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Summary. This paper examines the relationship between dynamic stability and the presence of Giffen goods in a standard two-sector growth model. We show that a steady state may take the form of a saddle point even when a labor-intensive good becomes a Giffen good at the steady state. The results highlight that the stability of equilibria is shaped not by the presence of Giffen behavior per se, but by the strength of the income effect associated with inferior goods. When this effect is sufficiently large, steady states can become unstable; otherwise, stability is preserved. These findings clarify the conditions under which Giffen behavior interacts with dynamic equilibria, and emphasize the central role of income elasticity in determining stability outcomes.

Key words: Giffen goods, Dynamic stability, Two-sector model, Income elasticity

JEL Classification Numbers: D11, E13, E21

1 Introduction

In classical dynamic models such as those of Walras and Marshall, it has long been recognized that the presence of Giffen goods may lead to instability, since the upward-sloping demand curve implied by Giffen behavior appears inconsistent with stable market adjustment.[1] Based on some experiments, Plott (2008) argued that which of the two models can appropriately derive the conditions under which equilibrium becomes unstable depends on the cause of an upward-sloping demand. While much research has been devoted to Giffen goods in static settings, the implications of Giffen behavior for stability in dynamic general equilibrium frameworks have received little attention.[2]

In this paper, we construct a standard two-sector model, where there are two factors of production, capital and labor, and assume that households accumulate their capital stocks, while their labor supply is constant. Then, we demonstrate that a steady state in the dynamic model can be a saddle point, even when a labor intensive good becomes a Giffen good at the steady state. Also, we show that steady states may become unstable, if the labor intensive good is inferior and has a large magnitude of the income effect of demand. This occurs because if the demand for the labor intensive good significantly declines in response to rising income and the demand for the other good (a capital intensive good) substantially increases, then the rental rate will rise in line with the economy's capital accumulation.

These two results may seem contradictory at first, but they are not. Because Giffen goods do not necessarily imply that they have a large magnitude of the negative income effect. It is apparent from the Slutsky equation that an inferior good becomes a Giffen good if the income effect dominates the substitution effect, and hence both the magnitude of the effects may be small.[3]

This paper is organized as follows. Section 2 presents a standard two-sector model. Section 3 derives the steady state equilibria with a Giffen good. Section 4 examines the stability of the steady state and market adjustment to economic shocks. Section 5 offers some concluding remarks.

2 The Two-Sector Model with Two Types of Consumption Goods

To examine the case where one good is inferior for some income levels, we formulate a two-sector model with two types of consumption goods and non-homothetic preferences. The first good is a pure consumption good, and the second good is a general good that can be used for consumption

[1] In the Walrasian (Marshallian) model, market equilibrium becomes unstable if both the demand and supply curves are upward-sloping and the former (latter) is less steep than the latter (former).

[2] See, for instance, Nachbar (1998).

[3] Indeed, the substitution effect is very small in the utility functions presented in the existing literature as those for which Giffen behavior can arise. This is consistent with the discussion in Fujimoto (2018) as follows: Giffen behavior can be categorized into two types, subsistence-driven and satiation-driven.

and investment. Under conditions of perfect competition and constant returns to scale, both goods are produced using capital and labor. We normalize the population to be one, and assume that each household supplies L units of labor inelastically and has a concave utility function u defined over consumption of goods 1 and 2, C_1 and C_2.

In order to utilize the results of Bond et al. (2012), we make the following assumptions, which are the same as those in their study.

Assumption 1: The production function in each sector is quasi-concave and linearly homogeneous. Both factors are indispensable for producing and pure consumption good 1 is labor intensive.

Assumption 2: The utility function is strictly concave, with $u_{11} < 0$ and $D \equiv u_{11}u_{22} - (u_{12})^2 > 0$ for any $(C_1, C_2) \in \{(C_1, C_2) \in \mathbb{R}^2_+ | u_i(C_1, C_2) > 0,\ i = 1, 2\}$, and satisfies $\lim_{C_i \to 0} u_i(C_1, C_2) = \infty$ $(i = 1, 2)$ for any C_j $(j \neq i)$.

First, we show that the factor prices are given by functions of the price of good 1 when both goods are produced. Let $a_i(r, w)$, $i = 1, 2$, denote the unit cost function in sector i, where r and w denote the rental on capital and the wage rate, respectively.[4] Then, the competitive profit conditions require that

$$a_1(r, w) = p, \tag{1}$$

$$a_2(r, w) = 1, \tag{2}$$

where good 2 is chosen as numeraire. From (1) and (2), we see that the rental on capital and the wage rate will be given by the functions of p, $r(p)$ and $w(p)$. Also, from the envelope theorem, we have the output of good 1, as follows.

$$Y_1(p, K) = r'(p)K + w'(p)L, \tag{3}$$

where K is the stock of capital and $r'(p) < 0 < w'(p)$ holds from the Stolper-Samuelson theorem.

Second, we analyze the optimization problem for a representative household who maximizes the discounted sum of its utilities

$$\max \int_0^\infty u(C_1, C_2) \exp(-\rho t) dt, \tag{4}$$

subject to its flow budget constraint

$$rK + wL = pC_1 + C_2 + \dot{K} + \delta K, \quad K_0 \text{ given}, \tag{5}$$

where δ is the rate of depreciation on capital and ρ is the discount rate.

From the first order conditions for this problem, we obtain the necessary conditions for the choice of consumption levels, the differential equation describing the evolution of the costate variable, λ :

$$u_1(C_1, C_2) = \lambda p, \quad u_2(C_1, C_2) = \lambda, \tag{6}$$

$$\dot{\lambda} = \lambda(\rho + \delta - r). \tag{7}$$

[4] We characterize the technology in sector i by the unit cost function.

Under Assumption 2, consumption of good i, $i = 1, 2$, becomes a function of p and λ, $C_i(p, \lambda)$. Let us define

$$E(p, \lambda) \equiv pC_1(p, \lambda) + C_2(p, \lambda).$$

The following lemma establishes some properties of these functions.

Lemma 1 *(i)* $\lambda C_{1\lambda} = pC_{1p} + C_{2p}$, *(ii)* $E_\lambda = pC_{1\lambda} + C_{2\lambda} < 0$, *(iii)* $C_{1p} < 0$, *(iv)* $E_p = C_1 + \lambda C_{1\lambda}$.

Proof. *See the Appendix.* ■

Third, from the market clearing condition for good 1, we show that the equilibrium price of good 1 becomes a function of K and λ. Let us define the excess demand function of good 1 as follows.

$$Z_1(p, K, \lambda) \equiv C_1(p, \lambda) - Y_1(p, K).$$

Since $Z_1 = 0$ holds, we have

$$dZ_1 = (C_{1p} - Y_{1p})dp - Y_{1K}dK + C_{1\lambda}d\lambda = 0,$$

where $Y_{1p} > 0$, $Y_{1K} = r' < 0$, and $C_{1\lambda}$ is negative (positive) if good 1 is normal (inferior). Thus, the price of good 1 is given by the function of K and λ with

$$\frac{\partial p}{\partial K} = \frac{r'}{Z_{1p}} > 0 \text{ and } \frac{\partial p}{\partial \lambda} = -\frac{C_{1\lambda}}{Z_{1p}}. \tag{8}$$

Based on the above, the dynamic system of the model is as follows.

$$\dot{K} = r(p(K, \lambda))K + w(p(K, \lambda))L - E(p(K, \lambda), \lambda) - \delta K, \tag{9}$$

$$\dot{\lambda} = \lambda \left[\rho + \delta - r(p(K, \lambda))\right]. \tag{10}$$

3 The Existence of a Steady State

First, we assume

Assumption 3: $\inf\{r | a_2(w, r) = 1\} < \rho + \delta < \sup\{r | a_2(w, r) = 1\}$.

This condition is necessary for the existence of a steady state, because $\dot{K} = 0$ implies that both goods are produced at any steady state under Assumption 2, and hence $\dot{\lambda} = 0$ requires that there is some $p^* > 0$ such that $r(p^*) = \rho + \delta$ holds.[5]

[5] Notice that for any value of $\rho + \delta$, the Cobb-Douglas technologies satisfy Assumption 3.

3.1 Determination of a steady state

Let us define the steady state budget constraint,

$$p^* C_1 + C_2 = [r(p^*) - \delta] K + w(p^*)L = \rho K + w(p^*)L, \tag{11}$$

which implies that $\dot{K} = 0$ holds. Then, the intersection between (11) and the income expansion path with $p = p^*$ denotes the optimal consumption bundle in a steady state if the steady state capital stock is K.

To find the outputs of two goods in the steady state that can be used for consumption after deducting investment, δK, Bond et al. (2012) define the steady state Rybczynski line as follows:

$$\left[p^* - \frac{\rho}{r'(p^*)}\right] C_1 + C_2 = \left[w(p^*) - \frac{\rho w'(p^*)}{r'(p^*)}\right] L. \tag{12}$$

And, they demonstrated that the intersection between (11) and (12) equals to the outputs. Therefore, if (11) passes through the intersection between the income expansion path with p^* and (12), then the market clearing condition is met.

Thus, the intersection corresponds to the steady state of this model. Note that it must be unique when both goods are normal. This uniqueness remains when a Giffen good exists and the income expansion path bends backward, if the slope of the steady state Rybczynski line is less steep than the income expansion path.

So, to guarantee the uniqueness of the intersection, we assume

Assumption 4: The slope of the steady state Rybczynski line is not so steep in the sense that the following condition is met for any λ that satisfies $C_{1\lambda}(p^*, \lambda) > 0$.[6]

$$-\left[p^* - \frac{\rho}{r'(p^*)}\right] > \frac{C_{2\lambda}(p^*, \lambda)}{C_{1\lambda}(p^*, \lambda)} \tag{13}$$

Note that the right-hand side of (13) denotes the slope of the income expansion path with $p = p^*$, and hence for any utility function with a Giffen good, we can find some ρ that satisfies (13), because as the value of ρ decreases while keeping the sum of ρ and δ fixed, the left-hand side of (13) goes to $-p^*$.[7]

Let us denote the unique intersection by (C_1^*, C_2^*). Then, the steady state values of K and λ are given by

$$K^* = \frac{p^* C_1^* + C_2^* - w(p^*)L}{\rho},$$
$$\lambda^* = u_2(C_1^*, C_2^*).$$

[6] As discussed in Bond et al. (2012), the intersection must be unique when capital intensive good 2 is inferior for some income levels.

[7] Any budget constraint and the corresponding income expansion path crosses only once when the utility function is strictly quasi-concave.

Notice that the market of good 1 clears only in this case, and that for $K > K^*$, there will be an excess demand for good 1, and vice versa (see Figure 1).

Thus, when the intersection is unique, the steady state is also uniquely determined.

3.2 The steady state with a Giffen good

To consider the steady state with a Giffen good, we specify the utility function as follows.[8]

$$u(C_1, C_2) = \alpha \ln C_1 + \beta \ln C_2 - \gamma C_1 C_2, \tag{14}$$

where $0 < \alpha < \beta < 2\alpha$, and $\gamma > 0$.

This utility function has been studied in detail in Doi et al. (2009) and is shown to have the following properties: (i) For any positive income and prices of goods 1 and 2, it has a unique interior solution to the utility maximization problem; (ii) For any fixed prices, as households' income increases, good 1 becomes an inferior good, eventually becoming a Giffen good.[9]

Figure 1 illustrates the income expansion path of (14). As labor supply L rises, the steady state consumption bundle moves upward along the income expansion path and good 1 becomes a Giffen good at some steady state with a sufficiently large value of L. Also, it is apparent from Figure 1 that the rise in L increases K^* and decreases λ^*.

4 Stability of the Steady State

As mentioned above, the excess demand for good 1 increases as K rises from K^* with satisfying the steady state conditions. In this case, the steady state becomes a saddle point. Intuitively, the saddle-point stability arises from the fact that if an increase in the capital stock yields an excess demand for the labor intensive good, then it results in a decrease in the rental rate, because the price of the good falls to clear the market.

4.1 The phase diagram

To examine the stability of the steady state, we derive the phase diagram around the steady state. From (3), (8), (9), and (10), we have

$$d\dot{K} = \left(r - \delta - \frac{r'\lambda C_{1\lambda}}{Z_{1p}}\right) dK + \left[-E_\lambda + \frac{\lambda (C_{1\lambda})^2}{Z_{1p}}\right] d\lambda, \tag{15}$$

$$d\dot{\lambda} = -\frac{\lambda (r')^2}{Z_{1p}} dK + \left(\rho + \delta - r + \frac{\lambda r' C_{1\lambda}}{Z_{1p}}\right) d\lambda, \tag{16}$$

[8] This specification is intended to yield clear results; when specifying other utility functions, additional conditions will be required.

[9] Also, the utility function satisfies Assumption 2.

where we utilize $E_p = C_1 + \lambda C_{1\lambda}$, and the coefficient of $d\lambda$ in (15) and that of dK in (16) are always positive.[10]

Then, we see that the slope of the locus of $\dot{K} = 0$ in (K, λ) space is given by

$$\left.\frac{d\lambda}{dK}\right|_{\dot{K}=0} = \frac{(r-\delta)Z_{1p} - r'\lambda C_{1\lambda}}{E_\lambda Z_{1p} - \lambda (C_{1\lambda})^2}, \tag{17}$$

which is negative if good 1 is normal, that is, $C_{1\lambda} < 0$ holds at the steady state, while it may be positive otherwise.

Since $\dot{\lambda} = 0$ implies that $p = p^*$, the coefficient of $d\lambda$ in (16) becomes $\lambda r' C_{1\lambda}/Z_{1p}$ along the locus of $\dot{\lambda} = 0$, and therefore its slope is given by

$$\left.\frac{d\lambda}{dK}\right|_{\dot{\lambda}=0} = \frac{r'}{C_{1\lambda}}, \tag{18}$$

which is positive (negative), if good 1 is normal (inferior) at the steady state.

So, in the case where good 1 is normal at the steady state, the steady state must be a saddle point, since

$$\left.\frac{d\lambda}{dK}\right|_{\dot{K}=0} < 0 < \left.\frac{d\lambda}{dK}\right|_{\dot{\lambda}=0}$$

holds at the steady state.

However, in the case of the steady state with $C_{1\lambda} > 0$, it becomes a saddle point if

$$\left.\frac{d\lambda}{dK}\right|_{\dot{K}=0} > \left.\frac{d\lambda}{dK}\right|_{\dot{\lambda}=0} \tag{19}$$

holds at the steady state, while it is unstable otherwise (see Figures 2 and 3).

We see from (17) and (18) that (19) is equivalent to (13) with $\lambda = \lambda^*$, and hence we have

Proposition 1 *Under condition (14), the unique steady state is a saddle point in the presence of a Giffen good.*

Remark 1 *Note that as long as condition (13) holds, the steady state with a Giffen good is a saddle point, regardless of the specific form of the utility function.*

Thus, we demonstrate that the steady state with a Giffen good can be a saddle point. The stability of steady states in this standard dynamic model does not depend on whether the labor intensive good is a Giffen good or not. What is important for the stability is the magnitude of the income elasticity of demand for the inferior good.

This can easily be clarified by examining the case where $C_{1\lambda}$ is positive and large enough to violate (13) at some steady state. Then,

$$\left.\frac{d\lambda}{dK}\right|_{\dot{K}=0} < \left.\frac{d\lambda}{dK}\right|_{\dot{\lambda}=0}$$

[10] See the Appendix.

holds at the steady state as in Figure 3, and hence it becomes unstable.[11]

4.2 Market adjustment to economic shocks

In dynamic models, shocks in the economy, excluding changes in state variables, results in shifts of the steady state. In this subsection, we examine the effects of changes in state variable K and changes in non-state variable L to the market.

Let the economy be initially in the steady state with a Giffen good and capital stocks fall below the steady state level.

In response to the reduction in K, the output of labor intensive good 1 increases if $p = p^*$. Also, households choose a higher shadow value of capital, λ, by the reduction, and hence the demand for inferior good 1 will rise. Although the supply and demand for good 1 both increase above the steady state levels, the market for good 1 will clear at a lower price. This is illustrated by point A in Figure 2. So, the rental rate will become higher than the steady state level and the economy converges to the steady state.

Next, let us consider the effect of the reduction in labor supply, L.

It shifts the steady state Rybczynski line downward, and hence the intersection with the income expansion path moves downward along the path. So, the reduction in L shifts the steady state to a new one with a lower capital stock and a higher shadow value. At the new steady state, the supply and demand for good 1 are both lower than they were at the previous steady state.

Since the locus of $\dot{\lambda} = 0$ is given by

$$K = \frac{C_1(p^*, \lambda) - w'(p^*)L}{r'(p^*)},$$

that decrease in L shifts the locus leftward in (K, λ) space. Therefore, the locus of $\dot{K} = 0$ moves rightward because the steady state must move to the upper left. Figure 4 illustrates these shifts in (K, λ) space, where point N denotes the new steady state.

We see from Figure 4 that the effect of that decrease on the shadow value is unclear, but the price of good 1 rises to increase the output of labor intensive good 1, which must decrease with $p = p^*$ due to the reduction in L. Therefore, the rental rate falls below the steady state level and the economy converges from the previous steady state to the new one.

[11] Let us consider the case where the steady state Rybczynski line is steep enough to have three intersections with the income expansion path for some range of values of L. When three steady states exist with the utility function (14), it is most likely to occur that the labor intensive good is normal at the lowest steady state, inferior but not a Giffen good at the middle one, and a Giffen good at the highest one. However, the lowest and highest steady states are saddle points, while the middle one is unstable.

4.3 Unstable outcome by income effect

As demonstrated above, Giffen goods do not necessarily lead to an unstable market outcome in this dynamic model. This occurs from the fact that Giffen goods do not necessarily imply that they are ultra inferior goods and have large income effects.

To consider unstable outcome due to a large magnitude of the income effect, we examine the case where $C_{1\lambda}$ is positive and large enough to violate (13) at some steady state.

Let the economy be in the unstable steady state and capital stocks fall below the steady state level. In response to the reduction in K, the output of labor intensive good 1 increases if $p = p^*$. Also, households chose a higher shadow value of capital, λ, by the reduction, and hence the demand for inferior good 1 rises. Although the supply and demand for good 1 both increase above the steady state levels as well as in the previous case with (13), the market for good 1 will clear at a *higher* price in this case, because the magnitude of the income effect of demand for good 1 is so large. This is illustrated by point A in Figure 3. Then, the rental rate falls below the steady state level and the economy will diverge from the steady state.

Also, we examine the effect of a decrease in L. It will raise the steady state value of K and decrease that of λ, in contrast to the previous case where (13) is satisfied. This occurs because the decrease in the demand for inferior good 1 is large enough to clear the market, even though both the decrease in L and the increase in K reduce the output of labor intensive good 1.

Then, the steady state moves to the lower right in (K, λ) space. But the price of good 1 rises to increase the output of labor intensive good 1 as well as in the previous case. Therefore, the rental rate falls below the steady state level although the capital stock is lower than the new steady state level. Thus, the economy diverges from the new steady state due to the reduction in L.

5 Concluding Remarks

In this paper, we have examined the relationship between dynamic stability and Giffen goods. Then, we have demonstrated that a steady state in a standard two-sector model can be a saddle point, even when one good becomes a Giffen good at the steady state. In contrast to static frameworks such as the Walrasian and Marshallian models, the existence of Giffen behavior does not affect the stability of the steady state equilibrium. Instead, we have shown that the magnitude of the income elasticity of demand for inferior goods is important for the stability. By doing so, the paper contributes to the broader literature on dynamic equilibria and provides a framework for interpreting the role of Giffen goods in growth models.

Appendix

We will prove Lemma 1 and verify the signs of the coefficients in (15) and (16).

First, totally differentiating of (6) yields

$$u_{11}dC_1 + u_{12}dC_2 = pd\lambda + \lambda dp,$$
$$u_{21}dC_1 + u_{22}dC_2 = d\lambda.$$

Then, we have

$$C_{1p} = \frac{\lambda u_{22}}{D}, \quad C_{1\lambda} = \frac{pu_{22} - u_{12}}{D}, \quad C_{2p} = -\frac{\lambda u_{12}}{D}, \quad \text{and} \quad C_{2\lambda} = \frac{u_{11} - pu_{12}}{D},$$

where $D = u_{11}u_{22} - (u_{12})^2 > 0$ due to Assumption 2. Also, we obtain

$$\begin{aligned} E_\lambda &= p\frac{pu_{22} - u_{12}}{D} + \frac{u_{11} - pu_{12}}{D} \\ &= \frac{u_{22}}{D}\left(p - \frac{u_{12}}{u_{22}}\right)^2 + \frac{1}{Du_{22}}. \end{aligned}$$

So, it can be easily shown that Lemma 1 holds.

Next, we verify the signs of the coefficients in (15) and (16).

Since $Z_{1p} = C_{1p} - Y_{1p}$ must be negative from Lemma 1 and $Y_{1p} > 0$, the coefficient of dK in (16) must be positive, while the signs of the coefficient of dK in (15) and that of $d\lambda$ in (16) are ambiguous. The coefficient of $d\lambda$ in (15) is also positive, because we see

$$\begin{aligned} -E_\lambda + \frac{\lambda(C_{1\lambda})^2}{Z_{1p}} &> -E_\lambda + \frac{\lambda(C_{1\lambda})^2}{C_{1p}} \\ &= -p\frac{pu_{22} - u_{12}}{D} - \frac{u_{11} - pu_{12}}{D} + \frac{(pu_{22} - u_{12})^2}{Du_{22}} \\ &= -\frac{1}{u_{22}} \\ &> 0. \end{aligned}$$

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Figure 1

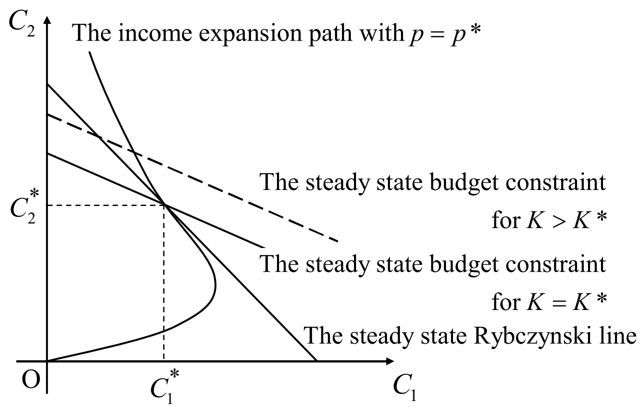

Figure 2

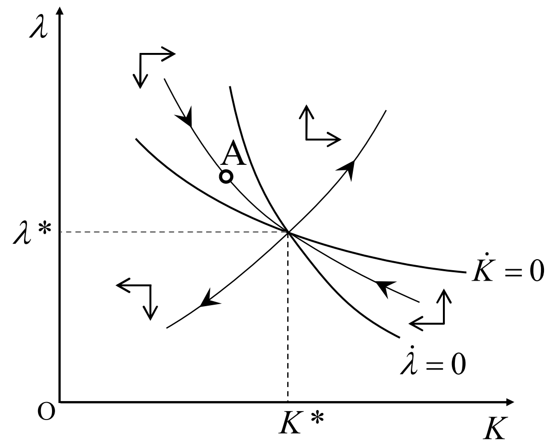

Figure 3

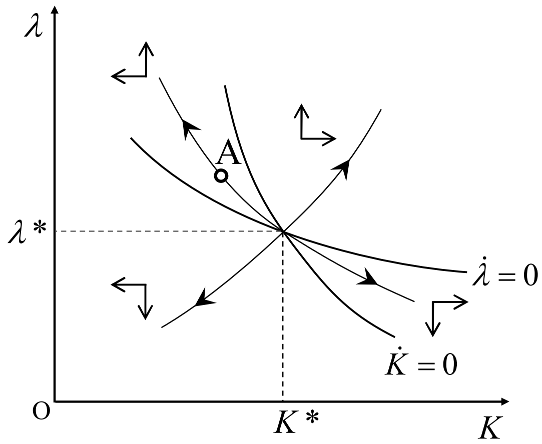

Figure 4