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**Aging, transitional dynamics, and gains from trade**

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# Aging, transitional dynamics, and gains from trade

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## Abstract

We formulate a two-country, two-good, two-factor, two-period-lived overlapping generations model to examine how population aging determines the pattern of and gains from trade. We obtain two main results. First, the aging country endogenously becomes a small country exporting the capital-intensive good, whereas the younger country endogenously dominates the world economy determining the world prices, in the free trade steady state. Second, although uncompensated free trade cannot be Pareto superior to autarky, there exists a compensation scheme applied within each country such that free trade is Pareto superior to autarky.

JEL classification: F43; J11; O41

Keywords: Aging and trade; Gains from trade; Overlapping generations model; Transitional dynamics; Compensation scheme

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# 1 Introduction

Population aging is in progress across many countries, but at different speeds. According to the United Nations (2007), in developed countries, the percentage of people aged 60 years or over is 20% in 2005, and it is expected to be 33% in 2050. In developing countries, on the other hand, that percentage is still 8% in 2005, but it is projected to go up to 20% in 2050. Aging has important consequences. It changes the composition of age groups with different saving patterns and demand behavior at the same time, which may increase the volume of international trade and capital flows.<sup>1</sup> For instance, in 2004, there were 3.62 million farmers in Japan, of which 57 percent were aged 65 or older, according to government statistics. From these numbers one might guess that soon Japan will have to open the rice market wider to foreign imports, unless Japanese consumers choose to eat significantly less rice. Moreover, trade has different distributional effects on the age groups within each country. Working-age people are likely to be affected by trade quite differently from retired people.<sup>2</sup> The purpose of this paper is to examine how the difference in the pace of population aging affects the pattern of trade, and whether people all over the world gain from trade; if not, what policies can be implemented to ensure that the largest number of people benefits from them.

We formulate a two-country, two-good, two-factor, two-period-lived overlapping generations model, in which the two countries are identical except for the exogenous rates of population growth. Although there are a number of multi-country, one-good overlapping generations models which investigate the relationship between demographic structure and the pattern of international borrowing and lending (e.g., Brooks, 2003; Momota and Futagami, 2005; Domeij and Flodén, 2006), little attention has been paid to the link between demographics and international trade. The only exception is Sayan (2005), who shows analytically that aging (i.e., an exogenous fall in the rate of population growth) makes the country relatively more capital-abundant (i.e., labor-scarce), thereby lowering the autarky steady state relative price of the capital-intensive good. He also presents numerical results that an

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<sup>1</sup>Horioka (1997) provides empirical evidence that the age structure of the population has a significant impact on the aggregate household saving rate in Japan.

<sup>2</sup>A former Fed expert argues that: "Differing patterns of demographic change in developing and developed countries may also affect international trade and capital flows... In developed countries, these changes are likely to benefit most citizens but hurt some workers. Policymakers in developed countries may wish to compensate these workers, while retaining the overall benefits of free trade." (Johnson, 2004, p. 56)

aging country tends to export the capital-intensive good, and more importantly, that the representative household in the aging country tends to lose from trade in the free trade steady state.

Our model builds on Sayan (2005). We depart from him mainly on two points. First, we characterize the complete path of the free trade equilibrium analytically rather than numerically. The fact that the two countries have different exogenous rates of population growth means that the countries' population shares keep on changing independently of market outcomes, and hence the world economy will experience very long periods of transition. One of our theoretical contributions is to carefully analyze the transitional dynamics as well as the steady state of the free trade equilibrium. Second, we pursue the possibility that free trade is potentially Pareto superior to autarky. Although generally someone may lose from trade in some situations (recall an elementary example of capitalists versus workers in the standard Heckscher-Ohlin model), there may often be some compensation mechanisms under which everyone can be made better off with trade (e.g., Wong, 1995, Part IV). With regard to the overlapping generations model, Kemp and Wong (1995) construct a compensation scheme for a small economy, which we apply to our two-country model.

We obtain the following main results. First, the aging country endogenously becomes a small country exporting the capital-intensive good, whereas the younger country endogenously dominates the world economy determining the world prices, in the free trade steady state. While this result appears surprising at first glance, it is a rather natural theoretical property: as long as we identify the aging country as the one with the lower exogenous rate of population growth, the population share of the aging country can be constant if and only if it approaches zero. Second, although uncompensated free trade cannot (stronger than "may not") be Pareto superior to autarky, there exists a compensation scheme applied within each country such that free trade is Pareto superior to autarky. The key ingredient of the compensation scheme, constructed by Kemp and Wong (1995), is a savings subsidy which replicates the path of savings in autarky. Once the path of factor supplies in autarky is replicated in free trade, we obtain the production gain evaluated at the world prices in every period, which can be redistributed to everyone living in that period. This thus overturns Sayan's (2005) pessimistic result of losses from trade due to population aging.

The rest of this paper is organized as follows. Section 2 sets up the model in autarky. Section 3 gives a positive analysis of the free trade equilibrium. Section 4 examines the normative side of trade without and with compensation. Section 5 concludes.

## 2 Autarky

In this section, we consider a single closed economy that has two goods, two factors, and two generations in each period. Time is discrete, and extends from zero to infinity. Good 1 is a capital good, which is either invested or consumed. On the other hand, good 2 is a pure consumption good. We take good 1 as the numeraire. The representative household in generation  $t$  is born in period  $t$ , and lives for two periods. When young in period  $t$ , he/she supplies one unit of labor inelastically, and allocates his/her wage income to consumption of the two goods and savings. Getting old in period  $t + 1$ , he/she spends his/her gross return from savings on consumption of the two goods. In each period, the representative firm in sector  $j$  ( $j = 1, 2$ ) uses capital and labor to produce good  $j$  under constant returns to scale technology, and pays rental and wage to the old and the young generations living in that period, respectively. Assuming complete capital depreciation in one period, the gross rate of return to savings equals the rental rate of capital. All households and firms take prices as given.

### 2.1 Firms

In sector  $j$ , the representative firm maximizes its profit  $\Pi_j = p_j Y_j - rK_j - wL_j$ , subject to the production function  $Y_j = F_j(K_j, L_j)$ , where  $p_j$  is the price of good  $j$ ;  $Y_j$  is the output;  $r$  is the rental rate;  $K_j$  is the demand for capital;  $w$  is the wage rate; and  $L_j$  is the demand for labor. It is assumed that  $F_j(\cdot)$  is increasing, concave, linearly homogeneous, and twice continuously differentiable. Let us define the unit cost function as  $\gamma_j(r, w) \equiv \min_{a_{Kj}, a_{Lj}} \{ra_{Kj} + wa_{Lj} : 1 \leq F_j(a_{Kj}, a_{Lj})\}$ , where  $a_{Kj} \equiv K_j/Y_j$  and  $a_{Lj} \equiv L_j/Y_j$  are demands for capital and labor per unit of output, respectively. It is easily verified that  $\gamma_j(\cdot)$  is increasing, concave, linearly homogeneous, and twice continuously differentiable. Assuming incomplete specialization, the first-order condition for profit maximization is given by:

$$p_j = \gamma_j(r, w), \quad (1)$$

which also implies that firms make zero profits. With  $p$  denoting the relative price of good 2 to good 1, we have  $p_1 \equiv 1$  and  $p_2 \equiv p$ .

### 2.2 Households

The representative household in generation  $t$  maximizes his/her utility  $U_t = \ln c_t + [1/(1 + \rho)] \ln d_{t+1}$ , where  $\rho$  is the subjective discount rate;  $c$  is the con-

sumption index when young; and  $d$  is the consumption index when old. We omit the time subscripts whenever there is no confusion. To allow for different tastes between the young and the old generations, we use distinct functional forms for the consumption indexes:  $c = C(c_1, c_2)$  and  $d = D(d_1, d_2)$ , where  $c_j$  and  $d_j$  are consumption of good  $j$  when young and old, respectively. It is assumed that  $C(\cdot)$  and  $D(\cdot)$  are nondecreasing, concave, linearly homogeneous, and twice continuously differentiable. The budget constraints when young and old are given by, respectively:

$$s_t + c_{1t} + p_t c_{2t} = w_t, \quad (2)$$

$$d_{1t+1} + p_{t+1} d_{2t+1} = r_{t+1} s_t, \quad (3)$$

where  $s$  is savings.

The problem has both intratemporal and intertemporal components. First, intratemporal optimization behavior is summarized by the expenditure functions:  $E^c(p, c) \equiv \min_{c_1, c_2} \{c_1 + p c_2 : c \leq C(c_1, c_2)\} = e^c(p)c$  and  $E^d(p, d) \equiv \min_{d_1, d_2} \{d_1 + p d_2 : d \leq D(d_1, d_2)\} = e^d(p)d$ , where the linearity of  $E^c(\cdot)$  and  $E^d(\cdot)$  with respect to  $c$  and  $d$  follows from the linear homogeneity of  $C(\cdot)$  and  $D(\cdot)$ , respectively. Just like the unit cost functions, the unit expenditure functions  $e^c(p)$  and  $e^d(p)$  are nondecreasing, concave, and twice continuously differentiable:  $e_p^c(p) \geq 0$ ,  $e_{pp}^c(p) \leq 0$ ,  $e_p^d(p) \geq 0$ , and  $e_{pp}^d(p) \leq 0$ , where a subscript represents partial differentiation.

Second, making use of the expenditure functions and the budget constraints (2) and (3), utility is rewritten as  $U_t = \ln[(w_t - s_t)/e^c(p_t)] + [1/(1 + \rho)] \ln(r_{t+1} s_t/e^d(p_{t+1}))$ . Intertemporal optimization yields the following savings function:

$$s_t = w_t/(2 + \rho), \quad (4)$$

where the independence of the optimal savings from its gross rate of return is because of the log-linearity of the utility function. Substituting Eq. (4) back into Eqs. (2) and (3), the optimal values of  $c_t$  and  $d_{t+1}$  are expressed as:

$$c_t = (w_t/e^c(p_t))[1 + \rho]/(2 + \rho), \quad (5)$$

$$d_{t+1} = (r_{t+1}/e^d(p_{t+1}))[w_t/(2 + \rho)]. \quad (6)$$

### 2.3 Equilibrium

The market-clearing conditions for goods and factors are as follows:

$$Y_{1t} = L_{t-1}d_{1t} + L_t c_{1t} + K_{t+1}, \quad (7)$$

$$Y_{2t} = L_{t-1}d_{2t} + L_t c_{2t}, \quad (8)$$

$$K_t = K_{1t} + K_{2t}, \quad (9)$$

$$L_t = L_{1t} + L_{2t}, \quad (10)$$

where  $L_t$  is the number of households in generation  $t$ ; and  $K_t$  is the capital stock in period  $t$ . Factor supplies evolve according to:

$$K_{t+1} = L_t s_t, \quad (11)$$

$$L_{t+1} = (1 + n)L_t, \quad (12)$$

where  $n$  is the rate of population growth, which we assume is exogenous and constant. Eq. (11) states that the capital stock is owned solely by the old generation with their savings. From Eqs. (1), (2), (3), and (11), we obtain Walras' law: the sum of the values of excess demands is identically zero. This implies that one of the four market-clearing conditions is redundant.

### 2.4 Dynamic system

Before deriving the dynamic system, it is convenient to define the GDP function:  $G(p, K, L) \equiv \max_{\{Y_j, K_j, L_j\}_{j=1}^2} \{Y_1 + pY_2 : F_j(K_j, L_j) \geq Y_j, K \geq K_1 + K_2, L \geq L_1 + L_2\}$ , or equivalently,  $G(p, K, L) \equiv \min_{r, w} \{rK + wL : 1 \leq \gamma_1(r, w), p \leq \gamma_2(r, w)\}$ . Assuming no factor intensity reversal, we solve the two constraints in the latter problem with equalities to obtain the Stolper-Samuelson relations:

$$r = r(p), w = w(p). \quad (13)$$

Substituting Eq. (13) into the objective function, we obtain  $G(p, K, L) = g(p, k)L$ , where  $g(p, k) \equiv r(p)k + w(p)$  is the GDP per worker; and  $k \equiv K/L$  is capital per worker. From the general properties of  $G(p, K, L)$ , we have  $g_p(p, k) \geq 0$ ,  $g_{pp}(p, k) \geq 0$ ,  $g_k(p, k) \geq 0$ , and  $g_{kk}(p, k) \leq 0$ . Moreover, the current setup implies that  $g_k(p, k) = r(p) > 0$  and  $g_{kk}(p, k) = 0$ .

From Eqs. (4), (11), (12), and (13), the evolution of  $k_{t+1}$  is governed by:

$$k_{t+1} = [1/(1+n)][w(p_t)/(2+\rho)]. \quad (14)$$

On the other hand, using Eq. (12), the expenditure functions, and the GDP function to rewrite Eq. (8),  $p_{t+1}$  is determined by:

$$g_p(p_{t+1}, k_{t+1}) = [1/(1+n)]e_p^d(p_{t+1})d_{t+1} + e_p^c(p_{t+1})c_{t+1}. \quad (15)$$

Substituting Eq. (13) into Eqs. (5) and (6),  $d_{t+1}$  and  $c_{t+1}$  are given by:

$$d_{t+1} = (r(p_{t+1})/e^d(p_{t+1}))[w(p_t)/(2+\rho)], \quad (16)$$

$$c_{t+1} = (w(p_{t+1})/e^c(p_{t+1}))[(1+\rho)/(2+\rho)]. \quad (17)$$

Eqs. (14), (15), (16), and (17) constitute the dynamic system. Eq. (7) is automatically satisfied because of Walras' law.

## 2.5 Steady state

A steady state is defined as a situation in which all prices and per-worker variables are constant over time. From Eqs. (14), (15), (16), and (17), a steady state is characterized by:

$$k = [1/(1+n)][w(p)/(2+\rho)], \quad (18)$$

$$g_p(p, k) = [1/(1+n)]e_p^d(p)d + e_p^c(p)c, \quad (19)$$

$$d = (r(p)/e^d(p))[w(p)/(2+\rho)], \quad (20)$$

$$c = (w(p)/e^c(p))[(1+\rho)/(2+\rho)], \quad (21)$$

where  $k = k_{t+1} = k_t$ , and so on. To examine the existence and uniqueness of steady states, we define the compensated excess demand function for good 2 per worker:

$$\tilde{z}_2(p, d, c, k) \equiv [1/(1+n)]e_p^d(p)d + e_p^c(p)c - g_p(p, k). \quad (22)$$

Substituting Eqs. (18), (20), and (21) into Eq. (22), we obtain the uncompensated excess demand function for good 2 per worker:  $z_2(p) \equiv \tilde{z}_2(p, d(p), c(p), k(p))$ . An autarky steady state relative price is determined by  $z_2(p^a) = 0$ , where superscript  $a$  represents autarky. Then the other steady-state variables are given by  $k^a = k(p^a)$ ,  $d^a = d(p^a)$ , and  $c^a = c(p^a)$ .

We derive the slope of  $z_2(p)$  against  $p$ . Totally differentiating Eqs. (18), (20), (21), and (22), using  $g_{pk} = r_p$ , and rearranging the terms, we obtain:



$$dz_2/dp = \zeta;$$

$$\zeta \equiv \sigma + \frac{1}{2 + \rho} \left[ -\frac{1}{1 + n} \left( r_p - r \frac{e_p^d}{e^d} \right) \left( w_p - w \frac{e_p^d}{e^d} \right) + \frac{e_p^c}{e^c} \left( w_p - w \frac{e_p^c}{e^c} \right) (1 + \rho) \right],$$

$$\sigma \equiv [1/(1 + n)]e_{pp}^d d + e_{pp}^c c - g_{pp} \leq 0.$$

In the definition of  $\zeta$ , the first term  $\sigma$  shows the sum of consumption and production substitution effects  $\partial \tilde{z}_2 / \partial p$ , which is necessarily nonpositive. In the brackets, the first term reflects the sum of the old generation's income effect  $(\partial \tilde{z}_2 / \partial d)(dd/dp)$  and the Rybczynski effect  $(\partial \tilde{z}_2 / \partial k)(dk/dp)$ , whereas the second term represents the young generation's income effect  $(\partial \tilde{z}_2 / \partial c)(dc/dp)$ . To see the signs of the latter terms, we use Eq. (1) to obtain:

$$r_p = -a_{L1}/a, w_p = a_{K1}/a, r_p - r/p = -a_{L2}/(pa), w_p - w/p = a_{K2}/(pa);$$

$$a \equiv a_{K1}a_{L2} - a_{L1}a_{K2} \begin{cases} > \\ < \end{cases} 0 \Leftrightarrow \frac{a_{K1}}{a_{L1}} \begin{cases} > \\ < \end{cases} \frac{a_{K2}}{a_{L2}}.$$

This indicates the Stolper-Samuelson theorem: a rise in the price of good 2 raises rental (or wage) more than proportionately but lowers wage (or rental) if and only if  $a <$  (or  $>$ )0; that is, good 2 is more capital-intensive (or labor-intensive) than good 1. Combining this with  $e^d \geq pe_p^d$  and  $e^c \geq pe_p^c$ , we have:

$$r_p - re_p^d/e^d \begin{cases} > \\ < \end{cases} 0, w_p - we_p^d/e^d \begin{cases} < \\ > \end{cases} 0, w_p - we_p^c/e^c \begin{cases} < \\ > \end{cases} 0 \Leftrightarrow a \begin{cases} < \\ > \end{cases} 0.$$

Therefore, the first term in the brackets in the definition of  $\zeta$  is always positive, whereas the second term depends on the factor intensity ranking.

Since our aim is to investigate the relationship between aging and trade in an environment as normal as possible, we make two assumptions that ensure the existence and uniqueness of the autarky steady state.

**Assumption 1**

$$dg_p(p, k(p))/dp = g_{pp} + g_{pk}k_p = g_{pp} + r_p[1/(1 + n)][w_p/(2 + \rho)] > 0.$$

This means that the supply of good 2 monotonically increases with its price, which occurs if and only if the production substitution effect outweighs

the Rybczynski effect. With this assumption, the domain of  $p$  for incomplete specialization is given by  $p \in (\underline{p}, \bar{p})$ , where  $\underline{p}$  and  $\bar{p}$  is implicitly defined as  $\lim_{p \rightarrow \underline{p}} Y_2 = 0$  and  $\lim_{p \rightarrow \bar{p}} Y_1 = 0$ , respectively. Then we have:

$$\lim_{p \rightarrow \underline{p}} z_2 > 0, \lim_{p \rightarrow \bar{p}} z_2 < 0,$$

where the latter follows from the fact that the excess demand for good 1 is positive for  $p \rightarrow \bar{p}$ , and Walras' law.

**Assumption 2**

$$\zeta < 0 \forall p \in (\underline{p}, \bar{p}).$$

This states that the uncompensated excess demand curve for good 2 per worker is downward-sloping everywhere, which is true if and only if the consumption and production substitution effects are dominant.

With Assumptions 1 and 2, we immediately obtain:

**Lemma 1** *There exists the unique  $p^a \in (\underline{p}, \bar{p})$ .*

## 2.6 Transitional dynamics

To examine the local dynamics around the autarky steady state, we totally differentiate Eqs. (14), (15), (16), and (17):

$$\begin{aligned} k_{t+1} - k^a &= [1/(1+n)][w_p/(2+\rho)](p_t - p^a), \\ 0 &= \sigma(p_{t+1} - p^a) + [1/(1+n)]e_p^d(d_{t+1} - d^a) + e_p^c(c_{t+1} - c^a) - r_p(k_{t+1} - k^a), \\ d_{t+1} - d^a &= (1/e^d)[1/(2+\rho)][(r_p - r e_p^d/e^d)w(p_{t+1} - p^a) + r w_p(p_t - p^a)], \\ c_{t+1} - c^a &= (1/e^c)[1/(2+\rho)](w_p - w e_p^c/e^c)(1+\rho)(p_{t+1} - p^a). \end{aligned}$$

Eliminating  $k_{t+1} - k^a$ ,  $d_{t+1} - d^a$ , and  $c_{t+1} - c^a$ , the path of  $p$  is given by:

$$\begin{aligned} p_{t+1} - p^a &= -(\zeta_+/\zeta_-)(p_t - p^a); \\ \zeta_- &\equiv \sigma + \frac{1}{2+\rho} \left[ \frac{1}{1+n} \left( r_p - r \frac{e_p^d}{e^d} \right) \frac{e_p^d}{e^d} w + \frac{e_p^c}{e^c} \left( w_p - w \frac{e_p^c}{e^c} \right) (1+\rho) \right], \\ \zeta_+ &\equiv -[1/(2+\rho)][1/(1+n)](r_p - r e_p^d/e^d)w_p > 0, \\ \zeta_- + \zeta_+ &= \zeta. \end{aligned}$$

Using this result, the path of  $k$  is solved as:

$$k_{t+1} - k^a = [1/(1+n)][w_p/(2+\rho)][-(\zeta_+/\zeta_-)(p_{t-1} - p^a)] = -(\zeta_+/\zeta_-)(k_t - k^a).$$

Since  $\zeta = \zeta_- + \zeta_+ < 0$  from Assumption 2 and  $\zeta_+ > 0$  from the Stolper-Samuelson theorem, we have  $0 < -(\zeta_+/\zeta_-) < 1$ . This implies that, given the initial condition  $k_0 > 0$ ,  $k_{t+1}$  monotonically converges to  $k^a$ . Finally,  $p_0$  is uniquely determined by  $k_0$  so that  $k_1 - k^a = [1/(1+n)][w_p/(2+\rho)](p_0 - p^a) = -(\zeta_+/\zeta_-)(k_0 - k^a)$  :

**Lemma 2** *The dynamic system is locally saddle-path stable around the autarky steady state.*

Assumption 2 is responsible for local saddle-path stability. It is also a sufficient condition for uniqueness of the steady state.

## 2.7 Demographic change

From Eqs. (18), (20), (21), and (22), the effect of a demographic shock on  $p^a$  is calculated as:

$$dp^a/dn = -\nu/\zeta; \nu \equiv [1/(1+n)^2][w/(2+\rho)](r_p - re_p^d/e^d).$$

$$dp^a/dn = -\nu/\zeta \left\{ \begin{array}{c} > \\ < \end{array} \right\} 0 \Leftrightarrow a \left\{ \begin{array}{c} < \\ > \end{array} \right\} 0.$$

In this model, aging is expressed as an exogenous decrease in the rate of population growth. Aging affects the market of good 2 per worker through two channels. First, it increases the demand for good 2 by the old generation per worker. Second, it increases capital per worker, which in turn changes the supply of good 2 through the Rybczynski theorem. When good 2 is more labor-intensive, both effects tend to raise the relative price of good 2. When good 2 is more capital-intensive, on the other hand, the Rybczynski magnification effect outweighs the demand effect of the old generation, thereby lowering the relative price of good 2. The following lemma summarizes our analysis:

**Lemma 3** *Aging lowers the autarky steady state relative price of the capital-intensive good.*

It is inferred from this lemma that an aging country exports the capital-intensive good. In the next section, we examine the free trade equilibrium analytically to check this prediction.

### 3 Free trade

Suppose that the world consists of two countries which are identical except the rates of population growth. Country A is an aging country, whereas country B is a younger country:  $n^A < n^B$ . Each country  $i$  ( $i = A, B$ ) was in its autarky steady state until period 0, and it opens to trade from period 1 on.

#### 3.1 Dynamic system

Note that Eqs. (1) - (6), and (9) - (12) still apply for each  $i$ . The only modification to the autarky model is about the market-clearing conditions for goods:

$$\sum_i Y_{1t}^i = \sum_i (L_{t-1}^i d_{1t}^i + L_t^i c_{1t}^i + K_{t+1}^i), \quad (23)$$

$$\sum_i Y_{2t}^i = \sum_i (L_{t-1}^i d_{2t}^i + L_t^i c_{2t}^i). \quad (24)$$

From Walras' law and the market-clearing conditions for factors, balance of trade is zero for each country.

We derive the dynamic system of the world economy. With identical technologies and free trade in goods, Eq. (1) implies that the factor prices are equalized even without international factor mobility:

$$r^i = r(p), w^i = w(p) \forall i. \quad (25)$$

With Eq. (25), Eqs. (14), (16), and (17) still apply for each  $i$ . On the other hand, Eq. (24), the market-clearing condition for good 2, is rewritten as:

$$\sum_i L_{t+1}^i g_p(p_{t+1}, k_{t+1}^i) = \sum_i L_{t+1}^i \{ [1/(1+n^i)] e_p^d(p_{t+1}) d_{t+1}^i + e_p^c(p_{t+1}) c_{t+1}^i \}. \quad (26)$$

Eqs. (14), (16), and (17) for  $i = A, B$ , and Eq. (26) constitute the dynamic system. Eq. (23) is automatically satisfied because of Walras' law.

#### 3.2 Transitional dynamics

We see from Eq. (26) that the world excess demand for good 2 per worker is the weighted average of the two countries' excess demands per worker, with the weights given by the shares of workers in the world. Since  $n^A < n^B$ , the worker share of the younger country is monotonically increasing toward

unity over time. Thus the free trade relative price continues to change until it reaches the autarky steady state relative price for the younger country. Taking this conjecture into account, we characterize the path of the free trade relative price.

Let us define the following functions:

$$\begin{aligned}\tilde{Z}_2^i(p_{t+1}, d_{t+1}^i, c_{t+1}^i, k_{t+1}^i) &\equiv [1/(1+n^i)]e_p^d(p_{t+1})d_{t+1}^i + e_p^c(p_{t+1})c_{t+1}^i - g_p(p_{t+1}, k_{t+1}^i), \\ Z_2^i(p_{t+1}, p_t^i) &\equiv \tilde{Z}_2^i(p_{t+1}, d_{t+1}^i(p_{t+1}, p_t^i), c_{t+1}^i(p_{t+1}, p_t^i), k_{t+1}^i(p_t^i)), \\ Z_2^W(p_{t+1}; p_t^A, p_t^B, \lambda_{t+1}^A) &\equiv \lambda_{t+1}^A Z_2^A(p_{t+1}, p_t^A) + (1 - \lambda_{t+1}^A) Z_2^B(p_{t+1}, p_t^B); \\ \lambda_{t+1}^A &\equiv L_{t+1}^A / (L_{t+1}^A + L_{t+1}^B) \in (0, 1).\end{aligned}$$

The first function is the compensated excess demand for good 2 per worker for country  $i$ . The second one, the uncompensated excess demand function for good 2 per worker for country  $i$ , is obtained by substituting Eqs. (14), (16), and (17) into  $\tilde{Z}_2^i(\cdot)$ . The third one is the uncompensated world excess demand function for good 2 per worker, where  $\lambda_{t+1}^A$  is the share of worker for country A. The free trade relative price in period  $t+1$  is determined by  $Z_2^W(p_{t+1}^*; p_t^A, p_t^B, \lambda_{t+1}^A) = 0$ , where an asterisk represents free trade. Comparing  $Z_2^i(p_{t+1}, p_t^i)$  with  $z_2^i(p^i)$  in the last section, we obtain the following properties:

1.  $Z_2^i(p^{ia}, p^{ia}) = z_2^i(p^{ia}) = 0$ . ( $\because$  definition of  $p^{ia}$ )
2.  $dZ_2^i/dp \approx \zeta^i < 0$  for  $dp = dp_{t+1} = dp_t^i$ . ( $\because$  Assumption 2)
3.  $\partial Z_2^i/\partial p_t^i \approx \zeta_+^i > 0$ . ( $\because$  Stolper-Samuelson theorem)
4.  $\partial Z_2^i/\partial p_{t+1} \approx \zeta_-^i < 0$ . ( $\because$   $\zeta^i = \zeta_-^i + \zeta_+^i$ , properties 2 and 3)

Finally, the autarky steady state relative prices work as the initial conditions:  $p_0^A = p^{Aa}$  and  $p_0^B = p^{Ba}$ .

The following proposition confirms our intuition that the free trade relative price should always lie between the two autarky steady state relative prices:

**Proposition 1** *There exists the unique  $\{p_{t+1}^*\}_{t=0}^\infty$  such that  $p_{t+1}^* \in (\min\{p^{Aa}, p^{Ba}\}, \max\{p^{Aa}, p^{Ba}\})$  for all  $t \in [0, \infty)$ .*

**Proof.** We prove this proposition only for  $a > 0$  (the proof for  $a < 0$  is straightforward by interchanging the roles of the two countries). Then

from Lemma 3, we have  $p^{Aa} > p^{Ba}$ . The following proof uses mathematical induction.

For  $t = 0$ ,  $Z_2^W(p_1; p_0^A, p_0^B, \lambda_1^A)$  satisfies the following relations:

$$\begin{aligned} Z_2^W(p^{Aa}; p^{Aa}, p^{Ba}, \lambda_1^A) &= \lambda_1^A Z_2^A(p^{Aa}, p^{Aa}) + (1 - \lambda_1^A) Z_2^B(p^{Aa}, p^{Ba}) < 0, \\ Z_2^W(p^{Ba}; p^{Aa}, p^{Ba}, \lambda_1^A) &= \lambda_1^A Z_2^A(p^{Ba}, p^{Aa}) + (1 - \lambda_1^A) Z_2^B(p^{Ba}, p^{Ba}) > 0, \end{aligned}$$

where properties 1 and 4 of  $Z_2^i(\cdot)$  are used. From the intermediate value theorem, there exists  $p_1^* \in (p^{Ba}, p^{Aa})$  such that  $Z_2^W(p_1^*; p^{Aa}, p^{Ba}, \lambda_1^A) = 0$ . From property 4,  $p_1^*$  is unique.

For  $t \geq 1$ , suppose that  $p_t^A = p_t^B = p_t^* \in (p^{Ba}, p^{Aa})$ . We have shown in the last paragraph that it is indeed true for  $t = 1$ . Property 3 implies that  $Z_2^W(p_{t+1}; p_t^*, p_t^*, \lambda_{t+1}^A)$  is bounded by:

$$Z_2^W(p_{t+1}; p^{Ba}, p^{Ba}, \lambda_{t+1}^A) < Z_2^W(p_{t+1}; p_t^*, p_t^*, \lambda_{t+1}^A) < Z_2^W(p_{t+1}; p^{Aa}, p^{Aa}, \lambda_{t+1}^A).$$

In the upper bound, we have  $Z_2^W(p^{Aa}; p^{Aa}, p^{Aa}, \lambda_{t+1}^A) = \lambda_{t+1}^A Z_2^A(p^{Aa}, p^{Aa}) + (1 - \lambda_{t+1}^A) Z_2^B(p^{Aa}, p^{Aa}) < 0$  from properties 1 and 2. Let  $\bar{p}_{t+1}^A (< p^{Aa})$  be the solution to  $Z_2^W(\bar{p}_{t+1}^A; p^{Aa}, p^{Aa}, \lambda_{t+1}^A) = 0$ . Similarly, in the lower bound, let  $\bar{p}_{t+1}^B (> p^{Ba})$  be the solution to  $Z_2^W(\bar{p}_{t+1}^B; p^{Ba}, p^{Ba}, \lambda_{t+1}^A) = 0$ . Since  $Z_2^W(\bar{p}_{t+1}^A; p^{Ba}, p^{Ba}, \lambda_{t+1}^A) < Z_2^W(\bar{p}_{t+1}^A; p^{Aa}, p^{Aa}, \lambda_{t+1}^A) = 0$ , we have  $\bar{p}_{t+1}^B < \bar{p}_{t+1}^A$ . In reference to  $\bar{p}_{t+1}^A$  and  $\bar{p}_{t+1}^B$ , the following relations hold:

$$\begin{aligned} Z_2^W(\bar{p}_{t+1}^A; p_t^*, p_t^*, \lambda_{t+1}^A) &< Z_2^W(\bar{p}_{t+1}^A; p^{Aa}, p^{Aa}, \lambda_{t+1}^A) = 0, \\ Z_2^W(\bar{p}_{t+1}^B; p_t^*, p_t^*, \lambda_{t+1}^A) &> Z_2^W(\bar{p}_{t+1}^B; p^{Ba}, p^{Ba}, \lambda_{t+1}^A) = 0. \end{aligned}$$

Hence there exists the unique  $p_{t+1}^* \in (\bar{p}_{t+1}^B, \bar{p}_{t+1}^A) \subset (p^{Ba}, p^{Aa})$  such that  $Z_2^W(p_{t+1}^*; p_t^*, p_t^*, \lambda_{t+1}^A) = 0$ . Since we have established that  $p_{t+1}^* \in (p^{Ba}, p^{Aa})$  if  $p_t^* \in (p^{Ba}, p^{Aa})$  for  $t \geq 1$ , and that  $p_1^* \in (p^{Ba}, p^{Aa})$ , the proof is done. ■

We next investigate how  $p_{t+1}^*$  behaves over time. The demographic assumption  $n^A < n^B$  implies that  $L_{t+1}^A/L_{t+1}^B$  is monotonically decreasing over time, and so is  $\lambda_{t+1}^A = (L_{t+1}^A/L_{t+1}^B)/(L_{t+1}^A/L_{t+1}^B + 1)$ . Thus it is expected that  $p_{t+1}^*$  monotonically approaches  $p^{Ba}$ . The following proposition gives indirect support to this argument:

**Proposition 2** *Let  $\bar{p}_{t+1}^A$  and  $\bar{p}_{t+1}^B$  be the equilibrium price bounds corresponding to  $p_t^A = p_t^B = p^{Aa}$  and  $p_t^A = p_t^B = p^{Ba}$ , respectively. For  $a > 0$  (or  $a < 0$ ), both  $\bar{p}_{t+1}^A$  and  $\bar{p}_{t+1}^B$  fall (or rise) monotonically over time.*

**Proof.** We prove this proposition only for  $a > 0$  (the proof for  $a < 0$  is straightforward by interchanging the roles of the two countries). In period  $t$ , noting that  $Z_2^W(p^{Aa}; p^{Aa}, p^{Aa}, \lambda_t^A) = \lambda_t^A Z_2^A(p^{Aa}, p^{Aa}) + (1 - \lambda_t^A) Z_2^B(p^{Aa}, p^{Aa}) < 0$  from properties 1 and 2 of  $Z_2^i(\cdot)$ , let  $\bar{p}_t^A (< p^{Aa})$  be the solution to  $Z_2^W(\bar{p}_t^A; p^{Aa}, p^{Aa}, \lambda_t^A) = \lambda_t^A Z_2^A(\bar{p}_t^A, p^{Aa}) + (1 - \lambda_t^A) Z_2^B(\bar{p}_t^A, p^{Aa}) = 0$ , where  $Z_2^A(\bar{p}_t^A, p^{Aa}) > 0$  from property 4, and hence  $Z_2^B(\bar{p}_t^A, p^{Aa}) < 0$ . In period  $t+1$ , since  $Z_2^W(\bar{p}_t^A; p^{Aa}, p^{Aa}, \lambda_{t+1}^A) = \lambda_{t+1}^A Z_2^A(\bar{p}_t^A, p^{Aa}) + (1 - \lambda_{t+1}^A) Z_2^B(\bar{p}_t^A, p^{Aa}) < \lambda_t^A Z_2^A(\bar{p}_t^A, p^{Aa}) + (1 - \lambda_t^A) Z_2^B(\bar{p}_t^A, p^{Aa}) = 0$ , we have  $\bar{p}_{t+1}^A < \bar{p}_t^A$ .

On the other hand, let  $\bar{p}_t^B (> p^{Ba})$  be the solution to  $Z_2^W(\bar{p}_t^B; p^{Ba}, p^{Ba}, \lambda_t^A) = \lambda_t^A Z_2^A(\bar{p}_t^B, p^{Ba}) + (1 - \lambda_t^A) Z_2^B(\bar{p}_t^B, p^{Ba}) = 0$ , where  $Z_2^B(\bar{p}_t^B, p^{Ba}) < 0$  from property 4, and hence  $Z_2^A(\bar{p}_t^B, p^{Ba}) > 0$ . Then  $Z_2^W(\bar{p}_t^B; p^{Ba}, p^{Ba}, \lambda_{t+1}^A) < Z_2^W(\bar{p}_t^B; p^{Ba}, p^{Ba}, \lambda_t^A) = 0$  implies that  $\bar{p}_{t+1}^B < \bar{p}_t^B$ . ■

For  $a > 0$  and for  $p_t^*$  such that  $Z_2^W(p_t^*; p_{t-1}^A, p_{t-1}^B, \lambda_t^A) = \lambda_t^A Z_2^A(p_t^*, p_{t-1}^A) + (1 - \lambda_t^A) Z_2^B(p_t^*, p_{t-1}^B) = 0$ , we would obtain the desired result  $p_{t+1}^* < p_t^*$  if  $Z_2^W(p_t^*; p_t^*, p_t^*, \lambda_{t+1}^A) = \lambda_{t+1}^A Z_2^A(p_t^*, p_t^*) + (1 - \lambda_{t+1}^A) Z_2^B(p_t^*, p_t^*) < 0$ . In fact, the sign of  $Z_2^W(p_t^*; p_t^*, p_t^*, \lambda_{t+1}^A)$  is ambiguous because  $Z_2^i(p_t^*, p_t^*) \neq Z_2^i(p_t^*, p_{t-1}^i)$  for all  $i$ . Although we cannot ensure that  $p_{t+1}^*$  changes monotonically, its upper and lower bounds always move toward  $p^{Ba}$ . This indicates that  $p_{t+1}^*$  also approaches  $p^{Ba}$  over time.

### 3.3 Steady state

The free trade steady state is given by  $Z_2^W(p^*; p^*, p^*, \lambda^{A*}) = 0$ , where  $\lambda^{A*}$  is a constant value of  $\lambda_{t+1}^A$ . The latter becomes constant if and only if it asymptotically approaches zero as  $t$  approaches infinity. Therefore,  $p^*$  is determined by  $Z_2^W(p^*; p^*, p^*, 0) = Z_2^B(p^*, p^*) = 0$ . Comparing this with  $z_2^B(p^{Ba}) = 0$ , we obtain  $p^* = p^{Ba}$ :

**Proposition 3** *The free trade steady state relative price is asymptotically equal to the autarky one for the younger country. In the free trade steady state, the aging country behaves like a small open economy, whereas the younger country behaves like a closed economy asymptotically.*

It might seem a bit strange from a practical point of view that the younger country would dominate the world economy, while the aging country would become small in the long run. This result also contrasts with Sayan (2005, bottom of Fig. 5), who does not examine the free trade equilibrium analytically, but shows numerically that the free trade steady state relative price lies strictly between the two autarky steady state relative prices. However, our proposition is rather a natural consequence of any multi-country

dynamic trade model in the absence of an adjustment mechanism on the countries' growth rates. It is the difference in the exogenous rates of population growth that is directly responsible for our result.

### 3.4 Pattern of trade

We check our inference given in the previous section that the aging country exports the capital-intensive good. Suppose first that  $a > 0$ . From Proposition 1 and property 4 of  $Z_2^i(\cdot)$ , we have  $Z_2^A(p_1^*, p_0^A) > Z_2^A(p^{Aa}, p^{Aa}) = 0$  in period 1. In the free trade steady state, Proposition 3 and property 2 imply that  $Z_2^A(p^*, p^*) > Z_2^A(p^{Aa}, p^{Aa}) = 0$ . During the transition, linearizing  $Z_2^A(p_{t+1}^*, p_t^*)$  around  $p_{t+1}^* = p_t^*$  yields  $Z_2^A(p_{t+1}^*, p_t^*) \approx Z_2^A(p_t^*, p_t^*) + \zeta_-^A(p_{t+1}^* - p_t^*)$ . The first term in the right-hand side is positive from Proposition 1 and property 2. The second term is also positive if  $p_{t+1}^* < p_t^*$  as expected from Proposition 2. Therefore,  $Z_2^A(p_{t+1}^*, p_t^*) > 0$  if  $p_{t+1}^* < p_t^*$ . From the fact that  $Z_2^A(p_{t+1}^*, p_t^A) > 0$  for all  $t \geq 0$  and the zero trade balance, country A always exports good 1, the capital-intensive good. The case in which  $a < 0$  is similarly analyzed by interchanging the roles of the two countries:

**Proposition 4** *The aging country exports the capital-intensive good both in the period of opening to trade and in the free trade steady state. Moreover, it is true for all transitional periods if  $p_{t+1}^*$  converges monotonically to  $p^* = p^{Ba}$ .*

We have so far characterized the free trade equilibrium analytically. In the next section, we turn to a normative analysis of the free trade equilibrium.

## 4 Gains from trade

### 4.1 Uncompensated trade

We examine how opening trade affects the utility of the representative household in each generation in each country. Since the sign of  $p_{t+1}^* - p^{ia}$  is definite for each country from Proposition 1, all generations born after opening trade are influenced by trade in qualitatively the same way. Substituting Eqs. (16) and (17) into the utility function and totally differentiating it, we obtain:



$$dU_t^i = \phi_t dp_t^i + \psi_{t+1} dp_{t+1}^i \forall i;$$

$$\phi_t \equiv (1/w_t)\{w_{p,t} - w_t e_{p,t}^c/e_t^c + [1/(1+\rho)]w_{p,t}\} \begin{cases} > \\ < \end{cases} 0 \Leftrightarrow a \begin{cases} > \\ < \end{cases} 0,$$

$$\psi_{t+1} \equiv [1/(1+\rho)](1/r_{t+1})(r_{p,t+1} - r_{t+1} e_{p,t+1}^d/e_{t+1}^d) \begin{cases} > \\ < \end{cases} 0 \Leftrightarrow a \begin{cases} < \\ > \end{cases} 0.$$

Any household has two roles as factor suppliers in his/her life cycle: he/she is a worker when young, and a capitalist when old. Because of the dual roles, price changes in the same direction have counteracting effects on his/her utility through the Stolper-Samuelson theorem. To be more precise, noting that  $a >$  (or  $<$ )  $0$  implies that  $p_{t+1}^* - p^{Aa} <$  (or  $>$ )  $0$  and  $p_{t+1}^* - p^{Ba} >$  (or  $<$ )  $0$ , we have  $\phi_t dp_t^A < 0$ ,  $\psi_{t+1} dp_{t+1}^A > 0$ ,  $\phi_t dp_t^B > 0$ , and  $\psi_{t+1} dp_{t+1}^B < 0$  regardless of the factor intensity ranking. Intuitively, the young generation as a worker loses but the old generation as a capitalist gains from trade in the more capital-abundant aging country, whereas the opposite is true in the younger country. Although it is ambiguous whether the generations born after opening trade gain or lose, we can clearly discuss it for the initial old generation, whose countries were still in autarky when this generation was young:

**Proposition 5** *In the period of opening trade, the old generation in the aging country gains from trade, whereas that in the younger country loses from trade.*

This proposition says that uncompensated trade cannot be Pareto superior to autarky. Sayan (2005) also demonstrates numerically that the utility of the representative household in the aging country is likely to fall in the long run. In contrast, we have shown rigorously that the initial old generation in the capital-scarce younger country necessarily loses from trade.

## 4.2 Compensated trade

The impossibility of gains from trade without government intervention motivates us to seek a compensation mechanism which ensures gains from trade for all generations for all countries. Kemp and Wong (1995) propose some gainful domestic compensation schemes for a small, two-good, two-factor overlapping generations economy. Here we apply one of the Kemp-Wong compensation schemes to our two-country model.

Suppose that in period  $t \geq 1$  the government in country  $i$  gives lump-sum net transfers (taxes if negative)  $b_t^{ci}$  and  $b_t^{di}$  to the young and the old generations living in that period, respectively, and it also provides a specific savings subsidy (tax if negative)  $\tau_t^i$ . Then the budget constraints for the young and the old generations become, respectively:

$$s_t^i + c_{1t}^i + p_t c_{2t}^i = w_t + b_t^{ci}, \quad (27)$$

$$d_{1t}^i + p_t d_{2t}^i = (r_t + \tau_t^i) s_{t-1}^i + b_t^{di}. \quad (28)$$

The savings function is now calculated as:

$$s_t^i = [1/(2+\rho)]\{w_t + b_t^{ci} - [(1+\rho)/(r_{t+1} + \tau_{t+1}^i)]b_{t+1}^{di}\} \equiv s_t^i(w_t + b_t^{ci}, r_{t+1} + \tau_{t+1}^i, b_{t+1}^{di}).$$

Consider the following compensation scheme:

$$b_t^{ci} = s_t^{ia} + c_{1t}^{ia} + p_t c_{2t}^{ia} - w_t + P_t^i / (2L_t^i), \quad (29)$$

$$b_t^{di} = d_{1t}^{ia} + p_t d_{2t}^{ia} - r_t s_{t-1}^{ia} - \tau_t^i s_{t-1}^i + P_t^i / (2L_{t-1}^i); \quad (30)$$

$$P_t^i \equiv Y_{1t}^i + p_t Y_{2t}^i - (Y_{1t}^{ia} + p_t Y_{2t}^{ia}), \quad (31)$$

$$\tau_t^i : s_{t-1}^{ia} = s_{t-1}^i (w_{t-1} + b_{t-1}^{ci}, r_t + \tau_t^i, b_t^{di}). \quad (32)$$

Remember that the variables with superscript  $a$  represent the autarky equilibrium values, which are already obtained in section 2. Eq. (31) can be called the production gain evaluated at the free trade relative price. Eqs. (29) and (30) indicate that the government divides the production gain evenly for the two age groups, and then redistributes it equally within each group. Eq. (32) states that the saving subsidy is chosen so that the subsidized saving replicates the autarky one. Note that all  $b_t^{ci}$ ,  $b_t^{di}$ ,  $P_t^i$ , and  $\tau_t^i$  are functions of the free trade relative prices, which all households, firms, and governments take as given. The path of the free trade relative price with compensation is endogenously determined just like section 3.2. The following proposition establishes gains from compensated trade:

**Proposition 6** *With the compensation scheme characterized by Eqs. (29) to (32) for all  $t \geq 1$  and for all  $i = A, B$ , if consumption substitution occurs for some households, then free trade is Pareto superior to autarky.*

**Proof.** We first verify that the compensation mechanism is feasible; that is, the government budget balance is zero in each period. Using Eqs. (29), (30), (31), and (32), it is given by:

$$\begin{aligned}
& L_t^i b_t^{c^i} + L_{t-1}^i b_t^{d^i} + L_{t-1}^i \tau_t^i s_{t-1}^i \\
&= L_t^i (s_t^{ia} + c_{1t}^{ia}) + p_t L_t^i c_{2t}^{ia} - w_t L_t^i + L_{t-1}^i d_{1t}^{ia} + p_t L_{t-1}^i d_{2t}^{ia} - r_t L_{t-1}^i s_{t-1}^{ia} \\
&+ Y_{1t}^i + p_t Y_{2t}^i - Y_{1t}^{ia} - p_t Y_{2t}^{ia} \\
&= Y_{1t}^i + p_t Y_{2t}^i - w_t L_t^i - r_t K_t^i (\cdot: (7), (8), (11), (32)) \\
&= 0. (\cdot: (1), (9), (10))
\end{aligned}$$

We next evaluate the consumption bundles in autarky and trade situations. Since Eqs. (11) and (32) ensure that factor supplies do not change with the opening of trade, the GDP function means that the production gain is nonnegative:  $P_t^i \geq 0$ . Substituting Eqs. (29), (30), and (32) into Eqs. (27) and (28), we obtain:

$$\begin{aligned}
c_{1t}^i + p_t c_{2t}^i &= c_{1t}^{ia} + p_t c_{2t}^{ia} + P_t^i / (2L_t^i) \geq c_{1t}^{ia} + p_t c_{2t}^{ia}, \\
d_{1t}^i + p_t d_{2t}^i &= d_{1t}^{ia} + p_t d_{2t}^{ia} + P_t^i / (2L_{t-1}^i) \geq d_{1t}^{ia} + p_t d_{2t}^{ia}.
\end{aligned}$$

These imply that, evaluated at the free trade relative price, the value of consumption in free trade is no less than that in autarky for both the young and the old generations, for all  $t \geq 1$ , and for all  $i = A, B$ . If the free trade consumption bundle is different from the autarky one for some household, then from the principle of revealed preference, his/her utility in free trade is higher than that in autarky. ■

The only obstacle to gains from trade in our dynamic world economy is that for some countries the supply of capital under free trade may be smaller than under autarky due to the decrease in savings. Our compensation scheme overcomes the obstacle by providing an offsetting savings subsidy. The only remaining task is to redistribute the guaranteed production gain to everyone.

## 5 Concluding remarks

Our model has important policy implications. First, as our theoretical results have demonstrated, faster aging in developed countries is a cause of trade in the early periods, but the incentives to trade for the younger developing countries diminish during the transition. This arises because the latter is becoming more and more influential in the world economy as its population share grows bigger. Second, although some people may lose from trade

under laissez-faire (it is definitely true for the old generation in developing countries at the very moment of opening to trade), free trade is potentially Pareto superior to autarky with proper compensation mechanisms. This gives new support to free trade even with population aging.

We suggest some directions for future research. First, the extreme dynamic results of our model are due to the assumption that the rates of population growth for the two countries are exogenous and different. In fact, economic activities including trade may affect the costs and benefits of raising children. We would be able to obtain more realistic results of aging and trade if countries' population growth rates can be endogenized and sometimes equalized in the long run. However, even in this case, if the population growth rate in the aging country stays lower than the other country, our results still remain robust. Second, it is assumed that the governments as well as households and firms take prices as given when implementing our compensation scheme. However, each government may have an incentive to manipulate the scheme to affect the terms of trade in its favor. It will be interesting to see whether gains from trade are retained with more strategic governments.

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