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Abstract

We study how tax and transfer policies affect economic growth and income inequality in a framework in which growth, market structure and time preferences are all endogenously determined. Firm-level investment in product quality drives economic growth, creating a demand for household savings to finance both market entry and in-house R&D. By distinguishing between affluent households that invest in financial assets and poor households that live hand-to-mouth, and linking the former's savings to an endogenously determined discount rate, we derive the conditions for a stable balanced growth path. We then explore the effects of taxing the wage income and asset income of affluent households, while redistributing the proceeds to poor households, and find that diminishing marginal impatience introduces a new channel where both higher growth and lower inequality can be achieved, through tax policies that influence market concentration.

Key Words: Endogenous Time Preferences; Diminishing Marginal Impatience; Endogenous Quality Growth; Wage Income Taxes; Interest Income Taxes

JEL Classifications: E00, O31, O41

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1 Introduction

While it is widely recognized that R&D is a key driver of economic growth, some scholars argue that there exists a trade-off between economic growth and income inequality (Aghion et al., 2019). This may indicate that an egalitarian tax policy by redistributing income from affluent to poor households reduces inequality only at the expense of slower growth. But, is this necessarily the case? In the present paper, we show that the trade-off between economic growth and income inequality may not be as severe as commonly suggested, especially when affluent households exhibit diminishing marginal impatience (DMI) in their consumption behavior, or equivalently increasing propensity to save in their savings behavior. Indeed, quite a number of empirical studies stress the importance of including DMI in the analysis of growth and inequality.[1] Our findings suggest that tax-cum-transfer fiscal policies have the potential to both enhance economic growth and reduce income inequality under DMI.

Specifically, we introduce a novel approach that extends the endogenous growth and endogenous market structure framework (Smulders and van de Klunder, 1995; Peretto, 1996; Aghion and Howitt, 1998; Kane and Peretto, 2020) to allow for endogenous time preferences. A key feature of the framework is heterogeneity in households: affluent households save and hold assets while poor households live 'hand to mouth' (Iwasa and Zhao, 2020). Under DMI, affluent households become more patient as their financial-asset wealth rises, leading to a falling intertemporal discount rate. Monopolistically competitive firms produce differentiated product varieties and also invest in in-house innovation to improve product quality. The model features a tension between market concentration and the rate of innovation: higher market concentration leads to larger firms that invest more in quality

[1]The assumption of DMI is supported by a number of empirical and theoretical studies showing that households discount the future at different rates, see for instance, Becker and Mulligan (1997), Barsky et al. (1997) and Hirose and Ikeda (2012). In particular, Lawrance (1991) and Warner and Pleeter (2001) find that more-educated households tend to have lower discount rates than less-educated ones. Also, some studies have found that the marginal propensity to save is considerably higher among wealthier households (Frederick et al., 2002; Mian et al., 2021).

innovation.

In equilibrium, a higher interest rate increases the cost of quality innovation, inducing firms to reduce R&D investment, which however raises operating profits, in turn attracting new entrants. Simultaneously, the higher interest rate directly increases market-entry costs, discouraging entry. These two forces generate the following mechanisms (as summarized in Lemma 1 later): when market-entry costs are more sensitive to interest-rate changes, higher interest rates lower market concentration and the rate of innovation; in contrast, when innovation costs are more sensitive, higher interest rates increase market concentration and the relationship between the interest rate and the innovation rate becomes convex.

We apply the framework to study the effects of redistributing income from affluent to poor households through the tax-cum-transfer of wage and interest incomes. Under DMI, raising the tax rate on wage income prompts affluent households to save more, increasing the supply of investment funds and thereby lowering the interest rate (whereas under constant marginal impatience, a wage income tax has no effect on saving behavior). A lower interest rate then leads to greater market concentration and a faster rate of innovation when the innovation effect dominates; when the market-entry cost effect dominates, however, the lower interest rate reduces market concentration and the innovation rate first falls before rising. Income inequality, as measured by the Gini coefficient, declines monotonically with the wage-income tax rate. As such, transferring wage income from affluent to poor households through taxation can reduce income inequality while increasing the long-run rate of economic growth–a result that is only possible with DMI.

Next, turning to the redistribution of affluent-household interest income to poor households, we find that an increase in the interest-income tax affects the saving behavior of affluent households through two channels: the lower net return on financial assets reduces the incentive to save, while the reduction in net income creates a countervailing incentive to save more. The disincentive of lower returns dominates, leading households to save less, which reduces the supply of investment funds and raises the interest rate. Thus, when the

innovation-cost effect dominates, higher taxes on interest income lower both market concentration and the innovation rate, and in this case, raising the tax rate decreases income equality but also reduces economic growth.

Conversely and more interestingly, when the market-entry cost effect dominates, higher interest-income taxes increase market concentration with the innovation rate first falling then rising, and in this case, income inequality becomes a convex function of the interest rate. As such, redistributing interest income from affluent to poor households brings three possible results: (i) reduces income inequality while lowering economic growth for low tax rates, (ii) reduces income inequality while increasing economic growth for mid-level tax rates, and (iii) raises both income inequality and the rate of growth for high tax rates. Intuitively, these cases are generated by the tension between lower market concentration and the sensitivity of innovation costs to interest rate adjustments in determining the innovation rate, coupled with the tension between the direct effect of the tax on interest income (negative) and the indirect effect of rising interest rates on affluent household income (positive).

Our analysis is closely related to the literature examining the relationship between income inequality and economic growth in models that focus on capital accumulation as the engine of growth (Galor and Zeira,1993; Aghion and Bolton, 1997; Matsuyama, 2000; Galor and Moav, 2004). Notably, Iwasa and Zhao (2020) develop a neoclassical growth model with affluent and poor households, and show that poor households may be better off in an unequal society under DMI. However, their analysis does not contain innovation.

Our work also builds on the antecedents that explore the general relationship between income inequality and innovation-based economic growth (Chou and Talmain, 1996; Foellmi and Zweimuller, 2006; Garcia-Penalosa and Wen, 2008; Aghion et al., 2019; Chu and Peretto, 2023).[2] In a closely related paper, Chu et al. (2025) introduce Schumpeterian

[2]Throughout history there has been a tension between technological progress and income inequality (Mokyr et al., 2015). A recent literature examines how income inequality may have supported innovation-based economic growth during industrial revolution (Madsen and Strulik, 2020, 2024; Chu et al., 2025).

growth with heterogeneous households and endogenous labor supply, showing that labor-income taxation does not affect long-run economic growth, while it may reduce consumption inequality but with ambiguous effects on income inequality. Our paper contributes to this literature by examining how egalitarian tax policy shapes the relationship between income inequality and economic growth in the long run.

The rest of the paper is organized as follows. In Section 2, we introduce our framework of both endogenous growth and endogenous market structure, as well as endogenous time preferences. Section 3 characterizes the long-run equilibrium and determines the conditions required for saddle-path stability. Section 4 solves for the long-run rate of quality innovation and investigates the effects of tax-cum-transfer policies on market concentration and economic growth. In Section 5, we calculate the Gini coefficient for the economy and examine how tax policy shapes the relationship between economic growth and income inequality. Section 6 concludes.

2 The Model Setup

This section introduces our model of endogenous growth and endogenous market structure with diminishing marginal impatience. At each moment in time, firms produce differentiated product varieties for household consumption and invest in innovation to improve the quality of products. Market entry for production requires the creation of a new product design and production capacity. Labor is the only factor of production, with each household supplying a single unit of labor inelastically. Households in the economy are separated into two types: Affluent households balance expenditure and savings as they earn labor income and invest in financial assets; Poor households spend all of their labor income on consumption and do not hold assets, i.e., they are the so-called 'hand-to-mouth' consumers.[3] We set the population share of affluent households exogenously to $s \in (0,1)$.

[3]Such consumers are more common than thought, even in rich countries. In 2023, 80.5 million workers age 16 and older in the U.S. were paid at hourly rates, representing 55.7 percent of all wage and salary workers. Among those paid by the hour, 81,000 workers earned exactly the prevailing federal minimum

Denoting the expenditure of affluent households by E and poor households by $\bar{E}$, the average expenditure of a household in the economy is $E_A = (1-s)\bar{E} + sE$. The structure of the model allows us to fix the time path of one nominal variable. Therefore, to simplify exposition, we set average household expenditure as the model numeraire; that is, $E_A = 1$ at all moments in time.

2.1 Affluent Households and Diminishing Marginal Impatience

Affluent households select an optimal expenditure path over an infinite time horizon. With time (t) flowing continuously, the lifetime utility of a typical affluent household is formulated as

$$U = \int_0^\infty \left(\frac{C(t)^{1-\mu}}{1-\mu} \right) e^{-\int_0^t \rho(A(\tau))d\tau} dt, \tag{1}$$

where C is household consumption and $\mu \neq 1$ measures the degree of relative risk aversion as in standard models.[4] Moreover, we incorporate endogenous time preferences, with a subjective discount rate of $\rho(A) = \theta A^{-\eta}$; that is, it is a function of household asset wealth (A), where $-\eta < 0$ is the wealth elasticity of time preference, and $\theta > 0$ is the exogenous rate of time preference that arises if time preference is perfectly wealth inelastic. In other words, if $\eta = 0$, then the rate of time preference collapses to the standard case where $\rho = \theta$ is a constant. Under this specification, households become more patient as they accumulate asset wealth; i.e., $d\rho(A)/dA < 0$. In the literature, Strulik (2012) uses the form $\rho(A) = \bar{\rho} + \theta A^{-\bar{\eta}}$, to examine cases where household asset wealth grows perpetually, driving the rate of time preference to the base level $\bar{\rho}$ in the long-run. For algebraic simplicity we set $\bar{\rho} = 0$. Nevertheless, in our framework affluent household wealth converges to a finite level in the steady state, ensuring that the rate of time preference remains endogenous in

wage of \$7.25 per hour. And in many developing countries such as China, physical 'flea' markets exist for hourly workers in different cities, where employers can count heads on the spot and hire workers by the hour.

[4]The empirical estimates of Hall (1988), Guvenen (2006), and Chiappori and Paiella (2011) suggest a value of $\mu = 2$ for the degree of relative risk aversion.

the long-run.

Affluent households maximize lifetime utility subject to a standard flow budget constraint:

$$\dot{A} = (1-\tau_A)rA + (1-\tau_L)w - E, \tag{2}$$

where r is the interest rate, w is the wage rate, and E is affluent household expenditure. Affluent households incur taxes on interest income at the rate τ_A and on wage income at the rate τ_L. In Appendix A, we derive the following Euler condition to describe the optimal saving-expenditure path for a typical affluent household:

$$\frac{\dot{E}}{E} = \frac{1}{\mu}\left((1-\tau_A)r - \rho - \frac{\eta E}{(\mu-1)A} + \eta\frac{\dot{A}}{A}\right) + \frac{(\mu-1)}{\mu}\frac{\dot{P}}{P}, \tag{3}$$

where P is the aggregate price index associated with household consumption. Expenditure dynamics reduce to the standard Euler condition when the wealth elasticity of time preference is set to zero; that is, when $\eta = 0$. If the wealth elasticity of substitution is negative ($-\eta < 0$), however, the household's optimal expenditure path includes two additional effects, as in Strulik (2012). The first is the negative, level-effect associated with current expenditure per unit of asset wealth. The second is a positive, growth-effect whereby an increase in the rate of growth in asset wealth ($\dot{A}/A$) leads to a steeper expenditure path as households become more patient, leading them to save more.

In contrast to affluent households, poor households spend all of their income on consumption: $\bar{E} = (1-\tau_L)w + T$, where T is a lump-sum transfer from the government to poor households, which is financed through the taxes on wage and interest income. We suppose that the wage income of poor households is taxed at the same rate as affluent households.

Given the expenditure path, both affluent and poor households allocate per-period expenditure across all of the currently available product varieties to maximize instantaneous utility. In particular, household consumption (C), and the corresponding aggregate price

index (P), can be written in a constant elasticity of substitution (CES) form:

$$C = \left(\int_0^{N(t)} (q(t,i)^{\alpha} c(t,i))^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad P = \left(\int_0^{N(t)} p(t,i)^{1-\varepsilon} q(t,i)^{\alpha(\varepsilon-1)} di \right)^{\frac{1}{1-\varepsilon}}, \quad (4)$$

where $p(i)$ and $c(i)$ are the price and consumption of a representative product i from the mass of varieties N currently available in the market. The quality of product variety i is denoted by $q(i)$ with α regulating the degree to which households value product quality. The constant elasticity of substitution across product varieties is denoted by $\varepsilon > 1$.

Households choose demand for a given product variety ($c(i)$) to maximize instantaneous utility (4) subject to a budget constraint. For affluent households, the budget constraint is given by $E = \int_0^N p(i)c(i)di$; For poor households, since they have no asset wealth, at each moment in time, they spend all of their labor income on consumption subject to the following budget constraint: $\bar{E} = (1-\tau_L)w + T = \int_0^N p(i)c(i)di$. Solving the static optimization problem for each household type, we obtain the combined aggregate demand for a representative product variety i from poor and affluent households as

$$x(i) = p(i)^{-\varepsilon} q(i)^{\alpha(\varepsilon-1)} P^{\varepsilon-1} L, \quad (5)$$

where our choice of average household expenditure as the model numeraire ($E_A = 1$) ensures that the aggregate market size for product varieties equals the population (L) of the economy.

2.2 Production

The production sector consists of a continuous mass of firms N that compete according to Dixit and Stiglitz (1977) monopolistic competition. Each firm produces a single product variety, ensuring an exact correspondence between the mass of firms and the mass of product varieties available at each moment in time. The production technology of a

representative firm i is

$$x(i) = l_X(i) - f, \tag{6}$$

where $x(i)$ is firm-level output, $l_X(i)$ is firm-level employment in production, and $f > 0$ is a per-period fixed cost measured in units of labor. Producing to meet the demand from both affluent and poor households (5), firms set price equal to a constant markup over unit cost to maximize per-period operating profit ($\pi(i) = p(i)x(i) - wl_X(i)$), giving an optimal price of $p(i) = \varepsilon w/(\varepsilon - 1)$. Note that a common industry-wide wage rate (w) ensures that all monopolistically competitive firms set the same optimal price.

2.3 Quality Innovation

Following the endogenous growth and endogenous market structure literature (Smulders and van de Klundert, 1995; Peretto, 1996, 2018; Kane and Peretto, 2020), firms invest in the improvement of product quality at each moment in time. The evolution of product quality for a representative firm i is then governed by

$$\dot{q}(i) = \xi q l_Q(i), \tag{7}$$

where $l_Q(i)$ is firm-level employment in innovation, $\xi > 0$ is a productivity parameter, and q is the average product quality of all market incumbents: $q = (\int_0^N q(i)di)/N$.

The innovation technology exhibits an intertemporal knowledge spillover (Romer, 1990). In particular, we assume that technical knowledge associated with product quality accumulates within the firm as a byproduct of investment in quality innovation. With a firm's stock of technical knowledge measured by its product quality $q(i)$, labor productivity in innovation is an increasing function of the weighted average of the technical knowledge of incumbent firms (q). As such, current efforts to improve product quality raise the labor productivity of future innovation efforts, generating endogenous quality growth.

Firms maximize firm value, effectively by setting employment. Denoting the per-period profit of firm i as $\Pi(i) = (p(i) - w)x(i) - wf - wl_Q(i)$, firm value equals the present discounted value of the future flow of profits:

$$v(i) = \int_0^{\infty} e^{-\int_0^t (r(t')+\delta)0dt'} \Pi(i)dt, \tag{8}$$

where $\delta \in (0,1)$ is a hazard rate measuring the probability that the firm is forced to exit the market due to an exogenous idiosyncratic shock at any given moment of time. The firm's optimal investment in quality innovation is captured by the following no-arbitrage condition:

$$r = \alpha(\varepsilon - 1)\xi(p - w)\frac{x}{w} + \frac{\dot{w}}{w} - \frac{\dot{q}}{q} - \delta, \tag{9}$$

where each firm disregards its own spillover associated with the effect of an improvement in firm-level quality ($q(i)$) on average quality (q), as the influence of each firm on industry-level values is negligible given its small market share under monopolistic competition. The first term on the right-hand side describes the marginal operating profit associated with the increase in demand that follows an improvement in product quality. The second and third terms capture the capital gains associated with changes in the cost of quality innovation, given the risk of being forced to exit the market at any given moment in time.

2.4 Product Market Entry

As is standard in the literature, firms incur costs as they design new products, create production capacity, and develop new supply networks. We assume that new firms enter the market with the average product quality of existing firms. In particular, following Peretto and Connolly (2007), we consider the following differential equation to describe

the dynamics of market entry and exit:

$$\dot{N} = \left(\frac{\beta N}{L}\right) L_R - \delta N, \tag{10}$$

where $\beta > 0$ is a productivity parameter and L_R is total employment in market entry. The first term on the right-hand side measures the total mass of new firms entering the market at each moment in time. We posit that market entry requires $L/(\beta N)$ units of labor. Specifically, with average household expenditure normalized to one, the population measures market size (L), with the cost of entry increasing in the market share of a representative firm (L/N). The basic intuition here is that the larger capacity and distribution requirements associated with a larger market share lead to greater entry costs. The second term measures the mass of firms that exit the market at each moment in time due to random idiosyncratic shocks.

Free market entry and exit reduce the value of the firm down to the cost of market entry:

$$v = \frac{wL}{\beta N}. \tag{11}$$

We take the time derivative of firm value (8), and use the free market entry condition (11) to pin down the following investment condition:

$$r = ((p-w)x - wf - wl_Q)\frac{\beta N}{wL} + \frac{\dot{w}}{w} - \frac{\dot{N}}{N} - \delta. \tag{12}$$

The first term on the right-hand side captures per-period profit relative to the cost of market entry. Together the second and third terms describe the capital gains associated with adjustments in market entry costs. The final term is the probability of exit due to an exogenous shock.

2.5 Asset Market Equilibrium

We first examine the asset market to derive an equilibrium condition for the determination of the interest rate, as a function of the population share of affluent households and the asset wealth of a representative affluent household. We then examine the level of product demand and the rate of quality growth that are consistent with the asset market equilibrium. Hereafter, we assume that product quality is symmetric across firms and drop the firm index i to simplify notation: $q(i) = q$.

The government assumes the role of taxation: taxing the asset income of affluent households as well as the wage income of all households, and transferring all proceeds to poor households. This tax and transfer system is operated under a balanced budget with no transaction costs. Thus, total tax revenue equals total transfers to poor households, $(1-s)TL = \tau_L wL + \tau_A rAsL$, generating the following transfer to each poor household:

$$T = \frac{\tau_A srA + \tau_L w}{1-s}. \tag{13}$$

Naturally, the size of the income transfer is increasing in the population share of affluent households, as well as asset and wage incomes.

At each moment in time, total household asset wealth equals total market capitalization ($sLA = Nv$). Thus, referencing the free market entry condition (11), the equilibrium value of asset wealth for a typical affluent household is directly linked with the wage rate as follows:

$$A = \frac{w}{s\beta}. \tag{14}$$

We employ (14) with the household flow budget constraint (2), the income transfer (13), and the model normalization $E_A = (1-s)((1-\tau_L)w + T) + sE = 1$ to solve for the interest

rate:

$$r = \tilde{r} + \frac{\dot{A}}{A}, \qquad \tilde{r} = \frac{1}{sA} - \beta, \tag{15}$$

where $\tilde{r}$ denotes the steady-state interest rate, with $A < 1/(s\beta)$ becoming a necessary condition for a positive interest rate ($\tilde{r} > 0$) in long-run equilibrium. Clearly, increases in either the population share of affluent households or their asset wealth lead to a decrease in the steady-state interest rate: $\partial \tilde{r}(A)/\partial A = -1/sA^2 < 0$; $\partial \tilde{r}(A)/\partial s = -1/s^2 A < 0$.

The conditions for asset market equilibrium allow us to describe product demand (5) in terms of market entry (N) and affluent household asset wealth (A):

$$x = \frac{(\varepsilon - 1)L}{s\varepsilon\beta AN}. \tag{16}$$

As affluent household wealth (A) is directly linked with the wage rate, a rise in A raises both labor income and product price. The price effect dominates, however, generating a negative relationship between product demand and affluent household wealth ($\partial x/\partial A < 0$). In addition, from (16) we see that product demand is naturally decreasing in the number of firms ($\partial x/\partial N < 0$), as more entry reduces the market share of each firm.

Similarly, combining the affluent household flow budget constraint (2) with the no-arbitrage condition for investment in quality innovation (9), and the asset market equilibrium conditions (14) and (15), we derive the rate of quality growth as a function of affluent household wealth (A) and market entry (N):

$$g \equiv \frac{\dot{q}}{q} = \frac{\alpha(\varepsilon - 1)\xi L}{s\varepsilon\beta AN} - \tilde{r} - \delta. \tag{17}$$

From this expression, one observes that an increase in affluent household wealth (A) has two opposing effects. On the one hand, the subsequent fall in the interest rate indicates a decrease in product demand that reduces the marginal return to investment in quality

improvements, but also lowers the cost of investment in innovation; An increase in market entry, on the other hand, straightforwardly lowers the representative firm's market share, reducing the marginal return to investment in innovation, and slowing the rate of quality growth.

The per-period firm profit associated with asset market equilibrium can be reduced to a function of affluent household wealth and market entry. Using the production technology (6), the innovation technology (7), product demand (16), and growth rate (17), we obtain

$$\Pi = \left(\frac{(1-\alpha(\varepsilon-1))L}{\varepsilon\beta sAN} - f + \frac{\tilde{r}+\delta}{\xi} \right) s\beta A. \tag{18}$$

An increase in market entry has a strictly negative effect on firm profit as market share is reduced ($\partial\Pi/\partial N < 0$). An increase in affluent household wealth, on the other hand, has three effects: the first is the negative effect of a fall in product demand; the second is the negative effect of a rise in innovation employment, as the fall in the interest rate stimulates investment in R&D; the third is the positive effect of an increase in the wage rate ($w = s\beta A$) that raises the overall scale and profit of each firm.

3 Long-run Equilibrium

In this section, we characterize the long-run balanced growth path of the economy. In particular, we study a steady state with constant values for both affluent household asset wealth ($\dot{A} = 0$) and market entry ($\dot{N} = 0$). The long-run equilibrium features a constant allocation of labor across production, quality innovation and market entry, and a constant positive rate of quality growth. The system is reduced to two differential equations to describe the evolution of affluent household wealth (A) and the dynamics of market entry and exit (N). We start with a benchmark dynamic system that arises with no taxes and no fiscal transfers between households (i.e., $\tau_A = \tau_L = 0$).

First, we combine (4), (14), and (15) with (3) to obtain the following differential equa-

tion to describe the evolution of affluent household asset wealth:

$$\frac{\dot{A}}{A} = \left(\rho - \tilde{r} + \frac{(1-(1-s)s\beta A)\eta}{(\mu-1)sA} + \alpha(\mu-1)g + \frac{(\mu-1)}{(\varepsilon-1)}\frac{\dot{N}}{N}\right)\left(\frac{1-s(1-s)}{\mu+(1-s(1-s))\eta}\right), \quad (19)$$

where the tax rates on asset and wage incomes have been set to zero ($\tau_A = \tau_L = 0$). The first two terms in parentheses on the right-hand side show that the rate of asset accumulation is increasing in the rate of time preference and decreasing in the interest rate. The third term captures the effects of changes in the ratio of affluent household expenditure to asset wealth (E/A), and depends on the asset elasticity of time preference (η). The fourth and fifth terms indicate that faster rates of quality growth and market entry always lead to a faster rate of asset accumulation.

Setting (19) equal to zero with $\dot{N} = 0$ yields the following steady-state locus for the long-run determination of affluent household wealth ($\dot{A} = 0$):

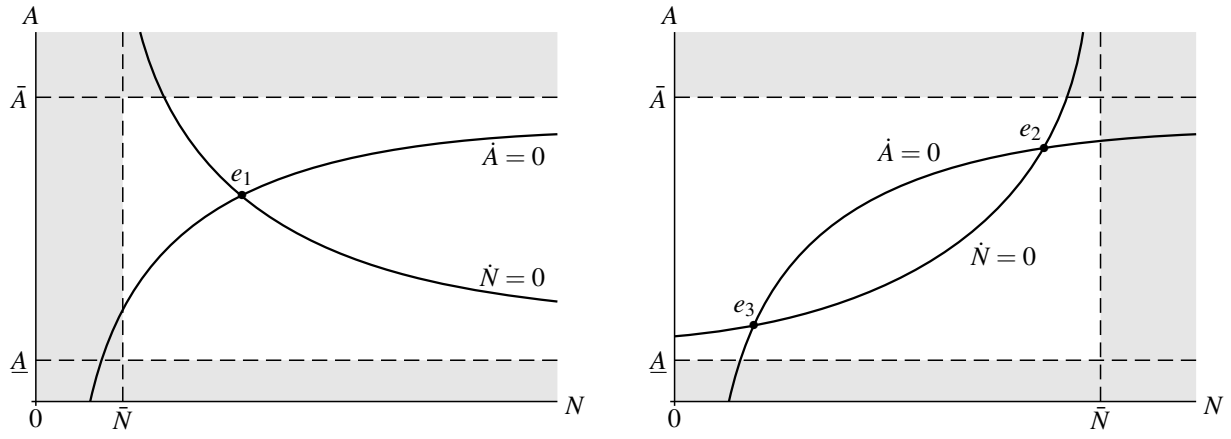
$$g = \frac{\tilde{r} - \rho}{\alpha(\mu-1)} - \frac{\eta(\tilde{r}+\beta s)}{\alpha(\mu-1)^2}, \quad (20)$$

where the rate of quality growth (g) is determined as a function of steady-state asset wealth ($\tilde{A}$) through (17), the steady-state interest rate depends on $\tilde{A}$ through (15), and the subjective discount rate is determined as a function of $\tilde{A}$ through $\rho(A) = \theta A^{-\eta}$.

On the left-hand side, an increase in the level of market entry slows the rate of quality growth, but a rise in affluent household wealth has an ambiguous effect. On the right-hand side, a rise in affluent household wealth lowers the interest rate with both positive and negative effects. In addition, the rate of time preference falls. In Appendix B, we show that the balance of these effects leads to a positive slope in N-A space, as illustrated by the $\dot{A} = 0$ curve in Figure 1.

Turning next to the dynamics of market entry, we substitute (7), (9), (14), (15), (16), (17), and (18) into the investment condition (12) to obtain the following differential equa-

Figure 1: Long-run Equilibrium



(a) $r_2 > r_1$ (b) $r_2 < r_1$

tion:

$$\frac{\dot{N}}{N} = \frac{(1-\alpha(\varepsilon-1))(\tilde{r}+\beta)}{\varepsilon} - \frac{(\xi f-\tilde{r}-\delta)\beta N}{\xi L} - (\tilde{r}+\delta). \tag{21}$$

Together the first and second terms on the right-hand side capture the ratio of industry profit to the total asset wealth of affluent households ($N\Pi/(sAL)$), and therefore describe the dividend yield. A rise in the steady-state interest rate ($\tilde{r}$) increases the dividend yield through two channels: a reduction in the wage rate that lowers the cost of market entry, and a decrease in optimal employment in innovation that raises per-period profit. The third term is the long-run risk-adjusted rate of return, with a rise in $\tilde{r}$ increasing the direct cost of financing market entry. A standard assumption employed in the literature to ensure stability of long-run equilibrium is that the rate of market entry is decreasing in the mass of market incumbents. As such, we focus our analysis on long-run equilibria that satisfy $\tilde{r} < r_1 \equiv \xi f - \delta$, as in Kane and Peretto (2020). Note that a zero bound on the steady-state interest rate implies $\xi f - \delta > 0$ as an additional parameter restriction.

Setting (21) to zero yields a steady-state condition for market entry, with the number of new market entrants just matching the number of firms forced to exit due to an exogenous

shock:

$$\tilde{N} = \frac{(1+\alpha)(\varepsilon-1)(r_2-\tilde{r})\xi L}{(r_1-\tilde{r})\varepsilon\beta}, \tag{22}$$

where $\tilde{r} = 1/(sA) - \beta$ and $r_2 \equiv (\beta - \alpha(\varepsilon-1)\beta - \varepsilon\delta)/((1+\alpha)(\varepsilon-1))$. The numerator captures the aggregate profit of the market, with an increase in r_2 improving profitability and raising the long-run number of firms. Hence, we find that $\tilde{r} < r_2$ is also required for a positive level of market entry in the long run, generating the additional parameter restriction $(1-\alpha(\varepsilon-1))\beta > \varepsilon\delta$, which is consistent with the parameter restriction adopted by Kane and Peretto (2020). The denominator describes the costs that firms incur each period. Thus, an increase in r_1 decreases the long-run number of firms.

Adjustments in the interest rate affect market entry through two channels. The numerator of the steady-state condition for market entry (22) indicates that a rise in the interest rate leads to an increase in the investment cost for market entry that lowers the number of firms. In contrast, the denominator shows that a rise in the interest rate increases the cost of investment in innovation, lowering firm-level employment in innovation and increasing the number of firms. Taking the derivative of the steady-state market entry condition with respect to affluent household asset wealth (A) yields

$$\frac{d\tilde{N}}{dA} = \frac{d\tilde{N}}{d\tilde{r}}\frac{d\tilde{r}}{dA}, \qquad \frac{d\tilde{N}}{d\tilde{r}} = \frac{(1+\alpha)(\varepsilon-1)(r_2-r_1)\xi L}{(r_1-\tilde{r})^2\varepsilon\beta}, \qquad \frac{d\tilde{r}}{dA} = -\frac{1}{sA^2} < 0.$$

Therefore, an increase in affluent household wealth decreases the interest rate ($d\tilde{r}/dA < 0$). Then, if $r_2 > r_1$, the innovation-cost effect dominates, and the fall in the interest rate induces greater investment in quality innovation, lowering per-period costs and reducing the number of firms ($d\tilde{N}/d\tilde{r} > 0$). As a result, in this case an increase in affluent household wealth leads to lower market entry ($d\tilde{N}/dA < 0$); Alternatively, if $r_2 < r_1$, the market-entry cost effect dominates, and the fall in the interest rate expands the number of firms ($d\tilde{N}/d\tilde{r} < 0$). In this case, an increase in affluent household wealth leads to higher market

entry ($d\tilde{N}/dA > 0$).

Figure 1 illustrates the long-run market entry condition in N-A space, as shown by the $\dot{N} = 0$ curve. Steady-state market entry is a hyperbolic function of affluent household asset wealth (A) with a horizontal transverse axis given by $\underline{A} = 1/(s(\xi f + \beta - \delta))$, and a vertical transverse axis given by $\bar{N} = (1+\alpha)(\varepsilon - 1)\xi L/(\varepsilon\beta)$. The shaded areas at the top and bottom of Figures 1(a) and 1(b) describe values of affluent household asset wealth that lie outside of the range of analysis; that is, for $A > \bar{A}$ we have $\tilde{r} < 0$, and for $A < \underline{A}$ we have $\tilde{r} > r_1$. As shown in Figure 1(a), the market entry condition has a negative slope ($dN/dA < 0$) if $r_2 > r_1$. Whereas, it has a positive slope ($dN/dA > 0$) if $r_2 < r_1$, as depicted in Figure 1(b).

In Appendix B, we evaluate a linear expansion of the dynamic system described by (21) and (19) around the steady state associated with (22) and (20), and obtain the following lemma, outlining two cases for a saddle-path stable long-run equilibrium.

Lemma 1 Without taxes and fiscal transfers between households ($\tau_A = \tau_L = 0$), a saddle-path stable long-run equilibrium arises for two cases:
(a) For $r_2 > r_1$, with $(dA/dN)|_{\dot{A}=0} - (dA/dN)|_{\dot{N}=0} > 0$, and $N > \bar{N}$;
(b) For $r_2 < r_1$, with $(dA/dN)|_{\dot{A}=0} - (dA/dN)|_{\dot{N}=0} < 0$, and $N < \bar{N}$;
where $r_1 \equiv \xi f - \delta$, $r_2 \equiv (\beta - \alpha(\varepsilon-1)\beta - \varepsilon\delta)/((1+\alpha)(\varepsilon-1))$, and $\bar{N} = (1+\alpha)(\varepsilon-1)\xi L/(\varepsilon\beta)$.
Proof: See Appendix B.

On the one hand, returning to Figure 1, we see that when the innovation cost effect dominates as in case (a), the market entry condition ($\dot{N} = 0$) has a negative slope, and there is a single stable long-run equilibrium at point e_1. On the other hand, when the market-entry cost effect dominates as in case (b), the market entry condition ($\dot{N} = 0$) has a positive slope and there may be two steady-state equilibria, as shown in panel (b). However, only the higher steady state at point e_2 satisfies the necessary slope ranking, ensuring a single stable long-run equilibrium.

4 Market Concentration and Quality Growth

After setting up the benchmark dynamic equilibrium in the previous section, we now study how changes in the tax rates on wage and interest incomes affect long-run quality growth. To simplify the analysis, we adopt the methodology in Smulders and van de Klundert (1995) to reduce the long-run equilibrium to two conditions that determine the steady-state values for the interest rate ($\tilde{r}$) and the rate of quality growth ($\tilde{g}$). The 'preference curve' describes the optimal saving behavior of affluent households and can be viewed as the supply schedule for savings. The 'technology curve' summarizes optimal investment of firms in quality innovation and market entry, and can be thought of as firms' demand schedule for savings.

4.1 The Preference Curve

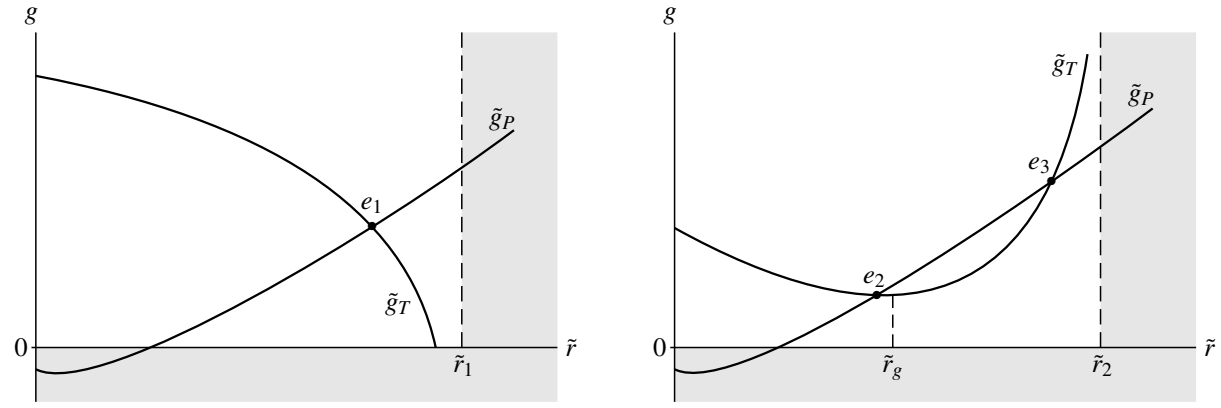
Beginning with the preference curve, we first solve (15) for asset wealth, $\tilde{A} = 1/(s(\tilde{r}+\beta))$, and then rewrite the rate of time preference as a function of the interest rate $\rho(\tilde{r}) = \theta(s(\tilde{r}+\beta))^{\eta}$. Substituting into (20) yields

$$\tilde{g}_P = \frac{1}{\alpha(\mu-1)}\left((1-\tau_A)\tilde{r} - \frac{\eta((1-\tau_A)\tilde{r}+(1-\tau_L)s\beta)}{\mu-1} - \theta(s(\tilde{r}+\beta))^{\eta}\right). \tag{23}$$

A rise in the interest rate has three effects on the supply of savings: (i) the direct increase in interest income; (ii) the rise in the ratio of expenditure to asset wealth ($E/A = (1-\tau_A)\tilde{r}+(1-\tau_L)s\beta$); (iii) the increase in the rate of time preference resulting from a fall in asset wealth.

Importantly, we suppose that the rate of time preference is wealth inelastic (i.e., $\eta \in [0,1)$) and that the degree of relative risk aversion satisfies $\mu > 1+\eta$. Under these parameter restrictions, the preference curve has a strictly positive slope for all economically

Figure 2: Long-run Quality Growth



(a) $r_2 > r_1$ (b) $r_2 < r_1$

relevant values of the interest rate ($\tilde{r}$) and the rate of quality growth ($\tilde{g}$):

$$\frac{d\tilde{g}_P}{d\tilde{r}} = \frac{\tilde{g}_P}{\tilde{r}+\beta} + \frac{[(\mu-\eta-1)(1-\tau_A)+\eta(1-\tau_L)s]\beta}{\alpha(\mu-1)^2(\tilde{r}+\beta)} + \frac{(1-\eta)\rho}{\alpha(\mu-1)(\tilde{r}+\beta)} > 0.$$

In Figure 2, the preference curve is a convex function of the interest rate ($\tilde{r}$), with a negative vertical intercept and a minimum determined implicitly at $(\tilde{r}+\beta)^{1-\eta} = (\mu-1)\theta\eta s^{\eta}/((1-\tau_A)(\mu-1-\eta)) > 0$. As such, the preference curve has a positive slope for all values of the interest rate that ensure a positive rate of quality growth. Note that the slope of the preference curve converges to a constant as the interest rate takes large values: $\lim_{\tilde{r}\to\infty} d\tilde{g}_P/d\tilde{r} = (1-\tau_A)(\mu-1-\eta)/(\alpha(\mu-1)^2) > 0)$.

4.2 The Technology Curve

Turning next to the technology curve, we substitute (15) into (17), and evoke (22), to obtain quality growth as a function of the interest rate:

$$\tilde{g}_T = \frac{\alpha(\varepsilon-1)(\tilde{r}+\beta)\xi L}{\varepsilon\beta\tilde{N}(\tilde{r})} - \tilde{r} - \delta. \tag{24}$$

A rise in the interest rate has three effects on the demand for savings: (i) an increase in product demand ($\tilde{r}+\beta$) raises the marginal value of quality improvement; (ii) a change in the level of market entry ($\tilde{N}$) may have a positive or negative impact on this marginal value, depending on the direction of the change in the number of firms; and, (iii) an increase in the direct cost of investing in innovation ($\tilde{r}+\delta$).

Next, taking the derivative of the technology curve (24) with respect to the interest rate, we have

$$\frac{d\tilde{g}_T}{d\tilde{r}} = -\left((\varepsilon-1)+\frac{((\beta-\varepsilon\delta)-(\varepsilon-1)\tilde{r})}{\tilde{N}}\frac{d\tilde{N}}{d\tilde{r}}\right)\frac{\xi L}{\varepsilon\beta\tilde{N}}.$$

The first term in parenthesis is strictly positive. For the second term, recalling that $\tilde{r}<\tilde{r}_2$ is required for a positive level of market entry ($\tilde{N}>0$), the numerator is strictly positive since $\tilde{r}_2<(\beta-\varepsilon\delta)/(\varepsilon-1)$. As such, the sign of this term depends on the direction of the change in market entry ($d\tilde{N}/d\tilde{r}$), and is therefore determined by the parameter condition (r_2-r_1), as summarized in Lemma 1. (i) For case (a), the innovation-cost effect dominates, and an increase in the interest rate raises the number of firms in the market ($d\tilde{N}/d\tilde{r}>0$), ensuring that the second term in parenthesis is positive ($\lim_{\tilde{r}\to\tilde{r}_1}(1/\tilde{N})(d\tilde{N}/d\tilde{r})=\infty$). Thus, as depicted in Figure 2(a), the slope of the technology curve is strictly negative. (ii) For case (b), the market-entry cost effect dominates, and an increase in the interest rate reduces the number of firms in the market ($d\tilde{N}/d\tilde{r}<0$), making the second term in parentheses negative. Then, as illustrated in Figure 2(b), the technology curve is a convex function of the interest rate with a minimum at $\tilde{r}_g$: For $\tilde{r}<\tilde{r}_g$, the first term dominates and the slope is negative; For $\tilde{r}>\tilde{r}_g$, the second term dominates and the slope turns positive as the influence of the adjustment in market entry increases ($\lim_{\tilde{r}\to\tilde{r}_2}(1/\tilde{N})(d\tilde{N}/d\tilde{r})=-\infty$).

4.3 Steady-state Quality Growth

The steady-state rate of quality growth is determined at the intersection of the technology and preference curves, with the long-run equilibria matching those illustrated in Figure 1. Matching the cases outlined in Lemma 1, for case (a), there is a single saddle-path stable long-run equilibrium at point e_1 as in Figure 2(a); For case (b), although multiple equilibria exist as in Figure 2(b), only the steady state at point e_2 satisfies the conditions for saddle-path stability, as outlined in Lemma 1. Note that the long-run rate of quality growth has the feature of scale neutrality, which is standard in frameworks of endogenous growth and endogenous market structure. As an increase in the size of the population (L) is fully absorbed by an increase in the steady-state mass of firms (22), there is no impact on the long-run interest rate ($\tilde{r}$) or the rate of quality growth ($\tilde{g}$).

4.4 Tax Policies

First, we consider the effects of a change in the tax rate on wage income.

Proposition 1 (Market concentration and quality growth with wage tax): An increase in the wage income tax rate lowers the interest rate ($d\tilde{r}/d\tau_L < 0$). Using Lemma 1, (a) if $r_2 > r_1$, market concentration increases ($d\tilde{N}/d\tau_L < 0$) and the rate of quality growth rises ($d\tilde{g}/d\tau_L > 0$); (b) If $r_2 < r_1$, market concentration decreases ($d\tilde{N}/d\tau_L > 0$), and the rate of quality growth falls ($d\tilde{g}/d\tau_L < 0$) for $\tilde{r} > \tilde{r}_g$, but then rises ($d\tilde{g}/d\tau_L > 0$) for $\tilde{r} < \tilde{r}_g$.
Proof: See Appendix C.

The economic intuition behind these results is clarified using Figure 2. From the preference curve, we obtain the following derivative:

$$\frac{\partial \tilde{g}_P}{\partial \tau_L} = \frac{\eta s \beta}{\alpha(\mu-1)^2} > 0.$$

Specifically, under DMI, the increase in the tax rate on wage income (τ_L) reduces the current level of expenditure per unit of asset wealth ($E/A = (1-\tau_A)\tilde{r} + (1-\tau_L)s\beta$), inducing

affluent households to increase their asset holdings. As a consequence, the supply of savings expands, causing an upward shift in the preference curve, and the size of this shift rises in the wealth elasticity of time preference (η). In contrast, when $\eta = 0$ and $\rho = \theta$, DMI disappears, and then adjustments in the tax rate on wage income have no effect on the economy.

Furthermore, the upward shift in the preference curve lowers the interest rate ($d\tilde{r}/d\tau_L < 0$), as affluent household asset wealth decreases. Following Lemma 1, in case (a), the number of firms falls ($d\tilde{N}/d\tau_L < 0$), leading to larger market shares and an increase in market concentration. Firm-level investment in innovation increases, and the rate of quality growth rises as the economy moves upwards along the technology curve in Figure 2(a). In contrast, for case (b), the fall in the interest rate leads to an increase in the number of firms ($d\tilde{N}/d\tau_L > 0$) that reduces market shares and lowers market concentration. The rate of quality growth initially falls, but then rises, as the economy moves leftward along the technology curve in Figure 2(b).

Next, we examine the effects of changes in the tax rate on interest income. The results are summarized in the following proposition.

Proposition 2 (Market concentration and quality growth with interest tax): An increase in the interest-income tax rate raises the interest rate ($d\tilde{r}/d\tau_A > 0$). (a) If $r_2 > r_1$, market concentration decreases ($d\tilde{N}/d\tau_A > 0$) and the rate of quality growth falls ($d\tilde{g}/d\tau_A < 0$). (b) If $r_2 < r_1$, market concentration increases ($d\tilde{N}/d\tau_A < 0$), and the rate of quality growth ($\tilde{g}$) falls ($d\tilde{g}/d\tau_A < 0$) for $\tilde{r} < \tilde{r}_g$, but then rises ($d\tilde{g}/d\tau_A > 0$) for $\tilde{r} > \tilde{r}_g$.
Proof: See Appendix C.

We examine these results using Figure 2, through a shift in the preference curve:

$$\frac{\partial \tilde{g}_P}{\partial \tau_A} = -\frac{(\mu - \eta - 1)\tilde{r}}{\alpha(\mu - 1)^2} < 0.$$

An increase in τ_A affects the supply of savings through two channels. The first is the

negative direct effect of a reduction in the net return on asset wealth ($(1-\tau_A)\tilde{r}$). The second is the positive indirect effect of a decrease in the expenditure to asset wealth ratio (E/A) of affluent households. As we have seen, DMI introduces a level effect into household saving behavior, whereby a decrease in (E/A) induces affluent households to increase their asset holdings, expanding the supply of savings. The strength of this second channel depends on the wealth elasticity of time preference (η), and therefore does not occur without DMI (i.e., when $\eta = 0$). Generally, with the parameter restriction $\mu - \eta - 1 > 0$, the negative direct effect always dominates, and the increase in the tax rate on interest income reduces the supply of savings, causing a downward shift in the preference curve.

After the downward shift in the preference curve, the interest rate ($\tilde{r}$) rises ($d\tilde{r}/d\tau_A > 0$) and the direction of the adjustments in market concentration and quality growth depend on the sign of the parameter condition $r_2 - r_1$, following Lemma 1. In case (a), the rise in the interest rate leads to an increase in the number of firms ($d\tilde{N}/d\tilde{r} > 0$) that lowers market shares and reduces market concentration. As illustrated in Figure 2(a), the economy moves rightward along the technology curve, reducing the rate of quality growth ($d\tilde{g}/d\tau_A < 0$), as firms lower their investment in innovation. In case (b), the rise in the interest rate leads to a decrease in the number of firms ($d\tilde{N}/d\tilde{r} < 0$) that raises market shares and increases market concentration. As shown in Figure 2(b), the economy moves rightward along the technology curve ($\tilde{g}_T$), and the rate of quality growth first falls ($d\tilde{g}/d\tau_A < 0$) for $\tilde{r} < \tilde{r}_g$, but then rises ($d\tilde{g}/d\tau_A > 0$) for $\tilde{r} > \tilde{r}_g$.

5 Income Inequality and Quality Growth

In this section, we derive the Gini coefficient for the economy, and then examine how tax-cum-transfer policies shape the relationship between income inequality and long-run quality growth. Along the balanced growth path, household income equals expenditure for both affluent households ($I = E$) and poor households ($\bar{I} = \bar{E}$). With only two income groups in the economy, the Gini coefficient is calculated as the difference between the total

income share and the population share of affluent households:

$$G = s(I-1), \qquad I = \frac{(1-\tau_A)\tilde{r} + (1-\tau_L)s\beta}{s(\tilde{r}+\beta)}, \tag{25}$$

where we have used the household budget constraint (2), the steady-state interest rate (15) and the model numeraire $E_A = I_A = sI + (1-s)\bar{I} = 1$. With average household income normalized to one, a positive Gini coefficient requires that affluent household income be greater than one ($I > 1$) and poor household income be less than one ($\bar{I} < 1$). Naturally, the Gini coefficient is an increasing function of affluent household income: $dG/dI = s > 0$.

We first investigate the relationship between income inequality and the interest rate:

$$\frac{dG}{d\tilde{r}} = \frac{(1-\tau_A-(1-\tau_L)s)\beta}{(\tilde{r}+\beta)^2} > 0. \tag{26}$$

A rise in the interest rate raises interest income but lowers wage income ($w = \beta/(\tilde{r}+\beta)$). Naturally, when the interest rate is equal to zero ($\tilde{r} = 0$), affluent households do not earn interest income and $I = (1-\tau_L)w < 1$. Because we are interested in how tax policy influences the relationship between income inequality and quality growth when affluent households have higher net incomes than poor households, we restrict our analysis to cases where $I > 1$ and thus $G > 0$, requiring $1-\tau_A-(1-\tau_L)s > 0$ and $\tilde{r} > \tau_L s\beta/(1-s-\tau_A)$. Then, a rise in the interest rate leads to greater income inequality ($dG/d\tilde{r} > 0$), which is intuitive since only affluent households have interest income.

Next, analyzing how taxes on wages influence income inequality and quality growth, we obtain

Proposition 3 (Income inequality and quality growth with wage tax) : (a) If $r_2 > r_1$, a rise in the wage income tax (τ_L) lowers income inequality ($dG/d\tau_L < 0$) while raising the rate of quality growth ($d\tilde{g}/d\tau_L > 0$). (b) If $r_2 < r_1$, income inequality falls ($dG/d\tau_L < 0$) while the rate of quality growth initially decreases ($d\tilde{g}/d\tau_L < 0$) for $\tilde{r} > \tilde{r}_g$ and then increases ($d\tilde{g}/d\tau_L > 0$) for $\tilde{r} < \tilde{r}_g$.

To examine these results in detail, we take the derivative of the Gini coefficient (25) with respect to the tax rate on wage income to obtain

$$\frac{dG}{d\tau_L} = -\frac{s\beta}{\tilde{r}+\beta} + \frac{(1-\tau_A-(1-\tau_L)s)\beta}{(\tilde{r}+\beta)^2}\frac{d\tilde{r}}{d\tau_L} < 0.$$

The first term on the right-hand side describes the direct decrease in income inequality that coincides with taxing the wage income of all households and transferring the revenue to poor households. Referencing Proposition 1, the second term is also negative and shows the indirect effect associated with the fall in the interest rate that occurs after an increase in wage taxes ($d\tilde{r}/d\tau_L < 0$). Since both the direct and indirect tax effects are negative, we conclude that an increase in the tax rate on wage income reduces income inequality ($dG/d\tau_L < 0$). Note that the tax rate on wage income has no influence on the interest rate without DMI.

Further, we examine how adjustments in the tax rate on wage income shape the relationship between quality growth and income inequality. While an increase in τ_L always reduces inequality, returning to Figure 1, we find several cases for the response of firm-level investment in quality innovation, depending on Lemma 1. In case (a), the rate of quality growth rises as the increase in τ_L shifts the preference curve upwards. Accordingly, higher taxes on wage income lead to lower income inequality and a faster rate of quality growth. In contrast, for case (b), the rate of quality growth initially decreases ($\tilde{r} > \tilde{r}_g$), before increasing for ($\tilde{r} < \tilde{r}_g$) as the preference curve shifts upwards, and income inequality is reduced.

Finally, studying how taxes on interest income impact inequality and quality growth, we obtain:

Proposition 4 (Income inequality and quality growth with interest tax): (a) If $r_2 > r_1$, an increase in the tax rate on interest income (τ_A) lowers income inequality ($dG/d\tau_A < 0$) while decreasing the rate of quality growth ($d\tilde{g}/d\tau_A < 0$). (b) If $r_2 < r_1$, the Gini coefficient

(G) is convex in τ_A with a minimum at $\tilde{r}_G$, yielding three cases as the interest rate rises: income inequality and quality growth both fall for $\tilde{r} < \tilde{r}_g$; income inequality falls while quality growth rises for $\tilde{r} \in (\tilde{r}_g, \tilde{r}_G)$; and income inequality and quality growth both rise for $\tilde{r} > \tilde{r}_G$.

Proof: See Appendix D.

This proposition gives rise to the important possibility that both economic growth and income equality can be achieved simultaneously. We provide detailed intuition below. The derivative of the Gini coefficient (25) with respect to the tax rate on interest income yields

$$\frac{dG}{d\tau_A} = -\frac{\tilde{r}}{\tilde{r}+\beta} + \frac{(1-\tau_A-(1-\tau_L)s)\beta}{(\tilde{r}+\beta)^2}\frac{d\tilde{r}}{d\tau_A}. \tag{27}$$

The first term on the right-hand side is negative and captures the direct reduction in income inequality that results from taxing the interest income of affluent households and transferring the proceeds to poor households. The second term is positive, and shows the indirect effect associated with the rise in the interest rate that follows an increase in the tax rate on interest income (i.e., $d\tilde{r}/\tau_A > 0$), referencing the results of Proposition 2.

In Appendix D, we demonstrate that the Gini coefficient is convex in the tax rate on interest income with a minimum at the tax rate (τ_A) that generates an interest rate of $\tilde{r} = \tilde{r}_G$. Accordingly, the direct effect dominates for $\tilde{r} < \tilde{r}_G$, with an increase in the tax rate on interest income lowering income inequality $(dG/d\tau_A < 0)$; alternatively, for $\tilde{r} > \tilde{r}_G$ income inequality rises $(dG/d\tau_A > 0)$.

Pairing the effects of changes in the interest income tax on income inequality and quality growth, we have $\tilde{r}_g < \tilde{r}_G$, which yields three cases: (i) an increase in τ_A raises the interest rate, and income inequality and the rate of quality growth initially fall for $\tilde{r} < \tilde{r}_g$; (ii) subsequently, income inequality continues to fall and the rate of quality growth rises for $\tilde{r} \in (\tilde{r}_g, \tilde{r}_G)$; (iii) lastly, both income inequality and the rate of quality growth increase for $\tilde{r} > \tilde{r}_G$. Nevertheless, in general the relationship between income inequality and quality

growth, which arises as the tax rate on interest income adjusts, depends on the current level of the interest rate.

6 Conclusion

In this paper, we have introduced a Schumpeterian growth model with diminishing marginal impatience (DMI) to examine whether egalitarian tax policy that redistributes income from affluent to poor households only reduces income inequality at the expense of slower economic growth. A key feature of the model is heterogeneity in households: affluent households save and hold assets while poor households live 'hand to mouth.' Under DMI, affluent households become more patient as they accumulate asset wealth. Monopolistically competitive firms produce differentiated product varieties and invest in in-house R&D to improve product quality. The model features a tension between market concentration and the rate of innovation: higher market concentration leads to larger firms that invest more in quality innovation.

With DMI, income tax policy influences the saving behavior of affluent households, leading to adjustments in the interest rate that affect firm-level investment in market entry and quality innovation. Specifically, an increase in the interest rate, (i) lowers market concentration and the rate of innovation when innovation costs are relatively sensitive to interest rate changes; (ii) alternatively, when the cost of market entry is relatively sensitive to interest rate adjustments, market concentration is increased and the innovation rate becomes convex in the interest rate.

Using the above mechanisms on the interest rate, we study the effects of transferring income from affluent to poor households through the tax-cum-transfer of wage and interest incomes, and find that an increase in either type of income tax can lower income inequality while raising the innovation rate, depending on the relative sensitivities of investment in market entry and quality innovation to interest rate adjustments. Importantly, the tax-cum-transfer of wage income can reduce income inequality while raising the rate of

economic growth—but only if affluent household saving behavior exhibits DMI. These results suggest that, under DMI, redistributive tax policies need not entail a trade-off between income inequality and economic growth.

Appendix A: Euler Equation for Household Expenditure

In this appendix we provide a derivation of the Euler equation used to describe optimal household saving behavior following the method introduced by Strulik (2012). The household's dynamic utility maximization problem is solved using the following Hamiltonian function:

$$H = \left(\frac{C^{1-\mu}}{1-\mu}\right) e^{\zeta} + \lambda_1((1-\tau_A)rA + (1-\tau_L)w - E) - \lambda_2\rho(A),$$

where λ_1 and λ_2 are the co-state variables associated with state variables A and ζ, and $\zeta = -\int_0^t \rho(A(t'))dt'$. The first order conditions for optimization are

$$\frac{\partial H}{\partial C} = C^{-\mu}e^{\zeta} - \lambda_1 P = 0, \quad \frac{\partial H}{\partial A} = \lambda_1(1-\tau_A)r - \lambda_2\frac{d\rho(A)}{dA} = -\dot{\lambda}_1, \quad \frac{\partial H}{\partial \zeta} = \left(\frac{C^{1-\mu}}{1-\mu}\right)e^{\zeta} = -\dot{\lambda}_2.$$

In addition, optimization satisfies the following transversality condition $\lim_{t\to\infty} H = 0$.

Noting that $\dot{\zeta} = -\rho(A)$, we use the first order conditions in the time derivative of the Hamiltonian function to show that

$$\frac{dH}{dt} = \frac{\partial H}{\partial C}\dot{C} + \frac{\partial H}{\partial A}\dot{A} + \frac{\partial H}{\partial \zeta}\dot{\zeta} + \frac{\partial H}{\partial \xi_1}\dot{\lambda}_1 + \frac{\partial H}{\partial \xi_2}\dot{\lambda}_2 = 0.$$

Then, from the transversality condition $\lim_{t\to\infty} H = 0$, we have $H = 0$ at all moments in time. Setting the Hamiltonian function equal to zero, and substituting in the first order

condition for consumption with $E = PC$, yields

$$\frac{\lambda_2}{\lambda_1} = \frac{1}{\rho(A)}\left(\frac{E}{1-\mu} + (1-\tau_A)rA + (1-\tau_L)w - E\right). \tag{A1}$$

In addition, the first order conditions for consumption and asset wealth imply

$$(1-\tau_A)r - \frac{\lambda_2}{\lambda_1}\frac{d\rho(A)}{dA} = \mu\frac{\dot{C}}{C} + \frac{\dot{P}}{P} + \rho. \tag{A2}$$

Combining (A1) and (A2) to remove the co-state variables (λ_2/λ_1), we have

$$(1-\tau_A)r - \left(\frac{E}{1-\mu} + (1-\tau_A)rA + (1-\tau_L)w - E\right)\frac{d\rho(A)}{\rho(A)dA} = \mu\frac{\dot{C}}{C} + \frac{\dot{P}}{P} + \rho(A).$$

Reorganizing the above results yields the Euler equation (3) for affluent household expenditure.

$$\frac{\dot{E}}{E} = \frac{1}{\mu}\left((1-\tau_A)r - \rho(A) - \frac{E}{(1-\mu)}\frac{d\rho(A)}{\rho(A)dA} - \frac{d\rho(A)}{dA}\frac{\dot{A}}{\rho(A)}\right) + \frac{(\mu-1)}{\mu}\frac{\dot{P}}{P}.$$

Appendix B: Local Stability Analysis

In this appendix, we evaluate a Taylor expansion of the dynamic system described by (19) and (21) around a steady state with a constant value for $(\dot{A} = 0)$ and a constant level of market entry $(\dot{N} = 0)$. The stability analysis focuses on the case with no financial transfers (i.e., $\tau_A = \tau_L = 0$). We rewrite the steady-state conditions for affluent household wealth (20) and market entry (22) as follows:

$$\Omega_1 = g + \frac{\eta(\tilde{r} + \beta s)}{\alpha(\mu-1)^2 sA} - \frac{\tilde{r} - \rho}{\alpha(\mu-1)},$$
$$\Omega_2 = \frac{\Pi N}{sAL} - \tilde{r} - \delta.$$

The partial derivatives of (19) and (21) with respect to A and N are

$$\begin{aligned}
\frac{\partial \dot{A}}{\partial A} &= \left((1-\eta)\rho + \alpha(\mu-1)(\beta-\delta) + \frac{((\mu-1)s+(1-s)\eta)\beta}{(\mu-1)s} + \frac{(\mu-1)A}{(\varepsilon-1)N}\frac{\partial \dot{N}}{\partial A} \right) \omega, \\
\frac{\partial \dot{A}}{\partial N} &= -\left(\frac{\alpha(\mu-1)\alpha(\varepsilon-1)\xi L}{\varepsilon\beta s N^2} - \frac{(\mu-1)A}{(\varepsilon-1)N}\frac{\partial \dot{N}}{\partial N} \right) \omega, \\
\frac{\partial \dot{N}}{\partial A} &= -\left(\frac{\varepsilon\beta N - (1+\alpha)(\varepsilon-1)\xi L}{\varepsilon\xi s A^2 L} \right) N, \\
\frac{\partial \dot{N}}{\partial N} &= \left(\frac{(1+\alpha)(\varepsilon-1)\xi f - (1-\alpha(\varepsilon-1))(\beta-\delta)}{\varepsilon\beta N - (1+\alpha)(\varepsilon-1)\xi L} \right) \beta N,
\end{aligned}$$

where $\omega = (1-s(1-s))/(\mu+(1-s(1-s))\eta) > 0$. Although the level of market entry (N) is a state variable, affluent household wealth (A) is linked directly with the wage rate (w) through the asset market (14), and is therefore a control variable. As such, with one state variable and one control variable, we require one positive and one negative characteristic root for a saddlepath stable long-run equilibrium. This will be the case when the determinant for the Jacobian matrix of the linearized system is negative:

$$|J| = \frac{\partial \dot{A}}{\partial A}\frac{\partial \dot{N}}{\partial N} - \frac{\partial \dot{A}}{\partial N}\frac{\partial \dot{N}}{\partial A} = \left(\frac{dA}{dN}\Big|_{\dot{A}=0} - \frac{dA}{dN}\Big|_{\dot{N}=0} \right) \frac{\partial \Omega_1}{\partial A}\frac{\partial \Omega_2}{\partial A}\frac{AN}{(\mu+\eta)} < 0,$$

where

$$\begin{aligned}
\frac{\partial \Omega_1}{\partial A} &= (1-\eta)\rho + \alpha(\mu-1)(\beta-\delta) + \frac{((\mu-1)s+(1-s)\eta)\beta}{(\mu-1)s} > 0, \\
\frac{\partial \Omega_2}{\partial A} &= \frac{(1+\alpha)(\varepsilon-1)\xi L - \varepsilon\beta N}{\varepsilon\xi s A^2 L}.
\end{aligned}$$

And,

$$\begin{aligned}
\frac{dA}{dN}\Big|_{\dot{A}=0} &= \frac{\alpha^2(\mu-1)^2(\varepsilon-1)\xi L}{[(1-\eta)(\mu-1)\rho + \alpha(\mu-1)^2(\beta-\delta) + ((\mu-1)+(1-s)\eta)\beta]\varepsilon\beta s N^2} > 0, \\
\frac{dA}{dN}\Big|_{\dot{N}=0} &= \frac{((1+\alpha)(\varepsilon-1)\xi f - (1-\alpha(\varepsilon-1))(\beta-\delta))\beta\varepsilon s A^2 L}{(\varepsilon\beta N - (1+\alpha)(\varepsilon-1)\xi L)^2}.
\end{aligned}$$

Appendix C: Steady-state Comparative Statics

This appendix provides a comparative static analysis of the effects of taxes on wage income (τ_L) and interest income (τ_A) on the long-run interest rate ($\tilde{r}$) and quality growth ($\tilde{g}$). Using (23) and (24), the comparative statics associated with changes in the wage income tax rate are calculated as follows:

$$\frac{d\tilde{g}}{d\tau_L} = \frac{\partial \tilde{g}_P}{\partial \tau_L}\frac{d\tilde{g}_T}{d\tilde{r}}\left(\frac{d\tilde{g}_T}{d\tilde{r}} - \frac{d\tilde{g}_P}{d\tilde{r}}\right)^{-1}, \qquad \frac{d\tilde{r}}{d\tau_L} = \frac{\partial \tilde{g}_P}{\partial \tau_L}\left(\frac{d\tilde{g}_T}{d\tilde{r}} - \frac{d\tilde{g}_P}{d\tilde{r}}\right)^{-1},$$

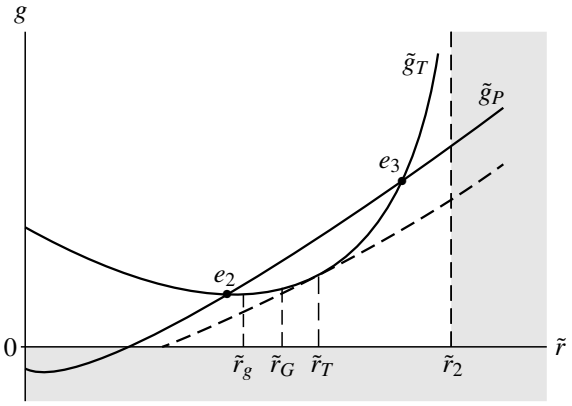
where $\partial \tilde{g}_P/\partial \tau_L > 0$. There are two cases for the signs of these comparative statics. In case (a) for $r_2 > r_1$, the slope of the technology curve is negative ($d\tilde{g}_T/d\tilde{r} < 0$), and $d\tilde{g}_T/d\tilde{r} - d\tilde{g}_P/d\tilde{r} < 0$. As such, we have $d\tilde{g}/d\tau_L > 0$ and $d\tilde{r}/d\tau_L < 0$. As $d\tilde{N}/d\tilde{r} > 0$, we have $d\tilde{N}/d\tau_L < 0$. In case (b) for $r_2 < r_1$, the long-run equilibrium is described by point e_2 in Figure 2(a). This long-run equilibrium is only stable if the preference curve ($\tilde{g}_P$) has a greater slope than the technology curve ($\tilde{g}_T$); that is, if $d\tilde{g}_T/d\tilde{r} - d\tilde{g}_P/d\tilde{r} < 0$. Thus, $d\tilde{g}/d\tau_L < 0$ when $d\tilde{g}_T/d\tilde{r} > 0$, and $d\tilde{g}/d\tau_L > 0$ when $d\tilde{g}_T/d\tilde{r} < 0$. Moreover, $d\tilde{r}/d\tau_L < 0$. Therefore, as $d\tilde{N}/d\tilde{r} < 0$ for $r_2 < r_1$, we have $d\tilde{N}/d\tau_L < 0$. This completes the proof of Proposition 1.

Next, using (23) and (24), we calculate the following comparative statics for changes in the interest income tax rate:

$$\frac{d\tilde{g}}{d\tau_A} = \frac{\partial \tilde{g}_P}{\partial \tau_A}\frac{d\tilde{g}_T}{d\tilde{r}}\left(\frac{d\tilde{g}_T}{d\tilde{r}} - \frac{d\tilde{g}_P}{d\tilde{r}}\right)^{-1}, \qquad \frac{d\tilde{r}}{d\tau_A} = \frac{\partial \tilde{g}_P}{\partial \tau_A}\left(\frac{d\tilde{g}_T}{d\tilde{r}} - \frac{d\tilde{g}_P}{d\tilde{r}}\right)^{-1}, \tag{C1}$$

where $\partial \tilde{g}_P/\partial \tau_A < 0$. There are once again two cases for the signs of these comparative statics. In case (a) for $r_2 > r_1$, the long-run equilibrium is described by point e_1 in Figure 2(a), and the slope of the technology curve is negative ($d\tilde{g}_T/d\tilde{r} < 0$), ensuring that $d\tilde{g}_T/d\tilde{r} - d\tilde{g}_P/d\tilde{r} < 0$. Accordingly, we have $d\tilde{g}/d\tau_A < 0$ and $d\tilde{r}/d\tau_A > 0$. For $r_2 > r_1$, as $d\tilde{N}/d\tilde{r} > 0$, we have $d\tilde{N}/d\tau_A > 0$. In case (b) for $r_2 < r_1$, the long-run equilibrium is only stable if

Figure 3: Proposition 3



the preference curve ($\tilde{g}_P$) has a greater slope than the technology curve ($\tilde{g}_T$); that is, if $d\tilde{g}_T/d\tilde{r} - d\tilde{g}_P/d\tilde{r} < 0$. Thus, we have $d\tilde{g}/d\tau_A < 0$ when $d\tilde{g}_T/d\tilde{r} < 0$, and $d\tilde{g}/d\tau_A > 0$ when $d\tilde{g}_T/d\tilde{r} < 0$. In addition, $d\tilde{r}/d\tau_A > 0$. Because $d\tilde{N}/d\tilde{r} < 0$ for $r_2 < r_1$, we have $d\tilde{N}/d\tau_A > 0$. This completes the proof of Proposition 2.

Appendix D: Income Inequality and Taxes on Interest Income

This appendix derives the results of Proposition 4. Substituting $d\tilde{r}/d\tau_A$ from (C1) into (27) and reorganizing yields

$$\frac{dG}{d\tau_A} = -\frac{\tilde{r}}{(\tilde{r}+\beta)^2}\left((\tilde{r}+\beta)\frac{d\tilde{g}_T}{d\tilde{r}} - \tilde{g} - \frac{(1-\tau_L)s\beta}{\alpha(\mu-1)} - \frac{(1-\eta)\rho}{\alpha(\mu-1)}\right)\left(\frac{d\tilde{g}_T}{d\tilde{r}} - \frac{d\tilde{g}_P}{d\tilde{r}}\right)^{-1},$$

where $d\tilde{g}_T/d\tilde{r} - d\tilde{g}_P/d\tilde{r} < 0$. In case (a), when $r_2 > r_1$, we have $d\tilde{g}_T/d\tilde{r} < 0$, and consequently $dG/d\tau_A < 0$ and $d\tilde{g}/d\tau_A < 0$. Alternatively, in case (b) when $r_2 < r_1$, the sign of $dG/d\tau_A$ depends on the size of $\tilde{r}$. Referencing Figure 3, we note that $d\tilde{g}_T/d\tilde{r} \leq 0$ for $\tilde{r} \leq \tilde{r}_g$ and $d\tilde{g}_T/d\tilde{r} > 0$ for $\tilde{r} > \tilde{r}_g$. Then, as

$$\left.\frac{dG}{d\tau_A}\right|_{\lim \tilde{r}\to\tilde{r}_T} = -\frac{(\mu-\eta-1)(1-\tau_A-(1-\tau_L)s)\tilde{r}\beta}{\alpha(\mu-1)^2}\left(\frac{d\tilde{g}_T}{d\tilde{r}} - \frac{d\tilde{g}_P}{d\tilde{r}}\right)^{-1} = \infty > 0,$$

since $d\tilde{g}_T/d\tilde{r} - d\tilde{g}_P/d\tilde{r}$ approaches zero as the increase in τ_A shifts the preference curve downwards to a point of tangency with the technology curve at $\tilde{r} = \tilde{r}_T$. Therefore, there must exist an intermediate value $\tilde{r}_G$ for the interest rate where $dG/d\tau_A = 0$. As such, we have the following results: $d\tilde{g}_T/d\tilde{r} < 0$ and $dG/d\tau_A < 0$ for $\tilde{r} < \tilde{r}_g$; $d\tilde{g}_T/d\tilde{r} > 0$ and $dG/d\tau_A < 0$ for $\tilde{r} \in (\tilde{r}_g, \tilde{r}_G)$; and $d\tilde{g}_T/d\tilde{r} > 0$ and $dG/d\tau_A > 0$ for $\tilde{r} > \tilde{r}_G$. This completes the proof of Proposition 4.

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