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Complementarities between Long-Term Relationships and Short-Term Contracts: Case of Early Modern Japan*

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Abstract

This paper examines the interaction between formal and relational enforcement in early modern Japan, focusing on financial relationships between *Daimyo* (regional lords) and merchants. Due to class distinctions, loans from merchants to *Daimyo* lacked legal enforceability, while contracts among merchants were court-enforceable. Some merchants built long-term self-enforcing relationships with *Daimyo* (becoming *Tachiiri*), whereas others provided short-term formal loans to underfunded *Tachiiri*. We develop a model with two markets—one that matches *Daimyo* with merchants, and the other that matches underfunded *Tachiiri* with lending merchants—and identify conditions for their co-existence in equilibrium. The analysis shows that merchants value becoming *Tachiiri* for long-term gains, and that the opportunities for short-term formal lending enhance the sustainability of relational contracts between *Daimyo* and *Tachiiri*.

JEL Classification Numbers: C73, D53, D83, D86, N25
Keywords: Relational contracts, Formal contracts, Matching markets, Financial relationships, Early modern Japan.

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1 Introduction

Relational contracts that are sustained by the value of future relationships are prevalent and play important roles, not only in developing or transition economies where legal protections are limited, but also in economies with well-developed legal systems. “The upshot is that private ordering is central to the performance of an economy whatever the conditions of lawfulness (Williamson, 2005, p.2).” When some but not all activities are formally enforced by the courts, formal and relational forms of contracts, governance, and enforcement can co-exist and interact with each other. While there is extensive theoretical literature that analyzes such interactions, empirical work providing convincing evidence is still limited (Gil and Zanarone, 2017, 2018).[1] Harris and Nguyen (2025) recently provide an empirical evidence that in the US truckload freight industry long-term relationships and short-term formal contracts co-exist and exhibit substitutes, consistent with the theoretical model of Kranton (1996). Are there cases where formal and informal forms of enforcement reinforce with each other?

In this paper we study financial contracting in the early modern period (from 17th until mid-19th centuries) in Japan to contribute to our further understanding of the interaction. The reason we focus on this particular period in Japan is that both formal and relational loan contracts co-existed because of the unique key feature of the Japanese society during this period: There existed a strict hierarchical class distinction between the ruling class (mostly warriors or *samurai*) and the ruled (farmers, merchants, townspeople).

While the Tokugawa shogunate ruled Japan, all the regions except for some domains that were directly controlled by the shogunate were governed by local loads (*Daimyo*). For reasons to be explained later, *Daimyo* had to rely on loans from merchants on regular basis. However, because of the class distinction, the loan contracts between *Daimyo* and merchants were unenforced by the shogunate. Some historical document cites examples of merchants who went bankrupt after lending money to *Daimyo* who defaulted on their loans, as well as examples of merchants who realized the riskiness of lending money to *Daimyo*. On the other hand, loan contracts between members of the ruled class were rigorously enforced by the courts established by the shogunate.

Some merchants established long-term financial relationships with *Daimyo* successfully. Such merchants were called *Tachiiri*. Other merchants took advantage of the formal enforcement by lending money to members of the ruled class, including *Tachiiri* when they could not secure funds to finance *Daimyo*. Why and when did merchants choose to lend money to *Daimyo* despite the lack of legal protection rather than to those of the ruled class through formal financial contracts? Why and when did *Daimyo* decide to repay the loan to *Tachiiri*? How did the opportunities for short-term formal loan contracting between merchants as lenders and *Tachiiri* as borrowers affect the *Daimyo*-*Tachiiri* relationships?

[1] Gil and Zanarone (2018, p.737) write, for example, as follows: “The analysis...is compelling, and yet we could not find any empirical study providing convincing related evidence.”

To answer these questions, we construct and analyze a model of *Daimyo*-*Tachiiri* relationships, along with two matching markets, one in which merchants are matched with *Daimyo*, and the other where they, as lenders, are matched with *Tachiiri*. There is a measure one of *Daimyo* and an infinite measure of merchants, both of whom live infinitely. At the beginning of each discrete period, *Daimyo* are either matched with *Tachiiri* or unmatched, and the latter enter the matching market to find merchants. The newly matched *Daimyo* must incur a fixed cost for contracting only in the first period of the relationship with the matched merchant.

Each merchant has funds available for financing with some probability, and is either matched with a *Daimyo* (that is, he is the *Tachiiri* of the *Daimyo*) or unmatched. *Tachiiri* with funds offer relational contracts to their *Daimyo*. Underfunded *Tachiiri* go to the short-term loan market to seek loans. Each unmatched merchant with funds chooses to participate in one of the three markets, the matching market to find a *Daimyo* and become the *Tachiiri*, the short-term loan market that matches him with an underfunded *Tachiiri*, and the exogenous outside market where he finances farmers and townspeople, for example. Unmatched merchants with no fund are inactive during the period and move to the next period.

In the baseline model we assume two matching markets are frictionless. In the *Daimyo* matching market, each merchant who is matched with a *Daimyo* starts their financial relationships by offering a relational contract that must be self-enforcing so that the *Daimyo* has incentives to repay the loan. The relationship is voluntarily dissolved if either the *Tachiiri* fails to finance *Daimyo* or the *Daimyo* who is able to repay the loan fails to do so. Their relationships may also be terminated by some exogenous shock such as natural disaster, business failure, and so on. In the short-term matching loan market, each matched lender offers a formal loan contract to the borrowing *Tachiiri*.

Based on the historical fact, we focus on the case in which there are less *Daimyo* than merchants in the *Daimyo* matching market, and derive sufficient conditions for the existence of a stationary equilibrium where two matching markets are viable. We first show that there is no stationary equilibrium in which both markets are viable if there are at least as many underfunded *Tachiiri* (borrowers) as lending merchants in the short-term loan market. We then consider the case where there are more lenders in the market and thus underfunded *Tachiiri* can find lenders for sure.

The sufficient conditions for the existence of a stationary equilibrium imply that involuntary separation is sufficiently unlikely to occur and the parties' discount factor is sufficiently high, and the fixed cost incurred by newly matched *Daimyo* is not too high nor too small. The stationary equilibrium is also more likely to exist as *Daimyo* are more productive and the outside market is less lucrative.

We then analyze the case in which the short-term loan matching market is not viable and show that the lack of the short-term loan market makes stationary equilibrium harder to exist, and provided it exists, becoming a *Tachiiri* is less attractive and the measure of *Daimyo*-*Tachiiri* relationships is smaller in the equilibrium than in the equilibrium with the viable short-term loan market.

Our model and analysis provide three main results. First, merchants find it attractive to become a *Tachiiri* whether or not the short-term loan market is viable, because of high future benefits from the relationship with *Daimyo* he is associated with. Second, the viable short-term loan market and the *Daimyo*-*Tachiiri* relationships exhibit complementarities, in the sense that the short-term loan market facilitates the existence of stationary equilibrium and helps the number of the relationships grow. Third, a stationary equilibrium is more likely to exist as involuntary separation is less likely to occur, the parties have more longer perspectives, *Daimyo* is more productive, and the outside market is less lucrative. We examine these results in the context of the early modern period of Japan, and argue that while we are unable to provide rigorous quantitative causal evidence, qualitative evidence is largely consistent with them.

Related Literature

Interactions between formal and relational contracts have been studied theoretically by Baker et al. (1994), Schmidt and Schnitzer (1995), Dixit (2004, Chapter 2), Battigalli and Maggi (2008), Kvaløy and Olsen (2009), Corts (2012), and Itoh and Morita (2015) among others, and surveyed by Malcomson (2013) and Gil and Zanarone (2017, 2018). In contrast to our model, they analyze a principal-agent relationship in isolation from markets. The interaction arises from the feature that contracts utilize both verifiable and unverifiable aspects of the relationship.

More closely related is the work that compares formal enforcement with informal enforcement in the setting with a large population of parties (see Wolitzky, 2025, for an overview). Earlier work such as Dixit (2004, Chapter 3) treats these forms of enforcement as substitutes (alternatives), and more recent work such as Acemoglu and Wolitzky (2020), Ali and Miller (2022), and Jackson and Xing (2024) show complementarities between formal and informal enforcement. All of these articles adopt variants of pairwise random matching games without long-term bilateral relationships. Acemoglu and Wolitzky (2020) introduce specialized formal enforcers, and analyze their incentives to engage in costly punishment. Ali and Miller (2022) analyze how incentives to communicate truthfully who has deviated are affected by formal and community enforcement. Jackson and Xing (2024) highlight what they call an incentive complementarity: Parties who have deviated in a formal market transaction are caught with some probability, and their names are made public and thus loose reputation and are ostracized in their communities. In contrast to these papers, we focus on interactions between long-term relationships and formal contracts in large populations, and enforcers' incentives, communication, and reputation play no role in our model. In particular, information concerning whether *Daimyo* and *Tachiiri* are separated voluntarily or not is not shared by other parties.

Long-term bilateral relationships in large populations have been analyzed, for example, by Shapiro and Stiglitz (1984), Ghosh and Ray (1996), and MacLeod and Malcomson (1998) in which the parties may voluntarily terminate their relationships. These articles only analyze informal forms of enforcement. Sobel (2006) analyzes formal and informal enforcement in a model in which a large population of players randomly form pairs and play the prisoner's dilemma. In his model, each pair can either continue or break off their relationship, but with some probability the value of the relationship decreases so much that it is efficient to terminate the current relationship and start a new one. Better formal enforcement may improve efficiency by discouraging continuation and making the relationship shorter. Kranton (1996) studies interactions between long-term relationships and formally enforced spot market exchange. In her model, if more parties choose market exchange, which makes the market thicker, it is more difficult to sustain relational contracts. If more parties choose long-term relationships, then the market becomes thinner and long-term relationships and market exchange can co-exist. Our model and analysis are distinct from both Sobel (2006) and Kranton (1996) in terms of the way two matching markets are interrelated, as well as our focus on complementarities.

In the field of economic history research, building on the seminal insight of North (1990), namely, that understanding the costs of third-party enforcement is indispensable for analyzing the crucial relationship between institutions, contract enforcement, and economic development, Greif (1989, 1993), Milgrom et al. (1990), and Greif et al. (1994) demonstrated through historical cases that, in the absence of formal state enforcement, contractual commitments in long-distance trade were maintained by private-order mechanisms such as merchant guilds and coalition-based arrangements (Hadfield, 2005). However, their studies did not provide evidence of situations in which public order and private order coexist and mutually influence each other. Our paper presents a concrete example from the financial markets of early modern Japan, analyzes the interplay between public order and private order, and offers an important case study for economic history research.

Roadmap

In Section 2 we summarize important information concerning Japan's financial market during the early modern period. In Section 3 we introduce our baseline model and derive stationary equilibrium in Section 4. The main results are presented in Section 5.

In Section 6 we consider several extensions of the baseline model to show that our main messages continue to hold with minor modifications. We introduce *Daimyo*'s hidden action so that the loan contract between *Daimyo* and *Tachiiri* must induce *Daimyo* to exert appropriate effort. We consider a random matching function that is more general than the frictionless matching in the baseline model. We relax the assumption in the baseline model that underfunded *Tachiiri* who are not repaid by *Daimyo* can pay back the loans to the lending merchants.

In the baseline model we assume that merchants are homogenous. This assumption helps us explain why some merchants who became *Tachiiri* experienced significant growth. However, there is also some evidence that the wealthy merchants tend to become *Tachiiri*. We thus in Section 7 introduce heterogeneity in terms of merchants' financing ability, and derive conditions for stationary equilibrium in which merchants enter the *Daimyo* matching market if and only if they are sufficiently likely to have funds for *Daimyo*. In Section 8 we summarize our main results and discuss future research.

In Appendix, we consider two alternative formulations of the baseline model. First, instead of infinitely lived merchants in the baseline model, we assume that they die with some probability and are replaced by newly born merchants. Second, we assume that building a new relationship takes time and thus it is costly to terminate the *Daimyo*-*Tachiiri* relationship, in contrast to the baseline model where we directly assume *Daimyo* incurs a fixed cost of starting a new relationship. We show that our main results continue to hold under these alternative formulations.

2 Background Information

Socio-Economic Structure of the Early Modern Japan

The model we describe below is inspired by the unique features of Japan's financial market during the early modern period. During this period (1603–1867), the Tokugawa shogunate held sway over Japan, with each region governed by a local lord (*Daimyo*). These lords, numbering in the hundreds, had the authority to rule specific domains. At the same time, the shogunate controlled its domains in various regions, including the large cities of Edo (now Tokyo), Osaka, Kyoto, and Nagasaki.

The shogun and *Daimyo* operated with independent financial systems. The shogun never directly imposed taxes on *Daimyo*. Each *Daimyo* as well as the shogunate, received tax payments in the form of rice from their domain. The primary income of *Daimyo* came from the rice tax collected from villages in autumn. They sent this annual tax rice to major cities like Osaka and sold it. However, expenses arose throughout the year, including extraordinary costs for dealing with disasters. As a result, they had no choice but to rely on external loans. At that time, Osaka was the most important financial market. Osaka merchants handled everything from rice acceptance to sales, payment management, and remittance, supporting the financial management of *Daimyo* (Takatsuki, 2022a, Chap.1).

Although merchants had provided financial services to *Daimyo* before this period, a significant change is known to have occurred in the early eighteenth century. For the ruling class, which relied on rice as tax revenue, a drop in the market price of rice meant less income. The early eighteenth century was exactly when this happened, and many *Daimyo* defaulted on their loans from merchants. However, merchants never filed civil lawsuits against *Daimyo*. This was because the shogun did not guarantee the fulfillment of contracts between *Daimyo* and merchants.

One of the most defining features of early-modern Japanese society was its clear class distinction. During the early modern period, a stark divide existed between the ruling class, predominantly composed of warriors (*samurai*) with executive power and military control, and the ruled, which included groups like farmers and townspeople. This class distinction also significantly influenced the financial market, particularly within the judicial system. The early modern period legal system's ramifications varied greatly depending on the classes of the lender and borrower. For instance, if *Daimyo* were to borrow money from a merchant and then default on the debt, the class dynamics were such that the shogunate would never order the *Daimyo* to make good on the payments, even if the lender submitted a complaint. Conversely, courts established by the shogunate enforced contracts between members of the subordinate classes. The courts in Osaka were primarily known for their quick judgments in favor of creditors, supporting economic transactions in Osaka, the largest commercial city at the time (Mandai, 2024a).

Relational Contracts between *Daimyo* and *Tachiiri*

The drop in rice prices during the early eighteenth century caused many *Daimyo* to face cash flow problems, making them refuse or delay repaying loans from merchants. However, the judicial system offered no protection in such cases. '*Chōnin Kōkenroku*,' established in 1718, is a document written by the head of the Mitsui Group, the largest business group at the time. It warns the descendants of the Mitsui family about the dangers of lending money to *Daimyo*. The document cites actual examples of Kyoto merchants who went bankrupt after lending money to *Daimyo* and defaulting on their loans. It says;

> Lending money to a local lord is like gambling. If you don't pull out once losses start, you'll try to recover them and fall into the *Daimyo*'s trap. No gambler begins expecting to lose. Lending money to townspeople won't bring much business, and even if they go bankrupt, you'll only get a small share of their assets through the courts. That's why merchants with capital tend to look for opportunities to lend to *Daimyo*. If you lend money to *Daimyo* and get repaid properly, it's an easy way to make money—if you keep good records, you can profit in your sleep. But every deal that seems too good to be true has hidden risks. This isn't just about dealing with *Daimyo*; it's a general rule to remember and always be cautious.

'*Chōnin Kōkenroku*' highlights the risks of lending money to *Daimyo*, but it only introduces one success story from Osaka.

> Among the moneylenders in Osaka, the most successful right now is Kōnoike Zen-e'mon. In recent years, many merchants who lent money to *Daimyo* have gone bankrupt one after another. Still, Kōnoike alone has managed his business skillfully and continues to increase his wealth.

Kōnoike and the merchants who learned from Kōnoike's methods established long-term relationships with *Daimyo*. While Kyoto merchants only lent money and did not sell *Daimyo*'s goods, Osaka merchants handled the sale of items *Daimyo* sent to Osaka, such as rice, sugar, and Japanese paper. Naturally, they recouped the loaned funds from the proceeds of these sales. Additionally, they offered full support for *Daimyo*'s financial management in Osaka, including providing fiscal advice (Takatsuki, 2024). Such merchants were called *Tachiiri*. *Tachiiri* literally means 'a merchant who enters the residence of *Daimyo*,' and was a term used specifically for merchants who were very close to *Daimyo*.

The term *Tachiiri* first appears in historical records in the mid-eighteenth century, which corresponds to the middle of the early modern period (Takatsuki, 2024). *Tachiiri* built long-term relationships with *Daimyo* and provided them with loans. They requested specific information about the financial situation of the *Daimyo* (such as rice harvests and spending plans). In return, they began offering loans at relatively low interest rates. They refused to lend money to *Daimyo*, who refused to provide such information or was unlikely to repay the loans. Of course, *Daimyo* were not legally obligated to repay these loans. However, maintaining honest dealings with merchants over the long term ultimately helped *Daimyo* reduce their capital costs. This is how merchants became deeply and permanently involved in *Daimyo* finances.

Some merchants who became *Tachiiri* experienced significant growth. Notable examples include Kōnoike, which became the precursor to the current Mitsubishi UFJ Bank, and Kajimaya Kyū-emon, which eventually led to Daido Life Insurance. However, the causal relationship between becoming *Tachiiri* and growing as a merchant is ambiguous. Many *Tachiiri* were already wealthy at the time when they became *Tachiiri*. In the 1863 ranking of Japan's wealthiest merchant houses, shown in Figure 1, 56 wealthy merchants were listed in the top tier, with 24 of them originating from Osaka, including Kōnoike Zen-e'mon and Kajimaya Kyū-emon. And at least 19 Osaka merchants of these 24 were well-known financial intermediates served for *Daimyo*.

Short-term Financial Market under the Shogunate's Jurisdiction

Since the mid-eighteenth century, several pairs of *Daimyo* and *Tachiiri* emerged, and it is known that relatively low-interest loan agreements began to be made within these pairs (Mori 1970). Recent research in economic history has shown that the *Daimyo* capable of forming such pairs were limited to those able to produce goods, such as high-quality rice, paper, sugar, candles, and so on, which generated steady cash flows in the Osaka market (Takatsuki, 2024). *Daimyo* borrowed funds to produce these goods from merchants in Osaka, especially *Tachiiri*. *Tachiiri* needed to fulfill *Daimyo*'s financial needs, and if he couldn't do so, he risked losing his position. Therefore, when *Daimyo* requested loans that *Tachiiri* couldn't cover with his own funds, *Tachiiri* sometimes raised money in the short-term financial market.

Merchants who were not *Tachiiri* did not lend directly to the local lord but instead loaned to

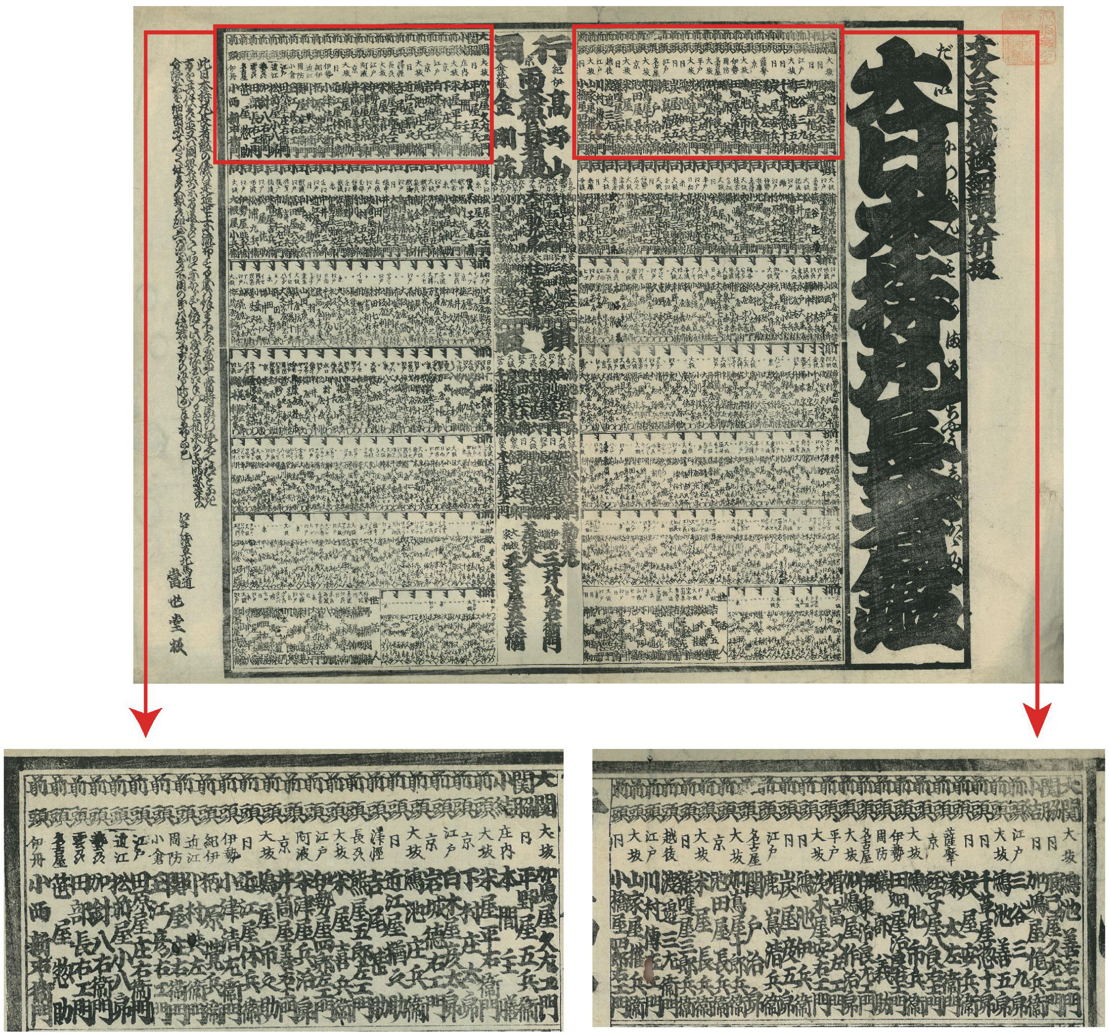

Figure 1: A ranking of wealthy merchant houses modeled after sumo wrestlers' rankings in 1863

Note: The original title of the ranking is '*Dainihon Mochimaru Chōja Kagami*'. The picture is reproduced with the kind permission of Osaka Chamber of Commerce and Industry. The top row on the list, surrounded by the right-side and left-side red frames, consists of 56 establishments that represent the most wealthy merchant houses. At the very right end of each side (the highest rank in sumo at the time, 'Ōzeki' (大関)) are the names Kōnoike Zen-e'mon (鴻池善右衛門) and Kajimaya Kyū-emon (加島屋久右衛門).

the *Tachiiri*, aiming to have the contract enforced by the shogun's court. As mentioned earlier, contracts between merchants in this short-term financial market were enforced by the shogun's judiciary. Therefore, this short-term financial market also involved merchants who were not specialized in finance. They provided loans to *Tachiiri* with an understanding of the special relationship between *Tachiiri* and *Daimyo*. While their loan agreements with *Tachiiri* were legally protected, if the relational contract between *Tachiiri* and *Daimyo* remained stable, loans to *Tachiiri* could be recovered smoothly without resorting to litigation. Figure 2 summarizes the structure of the financial market.

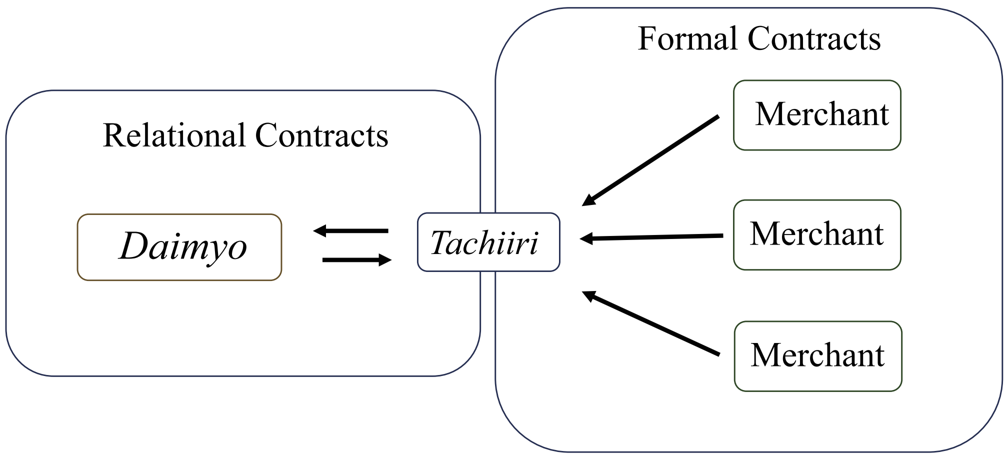

Figure 2: Structure of the financial market in Tokugawa Japan

3 Baseline Model

Players

Local lords (*Daimyo*) and merchants interact at each discrete period $t = 1, 2, \ldots$ with a common discount factor $\delta \in (0, 1)$. We assume that *Daimyo* are infinitely lived because there is not a single instance of a *Daimyo* being deposed by the Shogun for reasons like financial collapse. While we also assume that merchants are infinitely lived, we could alternatively assume that a fraction of existing merchants exit and are replaced by the same fraction of new merchants (see Section A1 in Appendix).

Daimyo

There is a measure 1 of *Daimyo*. At the beginning of every period each *Daimyo* (she)[2] has an investment project that requires funds $I > 0$. It yields return $R > 0$ with probability $p \in (0, 1)$ and

[2] While all the *Daimyo* were in fact males, we assume they are females for the purpose of identification only.

return 0 with probability $1-p$. We assume $pR > I$ and hence investment is efficient. This investment can be interpreted as referring to financing expenditures required up to the harvest period, or as an investment directed toward the cultivation or production of local specialty commodities. Examples of *Daimyo* obtaining loans from merchants to invest in commodity production include Satake's sericulture business, Uesugi's lacquer manufacturing, and Mōri's salt field development (Ito, 2011; Koseki, 2012; Kanamori, 2017).

The *Daimyo* has no initial wealth available for the project and hence must obtain loans from merchants. At the beginning of the period, each *Daimyo* is either matched or unmatched with a merchant. The merchant who is matched with a *Daimyo* is called *Tachiiri*. A *Daimyo* who does not have a *Tachiiri* participates in a market where she may be matched with a merchant (to be explained below). A *Daimyo* with a *Tachiiri*, who either finds a merchant in the matching market in the current period or has continued relationships from the previous periods, enters into a loan contract with the *Tachiiri* to borrow I.

Merchants

There is an infinite measure of merchants. At the beginning of every period, each merchant is either endowed with funds available for financing with probability $1 - q \in (0, 1)$, or no fund with probability q. Each merchant (he) is either matched with a *Daimyo* (that is, he has been the *Tachiiri* of the *Daimyo*) from the previous periods, or unmatched. The *Tachiiri* finances the *Daimyo* through a loan contract. If the *Tachiiri* is underfunded, he enters a short-term loan market to seek loans. The merchants other than *Tachiiri* decide to participate in one of the following markets, if they have funds available: *the Daimyo matching market* to be matched with a *Daimyo* and become the *Tachiiri* (and finance the matched *Daimyo*); *the short-term loan market* to become a lender if matched with an underfunded *Tachiiri*; and *the outside market*. The merchants who are not *Tachiiri* and have no fund are inactive and earn zero during the period, and move to the next period.

Markets and Contracts

In the model there are two matching markets, the *Daimyo* matching market and the short-term loan market, and the third, outside market. The merchants who participate in the outside market earn short-term benefit $\lambda > 0$. We assume $pR - I > \lambda$ to simplify the analysis. Below we explain each of the matching markets and contractual arrangements the participants agree on there.

Relational contracts in the *Daimyo* matching market

In the *Daimyo* matching market illustrated in Figure 3, each *Daimyo* is matched with a merchant with probability $\alpha_{\mathrm{D}} \in [0, 1]$, and each merchant is matched with a *Daimyo* and becomes a *Tachiiri* with probability $\alpha_{\mathrm{M}} \in [0,1]$. For simplicity, we assume one-to-one matching: No *Daimyo* is matched with more than one merchant, and no merchant is matched with more than one *Daimyo*. With the complementary probabilities ($1-\alpha_{\mathrm{D}}$ for the *Daimyo* and $1-\alpha_{\mathrm{M}}$ for the merchant) they are not matched, earn zero in the current period, and then move to the next period. We assume that these probabilities do not depend on the participants' past behavior because the financial condition of a *Daimyo* is insider information and is unobservable to those other than the *Tachiiri* of the *Daimyo*, and their past behavior is unlikely to be shared in the market. We aim at showing that a stationary equilibrium can exist without information sharing.

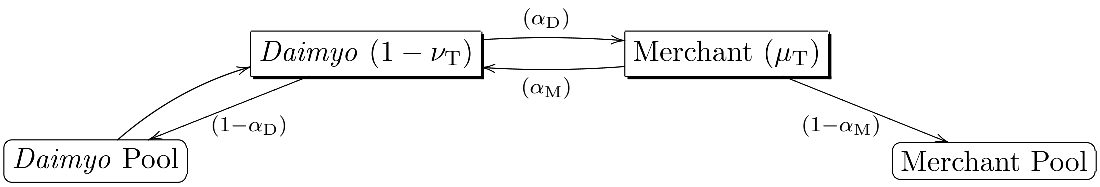

Figure 3: Transition in the *Daimyo* matching market

Note: $1-\nu_{\mathrm{T}}$ and μ_{T} are the measure of *Daimyo* without *Tachiiri* and that of merchants in the *Daimyo* matching market, respectively. Each *Daimyo* is matched with a merchant with probability α_{D}, and with probability $1-\alpha_{\mathrm{D}}$ she is not matched and moves to the next period. Each merchant is matched with a *Daimyo* with probability α_{M} and is not matched and moves to the next period with probability $1-\alpha_{\mathrm{M}}$.

We assume that the *Tachiiri*, who either is newly matched in the current period or has already been matched in the previous periods, offers a loan contract to the matched *Daimyo* in the take-or-leave-it fashion. The newly matched *Daimyo* has to spend plenty of time to negotiate with the *Tachiiri* concerning conditions for repayment such as timing, methods, and interest rate, as well as to prepare and submit a statement of future revenue and expenditure plan, by sending officers from their province to Osaka. The *Daimyo* also pays expenses for food and drink at restaurants where the negotiations are held. We thus assume that *Daimyo* incur a fixed cost $k > 0$ only in the first period of their relationships with *Tachiiri*. Compared with those costs, the costs incurred by the newly matched *Tachiiri* are small, and hence we simply ignore them.[3]

The *Tachiiri* and the *Daimyo* agree that the *Daimyo* pays R_{T} to the *Tachiiri* if and only if the project succeeds and generates R. Following the historical fact explained in Section 2, we assume that the outcome of the project is observable, but the contract is legally unenforceable, and must be a self-enforcing relational contract. They agree that the relationship is voluntarily dissolved if either the *Tachiiri* fails to finance the project[4] or the *Daimyo* fails to repay R_{T} despite project

[3]In Section A2 in Appendix, we analyze the model under an alternative assumption that neither *Daimyo* nor *Tachiiri* incur fixed costs, but building a new relationship takes time.

[4]This happens when and only when the *Tachiiri* from the previous period has no fund available and is not matched with a merchant in the short-term loan market.

success. Their relation is not voluntarily dissolved even if the project fails.

The relationship that is not dissolved voluntarily is terminated by some exogenous shock, such as abnormal climate, natural disaster, famine, business failure, and so on, that occurs with probability $s \in (0,1)$. The parties whose relationships are terminated voluntarily or involuntarily are unmatched at the beginning of the next period. Otherwise, they continue to be matched and move to the next period.

One case in which exogenous shock appears to have caused separation is that of merchant Masuya Hei-e'mon of Osaka, who was dismissed from his position as the *Tachiiri* of the Sendai Domain after experiencing a cash-flow shortfall in the aftermath of the Tempō famine in 1834 (Sato, 2020). In this case, both Masuya's limited financial capacity (related to q) and the sudden liquidity demand caused by the Tempō famine appear to have contributed to the separation. A case in which the relationship was dissolved for non-financial reasons is found in the early nineteenth century, when Kajimaya Sakubee, who had served as the Kumamoto Domain's *Tachiiri* for over half a century, was dismissed. His removal was neither due to a shortage of funds on the part of Kajimaya nor because the domain had secured another *Tachiiri* but seems to have been prompted by a memorandum submitted by a domain retainer criticizing him. Since he was reinstated shortly thereafter, the episode likely reflects the domain's temporary disfavor rather than any deterioration in his financial condition (Takatsuki, 2022b).

In our baseline model we consider the frictionless matching as follows. Let ν_{T} be the measure of *Daimyo* securing *Tachiiri*, which is equal to the measure of *Tachiiri*, and μ_{T} be the measure of merchants participating in the *Daimyo* matching market. Then if $1 - \nu_{\mathrm{T}} \leq \mu_{\mathrm{T}}$ holds, we define the matching probabilities as follows: $\alpha_{\mathrm{D}} = 1$ and $\alpha_{\mathrm{M}} = (1 - \nu_{\mathrm{T}})/\mu_{\mathrm{T}}$. If instead $1 - \nu_{\mathrm{T}} \geq \mu_{\mathrm{T}}$ holds, then the matching probabilities are defined as $\alpha_{\mathrm{D}} = \mu_{\mathrm{T}}/(1 - \nu_{\mathrm{T}})$ and $\alpha_{\mathrm{M}} = 1$. We show in Subsection 6.2 that the main results of the baseline model continue to hold under more general random matching.

Formal contracts in the short-term loan market

In the second matching market, borrowers (underfunded *Tachiiri*) and lenders, both are merchants, are matched. As shown in Figure 4, an underfunded *Tachiiri* is matched with a lender with probability $\beta_{\mathrm{T}} \in [0,1]$, and a lender is matched with an underfunded *Tachiiri* with probability $\beta_{\mathrm{L}} \in [0,1]$. Similar to the *Daimyo* matching market, we assume for simplicity that no borrower (lender) is matched with more than one lender (borrower, respectively). With probability $1 - \beta_{\mathrm{T}}$ a *Tachiiri* is not matched and cannot borrow, and hence the relation with the *Daimyo* is terminated. Both then earn zero in the current period, and move to the next period as unmatched. With probability $1 - \beta_{\mathrm{L}}$, a lender is not matched, earns zero in the current period, and moves to the next period as unmatched.

The matched lender offers a short-term loan contract to the matched, underfunded *Tachiiri*

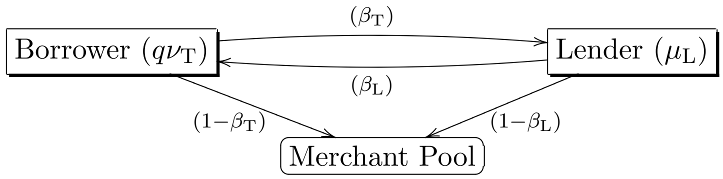

Figure 4: Transition in the short-term matching loan market

Note: $q\nu_{\mathrm{T}}$ and μ_{L} are the measure of underfunded *Tachiiri* and that of merchants in the short-term matching loan market, respectively. Each *Tachiiri* is matched with a lending merchant with probability β_{T}, and with probability $1-\beta_{\mathrm{T}}$ he is not matched, his relationship with the *Daimyo* is voluntarily terminated, and he moves to the next period. Each lending merchant is matched with an underfunded *Tachiiri* with probability β_{L} and is not matched and moves to the next period with probability $1-\beta_{\mathrm{L}}$.

in the take-or-leave-it fashion. Their short-term loan contract is legally enforceable. Note in this respect that it is not the *Daimyo* but the underfunded *Tachiiri* who seeks loans in the short-term market: If the *Daimyo* directly borrowed from a merchant other than the *Tachiiri*, then she would not repay the loan since their financial contract is not legally enforced, and hence merchants other than the *Tachiiri* would not choose to engage in financial contracting directly with the *Daimyo*.

Let R_{L} be the legally binding payment from the underfunded *Tachiiri* to the lender. For simplicity, we assume that *Tachiiri* who is not repaid by *Daimyo* in the same period can pay back R_{L}.[5] Similar to the *Daimyo* matching market, we consider the following frictionless matching. Let μ_{L} be the measure of merchants participating in the short-term loan matching market as lenders. Then if $\mu_{\mathrm{L}} \leq q\nu_{\mathrm{T}}$ holds, we define $\beta_{\mathrm{L}} = 1$ and $\beta_{\mathrm{T}} = \mu_{\mathrm{L}}/(q\nu_{\mathrm{T}})$. If instead $\mu_{\mathrm{L}} \geq q\nu_{\mathrm{T}}$, then we define $\beta_{\mathrm{L}} = q\nu_{\mathrm{T}}/\mu_{\mathrm{L}}$ and $\beta_{\mathrm{T}} = 1$.

Timing

The timing of each period goes as follows.

1. The fund availability of each merchant realizes.

2. Merchants with funds who are not *Tachiiri* decide which of three markets to participate in. *Daimyo* who do not have *Tachiiri* search for lenders. Underfunded *Tachiiri* search for short-term lenders.

3. Matching results are realized.

[5] We will relax this assumption later in Subsection 6.3 to show that the main results continue to be valid under an alternative assumption that *Tachiiri* cannot repay short-term debt when *Daimyo* did not pay to *Tachiiri*, and once a merchant falls behind in the repayment of short-term borrowings, he will not be able to take out any further short-term loans.

4. Each *Tachiiri* offers a contract to the matched *Daimyo* who decides whether or not to accept the contract. If she rejects the contract, their payoffs are zero in the period.

5. Each merchant who is matched with an underfunded *Tachiiri* offers a loan contract, and the *Tachiiri* decides whether or not to accept it. If the *Tachiiri* rejects the contract, the relationship with the *Daimyo* is dissolved, and the payoffs to the merchant and the *Tachiiri* are zero.

6. *Daimyo* who received loan execute projects.

7. Investment results are realized.

8. Each *Daimyo* decides whether or not to make the payment.

9. *Daimyo* and *Tachiiri* decide whether or not to dissolve relationship.

10. Some exogenous shock determines whether the relationship not voluntarily terminated is dissolved or not.

4 Stationary Equilibrium

In this section, we look for stationary equilibrium. In Subsection 4.1, we first consider the equilibrium in which two matching markets are both viable. We then consider the equilibrium in which the short-term loan matching market is not viable in Subsection 4.2. The main results are presented in the next section.

4.1 The Case of Two Viable Matching Markets

To study relational contracts between *Daimyo* and *Tachiiri*, denote by V_{D} the value of being a *Daimyo* without a *Tachiiri*, and by V_{T} the value of being a *Daimyo* with a *Tachiiri*, both evaluated at the beginning of each period. They are given as follows.

$$
\begin{aligned}
V_{\mathrm{D}} &= \alpha_{\mathrm{D}}\left\{p(R-R_{\mathrm{T}})-k+(1-s)\delta V_{\mathrm{T}}+s\delta V_{\mathrm{D}}\right\}+(1-\alpha_{\mathrm{D}})\delta V_{\mathrm{D}} \\
&= \alpha_{\mathrm{D}}v_k+\delta V_{\mathrm{D}}+\alpha_{\mathrm{D}}(1-s)\delta(V_{\mathrm{T}}-V_{\mathrm{D}}), \\
V_{\mathrm{T}} &= \{1-q(1-\beta_{\mathrm{T}})\}\{p(R-R_{\mathrm{T}})+(1-s)\delta V_{\mathrm{T}}\}+\{(1-q(1-\beta_{\mathrm{T}}))s+q(1-\beta_{\mathrm{T}})\}\,\delta V_{\mathrm{D}} \\
&= \phi v_0+\delta V_{\mathrm{D}}+\phi(1-s)\delta(V_{\mathrm{T}}-V_{\mathrm{D}}),
\end{aligned}
$$

where $v_0 = p(R-R_{\mathrm{T}})$, $v_k = p(R-R_{\mathrm{T}})-k = v_0-k$, and $\phi = 1-q(1-\beta_{\mathrm{T}})$. The last one is the probability that the *Tachiiri* has funds (either his own or borrowed from the matched merchant) and the *Daimyo* can exercise the project. V_{D} can be interpreted as follows. The expected payoff from the current period is $\alpha_{\mathrm{D}}v_k$. From the next period on, δV_{D} is assured, and their relationship extends to the next period with probability $\alpha_{\rm D}(1-s)$, and then the additional value is $\delta(V_{\rm T}-V_{\rm D})$. $V_{\rm T}$ can be interpreted in a similar fashion, following Figure 5 that illustrates transition in the *Daimyo-Tachiiri* relationship.

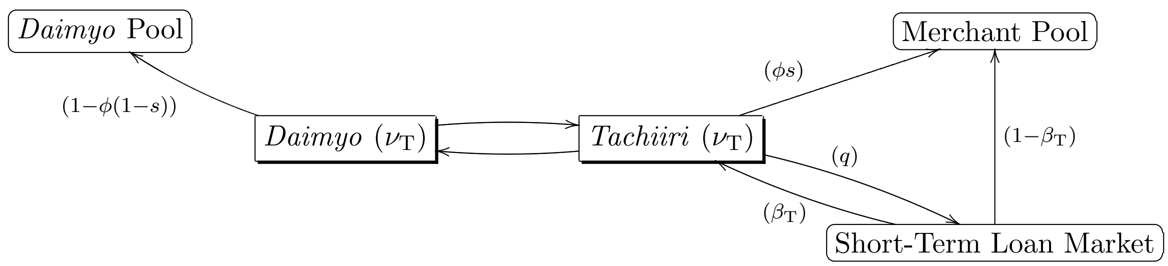

Figure 5: Transition in the *Daimyo-Tachiiri* relationship

Note: $\nu_{\rm T}$ is the measure of *Daimyo* with *Tachiiri* and hence equal to the measure of *Tachiiri.* Each *Tachiiri* is underfunded with probability q, and thus goes to the short-term matching loan market where he can borrow with probability $\beta_{\rm T}$. With probability $1-\beta_{\rm T}$, the *Tachiiri* is not matched, his relationship with the *Daimyo* is voluntarily terminated, and he moves to the next period. Each *Tachiiri* has enough fund with probability $1-q$, and hence he can offer a loan contract to the *Daimyo* with probability $\phi = 1-q(1-\beta_{\rm T})$. However, at the end of the period the relationship is involuntarily terminated with probability s. Each *Daimyo* goes to the *Daimyo* matching market in the next period with probability $q(1-\beta_{\rm T}) + (1-q(1-\beta_{\rm T}))s = 1-\phi(1-s)$.

Solving two equations given above, we obtain

$$\begin{aligned} V_{\rm T}-V_{\rm D} &= \frac{\phi v_0 - \alpha_{\rm D} v_k}{1+(\alpha_{\rm D}-\phi)(1-s)\delta}, \\ V_{\rm D} &= \delta V_{\rm D} + \alpha_{\rm D}\{v_k + (1-s)\delta(V_{\rm T}-V_{\rm D})\} = \frac{\alpha_{\rm D}}{1-\delta}\left[\frac{v_k+\phi(1-s)\delta k}{1+(\alpha_{\rm D}-\phi)(1-s)\delta}\right], \\ V_{\rm T} &= \delta V_{\rm D} + \phi\{v_0 + (1-s)\delta(V_{\rm T}-V_{\rm D})\}. \end{aligned}$$

The condition that the *Daimyo* chooses to repay the loan after the project succeeds is given by

$$R_{\rm T} \le (1-s)\delta(V_{\rm T}-V_{\rm D}) = \frac{(1-s)\delta(\phi v_0 - \alpha_{\rm D} v_k)}{1+(\alpha_{\rm D}-\phi)(1-s)\delta} = \frac{(1-s)\delta\left\{\alpha_{\rm D} k - (\alpha_{\rm D}-\phi)pR\right\}}{1+(\alpha_{\rm D}-\phi)(1-p)(1-s)\delta}.$$

Since the *Tachiiri* makes a take-it-or-leave-it offer, the amount of payment he proposes becomes

$$\text{[RT]} \quad R_{\rm T} = \frac{(1-s)\delta\left\{\alpha_{\rm D} k - (\alpha_{\rm D}-\phi)pR\right\}}{1+(\alpha_{\rm D}-\phi)(1-p)(1-s)\delta}.$$

Note $R_{\rm T}$ is increasing in ϕ: *Tachiiri* can demand more repayment if they are less likely to be short of money and unmatched to lenders.

The fixed cost of contracting, k, incurred by the *Daimyo* must satisfy the following conditions. First, since $V_{\mathrm{T}} > V_{\mathrm{D}}$ must hold for the *Daimyo* to pay R_{T} to *Tachiiri*,

$$k > \left[1 - \frac{\phi}{\alpha_{\mathrm{D}}}\right] pR \tag{1}$$

is necessary. Second, $V_{\mathrm{D}} \geq 0$ must hold for *Daimyo*'s participation in the *Daimyo* matching market, and hence

$$k \leq \overline{k} := \frac{1 + (\alpha_{\mathrm{D}} - \phi)(1-s)\delta}{\alpha_{\mathrm{D}} p(1-s)\delta + \{1 - \phi(1-s)\delta\}\{1 + (\alpha_{\mathrm{D}} - \phi)(1-p)(1-s)\delta\}} pR \tag{2}$$

is necessary.

We next consider the merchants. Let W_{M} be the value of being a merchant who is not a *Tachiiri*, and W_{T} be the value of being a *Tachiiri*, both evaluated at the beginning of each period. These are given as follows.

$$\begin{aligned} W_{\mathrm{M}} &= (1-q)\max\{X_{\mathrm{T}}, X_{\mathrm{L}}, X_{\mathrm{O}}\} + q\delta W_{\mathrm{M}}, \\ W_{\mathrm{T}} &= (1-q)\{pR_{\mathrm{T}} - I + (1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}\} \\ &\quad + q\beta_{\mathrm{T}}\{pR_{\mathrm{T}} - R_{\mathrm{L}} + (1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}\} + q(1-\beta_{\mathrm{T}})\delta W_{\mathrm{M}}, \end{aligned}$$

where X_{T}, X_{L}, and X_{O} are, given that the merchant has funds, the values of his participation in the *Daimyo* matching market, the short-term loan matching market as a lender, and the outside market, respectively, and are given as follows.

$$\begin{aligned} X_{\mathrm{T}} &= \alpha_{\mathrm{M}}(pR_{\mathrm{T}} - I + (1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}) + (1-\alpha_{\mathrm{M}})\delta W_{\mathrm{M}} \\ &= \alpha_{\mathrm{M}}\{pR_{\mathrm{T}} - I + (1-s)\delta(W_{\mathrm{T}} - W_{\mathrm{M}})\} + \delta W_{\mathrm{M}}, \\ X_{\mathrm{L}} &= \beta_{\mathrm{L}}(R_{\mathrm{L}} - I) + \delta W_{\mathrm{M}}, \\ X_{\mathrm{O}} &= \lambda + \delta W_{\mathrm{M}}. \end{aligned}$$

Assuming $X_{\mathrm{O}} = \max\{X_{\mathrm{T}}, X_{\mathrm{L}}, X_{\mathrm{O}}\}$ and solving for W_{M} and W_{T} yield

$$\begin{aligned} W_{\mathrm{M}} &= \frac{1-q}{1-\delta}\lambda, \\ W_{\mathrm{T}} &= \frac{(1-q)(pR_{\mathrm{T}} - I) - q\beta_{\mathrm{T}}(R_{\mathrm{L}} - pR_{\mathrm{T}}) - q(1-\beta_{\mathrm{T}})\delta\Delta - \{1 - q(1-\beta_{\mathrm{T}})\}s\delta\Delta}{1-\delta}, \end{aligned}$$

where

$$\Delta := W_{\mathrm{T}} - W_{\mathrm{M}} = \frac{(1-q)\{(pR_{\mathrm{T}} - I) - \lambda\} - q\beta_{\mathrm{T}}(R_{\mathrm{L}} - pR_{\mathrm{T}})}{(1-\delta) + q(1-\beta_{\mathrm{T}})\delta + \{1 - q(1-\beta_{\mathrm{T}})\}s\delta}.$$

The condition that the underfunded *Tachiiri* accepts a short-term loan contract offer is

$$\begin{aligned}
&pR_{\mathrm{T}} - R_{\mathrm{L}} + (1-s)\delta W_{\mathrm{T}} \geq (1-s)\delta W_{\mathrm{M}} \\
&\Leftrightarrow R_{\mathrm{L}} \leq pR_{\mathrm{T}} + \frac{(1-q)(1-s)\delta}{1-(1-q)(1-s)\delta}(pR_{\mathrm{T}} - I - \lambda).
\end{aligned}$$

Therefore, the lender offers the short-term loan contract with

$$[\mathrm{RL}] \quad R_{\mathrm{L}} = pR_{\mathrm{T}} + \frac{(1-q)(1-s)\delta}{1-(1-q)(1-s)\delta}(pR_{\mathrm{T}} - I - \lambda).$$

Substituting [RL] into Δ yields

$$\Delta = \frac{1-q}{1-(1-s)(1-q)\delta}(pR_{\mathrm{T}} - I - \lambda).$$

In order for *Tachiiri* not to voluntarily terminate the relationship, $\Delta \geq 0$, i.e., $pR_{\mathrm{T}} - I - \lambda \geq 0$ must hold. Taking this into consideration, [RL] shows that the lender obtains more than the *Tachiiri* receives from the *Daimyo* ($R_{\mathrm{L}} > pR_{\mathrm{T}}$), and instead of this, the *Tachiiri* obtains higher returns from future relationships ($\Delta = W_{\mathrm{T}} - W_{\mathrm{M}} > 0$). Using [RT], it is verified that $pR_{\mathrm{T}} - I - \lambda \geq 0$ is equivalent to

$$k \geq \underline{k} := \left[1 - \frac{\phi}{\alpha_{\mathrm{D}}}\right] pR + \frac{1 + (\alpha_{\mathrm{D}} - \phi)(1-p)(1-s)\delta}{\alpha_{\mathrm{D}} p(1-s)\delta}(I + \lambda). \tag{3}$$

This implies (1) is redundant.

The equilibrium also requires $X_{\mathrm{T}} = X_{\mathrm{O}}$ and $X_{\mathrm{L}} = X_{\mathrm{O}}$. These conditions lead to

$$\begin{aligned}
[\mathrm{OT}] \quad \lambda &= \alpha_{\mathrm{M}}[(pR_{\mathrm{T}} - I) + (1-s)\delta\Delta] \\
&= \alpha_{\mathrm{M}}\left[(pR_{\mathrm{T}} - I) + \frac{(1-q)(1-s)\delta}{1-(1-q)(1-s)\delta}\{(pR_{\mathrm{T}} - I) - \lambda\}\right],
\end{aligned}$$

and

$$\begin{aligned}
[\mathrm{OL}] \quad \lambda &= \beta_{\mathrm{L}}(R_{\mathrm{L}} - I) \\
&= \beta_{\mathrm{L}}\left[(pR_{\mathrm{T}} - I) + \frac{(1-q)(1-s)\delta}{1-(1-q)(1-s)\delta}\{(pR_{\mathrm{T}} - I) - \lambda\}\right],
\end{aligned}$$

respectively. They imply that $\alpha_{\mathrm{M}} = \beta_{\mathrm{L}}$ must hold: The merchant's probability of being matched with a *Daimyo* in the *Daimyo* matching market and that of being matched with an underfunded *Tachiiri* in the short-term matching market must be the same. Note that $\alpha_{\mathrm{M}} = \beta_{\mathrm{L}}$ is an artifact of the baseline model: Since in the short-term loan market the lender has all the bargaining power and thus the rent the borrower (underfunded *Tachiiri*) obtains from his relationship with the *Daimyo* is extracted. Then the payoff to the merchant given he is matched in the short-term market and that to the merchant given he is matched in the *Daimyo* matching market are equal, and thus the matching probabilities must be equal as well.[6]

The above equality condition is rewritten as

$$\frac{\mu_{\mathrm{L}}}{\mu_{\mathrm{T}}} = \frac{q\nu_{\mathrm{T}}}{1-\nu_{\mathrm{T}}}.$$

It implies that the ratio of merchants participating in the short-term loan market as lenders to those participating in the *Daimyo* matching market to become *Tachiiri* must be equal to the ratio of *Tachiiri* participating in the short-term loan market as borrowers to *Daimyo* participating in the *Daimyo* matching market.

Finally, the following stationary condition must hold.

$$[q(1-\beta_{\mathrm{T}}) + \{1 - q(1-\beta_{\mathrm{T}})\}s]\,\nu_{\mathrm{T}} = \alpha_{\mathrm{D}}(1-s)(1-\nu_{\mathrm{T}}).$$

The left-hand side represents the measure of *Daimyo* with *Tachiiri* whose relationships terminate voluntarily or involuntarily (outflow), and the right-hand side is the measure of *Daimyo* without *Tachiiri* who are matched and continue the relationships in the next period (inflow). Solving this, we obtain

$$\text{[S]} \quad \nu_{\mathrm{T}} = \frac{\alpha_{\mathrm{D}}(1-s)}{\alpha_{\mathrm{D}}(1-s) + q(1-\beta_{\mathrm{T}}) + \{1-q(1-\beta_{\mathrm{T}})\}s}.$$

A stationary equilibrium in the case of viable short-term loan market is defined as $(V_{\mathrm{D}}^*, V_{\mathrm{T}}^*, W_{\mathrm{M}}^*, W_{\mathrm{T}}^*, X_{\mathrm{T}}^*, X_{\mathrm{L}}^*, X_{\mathrm{O}}^*, R_{\mathrm{T}}^*, R_{\mathrm{L}}^*, \alpha_{\mathrm{D}}^*, \alpha_{\mathrm{M}}^*, \beta_{\mathrm{T}}^*, \beta_{\mathrm{L}}^*, \nu_{\mathrm{T}}^*, \mu_{\mathrm{T}}^*, \mu_{\mathrm{L}}^*)$ satisfying the above conditions.

4.2 The Case of No Viable Short-Term Loan Market

We next consider the equilibrium without the short-term loan matching market. The underfunded *Tachiiri* then cannot finance the *Daimyo*, and thus their relationship is voluntarily terminated. The value of being a *Daimyo* without a *Tachiiri*, and that of being a *Daimyo* with a *Tachiiri* are given as follows.

$$V_{\mathrm{D}} = \alpha_{\mathrm{D}} v_k + \delta V_{\mathrm{D}} + \alpha_{\mathrm{D}}(1-s)\delta(V_{\mathrm{T}} - V_{\mathrm{D}}),$$
$$V_{\mathrm{T}} = \phi v_0 + \delta V_{\mathrm{D}} + \phi(1-s)\delta(V_{\mathrm{T}} - V_{\mathrm{D}}).$$

Note that the essential difference between these values and the corresponding values in the case in which the short-term market is viable is that the probability of the *Tachiiri* having funds changes

[6] The equality also depends on other assumptions of the baseline model such that the underfunded *Tachiiri* who is not repaid by the *Daimyo* can pay back the loan, and merchants are homogeneous in terms of their financial ability. See Subsection 6.3 and Section 7, respectively.

from $\phi = 1 - q(1-\beta_\mathrm{T})$ to $\phi = 1 - q$, that is, the case in this subsection coincides with the case in the previous subsection with $\beta_\mathrm{T} = 0$. We thus obtain

$$V_\mathrm{T} - V_\mathrm{D} = \frac{\phi v_0 - \alpha_\mathrm{D} v_k}{1 + (\alpha_\mathrm{D} - \phi)(1-s)\delta}.$$

By the condition that the *Daimyo* repays the loan when the project succeeds, we obtain

$$\text{[RT]} \quad R_\mathrm{T} = \frac{(1-s)\delta\{\alpha_\mathrm{D} k - (\alpha_\mathrm{D} - \phi) pR\}}{1 + (\alpha_\mathrm{D} - \phi)(1-p)(1-s)\delta}.$$

The fixed cost must satisfy (2) and (3).

The value functions for merchants are given as follows.

$$\begin{aligned} W_\mathrm{M} &= (1-q)\max\{X_\mathrm{T}, X_\mathrm{O}\} + q\delta W_\mathrm{M}, \\ W_\mathrm{T} &= (1-q)\{pR_\mathrm{T} - I + (1-s)\delta W_\mathrm{T} + s\delta W_\mathrm{M}\} + q\delta W_\mathrm{M}, \\ X_\mathrm{T} &= \alpha_\mathrm{M}\delta W_\mathrm{T} + (1-\alpha_\mathrm{M})\delta W_\mathrm{M}, \\ X_\mathrm{O} &= \lambda + \delta W_\mathrm{M}. \end{aligned}$$

Repeating the analysis similar to the previous subsection, we can obtain

$$\begin{aligned} W_\mathrm{M} &= \frac{1-q}{1-\delta}\lambda, \\ W_\mathrm{T} &= \frac{(1-q)(pR_\mathrm{T} - I) - q\delta\Delta - (1-q)s\delta\Delta}{1-\delta}, \end{aligned}$$

where

$$\Delta = \frac{1-q}{1-(1-q)(1-s)\delta}(pR_\mathrm{T} - I - \lambda).$$

Condition [OT] is the same as the corresponding one in the previous subsection.

The stationary condition is

$$\{q + (1-q)s\}\,\nu_\mathrm{T} = \alpha_\mathrm{D}(1-s)(1-\nu_\mathrm{T}),$$

and therefore, we obtain the measure of *Daimyo-Tachiiri* relationships, ν_T, as

$$\text{[S]} \quad \nu_\mathrm{T} = \frac{\alpha_\mathrm{D}(1-s)}{\alpha_\mathrm{D}(1-s) + q + (1-q)s}.$$

A stationary equilibrium in the case of non-viable short-term loan market is defined as $(\hat{V}_\mathrm{D}, \hat{V}_\mathrm{T}, \hat{W}_\mathrm{M}, \hat{W}_\mathrm{T}, \hat{X}_\mathrm{T}, \hat{X}_\mathrm{O}, \hat{R}_\mathrm{T}, \hat{\alpha}_\mathrm{D}, \hat{\alpha}_\mathrm{M}, \hat{\nu}_\mathrm{T}, \hat{\mu}_\mathrm{T})$ satisfying the above conditions.

5 Basic Results

Based on the historical fact, we focus on the stationary equilibrium in which there are less *Daimyo* than merchants in the *Daimyo* matching market, that is, $1-\nu_{\mathrm{T}}^{*}<\mu_{\mathrm{T}}^{*}$ and $1-\hat{\nu}_{\mathrm{T}}<\hat{\mu}_{\mathrm{T}}$. In this case the matching probabilities are specified as $\alpha_{\mathrm{D}}^{*}=\hat{\alpha}_{\mathrm{D}}=1$, $\alpha_{\mathrm{M}}^{*}=(1-\nu_{\mathrm{T}}^{*})/\mu_{\mathrm{T}}^{*}<1$, and $\hat{\alpha}_{\mathrm{M}}=(1-\hat{\nu}_{\mathrm{T}})/\hat{\mu}_{\mathrm{T}}<1$: While *Daimyo* can always find merchants, some merchants may not be matched with *Daimyo* and wait for the market in the next period.

5.1 The Case of Two Viable Matching Markets

We first show that if there are at least as many underfunded *Tachiiri* (borrowers) as lending merchants in the short-term loan market, implying $\beta_{\mathrm{L}}^{*}=1$, there is no stationary equilibrium in which both matching markets are viable. To see this, suppose $\beta_{\mathrm{L}}^{*}=1$ holds. Then [OT] and [OL] require that $\alpha_{\mathrm{M}}^{*}=\beta_{\mathrm{L}}^{*}=1$ must hold, that contradicts $\alpha_{\mathrm{M}}^{*}<1$. In other words, if $\beta_{\mathrm{L}}^{*}=1$, the short-term loan matching market has become quite a lucrative market for lenders. Then if $\alpha_{\mathrm{M}}^{*}<1$, merchants would not choose to become *Tachiiri*.

Lemma 1. *Suppose there are less Daimyo than merchants in the Daimyo matching market ($1-\nu_{\mathrm{T}}^{*}<\mu_{\mathrm{T}}^{*}$). Then if $\mu_{\mathrm{L}}^{*}\leq q\nu_{\mathrm{T}}^{*}$, there is no stationary equilibrium in which both markets are viable.*

Based on Lemma 1, from now on, we consider the case in which there are more lenders than underfunded *Tachiiri* in the short-term loan market. Then $\beta_{\mathrm{T}}^{*}=1$ and $\beta_{\mathrm{L}}^{*}=q\nu_{\mathrm{T}}^{*}/\mu_{\mathrm{L}}^{*}<1$ hold: Underfunded *Tachiiri* can always find lenders, which implies $\phi^{*}=1$: The *Tachiiri* can always have funds for the *Daimyo* (either his own or those borrowed from the matched merchant).

Substituting $\alpha_{\mathrm{D}}^{*}=\phi^{*}=1$ yields $V_{\mathrm{T}}^{*}-V_{\mathrm{D}}^{*}=k$ and

$$[\mathrm{RT}]\quad R_{\mathrm{T}}^{*}=(1-s)\delta k.$$

The value of *Daimyo* with *Tachiiri* is higher than that without *Tachiiri* exactly by k, the fixed cost of contracting, and to induce the *Daimyo* to replay the loan, the *Tachiiri* can demand at most the expected future loss incurred by the *Daimyo* who reneges on the loan contract. Then the condition (2) boils down to

$$k\leq\overline{k}=\frac{1}{1-(1-p)(1-s)\delta}pR. \tag{4}$$

Under a stationary equilibrium, both [OT] and [OL] must hold, which become, respectively,

$$[\mathrm{OT}]\quad \lambda=\frac{\alpha_{\mathrm{M}}^{*}}{1-(1-q)(1-\alpha_{\mathrm{M}}^{*})(1-s)\delta}(pR_{\mathrm{T}}^{*}-I),$$

$$[\mathrm{OL}]\quad \lambda=\frac{\beta_{\mathrm{L}}^{*}}{1-(1-q)(1-\beta_{\mathrm{L}}^{*})(1-s)\delta}(pR_{\mathrm{T}}^{*}-I).$$

For these condition to be satisfied, $pR^*_{\mathrm{T}} = p(1-s)\delta k > I$ is necessary. Furthermore, $\alpha^*_{\mathrm{M}} = \beta^*_{\mathrm{L}} \in (0,1)$ satisfying [OT] and [OL] must exist. Under $p(1-s)\delta k - I > 0$, the right-hand side of [OT] and [OL] is increasing in α_{M} and β_{L} and smaller than λ as $\alpha_{\mathrm{M}} \to 0$ and $\beta_{\mathrm{L}} \to 0$, respectively. The right-hand side becomes larger than λ as $\alpha_{\mathrm{M}} \to 1$ and $\beta_{\mathrm{L}} \to 1$ if and only if (3) with strict inequality, or

$$k > \underline{k} = \frac{I+\lambda}{p(1-s)\delta} \tag{5}$$

is satisfied.

Note that if (5) holds, then $p(1-s)\delta k - I > 0$ is also satisfied. A stationary equilibrium exists if the fixed cost of contracting k satisfies both (4) and (5). The following proposition provides sufficient conditions for such k to exist.

Proposition 1. *Suppose that less Daimyo than merchants are in the Daimyo matching market. There exists $\overline{s} \in (0,1)$ such that if $s < \overline{s}$, then there exists $\underline{\delta} \in (0,1)$ such that k satisfying both (4) and (5) exists for $\delta > \underline{\delta}$.*

Proof. The conclusion is true if $\Delta_k := \overline{k} - \underline{k} > 0$. Δ_k goes to $-\infty$ as $\delta \to 0$, and is increasing in δ. As $\delta \to 1$, it approaches to

$$\frac{1}{1-(1-p)(1-s)} pR - \frac{1}{p(1-s)}(I+\lambda), \tag{6}$$

which is decreasing in s and goes to $-\infty$ as $s \to 1$. As $s \to 0$, it approaches to

$$\frac{1}{p}(pR - I - \lambda) > 0.$$

Therefore, there exists $\overline{s} \in (0,1)$ such that (6) becomes positive for $s < \overline{s}$. Then for such s there exists $\underline{\delta} \in (0,1)$ such that $\Delta_k > 0$ for $\delta > \underline{\delta}$. □

The proposition shows that sufficient conditions for the existence of a stationary equilibrium are that involuntary separation is sufficiently unlikely to occur and the parties have suffciently long-term perspectives. Furthermore, it is easy to verify that Δ_k is increasing in p and R, and decreasing in I and λ. Hence the stationary equilibrium in which both matching markets are viable is more likely to exist as *Daimyo*'s project is more productive and the outside market is less lucrative.

Along with $\beta^*_{\mathrm{T}} = 1$, the measure of *Tachiiri* becomes

$$\text{[S]} \quad \nu^*_{\mathrm{T}} = 1 - s.$$

Since the *Tachiiri* can always finance the *Daimyo* in the equilibrium, the measure of the *Daimyo-Tachiiri* relationship is equal to the probability that exogenous separation does not occur. Then the ratio of merchants participating in the short-term loan matching market as lenders to those participating in the *Daimyo* matching market becomes

$$\frac{\mu_{\rm L}^*}{\mu_{\rm T}^*} = \frac{q(1-s)}{s}.$$

implying that the relative measure of merchants in the short-term loan market is decreasing in the probability of exogenous shock (s), and increasing in the probability that the *Tachiiri* is short of funds (q).

Note that q does not appear in Δ_k, and hence has no effect on the existence of stationary equilibrium since under the assumption of the frictionless matching, underfunded *Tachiiri* can always find lenders in the short-term loan market ($\beta_{\rm T}^* = 1$). [7]

Note that [OT] and [OL] reveal that the matching probabilities of merchants in two matching markets satisfying these conditions must increase with q: A higher q implies that *Tachiiri* are more likely to be underfunded and the measure of borrowers in the short-term loan market is higher. Hence lending merchants are more likely to be matched, and going to that market becomes more attractive. Then in equilibrium, entering the *Daimyo* matching market must be more attractive as well.

5.2 The Case of No Viable Short-Term Loan Market

We next consider the case in which the short-term loan market is not viable. We continue to focus on the stationary equilibrium in which there are less *Daimyo* than merchants in the *Daimyo* matching market: $1 - \hat{\nu}_{\rm T} < \hat{\mu}_{\rm T}$, and thus $\hat{\alpha}_{\rm D} = 1$ and $\hat{\alpha}_{\rm M} = (1 - \hat{\nu}_{\rm T})/\hat{\mu}_{\rm T}$.

As we demonstrated in Subsection 4.2, the lack of the short-term loan matching market reduces the probability that *Tachiiri* have funds from $\phi^* = 1$ to $\hat{\phi} = 1 - q$, and hence the amount of repayment offered by *Tachiiri* when the project succeeds becomes

$$\text{[RT]} \quad \hat{R}_{\rm T} = \frac{(1-s)\delta(k - pqR)}{1 + (1-p)(1-s)q\delta},$$

which is smaller than $R_{\rm T}^*$. Then $\hat{W}_{\rm T}$, the value of being a *Tachiiri*, is smaller than $W_{\rm T}^*$, and hence $\hat{\Delta} < \Delta^*$.

Since *Tachiiri* are more likely to be short of money without the short-term loan market, *Daimyo* would deviate if *Tachiiri* demand a higher amount of payment. The fixed cost of contracting then has to satisfy $k > pqR$ and

$$k \leq \frac{1 + (1-s)q\delta}{p(1-s)\delta + \{1 - (1-q)(1-s)\delta\}\{1 + (1-p)(1-s)q\delta\}} pR \tag{7}$$

[7] We show in Subsection 6.2 that under general random matching q must be sufficiently small for the existence of stationary equilibrium with two viable markets.

The right-hand side of (7) is smaller than $\overline{k}$. Furthermore, [OT] and [OL] require $p\hat{R}_{\mathrm{T}} - I > 0$, and since $\hat{R}_{\mathrm{T}}$ is smaller than $R^*_{\mathrm{T}} = (1-s)\delta k$, the conditions on k for a stationary equilibrium to exist are more difficult to be satisfied.

Provided that a stationary equilibrium exists, the benefit from becoming *Tachiiri* decreases from Δ^* to $\hat{\Delta}$. Furthermore, the measure of *Daimyo*-*Tachiiri* relationships becomes

$$\hat{\nu}_{\mathrm{T}} = \frac{1-s}{1+q(1-s)},$$

which is smaller than ν^*_{T}.

Proposition 2. *Suppose that less Daimyo than merchants are in the Daimyo matching market. If a stationary equilibrium without the short-term loan market exists, then a stationary equilibrium with two viable matching markets also exist, but not vice versa. Provided that both equilibria exist,* $R^*_{\mathrm{T}} > \hat{R}_{\mathrm{T}}$*, which implies* $\Delta^* > \hat{\Delta}$ *and* $W^*_{\mathrm{T}} > \hat{W}_{\mathrm{T}}$*, as well as* $\nu^*_{\mathrm{T}} > \hat{\nu}_{\mathrm{T}}$*.*

5.3 Discussion

Our analysis of the baseline model provides three main results. First, merchants find it attractive to become *Tachiiri*, whether or not the short-term loan market is viable, because of high future benefits from the relationship ($\Delta^* > 0$ and $\hat{\Delta} > 0$). An example showing how attractive lending to *Daimyo* is to merchants is the entry of Konishi Shin-emon, a sake brewer from Itami whose enterprise still exists today. After achieving the pinnacle of success in the sake industry, Konishi sought to become a *Tachiiri* to the Kumamoto Domain. According to Takatsuki (2021a), he offered the clan a donation of fourteen storehouses and 150 kan of silver—equivalent to roughly 150 million yen in contemporary terms—for this purpose.

The large value generated from *Daimyo*-*Tachiiri* relationships can be illustrated by the case of the Tsuwano Domain and its *Tachiiri* merchant, Kajimaya Kyū-emon (hereafter *Kakyu*). The main provisions of their relational contract concluded in 1770 were as follows:

- All proceeds from the sale of handmade paper and candles (local products from the Tsuwano Domain) were to be deposited with *Kakyu*, who would deduct from these funds the repayment amounts owed by the Tsuwano Domain (both principal and interest). Any surplus funds would be retained by *Kakyu* and invested at an annual interest rate of 6%.

- Aside from debt repayments, all necessary expenditures of the Tsuwano Domain were to be made through *Kakyu*.

One of the advantages of becoming a *Tachiiri* was the acquisition of the right to sell the domain's specialty products. The terms of the contract between Tsuwano and *Kakyu* indicate that such (exclusive or oligopolistic) sales rights were granted primarily as collateral for the repayment of principal and interest. While in cases where a surplus remained after such payments, that surplus was credited to the domain's profit, the surplus was, at least initially, "deposited" in the account of the *Tachiiri*. The fact that sales from speciality goods and annual tax rice proceeds were first transferred to the *Tachiiri*'s account meant not only that repayment of principal and interest was assured, but also that the merchant enjoyed regular, substantial cash inflows. This offered significant managerial benefits for a *Tachiiri*, as it made it easier to secure repayment resources in the short-term credit market. Furthermore, control over sales proceeds also meant that the merchant could, to some extent, control the expenditures of the domain (and thereby impose fiscal discipline)—function only *Tachiiri* merchants were in a position to perform.

In summary, the benefits obtained by a *Tachiiri* merchant lay in two points: (i) securing a position that ensured more reliable interest income from the domain in the future, and (ii) obtaining regular, sizable cash inflows (which in turn strengthened repayment capacity in the short-term credit market).

Second, the viable short-term loan market and the *Daimyo-Tachiiri* relationships exhibit complementarities. The short-term loan market facilitates the existence of a stationary equilibrium with two viable matching markets, and helps the number of the relationships grow.

It is in general difficult to identify the period where the short-term loan market was absent, and hence to offer clear evidence of the effects of that market on the *Daimyo-Tachiiri* relationships. As we cited in Section 2, '*Chōnin Kōkenroku*,' compiled in 1718, warns of the dangers inherent in *Tachiiri* merchants borrowing third-party capital from the short-term loan market. This implies that while such a market already existed by the early eighteenth century, in this period merchants in Kyoto were unable to physically secure annual tax rice or specialty products, due to geographic constraints, and their relationships with *Daimyo* were weak and fragile as numerous bankruptcies of merchants are recorded in the '*Chōnin Kōkenroku*'. It is this period when Kōnoike Zen-e'mon of Osaka started growing by carefully constructing relational contracts and physically securing annual tax rice and specialty products.

From the mid-eighteenth century onward, records concerning the short-term loan market become more frequent in the historical sources. For example, in 1754, there is a documented case in which an outside merchant extended credit to an official purveyor (*goyotashi*) of the Saga Domain—who may also be regarded as a *Tachiiri*—using as collateral land belonging to the Saga Domain's Osaka warehouse, which was owned by the purveyor. Furthermore, a town ordinance (*machibure*) issued by the Osaka town magistrates in 1780 stipulates that, in the short-term laon market, when funds were lent to a *Tachiiri* with rice bills as collateral, such bills were to be invariably protected by the Osaka magistracy. This example represents the first instance in which the shogunate explicitly identified the two-tier structure of the *Daimyo* financial market. By this time, it appears that such a structure had become considerably widespread. It is also highly likely that these government policies served to stimulate further activity in the short-term loan market.

It is this period for the development of the short-term loan market when *Daimyo-Tachiiri* relationships expanded as well. Small and medium-sized Osaka money changers also entered the *Daimyo* financial market by incorporating third-party capital (Nakagawa, 2003). Merchants from other industries—such as drapers and sake brewers—entered the *Daimyo* lending market (Takatsuki, 2021b). It suggests that the development of the short-term loan market and that of the *Daimyo-Tachiiri* relationships went hand in hand.

The third result from the analysis of the baseline model is that a stationary equilibrium is more likely to exist as involuntary separation is less likely to occur, the parties have more long-term perspectives, *Daimyo* is more productive, and the outside market is less lucrative.

While we have explained in Section 3 the case of merchant Masuya Hei-e'mon, as an example of separation caused by exogenous shock, the fact that this is virtually the only such case identifiable in the sources suggests that the probability of exogenous separation was likely to be sufficiently small.

It was a common refrain among Osaka merchants that, unlike the warrior class, merchant households lacked stable sources of income and lived with persistent uncertainty, so that any deferment of principal-and-interest payments was a matter of life and death. It suggests that δ is unlikely to be small, at least for merchants.

In the pre-industrial world, it was difficult to obtain expected returns in other businesses (the outside market) that exceeded those from lending to *Daimyo*. This point is noted in the *Chonin kkenroku*, which emphasizes, precisely for this reason, there was no shortage of individuals entering the *Daimyo*-lending business despite being fully aware of its risks.

6 Extensions

In this section, we extend the baseline model to several directions, to show that our basic messages continue to be valid with minor modifications. In Subsection 6.1 we introduce *Daimyo*'s effort which is unobservable to merchants, and hence the loan contract between *Daimyo* and *Tachiiri* must induce *Daimyo* to exert appropriate effort. We show that the results of the baseline model continue to hold unless the hidden action problem is sufficiently serious. In Subsection 6.2 we consider a random matching function that is more general than the frictionless matching in the baseline model. In Subsection 6.3 we relax the assumption in the baseline model that underfunded *Tachiiri* who are not repaid by *Daimyo* can pay back the loans to the lending merchants.

6.1 *Daimyo*'s Hidden Action

In the baseline model, *Daimyo* do not exert effort to increase the success probability of their projects. In this subsection we introduce hidden action of each *Daimyo* as an important ingredient for the success of the project.[8]

The *Daimyo* chooses effort $e \in \{0, 1\}$, which is unobservable to the *Tachiiri* and other participants, at the time of the project execution. Let p_e be the probability of the success of the project and assume $0 \leq p_e \leq 1$, and $\Delta_p := p_1 - p_0 > 0$. The *Daimyo* enjoys private benefit $B(1-e)$ where $B > 0$. We assume $p_1 R > I + \lambda$ and $\Delta_p R > B$, so that executing the project with $e = 1$ is efficient.

When two matching markets are viable, the incentive compatibility condition is given by

$$R_{\mathrm{T}} \leq R - \frac{B}{\Delta_p}.$$

Taking into consideration the condition that the *Daimyo* repays the loan when the project succeeds, the *Tachiiri* offers the relational contract with

$$R_{\mathrm{T}} = \min \left\{ R - \frac{B}{\Delta_p}, \frac{(1-s)\delta \{\alpha_{\mathrm{D}} k - (\alpha_{\mathrm{D}} - \phi) p_1 R\}}{1 + (\alpha_{\mathrm{D}} - \phi)(1-p_1)(1-s)\delta} \right\}.$$

Then $R_{\mathrm{T}} = R - B/\Delta_p$ if

$$B > B^*_{\mathrm{IC}} := \frac{\{1 + (\alpha^*_{\mathrm{D}} - \phi^*)(1-s)\delta\} R - \alpha^*_{\mathrm{D}}(1-s)\delta k}{1 + (\alpha^*_{\mathrm{D}} - \phi^*)(1-p_1)(1-s)\delta} \Delta_p,$$

that is, if private benefit B from $e = 0$ is sufficiently large.

Supposing $B > B_{\mathrm{IC}}$ and $\alpha_{\mathrm{D}} = \phi = 1$, conditions for $V_{\mathrm{T}} > V_{\mathrm{D}}$ and $V_{\mathrm{D}} \geq 0$ become $k > 0$ and

$$k \leq \frac{p_1}{\{1 - (1-s)\delta\} \Delta_p} B. \tag{8}$$

A stationary equilibrium then exists if (8) and $p_1 R_{\mathrm{T}} - I > \lambda$, that is,

$$\frac{p_1}{\Delta_p} B > I + \lambda \tag{9}$$

holds.

For the case of the non-viable short-term loan market, $R_{\mathrm{T}} = B/\Delta_p$ if

$$B > \hat{B}_{\mathrm{IC}} := \frac{\{1 + (\hat{\alpha}_{\mathrm{D}} - \hat{\phi})(1-s)\delta\} R - \hat{\alpha}_{\mathrm{D}}(1-s)\delta k}{1 + (\hat{\alpha}_{\mathrm{D}} - \hat{\phi})(1-p_1)(1-s)\delta} \Delta_p.$$

The conditions for the existence of a stationary equilibrium become (9) and

$$\frac{(1-q)p_1}{\Delta_p} B < k \leq \frac{p_1}{\{1 - (1-q)(1-s)\delta\} \Delta_p} B,$$

[8] The example of the relationship between the Tsuwano Domain and its *Tachiiri* Kajimaya Kyū-emon, explained in Subsection 5.3, suggests that not only *Daimyo* but also *Tachiiri* exert effort to increase the probability of success. We can further extend the model to include *Tachiiri*'s effort as well.

which is harder to be satisfied than (8).

Note that given $\alpha_{\mathrm{D}}^* = \hat{\alpha}_{\mathrm{D}}$ and $\beta_{\mathrm{T}}^* > 0$, $\hat{B}_{\mathrm{IC}} > B_{\mathrm{IC}}^*$ holds. Hence there are three cases.

Case 1 ($B \leq B_{\mathrm{IC}}^*$)**:** The previous analysis is not affected.

Case 2 ($B_{\mathrm{IC}}^* < B \leq \hat{B}_{\mathrm{IC}}$)**:** $\hat{R}_{\mathrm{T}}$ is not altered. While R_{T}^* is smaller than the corresponding one in the previous analysis, $R_{\mathrm{T}}^* > \hat{R}_{\mathrm{T}}$, and thus $\Delta^* > \hat{\Delta}$ still holds.

Case 3 ($\hat{B}_{\mathrm{IC}} < B$)**:** For B large enough, $R_{\mathrm{T}}^* = \hat{R}_{\mathrm{T}} = B/\Delta_p$. Then we have $\Delta^* = \hat{\Delta}$.

The bottom line is that unless B is so large that Case 3 applies, the main messages from the analysis of the baseline model are still valid.

6.2 General Random Matching

In this section we consider a general random matching function, instead of the frictionless matching in the main model. In the *Daimyo* matching market, given $1 - \nu_{\mathrm{T}}$ and μ_{T}, the measure of *Daimyo-Tachiiri* pairs, α, is newly formed via a matching function A by

$$\alpha = A(1 - \nu_{\mathrm{T}}, \mu_{\mathrm{T}}).$$

The matching probabilities are then defined as follows.

$$\alpha_{\mathrm{D}} = \frac{\alpha}{1 - \nu_{\mathrm{T}}},$$
$$\alpha_{\mathrm{M}} = \frac{\alpha}{\mu_{\mathrm{T}}}.$$

Similarly, in the short-term loan market, given $q\nu_{\mathrm{T}}$ and μ_{L}, the measure of underfunded *Tachiiri*-merchant pairs, β, is newly formed via a matching function B by

$$\beta = B(q\nu_{\mathrm{T}}, \mu_{\mathrm{L}}),$$

and the matching probabilities are then defined as follows.

$$\beta_{\mathrm{T}} = \frac{\beta}{q\nu_{\mathrm{T}}},$$
$$\beta_{\mathrm{L}} = \frac{\beta}{\mu_{\mathrm{L}}}.$$

We consider the following class of matching functions.

Definition 1. *We calls a matching function $C(x, y)$ regular if it satisfies the following conditions.*

- *$C(x,y)$ is homogeneous of degree 1, or equivalently, there exist functions C_x and C_y such that*

$$\frac{C(x,y)}{x} = C_x(\theta),$$
$$\frac{C(x,y)}{y} = C_y(\theta).$$

where $\theta := x/y$.

- *C_x is continuous, strictly decreasing, and satisfies*

$$\lim_{\theta \to 0} C_x(\theta) = 1,$$
$$\lim_{\theta \to \infty} C_x(\theta) = 0.$$

- *C_y is continuous, strictly increasing, and satisfies*

$$\lim_{\theta \to 0} C_y(\theta) = 0,$$
$$\lim_{\theta \to \infty} C_y(\theta) = 1.$$

We assume both A and B are regular. This assumption is made to facilitate the existence of equilibrium. Especially, the homogeneity of the matching functions is often made in the labor search literature because it is empirically and theoretically justified in the literature. For details, see Pissarides (2000) and Petrongolo and Pissarides (2001). The assumption implies that in the *Daimyo* matching market, α_{D} is decreasing and α_{M} is increasing in $(1-\nu_{\mathrm{T}})/\mu_{\mathrm{T}}$, the ratio of *Daimyo* to merchants, and in the short-term loan market, β_{T} is decreasing and β_{L} is increasing in $q\nu_{\mathrm{T}}/\mu_{\mathrm{L}}$, the ratio of borrowers to lenders.[9]

The analysis in Section 4 is still valid under the general random matching functions since it does not depend upon the frictionless matching assumption.

Proposition 3. *Under the regular matching functions and the assumptions given above, there exist $\overline{q} \in (0,1)$ and $\overline{s} \in (0,1)$ such that if $q < \overline{q}$ and $s < \overline{s}$, then there exists $\underline{\delta} \in (0,1)$ such that a stationary equilibrium in which two matching markets are viable exists for $\delta > \underline{\delta}$.*

Proof. We define $\bar{\alpha}_{\mathrm{D}}$ as follows.

$$\bar{\alpha}_{\mathrm{D}}(\alpha_{\mathrm{M}}) := A_x \circ A_y^{-1}(\alpha_{\mathrm{M}}).$$

By the regularity of A, $\bar{\alpha}_{\mathrm{D}}$ is well-defined and strictly decreasing. By augmenting $\bar{\alpha}_{\mathrm{D}}$ with $\bar{\alpha}_{\mathrm{D}}(0) = 1$

[9]Note that the frictionless matching functions in the main model correspond to $A(1-\nu_{\mathrm{T}}, \mu_{\mathrm{T}}) = \min\{1-\nu_{\mathrm{T}}, \mu_{\mathrm{T}}\}$ and $B(q\nu_{\mathrm{T}}, \mu_{\mathrm{L}}) = \min\{q\nu_{\mathrm{T}}, \mu_{\mathrm{L}}\}$, and thus they are not regular.

and $\bar{\alpha}_{\mathrm{D}}(1) = 0$, the regularity of A also implies $\bar{\alpha}_{\mathrm{D}} : [0,1] \to [0,1]$ is continuous. $\bar{\beta}_{\mathrm{T}}$ is similarly defined.

We define

$$c_{\mathrm{T}}(\alpha_{\mathrm{M}}, \beta_{\mathrm{L}}) := \alpha_{\mathrm{M}} \left[(p\bar{R}_{\mathrm{T}}(\alpha_{\mathrm{M}}, \beta_{\mathrm{L}}) - I) + \frac{(1-q)(1-s)\delta}{1-(1-q)(1-s)\delta} \{ (p\bar{R}_{\mathrm{T}}(\alpha_{\mathrm{M}}, \beta_{\mathrm{L}}) - I) - \lambda \} \right] - \lambda,$$
$$c_{\mathrm{L}}(\alpha_{\mathrm{M}}, \beta_{\mathrm{L}}) := \beta_{\mathrm{L}} \left[(p\bar{R}_{\mathrm{T}}(\alpha_{\mathrm{M}}, \beta_{\mathrm{L}}) - I) + \frac{(1-q)(1-s)\delta}{1-(1-q)(1-s)\delta} \{ (p\bar{R}_{\mathrm{T}}(\alpha_{\mathrm{M}}, \beta_{\mathrm{L}}) - I) - \lambda \} \right] - \lambda,$$

where

$$\bar{R}_{\mathrm{T}}(\alpha_{\mathrm{M}}, \beta_{\mathrm{L}}) = \frac{(1-s)\delta \left\{ \bar{\alpha}_{\mathrm{D}}(\alpha_{\mathrm{M}})k - (\bar{\alpha}_{\mathrm{D}}(\alpha_{\mathrm{M}}) - \bar{\phi}(\beta_{\mathrm{L}}))pR \right\}}{1 + (\bar{\alpha}_{\mathrm{D}}(\alpha_{\mathrm{M}}) - \bar{\phi}(\beta_{\mathrm{L}}))(1-p)(1-s)\delta},$$
$$\bar{\phi}(\beta_{\mathrm{L}}) = 1 - q(1 - \bar{\beta}_{\mathrm{T}}(\beta_{\mathrm{L}})).$$

We then obtain $c_{\mathrm{T}}(0, \beta_{\mathrm{L}}) < 0$ and $c_{\mathrm{L}}(\alpha_{\mathrm{M}}, 0) < 0$. Furthermore, if $p\bar{R}_{\mathrm{T}}(\alpha_{\mathrm{M}}, \beta_{\mathrm{L}}) - I > \lambda$ holds for any $\alpha_{\mathrm{M}}, \beta_{\mathrm{L}} \in [0,1]$, we obtain $c_{\mathrm{T}}(1, \beta_{\mathrm{L}}) > 0$ and $c_{\mathrm{L}}(\alpha_{\mathrm{M}}, 1) > 0$. Therefore, it follows from Poincaré-Miranda Theorem that there exists $(\alpha^*_{\mathrm{M}}, \beta^*_{\mathrm{L}}) \in (0,1)^2$ satisfying $c_{\mathrm{T}}(\alpha^*_{\mathrm{M}}, \beta^*_{\mathrm{L}}) = c_{\mathrm{L}}(\alpha^*_{\mathrm{M}}, \beta^*_{\mathrm{L}}) = 0$. This implies $(\alpha^*_{\mathrm{D}}, \alpha^*_{\mathrm{M}}, \beta^*_{\mathrm{T}}, \beta^*_{\mathrm{L}}) \in (0,1)^4$ satisfies [OT] and [OL] where $\alpha^*_{\mathrm{D}} = \bar{\alpha}_{\mathrm{D}}(\alpha^*_{\mathrm{M}})$ and $\beta^*_{\mathrm{T}} = \bar{\beta}_{\mathrm{T}}(\beta^*_{\mathrm{L}})$. $(\alpha^*_{\mathrm{D}}, \alpha^*_{\mathrm{M}}, \beta^*_{\mathrm{T}}, \beta^*_{\mathrm{L}})$ determines ν^*_{T} via [S], which determines μ^*_{T} and μ^*_{L} via the matching functions.

Finally, we search for the condition that $\underline{k} < k \leq \overline{k}$ where $\overline{k}$ and $\underline{k}$ are defined in (2) and (3) respectively. It is verified that

$$\lim_{\delta \to 1, s \to 0, q \to 0} (\overline{k} - \underline{k}) = \frac{1 - (1-\alpha_{\mathrm{D}})(1-p)}{\alpha_{\mathrm{D}} p} (pR - I - \lambda),$$

which is strictly positive for any $\alpha_{\mathrm{D}} \in (0,1)$, and therefore $\overline{k} > \underline{k}$ as long as q and s are sufficiently small and δ is sufficiently large. □

In contrast to the corresponding result (Proposition 1) in the main text, the existence of stationary equilibrium with two viable matching markets depends on q: it exists if the probability that the *Tachiiri* is short of funds is sufficiently small and hence the *Daimyo* can exercise her projects with sufficiently high probability.

For the existence of stationary equilibrium where only the *Daimyo* matching market is viable, we can repeat the analysis similar to the one given above by setting $\beta = B(\cdot, \cdot) \equiv 0$ and $\hat{\phi} = \bar{\phi}(0) < \bar{\phi}(\beta^*_{\mathrm{L}})$. Instead of showing explicitly the sufficient conditions for the existence that are harder to satisfy, we show that our main results that becoming a *Tachiiri* is attractive and the short-term loan market complements the *Daimyo-Tachiiri* relationships continue to hold, provided that a stationary equilibrium exists whether or not the short-term loan market is viable.

Proposition 4. Suppose there exist stationary equilibria with and without viable short-term loan market. Then $R^*_{\mathrm{T}} > \hat{R}^*_{\mathrm{T}}$, which implies $\Delta^* > \hat{\Delta}$ and $W^*_{\mathrm{T}} > \hat{W}_{\mathrm{T}}$, holds. And $\nu^*_{\mathrm{T}} > \hat{\nu}_{\mathrm{T}}$ also holds.

Proof. We define $\bar{\alpha}_{\mathrm{M}}(\alpha_{\mathrm{D}}) := A_y \circ A_x^{-1}(\alpha_{\mathrm{D}})$ (i.e., $\bar{\alpha}_{\mathrm{M}}$ is the inverse of $\bar{\alpha}_{\mathrm{D}}$). Due to the regularity of A, $\bar{\alpha}_{\mathrm{M}}$ is well-defined and strictly decreasing.

Using $\bar{\alpha}_{\mathrm{M}}(\alpha_{\mathrm{D}})$, define

$$\begin{aligned} f_1(R_{\mathrm{T}}, \alpha_{\mathrm{D}}, \phi) :=& R_{\mathrm{T}} - \frac{(1-s)\delta \left\{\alpha_{\mathrm{D}} k - (\alpha_{\mathrm{D}} - \phi)pR\right\}}{1 + (\alpha_{\mathrm{D}} - \phi)(1-p)(1-s)\delta}, \\ f_2(R_{\mathrm{T}}, \alpha_{\mathrm{D}}, \phi) :=& \lambda - \bar{\alpha}_{\mathrm{M}}(\alpha_{\mathrm{D}}) \left[(pR_{\mathrm{T}} - I) + \frac{(1-q)(1-s)\delta}{1-(1-q)(1-s)\delta} \{(pR_{\mathrm{T}} - I) - \lambda\} \right], \end{aligned}$$

and $f := (f_1, f_2)$. By [RT] and [OT], in the case of the viable short-term loan market, a (partial) equilibrium condition is $f(R_{\mathrm{T}}^*, \alpha_{\mathrm{D}}^*, \phi^*) = 0$ where $\phi^* = 1 - q(1 - \beta_{\mathrm{T}}^*)$ for some $\beta_{\mathrm{T}}^* \in (0, 1)$. Similarly, in the case of the non-viable short-term loan market, a (partial) equilibrium condition is $f(\hat{R}_{\mathrm{T}}, \hat{\alpha}_{\mathrm{D}}, \hat{\phi}) = 0$ where $\hat{\phi} = 1 - q$. Note that $\phi^* > \hat{\phi}$.

Define $\overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}, \phi)$ by $f_1(\overline{R}_{\mathrm{T}}, \alpha_{\mathrm{D}}, \phi) = 0$. Our goal is to show $R_{\mathrm{T}}^* = \overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \phi^*) > \overline{R}_{\mathrm{T}}(\hat{\alpha}_{\mathrm{D}}, \hat{\phi}) = \hat{R}_{\mathrm{T}}$.

We first show $\alpha_{\mathrm{D}}^* > \hat{\alpha}_{\mathrm{D}}$. $\overline{R}_{\mathrm{T}}$ is strictly increasing in ϕ, and thus

$$\overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \phi^*) > \overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \hat{\phi})$$

holds. Because f_2 is strictly decreasing in R_{T} and independent of ϕ,

$$0 = f_2(\overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \phi^*), \alpha_{\mathrm{D}}^*, \phi^*) < f_2(\overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \hat{\phi}), \alpha_{\mathrm{D}}^*, \hat{\phi}),$$

and $f_2(\overline{R}_{\mathrm{T}}(\hat{\alpha}_{\mathrm{D}}, \hat{\phi}), \hat{\alpha}_{\mathrm{D}}, \hat{\phi}) = 0$ implies

$$f_2(\overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \hat{\phi}), \alpha_{\mathrm{D}}^*, \hat{\phi}) > f_2(\overline{R}_{\mathrm{T}}(\hat{\alpha}_{\mathrm{D}}, \hat{\phi}), \hat{\alpha}_{\mathrm{D}}, \hat{\phi}).$$

Then $\alpha_{\mathrm{D}}^* > \hat{\alpha}_{\mathrm{D}}$ must hold: Otherwise,

$$f_2(\overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \hat{\phi}), \alpha_{\mathrm{D}}^*, \hat{\phi}) \leq f_2(\overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \hat{\phi}), \hat{\alpha}_{\mathrm{D}}, \hat{\phi}) \leq f_2(\overline{R}_{\mathrm{T}}(\hat{\alpha}_{\mathrm{D}}, \hat{\phi}), \hat{\alpha}_{\mathrm{D}}, \hat{\phi}),$$

where the first inequality holds since f_2 is strictly increasing in α_{D}, and the second one follows from $\overline{R}_{\mathrm{T}}$ being decreasing in α_{D} for $\phi \in [\hat{\phi}, \phi^*]$ by using $\hat{V}_{\mathrm{D}} \geq 0$, and f_2 being strictly decreasing in R_{T}. A contradiction.

Now using the fact that f_2 is independent of ϕ yields

$$f_2(\overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \phi^*), \alpha_{\mathrm{D}}^*, \hat{\phi}) = f_2(\overline{R}_{\mathrm{T}}(\hat{\alpha}_{\mathrm{D}}, \hat{\phi}), \hat{\alpha}_{\mathrm{D}}, \hat{\phi}) = 0,$$

which implies $\overline{R}_{\mathrm{T}}(\alpha_{\mathrm{D}}^*, \phi^*) > \overline{R}_{\mathrm{T}}(\hat{\alpha}_{\mathrm{D}}, \hat{\phi})$ since f_2 is strictly increasing in α_{D} and strictly decreasing in R_{T}.

$\nu_{\mathrm{T}}^* > \hat{\nu}_{\mathrm{T}}$ follows from the fact that $\alpha_{\mathrm{D}}^* > \hat{\alpha}_{\mathrm{D}}$ and $\beta_{\mathrm{T}}^* > 0$. □

6.3 *Tachiiri*'s Fund Availability in the Short-Term Loan Market

In our baseline model, we assume *Tachiiri* can repay short-term debt at no additional cost even if *Daimyo* did not pay to *Tachiiri*. This is one extreme case. In this Subsection we consider the other extreme case in which *Tachiiri* cannot repay short-term debt when *Daimyo* did not pay to *Tachiiri*. We assume that, once a merchant falls behind in the repayment of short-term borrowings, he will not be able to take out any further short-term loans.

A reasonable situation is that *Daimyo* would expect *Tachiiri* to be unable to get short-term loans in the future, once *Daimyo* does not make repayments. Then the following three cases are possible, which are exhaustive, but may not be exclusive.

1. The *Daimyo* proposes to break off the relationship.

2. The *Tachiiri* proposes to break off the relationship.

3. Neither proposes to break off the relationship.

Hereafter in this Subsection, we focus on the third case, in other words, the case where neither *Daimyo* nor *Tachiiri* voluntarily terminates their relationship even after *Tachiiri* can no longer access to the short-run loan market in the future. For, in this case the value functions become the most complicated and it appears the most difficult to show the existence of equilibria.

We also assume that *Daimyo*'s payment is verifiable, and therefore short-term loans are forcibly collected whenever *Daimyo* makes a payment.

We focus on the case of $1-\nu_{\mathrm{T}}^* \leq \mu_{\mathrm{T}}^*$ (i.e., $\alpha_{\mathrm{D}}^* = 1$ and $\alpha_{\mathrm{M}}^* = (1-\nu_{\mathrm{T}}^*)/\mu_{\mathrm{T}}^*$) and $\mu_{\mathrm{L}}^* \geq q\nu_{\mathrm{T}}^*$ (i.e., $\beta_{\mathrm{T}}^* = 1$ and $\beta_{\mathrm{L}}^* = q\nu_{\mathrm{T}}^*/\mu_{\mathrm{L}}^*$).

We denote by $\check{x}$ a variable x for *Tachiiri* who cannot take short-term loans or *Daimyo* associated with them. Then the value functions for *Daimyo* become as follows:

$$
\begin{aligned}
V_{\mathrm{D}} &= p(R-R_{\mathrm{T}}) - k + (1-s)\delta V_{\mathrm{T}} + s\delta V_{\mathrm{D}} = v_k + \delta V_{\mathrm{D}} + (1-s)\delta(V_{\mathrm{T}} - V_{\mathrm{D}}),\\
V_{\mathrm{T}} &= (1-q)\{p(R-R_{\mathrm{T}}) + (1-s)\delta V_{\mathrm{T}} + s\delta V_{\mathrm{D}}\} + qp\{(R-R_{\mathrm{T}}) + (1-s)\delta V_{\mathrm{T}} + s\delta V_{\mathrm{D}}\}\\
&\quad + q(1-p)\{(1-s)\delta \check{V}_{\mathrm{T}} + s\delta V_{\mathrm{D}}\}\\
&= v_0 + \delta V_{\mathrm{D}} + (1-q(1-p))(1-s)\delta(V_{\mathrm{T}} - V_{\mathrm{D}}) + q(1-p)(1-s)\delta(\check{V}_{\mathrm{T}} - V_{\mathrm{D}}),\\
\check{V}_{\mathrm{T}} &= (1-q)\{p(R-\check{R}_{\mathrm{T}}) + (1-s)\delta \check{V}_{\mathrm{T}} + s\delta V_{\mathrm{D}}\} + q\delta V_{\mathrm{D}}\\
&= (1-q)\check{v}_0 + \delta V_{\mathrm{D}} + (1-q)(1-s)\delta(\check{V}_{\mathrm{T}} - V_{\mathrm{D}}).
\end{aligned}
$$

V_{T} changes from the baseline model when the *Tachiiri* is short of fund *and* the *Daimyo* cannot pay R_{T}, that happen with probability $q(1-p)$. Then the continuation value to *Daimyo* becomes $\check{V}_{\mathrm{T}}$ instead of V_{T}. And $\check{V}_{\mathrm{T}}$ is qualitatively the same as $\hat{V}_{\mathrm{T}}$ in the case where the short-term loan market is not viable.

Denoting $\Omega = V_{\mathrm{T}} - V_{\mathrm{D}}$ and $\check{\Omega} = \check{V}_{\mathrm{T}} - V_{\mathrm{D}}$ and using

$$R_{\mathrm{T}} = (1-s)\delta\Omega,$$
$$\check{R}_{\mathrm{T}} = (1-s)\delta\check{\Omega},$$

yield

$$V_{\mathrm{D}} = \frac{pR - k + (1-p)(1-s)\delta\Omega}{1-\delta},$$
$$\Omega = \frac{\{1-(1-2q)(1-p)(1-s)\delta\}\,k - q^2(1-p)(1-s)\delta pR}{1-(1-2q)(1-p)(1-s)\delta + q^2(1-p)^2(1-s)^2\delta^2},$$
$$\check{\Omega} = \frac{\{1-(1-q)(1-p)(1-s)\delta\}\,k - q\,\{1+q(1-p)(1-s)\delta\}\,pR}{1-(1-2q)(1-p)(1-s)\delta + q^2(1-p)^2(1-s)^2\delta^2}.$$

These must satisfy $\Omega \geq 0$ and $\check{\Omega} \geq 0$, in addition to $V_{\mathrm{D}} \geq 0$. Since $\Omega > \check{\Omega}$, the first condition is redundant. The third condition is equivalent to

$$k \leq \overline{k} := \frac{1-(1-2q)(1-p)(1-s)\delta}{\{1-(1-q)(1-p)(1-s)\delta\}^2} pR.$$

The values for merchants are given as follows.

$$\begin{aligned}
W_{\mathrm{M}} &= (1-q)\max\{X_{\mathrm{T}}, X_{\mathrm{L}}, X_{\mathrm{O}}\} + q\delta W_{\mathrm{M}},\\
W_{\mathrm{T}} &= (1-q)\{pR_{\mathrm{T}} - I + (1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}\}\\
&\quad + qp\{R_{\mathrm{T}} - R_{\mathrm{L}} + (1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}\} + q(1-p)\{(1-s)\delta\check{W}_{\mathrm{T}} + s\delta W_{\mathrm{M}}\},\\
\check{W}_{\mathrm{T}} &= (1-q)\{p\check{R}_{\mathrm{T}} - I + (1-s)\delta\check{W}_{\mathrm{T}} + s\delta W_{\mathrm{M}}\} + q\delta W_{\mathrm{M}},\\
X_{\mathrm{T}} &= \alpha_{\mathrm{M}}(pR_{\mathrm{T}} - I + (1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}) + (1-\alpha_{\mathrm{M}})\delta W_{\mathrm{M}},\\
X_{\mathrm{L}} &= \beta_{\mathrm{L}}(pR_{\mathrm{L}} - I) + \delta W_{\mathrm{M}},\\
X_{\mathrm{O}} &= \lambda + \delta W_{\mathrm{M}}.
\end{aligned}$$

Note that, since we search for the equilibrium with $X_{\mathrm{T}} = X_{\mathrm{L}} = X_{\mathrm{O}}$, $\check{W}_{\mathrm{M}} = W_{\mathrm{M}}$ must hold.

We denote $\Delta = W_{\mathrm{T}} - W_{\mathrm{M}}$ and $\check{\Delta} = \check{W}_{\mathrm{T}} - W_{\mathrm{M}}$. The condition that the underfunded *Tachiiri* accepts a short-term loan contract offer is

$$\begin{aligned}
&p\,\{R_{\mathrm{T}} - R_{\mathrm{L}} + (1-s)\delta W_{\mathrm{T}}\} + (1-p)(1-s)\delta\check{W}_{\mathrm{T}} \geq (1-s)\delta W_{\mathrm{M}}\\
&\Leftrightarrow R_{\mathrm{L}} \leq R_{\mathrm{T}} + (1-s)\delta\Delta + \frac{1-p}{p}(1-s)\delta\check{\Delta}.
\end{aligned}$$

The equality holds on the equilibrium path. By using this condition and assuming $X_{\mathrm{O}} =$

$\max\{X_{\mathrm{T}}, X_{\mathrm{L}}, X_{\mathrm{O}}\}$ yield

$$\begin{aligned}
W_{\mathrm{M}} &= \frac{1-q}{1-\delta}\lambda, \\
\Delta &= \frac{1-q}{1-(1-q)(1-s)\delta}(pR_{\mathrm{T}} - I - \lambda), \\
\check{\Delta} &= \frac{1-q}{1-(1-q)(1-s)\delta}(p\check{R}_{\mathrm{T}} - I - \lambda).
\end{aligned}$$

The incentive conditions are $\Delta \geq 0$ and $\check{\Delta} \geq 0$, and since $R_{\mathrm{T}} > \check{R}_{\mathrm{T}}$, the first condition is redundant. The second condition is equivalent to

$$k \geq \underline{k} := \frac{q^2(1-p)(1-s)\delta}{1-(1-q)(1-p)(1-s)\delta}pR + \frac{1-(1-2q)(1-p)(1-s)\delta + q^2(1-p)^2(1-s)^2\delta^2}{p(1-s)\delta\left\{1-(1-q)(1-p)(1-s)\delta\right\}}(I+\lambda).$$

The condition that $X_{\mathrm{T}} = X_{\mathrm{O}}$ holds yields

$$[\mathrm{OT}] \quad \lambda = \alpha_{\mathrm{M}}\left\{(pR_{\mathrm{T}} - I) + (1-s)\delta\Delta\right\}.$$

Also, the condition that $X_{\mathrm{L}} = X_{\mathrm{O}}$ holds yields

$$\begin{aligned}
[\mathrm{OL}] \quad \lambda &= \beta_{\mathrm{L}}(pR_{\mathrm{L}} - I) \\
&= \beta_{\mathrm{L}}\left\{(pR_{\mathrm{T}} - I) + p(1-s)\delta\Delta + (1-p)(1-s)\delta\check{\Delta}\right\}.
\end{aligned}$$

Therefore, the condition $\check{\Delta} \geq 0$ guarantees the existences of $\alpha^*_{\mathrm{M}}, \beta^*_{\mathrm{L}} \in (0,1)$ satisfying [OT] and [OL]. Note that in contrast to the baseline model, $\alpha^*_{\mathrm{M}} < \beta^*_{\mathrm{L}}$ must hold since $\Delta > \check{\Delta}$: The rent the lender extracts from the underfunded *Tachiiri* decreases because the value of being a *Tachiiri* falls after the project failure of his *Daimyo*. Then the matching probability for the lender in the short-term loan market must be higher than the matching probability for the merchant entering the *Daimyo* matching market.

It is verified that

$$\lim_{\delta\to 1, p\to 0, q\to 0} (\overline{k} - \underline{k}) = \frac{pR - I - \lambda}{p} > 0,$$

and therefore $\overline{k} > \underline{k}$ as long as q and s are sufficiently small and δ is sufficiently large.

On the other hand, the value functions in the case of no short-term contract market are the same as in the baseline model, i.e.,

$$\begin{aligned}
R_{\mathrm{T}} &= \frac{(1-s)\delta(k - qpR)}{1+q(1-p)(1-s)\delta}, \\
\Delta &= \frac{1-q}{1-(1-q)(1-s)\delta}(pR_{\mathrm{T}} - I - \lambda).
\end{aligned}$$

It is verified that $R^*_{\rm T} > \hat{R}_{\rm T}$, which implies $\Delta^* > \hat{\Delta}$ and $W^*_{\rm T} > \hat{W}_{\rm T}$.

7 Heterogenous Merchants

In this section we introduce the heterogeneity of merchants' financing ability q. Since there is an infinite measure of merchants, we assume the following distribution of q: q is distributed over $[\underline{q}, \overline{q}]$ where $0 < \underline{q} < \overline{q} < 1$. There is an infinite measure of merchants with $\overline{q}$, while there is only a finite measure of merchants with $[\underline{q}, \overline{q})$. Let $F(q)$ be the measure of merchants with less than q. We denote the density by $f(q) = F'(q)$ and assume $f(q) > 0$ for all $q \in [\underline{q}, \overline{q})$.

We assume q of each merchant is observable to *Daimyo* and other merchants once they are matched with him. This assumption aligns well with historical facts. Unlike modern corporations with publicly traded shares, the asset size and financial condition of early-modern Japanese merchants counted as private information. However, this data could be inferred from external indicators with a high degree of accuracy. For example, the type of real estate holdings, the number of employees, the number of *Daimyo* they traded with, and the volume of Shogunate bonds they purchased could be observed from outside. In fact, it is known that the Mitsui family collected this information when lending to merchants (Mandai, 2024b).

As we explained in Section 2, the wealthiest Osaka merchants tend to become *Tachiiri*. Based on this observation, we consider the stationary equilibrium in which there is a threshold $q^* \in (\underline{q}, \overline{q})$ such that merchants with $q \leq q^*$ enter the *Daimyo* matching market, while others enters the short-term loan matching market or the outside market. We denote the density function conditional on $q < q^*$ by f^*, that is,

$$f^*(q) = \frac{f(q)}{F(q^*)}.$$

We also focus on the case of $1 - \nu^*_{\rm T} \leq \mu^*_{\rm T}$ (i.e., $\alpha^*_{\rm D} = 1$ and $\alpha^*_{\rm M} = (1 - \nu^*_{\rm T})/\mu^*_{\rm T}$) and $\mu^*_{\rm L} \geq q\nu^*_{\rm T}$ (i.e., $\beta^*_{\rm T} = 1$ and $\beta^*_{\rm L} = q\nu^*_{\rm T}/\mu^*_{\rm L}$).

Since *Daimyo* funding needs are always met on the equilibrium path, the value of *Daimyo* does not depends upon *Tachiiri*'s q. It follows that

$$R_{\rm T} = (1 - s)\delta k.$$

On the other hand, the value functions for merchant with ability q become as follows:

$$\begin{aligned}
W_{\mathrm{M}}(q) &= (1-q)\max\{X_{\mathrm{T}}(q), X_{\mathrm{L}}(q), X_{\mathrm{O}}(q)\} + q\delta W_{\mathrm{M}}(q),\\
W_{\mathrm{T}}(q) &= (1-q)\{pR_{\mathrm{T}} - I + (1-s)\delta W_{\mathrm{T}}(q) + s\delta W_{\mathrm{M}}(q)\}\\
&\quad + q\{pR_{\mathrm{T}} - R_{\mathrm{L}}(q) + (1-s)\delta W_{\mathrm{T}}(q) + s\delta W_{\mathrm{M}}(q)\},\\
X_{\mathrm{T}}(q) &= \alpha_{\mathrm{M}}(pR_{\mathrm{T}} - I + (1-s)\delta W_{\mathrm{T}}(q) + s\delta W_{\mathrm{M}}(q)) + (1-\alpha_{\mathrm{M}})\delta W_{\mathrm{M}}(q),\\
X_{\mathrm{L}}(q) &= \beta_{\mathrm{L}}\left[\int_{\underline{q}}^{q^*} R_{\mathrm{L}}(q) f^*(q)dq - I\right] + \delta W_{\mathrm{M}}(q),\\
X_{\mathrm{O}}(q) &= \lambda + \delta W_{\mathrm{M}}(q).
\end{aligned}$$

For merchants with $q \leq q^*$, $X_{\mathrm{T}}(q) = \max\{X_{\mathrm{T}}(q), X_{\mathrm{L}}(q), X_{\mathrm{O}}(q)\}$ must hold, and thus we obtain

$$\begin{aligned}
W_{\mathrm{M}}(q) &= \frac{(1-q)\alpha_{\mathrm{M}}(pR_{\mathrm{T}} - I)}{1-\delta} + \frac{(1-q)\alpha_{\mathrm{M}}(1-s)\delta}{1-\delta}\Delta(q),\\
W_{\mathrm{T}}(q) &= \frac{(1-q)(pR_{\mathrm{T}} - I) - q(R_{\mathrm{L}}(q) - pR_{\mathrm{T}})}{1-\delta} - \frac{s\delta}{1-\delta}\Delta(q),
\end{aligned}$$

where

$$\Delta(q) = W_{\mathrm{T}}(q) - W_{\mathrm{M}}(q) = \frac{(1-q)(1-\alpha_{\mathrm{M}})(pR_{\mathrm{T}} - I) - q(R_{\mathrm{L}}(q) - pR_{\mathrm{T}})}{1 - (1-s)\left\{1-(1-q)\alpha_{\mathrm{M}}\right\}\delta}.$$

By the condition that the underfunded *Tachiiri* with q accepts a short-term loan contract offer, we obtain

$$R_{\mathrm{L}}(q) = pR_{\mathrm{T}} + \frac{(1-q)(1-\alpha_{\mathrm{M}})(1-s)\delta}{1-(1-q)(1-\alpha_{\mathrm{M}})(1-s)\delta}(pR_{\mathrm{T}} - I).$$

This implies

$$\begin{aligned}
\Delta(q) &= \frac{(1-q)(1-\alpha_{\mathrm{M}})}{1-(1-q)(1-\alpha_{\mathrm{M}})(1-s)\delta}(pR_{\mathrm{T}} - I),\\
W_{\mathrm{M}}(q) &= \frac{(1-q)\alpha_{\mathrm{M}}}{(1-\delta)\left\{1-(1-q)(1-\alpha_{\mathrm{M}})(1-s)\delta\right\}}(pR_{\mathrm{T}} - I),\\
W_{\mathrm{T}}(q) &= \frac{(1-q)(1-(1-\alpha_{\mathrm{M}})\delta)}{(1-\delta)\left\{1-(1-q)(1-\alpha_{\mathrm{M}})(1-s)\delta\right\}}(pR_{\mathrm{T}} - I).
\end{aligned}$$

Note $\Delta(q) > 0$ for $\alpha_{\mathrm{M}} < 1$.

On the other hand, for merchants with $q > q^*$, $X_{\mathrm{T}}(q) \leq X_{\mathrm{L}}(q) = X_{\mathrm{O}}(q)$ must hold. Since $X_{\mathrm{O}}(q) = \max\{X_{\mathrm{T}}(q), X_{\mathrm{L}}(q), X_{\mathrm{O}}(q)\}$, we obtain

$$\tilde{W}_{\mathrm{M}}(q) = \frac{1-q}{1-\delta}\lambda,$$

where $\tilde{W}_{\mathrm{M}}(q)$ is the value function of merchants with $q > q^*$ who do not participate in the *Daimyo* matching market, which should be distinguished from $W_{\mathrm{M}}(q)$ for $q \leq q^*$.

Using condition $X_{\mathrm{L}}(q) = X_{\mathrm{O}}(q)$ yields

$$\text{[OL]} \quad \lambda = \beta_{\mathrm{L}} \left[\int_{\underline{q}}^{q^*} R_{\mathrm{L}}(q) f^*(q) dq - I \right]$$
$$= \beta_{\mathrm{L}} \left[\int_{\underline{q}}^{q^*} \frac{1}{1-(1-q)(1-\alpha_{\mathrm{M}})(1-s)\delta} f^*(q) dq \right] (pR_{\mathrm{T}} - I),$$

Finally, merchants with $q > q^*$ cannot benefit from becoming *Tachiiri*, that is, $W_{\mathrm{T}}(q) \leq \tilde{W}_{\mathrm{M}}(q)$ must hold. A sufficient condition is $W_{\mathrm{T}}(q^*) = \tilde{W}_{\mathrm{M}}(q^*)$, which is rewritten as

$$\text{[TM]} \quad \lambda = \frac{1-(1-\alpha_{\mathrm{M}})\delta}{1-(1-q^*)(1-\alpha_{\mathrm{M}})(1-s)\delta} (pR_{\mathrm{T}} - I),$$

because as long as $pR_{\mathrm{T}} - I > 0$, it is verified that

$$W'_{\mathrm{T}}(q) < \tilde{W}'_{\mathrm{M}}(q) \quad \forall q \geq q^*.$$

By the stationary condition, we obtain

$$\nu_{\mathrm{T}} = 1 - s,$$

which implies

$$\text{[DMM]} \quad \alpha_{\mathrm{M}} = \frac{s}{F(q^*) - (1-s)}.$$

The stationary equilibrium is expressed by $(q^*, \alpha^*_{\mathrm{M}}, \beta^*_{\mathrm{L}})$ satisfying [OL], [TM], and [DMM]. If $pR^*_{\mathrm{T}} - I > \lambda$ holds, then there exists $\beta^*_{\mathrm{L}} \in (0,1)$ satisfying [OL] for any q^* and α_{M}, and therefore, it suffices to show that there exists $(q^*, \alpha^*_{\mathrm{M}})$ satisfying [TM] and [DMM].

Example 1. We consider the case in which $\underline{q} = 0$ and $f(q) = \sigma/\overline{q}$ where $\sigma > 1$.

In this case [DMM] implies $\alpha_{\mathrm{M}} = \overline{q}s/[\sigma q^* - \overline{q}(1-s)]$ for $q^* \in [(\overline{q}/\sigma), \overline{q}]$. Substituting this into [TM], we have

$$\lambda = G(q^*)(pR_{\mathrm{T}} - I),$$

where

$$G(q) := \frac{(1-\delta)\{\sigma q - \overline{q}(1-s)\} + \overline{q}s\delta}{\{\sigma q - \overline{q}(1-s)\} - (1-q)(\sigma q - \overline{q})(1-s)\delta}.$$

Note that

$$\lim_{q \downarrow \frac{\overline{q}}{\sigma}} G(q) = 1,$$

$$G(\overline{q}) = \frac{(1-\delta)(\sigma - 1 + s) + s\delta}{(\sigma - 1 + s) - (1-\overline{q})(\sigma - 1)(1-s)\delta} \underset{\overline{q}\uparrow 1}{\rightarrow} 1 - \delta + \frac{s\delta}{\sigma - 1 + s}.$$

Thus, if

$$1 > \frac{\lambda}{pR_{\mathrm{T}} - I} > 1 - \delta + \frac{s\delta}{\sigma - 1 + s}$$

holds, then, for sufficiently large $\overline{q}$, there exists $q^* \in ((\overline{q}/\sigma), \overline{q}]$ satisfying [TM] and there exists a stationary equilibrium with the features above.[10]

8 Concluding Remarks

This paper examines how formal and relational forms of enforcement interacted in early modern Japan, focusing on financial relationships between *Daimyo* (regional lords) and merchants. Because of class distinctions, loans from merchants to *Daimyo* lacked legal enforcement, while contracts among merchants were enforceable by courts. Some merchants formed long-term self-enforcing relationships with *Daimyo* (becoming *Tachiiri*), while others relied on formal lending to underfunded *Tachiiri*. We build a theoretical model with two matching markets: one where merchants and *Daimyo* form relational contracts, and another where underfunded *Tachiiri* borrow from merchants via short-term formal contracts. We analyze conditions under which both markets can coexist in a stationary equilibrium. We show that merchants still value becoming *Tachiiri* due to long-term benefits, regardless of the short-term loan market's viability, and that the short-term loan market and *Daimyo*-*Tachiiri* relationships complement each other: formal contracts make relational ones more sustainable.

This paper primarily focuses on the steady-state. Yet, historical evidence consistently shows that merchants who become *Tachiiri* tend to prosper over the long run. One example is Kajimaya Sakubē, who became the *Tachiiri* of Hosokawa, one of the major *Daimyo* at the time. Sakubē was virtually unknown in the early 18th century, but after becoming Hosokawa's *Tachiiri* in the mid-18th century, he gained recognition as one of Osaka's wealthy merchants (Takatsuki, 2021a). To account for this phenomenon, it is essential to analyze the dynamic mechanisms through which attaining *Tachiiri* status contributes to capital accumulation and reduces his probability of running out of funds, q. Addressing this dynamic process offers an important avenue for understanding the rise and characteristics of wealthy merchants in early modern Japan.

Furthermore, this paper abstracts from information exchange among merchants. Yet in a

[10]The conditions regarding k are the same as those in the baseline model.

broader and arguably more realistic setting, such exchange could play an important role. For example, by sharing information, merchants may alleviate informational asymmetries concerning the expected profitability and repayment capacity of *Daimyo*'s projects. In addition, exchanging information about past transactions could enhance the credibility and enforceability of long-term relational arrangements. A systematic analysis of these mechanisms would not only deepen our understanding of historical merchant networks but may also shed light on contemporary financial institutions, including the importance of interbank information exchange in Japan's Main Bank System (e.g., Aoki, 1994).

As discussed, a key finding of this paper is the complementarity between informal and formal contracts. In early modern Japan, rigid class distinctions created unequal access to legal enforcement, making informal mechanisms essential. Similar disparities persist today, whether in illegal transactions beyond judicial reach (e.g., Gambetta, 1993) or in developing economies where parties differ in transparency and traceability (e.g., Macchiavello, 2022). Yet the interaction between informal and formal contracts has received little attention in the literature. Recognizing this continuity shows that the interplay identified here is not merely historical but a structural feature of economic organization, with implications for policy—for example, curbing illegal transactions by regulating complementary formal market transactions, or supporting sustainable relationships by strengthening short-term formal markets in developing economies.

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Appendix

In this appendix, we consider two alternative formulations of the baseline model. In Section A1, we assume that merchants die with some probability and are replaced by newly born merchants. In

Section A2, we assume that building a new relationship takes time and thus it is costly to terminate the *Daimyo*-*Tachiiri* relationship.

A1 Alternative Formulation: Exogenous Separation Shock

In this Subsection we consider another formulation of exogenous separation shock that at the last stage of each period, a merchant dies with probability $(1-\rho)$ and is replaced by a newly born merchant. We focus on the case of $1-\nu_{\mathrm{T}}^* \leq \mu_{\mathrm{T}}^*$ (i.e., $\alpha_{\mathrm{D}}^* = 1$ and $\alpha_{\mathrm{M}}^* = (1-\nu_{\mathrm{T}}^*)/\mu_{\mathrm{T}}^*$) and $\mu_{\mathrm{L}}^* \geq q\nu_{\mathrm{T}}^*$ (i.e., $\beta_{\mathrm{T}}^* = 1$ and $\beta_{\mathrm{L}}^* = q\nu_{\mathrm{T}}^*/\mu_{\mathrm{L}}^*$).

In this setting the value function for *Daimyo* is

$$\begin{aligned} V_{\mathrm{D}} &= p(R-R_{\mathrm{T}}) - k + \rho\delta V_{\mathrm{T}} + (1-\rho)\delta V_{\mathrm{D}}, \\ V_{\mathrm{T}} &= p(R-R_{\mathrm{T}}) + \rho\delta V_{\mathrm{T}} + (1-\rho)\delta V_{\mathrm{D}}. \end{aligned}$$

Solving these, we obtain

$$\begin{aligned} V_{\mathrm{D}} &= \frac{p(R-R_{\mathrm{T}})}{1-\delta} - \frac{1-\rho\delta}{1-\delta}k, \\ V_{\mathrm{T}} &= \frac{p(R-R_{\mathrm{T}})}{1-\delta} - \frac{(1-\rho)\delta}{1-\delta}k \end{aligned}$$

By the condition for *Daimyo* to choose to repay the loan after the success of the project, we obtain

$$[\mathrm{RT}] \quad R_{\mathrm{T}} = \bar{\delta}k,$$

where $\bar{\delta} := \rho\delta$ is the effective discount factor. $V_{\mathrm{T}} > V_{\mathrm{D}}$ is equivalent to $k > 0$. Also, $V_{\mathrm{D}} \geq 0$ is equivalent to

$$k \leq \overline{k} := \frac{pR}{1-\bar{\delta}+p\bar{\delta}}.$$

On the other hand, the value functions for merchant are given as follows.

$$\begin{aligned} W_{\mathrm{M}} &= (1-q)\max\{X_{\mathrm{T}}, X_{\mathrm{L}}, X_{\mathrm{O}}\} + q\bar{\delta}W_{\mathrm{M}}, \\ W_{\mathrm{T}} &= (1-q)(pR_{\mathrm{T}} - I + \bar{\delta}W_{\mathrm{T}}) + q(pR_{\mathrm{T}} - R_{\mathrm{L}} + \bar{\delta}W_{\mathrm{T}}), \\ X_{\mathrm{T}} &= \alpha_{\mathrm{M}}(pR_{\mathrm{T}} - I + \bar{\delta}W_{\mathrm{T}}) + (1-\alpha_{\mathrm{M}})\bar{\delta}W_{\mathrm{M}}, \\ X_{\mathrm{L}} &= \beta_{\mathrm{L}}(R_{\mathrm{L}} - I) + \bar{\delta}W_{\mathrm{M}}, \\ X_{\mathrm{O}} &= \lambda + \bar{\delta}W_{\mathrm{M}}. \end{aligned}$$

Assuming $X_{\mathrm{O}} = \max\{X_{\mathrm{T}}, X_{\mathrm{L}}, X_{\mathrm{O}}\}$ and solving these, we obtain

$$W_{\mathrm{M}} = \frac{1-q}{1-\bar{\delta}}\lambda,$$
$$W_{\mathrm{T}} = \frac{(1-q)(pR_{\mathrm{T}} - I) - q(R_{\mathrm{L}} - pR_{\mathrm{T}})}{1-\bar{\delta}}.$$

By the condition that the underfunded *Tachiiri* accepts a short-term loan contract offer, we obtain

$$R_{\mathrm{L}} = pR_{\mathrm{T}} + \frac{(1-q)\bar{\delta}}{1-\bar{\delta}+q\bar{\delta}}(pR_{\mathrm{T}} - I - \lambda).$$

The condition that $X_{\mathrm{T}} = X_{\mathrm{O}}$ holds is

$$[\mathrm{OT}] \quad \lambda = \frac{\alpha_{\mathrm{M}}}{1-(1-q)(1-\alpha_{\mathrm{M}})\bar{\delta}}(pR_{\mathrm{T}} - I).$$

Also, the condition that $X_{\mathrm{L}} = X_{\mathrm{O}}$ holds is

$$[\mathrm{OL}] \quad \lambda = \frac{\beta_{\mathrm{L}}}{1-(1-q)(1-\beta_{\mathrm{L}})\bar{\delta}}(pR_{\mathrm{T}} - I).$$

By the stationary condition, we obtain

$$[\mathrm{S}] \quad \nu_{\mathrm{T}} = \rho.$$

Based on a similar analysis as the baseline model, it is verified that, if $pR_{\mathrm{T}}^{*} - I > \lambda$, or

$$k > \underline{k} := \frac{I+\lambda}{p\bar{\delta}}$$

is satisfied, there exist $\alpha_{\mathrm{M}}^{*}, \beta_{\mathrm{L}}^{*} \in (0,1)$ satisfying [OT] and [OL], and there exists $k \in (\underline{k}, \overline{k}]$ as long as $\bar{\delta}$ is sufficiently large.

Next, we consider the case in which the short-term loan matching market is not viable. We continue to focus on the case of $1 - \hat{\nu}_{\mathrm{T}} \leq \hat{\mu}_{\mathrm{T}}$ (i.e., $\hat{\alpha}_{\mathrm{D}} = 1$ and $\hat{\alpha}_{\mathrm{M}} = (1-\hat{\nu}_{\mathrm{T}})/\hat{\mu}_{\mathrm{T}}$).

The values of being *Daimyo* are given as follows.

$$V_{\mathrm{D}} = p(R - R_{\mathrm{T}}) - k + \rho\delta V_{\mathrm{T}} + (1-\rho)\delta V_{\mathrm{D}},$$
$$V_{\mathrm{T}} = (1-q)\{p(R - R_{\mathrm{T}}) + \rho\delta V_{\mathrm{T}} + (1-\rho)\delta V_{\mathrm{D}}\} + q\delta V_{\mathrm{D}}.$$

Solving these, we obtain

$$V_{\mathrm{D}} = \frac{p(R - R_{\mathrm{T}}) - k}{1-\delta} + \frac{\bar{\delta}}{1-\delta}\Omega,$$
$$V_{\mathrm{T}} = \frac{(1-q)p(R - R_{\mathrm{T}})}{1-\delta} - \frac{\delta}{1-\delta}\bar{\Omega} + \frac{(1-q)\bar{\delta}}{1-\delta}\bar{\Omega},$$

where

$$\bar{\Omega} = \frac{-qp(R - R_{\mathrm{T}}) + k}{1 + q\bar{\delta}}.$$

By the condition for *Daimyo* to choose to repay the loan after the success of the project, we obtain

$$[\mathrm{RT}] \quad R_{\mathrm{T}} = \frac{-qp\bar{\delta}R + \bar{\delta}k}{1 + q(1-p)\bar{\delta}}.$$

$V_{\mathrm{T}} > V_{\mathrm{D}}$ is equivalent to

$$k > qpR.$$

Also, $V_{\mathrm{D}} \geq 0$ is equivalent to

$$k \leq \frac{pR}{1 - (1-q)(1-p)\bar{\delta}}.$$

The values for merchants are given as follows.

$$W_{\mathrm{M}} = (1-q)\max\{X_{\mathrm{T}}, X_{\mathrm{O}}\} + q\bar{\delta}W_{\mathrm{M}},$$
$$W_{\mathrm{T}} = (1-q)(pR_{\mathrm{T}} - I + \bar{\delta}W_{\mathrm{T}}) + q\bar{\delta}W_{\mathrm{M}},$$
$$X_{\mathrm{T}} = \alpha_{\mathrm{M}}(pR_{\mathrm{T}} - I + \bar{\delta}W_{\mathrm{T}}) + (1-\alpha_{\mathrm{M}})\bar{\delta}W_{\mathrm{M}},$$
$$X_{\mathrm{O}} = \lambda + \bar{\delta}W_{\mathrm{M}}.$$

Assuming $X_{\mathrm{O}} = \max\{X_{\mathrm{T}}, X_{\mathrm{O}}\}$ and Solving these, we obtain

$$W_{\mathrm{M}} = \frac{1-q}{1-\bar{\delta}}\lambda,$$
$$W_{\mathrm{T}} = \frac{1-q}{1-\bar{\delta}}\left[(pR_{\mathrm{T}} - I) - \frac{q\bar{\delta}}{1-\bar{\delta}+q\bar{\delta}}(pR_{\mathrm{T}} - I - \lambda)\right].$$

The condition that $X_{\mathrm{T}} = X_{\mathrm{O}}$ holds is

$$[\mathrm{OT}] \quad \lambda = \frac{\alpha_{\mathrm{M}}}{1 - (1-\alpha_{\mathrm{M}})(1-q)\bar{\delta}}(pR_{\mathrm{T}} - I).$$

By the stationary condition, we obtain

$$[\text{S}] \quad \nu_{\text{T}} = (1-q)\rho.$$

Comparing these two cases yields $R_{\text{T}}^* > \hat{R}_{\text{T}}$, which implies $W_{\text{T}}^* > \hat{W}_{\text{T}}$, and $\nu_{\text{T}}^* > \hat{\nu}_{\text{T}}$.

A2 Alternative Formulation: Costs of Starting a New Relationship

In order for *Daimyo* to repay the loan to *Tachiiri*, terminating *Daimyo*-*Tachiiri* relationship must be costly for *Daimyo*. In the baseline model, we directly assume the *Daimyo* incurs a fixed cost k when she builds a new relationships with a *Tachiiri*. Another modeling strategy is to assume that building a new relationship takes time. In this subsection we pursue this line.

To be more precise, we assume loans can be made starting from the next period after a new match.[11] This can be seen as representing a situation where it takes time to investigate the financial status of the matched *Daimyo* or her nature of projects and create an appropriate relational contract.

We also assume the timing of merchant's market participation as follows.

1. Merchants who are not *Tachiiri* decide whether they participate in the *Daimyo* matching market or not.

2. The fund availability of each merchant realizes.

3. Merchants with funds who are not *Tachiiri* and do not participate in *Daimyo* matching market at the previous stage decide which the short-term loan market or the outside market to participate in.

4. Matching results are realized.

In other words, we assume merchants who did not find *Daimyo* in the *Daimyo* matching market cannot participate in the short-term loan market at that period.

Similarly in Section 5, we focus on the case of $1-\nu_{\text{T}}^* < \mu_{\text{T}}^*$ (i.e., $\alpha_{\text{D}}^* = 1$ and $\alpha_{\text{M}}^* = (1-\nu_{\text{T}}^*)/\mu_{\text{T}}^* < 1$) and $q\nu_{\text{T}}^* < \mu_{\text{L}}^*$ (i.e., $\beta_{\text{T}}^* = 1$ and $\beta_{\text{L}}^* = q\nu_{\text{T}}^*/\mu_{\text{L}}^* < 1$).

The value functions for *Daimyo* are given as follows.

$$V_{\text{D}} = (1-s)\delta V_{\text{T}} + s\delta V_{\text{D}},$$
$$V_{\text{T}} = p(R - R_{\text{T}}) + (1-s)\delta V_{\text{T}} + s\delta V_{\text{D}}.$$

[11] Alternatively, we could consider a matching function in which there is a positive probability that *Daimyo* cannot find a candidate for *Tachiiri* even when *Daimyo* are on the short side, which endogenously generates costs for terminating existing *Daimyo*-*Tachiiri* relationship.

Solving two equations given above, we obtain

$$V_{\mathrm{D}} = \frac{(1-s)\delta}{1-\delta} p(R - R_{\mathrm{T}}),$$
$$V_{\mathrm{T}} = \frac{1-s\delta}{1-\delta} p(R - R_{\mathrm{T}}).$$

The condition that the *Daimyo* chooses to repay the loan after the project succeeds yields

$$[\mathrm{RT}] \quad R_{\mathrm{T}} = \frac{p(1-s)\delta}{1+p(1-s)\delta} R.$$

On the other hand, the value functions for merchants are given as follow.

$$\begin{aligned}
W_{\mathrm{M}} &= \max\{X_{\mathrm{T}}, X_{\mathrm{N}}\}, \\
W_{\mathrm{T}} &= (1-q)\{pR_{\mathrm{T}} - I + (1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}\} + q\{pR_{\mathrm{T}} - R_{\mathrm{L}} + (1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}\}, \\
X_{\mathrm{T}} &= \alpha_{\mathrm{M}}\{(1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}\} + (1-\alpha_{\mathrm{M}})\delta W_{\mathrm{M}}, \\
X_{\mathrm{N}} &= (1-q)\max\{X_{\mathrm{L}}, X_{\mathrm{O}}\} + q\delta W_{\mathrm{M}}, \\
X_{\mathrm{L}} &= \beta_{\mathrm{L}}(R_{\mathrm{L}} - I) + \delta W_{\mathrm{M}}, \\
X_{\mathrm{O}} &= \lambda + \delta W_{\mathrm{M}},
\end{aligned}$$

where X_{N} is the value of merchants who are not *Tachiiri* and choose not to participate in the *Daimyo* matching market, evaluated before his fund availability realizes.

Assuming $X_{\mathrm{N}} = \max\{X_{\mathrm{T}}, X_{\mathrm{N}}\}$ and $X_{\mathrm{O}} = \max\{X_{\mathrm{L}}, X_{\mathrm{O}}\}$ and solving for W_{M} and W_{T} yield

$$W_{\mathrm{M}} = \frac{1-q}{1-\delta}\lambda,$$
$$W_{\mathrm{T}} = \frac{(1-q)(pR_{\mathrm{T}} - I) - q(R_{\mathrm{L}} - pR_{\mathrm{T}}) - s\delta\Delta}{1-\delta},$$

where

$$\Delta = W_{\mathrm{T}} - W_{\mathrm{M}} = \frac{(1-q)\{(pR_{\mathrm{T}} - I) - \lambda\} - q(R_{\mathrm{L}} - pR_{\mathrm{T}})}{1-(1-s)\delta}.$$

The condition that the underfunded *Tachiiri* accepts a short-term loan contract offer yields

$$[\mathrm{RL}] \quad R_{\mathrm{L}} = pR_{\mathrm{T}} + \frac{(1-s)(1-q)\delta}{1-(1-s)(1-q)\delta}(pR_{\mathrm{T}} - I - \lambda),$$

and

$$\Delta = \frac{1-q}{1-(1-s)(1-q)\delta}(pR_{\mathrm{T}} - I - \lambda).$$

Note that $\Delta \geq 0$ requires $pR_{\mathrm{T}} - I - \lambda \geq 0$.

The equilibrium requires $X_{\mathrm{L}} = X_{\mathrm{O}}$. This condition leads to

$$\begin{aligned}\lambda &= \beta_{\mathrm{L}}(R_{\mathrm{L}} - I) \\ &= \beta_{\mathrm{L}}\left[(pR_{\mathrm{T}} - I) + \frac{(1-s)(1-q)\delta}{1-(1-s)(1-q)\delta}(pR_{\mathrm{T}} - I - \lambda)\right],\end{aligned}$$

and therefore

$$[\mathrm{OL}] \quad \lambda = \frac{\beta_{\mathrm{L}}}{1-(1-\beta_{\mathrm{L}})(1-s)(1-q)\delta}(pR_{\mathrm{T}} - I).$$

The right-hand side of [OL] is increasing in β_{L} and becomes smaller than λ as $\beta_{\mathrm{L}} \to 0$. It becomes larger than λ as $\beta_{\mathrm{L}} \to 1$ if and only if $pR_{\mathrm{T}} - I - \lambda > 0$.

The equilibrium also requires $X_{\mathrm{T}} = X_{\mathrm{N}}$. This condition lead to

$$\begin{aligned}\lambda &= \alpha_{\mathrm{M}}\frac{(1-s)\delta}{1-q}\Delta \\ &= \alpha_{\mathrm{M}}\frac{(1-s)\delta}{1-(1-s)(1-q)\delta}(pR_{\mathrm{T}} - I - \lambda),\end{aligned}$$

and therefore

$$[\mathrm{NT}] \quad \lambda = \frac{\alpha_{\mathrm{M}}(1-s)\delta}{1-(1-q-\alpha_{\mathrm{M}})(1-s)\delta}(pR_{\mathrm{T}} - I).$$

The right-hand side of [NT] is increasing in α_{M} and becomes smaller than λ as $\alpha_{\mathrm{M}} \to 0$. It becomes larger than λ as $\alpha_{\mathrm{M}} \to 1$ if and only if

$$\frac{(1-s)\delta}{1+q(1-s)\delta}(pR_{\mathrm{T}} - I) > \lambda. \tag{A1}$$

Finally, the following stationarity condition yields

$$[\mathrm{S}] \quad \nu_{\mathrm{T}} = 1 - s.$$

Combining [RT] and (A1), we obtain the following sufficient condition for the existence of equilibrium.

$$\frac{(1-s)\delta}{1+q(1-s)\delta}\left[\frac{p(1-s)\delta}{1+p(1-s)\delta}pR - I\right] > \lambda.$$

It is verified that

$$\lim_{\delta\to 1, s\to 0, q\to 0} \frac{(1-s)\delta}{1+(1-s)q\delta}\left[\frac{p(1-s)\delta}{1+p(1-s)\delta}pR - I\right] = \frac{p^2}{1+p}R - I.$$

Thus, if R is sufficiently large, to be more precise, if $p^2R/(1+p) > I + \lambda$, then the equilibrium in which both matching markets are viable exists as long as q and s are sufficiently small and δ is sufficiently large.

Next, we consider the case in which the short-run loan matching market is not viable. We continue to focus on the case of $1-\hat{\nu}_{\mathrm{T}} < \hat{\mu}_{\mathrm{T}}$ (i.e., $\hat{\alpha}_{\mathrm{D}} = 1$ and $\hat{\alpha}_{\mathrm{M}} = (1-\hat{\nu}_{\mathrm{T}})/\hat{\mu}_{\mathrm{T}} < 1$).

The value functions for *Daimyo* are given as follows.

$$\begin{aligned} V_{\mathrm{D}} &= (1-s)\delta V_{\mathrm{T}} + s\delta V_{\mathrm{D}}, \\ V_{\mathrm{T}} &= (1-q)\left\{p(R-R_{\mathrm{T}}) + (1-s)\delta V_{\mathrm{T}} + s\delta V_{\mathrm{D}}\right\} + q\delta V_{\mathrm{D}}. \end{aligned}$$

Solving two equations given above, we obtain

$$\begin{aligned} V_{\mathrm{D}} &= \frac{(1-s)\delta}{1-\delta}\frac{(1-q)p(R-R_{\mathrm{T}})}{1+q(1-s)\delta}, \\ V_{\mathrm{T}} &= \frac{(1-q)p(R-R_{\mathrm{T}})}{1-\delta} - \frac{\{q+(1-q)s\}\delta}{1-\delta}\frac{(1-q)p(R-R_{\mathrm{T}})}{1+q(1-s)\delta}. \end{aligned}$$

The condition that the *Daimyo* chooses to repay the loan after the project succeeds yields

$$[\mathrm{RT}] \quad R_{\mathrm{T}} = \frac{(1-q)p(1-s)\delta}{1+\{q+(1-q)p\}(1-s)\delta}R.$$

On the other hand, the value functions for merchants are as follow.

$$\begin{aligned} W_{\mathrm{M}} &= \max\{X_{\mathrm{T}}, X_{\mathrm{N}}\}, \\ W_{\mathrm{T}} &= (1-q)\{pR_{\mathrm{T}} - I + (1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}\} + q\delta W_{\mathrm{M}}, \\ X_{\mathrm{T}} &= \alpha_{\mathrm{M}}\{(1-s)\delta W_{\mathrm{T}} + s\delta W_{\mathrm{M}}\} + (1-\alpha_{\mathrm{M}})\delta W_{\mathrm{M}}, \\ X_{\mathrm{N}} &= (1-q)\lambda + \delta W_{\mathrm{M}}. \end{aligned}$$

Assuming $X_{\mathrm{N}} = \max\{X_{\mathrm{T}}, X_{\mathrm{N}}\}$ and solving for W_{M}, and W_{T} yield

$$\begin{aligned} W_{\mathrm{M}} &= \frac{1-q}{1-\delta}\lambda, \\ W_{\mathrm{T}} &= \frac{(1-q)(pR_{\mathrm{T}} - I) - \{q+(1-q)s\}\delta\Delta}{1-\delta}, \end{aligned}$$

where

$$\Delta = \frac{1-q}{1-(1-q)(1-s)\delta}(pR_{\mathrm{T}} - I - \lambda).$$

The equilibrium requires $X_{\mathrm{T}} = X_{\mathrm{N}}$. This condition lead to

$$[\mathrm{NT}] \quad \lambda = \alpha_{\mathrm{M}} \frac{(1-s)\delta}{1-q} \Delta.$$

Finally, the following stationary condition yields

$$[\mathrm{S}] \quad \nu_{\mathrm{T}} = (1-q)(1-s).$$

Comparing the results with those with both matching markets being viable, we obtain $R^*_{\mathrm{T}} > \hat{R}_{\mathrm{T}}$, which implies $W^*_{\mathrm{T}} > \hat{W}_{\mathrm{T}}$, and $\nu^*_{\mathrm{T}} > \hat{\nu}_{\mathrm{T}}$.