Trade and Firm Heterogeneity In A Quality-Ladder Model of Growth

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Abstract

The present paper explores the effect of trade liberalization on the level of productivity as well as the rate of productivity growth in an R&D-based model with heterogeneous firms. We introduce new and plausible features that are absent in existing studies. First, technical progress takes the form of continual quality improvement of products over time. Second, firm entry and exit are endogenously determined due to creative destruction of products traded. In this framework, we demonstrate that a lower transport cost or export sunk cost unambiguously reallocates resources from non-exporting industries to R&D as well as exporting industries. This means that trade liberalization increases the level of manufacturing productivity and the rate of technical progress. These results are found to be robust in an extended model with population growth without scale effects. In extensions of the basic model, we also endogenize the ex ante distribution of firm heterogeneity and examine the effect of R&D subsidies.

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1 Introduction

Since the late 1990s, many empirical studies provide evidence regarding the microeconomic aspects of exporting firms (e.g. Bernard, Jensen, and Lawrence (1995) and Tybout (2003)). These firms are found to be bigger in employment, more skill- and capital-intensive, more productive and grow faster. To explain for those characteristics, firm heterogeneity is introduced into traditional models of trade. Pioneering studies are Eaton and Kortum (2002), Bernard, Jensen, and Kortum (2003) and Melitz (2003). However, those models are essentially static in nature. Therefore, although they are able to answer questions on changes in the *level* of productivity due to freer trade, they are not suitable to answer "dynamic" questions on changes in the *rate* of productivity growth. Insights on the latter issue are clearly important, given empirical evidence showing that exporting firms grow faster than non-exporters (e.g. Bernard and Jensen, 1999) and that trade and economic growth are positively correlated (e.g. Wacziarg and Welch, 2008). It is also imperative to inform policy makers of the growth effect of trade liberalization on the basis of rigorous theoretical reasoning.

The first objective of the present study is to fill this gap by developing an R&D-based model of international trade with heterogeneous firms. In the model, productivity growth is driven by endogenous technical progress in the form of continual quality improvement of products over time. This quality-ladder approach departs from existing studies, and the present paper represents the first attempt to introduce a quality-ladder model into the literature on trade with heterogeneous firms.[1] Closest to the present paper are Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2007), who develop R&D-based models of trade with heterogeneous firm. However, their studies are based on the variety expansion approach, pioneered by Melitz (2003).

The second objective of our paper is to generate new insights on the growth effect of trade liberalization in a model with heterogeneous firms. Baldwin and Robert-Nicoud (2008) demonstrate that freer trade encourages or discourages long-run growth, depending upon the structure of knowledge assumed. The model is extended in Gustafsson and Segerstrom (2007) by introducing pop-

[1] Quality-ladder models of growth are also called Shumpeterian growth models. See Segerstrom, Anant, and Dinopoulos (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) for early contributions.

ulation growth. They establish that globalization boosts productivity growth if the strength of knowledge spillovers is weak enough, but otherwise, productivity growth falls. Indeed, it seems difficult to reconcile their mixed results with many empirical studies which consistently show a positive relationship between trade and growth.[2] In addition, the "semi-endogenous" growth model of Gustafsson and Segerstrom (2007) also shows that trade liberalization does not affect the share of workers devoted to R&D in total working population.[3] It means that R&D incentives are independent of trade liberalization in the long run.

We re-examine those mixed results, using the quality-ladder framework of technical progress. More specifically, we will demonstrate that trade liberalization (i) reallocates resources from non-exporting industries to R&D as well as exporting industries, and (ii) unambiguously promotes technical progress in contrast with the above studies. We also derive conditions for a monotonic increase in welfare of trading economies (e.g., a sufficiently large market size) as they become more open to international trade. Those key results are found to be robust in an extended model incorporating population growth.

Turning to the description of the model, we consider two identical open economies, competing in the world market with transportation costs. We assume that the introduction of higher quality goods requires costly R&D, in addition to sunk costs for implementing innovation (or equivalently interpreted as entry cost into the domestic market) and beachhead costs for exporting. Manufacturing productivity is randomly drawn from a given ex ante probability distribution, as in Melitz (2003). A sufficiently advanced innovation, which results in a sufficiently low marginal cost of production along with a higher quality, enables entrepreneurs to capture monopoly rents in the foreign market as well as the domestic market. On the other hand, a less-advanced innovation only enables entrepreneurs to capture the domestic monopoly rents. A least-advanced innovation that results in a higher marginal cost than a cut-off level forces entrepreneurs to give up on implementation of innovation. After each innovation and subsequent implementation, product quality is improved,

[2] See Wacziarg and Welch (2008) for one of the latest studies on this issue. Indeed, Rodrik and Rodríguez (2000) point out the lack of credible evidence on a systematic negative correlation between openness and growth. Alesina, Spolaore, and Wacziarg (2005, p.1515) call it "a huge achievement" in the empirical literature.

[3] In semi-endogenous growth models, the rate of technical progress is pinned down by population growth in the long run, but the resource allocation to R&D is still endogenously determined. See Jones (2003) for a survey.

driving productivity growth in the long run.

Our model with quality improvement possesses several interesting features worth mention. First, in the literature, the exit of firms from the market in variety-based models is assumed to be an exogenous process. That is, no reason is given for why firms go out of business. In our model, in contrast, firm exits are endogenously determined, as quality improvement of a given product causes existing products obsolete. That is, firms exit form the market due to creative destruction caused by technical progress.

Second, quality improvement enables innovators to leapfrog incumbent firms. Because of this feature, the *direction* of trade in a given industry is reversed whenever innovation results in exportable products. This feature is consistent with the observation that innovation determines comparative advantage of products (e.g. cars and computers) and affects the direction of trade. In contrast, in variety-based models, the direction of trade of a given product remains unchanged until the firm exits from the market for an exogenous reason.

Third, in trade models with heterogeneous firms, a given industry can be *open* or *closed* to international trade due to export sunk costs. In addition to this feature, in our model, the state of a given industry (i.e. open or closed to trade) dynamically changes due to continual innovation in the world market. This arises because the state-of-the-art products may not be exported, in which case lower-quality goods are consumed in the non-innovating country. In this sense, there is asymmetry in terms of quality levels consumed in the two economies, although they are structurally symmetric.

The key result of our study concerns the effect of trade liberalization on the level of manufacturing productivity and the rate of technical progress. A lower transport cost or export cost is shown to induce inefficient firms to exit from the market. As a result, exporting industries expand and manufacturing productivity rises in consistent with many empirical studies. Regarding innovative activities, trade liberalization unambiguously reallocates resources to R&D, accelerating the rate of technical progress. The reason can be understood by identifying two channels that work to bring about this pro-growth result. First, trade liberalization, which expands exporting industries, increases ex ante sunk costs for developing a profitable product which includes costs for exporting. Resources are diverted from R&D through this sunk cost channel, discouraging R&D. Second, trade liberalization allows monopoly firms to raise the price-cost margin in the foreign market. Profits increase through this monopoly markup channel, boosting R&D incentives. In equilibrium, the monopoly markup channel always dominates the sunk cost channel, giving rise to our key result. In contrast, in Baldwin and Robert-Nicoud (2008), the monopoly markup channel disappears in equilibrium due to the CES production function used to model variety expansion. Through the remaining sunk cost channel, trade liberalization encourages or discourages technical progress, depending upon the structure of knowledge assumed. The same reason applies for the result of Gustafsson and Segerstrom (2007) that the share of workers devoted to R&D is unaffected by trade liberalization.

Robustness of these key results are checked by extending the basic model to incorporate population growth. We show that the proportion of workers devoted to R&D in total population increases due to globalization. In this sense, our key results established in the basic model with a constant population still hold in an arguably more plausible setting with population growth and without scale effects.

In existing trade models with heterogeneous firms, productivity is randomly drawn from a given *ex ante* distribution, and the *ex post* distribution of firm productivity is determined in equilibrium. We show how the ex ante distribution of firm productivity can be endogenized in an extended model. Our result indicates that an increase in manufacturing productivity is realized through the ex post distribution rather than changes in the ex ante distribution of firm heterogeneity. The issue is related to validity of the widely used assumption that the ex ante distribution is exogenously given.

We also investigate the policy impact of an R&D subsidy. The policy is shown to force inefficient firms to exit from both the domestic and foreign markets. That is, the subsidy increases the average productivity of operating firms, but it does not necessarily promote export. On the other hand, it is not clear whether R&D is promoted. This is partly because the policy makes it less likely that a given innovation is implemented, adversely affecting R&D incentives.

Product quality is often found to be important in the understanding of trade patterns. For example, recent studies emphasize product quality in explaining for price differences across countries. Richer countries are also found to export and import higher-quality products (e.g. Schott (2004), Hummels and Klenow (2005) and Hallak (2006)). For example, Baldwin and Harrigan

(2007) introduce product quality into a model a lá Melitz (2003). Other studies include Hallak and Sivadasan (2006), Helble and Okubo (2006) and Gervais (2008). Those models, however, are essentially static in nature and the level of quality is fixed in equilibrium. In our model, on the other hand, the quality of products improves over time due to costly innovative activities, and it is the source of productivity growth in the long run.

In terms of investment in innovative activity in an open economy framework, Atkeson and Burstein (2006) is closely related to our study. In their model, firms invest in cost reduction after entry until they exit for exogenous reasons. The authors examine the effect of freer trade on process innovation after market entry. On the other hand, the present study focuses on incentives for product innovation before firms enter the market. In addition, we analytically solve the model, whereas quantitative simulation is used in Atkeson and Burstein (2006).

The structure of the paper is as follows. Section 2 develops the basic model where quality improvement drives growth in the presence of heterogeneous firms. Steady state equilibrium is characterized in Section 3, and Section 4 analyzes the effect of trade liberalization on the distribution of firm productivity and technical progress. Section 5 introduces population growth in order to check the robustness of key results. It is followed by discussion on key differences between the present model and closely related studies in Section 6. In Section 7, we endogenize the ex ante distribution of firm heterogeneity, and the impact of R&D subsidies is also considered. Final remarks are given in Section 8.

2 The Basic Model Setup

2.1 Consumers

There are two identical economies, indexed by 1 and 2. The country index is suppressed unless otherwise ambiguity arises. In each economy, there are two production sectors, manufacturing and R&D with L number of workers. L is taken as fixed, until Section 5. The utility function of the representative consumer is

$$U = \int_0^\infty e^{-\rho t} \ln Y_t dt \tag{1}$$

where ρ is the subjective rate of time preference and Y denotes consumption of final output. Dynamic utility maximization requires

$$\frac{\dot{E}_t}{E_t} = r_t - \rho \tag{2}$$

where r is the rate of interest and E denotes consumption expenditure.

2.2 Production Technology

2.2.1 Final Output

Final output is produced under perfect competition by assembling a range of intermediate goods. Specifically, the production function takes the Cobb-Douglas form:

$$Y_t = \exp\left\{\int_0^1 \ln \lambda^{k_{it}} y_{it} di\right\}, \quad \lambda > 1, \quad k_{it} = 0, 1, 2... \tag{3}$$

where y_i denotes the quantity of intermediate goods i, and λ^{k_i} represents the quality level of the products. Given the Cobb-Douglas technology, the demand for intermediates good is given by

$$y_{it} = \frac{E_t}{p_{it}} \tag{4}$$

where $E = PY$, P is the price of final output and p_i is the price of y_i.

2.2.2 Intermediate Goods

Intermediate goods are produced, using labor only. We normalize the wage rate to one and assume that each intermediate good is produced by firms with marginal cost

$$a(c) = c_L + c, \quad 0 < c_L < \infty. \tag{5}$$

In addition, c is a random variable drawn from the distribution function

$$Z(c), \quad c \in (0, c_H), \quad 0 < c_H < \infty. \tag{6}$$

Intermediate goods are differentiated in quality within each industry. To become a monopoly, firms must first succeed in R&D to create a blueprint for the state-of-the-art products. The true value of c is revealed only after firms succeed in R&D.

In addition, there is an iceberg trade cost: $\tau > 1$ units of goods must be shipped in order for 1 unit to arrive in a foreign country. This means that the "effective" marginal cost of exported goods in the foreign market is $\tau a(c)$.

2.3 Price and Profits

Product quality improvement is the engine of growth in our model. Each innovation generates a blueprint for the product whose quality is higher than the state-of-the-art in the industry by a factor λ, irrespective of whether it occurs in country 1 or 2. That is, successful R&D firms always leapfrog the incumbent firm in quality.

Furthermore, we assume technology diffusion within an economy and between the two countries. Specifically, the second-highest quality goods can be competitively produced in any country with marginal cost c_L.[4] This means that the technological gap between two countries in a given industry, which are defined as $|k_{i1} - k_{i2}|$, is one at most.[5] An important consequence of such technological diffusion is that firms producing the top-quality goods always face price competition from competitive producers of the second-highest quality goods.

Because of this feature, firm's pricing behaviors are the same in all industries in the domestic and foreign markets. Since the price elasticity of demand is one, monopoly firms set the price of the state-of-the-art products at $c_L\lambda$, given that marginal cost of lower quality goods is c_L.

Firms that succeed in R&D can be grouped into three types, depending on the realized value of c. First, if c is too high, firms do not enter the market due to the presence of sunk costs required to implement innovation. Second, if c lies in the "middle" range, firms serve the domestic market only, with profits

$$\pi(c) = \left(1 - \frac{a(c)}{c_L\lambda}\right) E. \tag{7}$$

Third, for sufficiently low values of c, firms can serve both the domestic and foreign markets after paying a beachhead cost. It earns $\pi(c)$ in the domestic

[4] One could imagine a situation in which patents expire in the world market once higher-quality goods are created. In this case, the top-quality goods only are protected by patents.

[5] The same assumption is used in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) in a different context.

market and

$$\pi_x(c) = \left(1 - \frac{\tau a(c)}{c_L \lambda}\right) E \tag{8}$$

in the foreign market. Profits increase in the size of innovation λ, but decrease in the transport cost τ.

2.4 Entry Decisions

To enter the market, firms must incur two or three kinds of sunk costs, depending on firm types. First, **R&D costs** to create a blueprint for higher quality goods are required. R&D costs are sunk in the sense that it cannot be recovered, even if firms decide to exit. Note that marginal cost c is unknown during an R&D race. The true value of c becomes observable once firms succeed in R&D. Second, firms must also incur sunk costs f to implement the newly invented technology. **Implementation costs** are equivalently interpreted as costs of entry into the domestic market. Those costs must be incurred regardless of whether or not firms export their goods. Third, if firms decide to export their products, they must incur **export costs** f_x, which are additional sunk costs for exporters (i.e., beachhead costs).

To describe firm behaviors, consider first domestic firms. The present discounted values of profits in the domestic market (7) and in the foreign market (8) are denoted by $v(c)$ and $v_x(c)$, respectively. Firms pay implementation costs if $v_t(c) \geq f$, and become a monopoly in the domestic market, replacing the incumbent. However, if $v_t(c) < f$, firms do not enter, and the incumbent remains in the domestic market. In this case, even though new knowledge (a blueprint of higher quality goods) is invented, it is not implemented and the new good is not produced. We assume that the realized value of c is specific to the knowledge created and the industry to which it is applied, hence no other firms would find it profitable to produce the good. The cut-off value of c, denoted by C, at which firms are indifferent between paying and not paying implementation costs is defined by

$$v_t(C) = f, \qquad 0 < C \leq c_H. \tag{9}$$

Firms implement innovation for $c \leq C$, and not otherwise. Note that the implementation decision is a stable process because $\frac{\partial \pi(c)}{\partial c} < 0$ from (7), which means $\frac{\partial v(c)}{\partial c} < 0$.

Next, consider exporting firms, which earn profits (7) and (8) in the domestic and foreign markets, respectively. Similar reasonings used above apply here. Firms export their goods if the value of c is such that $v_x(c) \geq f_x$, but not otherwise. Therefore, the cut-off value of c, denoted by C_x, is defined by

$$v_x(C_x) = f_x, \quad 0 < C_x \leq c_H. \tag{10}$$

Firms export their products if $c \leq C_x$, and not otherwise. Note that the realized value of c is different across industries, though the cut-off values C and C_x are the same in all industries.

2.5 R&D Investment

The true value of c is unobservable during an R&D race. We use V_t to denote the ex ante value of innovation, which firms expect to achieve if they succeed in R&D. Innovation is assumed to arrive at a Poisson rate of

$$I_t = \frac{R_t}{f_r}, \qquad f_r > 0, \tag{11}$$

when R workers are used. Therefore, research firms choose the optimal number of R&D workers by solving

$$\max_{R_t} \; V_t I_t - (1 - s) R_t \tag{12}$$

where s is the rate of R&D subsidy. The first order condition is

$$V_t = (1 - s) f_r. \tag{13}$$

2.6 Industry Dynamics

Since the two countries are assumed to be structurally identical, a half of goods are produced in country 1, and the other half are produced in country 2. In addition, industries can be grouped into two types, depending on whether or not the state-of-the-art products are traded. They are termed type-A and type-B industries with key features summarized in Table 1.

Types of Industries	A	B
Measure of Industries	N_A	N_B
(i) Trade or Not	Top-quality products traded	No trade
(ii) Consumption	Top-quality products consumed in both countries	Top-quality consumed in the innovating country, and the 2nd highest quality in the other

Table 1: Types of industries.

Column A identifies industries in which firms produce the state-of-the-art products which are exported to the foreign country. The measure of such industries is denoted by N_A. Type-A industries are open to trade. This situation arises when innovation occurs with marginal cost being sufficiently small so that $0 \leq c \leq C_x$.

In column B, firms produce the top-quality goods, which are not exported because marginal costs is not sufficiently low, i.e., $C_x < c \leq C$. Therefore, the state-of-the-art products are consumed in the country where innovation take place, but not in the other country. On the other hand, because of technology diffusion, products consumed in the non-innovating country are second-highest in quality on the product line and competitively produced.

Given that the measure of industries is one, we have

$$1 = N_A + N_B. \tag{14}$$

Although this equation must hold at each moment of time, the type of a particular industry continually changes as innovation occurs either in country 1 or 2. That is, trade patterns in a given industry change as innovations occur. Such changes in industry types are described in Figure 1.

To explain for the meaning of arrows in the figure, consider an industry of type-A. If innovation takes place in either country with $1 \leq c \leq C_x$, the state-of-the-art products are still exported, and industry type does not change. In this case, the industry stays in the N_A group in the figure. However, suppose that innovation occurs with $C_x < c \leq C$ in either country. Since c is in the middle range, an innovating firm implements its latest technology and produces the state-of-the-art products for the domestic market only. That is, the industry moves from type-A to type-B, and this is represented by the rightward arrow. Since this transition occurs with the Poisson rate of $[Z(C) - Z(C_x)](2I)$, the number of type-A industries switching to type-B industries during an infinitesimal time period dt is $N_A [Z(C) - Z(C_x)](2I)$.

Next, consider an industry of type-B. Innovation with $C_x < c \leq C$ in either country does not change the type of industry, since an innovating firm does not export their top-quality products. In this case, the second-highest quality goods are competitively produced and consumed in an non-innovating country. On the other hand, innovation with $0 \leq c \leq C_x$ in either country moves the industry to type-A from type-B. The state-of-the-art products are consumed in both countries thanks to exporting. This case is captured by the leftward arrow in Figure 1. The term attached to the arrow shows the number of industries moving to group A from group B during a small time interval dt. In summary, not all countries can consume the highest quality products due to fixed costs of exporting. Only the state-of-the-art products with marginal cost below the cutoff level (C_x) are consumed in all countries.

Given the above discussion, the measure of type-A industries change according to

$$\dot{N}_{At} = N_{Bt} Z(C_x)(2I_t) - N_{At} [Z(C) - Z(C_x)](2I_t). \tag{15}$$

This equation implies that the direction of changes in N_A is determined by C, C_x and N_A, and independent of the arrival rate of innovation I.

2.7 Value of Firms and Innovation

Recall that $v(c)$ and $v_x(c)$ denote the expected present values of flow profits $\pi(c)$ and $\pi_x(c)$, respectively. Firms with successful innovation gain $v(c)$ in the domestic market. However, the value of $v(c)$ drops to zero if innovation with $0 \leq c \leq C$ occurs in either country. It is either because the second-highest quality goods are no longer demanded or because they are competitively produced. Therefore, $v_t(c)$ is defined by the following asset equation

$$r_t v_t(c) = \pi_t(c) + \dot{v}_t(c) - Z(C)(2I_t) v_t(c). \tag{16}$$

$v_x(c)$, which firms with successful innovation gain in the foreign market, is similarly defined by

$$r_t v_{xt}(c) = \pi_{xt}(c) + \dot{v}_{xt}(c) - Z(C)(2I_t) v_{xt}(c). \tag{17}$$

Note that $v(c)$ and $v_x(c)$ are ex post values in the sense that they are conditional on the realized value of c. Moreover, implementation costs must be incurred to obtain $v_t(c)$, and export costs must be sunk to gain $v_x(c)$. Therefore, the ex ante expected value of innovation in net (before c is revealed) is defined by

$$V_t = \int_0^C [v(c) - f] \, dZ(c) + \int_0^{C_x} [v_x(c) - f_x] \, dZ(c). \tag{18}$$

The first term represents expected net gains from domestic sales, and the second term captures additional gains from exports.

2.8 Labor Market Condition

Workers are used for four purposes: R&D, implementation of innovation, sunk costs for export and manufacturing. Given the measure of industries being one, the total number of R&D workers in a country is R.

During a small time interval, $Z(C)I$ is equivalent to the number of industries where R&D succeeds and innovation is implemented simultaneously. Since f workers are used for implementation in each industry, $fZ(C)I$ gives the total number of workers used for innovation implementation. Similarly, the total number of workers used for export sunk costs is $f_x Z(C_x) I$.

Turning to the manufacturing sector, first consider type-B industries, in which there are two sorts of firms. In one half of the industries, the domestic market is served with the state-of-the-art products, which are produced by local monopoly firms, but not exported. Thus, the labor demand of each of those manufacturing firms is $Ea(c)/c_L\lambda$, and the total labor demand of those firms is equivalent to

$$\ell_B = \frac{N_B}{2} \frac{E_t}{c_L \lambda} \int_{C_x}^{C} a(c) \frac{dZ(c)}{Z(C) - Z(C_x)}. \tag{19}$$

In the other half of type-B industries, the second-highest quality goods are competitively produced with marginal cost being c_L. Therefore, the labor demand of competitive firms is

$$\frac{N_B}{2}E_t. \tag{20}$$

Turning to type-A industries, firms in one half of the industries serve the domestic market with the state-of-the-art products and also export them to the foreign country. In the other half of the industries, the top-quality products are imported from abroad. Therefore, we only need to consider the half of type-A industries where domestic firms operate. Given the price of goods being $c_L\lambda$, the labor demand arising from domestic sales is given by

$$\ell_A = \frac{N_A}{2}\frac{E_t}{c_L\lambda}\int_0^{C_x} a(c)\,\frac{dZ(c)}{Z(C_x)}. \tag{21}$$

On the other hand, the marginal cost of producing exported goods is $\tau a(c)$, taking the transport cost into account. Therefore, the labor demand arising from foreign sales is

$$\tau\ell_A. \tag{22}$$

Then, the full employment condition in the labor market in an economy is

$$L = R_t + fZ(C)\,I_t + f_xZ(C_x)\,I_t + \ell_t \tag{23}$$

where $\ell \equiv \frac{N_B}{2}c_LE_t + \ell_B + (1+\tau)\,\ell_A$ is the total labor demand in manufacturing.

3 Steady State Equilibrium

3.1 Four Equilibrium Conditions

Using (2), (8), (7), (16) and (17), we derive the following conditions.

$$f = \frac{\left(1-\dfrac{a(C)}{\lambda}\right)E}{\rho + 2Z(C)\,R/f_r}, \qquad f_x = \frac{\left(1-\dfrac{\tau a(C_x)}{\lambda}\right)E}{\rho + 2Z(C)\,R/f_r} \tag{24}$$

where $r = \rho$ is also used. The right-hand sides of those equations define the expected present values of profits in the domestic and foreign markets when marginal costs take the threshold values C and C_x, respectively. Those conditions determine C and C_x, ceteris paribus. In this sense, (24) captures firms' decisions on implementation of innovation and entry into the foreign market.

The third equilibrium condition is related to firms' R&D decisions. The ex ante value of innovation during an R&D race is given by (18). In addition, free entry in R&D activity is captured by (13). Then, using those equations along with (13), (16), (17) and (18), one can derive the following condition:

$$\frac{\Lambda\left(C, C_x; \tau\right) E}{\rho+2 Z\left(C\right) R / f_r}=\tilde{F}\left(C, C_x; f_x, s\right) \tag{25}$$

where

$$\tilde{F}\left(C, C_x; f_x, s\right) \equiv \frac{(1-s) f_r}{Z\left(C\right)}+f+f_x \frac{Z\left(C_x\right)}{Z\left(C\right)} \tag{26}$$

and

$$\begin{aligned} \Lambda\left(C, C_x; \tau\right) \equiv & \int_0^C\left(1-\frac{a\left(c\right)}{\lambda}\right) \frac{d Z\left(c\right)}{Z\left(C\right)} \\ & +\frac{Z\left(C_x\right)}{Z\left(C\right)} \int_0^{C_x}\left(1-\frac{\tau a\left(c\right)}{\lambda}\right) \frac{d Z\left(c\right)}{Z\left(C_x\right)} \end{aligned} \tag{27}$$

(25) is the R&D incentive condition. Its left-hand side is the ex ante value of innovation, conditional on innovation being implementable. Its right-hand side is interpreted as the ex ante fixed costs of developing a profitable product. The first term of (26) is the average R&D costs incurred until R&D succeeds. The second term is the cost of implementation of innovation, and the third term is the expected cost of entry into the foreign market, given that innovation is implementable. Turning to (27), it is interpreted as the expected rate of monopoly markup over marginal cost, given that innovation is implementable. Its first and second terms concern the markup rate in the domestic and foreign markets, respectively. Note that $Z\left(C_x\right)/Z\left(C\right)$ is the probability that products based on implementable innovation are exported.

The final equilibrium condition is based on the full-employment condition (23). To derive it, note that the number of industries of each type in steady state can be calculated from (15). Steady state values of N_A and N_B are given by

$$N_A=\frac{Z\left(C_x\right)}{Z\left(C\right)}, \qquad N_B=\frac{Z\left(C\right)-Z\left(C_x\right)}{Z\left(C\right)}. \tag{28}$$

Condition (15) also shows that (28) is stable, given $Z\left(C_x\right)<Z\left(C\right)$ and $1> N_A>0$.[6] A convenient feature is that N_A and N_B depend on C and C_x

[6]Consider $N_A=0$. In this case, $\dot{N}_A>0$. On the other hand, we have $\dot{N}_A<0$ for $N_A=1$,

only, which in turn means that C and C_x must be constant in steady state. Note that N_A is equivalent to the probability that a newly invented product is exported, given that innovation is implementable. It is also useful to note that if the export cost f_x is so large that $C_x = 0$, then there is no exporting industry, i.e. $N_A = 0$, and all industries are of type-B. This case is equivalent to a closed economy, but with knowledge diffusion between countries.

Given (28), Appendix A shows that the total labor demand in manufacturing is given by

$$\ell = E - \frac{\Lambda\left(C, C_x; \tau\right) E}{2}. \tag{29}$$

The second term on the right-hand side is equivalent to total profits earned by firms in the economy. Indeed, $\Lambda\left(C, C_x; \tau\right) E$ is aggregate profits in the world, and a half of them accrues to firms based in one economy. Using (26) and (29), the labor market condition (23) can be re-expressed as

$$L = \frac{R}{f_r} Z\left(C\right) F\left(C, C_x; f_x\right) + \left(1 - \frac{\Lambda\left(C, C_x\right)}{2}\right) E. \tag{30}$$

where

$$F\left(C, C_x; f_x\right) \equiv \frac{f_r}{Z\left(C\right)} + f + f_x \frac{Z\left(C_x\right)}{Z\left(C\right)}. \tag{31}$$

Note that $F\left(C, C_x; f_x\right) = \tilde{F}\left(C, C_x; f_x, s\right)$ for $s = 0$. (24), (25) and (30) constitute the system of four equations with four unknowns, C, C_x, R and E.

3.2 Determination of Productivity Growth

In our model, productivity growth occurs due to quality improvement of products. To derive the rate of technical progress, let us identify the global technological frontier by the index constructed on the basis of the state-of-the-art products in all industries. Based on the production function (3), we consider the following index

$$Q_t^W = \exp\left(\int_0^1 \ln \lambda^{\tilde{k}_{it}} di\right) \tag{32}$$

where $\lambda^{\tilde{k}_{it}}$ is the highest quality achieved in the world economy as a whole. Note that the world technological frontier advances whenever innovation occurs as long as $c \leq C$. Therefore, the rate at which the world technological frontier

since $Z\left(C_x\right) < Z\left(C\right)$.

advances can be written as

$$g = (\ln \lambda) Z(C)(2I). \tag{33}$$

To understand this expression, note that each innovation improves the level of product quality by a factor λ in each industry, and such innovation arrives at the rate of $Z(C)(2I)$. Note that g and C are positively related. It is because technical progress depends on innovations which are implemented. Innovations which are not implemented do not count. Due to this property, the rate of quality improvement is inversely related to the level of manufacturing productivity.

Let us turn to the rate of technical progress from the perspective of each individual country. In general, the average *level* of quality of the products available in a given country is lower than the world technological frontier, since the second-highest quality products are competitively produced and consumed domestically in a half of type-B industries. However, note that unlike the world technological frontier, innovation raises the quality level of products by a factor λ or λ^2, depending on industry types. Because of this property, the ratio of the average quality level in a given economy to the world technological frontier is constant in equilibrium. Indeed, Appendix B shows that the rate at which quality improves in each country is equivalent to (33).

4 Effects of Trade Liberalization

In this section, we assume $s = 0$ (i.e. no R&D subsidy), as we are interested in the effect of trade liberalization. The effect of the industrial policy will be discussed in Section 7.2.

4.1 Manufacturing Productivity

Here, we consider the effect of trade liberalization on the threshold levels of marginal costs. Combine the equations in (24) to derive:

$$a(C) = \frac{\tau f}{f_x} a(C_x) + \left(1 - \frac{f}{f_x}\right)\lambda. \tag{34}$$

We assume $f_x > f$, which is consistent with empirical studies and ensures that $C > C_x$ in equilibrium. This equation defines the combination of C and C_x at which the threshold conditions (9) and (10) simultaneously hold. (34) shows that C and C_x are positively related. The intuition is simple. The condition basically determines the relative ex post values (i.e. after c is revealed) of the two threshold firms with marginal costs C and C_x. A higher C reduces the value of the firm indifferent between serving the domestic market only and shutting down. A constant relative profitability is maintained with a lower value of the firm indifferent to export, which is realized via a higher C_x. For later use, (34) is succinctly written as

$$C_x = C_x\left(C;\tau,f_x\right) \qquad \text{or} \qquad C = C\left(C_x;\tau,f_x\right). \tag{35}$$

Substituting the first equations of (24) and (35) into (25), the following condition can be derived:

$$f_r = D\left(C;\tau,f_x\right) \tag{36}$$

where

$$\begin{aligned} D\left(C;\tau,f_x\right) \equiv & f\int_0^{C}\left(\frac{1-\frac{a(c)}{c_L\lambda}}{1-\frac{a(C)}{c_L\lambda}}-1\right)dZ\left(c\right) \\ & +\int_0^{C_x(C;\tau,f_x)}\left(\frac{1-\frac{\tau(a(c))}{c_L\lambda}}{1-\frac{a(C)}{c_L\lambda}}f-f_x\right)dZ\left(c\right) \end{aligned} \tag{37}$$

$D\left(C\right)$ is monotonically increasing in C, as depicted in Figure 2. Intuitively, $D\left(C\right)$ is equivalent to the ex ante value of innovation during an R&D race. A higher C means that a given innovation becomes more likely to be implemented. This raises the value of innovation. In Figure 2, the equilibrium value of C is found at the intersection point between the D curve and a horizontal line at f_r.

We can conduct a similar analysis for the determination of C_x. Eliminate C_x in the R&D incentive condition (25) by making use of the second equations of (24) and (35) to obtain

$$f_r = X\left(C_x;\tau,f_x\right) \tag{38}$$

where

$$\begin{aligned} X\left(C_x;\tau,f_x\right) \equiv \int_0^{C(C_x;\tau,f_x)} \left(\frac{1-\frac{a(c)}{c_L\lambda}}{1-\frac{\tau a(C_x)}{c_L\lambda}} f_x - f\right) dZ\left(c\right) \\ + f_x \int_0^{C_x} \left(\frac{1-\frac{\tau(a(c))}{c_L\lambda}}{1-\frac{\tau a(C_x)}{c_L\lambda}} - 1\right) dZ\left(c\right). \end{aligned} \tag{39}$$

Figure 2 depicts an upward sloping curve representing $X\left(C_x\right)$. The intuition for the positive slope is basically the same as in the case of $D\left(C\right)$. That is, a higher C_x makes products more likely to be exported, increasing the value of innovation. The equilibrium value of C_x is given at the intersection point between the X curve and the horizontal line at f_r.

Having characterized the equilibrium, let us consider the effects of trade liberalization, which is captured by a lower transport cost τ and a lower beachhead cost f_x. It is straightforward to show that in response to a fall in either parameter, the D curve unambiguously shifts up, decreasing C. On the other hand, the X curve shifts down, increasing C_x. The result is summarized below:

Proposition 1. *A lower transport cost τ or export cost f_x decreases C and increases C_x.*

A higher C_x means that manufacturing firms are more likely to be exporters. A lower C means that innovation is less likely to be implemented. This means that entry into the domestic market becomes more difficult and that inefficient firms are driven out of the market. In this sense, resources are reallocated to exporting firms from non-exporting ones. It also means that the average manufacturing productivity of operating firms, which is given by

$$\int_0^C a\left(c\right)^{-1} \frac{dZ\left(c\right)}{Z\left(C\right)} \tag{40}$$

increases. Another aspect of these changes is that exporting firms earn higher profits than before (see (8)). Indeed, the result of Proposition 1 is found in the variety-expansion models of Melitz (2003), Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2007). The contribution of our paper is that the same result holds in the growth model of product *quality improvement*, which has not been modelled in the literature on trade and growth with heterogeneous firms.

An additional implication can be derived from Proposition 1. Since C falls and C_x rises due to trade liberalization, it should be clear that the measure of type-A industries increases (see (28)). Therefore, the following result is obvious:

Proposition 2. *A lower τ or f_x increases the measure of exporting industries, and decreases the measure of industries which are closed to trade.*

Proposition 1 implies that resources are reallocated to exporting industries due to trade liberalization. In addition, Proposition 2 means that trade liberalization makes more industries open to international trade. These predictions are consistent with many empirical results. For instance, Bernard, Jensen, and Schott (2003) find that low-productivity plants in U.S. manufacturing industries with falling trade costs are more likely to die. In a comprehensive survey of empirical studies, Tybout (2003) concludes that the general consensus of this literature is that foreign competition both reduces the domestic market share of import-competing firms and reallocates domestic market share from inefficient to efficient firms. More recently, Baldwin and Harrigan (2007) show that trade does not take place in many industries and the incidence of what they call "export zeros" is strongly correlated with distance, which is measured by the transport cost in the present model.

4.2 R&D and Technical Progress

One of the main issues that the present paper tackles concerns the effects of globalization on R&D incentives in the presence of heterogeneous firms. To obtain insights on the issue, combine (25) and (30) and eliminate E to derive the following condition

$$R = \frac{f_r}{2Z(C)} \left[\frac{\Lambda(C, C_x; \tau)}{F(C, C_x, f_x)} L - \left(1 - \frac{\Lambda(C, C_x; \tau)}{2} \right) \rho \right]. \tag{41}$$

This is the key equation of the present study. It determines the equilibrium number of R&D workers, given C and C_x. Using (41), we can establish the following proposition:

Proposition 3. *A lower transport cost τ or export cost f_x increases the number of R&D workers employed.*

Proof. Rewrite (36) as

$$\Lambda(C, C_x; \tau) = \frac{F(C, C_x)}{f} \left(1 - \frac{a(C)}{\theta \lambda} \right). \tag{42}$$

Differentiating both sides of this equation w.r.t. $m = \tau,\ f_x$ yields

$$-\frac{d\Lambda\left(C, C_x; \tau\right)}{dm} = -\frac{d}{dC}\left[\frac{F\left(C, C_x; f_x\right)}{f}\left(1 - \frac{a\left(C\right)}{\theta\lambda}\right)\right]\frac{dC}{dm} - \frac{d}{dC_x}\left[\frac{F\left(C, C_x; f_x\right)}{f}\left(1 - \frac{a\left(C\right)}{\theta\lambda}\right)\right]\frac{dC_x}{dm} > 0 \tag{43}$$

where $\frac{dC}{dm} > 0$ and $\frac{dC_x}{dm} < 0$ due to Proposition 1. Next, rewrite (42) as

$$\frac{\Lambda\left(C, C_x; \tau\right)}{F\left(C, C_x; f_x\right)} = \frac{1}{f}\left(1 - \frac{a\left(C\right)}{\theta\lambda}\right). \tag{44}$$

Differentiating both sides w.r.t. $m = \tau,\ f_x$ gives

$$-\frac{d}{dm}\left(\frac{\Lambda\left(C, C_x; \tau\right)}{F\left(C, C_x; f_x\right)}\right) = -\frac{1}{f}\frac{d}{dC}\left(1 - \frac{a\left(C\right)}{\theta\lambda}\right)\frac{dC}{dm} > 0. \tag{45}$$

Given that $Z(C)$ is increasing in C, differentiating (41) gives

$$-\frac{dR}{dm} > 0,$$

due to (43) and (45). ■

Trade liberalization unambiguously increases R&D workers. That is, resources are reallocated from manufacturing to R&D. To develop an intuition for the result, consider the transport cost. There are three effects that we can distinguish, and we consider each of them in turn. First, the direct effect of a lower τ works though $\Lambda\left(C, C_x; \tau\right)$, which is the expected rate of monopoly price markup over marginal cost. Since the transport cost enters as part of "effective" marginal cost, a fall in τ unambiguously increases the markup, boosting R&D incentives.

Second, the expected price markup $\Lambda\left(C, C_x; \tau\right)$ is also affected by τ indirectly via changes in the threshold marginal costs C and C_x. Proposition 1 shows that a lower transport cost increases C_x, which leads to an increase in the expected monopoly markup (see (27)). On the other hand, a lower C, caused by a lower transport cost, decreases the markup. Although these indirect effects work in opposite directions, the monopoly markup unambiguously increases as τ falls if its direct and indirect effects are combined together (see (43)).

Third, a lower transport cost affects $F\left(C, C_x\right)$, which is the ex ante fixed costs of developing a profitable product, via the threshold marginal costs C and C_x. A fall in C due to a lower τ makes it less likely for innovation to be implemented. This increases ex ante R&D costs $(f_r/Z(C))$ and the expected number of workers used as export costs $(f_x Z(C_x)/Z(C))$. In addition, a higher C_x also increases the export sunk costs $(f_x Z(C_x)/Z(C))$. That is, the indirect effect of a lower transport cost, which works via the ex ante fixed costs of a profitable product, increases as trade liberalization proceeds. However, Proposition 3 establishes that this negative effect of a lower τ is always dominated by the first and second effects combined.

Proposition 3 also shows that another aspect of trade liberalization, captured by a lower export cost f_x, causes resource reallocation to innovative activities. An intuitive account is similar to that of a lower transport cost. A key difference, however, is that the direct effect of f_x works through $F(C, C_x; f_x)$, i.e. the ex ante fixed costs of a profitable product.

Proposition 3 demonstrates that trade liberalization reallocates workers from manufacturing to R&D, reducing employment in manufacturing. On the other hand, Proposition 1 shows that workers are reallocated to exporting firms from non-exporters within the manufacturing sector. These propositions combined mean that freer trade releases workers from non-exporting firms and makes them available to both of exporting firms and research firms that conduct R&D.

Next, consider the rate of technical progress. (33) allows us to re-write (41) as

$$g = (\ln\lambda)\left[\frac{\Lambda(C, C_x; \tau)}{F(C, C_x, f_x)}L - \left(1 - \frac{\Lambda(C, C_x; \tau)}{2}\right)\rho\right], \tag{46}$$

which determines the rate of technical progress. Given Proposition 3, the next result is straightforward:

Proposition 4. *A lower transport cost τ or a lower export cost f_x promotes the rate of technical progress.*

Proof. It is obvious from (43) and (45). ∎

This proposition confirms that resources which are reallocated to R&D due to trade liberalization unambiguously translate into an accelerated rate of technical progress. (33) shows that technical progress depends on the threshold level of marginal cost C as well as the number of R&D workers. An increase in R&D workers tends to promote technical progress, but a lower threshold marginal cost C tends to decelerate it, since innovation becomes less likely to be implemented. Although these effects operate in opposite directions, Proposition 4 confirms that the positive effect a lower transport cost that work via R&D workers always outweighs the negative effect through the threshold marginal cost.

4.3 Welfare Effects

This section considers the impact of trade liberalization on welfare. Welfare is measured by consumer's intertemporal utility, which depends on consumption $Y = E/P$. Appendix C shows that

$$P = \frac{c_L \lambda^{\frac{1+N_A}{2}}}{Q} \tag{47}$$

where

$$Q = \exp\left\{ \int_0^1 \ln \lambda^{k_{it}} di \right\} \tag{48}$$

is the average quality level of intermediate goods across industries. It is also equivalent to the world technological frontier defined in (32). Since Q increases over time due to technical progress, the price of final output falls over time. Given that expenditure E and the measure of type-A industries N_A are both constant in equilibrium, quality improvement increases consumption at the rate of technical progress in the long run. In turn, this means that Proposition 4 applies to consumption growth. This result again comes in stark contrast with existing studies. Baldwin and Robert-Nicoud (2008) show that consumption growth can rise or fall in the long run with trade liberalization.[7]

On the other hand, the numerator of (47) represents the average price across industries. In type-A industries where products are exported, the price of intermediate goods is $c_L\lambda$. The same price is charged in a half of type-B industries where the top-quality goods are produced by domestic monopoly firms. In the other half of type-B industries, products are competitively produced at c_L. Therefore, the geometric average of prices is given by $c_L\lambda^{\frac{1+N_A}{2}}$.[8] It shows that the average price rises, as the exporting industry expands. This

[7] In Gustafsson and Segerstrom (2007), consumption growth is pinned down by population growth in the long run, because their model exhibits semi-endogenous growth (see Jones (1995)).

[8] To be more precise, the geometric average is $(c_L\lambda)^{N_A/2}\,(c_L\lambda)^{N_A/2}\,(c_L\lambda)^{(1-N_A)/2}\,c_L^{(1-N_A)/2} = c_L\lambda^{\frac{1+N_A}{2}}$.

is the source of an adverse welfare effect of trade liberalization in our model.

Now, plug (41) into (30) to derive the equilibrium level of consumption expenditure:

$$E = L + \rho \frac{F(C, C_x)}{2}. \tag{49}$$

In fact, it is equivalent to the representative consumer's intertemporal budget constraint in steady state. Consumption expenditure on the left-hand side is equal to the sum of labor income L and interest income from equity investment $\rho F(C, C_x)/2$. Note that the total value of assets in the world is given by $F(C, C_x)$ in (31), and a half of it is owned by the representative consumer in a country. In (31), the total asset is expressed in terms of costs associated with R&D, implementation of innovation and export. The second term of (31), f, captures the value of the threshold firm which is indifferent between implementing innovation and shutting down. The third term $f_x Z(C_x)/Z(C)$ is the expected value of the threshold firm indifferent between exporting and not exporting. The first term $f_r/Z(C)$, therefore, captures the combined "excess" values of firms with $0 \leq c < C$ over f in the domestic market and of firms with $0 \leq c < C_x$ over f_x in the foreign market.

Noting $g = (\ln \lambda)\dot{Q}/Q$ and using (47) and (49), the intertemporal utility function can be re-expressed as

$$\rho U = \overbrace{\ln\left(L + \rho\frac{F(C, C_x)}{2}\right)}^{\text{Expenditure Effect}} - \overbrace{\left(\frac{1 + N_A(C, C_x)}{2}\ln\lambda + \ln c_L\right)}^{\text{Price Effect}} + \overbrace{\frac{g(C, C_x)}{\rho}}^{\text{Growth Effect}}. \tag{50}$$

The first term on the right-hand side represents utility from the level of consumption expenditure, and the second and third terms combined pick up the effect due to the price of final output. The last term captures the effect of technical progress.[9]

Now, let us consider the effect of a lower transport cost τ. We know that both $F(C, C_x)$ and g increase, i.e. the consumption expenditure and the rate of technical progress rise with globalization. These two effects tend to increase welfare. On the other hand, trade liberalization increases the measure of type-A industries, reducing the number of industries where products are competitively produced. This has a negative impact on welfare.

[9]Strictly speaking, both the second and third terms come from the price of final output. The third term captures a continued fall in the price due to technical progress, and the second term represents the "normalized" level of the price.

Given the opposing effects, there are two possibilities, which are illustrated in Figure 3. Suppose that the transport cost τ is sufficiently large. As τ drops, welfare increases, since the positive effects of higher consumption expenditure and accelerated technical progress dominate the negative effect of a higher price level of final output. As the transport cost increases further, two cases can be distinguished. In the first case, welfare continues to increase until $\tau = 1$ is reached, as described by the thick curve in Figure 3. In the second case, on the other hand, welfare starts declining after reaching the maximum before $\tau = 1$. This case is shown as a dotted curve in the figure.

Then, under what conditions do those different cases arise? To answer this question, we explore two cases. In the first case, suppose that the export sunk cost is zero, i.e. $f_x = 0$. In this case, all products based on implementable innovation are exported. This means that all industries are of type-A with $N_A = 1$. That is, all products are sold at monopoly price, and the price effect in (50) becomes independent of the transport cost. Therefore, in this case, welfare monotonically increases as the transport cost falls. By continuity, the next proposition follows.

Proposition 5. *A lower transport cost monotonically increases welfare, if the export sunk cost f_x is sufficiently small.*

Entry into a foreign market involves different costs. Costs that do not vary with export volume (e.g. research on local regulatory environment and setting up new distribution channels) are captured by the export sunk costs in our model. On the other hand, per-unit costs (e.g. tariffs) are represented by the transport cost. Proposition 5 shows that a fall in per-unit costs improves welfare as long as costs independent of export volume are small enough.

In the second case, we exploit the fact that the threshold marginal costs C and C_x are independent of the size of population L. This means that the price effect in (50) does not vary with the size of population. On the other hand, the growth effect is positively related to L. This is what is known as scale effects in the literature on R&D-based models. The larger the size of the market, the greater the profit incentive for R&D. Indeed, this property also increases the marginal impact of a lower transport cost on welfare through the growth effect. The point can be understood by differentiating the third term in (50) with respect to τ:

$$-\frac{d}{d\tau}\left(\frac{g\left(C,C_x\right)}{\rho}\right)=-\left(\ln\lambda\right)\left[\frac{d}{d\tau}\left(\frac{\Lambda\left(C,C_x;\tau\right)}{F\left(C,C_x,f_x\right)}\right)\frac{L}{\rho}+\frac{1}{2}\frac{d\Lambda\left(C,C_x;\tau\right)}{d\tau}\right]>0. \tag{51}$$

This is positive, since the two derivatives on the right-hand side are negative due to (43) and (45). More importantly, the magnitude of the derivative, which captures the growth effect of a lower transport cost, increases with the size of population. Therefore, for a sufficiently large L, the price effect of a lower transport cost (the second term in (50)) is outweighed by the other two positive effects.[10] In this case, welfare monotonically increases as the transport cost falls. This case corresponds to a thick curve in Figure 3. On the other hand, if L is not sufficiently large, then welfare is maximized before $\tau = 1$. This is the case where the price effect is relatively large, corresponding to the dotted curve in Figure 3.

A similar analysis can be conducted regarding the welfare effect of a lower export cost f_x. Recalling Proposition 3, welfare monotonically increases with a lower f_x for a sufficiently large L. Therefore, an essentially same result as illustrated in Figure 3 applies to export costs f_x. The discussion above is summarized in the following proposition:

Proposition 6. *There are two possibilities regarding the normative effect of a lower transport cost τ or export cost f_x:*

1. *welfare monotonically improves if L is sufficiently large;*
2. *otherwise, welfare initially increases and then falls.*

5 Introducing Population Growth

Proposition 6 implies that the larger the economy, the more likely that it benefits from trade liberalization. However, this result depends on scale effects in the sense that the rate of technical progress increases with the size of an economy. This property, which is typical for the first-generation R&D-based models, is criticized by Jones (1995) as being inconsistent with data. Since Jones's criticism, several types of alternative R&D-based models are put for-

[10] A larger L lowers the expenditure effect. But, as long as the growth effect is sufficiently large, changes in welfare are positive.

ward in an effort to make them data-consistent.[11] Following the literature, we extend our basic model to introduce population growth to examine whether our key results survive in an arguably more plausible setting.

We maintain all assumptions of the basic model, except the following. Population grows at a rate of $n > 0$. Population is considered as a dynastic family, whose intertemporal utility function is now given by

$$U = \int_0^\infty e^{-(\rho-n)t} \ln h_t dt \tag{52}$$

where h is consumption per person. Interpreting E as expenditure per person, the Euler condition (2) still holds. To remove scale effects, we assume that R&D becomes increasingly more difficult, as the world technological frontier advances. To capture this insight, we follow Segerstrom (1998) in assuming that the Poisson arrival rate of innovation is given by

$$I_t = \frac{R_t}{f_r K_t}, \qquad \text{where } \dot{K}_t = \kappa Z\left(C\right) I_t K_t, \quad \kappa > 0, \tag{53}$$

which replaces (11). K is the index of R&D difficulty, which captures the property that R&D productivity tends to fall as more implementable innovations are created. A parameter κ governs how fast R&D becomes more difficult. Given this assumption, free entry into R&D leads to

$$V_t = f_r K_t, \tag{54}$$

which replaces (13). Note that (53) means that R&D costs increase as the technological frontier advances. Similarly, costs of implementing innovation and entry into the foreign market are also assumed to increase, as the quality level of products rises because of, e.g. increasing complexity of higher-quality products. This is captured by assuming that costs of innovation implementation and foreign market entry are fK and $f_x K$, respectively. Given these assumptions, the threshold marginal costs C and C_x are determined by

$$v_t\left(C\right) = fK_t, \qquad v_{xt}\left(C_x\right) = f_x K_t, \tag{55}$$

which replace (9) and (10). Turning to the labor market condition, it is still

[11] Those studies include Young (1998), Segerstrom (1998) and Li (2000). See Jones (2005) for a literature survey.

given by (23). Remember that $Z(C)I$ is equivalent to the number of industries where implementable innovation occurs during a time interval dt. Therefore, the total number of workers used to implement innovation is $Z(C)I$ times fK, which is $fZ(C)R/f_r$. Workers required to export goods can also be calculated in a similar manner.

Now, we are in a position to solve the model in steady state. It is easy to confirm that the new assumptions introduced above do not change equilibrium conditions (35), (36) and (38) regarding the determination of the threshold marginal costs, C and C_x. Therefore, the following result holds:

Proposition 7. *Introducing population growth does not alter the properties of the threshold marginal costs C and C_x in equilibrium, i.e. Propositions 1 and 2 are valid when population grows.*

Turning to R&D, let us rewrite the Poisson arrival rate of innovation (53) as

$$I_t = \frac{R_t}{L_t} \cdot \frac{L_t}{fK_t}. \tag{56}$$

In steady state, labor allocation across different sectors must be constant. That is, the share of workers devoted to R&D, R/L, must be time-invariant. In addition, the Poisson arrival rate of innovation (56) must be constant in the long run. These properties together mean that the index of R&D difficulty must grow at the rate of population growth, i.e. $\dot{K}/K = n$. From this property and the equation of motion of K in (53), we can derive the following result:

$$Z(C)I = \frac{n}{\kappa} \qquad \Rightarrow \qquad g = 2(\ln\lambda)\frac{n}{\kappa} \tag{57}$$

where the second equation uses (33). The first equation says that the arrival rate of implementable innovation is pinned down by population growth. The second equation means that the rate of technical progress depends on neither the transport cost nor the export costs. Therefore, the rate of technical progress is independent of trade liberalization. This is the same result as in Gustafsson and Segerstrom (2007).

In this type of R&D-based models, the proportion of workers devoted to R&D is endogenously determined. To calculate it, note that the values of the threshold firms with C and C_x, which are defined by (55), grow at the rate of population growth. Taking this into account, the following condition can be derived:

$$\frac{R}{L} = \frac{f_r}{Z(C)} \cdot \frac{\dfrac{\Lambda(C, C_x; \tau)}{F(C, C_x; f_x)}}{2 + \left(1 - \dfrac{\Lambda(C, C_x; \tau)}{2}\right) \dfrac{\kappa}{n} (\rho - n)}. \tag{58}$$

This condition determines the share of workers used in R&D, taking the threshold marginal costs C and C_x as given. Then, we can establish the following result:

Proposition 8. *A lower transport cost or export sunk cost unambiguously increases the share of workers devoted to R&D in the total working population.*

Proof. It is obvious from (43) and (45). ■

Proposition 3 demonstrates that trade liberalization boosts R&D incentives, and as a result, the economy reallocates more workers to innovative activities. Proposition 8 shows that the result is robust even in the presence of population growth without scale effects.

Next, let us examine the impact of trade liberalization on welfare. To this end, we use (23), (56), the first equation of (57) and (58) to derive consumption expenditure per person

$$E = 1 + \frac{\rho - n}{2} \cdot \underbrace{\frac{\kappa \Lambda(C, C_x; \tau)}{2 + \left(1 - \dfrac{\Lambda(C, C_x)}{2}\right) \kappa (\rho - n)}}_{W(C, C_x; \tau)} \tag{59}$$

where $W(C, C_x; \tau)$ is the value of assets held by each consumer. Given this and following the calculation procedure in Appendix C, the intertemporal utility function can be expressed as

$$\begin{aligned}(\rho - n) U = & \overbrace{\ln\left(1 + (\rho - n) \frac{W(C, C_x; \tau)}{2}\right)}^{\text{Expenditure Effect}} \\ & - \underbrace{\left(\frac{1 + N_A(C, C_x)}{2} \ln \lambda + \ln c_L\right)}_{\text{Price Effect}} + \underbrace{\frac{g}{\rho - n}}_{\text{Growth Effect}}.\end{aligned} \tag{60}$$

Now, let us consider the effect of a lower transport cost. It increases the term called the expenditure effect in (60) for the following reason. Consumption expenditure depends on the value of assets held, which increases with the rate of monopoly markup $\Lambda(\cdot)$. Since a lower transport cost raises monopoly markup through the direct and indirect channels identified above, consumption expenditure increases. The terms which capture the price effect in (60) is the same as in the basic model. Due to this effect, trade liberalization tends to reduce welfare, as the number of competitive industries drops. On the other hand, the growth effect is independent of the transport cost and export cost, given (57). Therefore, the expenditure and price effects only determine how welfare changes as trade becomes less restrictive. Since those two effects operate in opposite directions, there are still two possibilities, as illustrated in Figure 3.

Then, under what conditions does welfare monotonically increase with a lower transport cost? Again, we consider two cases. In the first case, the export sunk cost is taken to be zero. In this case, as before, the price effect is independent of the transport cost. Therefore, welfare monotonically increases with a lower transport cost. By continuity, Proposition 5 holds in this extended model.

In the second case, we exploit the fact that the threshold marginal costs C and C_x are independent of a parameter κ, which captures the rate at which R&D difficulty increases. This means that the price effect is not affected by the parameter. On the other hand, the magnitude of the expenditure effect increases in κ. This can be verified by differentiating (59) with respect to τ:

$$-\frac{dE}{d\tau} = -\frac{1}{2}\frac{\frac{2}{\kappa(\rho-n)}+1}{\left[\frac{2}{\kappa(\rho-n)}+1-\frac{\Lambda(C,C_x;\tau)}{2}\right]^2}\cdot\frac{d\Lambda(C,C_x;\tau)}{d\tau} > 0. \tag{61}$$

It is easy to see that this marginal effect is increasing in κ, given that the derivatives on the left-hand side is negative due to (43). Therefore, a sufficiently large κ can give rise to the situation where the expenditure effect dominates the price effect as long as the price effect is not too large (i.e. f_x is small).[12] In summary;

Proposition 9. *Regarding the welfare effect of a lower transport cost τ, it is ambiguous in general. However,*

1. *welfare can monotonically increase if κ is sufficiently large and the price*

[12] In (61), there is an upper limit on the derivative, as κ goes to infinity. In this sense, the expenditure effect can dominate the price effect if the latter is not too large.

effect is not too large;

2. *otherwise, welfare initially increases and then falls.*

The intuition for this result goes as follows. A higher κ means a lower Poisson arrival rate of implementable innovation in the steady state (see (57)). This tends to raise the value of firms, hence that of assets held by consumers. Because of this, the expenditure effect of a lower transport cost becomes so large that it can dominate the price effect.

Some comments are in order. First, the channel through which a higher κ magnifies the expenditure effect is the value of assets held. On the other hand, in the basic model with scale effects, a monotonically increasing welfare due to trade liberalization arises for a sufficiently large market size (L). In this case, a greater L increases labor income, leading to a higher consumption expenditure. Therefore, the two parameters affect two different sources of income.

Second, the R&D productivity of a given firm is affected unintentionally by other firms' investment in R&D in the present as well in the past. In this sense, the parameter κ, which governs how fast R&D difficulty increases, captures a negative externality in R&D. Using this interpretation, Proposition 9 means that the larger the negative externality in R&D, the more likely that welfare monotonically increases as trade becomes less restrictive. Indeed, Gustafsson and Segerstrom (2007) reports a similar result. In their model, consumers become better-off in the long run if what they call the knowledge spillover effect is small enough. The knowledge spillover effect in their model combines (i) the "standing on shoulders effect" in the sense that R&D productivity improves with the stock of knowledge created in the past, and (ii) the "stepping on toes" effect of R&D which reduces R&D productivity due to duplication. In Gustafsson and Segerstrom (2007), if the second negative effect is dominant, welfare monotonically increases with trade liberalization. In this sense, Proposition 9 is consistent with their result. Note that Gustafsson and Segerstrom (2007) argue that the negative effect is likely to be dominant on the basis of empirical data, including a falling trend of patents per researcher.

6 Discussion

Propositions 3, 4 and 8 sharply contrast with the results reported in Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2007). In the first paper, R&D workers as well as growth of variety expansion can rise or fall due to trade liberalization. In the second paper with population growth, trade liberalization has no effect on the share of workers devoted to R&D in total population and causes a temporary slowdown in variety growth in the short run. Why does the present model generate contrasting results?

The key difference between our model and the above-mentioned studies lies in the type of technical progress. Because of this, different types of production functions are assumed. In our model of quality improvement, the production function (3) is of a Cobb-Douglas type. On the other hand, CES production functions are used in Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2007), who follow the variety-based approach of Melitz (2003). This difference is crucial in understanding contrasting results.

To be more specific, note that in our model, there are two channels through which the transport cost affects the number of R&D workers employed in equilibrium; (1) aggregate profits, and (2) the ex ante fixed costs of developing a profitable product. Let us explain those channels in turn.

In the present study, aggregate profits are given by

$$\frac{\Lambda\left(C, C_x; \tau\right)}{2}\tilde{E} \tag{62}$$

where $\tilde{E} = E$ in the basic model and $\tilde{E} = EL$ in the model extended with population growth. The transport cost affects the rate of monopoly markup or $\Lambda\left(C, C_x; \tau\right)$ directly and indirectly through the threshold marginal costs C and C_x. Remember that $\Lambda\left(C, C_x; \tau\right)$ unambiguously increases through these channels as the transport cost drops (see (43)).

Note that the aggregate profits are linearly related to the ex ante expected value of innovation (see (25)). This means that the transport cost affects aggregate profits, hence the value of innovation. Through this R&D incentive effect, a lower transport cost induces R&D firms to employ more workers. Next, note that the labor demand in manufacturing is negatively related to aggregate profits, as (29) shows. That is, changes in the transport cost cause labor reallocation between manufacturing and R&D. Through this general equilibrium effect, a lower transport cost reduces labor demand in manufacturing, making more workers available to R&D. On the other hand, in the variety-based models of Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2007), aggregate profits are independent of the transport cost in equilibrium, because of the CES production functions assumed.[13] That is, changes in the transport cost neither affect the R&D incentives nor cause the general equilibrium effect through aggregate profits in the long run. Indeed, because of this property, the share of workers devoted to R&D becomes independent of the transport cost in Gustafsson and Segerstrom (2007).

Next, let us turn to the ex ante fixed costs of developing a profitable products. This channel is captured by $F(C, C_x; f_x)$ in our model. Note that it increases with a lower transport cost or export cost. Put differently, trade liberalization raises the ex ante fixed costs. Because of this effect, R&D incentives are negatively affected, and less workers are made available to R&D in the labor market. In our model, however, this negative effect is always dominated by the effects that operate through aggregate profits. In Baldwin and Robert-Nicoud (2008), in contrast, the ex ante sunk cost is the only channel that operates in response to trade liberalization. In their model, the ex ante fixed costs can rise or fall, depending on the structure of knowledge assumed. Hence, trade liberalization can be pro- or anti-growth in the long run.

Two questions still remain to be answered. Li (2001) develops a quality-ladder model with the assumption of the CES production function. The model is also extended to include population growth in Li (2000). Those two models essentially come in between the present model on one hand and Baldwin and Robert-Nicoud (2008) and Gustafsson and Segerstrom (2007) on the other. Then, the first question is "How do our key results change if the present model is developed in the framework of the CES production function?" The issue is explored in Haruyama and Koléda (2008).

Baldwin and Robert-Nicoud (2008) shows that the impact of trade liberalization on growth depends upon the structure of knowledge. This result partly owes to the property of variety-based models which require knowledge to be expressed in explicit forms. Then, the second question is "Are our key results sensitive to the knowledge structure assumed?" In the present model, knowledge is implicitly incorporated in the quality index λ^k.[14] In this sense, modelling knowledge is limited in our model. However, a quality-ladder model with the CES production function allows one to assume different forms of

[13]See equations (6) and (10) of Baldwin and Robert-Nicoud (2008), and equations (11), (16) and (23) of Gustafsson and Segerstrom (2007).

[14]Note that if successful in R&D, firms can leapfrog the incumbent firm in the quality level without re-inventing lower quality products. This captures the intertemporal spillover of knowledge.

knowledge. Research on the issue is tackled in Haruyama and Koléda (2008).

7 Further Analysis

7.1 Endogenizing Ex Ante Heterogeneity

In existing trade models of heterogeneous firms, the distribution of firm heterogeneity is determined based on sunk costs, taking the *ex ante* distribution of firm productivity as given. The purpose of this section is to demonstrate that the ex ante distribution of firm heterogeneity can be endogenized in our framework. Another objective is to explore the issue of whether or not benefits of trade liberalization come from changes in the ex ante distribution of firm productivity.

We assume that marginal costs are drawn from the following Pareto density function:

$$\begin{aligned} &z(c; f_r) = \mu \frac{c^{\mu-1}}{c_H(f_r)^{\mu}}, \qquad c \in (0, c_H(f_r)), \\ &0 < c_H(f_r) < \infty, \quad c_H'(f_r) < 0, \quad \mu > 1, \end{aligned} \tag{63}$$

which replaces (6). The derivative implies that a higher probability of lower marginal costs is realized if a higher f_r is chosen. This introduces a trade-off facing research firms. Now, research firms solve the following problem:

$$\max_{R, f_r} V(f_r) \frac{R}{f_r} - (1-s) R \tag{64}$$

where

$$V(f_r) = \int_0^C [v(c) - f] z(c; f_r) dc + \int_0^{C_x} [v_x(c) - f_x] z(c; f_r) dc \tag{65}$$

is the ex ante value of innovation, which is now increasing in f_r. The number of R&D workers is still determined by the free entry condition (13). Regarding the choice of f_r, research firms face a trade-off between a greater ex ante value of innovation and a lower probability of R&D success. Appendix D shows that the first-order condition can be rearranged into

$$1 = -\mu\eta(f_r) \qquad \text{where } \eta(f_r) \equiv \frac{f_r}{c_H(f_r)} \frac{dc_H(f_r)}{df_r}. \tag{66}$$

The second-order condition is satisfied when $\eta'(\delta) < 0$ where $\delta \equiv 1/f_r$, which is assumed. Given R&D productivity f_r chosen by firms, the ex ante distribution of manufacturing productivity is determined. Note that f_r in (66) is determined, independent of other endogenous variables. Therefore, the following result is obvious.

Proposition 10. *Suppose that the density function of marginal costs is given by the Pareto distribution (63). Then, τ, f_x and R&D subsidies do not affect R&D productivity and the ex ante distribution of marginal costs.*

Note that the Pareto distribution is often found to be a plausible approximation of the distribution of manufacturing productivity in many empirical studies. The present model predicts that trade liberalization has no impact on the *ex ante* distribution of firm productivity. This means that an increase in manufacturing productivity due to freer trade comes mainly from changes in the *ex post* distribution of productivity.

7.2 R&D Subsidy

In the literature on endogenous technical progress, it is well known that subsidies to R&D increase the number of R&D workers employed and promote technical progress, as costs of innovative activities are reduced. The policy is often analyzed as a means to restore Pareto efficiency, which is not achieved in R&D-based models where external effects are the driving force of long-run growth. In this section, we re-examine whether or not R&D subsidies are still useful in boosting R&D incentives. We are also interested in the issue of how the industrial policy affects the distribution of manufacturing productivity of firms, which benefit from R&D subsidies.

Equilibrium conditions (36) and (38), which determine threshold marginal costs C and C_x, are now given by

$$(1-s)\, f_r = D\left(C; \tau, f_x\right), \qquad (1-s)\, f_r = X\left(C_x; \tau, f_x\right). \tag{67}$$

They are still depicted in Figure 2. As the rate of R&D subsidy increases, the horizontal line in the figure shifts downward. It should be obvious that both threshold marginal costs C and C_x fall in response. The intuition is simple. The subsidy reduces the cost of R&D, inducing more workers to be employed in innovative activity. In turn, this leads to a fall in the ex ante value of innovation. Such a change is realized partly by a fall in C (as it makes it less likely that innovation is implemented), and partly by a fall in C_x (as it makes it more difficult for products to be exported).

There are two implications of this result. First, the number of exporting industries, measured by N_A, ambiguously changes. In this sense, R&D subsidies are not necessarily trade-promoting. Second, a fall in C means that the average productivity of operating firms rises (see (40)). That is, the subsidy reinforces the productivity-improving effect of globalization.

The rate of technical progress is now given by

$$g = \frac{\dfrac{\Lambda\left(C, C_x\right)}{\tilde{F}\left(C, C_x; s\right)} L - \left(1 - \dfrac{\Lambda\left(C, C_x\right)}{2}\right)\rho}{\dfrac{\Lambda\left(C, C_x\right)}{2} \dfrac{F\left(C, C_x\right)}{\tilde{F}\left(C, C_x; s\right)} + \left(1 - \dfrac{\Lambda\left(C, C_x\right)}{2}\right)}. \tag{68}$$

It is reduced to (46) when $s = 0$, as $F\left(C, C_x\right) = \tilde{F}\left(C, C_x\right)$. The direct effect of the policy works via the term $\tilde{F}\left(C, C_x; s\right)$. It is easy to show that g is falling in $\tilde{F}\left(C, C_x; s\right)$, which means that g is increasing in s through this direct channel. However, as discussed above, R&D subsidies also affect threshold marginal costs C and C_x, which indirectly impact on technical progress. Since both of them fall in response to the policy, the indirect effect is ambiguous in general. One reason is that through the indirect channel, the expected rate of monopoly markup drops, while the ex post fixed costs of developing a profitable product changes ambiguously. As a result, the rate of technical progress changes ambiguously in response to R&D subsidies. The same conclusion can be drawn regarding the number of R&D workers, given (33). In summary,

Proposition 11. *As R&D subsidies are applied,*

1. *the threshold marginal costs C and C_x fall*

2. *the number of R&D workers and the rate of technical progress change ambiguously in general.*

The implication of Result 2 of this proposition is that a clear-cut result regarding the growth effect of R&D subsidies in existing R&D-based does not necessarily hold in the presence of heterogeneous firms. It also means that R&D subsidies may not be enough to restore social optimum when the market fails due to externalities associated with knowledge creation.

8 Concluding Remarks

The present paper is the first attempt in the literature to introduce the continual improvement of product quality into an international trade model with heterogeneous firms. Our modelling approach departs from most of the existing studies which are founded on the variety-based model of Melitz (2003). One advantage of our quality-ladder model over variety-based models is that the exit of firms from the market is endogenously determined due to the process of creative destruction.

Using our framework, several interesting results are established. First, trade liberalization, captured by a lower transport cost or export sunk cost, drives less inefficient firms out of the market and reallocates resources to exporting industries from non-exporting ones. That is, less restrictive trade improves the level of manufacturing productivity, expanding the exporting sector. This widely accepted result is shown to hold even if continual quality enhancement drives grow with and without population growth.

Second, trade liberalization is found to promote long-run growth. Freer trade increases profit incentives for innovative activities, and consequently, the employment of R&D workers expands. This translates into a higher rate of technical progress and consumption growth. Robustness of this key result is checked in an extended model with population growth. Trade liberalization is shown to increase the proportion of R&D workers in total population. These results, together with the first result, mean that resources are reallocated to R&D and exporting industries from non-exporting industries. These pro-R&D results come in stark contrast with existing studies which essentially show the opposite. In addition, we also establish that welfare unambiguously improves under certain conditions.

Third, we endogenize the ex ante distribution of firm heterogeneity, which is assumed to be exogenous in existing studies. Our result shows that an increase in manufacturing productivity due to trade liberalization is realized through the ex post rather than ex ante distribution of firm heterogeneity. In this sense, the assumption of an exogenous ex ante distribution in existing studies is justified. As part of an extended analysis, we also consider the effect of R&D subsidies, which is often discussed in the literature on endogenous technical progress. This industrial policy is found to induce less efficient firms to exit from both the domestic and foreign markets. In this sense, the policy boosts the level of manufacturing productivity, though it does not necessarily promote export. On the other hand, it is not clear whether or not subsidizing R&D promotes technical progress.

We believe that these results represent an important improvement in the understanding of the effects of trade liberalization on the level as well as growth rate of firm productivity.

Appendix A

This appendix derives (29). In the intermediate goods sector, the total number of workers employed can be rewritten as

$$
\begin{aligned}
\ell =& \frac{N_B}{2}E_t + \frac{N_B}{2}\frac{E}{\lambda c_L}\int_{C_x}^{C} a(c)\,\frac{dZ(C)}{Z(C)-Z(C_x)} \\
&+ \frac{N_A}{2}\frac{E}{\lambda c_L}\int_0^{C_x} a(c)\,\frac{dZ(C)}{Z(C_x)} + \frac{N_A}{2}\frac{\tau E}{\lambda c_L}\int_0^{C_x} a(c)\,\frac{dZ(C)}{Z(C_x)} \\
=& \frac{N_B}{2}E\int_{C_x}^{C}\frac{dZ(C)}{Z(C)-Z(C_x)} + \frac{N_B}{2}\int_{C_x}^{C}\left(1-1+\frac{a(c)}{\lambda c_L}\right)E\frac{dZ(C)}{Z(C)-Z(C_x)} \\
&+ \frac{N_A}{2}\int_0^{C_x}\left(1-1+\frac{a(c)}{\lambda c_L}\right)E\frac{dZ(C)}{Z(C_x)} + \frac{N_A}{2}\int_0^{C_x}\left(1-1+\frac{\tau a(c)}{\lambda c_L}\right)E\frac{dZ(C)}{Z(C_x)} \\
=& E\left(\frac{N_B}{2}\int_{C_x}^{C}\frac{dZ(C)}{Z(C)-Z(C_x)} + \frac{N_B}{2}\int_{C_x}^{C}\frac{dZ(C)}{Z(C)-Z(C_x)}\right. \\
&\left.+\frac{N_A}{2}\int_0^{C_x}\frac{dZ(C)}{Z(C_x)} + \frac{N_A}{2}\int_0^{C_x}\frac{dZ(C)}{Z(C_x)}\right) \\
&- \frac{N_B}{2}\int_{C_x}^{C}\left(1-\frac{a(c)}{\lambda c_L}\right)E\frac{dZ(C)}{Z(C)-Z(C_x)} \\
&- \frac{N_A}{2}\int_0^{C_x}\left(1-\frac{a(c)}{\lambda c_L}\right)E\frac{dZ(C)}{Z(C_x)} - \frac{N_A}{2}\int_0^{C_x}\left(1-\frac{\tau a(c)}{\lambda c_L}\right)E\frac{dZ(C)}{Z(C_x)} \\
=& E - \frac{\Lambda(C, C_x)}{2}E
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{\Lambda(C, C_x)}{2} =& \frac{N_B}{2} \int_{C_x}^{C} \left(1 - \frac{a(c)}{\lambda c_L}\right) \frac{dZ(C)}{Z(C) - Z(C_x)} \\
&+ \frac{N_A}{2} \int_0^{C_x} \left(1 - \frac{a(c)}{\lambda c_L}\right) \frac{dZ(C)}{Z(C_x)} \\
&+ \frac{N_A}{2} \int_0^{C_x} \left(1 - \frac{\tau a(c)}{\theta \lambda c_L}\right) \frac{dZ(C)}{Z(C_x)} \\
=& \frac{1}{2} \int_{C_x}^{C} \left(1 - \frac{a(c)}{\theta \lambda c_L}\right) \frac{dZ(C)}{Z(C)} + \frac{1}{2} \int_0^{C_x} \left(1 - \frac{a(c)}{\theta \lambda c_L}\right) E \frac{dZ(C)}{Z(C)} \\
&+ \frac{N_A}{2} \int_0^{C_x} \left(1 - \frac{\tau a(c)}{\theta \lambda c_L}\right) \frac{dZ(C)}{Z(C_x)} \\
=& \frac{1}{2} \left[\int_0^{C} \left(1 - \frac{a(c)}{\theta \lambda c_L}\right) \frac{dZ(C)}{Z(C)} + N_A \int_0^{C_x} \left(1 - \frac{\tau a(c)}{\theta \lambda c_L}\right) \frac{dZ(C)}{Z(C_x)} \right].
\end{aligned}
\tag{A1}
$$

The last line is equivalent to (27).

Appendix B

This appendix shows that utility growth of the representative consumer in a country is equivalent to (32). Consider Country 1 and define N_A^X, N_A^{IM}, N_B^M and N_B^C as a measure of industries where (i) the state-of-the-art products are exported, (ii) the top-quality goods are imported, (iii) the top-quality products are domestically produced by a monopoly, but not exported, and (iv) the second-highest quality products are competitively produced, and not exported, respectively. Then, (14) is rewritten as

$$1 = N_A^X + N_A^{IM} + N_B^M + N_B^C.$$

Using these definitions, Figure 4 shows the directions of a movement of industries as innovation occurs. Each term attached to the arrows indicates the Poisson arrival rate of such movement. Now, rewrite (32) as

$$Q_t = N_A^X Q_A^X + N_A^{IM} Q_A^{IM} + N_B^M Q_B^M + N_B^C Q_B^C \tag{B1}$$

where

$$Q_m^b = \frac{1}{N_m^b} \int_0^{N_m^b} \ln \lambda^{k_i} di, \qquad m = A, B, \qquad b = X, IM, M, C$$

is the average of $\ln \lambda^k$ in each measure of industries. Given N_m^b, $m = A, B$, $b = X, IM, M, C$, are all constant in steady state, (B1) means

$$g \equiv \frac{\dot{Q}_t}{Q_t} = N_A^X \dot{Q}_A^X + N_A^{IM} \dot{Q}_A^{IM} + N_B^M \dot{Q}_B^M + N_B^C \dot{Q}_B^C \tag{B2}$$

where $\dot{Q}_m^b$ is the average change in $\ln \lambda^k$ in each type of industries. To rewrite $\dot{Q}_m^b$, note that the quality level of products λ^{k_i} increases as innovation moves industries along the arrows in Figure 4. Especially, three cases can be distinguished; (i) thick arrows show the case where the quality level of products rises by λ, (ii) thick dotted arrows indicate the case where the quality level increases by λ^2, and (iii) thin dotted arrows correspond to the case where the quality level does not change. Therefore, we can write

$$\dot{Q}_m^b = \frac{1}{N_m^b} \int_0^{N_m^b} \left(\ln \lambda^{k_i+1} - \ln \lambda^{k_i} \right) \phi_m^b \left(I, C, C_x \right) di$$

for $m = A, B$ and $b = X, IM, M$, and

$$\begin{aligned} \dot{Q}_B^C = & \frac{1}{N_B^C} \int_0^{N_B^C} \left[\left(\ln \lambda^{k_i+1} - \ln \lambda^{k_i} \right) \phi_B^C \left(I, C, C_x \right) \right. \\ & \left. + \left(\ln \lambda^{k_i+2} - \ln \lambda^{k_i} \right) \hat{\phi}_B^C \left(I, C, C_x \right) \right] di \end{aligned}$$

where ϕ's are the Poisson arrival rates of innovation, given that innovation is implementable and, if relevant, goods are exported. Those arrival rates are given by

$$\begin{aligned} \phi_A^X \left(I, C, C_x \right) = & Z \left(C_x \right) I + Z \left(C_x \right) I^* + \left[Z \left(C \right) - Z \left(C_x \right) \right] I, \\ \phi_A^{IM} \left(I, C, C_x \right) = & Z \left(C_x \right) I + Z \left(C_x \right) I^* + \left[Z \left(C \right) - Z \left(C_x \right) \right] I^*, \\ \phi_B^M \left(I, C, C_x \right) = & Z \left(C_x \right) I + Z \left(C_x \right) I^* + \left[Z \left(C \right) - Z \left(C_x \right) \right] I, \\ \phi_B^C \left(I, C, C_x \right) = & \left[Z \left(C \right) - Z \left(C_x \right) \right] I^*, \\ \hat{\phi}_B^C \left(I, C, C_x \right) = & Z \left(C_x \right) I + Z \left(C_x \right) I^* + \left[Z \left(C \right) - Z \left(C_x \right) \right] I \end{aligned}$$

where asterisks indicate foreign variables. Therefore,

$$\dot{Q}_A^X = \dot{Q}_A^{IM} = \dot{Q}_B^M = \left(\ln \lambda \right) \left[Z \left(C \right) + Z \left(C_x \right) \right] I, \tag{B3}$$

$$\dot{Q}_B^C = \left(\ln \lambda \right) \left[3Z \left(C \right) + Z \left(C_x \right) \right] I, \tag{B4}$$

using $I = I^*$. Noting $N_A^X = N_A^{IM} = N_A/2$ and $N_B^M = N_B^C = N_B/2$ and substituting (B3) and (B4) into (B2), we can derive (32).

Appendix C

C.1 Price of Final Output

Using the production function (3) and the demand function (4), one can derive the price of final output in terms of the prices of intermediate goods:

$$P_t = \exp\left\{\int_0^1 \ln \frac{\lambda^{k_{it}}}{p_{it}} di\right\}.$$

This price index can be rewritten as

$$\begin{aligned} P_t =& \frac{1}{Q}\exp\left\{\int_0^1 \ln p_{it} di\right\} \\ =& \frac{1}{Q}\exp\left\{\int_0^{N_A/2} \ln \lambda c_L di + \int_0^{N_A/2} \ln \lambda c_L di \right. \\ & \left. + \int_0^{(1-N_A)/2} \ln \lambda c_L di + \int_0^{(1-N_A)/2} \ln c_L di\right\} \\ =& \frac{c_L \lambda^{\frac{1+N_A}{2}}}{Q} \end{aligned} \tag{C1}$$

taking into account that some industries are competitive and others are monopoly.

C.2 Intertemporal Utility Function

Remembering $Y = E/P$, substitute (C1) and (49) into the intertemporal utility function to obtain

$$U = \int_0^\infty e^{-\rho t}\left(\ln L + \rho \frac{F(C, C_x)}{2} - \ln c_L \lambda^{\frac{1+N_A}{2}} + \ln Q\right) dt$$

which can be reduced to (50).

Appendix D

Letting $\delta \equiv 1/f_r$, (63) is equivalent to $z(c;\delta)$. Note that

$$\frac{\delta z_\delta (c;\delta)}{z(c;\delta)} = -\mu \frac{\delta}{c_H}\frac{dc_H}{d\delta} = -\mu\eta(\delta) < 0 \tag{D1}$$

where $z_\delta \equiv \frac{z(c;\delta)}{d\delta}$ and $\eta(\delta) \equiv \frac{\delta}{c_H}\frac{dc_H}{d\delta}$. Note that this elasticity is independent of c, given (63). (64) is equivalent to $\max_{R,\delta} V(\delta)\delta R - (1-s) R$. Differentiating this w.r.t. δ gives

$$\begin{aligned} \frac{dV(\delta)\delta}{d\delta} =& \int_0^C [v(c) - f][z(c;\delta) + \delta z_\delta (c;\delta)] dc \\ & + \int_0^{C_x} [v_x(c) - f_x][z(c;\delta) + \delta z_\delta(c;\delta)] dc. \end{aligned} \tag{D2}$$

Therefore, the F.O.C. is

$$0 = [1 - \mu\eta(\delta)] \left(\int_{c_L}^{C} [v(c) - f] \, z(c;\delta) \, dc + \int_{c_L}^{C_x} [v_x(c) - f_x] \, z(c;\delta) \, dc \right)$$

or

$$1 = \mu\eta(\delta) \quad \Rightarrow \quad 1 = -\mu\eta(f_r)$$

using (D1). This is equivalent to (66). To calculate the S.O.C., differentiate (D2) to obtain

$$\begin{aligned} \frac{d^2 V(\delta)\,\delta}{d\delta^2} = & \int_0^{C} [v(c) - f] \, z_\delta(c) \left[2 + \frac{\delta z_{\delta\delta}(c)}{z_\delta(c)} \right] dc \\ & + \int_0^{C_x} [v_x(c) - f_x] \, z_\delta(c) \left[2 + \frac{\delta z_{\delta\delta}(c)}{z_\delta(c)} \right] dc. \end{aligned}$$

For this to be negative, we must have

$$2 + \frac{\delta z_{\delta\delta}(c)}{z_\delta(c)} < 0. \tag{D3}$$

From (63), we have

$$z_\delta(c;\delta) = -\,\mu\eta(\delta)\,\frac{z(c;\delta)}{\delta}, \tag{D4}$$

$$z_{\delta\delta}(c;\delta) = 2\mu\eta(\delta)\,\frac{z(c;\delta)}{\delta^2} - \mu\eta'(\delta)\,\frac{z(c;\delta)}{\delta}. \tag{D5}$$

Substituting (D4) and (D5) into (D3) gives

$$2 + \frac{\delta z_{\delta\delta}(c)}{z_\delta(c)} = \frac{\delta\eta'(\delta)}{\eta(\delta)} \tag{D6}$$

which is negative if and only if $\eta'(\delta) < 0$.

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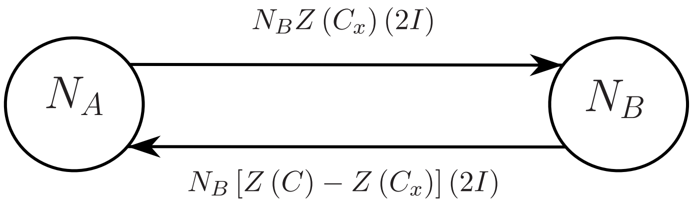

Figure 1: Continual changes in types of industries due to innovation.

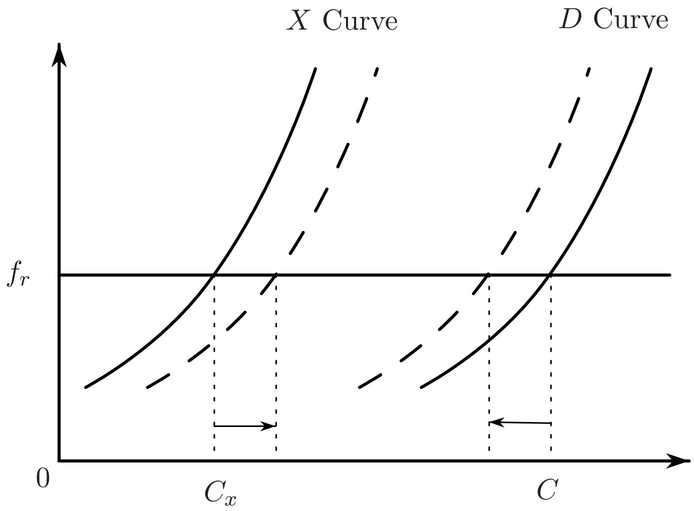

Figure 2: Determination of the threshold marginal costs and the effect of trade liberalization.

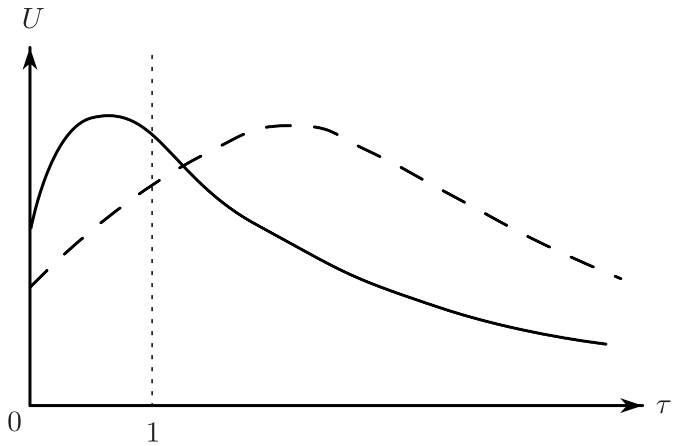

Figure 3: Welfare effects of a lower transport cost.

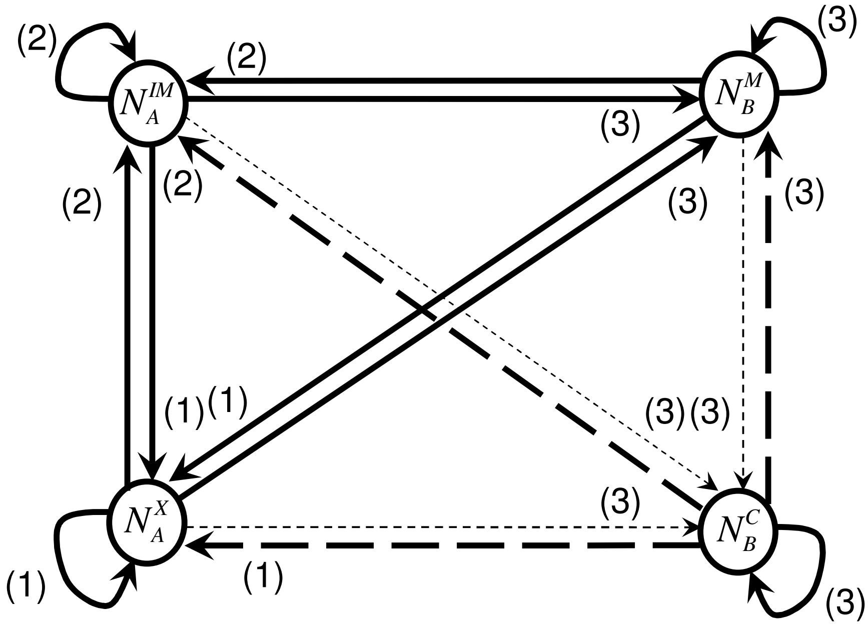

Figure 4: In the figure, the arrows indicate the directions of industry dynamics due to innovation. Interpretations of the arrows are as follows: (i) thick arrows shows the cases where the quality level rises by λ, (ii) thick dotted arrows indicate the cases where the quality level increases by λ^2, and (iii) thin dotted arrows correspond to the cases where the quality level does not change. In addition, (1)-(3) indicate the Poisson arrival rates of innovation along their associated arrows, and they are defined as follows: (1) $Z(C_x)I$, (2) $Z(C_x)I^*$, (3) $[Z(C) - Z(C_x)]I$ where asterisks indicate a foreign variable.