An Evolutionary Analysis of Pre-Play Communication and Efficiency in Games

Kenichi Amaya*
Kobe University

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Abstract

This paper studies the effects of pre-play communication on equilibrium selection in 2×2 symmetric coordination games. The players repeatedly play a coordination game preceded by an opportunity to exchange payoff irrelevant messages and gradually adjust their behavior. In short run, the players' access to the actions of the coordination game may be restricted. While the players can revise the set of accessible actions only occasionally, they frequently adjust their behavior in the cheap-talk game, taking the set of currently available actions as given. We obtain an efficient-equilibrium-selection result if the underlying coordination game satisfies the self-signalling condition. On the other hand, if the game is not self-signalling, both the efficient and the inefficient equilibrium outcomes are stable.

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*Research Institute for Economics and Business Administration, Kobe University. 2-1 Rokkodai-cho Nada-ku Kobe 657-8501 Japan. Email: amaya@rieb.kobe-u.ac.jp

1 Introduction

There are many interesting economic and social problems that can be analyzed as coordination games. Since coordination games have multiple equilibria, the question of equilibrium selection has attracted much interest. For example, the literature in stochastic evolution (e.g. Kandori, Mailath and Rob [9] and Young [17]) and the literature in robustness to incomplete information (e.g. Carlsson and van Damme [4]) have selected risk dominant equilibria (Harsanyi and Selten [8]).

There has been a substantial literature in evolutionary game theory which argues that if players can communicate each other before playing the game, then the evolutionary force leads to Pareto efficient equilibria (e.g. Robson [13], Wärneryd [16], Matsui [11], Kim and Sobel [10]).

The essential idea in evolutionary game theory is that the players play the same game repeatedly and gradually adjust their behavior. In the evolutionary analysis of pre-play communication, the game to be repeated is a *cheap-talk game*, which consists of two stages. In the first stage, each player sends a message. In the second stage, each player chooses an action of the coordination game. The second stage actions can be contingent on the messages observed in the first stage. Therefore, a *strategy* of the cheap talk game consists of two components: (i) a messege to send in the first stage and (ii) a *decision rule*, which specifies an action to play in the second stage for each possible outcome in the first stage. If we fix the set of strategies in the coordination game and the message space, then the strategy space of the cheap talk game is determined accordingly.

The existing literature on evolution and pre-play communication has assumed (explicitly or in some indirect way) that whenever a player faces an opportunity to adjust her behavior, she can freely adopt anything in the strategy space of the cheap talk game. In this paper, we claim that this assumption is not very realistic in many economic applications. We show that if we modify this assumption in a certain reasonable way, the efficient-equilibrium-selection result does not hold any more.

Consider the following example which illustrates the motivation for our approach. Suppose there is a society with many people. People are randomly matched into pairs and engage in some productive activity. Each player chooses between "Big Project" and "Small Project". The Big Project succeeds only when the two players collaborates. If only one player chooses the Big Project, she incurs a cost in vain. This game has two pure strategy Nash equilibria; one in which both players choose "Big Project" and the other in which both players choose "Small Project". Suppose the former equilibrium Pareto dominates the other. In other words, this is a coordination game. In the cheap-talk extension of this game, each player first sends a message and then chooses an action of the coordination game.

Now suppose that in order to work on a project, one needs to have a project-specific equipment (or know-how, or ability) in his hand. Only occasionally does a player have an opportunity to acquire a new equipment or abondon an existing equipment. In contrast, a player frequently faces an opportunity to adjust the behavior in the cheap talk game, taking the set of equipment in hand as given. For example, if a player has equipment for both projects, he can choose anything in the strategy space of the cheap talk game. On the other hand, if he has only the equipment for "Small Project", then he is forced to choose "Small Project" in the second stage of the cheap talk game. However, he still has a freedom in the choice of his message in the first stage.

This story sounds very reasonable if we consider real economic problems. A manufacturing firm makes decisions of building a new factory or closing an existing factory only occasionally. In contrast, it adjusts the project to implement and how to communicate with other firms with much mobility.

Based on the above motivation, this paper analyzes the effects of pre-play communication on equilibrium selection under an evolutionary adjustment process with the following properties: (i) The players adjust the set of accessible actions of the coordination game only occasionally. (ii) The players frequently adjust their behaviors in the cheap-talk game, taking the set of accessible actions as given.[1]

We analyze 2×2 symmetric coordination games with two symmetric pure strategy Nash equilibria that are Pareto ranked. We show that whether we obtain an efficient-equilibrium-selection result depends on the structure of the underlying coordination game, i.e., whether the game is *self-signalling* or not.

[1] We call strategies in the cheap talk game *behaviors* to avoid confusion, because we will label another object as *strategy*.

	a	b
a	$9,9$	$0,6$
b	$6,0$	$7,7$

Figure 1.

	a	b
a	$9,9$	$0,8$
b	$8,0$	$7,7$

Figure 2.

A game is said to be self-signalling if a player has a right incentive to reveal her intention of play. Consider the game of Figure 1. If the row player is planning to play b, then she has a right incentive to reveal her intention. If she can successfully convince the column player of her intention of playing b, then the column player's rational reaction is to play b, and the row player receives 7. On the other hand, if the row player misleads the column player to believe she will play a, this leads the column player to play a, which gives the row player the payoff of 6. Similarly, if the row player is planning to play a, then she has a right incentive to reveal it. Therefore this game is self-signalling.

In contrast, in the game of Figure 2, if the row player is planning to play b, then she does not have a right incentive to reveal her intention. If she convinces the column player that she will indeed play b, this leads the column player to play b, and her payoff will be 7. If the row player deceives the column player and lets him believe she will play a, then the column player plays a and the row player's payoff will be 8. Therefore this game is not self-signalling.

For general 2×2 symmetric coordination games, if we call the efficient equilibrium strategy a and the inefficient equilibrium strategy b, then the game is self-signalling if and only if (b, b) gives a higher payoff than (b, a) to the row player.[2]

We show that if the underlying coordination game is self-signalling, then only the Pareto efficient equilibrium outcome is stable. If the underlying game is not self-signalling, then both the efficient equilibrium outcome and the inefficient equilibrium outcome are stable. Therefore an efficient-equilibrium-selection result is obtained if and only if the underlying game

[2] For more discussion on the self-signalling condition, see a survey of the cheap talk literature by Farrell and Rabin [6] .

is self-signalling. We emphasize here that these results have nothing to do with the risks of the equilibria. Both in the games of Figure 1 and Figure 2, the inefficient equilibrium (b, b) is risk dominant. If we increase the payoffs in the efficient equilibrium (a, a) to 50, then this equilibirum becomes risk dominant while the self-signalling condition is unchanged.

The result which should be highlighted in comparison with the existing literature is that the inefficient equilibrium outcome is now stable if the game is not self-signalling. Let us give an intuition for this result here.

The argument in the existing literature is as the following. Suppose that initially all the players are playing the inefficient equilibrium action (action b). Now a small population of mutants enters. These mutants send a new message that is not used by the incumbents, and plays the efficient equilibirum action (action a) if and only if the opponent also sends this new message. They play b otherwise. After the entry of these mutants, an incumbent player always receives the inefficient equilibrium payoff. On the other hand, a mutant player receives the inefficient equilibrium payoff when she is matched with an incumbent, and receives the efficient equilibrium payoff when matched with another mutant. Therefore, the mutants receive a higher payoff on average and invade the population. The key point here is that the mutants can separate themselves away from the incumbents through messages and can coordinate on the efficient equilibrium only among themselves. This is so called "secret handshake" effect of communication.

Now consider what happens in our analysis. There are three possible types of players in terms of accessibility to the actions; type a, who has access only to action a, type b, who has access only to action b, and type s, who can access both strategies.[3] Imagine initially only type b players exist in the population. Suppose now a small population of mutants enters. These mutants are type s and plays the "secret handshake" behavior as described in the previous analysis. Now, before these type s mutants thrive in the population, the incumbents can immediately adjust their messages, because we are assuming that the adjustment of behaviors under a fixed type distribution occurs much more frequently than the evolution of types. If these mutants enter, the incumbents always try to send the same message

[3] Of course, we can think of another type who can access neither action. We can consider a model with this type by properly defining the payoff from playing neither action, i.e., exiting the game. We expect this consideration will not change our result.

as the munants so as to induce the mutants to play action a. Therefore, the mutants can never successfully separate themselves out from the incumbent through messages. Here, a secret handshake does not work. In fact, the incumbents have such an incentive to pool with the mutants if and only if the game is not self-signalling.

Let us turn to a discussion of our modelling methodology. In our model, two things evolve over time. First, the population distribution of types evolves. Second, the behaviors of the players in the cheap-talk game evolve. Since we assume the behaviors of the players are adjusted much more frequently than the evolution of types, we separate these two evolutions in the following way. For each fixed type distribution, we look for stable outcomes in the adjustment of behaviors and examine how well each type performs in the stable outcomes. We model the evolution of the type distribution in the way that the type earning a higher payoff in the stable outcomes thrives better. This is the same approach as the literature in evolutoin of preferences (e.g. Ely and Yilankaya [5] and Sandholm [14]). In fact, we can find much analogy between our problem and evolution of preferences. In the study of evolution of preferences, the players' preferences evolve slowly, while they adjust their behavior in games taking the current preference as exogenously fixed.

The rest of the paper is organized as the following. Section 2 describes the cheap talk game, which is to be repeated. Section 3 defines the types of the players and the equilibrium under a fixed type distribution. Section 4 studies the evolution of the type distribution. Section 5 considers the evolution of behaviors under fixed type distributions. Section 6 discusses how our result may be changed if we alter some of our assumptions. Section 7 surveys the related literature. Section 8 concludes.

2 The Cheap Talk Game

Let G be the base game. We assume G is a 2×2 symmetric coordination game. The set of pure strategies is $\{a, b\}$ and π_{ij}, $i, j = a, b$ is the payoff when a player plays i and the opponent plays j. Thus the payoff matrix is given by Figure 3.

	a	b
a	π_{aa}, π_{aa}	π_{ab}, π_{ba}
b	π_{ba}, π_{ab}	π_{bb}, π_{bb}

Figure 3.

Assume $\pi_{aa} > \pi_{ba}$, $\pi_{bb} > \pi_{ab}$, and $\pi_{aa} > \pi_{bb}$. By the first two inequalities the game has two symmetric pure strategy Nash equilibria, namely both playing a and both playing b. The last inequality says the former equilibrium is Pareto efficient. Also assume there is no tie in payoffs. The game is said to be *self-signalling* if $\pi_{bb} > \pi_{ba}$. To avoid confusion later, we call the strategies in the base game *actions*.

Players are randomly matched into pairs and play the cheap talk game. The cheap talk game consists of two stages. In the first stage, both players send a message $m \in M = \{m_1, m_2\}$ simultaneously. In the second stage, both players play either a or b of the base game simultaneously, having observed the messages sent in the first stage. The play in the second stage can be contingent on the message sent by the opponent in the first stage. The payoff from the cheap-talk game is determined by the actual action chosen in the second stage. The first stage messages do not directly affect the players' payoffs.

To avoid confusion later, we call the strategies in the cheap talk game *behaviors*. A behavior $\sigma = (\mu, f)$ in the cheap talk game consists of two components. The first component $\mu \in M$ specifies which message to send in the first stage. The second component f, which we call a *decision rule*, specifies an action of the base game to play in the second stage for each realization of the opponent's message in the first period. Therefore, a decision rule is a mapping from the message space to the action space. There are four possible (pure) decision rules f_a, f_b, f_c and f_d.

$$\begin{aligned} f_a(m_1) = f_a(m_2) &= a, \\ f_b(m_1) = f_b(m_2) &= b, \\ f_c(m_1) = a, \quad f_c(m_2) &= b, \\ f_d(m_1) = b, \quad f_d(m_2) &= a. \end{aligned}$$

We denote the set of all decision rules by $F = \{f_a, f_b, f_c, f_d\}$.

3 Types of the Players and Short Run Games

The *type* θ of a player describes which actions of the base game are available for the player. The set of possible types is $\Theta \equiv \{a, b, s\}$. A player of type a can play only action a, and a player of type b can play only action b. There is a third type, called type s, who can choose between actions a and b.

Since a type a player can access only action a, the set of behaviors available for type a players is $\Sigma_a = \{(m_1, f_a), (m_2, f_a)\}$. Similarly, the set of behaviors available for type b players is $\Sigma_b = \{(m_1, f_b), (m_2, f_b)\}$. Since type s players can follow any of the decision rules in F, the set of behaviors available for them is $\Sigma_s = M \times F$.

Let $x = (x(a), x(b), x(s))$ be a vector representing the population ratio of each type in the whole population, where $x(\theta)$ is the proportion of type θ. Naturally, we require $x \in \Delta\Theta$ where

$$\Delta\Theta \equiv \{(x(a), x(b), x(s)) \in R^3 : \quad x(\theta) \in [0, 1], \ \sum_{\theta} x(\theta) = 1\}.$$

In the rest of the paper, we refer to x as a *type population state (TPS)*.

The idea of this paper is that in the short run, each player takes the set of accessible actions as given and adjust the behavior in the cheap-talk game. In the terminology of our model, the type of each player is exogenously fixed (and thus the TPS is fixed) and the players choose their behaviors. We will call this strategic interaction as a *short run game*. We assume that the adjustment of behaviors is sufficiently fast so that an equilibrium of the short run game is always played. To properly define the equilibrium concept of the short run game, we need to specify the information structure. We assume that each player knows her own type but does not know the type of the player whom she is matched with. However, the players know the current TPS and also know that the random matching obeys the uniform distribution. Therefore, the short run game can be described as a game of incomplete information.

We denote a strategy of the short run game by y. A strategy y specifies a probability distribution over Σ_θ for each type θ. Formally, let $\Delta\Sigma_\theta$ be the set of probability distributions over Σ_θ and

$$y : \Theta \to \cup_{\theta \in \Theta} \Delta\Sigma_\theta,$$

where $y_\theta \in \Delta\Sigma_\theta$ for all θ. Here $y_\theta(\sigma)$ is the probability of playing strategy σ when the type is θ, and $y_\theta = \{y_\theta(\sigma)\}_{\sigma \in \Sigma_s}$.

Here we limit our attention to symmetric equilibria, where all the players are playing the same strategy y. This is without any loss of generality. It can be easily verified that if there is an asymmetric Bayesian Nash equilibrium of the short run incomplete information game where proportion α of players play strategy y and proportion $1-\alpha$ of players play strategy y', then the strategy profile in which all the players are playing $\alpha y + (1-\alpha)y'$ is also a Bayesian Nash equilibrium.

Let $u(\sigma, \tilde{\sigma})$ be the payoff in the cheap talk game to a player when she plays behavior $\sigma = (\mu, f)$ and the opponent plays $\tilde{\sigma} = (\tilde{\mu}, \tilde{f})$. Therefore,

$$u(\sigma, \tilde{\sigma}) = \pi_{f(\tilde{\mu}), \tilde{f}(\mu)}.$$

We assume that a player is randomly matched with another player according to the uniform distribution. Therefore, under the TPS x, if all the other players are following strategy y, the expected payoff to a player with behavior σ is given by

$$v(\sigma|x, y) = \sum_{\tilde{\theta} \in \Theta} \sum_{\tilde{\sigma} \in \Sigma_\theta} u(\sigma, \tilde{\sigma}) y_{\tilde{\theta}}(\tilde{\sigma}) x(\tilde{\theta}).$$

Notice that the expected payoff to a player depends on her behavior, but not on her type. A behavior $\sigma \in \Sigma_\theta$ is called a best response for a type θ player against strategy y under the TPS x, if it gives the highest expected payoff among the behaviors in Σ_θ. The set of pure best responses to a type θ player against y under the TPS x is denoted by

$$BR_\theta(x, y) = \text{argmax}_{\sigma \in \Sigma_\theta} v(\sigma|x, y).$$

Let $\overline{BR}_\theta(x, y)$ be the set of mixed best responses to a type θ player against y under x, i.e., the set of probability distributions on Σ_θ which puts positive probabilities only on the members of $BR_\theta(x, y)$. We write $y' \in \overline{BR}(x, y)$ if for all $\theta \in \Theta$, $y'_\theta \in \overline{BR}_\theta(x, y)$. The equilibrium concept of the short run game can be defined as the following.

Definition A strategy y is a *Bayesian Nash Equilibrium (BNE)* under the type population state (TPS) x if

$$y \in \overline{BR}(x, y).$$

4 Stability of Type Distributions

4.1 The Stability Concept

Now, we define the stability concept of type population states (TPS). The stability concept we use here borrows the backbones from the traditional stability concepts in evolutionary game theory. To illustrate the idea, consider the stability of a TPS where all players are of the same type θ. Suppose that initially all the players are of type θ. Now inject a small populaiton share ϵ of type θ' individuals ($\theta' \neq \theta$). The players immediately adjust to play a BNE of the short run game under the ex-post TPS $(1-\epsilon)\theta + \epsilon\theta'$. The initial TPS is not invaded by an injection of θ' if, for sufficiently small ϵ, type θ' does not receive higher payoff than type θ in *any* BNE under the ex-post TPS. The initial TPS is stable if it is not invaded by any small population of mutants, where the mutants may be a mixture of multiple types. The following definition generalizes this idea. This formal definition allows both the incumbent population and the mutant population be a mixture of multiple types.

Definition A TPS $x \in \Delta\Theta$ is *stable* if

for all $x' \in \Delta\Theta$, there exists $\bar{\epsilon} \in (0,1)$, such that $\forall \epsilon \in (0, \bar{\epsilon})$,

$$\begin{aligned}
&\sum_{\theta \in \Theta} \Big(\sum_{\sigma \in \Sigma_\theta} v(\sigma | (1-\epsilon)x + \epsilon x', y) y_\theta(\sigma) \Big) x(\theta) \\
\geq &\sum_{\theta \in \Theta} \Big(\sum_{\sigma \in \Sigma_\theta} v(\sigma | (1-\epsilon)x + \epsilon x', y) y_\theta(\sigma) \Big) x'(\theta)
\end{aligned}$$

for all y such that y is a BNE under $(1-\epsilon)x + \epsilon x'$.

Notice the same expression

$$\sum_{\sigma \in \Sigma_\theta} v(\sigma | (1-\epsilon)x + \epsilon x', y) y_\theta(\sigma)$$

appears in both side of the inequality. This is the expected payoff of a type θ individual in the BNE y under the post entry TPS $(1-\epsilon)x + \epsilon x'$. Therefore, the left hand side of the equation is the average post-entry payoff of the incumbents, and the right hand side is the average post-entry payoff of the mutants.

Two remarks must be stated about this definition of stability. First, so far, we are staying away from the issue of equilibrium selection in the short run games. This is of course an important issue and we will discuss it in the next section. However, we can claim the following. If we find that a TPS is stable, then we do not need to worry about the equilibium selection in the short run games. This is because the stability in this definition means that no matter what equilibrium is played in the post-entry short run game, the mutants cannot earn a higher average payoff than the incumbents. On the other hand, if we find that a TPS is not stable, we need to be careful about the equilibrium selection in the short run game. Instability here means only that there exists *some* equilibrium in which the mutants' average payoff exceeds the incumbents' average payoff. But this equilibrium may be an unreasonable prediction if we think about the equilibrium selection issue. It is possible that in "reasonable" equilibria the mutants can never earn a higher payoff than the incumbents.

Second, notice the condition is a weak inequality. It only requires that no mutant thrives in the sense of earning a strictly higher payoff than the incumbent. This corresponds to the concept of neutrally stable strategy (NSS) in the traditional evolutionary game theory. One may think that it is more reasonable to require the condition to hold in a strict inequality, which corresponds to the concept of evolutionarily stable strategy (ESS). Such a condition requires that no mutant persist in the sense of earning an equal or higher payoff than the incumbent. Our result does not hold any longer if we employ such an ESS-like stability concept.

4.2 Results

Now we state our results. We do not fully characterize the set of stable TPSs. Instead, we investigate if there is a stable TPS with an outcome such that all players play the same action of the base game, i.e., an outcome corresponding to a Nash equilibirum of the base game.

Our first finding is that the Pareto efficient equilibrium outcome is stable.

Proposition 1 Let x^s be the TPS such that $x^s(s) = 1$ and $x^s(a) = x^s(b) = 0$. Then, x^s is stable.

Proof See Appendix.

Of course, this proposition alone does not tell us the Pareto efficient equilibrium outcome is stable. It is supplemented by Proposition 4 in Section 5. Proposition 1 says that the TPS where only type s is present is stable. In this TPS, the short run game has multiple equilibria, in particular an equilibrium in which everyone plays a and another one where everyone plays b. Proposition 4 shows that if we consider evolution in the short run game, the unique stable outcome is such that everyone plays a.

The intuition for the proof of Proposition 1 is straightforward. Since a type s player has more freedom in choosing behaviors than other types, she can imitate the behavior of a player of type a or b. Therefore, in any equilibrium of the short run game, type s's payoff is at least as good as other types' payoffs. Hence, a mutant's payoff can never exceed the payoff of the type s incumbents.

Secondly, we show that if the base game satisfies the self-signalling condition, then the Pareto inefficient equilibrium outcome is unstable. The only candidate TPSs in which all the players are playing action b are the TPSs where only type b and type s are present. Because Proposition 4 in the next section rules out x^s from the candidates, it suffices to show that the TPS with a positive share of type b is unstable. Proposition 2 shows this.

Proposition 2 For $\beta \in (0,1]$, let x^β be the TPS such that $x^\beta(a) = 0$, $x^\beta(b) = \beta$ and $x^\beta(s) = 1-\beta$. If the base game is self-signalling, then for all $\beta \in (0,1]$, x^β is not stable.

Proof See Appendix.

To prove this, we show that the itinial TPS is unstable against a injection of type s mutants. In the post-entry TPS, both type s and type b have a strictly positive population share and there are no type a players. Lemma 1 shows that for such an TPS there exists an equilibrium in which type s receives strictly higher payoff than type b.

Lemma 1 For all $\beta \in (0,1)$, there exists a strategy of the short-run game y^β such that y^β is a BNE under x^β and

$$\sum_{\sigma\in\Sigma_s} v(\sigma|x^\beta, y^\beta) y^\beta_s(\sigma) > \sum_{\sigma\in\Sigma_b} v(\sigma|x^\beta, y^\beta) y^\beta_b(\sigma).$$

Since the incumbents are a mixture of type s and type b and the mutants are type s, the mutants receive a higher payoff than the incumbents on average.

As we discussed in the remark following the definition of stability, we need to be careful about whether this instability result is robust to the equilibrium selection issue in the short run games. In other words, it must be shown that the equilibrium in Lemma 1 is a reasonable one. Proposition 5 in the next section shows that there indeed exists an equilibrium satisfying the condition in Lemma 1, which is stable with respect to evolution in the short run games.

Lastly, we show that if the base game is not self-signalling, then the Pareto inefficient equilibrium outcome is stable.

Proposition 3 Let x^b be the TPS such that $x^b(b) = 1$ and $x^b(a) = x^b(s) = 0$. If the base game is not self-signalling, then, x^b is stable.

Proof See Appendix.

To prove this, we show that for any post-entry TPS where the population ratios of type a and type s are sufficiently small, there is no Bayesian Nash equilibrium of the short run game in which type a or type s receives a strictly higher average payoff than type b. This result shows that if the coordination game is not self-signalling, a secret handshake cannot happen in any equilibrium of the post-entry short run games.

5 Equilibrium Selection in Short Run Games

This section considers the equilibrium selection problems in short run games. In the previous section, we were assuming that under each TPS *some* equilibrium of the short run game is played, and we did not discuss which equilibrium should be played if the short run game has multiple equilibria. Now we explicitly examine this issue by an evolutionary approach.

This section has two goals. First, we show that in the TPS x^s, which was shown to be stable in Proposition 1, there is a unique outcome that is stable in terms of evolution in the short run game, where all players play action a of the base game.

Second, we show that Proposition 2 is robust with respect to equilibrium selection in short run games. Proposition 2 says that for $\beta \in (0, 1]$ the TPS x^β is unstable in the sense that if a certain kind of mutants enters the population, then the mutants receive a higher payoff than the incumbents in *some* equilibrium in the post-entry short run game. In particular, the mutants receive a higher payoff than the incumbents in an equilibrium described in Lemma 1. Now we show that this equilibrium is indeed reasonable, i.e., this equilibrium is stable in terms of evolution in the short run game.

Our approach here is as the following. We fix a TPS x and asks whether a set of Bayesian Nash equilibria under x is stable or not. The stability concepts we use here are *cyclically stable set* (CSS) proposed by Gilboa and Matsui [7] and *equilibrium evolutionarily stable set* (EES set) proposed by Swinkels [15]. Since Matsui [12] showed the equivalence of these two concepts, we are essentially working with only one solution concept. However, we use the names of both solution concepts just to make our argument simple and clear.

5.1 Stable Outcome under the TPS x^s

Under the TPS x^s, all the players have an access to both action a and action b of the base game. Intuitively speaking, when a player faces an opportunity to revise her behavior, she can simultaneously choose any message and decision rule. This is exactly the situation analyzed in the existing literature in evolutionary analysis of equilibrium selection with pre-play communication. In particular, The environment Matsui [11] analyzes coincides exactly with the short run game under the TPS x^s in our model. Matsui considers the case where the base game is a 2×2 symmetric game with two symmetric pure strategy Nash equilibria that are Pareto ranked and the size of message space is two. Matsui showed that there is a unique CSS and every strategy distribution in the CSS yields the outcome that all players play the Pareto efficient equilibrium action. We can simply apply Matsui's result to our analysis.

Proposition 4 Under the TPS x^s, there is a unique CSS in the short run game and every strategy distribution in the CSS yields the outcome that all players play a.

Proof See Matsui [11].

5.2 Robustness of Proposition 2

Here we show that for all $\beta \in (0, 1)$, there exists a "stable" equilibrium of the short run game which satisfies the condition in Lemma 1. We use EES set as our solution concept. Let Δ denote the strategy space of the short run game.

Definition $Y \subset \Delta$ is *an equilibrium evolutionarily stable (EES) set under the TPS* x if it is minimal with respect to the following property: (i) Y is a nonempty and closed set of Bayesian Nash equilibria under x and (ii) there exists $\bar{\epsilon} > 0$ such that $\forall \epsilon \in (0, \bar{\epsilon})$, $\forall y \in Y$ and $\forall y' \in \Delta$, $y' \in \overline{BR}(x, (1-\epsilon)y + \epsilon y')$ implies $(1-\epsilon)y + \epsilon y' \in Y$.

Proposition 5 Suppose the base game is self-signalling. Let y^* and y^{**} be the strategies in short run games defined below:

- $y^*_a(m_1, f_a) = y^*_b(m_2, f_b) = y^*_s(m_1, f_c) = 1$.
- $y^{**}_a(m_2, f_a) = y^{**}_b(m_1, f_b) = y^{**}_s(m_2, f_d) = 1$.

(i) For all $\beta \in (0, 1)$, $\{y^*\}$ and $\{y^{**}\}$ are (singleton) EES sets under the TPS x^β. (ii) Furthermore, for all $\beta \in (0, 1)$, both y^* and y^{**} satisfy the inequality in Lemma 1.

Proof See Appendix.

6 Discussion

This section discusses the robustness of our result with respect to the assumptions made in our model and possible extensions. In particular, we consider the issue of the solution concept and the size of message spaces. Also, we make an informal discussion for the reverse case, i.e., the case where the messages evolve slowly.

6.1 Solution Concept

As we mentioned in Section 4.1, our result does not remain true if we use a stronger stability concept which requires that the incumbents receive a strictly higher payoff than the mutants. In particular, Proposition 3 does not hold any more. If a small population of type s mutants enters, they receive the same payoff as the type b incumbents, and thus they stay in the population, although they do not grow. One may feel Proposition 3 is an unsatisfactory result beause an accumulation of type s mutants may increase their population share gradually and they may eventually perform better than type b. We can overcome this problem by slightly modifying our model. Suppose there are small fixed costs of keeping actions accessible. For example, these costs are the maintenance costs of factories in the example in Introduction. These costs enter the utility in a lexicographic manner. We say a player does better than another if either he receives a strictly higher payoff from the game or he receives the same payoff and incurs less cost of keeping actions. With these costs, the state x^b becomes stable under the stronger stability concept. If a small population of type s mutants enter, then they receive the same payoff as the incumbents and incur more fixed costs. The readers may feel the lexicographic representation of the fixed costs is too artificial and unrealistic. We claim it is not. In the real world, very many but only finite people are interacting each other. A continuous population model is only an approximation for such a society. Suppose the fixed cost is so small that one player's payoff may be increased or decreased by more than the fixed cost if only one of the other players changes the behavior. In such a case, the entry of the smallest possible population of mutants may have a larger impact than the fixed cost. However, when we consider stability in a continuous population model, the population share of the mutants can be taken arbitrarily small so that the impact of the mutants' entry can never dominate the fixed cost. If we remain working with a continuous population model and want to incorporate the possibility that the minimal mutants' effect dominate the fixed cost, it is fairly reasonable to model the fixed cost in a lexicographic manner.

6.2 The Size of The Message Space

We worked on a model with the message space of size two, primarily for notational simplicity. Here we argue that our results remain to hold when the size of the message space is larger. Generally speaking, the secret handshake becomes easier as the size of the message space becomes larger, because it becomes easier for the mutants to find a message that is not used by the incumbents. Thus we are looking at the environment where destabilizing the inefficient equilibrium outcome is the most difficult. Proposition 2 and 4 show that even in such an environment, we can still destabilize the inefficient equilibrium. It is natural to guess that the same result can be obtained for the cases with larger message spaces. On the other hand, Proposition 3 may be regarded as a weak result because it only shows that the secret handshake is impossible in the most difficult environment. However, this result does not depend on the smallness of the message space. We can easily extend the result to the cases with larger message spaces.

6.3 Slow Adjustment of Messages

Consider the opposite case where the players change their messages only occasionally. In short run, each player takes his message as given and adjust their decision rules. This consideration applies to some biological evolution issues. Here a message is interpreted as a physical trait of an animal. A physical trait is inherited from ancestors and each individual animal takes it as exogenously given. Each individual animal frequently adjust which action to play in the coordination game, and the action may depend on the opponent's appearance, i.e., the message. With this interpretation, it is reasonable to assume it is the message of each individual player that evolves slowly, rather than the set of available messages.

Consider the cheap talk game defined in Section 2. There are two types of players, $i \in \{1, 2\}$. A type i player is forced to send message m_i in the first stage. However, he can adopt any decision rule in $F = \{f_a, f_b, f_c, f_d\}$.

Suppose initially only type 1 players are present and they are playing action b. We investigate if an injection of type 2 mutants can destabilize the inefficient equilibrium outcome. Suppose the type 1 incumbents are using the decision rule f_b, i.e., they always play action b. In this case, the type 2 mutants can do better than the incumbents if they adopt f_d, i.e., play action b when observing message m_1 and play action a when observing message m_2. After the entry of these mutants, a type 1 incumbent always receives the inefficient equilibrium payoff. A type 2 mutant receives the inefficient equilibirum payoff when she is matched with an incumbent, and receives the efficient equilibrium payoff when matched with another mutant. A type 1 incumbent has no incentive to change her decision rule if all the other type 1 players are following f_b and all type 2 players are following f_d In fact, it is an equilibrium of the short run game. Therefore, type 2 thrives and eventually dominates the population, and they play the efficient equilibrium. The same logic as the existing literature works here and the efficient equilibrium is selected no matter whether the game is self-signalling or not. One may worry the mutants cannot separate themselves through messages if both type 1 and 2 are initially present and they are playing the inefficient equilibrium. We can overcome this problem by just applying the argument of drift in the existing literature.

7 Related Literature

Our argument that the self-signalling condition is necessary for a communication to help achieving the efficient equilibrium has an analogy with Aumann's [1] discussion. Aumann raises a question to the old justification for Nash equilibrium as "self-enforcing agreements", which claims that a pre-play agreement to play a certain strategy profile will be kept if and only if it is a Nash equilibrium. Aumann considers the game of Figure 2 and starts from assuming that players are cautious so that they are likely to play the risk dominant equilibrium (b, b) without communication. Aumann asks if a pre-play agreement to play (a, a) will be kept. Aumann argues that even if a player is very cautious and therefore is planning to play b in any case, she wants to agree on playing (a, a) because it leads the other player to choose a and induces her a higher payoff. Therefore, the fact that an agreement is achieved contains no information about the intention of the opponent and thus there is no reason to keep the agreement.

Baliga and Morris [2] discuss the relevance of Aumann's intuition to two player games with one-sided incomplete information. In Baliga and Morris's model, one player has a payoff relevant private information. In the cheap-talk game, the informed player sends a message and then the two players choose an action. They ask (i) when there is full communication, in the sense that the informed player truthfully reveals his type and the players then play a Nash equilibrium of the underlying complete information game and (ii) when there is no communication, so that the equilibria of the cheap talk game are outcome equivalent to equilibria where cheap talk is not allowed. Baliga and Morris show that if the uninformed player has only two actions, then a failure of self-signalling implies that no communication is possible. The short run game in our model can be interpreted as an extension of Baliga and Morris's analysis to two-sided incomplete information.[4] Although we do not know

[4] In Baliga and Morris's model, the action space of a player is independent of his type. Our model fits their framework if we assume that a type a (type b) player can acess action

in general how Baliga and Morris's results can be extended to games with two-sided incomplete information, our result suggests similar results may be obtained.

Blume [3] demonstrates a possibility that evolution and communication may fail to select the efficient equilibrium outcome, by a different approach from ours. Blume points out that the previous literature in evolution and communication considered models in which the dynamics are gradual, especially an arrival of mutants affects only a small fraction of the entire population. Blume proposes a class of population dynamics which permit simultaneous adjustment of strategies of large fractions of the population and shows that whether the efficient-equilibrium-selection result is obtained depends on the risks of the underlying game and the size of the message space. Our analysis does not depart from the previous literature in the sense that the adjustment is only gradual, and our result has nothing to do with the risks. On the other hand, Blume keeps the same assumption with the previous literature that whenever a player faces an opportunity to adjust her behavior, she can freely adopt anything in the strategy space of the cheap talk game. The self-signalling condition does not distinguish the result in Blume's model.

8 Conclusion

In many economic problems, the set of available alternatives in game theoretic situations is determined endogenously. In particular, long term decisions such as opening and closing a factory may restrict the flexibility of short term decisions such as choosing which product to produce. Therefore it is worthwhile to analyze an evolutionary model in which the set of available actions is adjusted slowly and the short term adjustments are restricted by the availability. This paper studied the effect of pre-play communication on equilibrium selection in coordination games and showed that the existing efficiency result is fragile to the consideration of the difference in the speed of evolution. The incentive of a player to pool with players with different availability is the main obstruction in achieving efficiency.

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b (action a) but this action is very costly so that action a (action b) is the dominant strategy.

A Appendix: Proofs

A.1 Proof of Proposition 1

Fix $x' \in \Delta\Theta$ and $\epsilon \in (0,1)$ arbitrarily. Let y be a BNE under $(1-\epsilon)x^s + \epsilon x'$. From the definition of BNE,

$$\sum_{\sigma \in \Sigma_\theta} v(\sigma|(1-\epsilon)x^s + \epsilon x', y)y_\theta(\sigma) = \max_{\sigma \in \Sigma_\theta} v(\sigma|(1-\epsilon)x^s + \epsilon x', y)$$

for all $\theta \in \Theta$. Since $\Sigma_a \subset \Sigma_s$ and $\Sigma_b \subset \Sigma_s$,

$$\max_{\sigma \in \Sigma_s} v(\sigma|(1-\epsilon)x^s + \epsilon x', y) \geq \max_{\sigma \in \Sigma_\theta} v(\sigma|(1-\epsilon)x^s + \epsilon x', y),$$

for $\theta \in \{a, b\}$. Now,

$$\begin{aligned}
&\sum_{\theta \in \Theta} \Big(\sum_{\sigma \in \Sigma_\theta} v(\sigma|(1-\epsilon)x^s + \epsilon x', y)y_\theta(\sigma) \Big) (x^s(\theta) - x'(\theta)) \\
&= \sum_{\theta \in \Theta} \Big(\max_{\sigma \in \Sigma_\theta} v(\sigma|(1-\epsilon)x^s + \epsilon x', y) \Big) (x^s(\theta) - x'(\theta)) \\
&= \sum_{\theta \in \{a,b\}} \Big(\max_{\sigma \in \Sigma_s} v(\sigma|(1-\epsilon)x^s + \epsilon x', y) - \max_{\sigma \in \Sigma_\theta} v(\sigma|(1-\epsilon)x^s + \epsilon x', y) \Big) x'(\theta) \\
&\geq 0,
\end{aligned}$$

which proves the proposition.

A.2 Proof of Proposition 2

Lemma 1 For all $\beta \in (0,1)$, there exists a strategy of the short-run game y^β such that y^β is a BNE under x^β and

$$\sum_{\sigma \in \Sigma_s} v(\sigma|x^\beta, y^\beta)y_s^\beta(\sigma) > \sum_{\sigma \in \Sigma_b} v(\sigma|x^\beta, y^\beta)y_b^\beta(\sigma).$$

Proof For all $\beta \in (0,1)$, let a strategy y^β satisfy $y_s^\beta(m_1, f_c) = 1$, $y_a^\beta(m_1, f_a) = 1$ and $y_b^\beta(m_2, f_b) = 1$. It can be easily verified that this is indeed a BNE if the base game is self-signalling. The left hand side of the inequality is $(1-\beta)\pi_{aa} + \beta\pi_{bb}$ and the right hand side is π_{bb}. Therefore, the inequality holds.

Proof of Proposition 2

Let $x = x^{\beta}$ and $x' = x^{s}$. Fix any $\epsilon \in (0,1)$. Then, $(1-\epsilon)x + \epsilon x' = x^{(1-\epsilon)\beta}$. From Lemma 1, there exsists $y^{(1-\epsilon)\beta}$ such that $y^{(1-\epsilon)\beta}$ is a BNE under $x^{(1-\epsilon)\beta}$ and

$$\sum_{\sigma\in\Sigma_s} v(\sigma|x^{(1-\epsilon)\beta}, y^{(1-\epsilon)\beta}) y_s^{(1-\epsilon)\beta}(\sigma) > \sum_{\sigma\in\Sigma_b} v(\sigma|x^{(1-\epsilon)\beta}, y^{(1-\epsilon)\beta}) y_b^{(1-\epsilon)\beta}(\sigma).$$

This implies that

$$\begin{aligned} &\sum_{\theta\in\Theta}\Big(\sum_{\sigma\in\Sigma_\theta} v(\sigma|x^{(1-\epsilon)\beta}, y^{(1-\epsilon)\beta}) y_s^{(1-\epsilon)\beta}(\sigma)\Big) x'(\theta) \\ > &\sum_{\theta\in\Theta}\Big(\sum_{\sigma\in\Sigma_\theta} v(\sigma|x^{(1-\epsilon)\beta}, y^{(1-\epsilon)\beta}) y_b^{(1-\epsilon)\beta}(\sigma)\Big) x(\theta). \end{aligned}$$

Therefore, x^{β} is not stable.

A.3 Proof of Proposition 3

Let $\eta > 0$ be a positive constant. We claim that if η is sufficiently small, then under any TPS x with $x(b) \geq 1-\eta$, there exists no BNE of the short run game in which either type a or type s receives a higher payoff than type b. Once the claim is established, proof of the proposition is straightforward.

Now we prove the claim. First we show that in any equilibrium type a does not receive a higher payoff than type b. A type a player is matched with type b and receives a payoff π_{ab} with at least probability $1-\eta$. Therefore, a type a player's average payoff is no greater than $(1-\eta)\pi_{ab} + \eta\pi_{aa}$. A type b player is matched with another type b and receives a payoff π_{bb} with at least probability $1-\eta$. Therefore, a type b player's average payoff is no less than $(1-\eta)\pi_{bb} + \eta\pi_{ba}$. For η sufficiently small, $(1-\eta)\pi_{ab} + \eta\pi_{aa} < (1-\eta)\pi_{bb} + \eta\pi_{ba}$ holds. Thus, type a never receives a higher payoff than type b.

Next we show that there does not exist any equilibrium in which type s receives a strictly higher payoff than type b. Suppose such an equilibirum exists. The equilibrium is denoted by y. First, in this equilibrium the decision rule f_a is not chosen with positive probability by type s. This is because the decision rule f_a can never yield a higher payoff than type b players by the same argument as above. Second, if type s chooses the decision rule f_b with probability 1, then type s and type b receive the same payoff. Therefore, in equilibrium y, either f_c or f_d must be chosen with positive probability by type s.

- Consider the case where f_c is chosen with higher probability than f_d. In this case, a type b player's best response is only (m_1, f_b), and thus we have $y_b(m_1, f_b) = 1$. Here, a type s player employing the decision rule f_c receives a payoff π_{ab} when she is matched with a type b player. Therefore, her payoff is no greater than $(1 - \eta)\pi_{ab} + \eta\pi_{aa}$. For the same reason as the previous argument about type a players, here a type s player does not receive a higher payoff than a type b player. Contradiction.

- The same argument applies to the case where f_d is chosen with higher probability than f_c.

- The remaining case is where type s chooses decision rules f_c and f_d with a posive and equal probability. This case can be divided into two sub-cases. First, consider the case where $y_b(m_1, f_b) \geq \frac{1}{2}$. When a type s player with the decision rule f_c is matched with a type b player, her expected payoff is $y_b(m_1, f_b)\pi_{ab} + y_b(m_2, f_b)\pi_{bb}$, which is less than or equal to $\frac{1}{2}(\pi_{ab} + \pi_{bb})$. Therefore the payoff for a type s player with the decision rule f_c is no greater than $(1-\eta)\frac{1}{2}(\pi_{ab}+\pi_{bb})+\eta\pi_{aa}$. On the other hand, a type b player's average payoff is no less than $(1 - \eta)\pi_{bb} + \eta\pi_{ba}$. Therefore, for a sufficiently small η, a type b player receives a higher payoff than a type s player with decision rule f_c. This contradicts with the assumption that y is a BNE where type s receives a higher payoff than type b and f_c is chosen by type s with positive probability by type s. For the opposite case where $y_b(m_2, f_b) \geq \frac{1}{2}$, we can make the same argument by looking at the payoff of a type s player with the decision rule f_d.

We started from the assumption that there exists an equilibrium in which type s receives a strictly higher payoff than type b and reached to a contradiction for all possible cases. Therefore we can conclude that such an equilibrium does not exist.

A.4 Proof of Proposition 5

The second statement of the theorem can be easily verified and thus the proof if omitted. Here we prove the first statement only for $\{y^*\}$, because the proof for $\{y^{**}\}$ is essentially the same.

Since $\{y^*\}$ is a singleton, it is obviously closed and has no proper nonempty subset. Therefore, it suffices to show that y^* is a BNE and satisfies condition (ii) in the definition of EES set.

It can be easily verified that for all $\beta \in (0, 1)$,

$$BR_a(x^\beta, y^*) = \{(m_1, f_a)\},$$
$$BR_b(x^\beta, y^*) = \{(m_2, f_b)\},$$
$$BR_s(x^\beta, y^*) = \{(m_1, f_c)\}.$$

This establishes that y^* is a Bayesian Nash equilibrium under x^β.

Since each type has a unique best response, i.e., any other behavior gives strictly lower payoff, the best response remains the unique best response when other players' behaviors are slightly perturbed. In other words, for all $\beta \in (0, 1)$, there exists a sufficiently small $\bar{\epsilon}_\beta > 0$ such that for all $\epsilon \in (0, \bar{\epsilon}_\beta)$ and for all $y' \in \Delta$,

$$BR_a(x^\beta, (1-\epsilon)y^* + \epsilon y') = \{(m_1, f_a)\},$$
$$BR_b(x^\beta, (1-\epsilon)y^* + \epsilon y') = \{(m_2, f_b)\},$$
$$BR_s(x^\beta, (1-\epsilon)y^* + \epsilon y') = \{(m_1, f_c)\}.$$

Hence, for all $\theta \in \Theta$, $y''_a \in \overline{BR}_a(x^\beta, (1-\epsilon)y^* + \epsilon y')$ implies $y''_a = y^*$. Therefore, $y' \in \overline{BR}(x^\beta, (1-\epsilon)y^* + \epsilon y')$ implies $y' = y^*$, and thus $(1-\epsilon)y + \epsilon y' \in Y$.

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