields, respectively,

\[ du^i = -(\Lambda^i)^T d\varepsilon^i \]

(8)
and

\[ 0 = -\sum_{s=1}^{n} [(E^s_u(\bar{P} + \Lambda^s, \bar{u}^s) - (\Lambda^s)^T E^s_u(\bar{P} + \Lambda^s, \bar{u}^s))] I_{m,m} + E^s_u(\bar{P} + \Lambda^s, \bar{u}^s)(\Lambda^s)^T] \epsilon^s \]

\[ = -\sum_{s=1}^{n} \left[ \begin{array}{cccc} E^s_u - \Sigma_{j \neq 1,j=2}^m E^s_{uP} & \tau^s_1 E^s_{uP1} & \cdots & \tau^s_m E^s_{uPm} \\ \tau^s_1 E^s_{up2} & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \vdots \\ \tau^s_1 E^s_{upm} & \cdots & \cdots & E_u - \Sigma_{j=1,j \neq m}^m \tau^s_m E^s_{uP} \end{array} \right] \epsilon^s \]

\[ = -\sum_{s=1}^{n} \sum_{h=1}^{m} \sum_{s=1}^{m} \left[ \begin{array}{c} E^s_u - \Sigma_{j=1,j \neq h}^m \tau^s_j E^s_{uP} \\ \tau^s_h E^s_{up1} \\ \vdots \\ \tau^s_h E^s_{upm} \end{array} \right] \epsilon^s_h \]

where \( I_{m,m} \) is the \( m \)-dimensional identity matrix. 

Let us assume that for Good \( j, j = 1, \ldots, m, \) a club country \( i(j) \) in \( \Delta \) that imports Good \( j \) reduces \( \epsilon_{i(j)}^j(t) \) (i.e., \( \epsilon_{i(j)}^j(t) < 0 \)) and a club country \( i^*(j) \) in \( \Delta^* \) that exports Good \( j \) raises \( \epsilon_{i^*(j)}^j(t) \) (i.e., \( \epsilon_{i^*(j)}^j(t) > 0 \)), while all other \( \epsilon_{i(j)}^j \)’s are kept to be zero. It follows from (8) and Assumption 3 that \( du^i > 0 \) for any \( i \in \Delta \cup \Delta^* \), while \( du^i = 0 \) for any \( i \in \{0, 1, \ldots, n\} - \Delta \cup \Delta^* \).

Thus, what remains is to show that there exists two vectors,

\[ d\Xi^T = (d\epsilon^1_1, d\epsilon^2_2, \ldots, d\epsilon^m_m)^T < 0_m \text{ and } (d\Xi^*)^T = (d\epsilon^1_1, d\epsilon^2_2, \ldots, d\epsilon^m_m)^T > 0_m, \]

that satisfy (9), i.e.,

\[ \Gamma d\Xi + \Gamma^* d\Xi^* = 0, \]

where

\[ \Gamma \equiv \left[ \begin{array}{cccc} E^1_u - \Sigma_{j \neq 1}^m \tau^1_j E^1_{uP} & \tau^1_1 E^1_{uP1} & \cdots & \tau^1_m E^1_{uPm} \\ \tau^2_1 E^2_{uP1} & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \vdots \\ \tau^m_1 E^m_{uPm} & \cdots & \cdots & E_u - \Sigma_{j=1,j \neq m}^m \tau^m_m E^m_{uP} \end{array} \right] \]

\[ \Gamma^* \equiv \left[ \begin{array}{cccc} E^1_u - \Sigma_{j \neq 1}^m \tau^1_j E^1_{uP} & \tau^1_1 E^1_{uP1} & \cdots & \tau^1_m E^1_{uPm} \\ \tau^2_1 E^2_{uP1} & \cdots & \cdots & \cdots \\ \vdots & \ddots & \ddots & \vdots \\ \tau^m_1 E^m_{uPm} & \cdots & \cdots & E_u - \Sigma_{j=1,j \neq m}^m \tau^m_m E^m_{uP} \end{array} \right] \]
Since we assume away inferior goods, it is clear from \( \sigma_i^{(j)} > 0 \) that \( \sigma_i^{(j)} E^{(j)}_{i} > 0 \) for any \( j, h = 1, \ldots, m \). Moreover, we see from the linear homogeneity of \( E_u^i(p_0, p_1 + \tau_1^i, \ldots, p_m + \tau_m^i, u_i) \) with respect to \( (p_0, p_1, \ldots, p_m) \) that

\[
E_u^i = \sum_{j \neq h, j=1}^m \tau_j^{(h)} E_{upj}^i = \begin{pmatrix} p_0 E_{up0} \sum_{j=1}^m (\bar{p_j} + \tau_j^{(h)}) E_{upj}^i \end{pmatrix} - \sum_{j \neq h, j=1}^m \tau_j^{(h)} E_{upj}^i
\]

Next, let us consider the matrix \( \Gamma^* \). It is clear that all the diagonal elements are positive while all off-diagonal elements are negative. Now, take the \( h \)th column of the matrix and sum all its elements.

\[
\left| E_u^i(h) - \sum_{j \neq h, j=1}^m \tau_j^{(h)} E_{upj}^i \right| + \tau_h \sum_{j \neq h, j=1}^m E_{upj}^i
\]

which is positive if \( \tau_h^{(i)} \) is smaller than \( \bar{p_j} \) for any \( j, h = 1, \ldots, m \). It follows from the Frobenius Theorem (e.g., Takayama (1984), Theorem 4.C.9 on page 387) that \( \Gamma^* \) is non-singular with the positive inverse matrix \( (\Gamma^*)^{-1} \) > 0 for any \( m, m \). Therefore, for any negative vector \( d \Xi \),

\[
d \Xi^* = -\Gamma^{-1} \Gamma d \Xi > 0
\]

That is, there exists a pair \( (-d \Xi, d \Xi^*) > (0_m, 0_m) \) that satisfies (10), as was to be proved. (QED)

# 4 An Example: 3 X 3 Model

## 4.1 The Assumptions

Let me construct a 3 by 3 model satisfying the following assumptions.

**Assumption 4:** Country A and Country B are going to form a trading club and Country C is the rest of the world.

**Assumption 5:** There are three goods, a, b, c, and good c serves as the numeraire good. Country A exports good a and imports good b and good c. Country B exports good b and imports good a and good c. Country C exports good c and imports good a and good b.
Assumption 6: Initially, Country A imposes tariff on imports of good b and Country B imposes tariff on imports of good a. More specifically, we assume that at the pre-club equilibrium
- Country A imposes positive import tariffs on good b. Let us denote the tariff rate by $t^A_b$. Country B imposes positive import tariffs on good a. Let us denote the tariff rate by $t^B_a$.
- Country A and Country B impose zero tax/subsidy on their exports, i.e., $t^A_a = t^B_b = 0$.
- Country C is assumed to be a free-trade country.

Assumption 7: The income effect of each good is positive.

4.2 The Model

Let us describe the model as.

$$E^A(p_a + t^A_a, p_b + t^A_b, 1, u^A) - F^A(p_a + t^A_a, p_b + t^A_b, 1) = t^A_a[E^A - F^A_A] + t^A_b[E^A - F^A_B]$$ (11)

$$E^B(p_a + t^B_a, p_b + t^B_b, 1, u^B) - F^B(p_a + t^B_a, p_b + t^B_b, 1) = t^B_a[E^B - F^B_A] + t^B_b[E^B - F^B_B]$$ (12)

$$E^C(p_a, p_b, 1, u^C) = F^C(p_a, p_b, 1)$$ (13)

$$E^A_a - F^A_a + E^B_a - F^B_a + E^C_a - F^C_a = 0$$ (14)

$$E^A_b - F^A_b + E^B_b - F^B_b + E^C_b - F^C_b = 0$$ (15)

where $E^i_j = \frac{\partial E^i}{\partial p_j}$, $F^i_j = \frac{\partial F^i}{\partial p_j}$. The five equations (11)-(15) determine the five unknowns, $u^i, i = A, B, C$, and $p_j, j = a, b$, for given initial tariff rates, $t^A_a, t^A_b, t^B_a,$ and $t^B_b$.

Starting from a given set of tariffs $\{t^A_a, t^A_b, t^B_a, t^B_b\}$, where $t^A_a = t^B_b = 0$ initially (See Assumption 6), we can derive the above system. The above system is the starting point of our tariff reform analysis.

In order to avoid a possible confusion, we shall denote the initial levels of tariffs and equilibrium prices before forming a trading club by

$$t^A_a, t^A_b, t^B_a, t^B_b, p^A_a, p^A_b$$
4.3 A Pareto-Improving Trading Club

Since Assumption 6 means that
\[ t_{a}^{Ae} = 0, \quad t_{b}^{Ae} > 0, \quad t_{a}^{Re} > 0, \quad t_{b}^{Re} = 0, \]

Given the pre-club equilibrium, Country A and Country B form a club and adjust their import and export tariffs. The tariff adjustment scheme is as follows
\[
\begin{bmatrix}
  t_{a}^{i}(\varepsilon_{a}^{i}, \varepsilon_{b}^{i}) \\
  t_{b}^{i}(\varepsilon_{a}^{i}, \varepsilon_{b}^{i})
\end{bmatrix} = \begin{bmatrix}
  t_{a}^{ic} \\
  t_{b}^{ic}
\end{bmatrix}
\]
\[
-(E_{a}^{i} - t_{a}^{ic}E_{ua}^{i} - t_{b}^{ic}E_{ub}^{i}) \left( \begin{array}{cc}
  E_{aa}^{i} - F_{aa}^{i} & E_{ab}^{i} - F_{ab}^{i} \\
  E_{ba}^{i} - F_{ba}^{i} & E_{bb}^{i} - F_{bb}^{i}
\end{array} \right)^{-1} \begin{bmatrix}
  \varepsilon_{a}^{i} \\
  \varepsilon_{b}^{i}
\end{bmatrix}
\]
\[ i = A, B, \]

(16)

Totally differentiating \( t_{b}^{i}(\varepsilon_{a}^{i}, \varepsilon_{b}^{i}) \) and \( t_{b}^{i}(\varepsilon_{a}^{i}, \varepsilon_{b}^{i}) \) with respect to \( \varepsilon_{a}^{i} \) and \( \varepsilon_{b}^{i} \) at \( (\varepsilon_{a}^{i}, \varepsilon_{b}^{i}) = (0, 0) \), we derive
\[
\begin{bmatrix}
  dt_{a}^{i} \\
  dt_{b}^{i}
\end{bmatrix} = -(E_{a}^{i} - t_{a}^{ic}E_{ua}^{i} - t_{b}^{ic}E_{ub}^{i}) \left( \begin{array}{cc}
  E_{aa}^{i} - F_{aa}^{i} & E_{ab}^{i} - F_{ab}^{i} \\
  E_{ba}^{i} - F_{ba}^{i} & E_{bb}^{i} - F_{bb}^{i}
\end{array} \right)^{-1} \begin{bmatrix}
  d\varepsilon_{a}^{i} \\
  d\varepsilon_{b}^{i}
\end{bmatrix}
\]
\[ i = A, B, \]

(17)

Remark 1: Note that both the inverse matrix
\[
\left( \begin{array}{cc}
  E_{aa}^{i} - F_{aa}^{i} & E_{ab}^{i} - F_{ab}^{i} \\
  E_{ba}^{i} - F_{ba}^{i} & E_{bb}^{i} - F_{bb}^{i}
\end{array} \right)^{-1}
\]
and the term \( (E_{a}^{i} - t_{a}^{ic}E_{ua}^{i} - t_{b}^{ic}E_{ub}^{i}) \) are evaluated at the pre-club equilibrium domestic prices and utilities. Therefore, those terms do not depend on \( \varepsilon_{a}^{i} \) and \( \varepsilon_{b}^{i} \), which means that the tariff adjustment mechanism of Country X, (16), is a linear function of \( \varepsilon_{a}^{i} \) and \( \varepsilon_{b}^{i} \).

Now, making a parallel argument to the calculations for the \( n \) by \( m \) case, we obtain
\[
du^{A} = - \left( \begin{bmatrix}
  t_{a}^{Ae} \\
  t_{b}^{Ae}
\end{bmatrix} \begin{bmatrix}
  d\varepsilon_{a}^{A} \\
  d\varepsilon_{b}^{A}
\end{bmatrix}
\right)
\]
\[ \quad (19) \]
\[
du^{B} = - \left( \begin{bmatrix}
  t_{a}^{Re} \\
  t_{b}^{Re}
\end{bmatrix} \begin{bmatrix}
  d\varepsilon_{a}^{B} \\
  d\varepsilon_{b}^{B}
\end{bmatrix}
\right)
\]
\[ \quad (20) \]
\[
\begin{pmatrix}
E_u^A - t_b^a E_{ub}^A \\
\lambda_A E_u^A - t_a^a E_{ua}^A
\end{pmatrix}
= 
\begin{pmatrix}
\frac{\partial}{\partial d_{ca}} \\
\frac{\partial}{\partial d_{cb}}
\end{pmatrix}
+ \begin{pmatrix}
E_b^A - t_b^b E_{ub}^A \\
\lambda_B E_u^A - t_a^a E_{ua}^A
\end{pmatrix}
\]

Note that the adjustments \( d\epsilon_i \), \( i = A, B, j = a, b \), has to satisfy (21) in order that the adjustments keep the trade volumes of three goods with Country C unchanged, in which case the international prices are also unchanged and so is the welfare level of Country C.

**Lemma 1:** If
\[
p_i^e + t_j^e > 0 \text{ and } p_j^e + t_k^e > 0, \quad i = A, B, j, k = a, b, j \neq k,
\]
then each diagonal element and column sums of the two matrices in (21) are positive,
\[
E_u^i - t_j^e E_{uj}^i > 0, \quad i = A, B, j, k = a, b, j \neq k.
\]

**Proof:** Since \( E_u^i \) is linearly homogeneous in three prices, we have
\[
E_u^i = (p_j^e + t_j^e) E_{ju}^i + (p_k^e + t_k^e) E_{ku}^i + 1 \cdot E_{cu}^i
\]
Therefore,
\[
E_u^i - t_j^e E_{uj}^i = (p_j^e + t_j^e) E_{ju}^i + (p_k^e + t_k^e) E_{ku}^i + 1 \cdot E_{cu}^i - t_j^e E_{uj}^i
\]
\[
= p_j^e E_{ju}^i + (p_k^e + t_k^e) E_{ku}^i + 1 \cdot E_{cu}^i,
\]
which is positive as long as positive income effects prevail and under (22). (QED)

Now, we know that in the present case
\[
t_b^Be > 0, \quad t_b^Ae > 0,
\]
and
\[
t_a^Ae = 0, \quad t_b^Be = 0
\]
Having these sign patterns in mind, let me rearrange (21) in the following way,

\[
\begin{pmatrix}
E^A_a - t_b^A E^A_{ub} & 0 \\
0 & d^A_a
\end{pmatrix} d^A_B + \begin{pmatrix}
t_b^A E^A_{ua} & E^A_a \\
E^B_a - t_b^B E^B_{ua} & 0
\end{pmatrix} d^B_B
= \begin{pmatrix}
E^B_a & t_b^B E^B_{ub} \\
t^A_a E^A_{ub} & E^A_a
\end{pmatrix}
d^A_a + \begin{pmatrix}
t^B_e E^A_{ua} & 0 \\
E^B_a - t_b^B E^B_{ua} & 0
\end{pmatrix} d^B_B
= \begin{pmatrix}
E^B_a - t_b^A E^A_{ub} & 0 \\
t^A_a E^A_{ub} & E^A_a
\end{pmatrix}
d^A_a + \begin{pmatrix}
0 & E^B_a - t_b^B E^B_{ua} \\
E^B_a - t_b^B E^B_{ua} & 0
\end{pmatrix} d^B_B
\]

That is, we derive

\[
\begin{pmatrix}
E^B_a - t_b^B E^B_{ub} & t_b^A E^A_{ub} \\
t^A_a E^A_{ub} & E^A_a
\end{pmatrix}
d^A_a + \begin{pmatrix}
t^B_e E^A_{ua} & 0 \\
E^B_a - t_b^B E^B_{ua} & 0
\end{pmatrix} d^B_B
= \begin{pmatrix}
E^B_a & t_b^B E^B_{ub} \\
t^A_a E^A_{ub} & E^A_a
\end{pmatrix}
d^A_a + \begin{pmatrix}
0 & E^B_a - t_b^B E^B_{ua} \\
E^B_a - t_b^B E^B_{ua} & 0
\end{pmatrix} d^B_B
\]

which corresponds to (10). It follows Lemma 1 that all elements of the matrix at the LHS of (26) are positive, and all elements of the inverse matrix

\[
\begin{pmatrix}
E^A_a - t_b^A E^A_{ub} & 0 \\
0 & E^B_a - t_b^B E^B_{ua}
\end{pmatrix}^{-1}
\]

are also non-negative. Since

\[
\begin{pmatrix}
d^A_a \\
d^B_B
\end{pmatrix} = - \begin{pmatrix}
E^A_a - t_b^A E^A_{ub} & 0 \\
0 & E^B_a - t_b^B E^B_{ua}
\end{pmatrix}^{-1} \begin{pmatrix}
E^B_a & t_b^B E^B_{ub} \\
t^A_a E^A_{ub} & E^A_a
\end{pmatrix}
\begin{pmatrix}
d^A_a \\
d^B_B
\end{pmatrix}
\]

it follows that if \(d^A_a\) and \(d^B_B\) are chosen so that (27) is satisfied for any \(d^A_a < 0\) and \(d^B_B < 0\), then \(d^A_a > 0\) and \(d^B_B > 0\) and the tariff adjustments leave the club’s trade volumes with Country C unchanged and the international prices do not change, which means that Country C’s welfare is not affected by the tariff adjustments. Moreover, combining

\[
d^A_a < 0, \quad d^B_B < 0, \quad d^A_a > 0, \quad d^B_B > 0
\]

with

\[
t^B_e > 0, \quad t^A_a > 0, \quad t^A_e = 0, \quad t^B_e = 0,
\]

10
we see that

\[ du^A = -[t_a^A d\varepsilon_a^A + t_b^A d\varepsilon_b^A] \]
\[ = -[(0)(+) + (+)(-)] > 0 \]

\[ du^B = -[t_a^B d\varepsilon_a^B + t_b^B d\varepsilon_b^B] \]
\[ = -[(+)(-) + (0)(+)] > 0 \]

Hence both \( du^A \) and \( du^B \) are positive.

**Proposition:** If the initial tariff-ridden equilibrium satisfies (??) and (??), then the implementation of the tariff adjustment scheme (??) (or one could say (17)) makes Country A and Country B better off without hurting Country C.

**Remark 2:** Since

\[
\begin{pmatrix}
\varepsilon_a^i (0, 0) \\
\varepsilon_b^i (0, 0)
\end{pmatrix} = \begin{pmatrix}
\varepsilon_a^i \\
\varepsilon_b^i
\end{pmatrix}
\]

the tariff adjustments are expressed by a small change in tariffs \((d\varepsilon_a^i, d\varepsilon_b^i)\) from their pre-club levels.

**Remark 3** Consider the direction of tariff adjustment, determined by

\[
\begin{pmatrix}
dt_a^i \\
dt_b^i
\end{pmatrix} = \begin{pmatrix}
dt_a^i (\varepsilon_a^i, \varepsilon_b^i) \\
dt_b^i (\varepsilon_a^i, \varepsilon_b^i)
\end{pmatrix}
\]

\[ = -(E_a^i - t_a^i E_{aa} - t_b^i E_{ab})(E_{ba} - F_{bb})^{-1} \begin{pmatrix} d\varepsilon_a^i \\ d\varepsilon_b^i \end{pmatrix} \]

If

\[
\begin{pmatrix} E_a^i - F_{aa} \\ E_{ba} - F_{ba} \end{pmatrix}^{-1} = \begin{pmatrix} (-) & (+) \\ (+) & (-) \end{pmatrix}
\]

then we find \( dt_a^A > 0, dt_a^B < 0, dt_b^A < 0, dt_b^B > 0 \), that is, tariffs are adjusted in the direction to level them (recall that \( t_a^A = 0, t_b^A > 0, t_a^B > 0, t_b^B = 0 \)).

However, in general, the signs of elements in this inverse matrix are ambiguous, and so the signs of \( d\varepsilon_j^i \) \((i = A, B; j = a, b)\).