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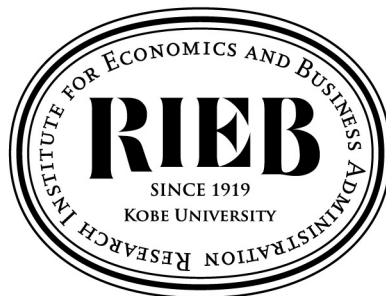
Kobe University

DP2026-12

**Unemployment Fluctuation and
Referral Hiring**

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March 30, 2026



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Unemployment fluctuation and referral hiring*

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April 6, 2026

Abstract

Referral hiring has a similar nature to unemployment insurance. An additional channel provided by referrals can shorten workers' unemployment duration due to an increase in the matching probability. Accordingly, referral hiring has the potential to contribute to business cycle stability. This study examines to what extent referrals affect cyclical properties of the economy. Using two representative models with referral processes, I propose a comparison of the dynamics between models with and without referrals. From the impulse response to a productivity shock, it is found that referral hiring does not necessarily reduce the business cycle fluctuations. The key structure leading to the result is whether the referral process passes through the labor market; in particular, there are significant shifts in the dynamics of the unemployment rate.

JEL Classification: E24, E32, J64;

Keywords: Referral hiring; business cycle; unemployment.

*I acknowledge financial support from the Japan Society for the Promotion of Science (JSPS KAKENHI Grant Number 24K22648) and the Doctoral Student Fellowship at Kobe University. I am very grateful for discussions at CASE, the Kobe Macroeconomics Study Group, the DSGE Conference 2024, and the International Symposium on Trade and Employment. I also thank Kazuhiro Teramoto and Tetsuaki Takano for their insightful comments.

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1 Introduction

Referral hiring is used by many people. According to Holzer (1988), about 50% of people use referrals as one of the major job finding methods in the US. The referral gives job seekers an additional option. This additional channel improves the matching efficiency between workers and jobs, although it can exacerbate social divisions in the economy. The embedded mechanism leading to these effects stems from the nature of referral hiring, like unemployment insurance.

Referral hiring occurs through the social network in which nodes and edges are people and social relationships among them. When workers in social networks seek jobs, they can access not only the labor market, but also referrals from their friends. The referral channel added to labor market matching decreases workers' unemployment durations thanks to a decline in the unemployment probability, and several empirical works support this deduction (e.g., Brown et al., 2016; Glitz, 2017). In this sense, referrals are expected to contribute to business cycle stability.

Most models that incorporate referral hiring are formulated based on the search and matching model. In the literature, the proposed models are roughly divided into two types. First, referral hiring occurs when both employed and unemployed workers can receive job information (e.g., Calvó-Armengol and Zenou, 2005; Ioannides and Soetevent, 2006; Fontaine, 2007). Job information arriving at employed workers is passed to unemployed workers via social network ties. Second, referrals occur when jobs in production expand (e.g., Galenianos, 2014, 2021). The newly created positions from job expansion are sent to unemployed workers through social networks. Regardless of these different processes, it is anticipated that referral hiring works as a stabilizer for the business cycle if it behaves like unemployment insurance.

This paper examines whether referral hiring in the economy affects the business cycle fluctuations. Two theoretical models, based on the search and matching model, are proposed for different referral procedures. In the first model, called the market search model, referrals are specified such that employed workers who join the labor market send job information to unemployed workers. In the second model, called the job expansion model, referrals occur only when jobs expand. By comparing these models with the non-referral economy, I reveal the contribution of referrals to the business cycle.

The main finding in this paper is that referral hiring does not necessarily reduce the business cycle fluctuations. The model calibrated to the US economy, mainly borrowing from Fujita and Ramey (2012) parameters, shows amplification of the business cycle in response to a productivity shock in the market search model, although dampening occurs in the job expansion model. The key factor for understanding the model response is the reaction of the job arrival rate driven by the matching function. In the market search model, the job arrival rate via referrals depends on the job arrival rate via the labor market, while in the job expansion model, the job arrival rate is independent of the market arrival rate, at least directly. This difference in the specifications, whether referral jobs pass through the labor market or not, leads to completely opposite results for cyclical properties in the economy.

This paper contributes to the two strands of literature. First, the study yields additional insights into the relationship between referral and the business cycle in the referral hiring literature. Starting from the seminal works proposing tractable models in the search and matching situation (e.g., Calvó-Armengol and Zenou, 2005; Fontaine, 2007; Galenianos, 2014; Ioannides and Soetevent, 2006), there are many works using both theoretical and empirical approaches to investigate the effects of referral on the inequality

(e.g., Fontaine, 2008; Galenianos, 2021; Hellerstein et al., 2014; Horváth and Zhang, 2018; Miller and Schmutte, 2023; Tassier and Menczer, 2008; Zaharieva, 2013), occupational mismatches (e.g., Alaverdyan and Zaharieva, 2022; Horváth, 2014; Zaharieva, 2018), match qualities (e.g., Brown et al., 2016; Burks et al., 2015; Galenianos, 2013), and efficiency (e.g., Galenianos, 2014; Igarashi, 2016). However, except for several works (e.g., Galeotti and Merlino, 2014), there is not a large body of research examining the business cycle properties related to referral hiring. This paper opens up a new avenue in the referral hiring literature by providing the dynamic relevance between referral hiring and the economy.

Second, the research offers a new channel to fill part of the gap between data and models based on the search and matching framework. It is difficult to capture the cyclical properties observed in the economy using simulations based on the standard search and matching model Shimer (2005); Hagedorn and Manovskii (2008). There are various approaches to addressing this issue, for instance, by introducing on-the-job search (e.g., Fujita and Ramey, 2012; Lise and Robin, 2017; Lise et al., 2016), and labor-leisure choice with the borrowing constraint (e.g., Nakajima, 2012). For on-the-job search models, referral hiring has the potential to produce richer dynamics in the economy because it can be a major job finding method in on-the-job search through the headhunting or direct outreach market. For the literature on the business cycle with borrowing constraint, referral, as a form of unemployment insurance, eases the strict conditions for marginal workers without sufficient assets. While these challenges are beyond the purpose of this research, the work here can be positioned as a starting point for further examination.

The rest of this paper is organized as follows. In Sec. 2, I propose two models with different referral procedures and give equilibrium definitions. In Sec. 3, I describe the steady state properties and provide the basic comparison between the two models, and deal with the primary analyses by computing the impulse response functions and running simulations. Finally, the discussion is summarized in Sec. 4.

2 Model

2.1 Matching

There is no on-the-job search. A match is composed of a worker and a job. A match is created in two ways: through the labor market and referrals. Let $p_{m,t}$ and $p_{r,t}^s$ be the job arrival rates to workers through the labor market and referrals in period t , respectively. The aggregate job arrival rate is

$$p_t^s = p_{m,t} + p_{r,t}^s, \quad (1)$$

and the number of aggregate matches, denoted by M^s , is given by

$$M_t^s = p_t^s u_t^s N, \quad (2)$$

where u_t^s and N are the unemployment rate in period t and the number of workers, respectively, and $s \in \{M, J\}$ is an identifier for the market search model (M) and the job expansion model (J) described below. In the labor market, the matching probability is assumed to be a Cobb-Douglas function $A(v_t^s)^\alpha (u_t^s)^{1-\alpha}$, where v_t^s is the vacancy rate in period t , and $A > 0$ and $\alpha \in (0, 1)$ are parameters, implying

$$p_{m,t}^s = A \left(\frac{v_t^s}{u_t^s} \right)^\alpha.$$

The job arrival rate via referrals can take different forms depending on concrete referral procedures. I consider two types of referral procedures adopted in the literature, which are discussed below. Each match is destroyed with probability $\delta \in (0, 1)$ at the end of each period.

2.2 Production

Let y_t be the amount of output from a match, assumed to be a stochastic variable and interpreted as labor productivity, following

$$y_t = z_t y^*, \quad (y^* > 0) \quad (3)$$

and the logarithm of z_t follows an AR(1) process

$$\begin{aligned} \log z_t &= \rho \log z_{t-1} + \epsilon_t, \\ \epsilon_t &\sim \mathcal{N}(0, \sigma^2). \end{aligned} \quad (4)$$

This formulation is standard in the studies related to business cycles (e.g., Fujita and Ramey, 2012; Hagedorn and Manovskii, 2008).

2.3 Workers

Workers are risk neutral and form a social network \mathcal{G} , which is an undirected, unweighted, and random regular network that does not change over time. Let G be the adjacency matrix associated with \mathcal{G} in which the (i, j) element of G , denoted by g_{ij} , is 1 if workers i and j are connected, otherwise 0. Because the social network is a regular network, each worker has the same number of friends, denoted by d , implying that $\sum_j g_{ij} = d \forall i$.

When a worker is matched with (or employed in) a job, the worker supplies a unit of labor and receives wage w_t^s . When a worker is unmatched (or unemployed), the worker seeks a job in the labor market and through referrals. Let W_t^s and U_t^s be the value functions of the employed and unemployed workers, respectively. The value functions of a worker satisfy

$$W_t^s = w_t^s + \beta E_t [(1 - \delta)W_{t+1}^s + \delta U_{t+1}^s], \quad (5)$$

$$U_t^s = b + \beta E_t [p_{t+1}^s W_{t+1}^s + (1 - p_{t+1}^s)U_{t+1}^s], \quad (6)$$

where $\beta \in (0, 1)$ is the discount factor, and $b > 0$ is home production.

2.4 Market search model

2.4.1 Jobs

One specification for referrals is that both employed and unemployed workers receive job positions in the labor market, and these job positions are passed to unemployed workers (e.g., Calvó-Armengol and Zenou, 2005; Ioannides and Soetevent, 2006). I call the model based on this specification the market search model.

A job matched with a worker is filled; otherwise, it is vacant. A filled job produces an output of y_t and pays wage w_t^M to the matched worker. Vacant jobs incur a cost of $c > 0$ for keeping them open. Let J_t^M and V_t^M be the value functions of the job matched with a worker and vacant jobs in the market search model, respectively. J_t^M and V_t^M satisfy the following conditions:

$$J_t^M = y_t - w_t^M + \beta E_t [(1 - \delta)J_{t+1}^M + \delta V_{t+1}^M], \quad (7)$$

$$V_t^M = -c + \beta E_t [q_{t+1}^M J_{t+1}^M + (1 - q_{t+1}^M)V_{t+1}^M], \quad (8)$$

where $q_t^M \equiv M_t^M / (Nv_t^M)$ is the worker arrival rate in the market search model.

2.4.2 Referrals

In the market search model, job offers are randomly sent to workers regardless of employment states. If an unemployed worker receives a job offer, the worker accepts the job offer and becomes employed. If an employed worker receives an offer, the worker passes it to a randomly chosen unemployed friend with probability $\psi \in [0, 1]$. Thus, an unemployed worker who fails to receive an offer in the labor market, has the possibility of receiving an offer from friends.

An unemployed worker, denoted by i , receives a job offer from one contact, denoted by j , with probability $P_{j,t}^M$, which is given by

$$\begin{aligned} P_{j,t}^M &= \psi p_t^M (1 - u_t^M) \sum_{\ell=0}^{d-1} \frac{1}{1 + \ell} \binom{d-1}{\ell} (u_t^M)^\ell (1 - u_t^M)^{d-\ell-1} \\ &= \psi p_t^M (1 - u_t^M) \frac{1 - (1 - u_t^M)^d}{u_t^M d}, \end{aligned} \quad (9)$$

In this formulation, $\psi p_t^M (1 - u_t^M)$ corresponds to the joint probability that worker j is employed, obtains an offer, and sends it to an unemployed friend. The remaining part is the average probability that worker j selects the originating unemployed worker i as the receiver. If worker j has ℓ unemployed friends other than worker i , the probability that worker i receives the offer is $1/(1 + \ell)$. Given the unemployment rate u_t , the probability that worker j has ℓ unemployed friends other than i can be computed based on a binomial distribution with the mean-field approximation. I omit j from $P_{j,t}^M$, meaning that $P_t^M \equiv P_{j,t}^M$, because $P_{j,t}^M$ is the same for all workers.

Let $p_{r,t}^M$ denote the probability that unemployed worker i receives an offer via referrals in the market search model, which is calculated by

$$p_{r,t}^M = dP_t^M. \quad (10)$$

Note that the model assumes that the job arrival rate via referrals is approximated by a linear function of the number of friends, d . This specification makes analysis tractable, and does not make a large difference compared with a non-linear specification (i.e., $(1 - P_t^M)^d$) around the equilibrium.

2.5 Job expansion model

2.5.1 Jobs

The other specification for referrals is that new job positions are created as jobs expand, and these expanded positions are filled through referrals from workers who are matched with the existing jobs (e.g., Galenianos, 2014, 2021). I call the model based on this specification the job expansion model.

At the beginning of the period, the new job position is created by the expansion of an existing job with probability $\mu \in [0, 1]$. The new position is immediately sold to a new entrepreneur, and the existing job receives a ratio of $\gamma \in [0, 1]$ from the value of the new job position, denoted by X_t . If the new position can match with a worker through referral, it becomes a filled job; otherwise, it becomes a vacant job. A worker matched with an

existing job refers the new expanding position to an unemployed friend chosen randomly. Thus, the job value generated by expansion satisfies the condition,

$$X_t = V_t^J + du_t^J(J_t^J - V_t^J). \quad (11)$$

Let J_t^J and V_t^J be the value functions of the matched and vacant jobs in the job expansion model, respectively. J_t^J and V_t^J satisfy the following conditions:

$$J_t^J = y_t - w_t^J + \beta E_t [(1 - \delta)J_{t+1}^J + \delta V_{t+1}^J + \gamma \mu X_{t+1}], \quad (12)$$

$$V_t^J = -c + \beta E_t [q_{t+1}^J J_{t+1}^J + (1 - q_{t+1}^J) V_{t+1}^J], \quad (13)$$

where $q_t^J \equiv M_t^J / (N v_t^J)$ is the worker arrival rate.

2.5.2 Referrals

In the job expansion model, when an employer asks for a referral search, an offer is sent to a randomly chosen unemployed friend. Unemployed worker i receives a job offer from contact j with probability

$$P_{j,t}^J = \mu (1 - u_t^J) \frac{1 - (1 - u_t^J)^d}{u_t^J d}. \quad (14)$$

Note that the referral process is slightly different from representative works (e.g., Galenianos, 2014), in which a referrer chooses a referee among all friends, including employed workers, to enable a comprehensive comparison with the market search model. It is confirmed that the difference does not change analytical results significantly. Again, I omit j from $P_{j,t}^J$ (i.e., $P_t^J = P_{j,t}^J$). In the job expansion model, the job arrival rate via referrals is

$$p_{r,t}^J = d P_t^J. \quad (15)$$

2.6 Equilibrium

The equilibrium for each model is commonly defined as follows.

Definition 2.1. *The equilibrium path is a set of $\{u_t^s, v_t^s, W_t^s, U_t^s, J_t^s, V_t^s, w_t^s\}_{t=0}^\infty$ with a given path of a stochastic process $\{z_t\}_{t=0}^\infty$, which satisfies the following conditions:*

(i) *the wage is determined by Nash bargaining,*

$$w_t^s = \arg \max_{w_t^s} (W_t^s - U_t^s)^\eta (J_t^s - V_t^s)^{1-\eta}, \quad (16)$$

where $\eta \in (0, 1)$ is the bargaining power of workers,

(ii) *the free entry condition is satisfied;*

$$V_t^s = 0 \quad \forall t, \quad (17)$$

(iii) *for given u_0 , the law of motion of the unemployment rate is given by*

$$u_{t+1}^s = (1 - u_t^s)\delta + u_t^s(1 - p_t^s). \quad (18)$$

Let $S_t^s \equiv W_t^s - U_t^s + J_t^s - V_t^s$ be a match surplus. The dynamics of S_t^s is different for the market search and job expansion models due to the difference in filled job values. For each model, the match surpluses follow

$$S_t^M = y_t - b + \beta E_t [(1 - \delta - \eta p_{t+1}^M) S_{t+1}^M], \quad (19)$$

$$S_t^J = y_t - b + \beta E_t [(1 - \delta - \eta p_{t+1}^J + \mu \gamma d u_t^J) S_{t+1}^J], \quad (20)$$

The Nash bargaining of wages provides the dividend of the match surplus as follows:

$$W_t^s - U_t^s = \eta S_t^s, \quad (21)$$

$$J_t^s = (1 - \eta) S_t^s, \quad (22)$$

where Eq. (17) is applied in the bargaining result. Eqs. (13) and (22) lead to

$$c = \beta(1 - \eta) E_t [q_{t+1}^s S_{t+1}^s]. \quad (23)$$

This equation determines the equilibrium vacancy rate via q_{t+1}^s .

Both systems of Eqs. (19) and (20) can be solved numerically with Eq. (23) using backward substitution, which is a procedure similar to that of Fujita and Ramey (2012). The difference is that the system is solved for v_t for a given u_t in this model, rather than for the tightness of the labor market, v_t/u_t , in Fujita and Ramey (2012).

2.7 Timeline

The timeline of events is mainly identical for both models, except for the job expansion event. The timeline is summarized as follows:

- (i) At the beginning of period t , productivity z_t is observed.
- (ii) Jobs enter the market, and the vacancy rate is determined.
 - (a) In the job expansion model, the job expands with a given probability after the entry.
- (iii) Job information arrives at workers and
 - (a) it is retained if the worker is unemployed, and
 - (b) it is passed to one of the unemployed friends chosen at random if the worker is employed.
- (iv) Existing matches are separated with probability δ .
- (v) New matches are created if workers hold job information.
- (vi) Job-worker matches produce y_t of output, and jobs pay wages to matched workers. Unemployed workers also produce b of home production.
- (vii) Return to (i) with t incremented by 1.

3 Analyses

3.1 Steady state properties

I propose comparisons between the market search and job expansion models when there are no shocks. I describe the situation with the fixed productivity $z^* > 0$, and then obtain the following propositions.

Proposition 3.1. Assume that $z_t = z^*$, and $u < 1 - [(\mu - \delta)/\mu]^{\frac{1}{d}}$ for the job expansion model. Then, there is at least one equilibrium in a steady state for each model.

Proof. See Appendix A. □

Proposition 3.2. Assume that $z_t = z^*$, and $u < 1 - [(\mu - \delta)/\mu]^{\frac{1}{d}}$ for the job expansion model. If an equilibrium exists around $u = 0$, it is unique for each model.

Proof. See Appendix B. □

The next proposition holds.

Proposition 3.3. Assume $u^M = u^J$, and $v^M = v^J$, then $w^M < w^J$.

Proof. With simple algebra, equilibrium wages in the market search model and the job expansion model can be obtained as

$$\begin{aligned} w^M &= y\eta + (1 - \eta)b + \eta c \frac{v^M}{u^M}, \\ w^J &= y\eta + (1 - \eta)b + \eta c \frac{v^J}{u^J} + \frac{\eta c \gamma \mu d u^J}{q^J}, \end{aligned}$$

Given $v^M = v^J$ and $u^M = u^J$, it is obvious that $w^J - w^M = \eta c \gamma \mu d u^J / q^J > 0$. □

The wage in the job expansion model increases due to an increase in the match surplus, while the wage in the market search model does not. In the job expansion model, workers can receive part of the increased surplus as an additional wage through bargaining.

3.2 Calibration

I propose a comparison among the proposed models, the market search model and the job expansion model, and the model without referral, which can be obtained immediately from the market search model by setting $\psi = 0$, called the no referral model. The no referral model is set as the benchmark model. Due to the difference in model specifications, the steady state values for each model slightly differ if each model uses identical parameters. Because the focus is on dynamic properties, I adjust some parameters such that the unemployment and vacancy rates in the market search model and the job expansion model are consistent with the ones obtained in the no referral model.

The calibrated parameters are shown in Tab. 1. The calibration strategy mainly mimics that proposed by Fujita and Ramey (2012). The frequency is weekly, and the discount factor is calculated based on a 4% annual interest rate. The average output y^* is normalized to one. The home production b and the elasticity parameter of the matching function α are set to 0.7 and 0.3, respectively. The parameters for aggregate shocks ρ and σ are set to 0.9895 and 0.0034, based on Hagedorn and Manovskii (2008), which are the same values taken in Fujita and Ramey (2012). These are standard benchmark choices.

Tab. 1. Parameters for numerical calculations.

Parameters		NO Referral	Market Search	Job Expansion
β	discount factor	0.9992	0.9992	0.9992
y^*	average output	1	1	1
b	home production	0.7	0.7	0.7
α	elasticity of market matching	0.3	0.3	0.3
ρ	autocorrelation of $\log z$	0.9895	0.9895	0.9895
σ	standard deviation of $\log z$	0.0034	0.0034	0.0034
δ	job destruction rate	0.005	0.005	0.005
d	number of contacts	10	10	10
γ	share of expanding surplus to originated job	–	–	0.25
η	workers' bargaining power	0.7000	0.7000	0.7028
c	job opening cost	0.1700	0.1700	0.1684
A	efficiency in market matching	0.095	0.0475	0.0475
ψ	efficiency in referral hiring	–	0.1351	–
μ	job expanding probability	–	–	0.0057

Notes: the no referral model is a replication of the constant separation model in Fujita and Ramey (2012). The market search model and the job expansion model are calibrated such that (u, v) is the same to that in the no referral model in the steady state.

Tab. 2. Results in steady state.

	No Referral	Market Search	Job Expansion
u	0.0556	0.0556	0.0556
v	0.0383	0.0383	0.0383
p	0.0849	0.0849	0.0849
w	0.9920	0.9920	0.9931

Notes: u and v are the same for all models due to the calibration. In addition to them, p is equivalent for all models. The wage, w , in the job expansion model is larger than in the other models, consistent with Proposition 3.3.

The number of friends can lie within a relatively wide range, about 3 to 80 as provided in some data repositories (e.g., Leskovec and Krevl, 2014; Rossi and Ahmed, 2015), due to the difficulty in capturing a *true* social network. I calculate the distributions for the number of contacts of the Facebook and Twitter ego networks, provided by Leskovec and McAuley (2012). The Facebook network has 11 contacts for the median, and the 1st-3rd quartiles are 4.5-28.0. The Twitter network has 13 contacts for the median, and the 1st-3rd quartiles are 5.0-33.0. Based on these observations, I choose $d = 10$ as a benchmark in the paper.

In the job expansion model, there are some options for calibrating the surplus share allocated to the existing job γ . One approach adopted here is to use the share of cash flow rights in M&A. Kaplan and Strömberg (2003) report that residual cash flow rights to founders typically range between 20% and 30% in venture capital settings. Accordingly, I set $\gamma = 0.25$, the midpoint of the range.

The worker's bargaining power η and the vacancy opening cost c basically follow Fujita and Ramey (2012), meaning $\eta = 0.7$, and $c = 0.17$ in the no referral model and the market search model. The worker's bargaining power is a standard choice. The job opening cost is calculated as 17 percent of a 40-hour work week, as discussed in Fujita and Ramey (2012). In the job expansion model, η and c are slightly revised to be consistent with the unemployment and vacancy rate in the no referral model, leading to 0.7028 and 0.1684.

The remaining parameters, efficiency in the market matching function A , efficiency in referral hiring in the market search model ψ , and job expanding probability in the job

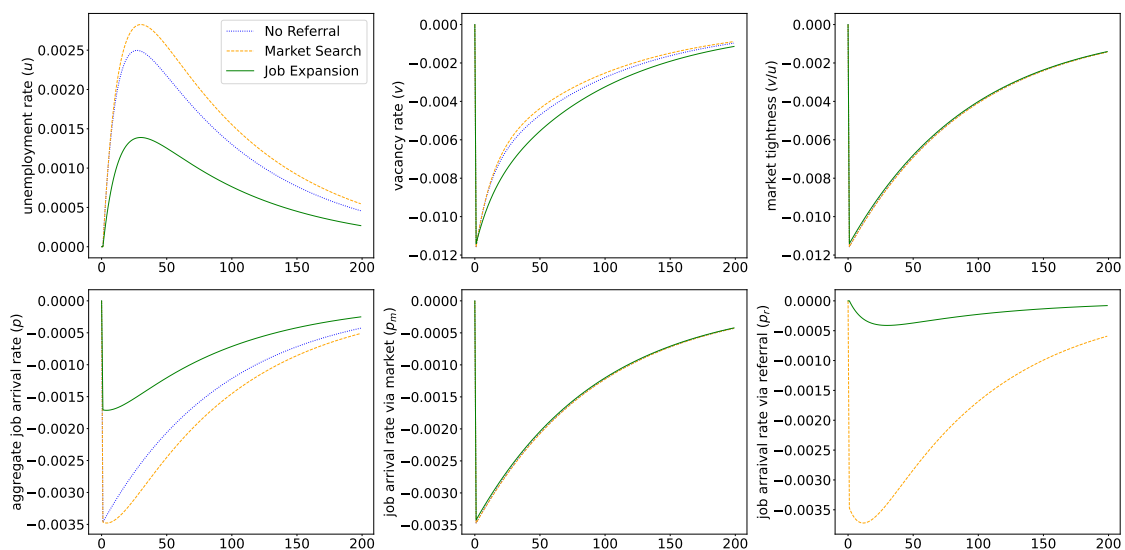


Fig. 1. Impulse response functions. In the market search model, unemployment fluctuations are amplified, while in the job expansion model, they are stabilized. The difference in the responses of u stems from the difference in p . In the job expansion model, the impulse response of p is substantially dampened. This reflects the significant impact of referrals that bypass the market.

expansion model μ , are calibrated to some targets. The parameter A in the no referral model is set to 0.095 borrowed from Fujita and Ramey (2012) to ensure consistency. In the market search model and the job expansion model, these parameters are chosen such that the ratio of referral matching to all matching is 50%. The ratio shown in Holzer (1988) is sometimes used for the calibration target in the referral hiring literature (e.g., Igarashi, 2016). This target gives $p_r^s/p^s = 0.5$, and heads and tails of a coin, $p_m^s/p^s = 0.5$ as well. Given a steady state value of unemployment, vacancy, and job arrival rate in the no referral model, denoted by u^N , v^N , and p^N , respectively, the second constraint imposes $A = 0.5p^N(u^N/v^N)^\alpha = 0.0475$. The parameters $\psi = 0.1351$ and $\mu = 0.0057$ are computed from the first constraint.

The calculated endogenous values in the steady state are summarized in Tab. 2. The targeted values, unemployment rate u and vacancy rate v , are forced to be identical. The identical u ensures an identical job arrival rate p for every model (see (18)) in the steady state. The difference arises in wage level w , which is consistent with Proposition 3.3.

3.3 Impulse response

To analyze the impacts of referral on the dynamics, I compute the impulse response functions for each model. Starting from the steady state, I apply an unexpected negative shock of one standard deviation to the productivity and plot the impulse responses in Fig. 1. The unemployment rate shifts its path due to referrals, while the direction of the shift is clearly opposite depending on how the referral process is introduced into the model. The difference is mainly attributable to the response of the aggregate job arrival rate. The impulse magnitude of the aggregate job arrival rate (p) in the job expansion model is almost half of that of the other models. The difference in the trajectory of the aggregate job arrival rate is attributed to the difference in the job arrival rate via referrals (p_r) rather than via the labor market (p_m). In the vacancy rate panel, the job expansion

model falls below the other models to compensate for its sticky response of the job arrival rate. The impulses of unemployment and vacancy rate are almost canceled out in response to the market tightness (v/u).

Given an identical steady state, the situation under the job expansion model is obviously more robust to shocks than the other models. In contrast, there is little discrepancy in the initial responses to a shock between the market search model and the no referral model. In the latter comparison, the difference emerges after the initial responses. Referral generates a hump-shaped pattern in the aggregate job arrival rate, meaning that the delay in the adjustment process results in a longer recession experience caused by a cumulative effect on the unemployment rate.

The key factor for understanding the difference in the responses between the market search model and the job expansion model is the specification of the referral process. In the market search model, referral jobs are already posted ones, but they fail to match in the labor market. In this situation, potential entrepreneurs have to enter the market if they utilize the referral channel to meet workers. In the job expansion model, however, referral jobs are not posted in the market. Referral occurs immediately when existing jobs expand. Accordingly, potential entrepreneurs do not have to be in the market to wait for referrals. In other words, the potential entrepreneurs (or jobs) are privileged to have an additional option of waiting for referrals out of the market without incurring the job opening cost in the job expansion model. The backdoor for the matching systematically lowers the effects of firm exit-entry on the matching rate. This mechanical difference results in a difference in the stability of the unemployment rate.

These results bring one simple implication. Despite referral hiring having an aspect of unemployment insurance, the effects on the stability of the unemployment rate are completely different depending on the referral procedure. The watershed is whether the referral process is included in the labor market. If entrepreneurs (or jobs) anticipate that the referral process passes through the labor market, referrals amplify business cycle fluctuations. If entrepreneurs do not expect the referral process to bypass the labor market, referrals contribute to stabilizing the labor market.

3.4 Simulation

In this subsection, I compute the second order moments for each model by simulated streams to examine unique properties arising from business cycle amplification and stabilization through referral hiring. To solve the full models with the expectation over shocks, the backward substitution method is utilized as in Fujita and Ramey (2012). For computation, the AR(1) process, Eq. (4), is discretized into a Markov chain with a state space having 13 states associated with the transition probability matrix by the method of Tauchen (1986).

Given $z_1 = 0$, a stream of productivity, $\{z_t\}_{t=1}^T$ is generated for each simulation. Starting from the steady state, endogenous variables are calculated based on the model one by one. The time length of a simulation, T , is set to 3,840 weeks. The observation is averaged to quarterly frequency. Accordingly, the sample size consists of 320 quarterly observations. The first 200 observations are discarded for excluding the initial condition dependency, and the moments are computed by the last 120 observations after taking the logarithm and HP-filter with a smoothing parameter of 1,600.

Tab. 3 summarizes computational results, which consist of data from Fujita and Ramey (2012) (Panel A), the no referral model (Panel B), the market search model (Panel C), and the job expansion model (Panel D). The no referral model is actually a replication of the constant separation model provided by Fujita and Ramey (2012), which is the baseline.

Tab. 3. Second moments of business cycle statistics

	u_t	p_t	UE_t	EU_t	v_t	v_t/u_t
<i>Panel A: Data (Fujita and Ramey, 2012)</i>						
σ_X	0.096	0.077	0.042	0.052	0.126	0.218
$\text{cor}(y_t, X_t)$	-0.460	0.369	-0.337	-0.521	0.564	0.527
$\text{cov}(y_t, X_t)/\sigma_y^2$	-5.914	3.786	-1.879	-3.644	9.524	15.437
$\text{cor}(X_t, X_{t-1})$	0.926	0.804	0.416	0.560	0.920	0.930
<i>Panel B: No Referral Model</i>						
σ_X	0.011	0.013	0.006	0.001	0.034	0.043
$\text{cor}(y_t, X_t)$	-0.883	0.999	0.539	0.882	0.988	0.999
$\text{cov}(y_t, X_t)/\sigma_y^2$	-0.714	0.957	0.243	0.042	2.475	3.188
$\text{cor}(X_t, X_{t-1})$	0.860	0.762	0.382	0.860	0.698	0.762
<i>Panel C: Market Search Model</i>						
σ_X	0.012	0.015	0.006	0.001	0.033	0.043
$\text{cor}(y_t, X_t)$	-0.861	0.997	0.577	0.860	0.981	0.999
$\text{cov}(y_t, X_t)/\sigma_y^2$	-0.796	1.078	0.281	0.047	2.404	3.200
$\text{cor}(X_t, X_{t-1})$	0.870	0.786	0.425	0.870	0.684	0.763
<i>Panel D: Job Expansion Model</i>						
σ_X	0.006	0.007	0.003	0.000	0.037	0.042
$\text{cor}(y_t, X_t)$	-0.861	0.997	0.580	0.861	0.996	0.999
$\text{cov}(y_t, X_t)/\sigma_y^2$	-0.385	0.522	0.137	0.023	2.718	3.102
$\text{cor}(X_t, X_{t-1})$	0.870	0.786	0.425	0.870	0.730	0.763

Notes: σ_X is the standard deviation of the HP-filtered (smoothing parameter 1600) log variable. $\text{cor}(y, X)$ is the correlation with output y . $\text{cov}(y, X)/\sigma_y^2$ is the covariance with y normalized by the variance of y . $\text{cor}(X', X)$ is the first-order autocorrelation. UE_t and EU_t are defined such that $UE_t \equiv u_t p_t$ and $EU_t \equiv \delta(1 - u_t)$. No referral model (Panel B) is a replication of the constant separation model in Fujita and Ramey (2012). All statistics are averages across 1,000 simulation samples.

To align with several prior studies, I compute the standard deviation (σ_X), the correlation with output ($\text{cor}(y_t, X_t)$), the elasticity to labor productivity ($\text{cov}(y_t, X_t)/\sigma_y^2$), and the first-order autocorrelation ($\text{cor}(X_t, X_{t-1})$) for each $X_t \in \{u_t, p_t, UE_t, EU_t, v_t, v_t/u_t\}$, where UE_t and EU_t are worker flows from unemployment to employment pools and from employment to unemployment pools, respectively. The vacancy-unemployment ratio, v_t/u_t , is the labor market tightness. The results in Panel B to D suffer from the well known difficulties that the basic search and matching situation cannot replicate the actual data (Shimer, 2005), although the comparison of Panel B with C and D can capture the economic impact through referral hiring.

Based on the standard deviation, the first row, referral hiring in the market search model amplifies fluctuations in the unemployment rate and stabilizes fluctuations in the vacancy rate, while referral hiring in the job expansion model stabilizes fluctuations in the unemployment rate and amplifies fluctuations in the vacancy rate. Remembering the hump-shaped impulse response functions in the unemployment and job arrival rates, the impacts in these variables persist during the running of the economy. On the one hand, this persistence leads to the amplification of the standard deviation of u_t in the market search model. The more volatile fluctuations in hiring conditions discourage jobs from entry into the market, resulting in the lower fluctuation of the vacancy rate; vacancy can be filled easily even if there is only a little entry. The market search model shows about 10% and 15% increase in the unemployment and job arrival rates compared with the no referral model. This could have a significant effect in alternative model formulations, for example, incorporating on-the-job search or endogenous separations as examined in

Fujita and Ramey (2012). On the other hand, the persistence effect is overwhelmed by the bypass effect given by referrals in the job expansion model. The bypass, referrals out of the market, lessens reliance on the market search, and brings substantially lower fluctuations in u_t and p_t , decreases of 45% and 47%, respectively. The fluctuations of worker flows, UE_t and EU_t , are consistent with such stories; however, the labor market tightness, v_t/u_t , is almost the same for all models.

The second to fourth rows in each panel report the correlation with productivity, the elasticity to the productivity, and the autocorrelation, respectively. In these moments, there are two notable differences from the no referral model. First, the elasticity in the market search model is 1.078, which is a higher value than the maximum value from Fujita and Ramey (2012) with endogenous separation and on-the-job search, which is 1.020. This implies that referral hiring through the market holds potential to fill part of the gap from empirics to theory. Second, in the job expansion model, the market stabilization effect substantially suppresses the magnitude of the reliance on productivity, except for the vacancy rate. According to these comparisons, the mechanism embedded in the market search model may dominate the one in the job expansion model, albeit no more than stylized models.

4 Conclusion

In this paper, I examine the effects of referral hiring on business cycle fluctuations, especially focusing on the unemployment rate, using two types of referral specifications. Referral hiring works as a form of unemployment insurance, but it does not generally stabilize the business cycle. The key structure is whether the referral process is internalized in the market or not. On the one hand, in the situation where referrals are made in the market, referrals could make the business cycle volatile. On the other hand, if entrepreneurs receive new jobs out of the market, these entries do not affect the business cycle directly. In such a situation, referrals provide a bypass to the market, and work as a stabilizer for the business cycle. In reality, because the referral match could be made either in or out of the market, a distinct measure of the referral process can help clarify the relationship between referral hiring and the business cycle. In particular, given that the referral hiring has an aspect of social security, such empirical measurements of referral based on the business cycle property can encourage better policymaking.

This study has room for deeper insight through further analysis. First, endogenous separation with match-specific productivity could be introduced. It is known that endogenous separation magnifies the effects of business cycles on the fluctuations of equilibrium variables (Fujita and Ramey, 2012). It is shown by Fujita and Ramey (2012) that the endogenous separation model does not replicate the second moments observed in the data, especially the correlation between vacancy rate and productivity. Even with on-the-job search, the magnitude of fluctuations is not fully matched, suggesting that introducing referrals could potentially contribute to closing this gap.

Second, endogenous social networks could be introduced. Previous works in the referral hiring literature reveal the economic effects of endogenous social networks in the steady state (e.g., Galenianos, 2021; Galeotti and Merlino, 2014; Ioannides and Soetevent, 2007; Merlino, 2019). Social networking is an investment in future jobs, but it is a time-consuming activity. It is necessary for workers to make labor-networking choices. This challenge aligns with the labor-leisure choice in business cycle studies (e.g., Nakajima, 2012), and is expected to generate complicated behavior in equilibrium dynamics.

Finally, the limitations of the model are noted. The proposed model cannot identify

the effects of network structure due to the regular network assumption. The effects of social network structure on the economy receive attention in the literature as summarized in Jackson et al. (2017). By introducing the network structure, the model could generate richer implications, although it loses analytical tractability. Analyses incorporating network structural effects would be complementary to this research.

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Appendix

A Existence of the equilibrium

In this appendix, proofs are independently given for each model; hence, the identifier $s \in \{M, J\}$ is omitted.

A.1 Market search model

The economy is in the steady state. In the equilibrium, then,

$$W - U = \frac{w - b}{1 - \beta(1 - \delta - p)}$$

$$J = \frac{y - w}{1 - \beta(1 - \delta)}$$

Using the Nash bargaining solution, $\eta(W - U) = (1 - \eta)J$, I obtain

$$[1 - \beta(1 - \delta)]w = [1 - \beta(1 - \delta)][y\eta + (1 - \eta)b] + \eta\beta p(y - w). \quad (24)$$

The value of a filled job, combined with the free entry condition $V = 0$, leads to

$$y - w = \frac{c[1 - \beta(1 - \delta)]}{\beta q}, \quad (25)$$

where

$$q = A \left(\frac{u}{v}\right)^{1-\alpha} \left[1 + \psi(1 - u) \frac{1 - (1 - u)^d}{u}\right].$$

Substituting Eq. (25) into (24), and after some calculation,

$$w = y\eta + (1 - \eta)b + \eta c \frac{v}{u}$$

and based on that,

$$y = b + \frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)q} + \frac{\eta c}{1 - \eta} \frac{v}{u} \quad (26)$$

Eq. (26) is called the job creation curve, which is one equation of the system.

The other constraint is the Beveridge curve, derived from $p = (1 - u)\delta/u$. By some substitutions, it is obtained that

$$v = \left[\frac{(1 - u)\delta}{A \left[u + \psi \left\{ 1 - u - (1 - u)^{d+1} \right\} \right]} \right]^{\frac{1}{\alpha}} u \quad (27)$$

Consider the polarized cases of the job creation curve. Substituting q into (26),

$$y = b + \frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)A \left(\frac{v}{u}\right)^{1-\alpha} \left[1 + \psi(1 - u) \frac{1 - (1 - u)^d}{u}\right]} + \frac{\eta c}{1 - \eta} \frac{v}{u}.$$

Given that

$$\psi(1 - u) \frac{1 - (1 - u)^d}{u} = \psi \sum_{i=1}^d (1 - u)^i \rightarrow \psi d \quad \text{as } u \rightarrow 0,$$

the RHS of the job creation curve can be regarded as a function of v/u . Therefore, to keep the LHS constant (y), v/u necessarily remains constant, meaning that $v \propto u$ needs to be satisfied. Correspondingly, when $u \rightarrow 0$, $v \rightarrow 0$.

Given that

$$\psi(1 - u) \frac{1 - (1 - u)^d}{u} \rightarrow 0 \quad \text{as } u \rightarrow 1,$$

the RHS of the job creation curve is

$$y = b + \frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)A} v^{1-\alpha} + \frac{\eta c}{1 - \eta} v.$$

The RHS increases in v starting from $b < y$, and has a unique solution of $v = v^* > 0$. Accordingly, when $u \rightarrow 1$, $v \rightarrow v^* > 0$.

Consider the Beveridge curve. Rearranging Eq. (27), I get

$$v = \left[\frac{\delta \left(\frac{1}{u^{1-\alpha}} - u^\alpha\right)}{A \left[1 + \psi \sum_{i=1}^d (1 - u)^i\right]} \right]^{\frac{1}{\alpha}}. \quad (28)$$

Based on the equation, when $u \rightarrow 0$, $v \rightarrow \infty$, and when $u \rightarrow 1$, $v \rightarrow 0$.

By comparing the $u \rightarrow 0$ and $u \rightarrow 1$ cases between the job creation curve and the Beveridge curve on (u, v) plane, the job creation curve is lower than the Beveridge curve for $u \rightarrow 0$, while the former is higher than the latter for $u \rightarrow 1$. By the intermediate value theorem, there exists at least one equilibrium between $u \in (0, 1)$.

A.2 Job expansion model

In the steady state, the filled job value in the job expansion model is

$$J = \frac{y - w}{1 - \beta(1 - \delta) - \gamma \mu du}$$

By using the Nash bargaining solution $\eta(W - U) = (1 - \eta)J$,

$$[1 - \beta(1 - \delta)]w = [1 - \beta(1 - \delta)][y\eta + (1 - \eta)b] + \eta\beta p(y - w) + (1 - \eta)\beta\gamma\mu du(w - b) \quad (29)$$

With the filled value combined with the free entry condition,

$$y - w = \frac{c[1 - \beta(1 - \delta) - \beta\gamma\mu du]}{\beta q} \quad (30)$$

and with some algebra it is obtained that

$$w - b = \frac{\eta c}{1 - \eta} \frac{1 - \beta(1 - \delta) + \beta p}{\beta q}, \quad (31)$$

where

$$q = A \left(\frac{u}{v} \right)^{1-\alpha} + \mu(1-u) \frac{1 - (1-u)^d}{v}.$$

Substituting Eqs. (30) and (31) into (29),

$$w = y\eta + (1 - \eta)b + \eta c \frac{v}{u} + \frac{\eta c \gamma \mu du}{q},$$

or, in another form, the job creation curve is obtained such that

$$y = b + \frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)q} + \frac{\eta c}{1 - \eta} \frac{v}{u} + \frac{\eta c \gamma \mu du}{(1 - \eta)q}. \quad (32)$$

The Beveridge curve, another condition for the equilibrium, is given by

$$v = \left[\frac{(1-u)}{A u^{1-\alpha}} \left[\delta - \mu + \mu(1-u)^d \right] \right]^{\frac{1}{\alpha}}. \quad (33)$$

Since $v > 0$, u has a constraint

$$\delta - \mu + \mu(1-u)^d > 0 \iff u < 1 - \left(\frac{\mu - \delta}{\mu} \right)^{\frac{1}{d}} \equiv \bar{u},$$

and for $u \in (0, \bar{u})$, it is obvious that the slope of the Beveridge curve is negative, $dv/du < 0$. Furthermore, for $u \rightarrow 0$, $v \rightarrow \infty$, and for $u \rightarrow \bar{u}$, $v \rightarrow 0$.

The job creation curve (32) is revised by substitution of q ,

$$y = b + \frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)A \left(\frac{u}{v} \right)^{1-\alpha} + \mu \left(\frac{u}{v} \right) \sum_{i=1}^d (1-u)^i} + \frac{\eta c}{1 - \eta} \frac{v}{u} + \frac{\eta c \gamma \mu dv}{(1 - \eta)A \left(\frac{u}{v} \right)^{\alpha} + \mu \sum_{i=1}^d (1-u)^i}$$

When $u \rightarrow 0$, assuming $v \rightarrow \infty$ or $v \rightarrow v^* > 0$, it is obvious that the RHS diverges for both cases. This is a contradiction to a constant LHS. When $u \rightarrow 0$, assumed that $v \rightarrow 0$, there are three cases of the limit of v/u ; it goes to 0, ∞ , or remains constant. If $v/u \rightarrow 0$, $b = y$ holds, which violates the parameter constraint $y > b$. If $v/u \rightarrow \infty$ the RHS diverges, and it is a contradiction to y of the LHS. As a consequence, the possible case for $u \rightarrow 0$ is that $v \rightarrow 0$ while v/u is constant, meaning that $v \propto u \rightarrow 0$.

For $u \rightarrow \bar{u}$, the job creation curve goes to

$$y = b + \frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)\bar{q}} + \frac{\eta c}{1 - \eta} \frac{v}{\bar{u}} + \frac{\eta c \gamma \mu d \bar{u}}{(1 - \eta)\bar{q}},$$

where

$$\bar{q} \equiv A \left(\frac{\bar{u}}{v} \right)^{1-\alpha} + \mu(1 - \bar{u}) \frac{1 - (1 - \bar{u})^d}{v}.$$

It is easily confirmed that $d\bar{q}/dv < 0$, $\bar{q} \rightarrow \infty$ as $v \rightarrow 0$, and $\bar{q} \rightarrow 0$ as $v \rightarrow 1$. The limit of the job creation curve can be rearranged to

$$(y - b)\bar{q} = \frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)} + \frac{\eta c}{1 - \eta} \bar{p} + \frac{\eta c \gamma \mu d \bar{u}}{(1 - \eta)}, \quad (34)$$

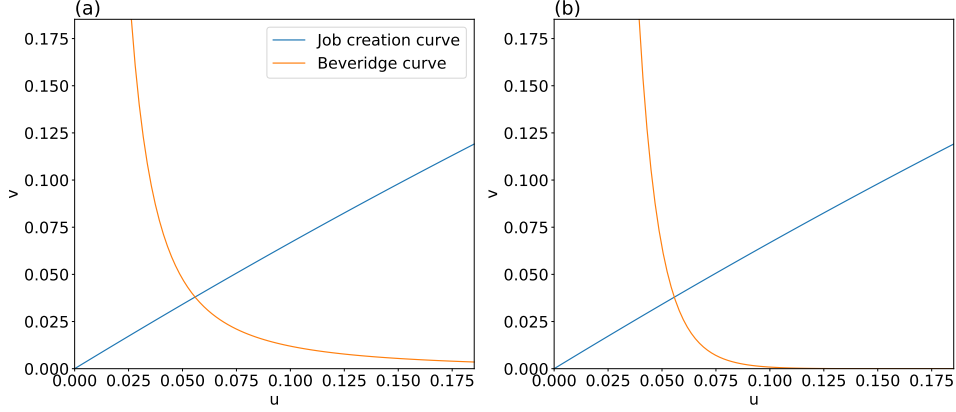


Fig. 2. Local uniqueness for (a) market search model and (b) job expansion model under calibrated parameters.

where

$$\bar{p} \equiv \frac{v}{\bar{u}}q = A \left(\frac{v}{\bar{u}} \right)^\alpha + \mu(1 - \bar{u}) \frac{1 - (1 - \bar{u})^d}{\bar{u}},$$

and obviously $d\bar{p}/dv > 0$, $\bar{p} \rightarrow 0$ as $v \rightarrow 0$, and $\bar{p} \rightarrow \infty$ as $v \rightarrow \infty$. In summary, the LHS of Eq. (34) is an increasing function of v , and the RHS is a decreasing function of v . Combining the fact that, for $v \rightarrow 0$, $LHS \rightarrow \infty$ and $RHS \rightarrow 0$, the Eq. (34) has a unique solution for a $v^* > 0$ on (u, v) plane. As a consequence, as $u \rightarrow \bar{u}$, $v \rightarrow v^* > 0$ on the job creation curve.

Since the job creation curve is lower than the Beveridge curve for $u \rightarrow 0$ and the former is higher than the latter for $u \rightarrow \bar{u}$, the model has at least one equilibrium in $u \in (0, \bar{u})$ by the intermediate value theorem.

B Local uniqueness

B.1 Market search model

Let the job creation curve be

$$y = b + \frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)q} + \frac{\eta c}{1 - \eta} \frac{v}{u} \equiv G(u, v)$$

On (u, v) plane, the implicit function theorem implies

$$\begin{aligned} \frac{dv}{du} &= - \frac{\frac{\partial G(u, v)}{\partial u}}{\frac{\partial G(u, v)}{\partial v}} \\ &= - \frac{-\frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)q^2} \frac{\partial q}{\partial u} - \frac{\eta c}{1 - \eta} \frac{v}{u^2}}{-\frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)q^2} \frac{\partial q}{\partial v} + \frac{\eta c}{1 - \eta} \frac{1}{u}} \end{aligned}$$

The key for determining the slope is derivatives of q with respect to u and v . With linear approximation around $u = 0$,

$$\begin{aligned} q &= A \left(\frac{u}{v}\right)^{1-\alpha} \left[1 + \psi \frac{1-u-(1-u)^{d+1}}{u} \right] \\ &\approx A \left(\frac{u}{v}\right)^{1-\alpha} \left[1 + \psi \frac{1-u-\{1-(d+1)u\}}{u} \right] \\ &= A(1+\psi d) \left(\frac{u}{v}\right)^{1-\alpha} \end{aligned}$$

Then,

$$\frac{\partial q}{\partial v} = -\frac{1-\alpha}{v} A(1+\psi d) \left(\frac{u}{v}\right)^{1-\alpha} < 0,$$

and

$$\frac{\partial q}{\partial u} = \frac{1-\alpha}{u} A(1+\psi d) \left(\frac{u}{v}\right)^{1-\alpha} > 0.$$

Therefore, around $u = 0$, the slope of the job creation curve satisfies

$$\frac{dv}{du} > 0. \quad (35)$$

The slope of the Beveridge curve (27) is

$$\frac{dv}{du} = \left(\frac{\delta}{A}\right)^{\frac{1}{\alpha}} \left[\frac{(1-u)}{u + \psi \{1-u-(1-u)^{d+1}\}} \right]^{\frac{1}{\alpha}} \left[-\frac{\psi d(1-u)^{d+1} + 1}{u + \psi \{1-u-(1-u)^{d+1}\}} \frac{u}{\alpha(1-u)} + 1 \right]$$

The sign of this equation depends on the last bracket; therefore, with linear approximations,

$$\begin{aligned} \frac{dv}{du} &\propto -\frac{\psi d(1-u)^{d+1} + 1}{u + \psi \{1-u-(1-u)^{d+1}\}} \frac{u}{\alpha(1-u)} + 1 \\ &\approx -\frac{\psi d(1-(d+1)u) + 1}{u + \psi \{1-u-(1-(d+1)u)\}} \frac{u}{\alpha(1-u)} + 1 \\ &= -\frac{\psi d + 1 - \alpha - \alpha\psi d}{\alpha(1+\psi d)(1-u)} - \left[\frac{-\psi d - \psi d^2 + \alpha + \alpha\psi d}{\alpha(1+\psi d)} \right] \frac{u}{1-u} \\ &= -\frac{1-\alpha}{\alpha(1-u)} < 0, \end{aligned}$$

where $u/(1-u) \approx 0$ for a small u is used. Accordingly, the slope of the Beveridge curve satisfies

$$\frac{dv}{du} < 0, \quad (36)$$

near a small u .

With the fact that the job creation curve is lower than the Beveridge curve for $u \rightarrow 0$ on (u, v) plane, the job creation curve with positive slope (35) and the Beveridge curve with negative slope (36) cross only once around $u = 0$. The equilibrium $u^* = 0.0556$ satisfies such a condition in Fig. 2a.

B.2 Job expansion model

Let the job creation curve be

$$y = b + \frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)q} + \frac{\eta c}{1 - \eta} \frac{v}{u} + \frac{\eta c \gamma \mu d u}{(1 - \eta)q}.$$

By the implicit function theorem,

$$\begin{aligned} \frac{dv}{du} &= - \frac{\frac{\partial G(u,v)}{\partial u}}{\frac{\partial G(u,v)}{\partial v}} \\ &= - \frac{-\frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)q^2} \frac{\partial q}{\partial u} - \frac{\eta c}{1 - \eta} \frac{v}{u^2} + \frac{\eta c \gamma \mu d}{1 - \eta} \left(\frac{1}{q} - \frac{u}{q} \frac{\partial q}{\partial u} \right)}{-\frac{c[1 - \beta(1 - \delta)]}{\beta(1 - \eta)q^2} \frac{\partial q}{\partial v} + \frac{\eta c}{1 - \eta} \frac{1}{u} + \frac{\eta c \gamma \mu d u}{(1 - \eta)q^2} \frac{\partial q}{\partial v}} \end{aligned}$$

The key for determining the slope is derivatives of q with respect to u and v . The function of q is given by

$$q = A \left(\frac{u}{v} \right)^{1 - \alpha} + \mu(1 - u) \frac{1 - (1 - u)^d}{v}.$$

Then,

$$\frac{\partial q}{\partial v} = -\frac{1 - \alpha}{v} A \left(\frac{u}{v} \right)^{1 - \alpha} - \mu(1 - u) \frac{1 - (1 - u)^d}{v^2} < 0,$$

and

$$\frac{\partial q}{\partial u} = \frac{1 - \alpha}{u} A \left(\frac{u}{v} \right)^{1 - \alpha} + \frac{\mu}{v} [-1 + (d + 1)(1 - u)^d].$$

Using linear approximations around $u = 0$, I obtain

$$\begin{aligned} \frac{\partial q}{\partial u} &\approx \frac{1 - \alpha}{u} A \left(\frac{u}{v} \right)^{1 - \alpha} + \frac{\mu}{v} [-1 + (d + 1)(1 - ud)] \\ &= \frac{1 - \alpha}{u} A \left(\frac{u}{v} \right)^{1 - \alpha} + \frac{\mu d}{v} [1 - (d + 1)u] \end{aligned}$$

and when $u < 1/(d + 1)$,

$$\frac{\partial q}{\partial u} > 0,$$

Setting $d = 10$ gives $u^* = 0.0556 < 0.09 = 1/(1 + d)$. In this case,

$$\frac{dv}{du} > 0. \tag{37}$$

As shown in the proof of existence, the Beveridge curve (33) satisfies $dv/du < 0$ for $u \in (0, \bar{u})$.

The slope of the job creation curve (37) and that of the Beveridge curve around $u = 0$, along with the comparison of their values for $u \rightarrow 0$, ensure that the equilibrium is locally unique. Fig. 2b shows the equilibrium under calibrated parameters.