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Abstract

Constructing a two-country, two-good, two-factor model of international trade under quasi-linear utility functions, we obtain a Modified Heckscher-Ohlin (MHO) Theorem that relates the trade pattern to the international distribution of factor endowments. We also show that the MHO Theorem survives imperfect competition and increasing returns.

Keywords: The Modified Heckscher-Ohlin Theorem, quasi-linear utility function, imperfect competition, increasing returns to scale.

JEL Classification Numbers: F 10, F 12.

1 Introduction

The determination of trade pattern is a central topic of trade theory. For a long time, the two(-country) by two(-good) by two(-factor) Heckscher-Ohlin (HO) model and its various extensions have been the standard general equilibrium framework that explains the pattern of international trade in terms of a difference in factor endowments among countries.

Most HO models[1] commonly assumes homothetic utility functions, which seems to have an unrealistic implication such that the income elasticity of the demand for each good is unity. One may naturally ask whether we could establish a trade-pattern theorem under non-homothetic utility functions.

In this paper we derive such a new theorem, replacing homothetic utility functions by a quasi-linear one

$$u(C_1, C_2) = v(C_1) + aC_2,$$

where $v(C_1)$ is an increasing and strictly-concave function, C_i, $i = 1, 2$, is the consumption of Good i, and a is positive and constant. The quasi-linearity assumption implies that the income elasticity of Good 1 is zero. Under this assumption, we shall establish a Modified Heckscher-Ohlin (MHO) Theorem that relates the international distribution of factor endowments to the pattern of international trade between two countries in a different manner from the standard HO (SHO) Theorem. .

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[1] Wong (1995, Chapters 2, 6 and 7) provides a survey of perfectly and imperfectly competitive HO models. extensions.

As is well known, the SHO Theorem states that each country exports the good the production of which intensively uses the factor of production that is *relatively abundant* in the country. So does the MHO Theorem. The difference between the two Theorems is in the meaning of "*relatively abundant*". See Figure 1, where K and L denote capital and labor endowments of a country, say Home. In the SHO model the straight line OEM is the border of relative factor abundance in the sense that if the factor endowment point of the other country, say Foreign, (K^*, L^*), is below (resp. above) the border line like E^* (resp. $\overline{E}^*$), Foreign is relatively labor (resp. capital) abundant. On the other hand, in the MHO Theorem the border line is either $A'EM'$ or $A''EM''$, the slope of which is equal to the equilibrium factor intensity of Good 2 that has positive income effects under the assumption that the factor endowments in Foreign, (K^*, L^*), are exactly equal to those in Home, (K, L).[2]

Considering that reality is between the quasi-linearity and homotheticity assumptions, the MHO Theorem implies that the HO relationship between trade pattern and the international distribution of factor endowments roughly holds in the realistic case such that commodities have positive but different income elasticities.

Moreover, what we would like to emphasize is that the MHO Theorem holds not only in a competitive trade model but also in a monopoly trade model studied by Melvin and Warne (1973) and Markusen (1981). Assume that Good 1 is produced in a monopoly sector with restrictive entry and that Good 2 is competitively produced in each of the two countries and that increasing returns to scale prevail in the monopoly sector. We show that the MHO Theorem exactly holds in this duopolistic world equilibrium. We believe that the MHO Theorem can be thought of as a contribution to the literature on imperfectly comeptitive general equilibrium models of trade, since so far we have no trade-pattern theorem under the homotheticity assumption which takes into account both *arbitrary* difference in factor endowment ratios between countries and increasing-returns-to-scale monopoly industry with restrictive entry.[3]

The plan of the paper is as follows. Section 2 presents the MHO Theorem in a perfectly-competitive world equilibrium. Section 3 shows that it holds even in an oligopolistic model of international trade. Section 4 provides some concluding remarks.

2 The MHO Theorem in a Competitive Model

The model has a familiar two-country (Home and Foreign), two-good (Goods 1 and 2), two-factor (Capital and Labor) framework. In this section, Goods 1 and 2 are perfectly-competitive and constant-returns-to-scale goods. Good 2 serves as the numeraire. Capital and Labor are inelastically supplied and fully

[2] Which of AEM' and BEM'' holds depends on whether Good 2 is more capital-intensive than Good 1.or not.

[3] Markusen (1981) derives the well-known trade-pattern proposition such that, other things being equal, the country with larger factor endowments imports the monopoly good. So do Kemp and Shimomura (2002), but under a definition of the representative agent which is stricter than usually assumed. On the other hand, Markusen's proposition does not cover the case such that the factor endowment ratio *arbitrarily* differs between the two countries. Moreover, it may not survive increasing returns to scale in the monopoly sector.

employed. The production function in each industry is given by

$$\begin{aligned} Y_1 &= f^1(K_1, L_1) \quad (1) \\ Y_2 &= f^2(K_2, L_2), \quad (2) \end{aligned}$$

where $Y_i, i = 1, 2$ is the home output of each good while K_i and L_i are the capital and labor input in each industry. $f^i(\cdot)$ is an increasing, continuously-differentiable, strictly quasi-concave and linearly homogeneous function in K_i and L_i.

As already mentioned, Home's preference is represented by a quasi-linear utility function:

$$u = v(C_1) + aC_2, \ a > 0, \quad (3)$$

Hence, the Marshallian demand function of Good 1 is derived as

$$C_1 = D(p), \quad (4)$$

where $D(\cdot) \equiv v'^{-1}(\cdot)$. Foreign's preference is defined by the same function as (3). Thus, the market-clearing condition is

$$2D(p) = Y_1 + Y_1^*, \quad (5)$$

where Y_1^* is the foreign output of Good 1[4]. The market-clearing condition immediately implies the following lemma.

Lemma 1. *Home exports (resp. imports) Good 1 iff* $Y_1 >$ *(resp.* $<$*)*Y_1^*.

Let us turn to the production side. The production possibility frontier (PPF) is defined as

$$\begin{aligned} Y_2 &= G(Y_1, K, L) \\ &\equiv \max_{K_i, L_i, i=1,2} f^2(K_2, L_2) \\ &\quad \text{subjct to} \end{aligned}$$

$$\begin{aligned} f^1(K_1, L_1) &\geq Y_1 \\ K_1 + K_2 &\leq K \\ L_1 + L_2 &\leq L, \end{aligned}$$

It is well known that the PPF has the following property:

$$\frac{\partial G(Y_1, K, L)}{\partial Y_1} \equiv G_1(Y_1, K, L) = -\frac{\Lambda^1(w, r)}{\Lambda^2(w, r)}, \quad (6)$$

where $\Lambda^i(w, r)$, $i = 1, 2$, is the average cost of Good i, and w and r are the wage and rental rates.

[4] In what follows, we attach an asterisk (*) to all foreign variables.

Since prices are equal to average costs under perfect competition, we have the system of equations that describes the two-country model[5]:

$$\begin{aligned} p &= -G_1(Y_1, K, L) \\ p &= -G_1(Y_1^*, K^*, L^*) \\ p &= \Lambda^1(w, r) \\ 1 &= \Lambda^2(w, r) \\ 2D(p) &= Y_1 + Y_1^*. \end{aligned}$$

Inspecting a familiar box diagram, we can check which of Y_1 and Y_1^* is larger. See Figure 2, where the segments OK and OL of the box $OLEK$ measure Home's capital and labor endowments, B is the resource allocation point in Home, and the curve lBl' is an iso-quant curve of Good 1. The solid concave curve OBE is the efficiency locus and located above the diagonal line connecting O and E, which means that Good 1 is assumed to be capital-intensive in the figure. B and lBl' correspond to Home's autarchic equilibrium output $\overline{Y}_1$ determined by

$$\Gamma(Y_1) = -G_1(Y_1, K, L),$$

where $\Gamma(\cdot)$ is the inverse function of $D(\cdot)$.

Now, suppose that Foreign's factor endowment point (K^*, L^*) is E^* i.e., below the line $A'BEM'$, the slope of which is equal to the factor intensity of Good 2. Figure 2 shows that at Home's autarchic equilibrium price $\overline{p} \equiv \Gamma(\overline{Y}_1)$ the foreign production point is E^*, which means that Foreign's output of Good 2 is larger than Home's autarchic equilibrium output. Since the price-output relationship is normal in the present constant-returns-to-scale competitive model and Home's and Foreign's demand functions are exactly identical with each other, it follows that Foreign's autarchic equilibrium price is higher than $\overline{p}$: Home has the comparative advantage concerning Good 1 and exports it.

If the foreign factor endowment point is above $A'BEM'$ like $\overline{E}^*$, the foreign production point is $\overline{B}^*$, which means that the foreign output of Good 2 is smaller than the home autarchic level. The foreign country has the comparative advantage concerning Good 1 and exports it.

Figure 2 assumes that Good 1 is more capital-intensive than Good 2. We can make a parallel argument when Good 1 is labor-intensive. We arrive at the MHO Theorem.

The MHO Theorem. *Each country exports the good the production of which intensively uses the factor of production which is relatively abundant (in the above modified sense) in the country.*

[5] The factor price equalization holds: $w^* = w$ and $r = r^*$.

3 The MHO Theorem in an Oligopolistic Model

3.1 The Main Assumptions

Let us turn to an oligopolistic model. The model employed in this section is similar to the ones in Melvin and Warne (1973) and Markusen (1981). Let Good 1 be an imperfectly-competitive good produced by using increasing-returns-to-scale technologies. Specifically, we assume that the production function of Good 1 is homothetic:

$$Y_1 = F(f^1(K_1, L_1)),$$

where it is assumed that $F(\cdot)$ is an increasing and strictly convex function and that $f^1(K_1, L_1)$ is a twice-differentiable, linearly homogeneous, strictly quasi-concave, and increasing funciton of K_1 and L_1[6]. On the other hand, we keep assuming that Good 2 is a perfectly-competitive and constant-returns-to-scale good. Henceforth, we assume that entry to the sector producing Good 1 is restricted and the number of the imperfectly-competitive firms is normalized to unity.

Let us denote by $\phi(\cdot)$ the inverse function of $F(\cdot)$.The above produciton function is rewritten as

$$\phi(Y_1) = f^1(K_1, L_1)$$

Considering the assumed properties of the function $f^1(K_1, L_1)$, we can write the PPF as $G(\phi(Y_1), K, L)$.

Due to the homothetic production function, the cost function of Good 1 is multiplicatively separble as follows.

$$\Lambda^1(w, r)\phi(Y_1),$$

Making use of this cost function and the inverse demand function, $\Gamma(\cdot)$, we can write Home and Foreign imperfectly-competitive firms' profits as

$$\Gamma\left(\frac{Y_1 + Y_1^*}{2}\right) Y_1 - \Lambda^1(w, r)\phi(Y_1)$$

$$\Gamma\left(\frac{Y_1 + Y_1^*}{2}\right) Y_1^* - \Lambda^1(w^*, r^*)\phi(Y_1^*).$$

The two firms play a non-cooperative duopoly game in the international market of Good 1 in which outputs are their strategic variables.

Following Melvin and Warne (1973) and Markusen (1981), we assume that each duopolist thinks that his decision may not affect the factor prices. Therefore, the first-order conditions for profit maximization are

$$\Psi(Y_1, Y_1^*; K, L) \equiv MR(Y_1, Y_1^*) + G_\phi(\phi(Y_1), K, L)\phi'(Y_1) = 0 \qquad (7)$$

and

$$\Psi^*(Y_1, Y_1^*; K^*, L^*) \equiv MR^*(Y_1, Y_1^*) + G_\phi(\phi(Y_1^*), K^*, L^*)\phi'(Y_1^*) = 0, \qquad (8)$$

[6] Thus, it is formally identical to eq. (1), and, without loss, we can use the same notatoin.

where $G_\phi(\phi(Y_1), K, L)$ is the partial derivative of $G(\cdot)$ with respect to ϕ,and

$$MR(Y_1, Y_1^*) \equiv \Gamma'\left(\frac{Y_1 + Y_1^*}{2}\right)\frac{Y_1}{2} + \Gamma\left(\frac{Y_1 + Y_1^*}{2}\right)$$

and

$$MR^*(Y_1, Y_1^*) \equiv \Gamma'\left(\frac{Y_1 + Y_1^*}{2}\right)\frac{Y_1^*}{2} + \Gamma\left(\frac{Y_1 + Y_1^*}{2}\right)$$

are Home and Foreign marginal revenues. In what follows, we make the following assumptions.

Assumption 1: *There is a neighborhood of (K, L) in the R_+^2-space, $V(K, L)$, such that for any (K^*, L^*) in $V(K, L)$ the system of equations (7) and (8) has a unique solution pair (Y_1, Y_1^*) that satisfies $0 < Y_1 < F(f^1(K, L))$ and $0 < Y_1^* < F(f^1(K^*, L^*))$.*

Assumption 2: *The solution pair satisfies the second-order conditions:*

$$\frac{\partial}{\partial Y_1} MR(Y_1, Y_1^*) + G_\phi(\phi(Y_1), K, L)\phi''(Y_1) < 0 \tag{9}$$

$$\frac{\partial}{\partial Y_1^*} MR^*(Y_1, Y_1^*) + G_\phi(\phi(Y_1^*), K^*, L^*)\phi''(Y_1^*) < 0. \tag{10}$$

Assumption 3: *The solution pair is stable in the standard sense, i.e., it satisfies*

$$\left|\frac{dY_1^*}{dY_1}\right|_{(7)} > \left|\frac{dY_1^*}{dY_1}\right|_{(8)}. \tag{11}$$

Since the second partial derivative of G with respect to ϕ is negative, we see that the second-order conditions (9) and (10) imply that

$$\frac{\partial}{\partial Y_1}\Psi(Y_1, Y_1^*, K, L) < 0 \quad \text{and} \quad \frac{\partial}{\partial Y_1^*}\Psi^*(Y_1, Y_1^*, K^*, L^*) < 0. \tag{12}$$

3.2 Deriving the MHO Theorem

Let us show that the MHO Theorem holds in the duopolistic model. First, consider the following system:

$$\Psi(Y_1, Y_1^*, K, L) = 0 \tag{13}$$

$$\Psi^*(Y_1, Y_1^*, K, L) = 0. \tag{14}$$

Under Assumptions 1-3, (13) and (14) have a unique solution $(Y_1, Y_1^*) = (y_0, y_0)$ on the 45^0-line. See Figure 3.

Now, let us change (K, L) in (14) to (K^*, L^*). The locus of (14) has to shift up or shift down. The direction depends on whether $G_\phi(\phi(y_0), K^*, L^*)$ is greater or smaller than $G_\phi(\phi(y_0), K, L)$: Suppose that

$G_\phi(\phi(y_0), K^*, L^*) > G_\phi(\phi(y_0), K, L)$. Then, by choosing an appropriate value of positive Δ, we have

$$\begin{aligned} 0 &= \Psi^*(y_0, y_0, K, L) \\ &= MR^*(y_0, y_0) + G_\phi(\phi(y_0), K, L)\phi'(y_0) \\ &< MR^*(y_0, y_0) + G_\phi(\phi(y_0), K^*, L^*)\phi'(y_0) \\ &> MR^*(y_0, y_0 + \Delta) + G_\phi(\phi(y_0 + \Delta), K^*, L^*)\phi'(y_0 + \Delta) \\ &= 0, \end{aligned}$$

where the last inequality is implied by (12). Therefore, Foreign's reaction curve shifts up to $a\Theta_a a'$ and point Θ_a is the equilibrium point: We have $Y_1 < Y_1^*$ there. Making a parallel argument, we see that if $G_\phi(\phi(y_0), K^*, L^*) < G_\phi(\phi(y_0), K, L)$, then Foreign's reaction curve shifts down to $\alpha\Theta_\alpha\alpha'$ and point Θ_α is the equilibrium point: We have $Y_1 > Y_1^*$ there.

Based on the foregoing argument, we have the following lemma.

Lemma 2. *If $G_\phi(\phi(y_0), K^*, L^*) - G_\phi(\phi(y_0), K, L) >$ (resp. $<$) 0, then $Y_1 <$ (resp. $>$) Y_1^* in the equilirium.*

Second, let us examine how the sign of $[G_\phi(\phi(y_0), K^*, L^*) - G_\phi(\phi(y_0), K, L)]$ is related to (K^*, L^*). See the box diagram in Figure 4. The curve $\ell BS\ell'$ corresponds to the iso-quant curve of Good 1,

$$\phi(y_0) = f^1(K_1, L_1).$$

Suppose that point B is the intersection of the iso-quant curve and the efficiency locus. If Home and Foreign are endowed with the same factor endowments (K, L), then, B exhibits the equilibrium resource allocation in both countries under which y_0 amounts of Good 1 are commonly produced. That is, point B in Figure 4 corresponds to point B in Figure 3.

Now, let us assume that Foreign's factor endowment point E_0^* $[= (K^*, L^*)]$ is located below the dotted line $A'BEM'$, as is depicted in Figure 4. Then, the slope of BE_0^* $(= \angle K^* E_0^* B)$ is smaller than the slope of BE $(= \angle KEB)$. Due to the textbook relationship between factor intensity and the marginal rate of substitution in a neo-classial production function, it follows that the iso-quant curve of Good 2 $(\beta B\beta')$ whose origin is point E_0^* is less steeper than the iso-quant curve (bBb') whose origin is point E. Hence, Foreign's efficiency locus connecting O_1 and E_0^* has to cross the iso-quant curve, $\phi(y_0) = f^1(K_1, L_1)$, somewhere between B and ℓ', like point S. Since the slope of the iso-quant curve is the factor price ratio w/r, we see that $(w/r)_B > (w/r)_S$. Since Good 1 is assumed to be more capital intensive than Good 2 in Figure 4, it follows that the Stolper-Samuelson Theorem ensures us that $-G_\phi(\phi(y_0), K^*, L^*) > -G_\phi(\phi(y_0), K, L)$, or

$$G_\phi(\phi(y_0), K^*, L^*) < G_\phi(\phi(y_0), K, L).$$

If Good 1 is more labor-intensive than Good 2, the Stolper-Samuelson Theorem ensures us that $(w/r)_B > (w/r)_S$ implies $G_\phi(\phi(y_0), K^*, L^*) > G_\phi(\phi(y_0), K, L)$.

Making a parallel argument, we find that if the foreign factor endowment point is above the line $A'BEM'$,

$$G_\phi(\phi(y_0), K^*, L^*) > (\text{resp.} <) G_\phi(\phi(y_0), K, L),$$

if Good 1 is more capital- (resp. labor-)intensive than Good 2.

Combining these results with Lemmas 1 and 2, we conclude that if Good 1 is more capital-intensive than Good 2 and if Foreign's factor endowment point (K^*, L^*) is below (resp. above) the line $A'BEM'$, then $G_\phi(\phi(y_0), K^*, L^*) < (resp. >) G_\phi(\phi(y_0), K, L)$, which implies that Home exports (resp. imports) Good 1, the capital-intensive good. If Good 1 is more labor-intensive than Good 2 and if Foreign's factor endowment point is below the line, then Home exports Good 2, the capital-intensive good. In any case, each country exports the good that intensively uses the factor of production relatively abundant in the country, where "relative factor abundant" is defined by the line $A'BEM'$ in Figure 4 ($= A'EM'$ in Figure 1) whose slope is the factor intensity of Good 2 evaluated at the equilibrium under the condition that factor endowments in Foreign are identical to those in Home, i.e., $(K^*, L^*) = (K, L)$.

Proposition. *The MHO Theorem holds in the above duopolistic trade model as well.*

4 A Concluding Remark

We have established a new theorem on the pattern of international trade under the assumption that utility functions are quasi-linear. While the new theorem relates the pattern of trade to the international distribution of factor endowments roughly in the HO manner, the difference in the border of relative factor abundance between the traditional and new theorems has an interesting implication for empirical studies of trade patterns.

Let us consider, for example, the case that Foreign's factor endowment point is E_0^* in Figure 1. Since it is above OEM, Foreign is a capital-abundant country in the standard sense. Suppose that in reality Foreign imports a capital-intensive good. This "paradox" can be resolved by the MHO Theorem if empirical evidences show that the income effect on the demand for the capital-intensive good is small. For, the MHO Theorem suggests us that the empirical evidences mean that the border of relative factor abundance is not OEM but $A'EM'$. Hence, Foreign is a labor-abundant country and imports the capital-intensive good.

Thus, following the MHO Theorem, we see that what is crucial to the explanation of trade pattern is not the comparison between K/L and K^*/L^* but the comparison between K/L and $|K^* - K|/|L^* - L|$. It is one of our future research agenda to investigate such implications of the MHO Theorem for empirical studies of trade pattern.

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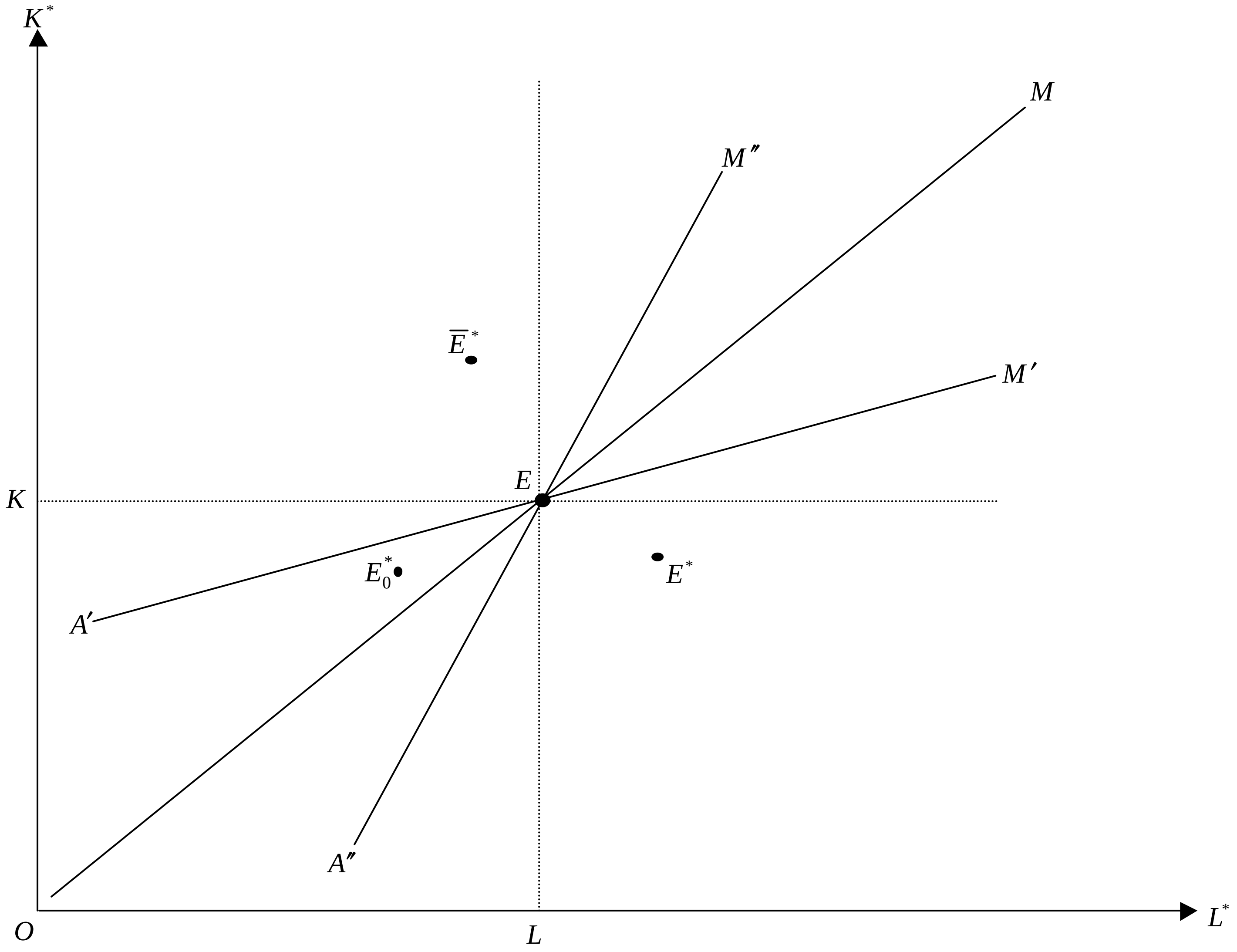

Figure 1 : the border of relative factor abundance

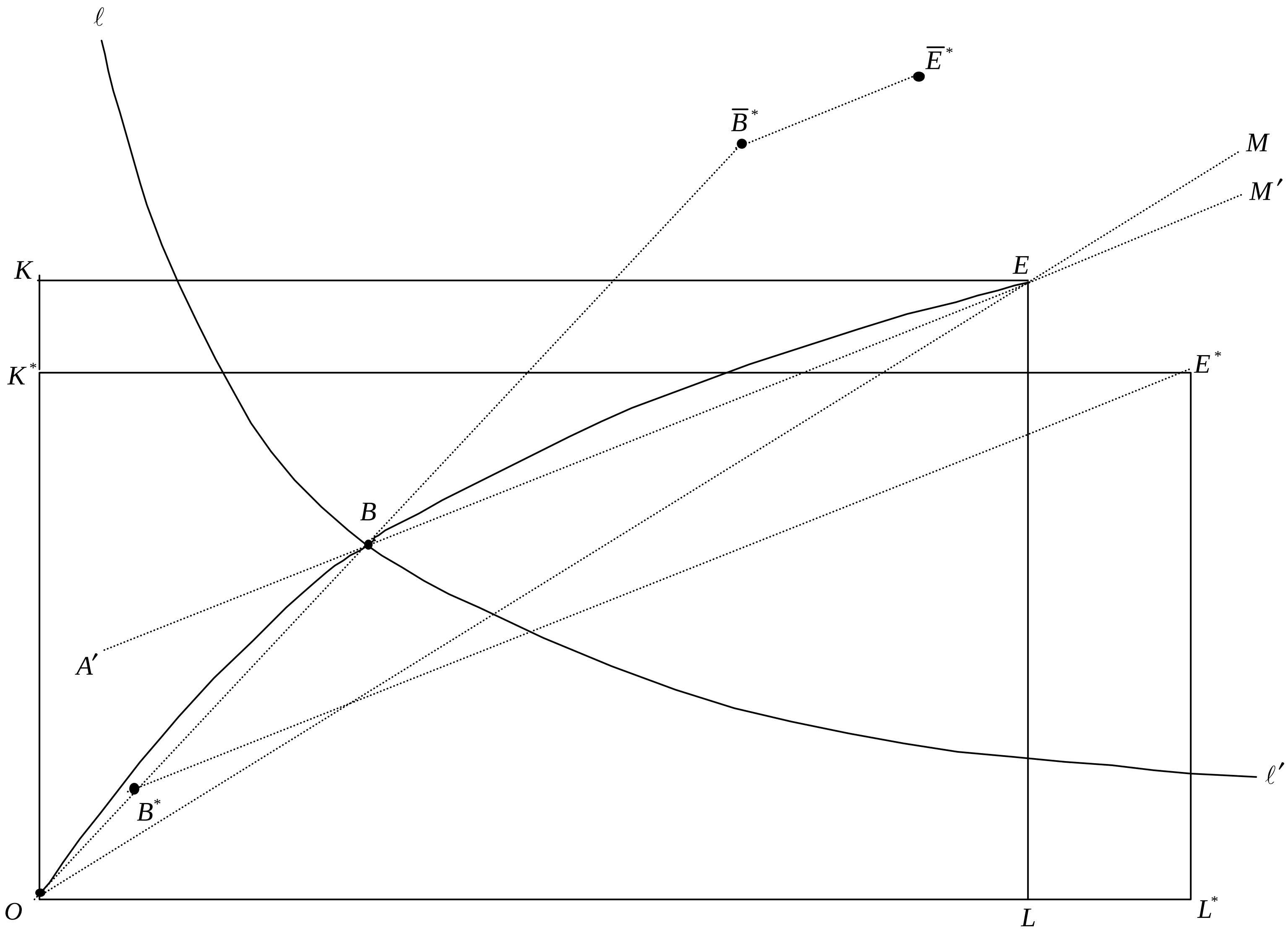

Figure 2 : Home and Foreign box diagrams in the case that Good 2 is labor-intensive

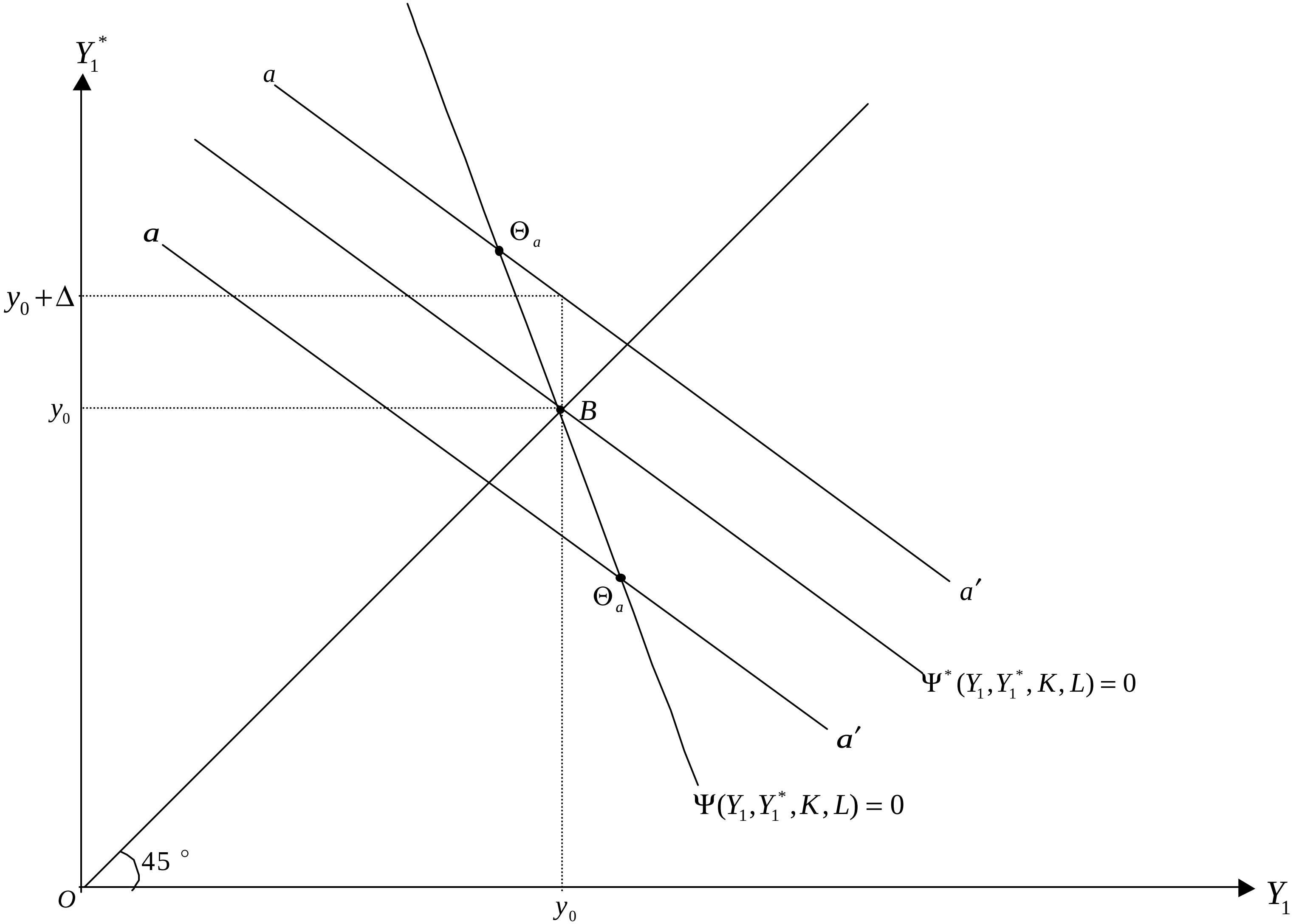

Figure 3 : Home and Foreign reaction curves

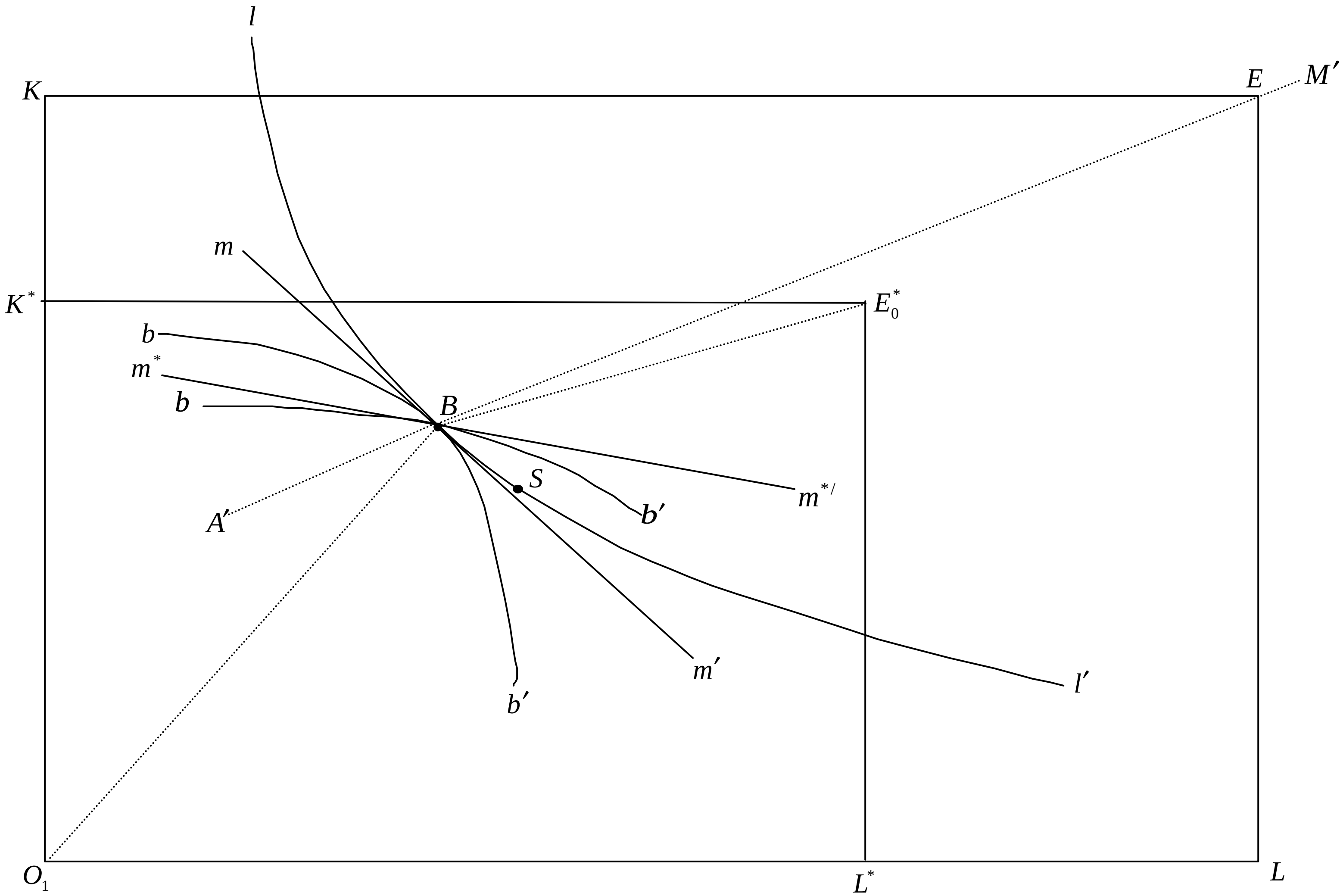

Figure 4 : $\angle KEB > \angle K^*E^*B$, which implies $\angle mBm' > \angle bBb'$ at B. Note that $(w/r)_B > (w/r)_S$