Discussion Paper Series

RIEB

Kobe University

DP2025-12

Assistance-proofness*

RIEB Junior Research Fellow

Ryoga DOI

June 4, 2025

* This Discussion Paper won the Kanematsu Prize (FY 2024).

Research Institute for Economics and Business Administration

Kobe University

2-1 Rokkodai, Nada, Kobe 657-8501 JAPAN

Assistance-proofness*

Ryoga Doi†

June 4, 2025

Abstract

We consider situations in which the final ranking of candidates is determined by rankings of multiple factors. For example, in Formula 1 racing, the annual ranking is determined by the results of many races. In sport climbing, the final ranking is determined by combining the results of two or three events. Dependent on rules that aggregate rankings across multiple factors, a candidate can improve the final position of a fellow candidate by holding back her performance without dropping the final position. We call the property of rules that prevent this kind of strategic manipulation *assistance-proofness*. We show that when there are four or more events, no scoring rule other than the null rule satisfies *assistance-proofness*. However, when there are two events, all dichotomous scoring rules satisfy *assistance-proofness*. For three events, we characterize a subclass of dichotomous scoring rules that satisfy *assistance-proofness*.

Keywords: Assistance-proofness, Scoring rules, Voting, Strategic manipulation, Collusion.

JEL: D71, D72.

---

*This paper won the Kanematsu Prize from the Research Institution for Economics and Business Administration, Kobe University, in 2024. The author is grateful to Toyotaka Sakai for helpful discussions and support, and would also like to thank Noriaki Okamoto, Susumu Cato, Tsuyoshi Adachi, Takako Fujiwara-Greve, Toru Hokari, Shigehiro Serizawa and the participants in workshops at Jeju National University, the University of Tokyo and Kobe University for their helpful comments.

†Graduate School of Economics, Keio University, Tokyo 108-8345, Japan. Email:doiryouga@keio.jp.

1 Introduction

At the 1991 Suzuka Circuit, Ayrton Senna of the McLaren Honda team led the race on the final lap but relinquished the lead to his teammate, Gerhard Berger, just prior to reaching the finish line. It is believed that Senna's decision to cede the lead was to ensure that Berger would finish the season at a higher position, because Senna's title as the champion of the year was guaranteed. Although this incident may sound as a good story, it can also be viewed as a strategic maneuver by Senna, which undermines the merit-based determination of final rankings. Similar situations can arise in other sports in which final rankings are determined by multiple races or events such as sport climbing. This study introduces a robustness condition against such strategic maneuvers called *assistance-proofness*. We analyze the existence and properties of the ranking rules that satisfy it.

As an example, consider the following situation. Events 1, 2, and 3 are held with three competitors, namely, A, B, and C. For each event, the competitors obtain scores according to their ranking. The final ranking is determined by their total scores.

	Event 1	Event 2	Event 3	Final ranking
First (10pts)	C	A	A	A (25pts)
Second (5pts)	A	B	B	C (16pts)
Third (3pts)	B	C	C	B (13pts)

In this situation, if competitor A makes a worse performance in Event 1 as indicated in the following table, then Competitor A can manipulate the final ranking as follows:

	Event 1	Event 2	Event 3	Final ranking
First (10pts)	C	A	~~A~~ B	A (20pts)
Second (5pts)	A	B	~~B~~ A	B (18pts)
Third (3pts)	B	C	C	C (16pts)

This manipulation exhibits the following features: (i) Competitor A reverses the final rankings of B and C; (ii) The reversal was made by holding back A's own performance; (iii) But the holding back did not drop A's final ranking. This type of manipulation may occur when Competitors A and B have a close relationship (e.g., friends or teammate), or when A is not in favor of C. Moreover, even if A and B have no relationship, this type of manipulation could be accomplished by a bribe from B to A. However, in a competition in which participants should compete on equal footing, this maneuver harms fairness in competition.

This study focuses on scoring rules, which are widely used in the context of sports. Specifically, we examine which scoring rules are *assistance-proof*. The null rule, which always equally ranks all competitors, trivially satisfies *assistance-proofness*. We show that in the case of four or more events, no scoring rule other than the null rule is *assistance-proof*. We also show that, when there are two events, all scoring rules with only two distinct scores satisfy *assistance-proofness*. Lastly, for three events, we characterize a subclass of such dichotomous rules that satisfy *assistance-proofness*.

A number of studies examine certain team manipulation in sports competitions. For example, Duggan and Levvit (2002) discuss the possibility of corruption in sumo wrestling using data. Preston and Szymanski (2003) theoretically analyze match-fixing in cricket.

Our multi event ranking model can also be considered a voting model by considering events as voters and competitors as candidates. In particular, the ranking rules aggregate multiple rankings in events into a single final ranking; thus, our model is formally equivalent to that of Arrovian social choice. The new axiom *assistance-proofness* is implied by the well-known *binary independence* of Arrow (1951).

Although within a different framework, Dutta, Jackson, and Le Breton (2001) examine the manipulation of noncontending candidates to change an outcome by entering or exiting an election. Although they focus on social choice functions, the current study is similar to theirs in that we consider the manipulation of candidates. *Assistance-proofness* is also somewhat similar to *non-bossiness* (Satterthwaite and Sonnenschein, 1981) in the literatures of mechanism design and fair allocation. Non-bossiness states that if a change in the preferences of an agent does not alter his or her assignment, then it does not alter the assignments of others (see, Thomson (2016) for a survey). As previously mentioned, the assist considered in the current study may occur through bribes. Schummer (2000a, 2000b) discusses the manipulation of allocation rules through bribes. His results exhibit difficulties in achieving robust against manipulation through bribes. Moreover, Eso and Schummer (2004) show that bribing also occurs in second price auctions.

The remainder of the paper is structured as follows: Section 2 introduces the model, while Section 3 presents the main results. Section 4 offers a few discussions, and Section 5 concludes.

2 Model

Let $N = \{1, 2, ..., n\}$ and $M = \{1, 2, ..., m\}$ be finite sets of competitors and events, respectively. We assume that $n \geq 3$ and $m \geq 2$. Let $\mathscr{R}$ and $\mathscr{P}$ denote the sets of orderings[1] and linear orderings on N, respectively. A **ranking profile** is a list $\succ \equiv (\succ_1, ..., \succ_m) \in \mathscr{P}^m$, where $\succ_h$ denotes a linear ordering of competitors in event $h \in M$. A **ranking rule** is a function $R : \mathscr{P}^m \to \mathscr{R}$ which maps each ranking profile to a final ranking. For all $i, j \in N$ and for all $\succ \in \mathscr{P}^m$, we write $iR(\succ)j$ if i is finally ranked at least as high as j.[2] Let $P(\succ)$ denote the asymmetric part of $R(\succ)$.

For all $i \in N$, her position in $h \in M$ at $\succ \in \mathscr{P}^m$ is denoted by $p_h(\succ, i) \equiv |\{j \in N | j \succ_h i\}| + 1 \in \{1, ..., n\}$. For all $i \in N$, for all $k \in \{1, ..., n\}$, and for all $\succ \in \mathscr{P}^m$, let $s_k(\succ, i) \equiv |\{h \in M | p_h(\succ, i) = k\}|$ be the number of times in which i obtains the k-th place at $\succ$.

Definition 1. *A ranking rule R is a **scoring rule** if there exists an n-tuple of scores $a = (a_1, ..., a_n) \in \mathbb{R}^n$ such that $a_1 \geq a_2 \geq ... \geq a_n$ and for all $\succ \in \mathscr{P}^m$ and for all $i, j \in N$,*

$$iR(\succ)j \iff \sum_{k=1}^{n} a_k \cdot s_k(\succ, i) \geq \sum_{k=1}^{n} a_k \cdot s_k(\succ, j).$$

Let $score_R(\succ, i) \equiv \sum_{k=1}^{n} a_k \cdot s_k(\succ, i)$ denote i's score associated with R at $\succ$. We will omit subscript R if there is no risk of confusion. This study considers scenarios in which competitors can only underperform relative to their true abilities.

Definition 2. *For all $i \in N$ and for all $\succ \in \mathscr{P}^m$, i's **lower performance set** at $\succ$ is as follows:*

$$\mathscr{L}(\succ, i) \equiv \left\{ \succ' \in \mathscr{P}^m \,\middle|\, \begin{array}{l} \forall h \in M, \forall j, k \in N \backslash \{i\}, \\ [p_h(\succ, i) \leq p_h(\succ', i)] \text{ and } [j \succ_h k \iff j \succ'_h k] \end{array} \right\}.$$

We are now in a position to introduce our new axiom. It states that for all $i \in N$ and for all $\succ \in \mathscr{P}^m$, if i's final rank does not drop by holding back i's

[1] An ordering is a complete, reflexive, and transitive binary relation. A linear ordering is an antisymmetric ordering.

[2] To be precise, we write $iR(\succ)j$ if $(i, j) \in R(\succ)$.

performance in some event, then i cannot assist anyone. A ranking rule that allows such manipulation will unlikely reflect the true ability of the competitors in the final rankings and may result in the unfair downgrade of a few competitors due to personal relationships with the other competitors or bribes.

Definition 3. *A ranking rule R is **assistance-proof** if for all $i \in N$, for all $j, k \in N \backslash \{i\}$, for all $\succ \in \mathscr{P}^m$, and for all $\succ' \in \mathscr{L}(\succ, i)$,*

$$[\forall l \in N \backslash \{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l] \Longrightarrow [jR(\succ)k \Longleftrightarrow jR(\succ')k].$$

Does an *assistance-proof* scoring rule exist? The following rule is a trivial one:

Definition 4. *A ranking rule R is the **null rule** if for all $i, j \in N$ and for all $\succ \in \mathscr{P}^m$, $iR(\succ)j$.*

The null rule is a scoring rule with $a_1 = a_2 = \cdots = a_n$ and satisfies *assistance-proofness*. However, this rule totally fails to distinguish any competitors. We then pose another question: is there any nonnull scoring rule that satisfies *assistance-proofness*? The answer to this question is somewhat complicated and dependent on the number of events.

3 Main Results

Unfortunately, if there are at least four events, there exists no scoring rule that satisfies *assistance-proofness* other than the null rule.

Theorem 1. *Suppose $m \geq 4$. The null rule is the only scoring rule satisfying assistance-proofness.*

Proof. See the Appendix. □

We then consider the case of two events. The range of scoring rules that satisfy *assistance-proofness* slightly expands. Specifically, dichotomous rules (i.e., scoring rules with only two distinct scores) are *assistance-proof.*[3]

[3]Regarding events as voters, dichotomous rules can be interpreted as those that require voters to express dichotomous preferences. For discussions on Arrovian social choice in which voter preferences are dichotomous, see Sakai and Shimoji (2006).

Definition 5. *A ranking rule R is a* ***dichotomous rule*** *if R is a scoring rule and there exists two distinct scores $a_{\text{high}}, a_{\text{low}}$ and $\theta \in \{1, ..., n-1\}$ such that*

$$|\{k \in N | a_k = a_{\text{high}}\}| = \theta \text{ and } |\{k \in N | a_k = a_{\text{low}}\}| = n - \theta.$$

Theorem 2. *Suppose $m = 2$. A scoring rule R is assistance-proof if and only if R is either null or dichotomous.*

Proof. See the Appendix. □

Finally, consider the case with three events. In this case, the value of m lies between those of the previous cases, and correspondingly, the range of ranking rules that satisfy *assistance-proofness* also falls between those of the earlier cases. Specifically, we characterize the class of *assistance-proof* dichotomous rules in terms of θ.

Theorem 3. *Suppose $m = 3$. A scoring rule R is assistance-proof if and only if R is the null rule or a dichotomous rule with $\theta \in (\frac{n+1}{3}, \frac{2n-1}{3})$.*

Proof. See the Appendix. □

Note that such θ does not exist if $n = 3$ or 5, because θ is an integer. Thus, the following corollary holds.

Corollary 1. *Suppose $(n, m) = (3, 3)$ or $(5, 3)$. The null rule is the only scoring rule satisfying assistance-proofness.*

4 Discussion

4.1 Alternative definitions of assistance-proofness

We introduce the notion of partition[4] to clarify the structure of the teams. Let $\mathscr{T}$ denote the set of all partitions of N. For all $T \in \mathscr{T}$ and for all $i \in N$, let $T[i]$ denote the member of T that contains i as an element. We can then restate the definition of *assistance-proofness*. In the following new definition, assistance is explicitly represented by treating individuals within the same partition as members of the same team.

[4]A partition T of N is a family of subsets of N such that $\emptyset \notin T$, $\bigcup_{t \in T} t = N$, and $t_1 \cap t_2 = \emptyset$ $\forall t_1 \neq t_2 \in T$.

Proposition 1. *The following two statements on any ranking rule R are equivalent:*

(i) R is assistance-proof,

(ii) for all $T \in \mathscr{T}$, for all $i \in N$, for all $j \in T[i] \backslash \{i\}$, for all $k \in N \backslash T[i]$, for all $\succ \in \mathscr{P}^m$, and for all $\succ' \in \mathscr{L}(\succ, i)$,

$$[\forall l \in N\backslash\{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l] \\ \Longrightarrow [kR(\succ)j \Longrightarrow kR(\succ')j \text{ and } kP(\succ)j \Longrightarrow kP(\succ')j].$$

Proof. First, we show (ii) $\Longrightarrow$ (i). Take any R. Assume (ii). Take any $i \in N$, $j, k \in N\backslash\{i\}$, $\succ \in \mathscr{P}^m$, and $\succ' \in \mathscr{L}(\succ, i)$. Suppose that

$$\forall l \in N\backslash\{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l.$$

Then, by taking $T \in \mathscr{T}$ with $T[i] = \{i, j\}$, we have $kP(\succ)j \Longrightarrow kP(\succ')j$. Hence

$$jR(\succ')k \Longrightarrow jR(\succ)k.$$

In addition, by taking $T' \in \mathscr{T}$ with $T'[i] = \{i, k\}$, we have

$$jR(\succ)k \Longrightarrow jR(\succ')k.$$

Therefore, we have

$$jR(\succ)k \Longleftrightarrow jR(\succ')k.$$

Second, we show (i) $\Longrightarrow$ (ii). Take any *assistance-proof* R. Take any $T \in \mathscr{T}$, $i \in N$, $j \in T[i]\backslash\{i\}$, $k \in N\backslash T[i]$, $\succ \in \mathscr{P}^m$, and $\succ' \in \mathscr{L}(\succ, i)$. Suppose that

$$\forall l \in N\backslash\{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l.$$

Then, since $j, k \in N\backslash\{i\}$ and R is *assistance-proof*, we have $jR(\succ)k \Longleftrightarrow jR(\succ')k$ and $kR(\succ)j \Longleftrightarrow kR(\succ')j$, hence

$$kR(\succ)j \Longrightarrow kR(\succ')j \text{ and } kP(\succ)j \Longrightarrow kP(\succ')j.$$

□

Next, we restrict the possible manipulations of competitors. Specifically, we consider a case in which competitors can only manipulate profiles in an event and adjacent to the true profile.[5]

[5]For this type of voter manipulation, see Sato (2013).

Definition 6. *For all $i \in N$ and for all $\succ \in \mathscr{P}^m$, i's **adjacent lower performance set** at $\succ$ is as follows:*

$$\mathscr{A}(\succ, i) \equiv \left\{ \succ' \in \mathscr{P}^m \;\middle|\; \begin{array}{l} \exists! h \in M, \forall g \in M \backslash \{h\}, \forall j,k \in N \backslash \{i\}, \\ [p_h(\succ, i) + 1 = p_h(\succ', i)], [j \succ_h k \Longleftrightarrow j \succ'_h k], \text{ and } [\succ_g = \succ'_g]. \end{array} \right\}.$$

Note that, for all $i \in N$ and for all $\succ \in \mathscr{P}^m$, $\mathscr{A}(\succ, i) \subset \mathscr{L}(\succ, i)$. However, if we replace $\mathscr{L}(\succ, i)$ of the definition of *assistance-proofness* with $\mathscr{A}(\succ, i)$, then the same results as Theorems 1, 2, and 3 hold.

Proposition 2. *The following two statements on any scoring rule R are equivalent:*

(i) R is assistance-proof,

(ii) for all $i \in N$, for all $j, k \in N \backslash \{i\}$, for all $\succ \in \mathscr{P}^m$, and for all $\succ' \in \mathscr{A}(\succ, i)$,

$$[\forall l \in N \backslash \{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l] \Longrightarrow [jR(\succ)k \Longleftrightarrow jR(\succ')k].$$

Proof. Since $\mathscr{A}(\succ, i) \subset \mathscr{L}(\succ, i)$, (i) $\Longrightarrow$ (ii) is obvious. We show (ii) $\Longrightarrow$ (i) by showing the contraposition. Take any R and assume that R is not *assistance-proof.* Then there exist $i \in N$, $j, k \in N \backslash \{i\}$, $\succ \in \mathscr{P}^m$, and $\succ' \in \mathscr{L}(\succ, i)$ such that

$$\forall l \in N \backslash \{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l$$

and

$$\neg[jR(\succ)k \Longleftrightarrow jR(\succ')k].$$

By definitions of $\mathscr{A}$ and $\mathscr{L}$, we can take $t \in \mathbb{N}$ and t profiles $\succ = \succ^1, \succ^2, ..., \succ^t = \succ'$ such that for all $s \in \{1, ..., t-1\}$,

$$\succ^{s+1} \in \mathscr{A}(\succ^s, i).$$

By definition of scoring rules, i's score is non-increasing and others' scores are non-decreasing in s. Thus, for all $s \in \{1, ..., t\}$ and $l \in N \backslash \{i\}$,

$$iR(\succ')l \Longrightarrow iR(\succ^s)l \text{ and } iR(\succ^s)l \Longrightarrow iR(\succ)l,$$

and

$$iP(\succ')l \Longrightarrow iP(\succ^s)l \text{ and } iP(\succ^s)l \Longrightarrow iP(\succ)l.$$

Therefore, the fact that

$$\forall l \in N \backslash \{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l$$

implies that for all $s \in \{1, ..., t-1\}$,

$$\forall l \in N \backslash \{i\}, iR(\succ^s)l \Longrightarrow iR(\succ^{s+1})l \text{ and } iP(\succ^s)l \Longrightarrow iP(\succ^{s+1})l.$$

However, since $\neg[jR(\succ)k \Longleftrightarrow jR(\succ')k]$ for some j, k, there exists some $s \in \{1, ..., t-1\}$ such that $\neg[jR(\succ^s)k \Longleftrightarrow jR(\succ^{s+1})k]$ for some j, k. Thus, (ii) is not true. $\square$

Note that Proposition 2 is established for the scoring rules. If scoring rules are replaced with ranking rules in the statement of Proposition 2, then (i) $\Longrightarrow$ (ii) holds; however, whether or not the converse holds remains unknown.

4.2 Arrovian social choice

We introduced the axiom of *assistance-proofness* from the perspective of strategic manipulation. However, as Proposition 3 demonstrates, *assistance-proofness* is a weaker version of binary independence. Consequently, a few of the results presented in the previous section can be interpreted as circumventing impossibility outcomes by relaxing the binary independence condition in Arrow's impossibility theorem. Numerous studies (e.g., Baigent 1987; Campbell and Kelly 2000, 2007; Cato 2014; Fleurbaey et al. 2005) have derived possibility or impossibility results by weakening binary independence. In this subsection, we elucidate the relationship between our theorems and the foundational results in Arrovian social choice theory. First, we review the definition of binary independence and point out that it implies *assistance-proofness.*

Definition 7. *A ranking rule R satisfies* ***binary independence*** *if for all $i, j \in N$ and for all $\succ, \succ' \in \mathscr{P}^m$,*

$$\succ|_{\{i,j\}} = \succ'|_{\{i,j\}} \Longrightarrow [iR(\succ)j \Longleftrightarrow iR(\succ')j].$$

Proposition 3. *If a ranking rule R satisfies binary independence, then R satisfies assistance-proofness.*

Proof. Take any R satisfying binary independence. Take any $i \in N$, $j, k \in N \backslash \{i\}$, $\succ \in \mathscr{P}^m$, and $\succ' \in \mathscr{L}(\succ, i)$. By definition of $\mathscr{L}(\succ, i)$, it follows that

$$\succ|_{\{j,k\}} = \succ'|_{\{j,k\}}.$$

Since R satisfies binary independence, we have $jR(\succ)k \Longleftrightarrow jR(\succ')k$, hence R is *assistance-proof.* $\square$

In addition, a theorem similar to that of Murakami (1968) also holds in this environment.[6]

Definition 8. *A ranking rule R satisfies **non-imposition** if for all $i, j \in N$, there exists $\succ \in \mathscr{P}^m$ such that $iR(\succ)j$.*

Definition 9. *A ranking rule R is (inversely) **dictatorial** if there exists $k \in M$ such that for all $i, j \in N$ and $\succ \in \mathscr{P}^m$,*

$$i \succ_k j \Longrightarrow iP(\succ)j(jP(\succ)i).$$

Theorem 4 (Murakami, 1968)**.** *If a ranking rule R satisfies binary independence and non-imposition, then R is either null, dictatorial, or inversely dictatorial.*

Evidently, any scoring rule satisfies non-imposition, and is neither dictatorial nor inversely dictatorial. Thus, we have the following proposition.

Proposition 4. *If a scoring rule R satisfies binary independence, then R is null.*

The results presented in the previous section indicate that the impossibility in Theorem 4 and Proposition 4 can be resolved by weakening binary independence to *assistance-proofness.* We restate theorems 1,2 and 3 below for convenience.

- **Theorem 1.** *Suppose $m \geq 4$. The null rule is the only scoring rule satisfying assistance-proofness.*
- **Theorem 2.** *Suppose $m = 2$. A scoring rule R is assistance-proof if and only if R is either null or dichotomous.*
- **Theorem 3.** *Suppose $m = 3$. A scoring rule R is assistance-proof if and only if R is the null rule or a dichotomous rule with $\theta \in (\frac{n+1}{3}, \frac{2n-1}{3})$.*

However, dichotomous rules do not satisfy weak Pareto[7]; it is unknown if there is any ranking rule satisfying both weak Pareto and *assistance-proofness.*

[6] For the relationship between the results of Murakami (1968) and those of Wilson (1972), see Malawski and Zhou (1994) and Holliday and Kelley (2020).

[7] *Weak Pareto*: for all $\succ \in \mathscr{P}^m$, for all $i, j \in N$, $[i \succ_k j \forall k \in M] \Longrightarrow iP(\succ)j$.

4.3 Asymmetric case

Thus far, we have implicitly assumed that all events are equally important. However, in certain situations, one may wish to give asymmetric weights over the events due to the varying degrees of importance of events. Here, we discuss the possibility of *assistance-proof* scoring rules when events are given such asymmetric weights.[8]

Definition 10. *A ranking rule R is a* ***generalized scoring rule*** *if there exists an m-tuple of weights $w = (w_1, ..., w_m) \in \mathbb{R}^m$ and n-tuple of scores $a = (a_1, ..., a_n) \in \mathbb{R}^n$ such that $w_1 \geq w_2 \geq ... \geq w_m$, $a_1 \geq a_2 \geq ... \geq a_n$, and for all $\succ \in \mathscr{P}^m$ and for all $i, j \in N$,*

$$iR(\succ)j \Longleftrightarrow \sum_{k=1}^{m} w_k a_{p_k(\succ,i)} \geq \sum_{k=1}^{m} w_k a_{p_k(\succ,j)}.$$

Note that, if $w_1 = w_2 = ... = w_m$, then R is a scoring rule. We derived the necessary and sufficient conditions for the generalized scoring rule to satisfy *assistance-proofness* in the case of $(n, m) = (3, 2)$.

Theorem 5. *Suppose $(n, m) = (3, 2)$. A generalized scoring rule R is assistance-proof if and only if R is either null, dichotomous, or dictatorial.*

Proof. See the Appendix. □

5 Conclusion

The major contribution of this study is the introduction of *assistance-proofness.* In addition, it demonstrated that only a limited class of scoring rules are *assistance-proof.* As future research, it is interesting to investigate ranking rules other than scoring rules. Furthermore, it is also interesting to examine situations in which not only ranking but also performances (e.g., times or distances) per event, are observable.

Appendix

Proof of Theorem 1:

The null rule is a scoring rule satisfying *assistance-proofness.* Thus, we take any

[8] A related example is the method used to determine the world ranking of tennis.

nonnull scoring rule R, and show that R is not *assistance-proof*. Let $a = (a_1, ..., a_n)$ be a score vector associated with R. Without loss of generality, assume $a_1 = 1$ and $a_n = 0$.[9] Let $t \in \{1, ..., n-1\}$ denote the largest integer satisfying $a_t > 0$.

Case 1. Consider the case where m is an even number and $t = n-1$. Since m is an even number and $m \geq 4$, we can take $m' \geq 2$ such that $m = 2m'$. Let $\succ$ be a ranking profile such that

$p_1(\succ, 3) = n-2, p_1(\succ, 1) = n-1, p_1(\succ, 2) = n,$
$p_h(\succ, 3) = n-2, p_h(\succ, 2) = n-1, p_h(\succ, 1) = n$ for all $h \in \{2, ..., m'\}$,
$p_h(\succ, 2) = n-2, p_h(\succ, 3) = n-1, p_h(\succ, 1) = n$ for all $h \in \{m'+1, ..., 2m'-1\}$, and
$p_{2m'}(\succ, 2) = n-2, p_{2m'}(\succ, 1) = n-1, p_{2m'}(\succ, 3) = n.$

score\event	1	2	$\cdots$	m'	$m'+1$	$\cdots$	$2m'-1$	$2m'$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$
a_{n-2}	3	3	$\cdots$	3	2	$\cdots$	2	2
$a_{n-1} > 0$	1	2	$\cdots$	2	3	$\cdots$	3	1
$a_n = 0$	2	1	$\cdots$	1	1	$\cdots$	1	3

In addition, consider $\succ' \in \mathscr{P}^m$ such that

$$p_1(\succ', 2) = n-1, p_1(\succ', 1) = n, \text{ and}$$
$$p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M \backslash \{(1,1), (2,1)\}.$$

score\event	1	2	$\cdots$	m'	$m'+1$	$\cdots$	$2m'-1$	$2m'$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$
a_{n-2}	3	3	$\cdots$	3	2	$\cdots$	2	2
$a_{n-1} > 0$	$\cancel{1}$ 2	2	$\cdots$	2	3	$\cdots$	3	1
$a_n = 0$	$\cancel{2}$ 1	1	$\cdots$	1	1	$\cdots$	1	3

Then we have

$$\forall j, k \in N \backslash \{1\}, \forall h \in M, [p_h(\succ, 1) \leq p_h(\succ', 1)] \text{ and } [j \succ_h k \Longleftrightarrow j \succ'_h k],$$

[9]If $a_1 \neq 1$ or $a_n \neq 0$, we can obtain a new score vector a' without changing the final ranking so that $a'_1 = 1$ and $a'_n = 0$ by subtracting a_n from all scores and then dividing by $a_1 - a_n$.

and hence $\succ' \in \mathscr{L}(\succ, 1)$. Furthermore, for all $l \in N \backslash \{1\}$, it follows that

$$score(\succ, l) \geq (2m' - 1)a_{n-1} > 2a_{n-1} = score(\succ, 1).$$

Thus, for all $l \in N \backslash \{1\}$, $lP(\succ)1$ and hence

$$\forall l \in N \backslash \{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, since

$$score(\succ, 2) = score(\succ, 3) = m'a_{n-2} + (m' - 1)a_{n-1}$$

and

$$score(\succ', 2) = m'a_{n-2} + m'a_{n-1} > m'a_{n-2} + (m' - 1)a_{n-1} = score(\succ', 3),$$

it follows that $3R(\succ)2$, but $3R(\succ')2$ does not hold. Thus, R is not *assistance-proof.*

Case 2. Consider the case where m is an odd number and $t = n - 1$. Since m is an odd number, we can take $m' \geq 2$ such that $m = 2m' + 1$. Let $\succ$ be a ranking profile such that

$p_1(\succ, 3) = n - 2, p_1(\succ, 1) = n - 1, p_1(\succ, 2) = n,$
$p_h(\succ, 3) = n - 2, p_h(\succ, 2) = n - 1, p_h(\succ, 1) = n$ for all $h \in \{2, ..., m'\}$,
$p_h(\succ, 2) = n - 2, p_h(\succ, 3) = n - 1, p_h(\succ, 1) = n$ for all $h \in \{m' + 1, ..., 2m'\}$, and
$p_{2m'+1}(\succ, 1) = n - 2, p_{2m'+1}(\succ, 2) = n - 1, p_{2m'+1}(\succ, 3) = n.$

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$p_1(\succ', 2) = n - 1, p_1(\succ', 1) = n,$ and
$p_h(\succ, i) = p_h(\succ', i)$ for all $(i, h) \in N \times M \backslash \{(1, 1), (2, 1)\}$.

score\event	1	2	$\cdots$	m'	$m' + 1$	$\cdots$	$2m'$	$2m' + 1$
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$		$\vdots$	$\vdots$
$a_{n-2} > 0$	3	3	$\cdots$	3	2	$\cdots$	2	1
$a_{n-1} > 0$	$\cancel{1}$ 2	2	$\cdots$	2	3	$\cdots$	3	2
$a_n = 0$	$\cancel{2}$ 1	1	$\cdots$	1	1	$\cdots$	1	3

Then, the scores of each competitor are as follows:

$$score(\succ, 1) = a_{n-2} + a_{n-1}, score(\succ', 1) = a_{n-2},$$
$$score(\succ, 2) = m'a_{n-2} + m'a_{n-1}, score(\succ', 2) = m'a_{n-2} + (m' + 1)a_{n-1},$$
$$score(\succ, 3) = m'a_{n-2} + m'a_{n-1}, score(\succ', 3) = m'a_{n-2} + m'a_{n-1},$$
$$score(\succ, i) = score(\succ', i) \geq (2m' + 1)a_{n-2} \text{ for all } i \in N\backslash\{1, 2, 3\}.$$

Thus, we have

$$\forall l \in N\backslash\{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, $3R(\succ)2$ holds but $3R(\succ')2$ does not hold, hence R is not *assistance-proof.*

Case 3. Consider the case with $t < n - 1$. Let $\succ$ be a ranking profile such that

$$p_h(\succ, 1) = t, p_h(\succ, 2) = t + 1, p_h(\succ, 3) = t + 2 \text{ for all } h \in \{1, ..., m - 1\},$$
$$p_m(\succ, 3) = t, p_m(\succ, 1) = t + 1, p_m(\succ, 2) = t + 2, \text{ and}$$
$$p_h(\succ, i) = p_{h'}(\succ, i) \text{ for all } h, h' \in M \text{ and } i \in N\backslash\{1, 2, 3\}.$$

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$p_1(\succ', 2) = t, p_1(\succ', 1) = t + 1, \text{ and}$$
$$p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M\backslash\{(1, 1), (2, 1)\}.$$

score\event	1	2	$\cdots$	$m-1$	m
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
$a_t > 0$	$\cancel{1}$ 2	1	$\cdots$	1	3
$a_{t+1} = 0$	$\cancel{2}$ 1	2	$\cdots$	2	1
$a_{t+2} = 0$	3	3	$\cdots$	3	2
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$

Then, the scores of each competitor are as follows:

$$score(\succ, 1) = (m - 1)a_t, score(\succ', 1) = (m - 2)a_t,$$
$$score(\succ, 2) = 0, score(\succ', 2) = a_t,$$
$$score(\succ, 3) = a_t, score(\succ', 3) = a_t,$$
$$score(\succ, i) = score(\succ', i) \geq ma_t \text{ for all } i \in \{j \in N | p_1(\succ, j) < t\}, \text{ and}$$
$$score(\succ, i) = score(\succ', i) = 0 \text{ for all } i \in \{j \in N | p_1(\succ, j) > t + 2\}.$$

Thus, we have

$$\forall l \in N\backslash\{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, $2R(\succ')3$ holds but $2R(\succ)3$ does not hold, hence R is not *assistance-proof.*

□

Proof of Theorem 2:

First, we show the "if" part. The null rule is a scoring rule satisfying *assistance-proofness.* We show that any dichotomous rule satisfies *assistance-proofness.* Take any dichotomous rule R. Without loss of generality, assume that $a_{\text{high}} = 1$, and $a_{\text{low}} = 0$. Take any $i \in N$, $j, k \in N\backslash\{i\}$, $\succ \in \mathscr{P}^m$, and $\succ' \in \mathscr{L}(\succ, i)$.

Case 1. Consider the case where $score(\succ, i) = 2$. If $score(\succ', i) = 2$, then

$$jR(\succ)k \Longleftrightarrow jR(\succ')k$$

holds, since no one's score changes. If $score(\succ', i) = 1$, then there exists $l \in N\backslash\{i\}$ such that

$$[score(\succ, l) = 1 \text{ and } score(\succ', l) = 2] \text{ or } [score(\succ, l) = 0 \text{ and } score(\succ', l) = 1].$$

In both cases,

$$iP(\succ)l \Longrightarrow iP(\succ')l$$

does not hold. Similarly, if $score(\succ', i) = 0$, then $iP(\succ)l \Longrightarrow iP(\succ')l$ does not hold. Thus, it follows that

$$[\forall l \in N\backslash\{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l] \Longrightarrow [jR(\succ)k \Longleftrightarrow jR(\succ')k],$$

and hence R is *assistance-proof.*

Case 2. Consider the case where $score(\succ, i) = 1$. If $score(\succ', i) = 1$, then

$$jR(\succ)k \Longleftrightarrow jR(\succ')k$$

since no one's score changes. If $score(\succ', i) = 0$, then there exists $l \in N\backslash\{i\}$ such that

$$[score(\succ, l) = 1 \text{ and } score(\succ', l) = 2] \text{ or } [score(\succ, l) = 0 \text{ and } score(\succ', l) = 1].$$

In both cases,

$$iR(\succ)l \Longrightarrow iR(\succ')l$$

does not hold. Thus, it follows that

$$[\forall l \in N\backslash\{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l] \Longrightarrow [jR(\succ)k \Longleftrightarrow jR(\succ')k],$$

and hence R is *assistance-proof.*

Case 3. Consider the case where $score(\succ, i) = 0$. Then $score(\succ', i) = 0$ and hence

$$jR(\succ)k \Longleftrightarrow jR(\succ')k$$

holds, since no one's score changes. Thus, it follows that

$$[\forall l \in N\backslash\{i\}, iR(\succ)l \Longrightarrow iR(\succ')l \text{ and } iP(\succ)l \Longrightarrow iP(\succ')l] \Longrightarrow [jR(\succ)k \Longleftrightarrow jR(\succ')k],$$

and hence R is *assistance-proof.*

Next, we show the "only if" part by showing the contraposition. Take any scoring rule R which is neither null nor dichotomous. We show that R is not *assistance-proof.* Without loss of generality, assume that $a_n = 0$ and the third-lowest score is 1. Let a^s denote the second-lowest score, and $t_1 \in \{1, ..., n-2\}$ (resp. $t_2 \in \{2, ..., n-1\}$) be the largest integer satisfying $a_{t_1} = 1$ (resp. $a_{t_2} = a^s$).

Case 1. Consider the case where $|\{q \in \{1, ..., n\}|a_q = 1\}| \leq |\{q \in \{1, ..., n\}|a_q = 0\}|$. Let $\succ$ be a ranking profile such that

$$\begin{aligned}
&p_1(\succ, 1) = t_1, p_1(\succ, 2) = t_1 + 1, p_1(\succ, 3) = t_2 + 1,\\
&p_2(\succ, 1) = t_1, p_2(\succ, 3) = t_1 + 1, p_2(\succ, 2) = t_2 + 1,\\
&a_{p_2(\succ,i)} = 0 \text{ for all } i \in \{j \in N\backslash\{1,2,3\}|a_{p_1(\succ,j)} = 1\},\\
&a_{p_1(\succ,i)} = 0 \text{ for all } i \in \{j \in N\backslash\{1,2,3\}|a_{p_2(\succ,j)} = 1\}, \text{ and}\\
&p_1(\succ, i) = p_2(\succ, i) \text{ for all } i \in \{j \in N\backslash\{1,2,3\}|a_{p_1(\succ,j)} \neq 1 \text{ and } a_{p_2(\succ,j)} \neq 1\}.
\end{aligned}$$

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$\begin{aligned}
&p_1(\succ', 2) = t_1, p_1(\succ', 1) = t_1 + 1, \text{ and}\\
&p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M\backslash\{(1,1),(2,1)\}.
\end{aligned}$$

score\event	1	2
$\vdots$	$\vdots$	$\vdots$
$a_{t_1} = 1$	$\cancel{1}$ 2	1
$a_{t_1+1} = a^s$	$\cancel{2}$ 1	3
$\vdots$	$\vdots$	$\vdots$
$a_{t_2+1} = 0$	3	2
$\vdots$	$\vdots$	$\vdots$

Then, the scores of each competitor are as follows:

$score(\succ, 1) = 2, score(\succ', 1) = 1 + a^s,$

$score(\succ, 2) = a^s, score(\succ', 2) = 1,$

$score(\succ, 3) = a^s, score(\succ', 3) = a^s,$

$score(\succ, i) = score(\succ', i) = 1$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ, j)} = 1$ or $a_{p_2(\succ, j)} = 1\}$,

$score(\succ, i) = score(\succ', i) = 0$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ, j)} = a_{p_2(\succ, j)} = 0\}$,

$score(\succ, i) = score(\succ', i) = 2a^s$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ, j)} = a^s\}$, and

$score(\succ, i) = score(\succ', i) > 2$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ, j)} > 1\}$.

Thus, we have

$$\forall l \in N \backslash \{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, $3R(\succ)2$ holds but $3R(\succ')2$ does not hold, hence R is not *assistance-proof*.

Case 2. Consider the case where $|\{q \in \{1, ..., n\} | a_q = 1\}| > |\{q \in \{1, ..., n\} | a_q = 0\}|$. Let $\succ$ be a ranking profile such that

$p_1(\succ, 2) = t_1, p_1(\succ, 3) = t_2, p_1(\succ, 1) = t_2 + 1,$

$p_2(\succ, 3) = t_1, p_2(\succ, 1) = t_2, p_2(\succ, 2) = t_2 + 1,$

$a_{p_2(\succ, i)} = 1$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ, j)} = 0\}$,

$a_{p_1(\succ, i)} = 1$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_2(\succ, j)} = 0\}$, and

$p_1(\succ, i) = p_2(\succ, i)$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ, j)} \neq 0$ and $a_{p_2(\succ, j)} \neq 0\}$.

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$p_2(\succ', 2) = t_2, p_2(\succ', 1) = t_2 + 1, \text{ and}$$
$$p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M \backslash \{(1, 2), (2, 2)\}.$$

score\event	1	2
$\vdots$	$\vdots$	$\vdots$
$a_{t_1} = 1$	2	3
$\vdots$	$\vdots$	$\vdots$
$a_{t_2} = a^s$	3	$\cancel{1}$ 2
$a_{t_2+1} = 0$	1	$\cancel{2}$ 1
$\vdots$	$\vdots$	$\vdots$

Then, the scores of each competitor are as follows:

$score(\succ, 1) = a^s, score(\succ', 1) = 0,$

$score(\succ, 2) = 1, score(\succ', 2) = 1 + a^s,$

$score(\succ, 3) = 1 + a^s, score(\succ', 3) = 1 + a^s,$

$score(\succ, i) = score(\succ', i) = 1$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ,j)} = 0$ or $a_{p_2(\succ,j)} = 0\},$

$score(\succ, i) = score(\succ', i) = 2$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ,j)} = a_{p_2(\succ,j)} = 1\},$

$score(\succ, i) = score(\succ', i) = 2a^s$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ,j)} = a^s\},$ and

$score(\succ, i) = score(\succ', i) > 2$ for all $i \in \{j \in N \backslash \{1, 2, 3\} | a_{p_1(\succ,j)} > 1\}.$

Thus, we have

$$\forall l \in N \backslash \{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, $2R(\succ')3$ holds but $2R(\succ)3$ does not hold, hence R is not *assistance-proof.*

□

Proof of Theorem 3:

First, we show that the statement holds if $n = 3$. Since there exists no integer in $(\frac{4}{3}, \frac{5}{3})$, it suffices to show that any nonnull scoring rule is not *assistance-proof.* Take any nonnull scoring rule R. Without loss of generality, assume $a_1 = 1$ and $a_3 = 0$.

Case 1. Consider the case with $a_2 < \frac{1}{2}$. Let $\succ$ be a ranking profile such that

$$p_1(\succ, 1) = 1, p_1(\succ, 2) = 2, p_1(\succ, 3) = 3,$$
$$p_2(\succ, 1) = 1, p_2(\succ, 2) = 2, p_2(\succ, 3) = 3, \text{ and}$$
$$p_3(\succ, 3) = 1, p_3(\succ, 1) = 2, p_3(\succ, 2) = 3.$$

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$p_1(\succ', 2) = 1, p_1(\succ', 1) = 2, \text{ and}$$
$$p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M \backslash \{(1, 1), (2, 1)\}.$$

score\event	1	2	3
$a_1 = 1$	$\cancel{1}$ 2	1	3
a_2	$\cancel{2}$ 1	2	1
$a_3 = 0$	3	3	2

Then, the scores of each competitor are as follows:

$$\begin{aligned}
&score(\succ, 1) = 2 + a_2, score(\succ', 1) = 1 + 2a_2,\\
&score(\succ, 2) = 2a_2, score(\succ', 2) = 1 + a_2, \text{ and}\\
&score(\succ, 3) = score(\succ', 3) = 1.
\end{aligned}$$

Thus, we have

$$\forall l \in N\backslash\{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, $2R(\succ')3$ holds but $2R(\succ)3$ does not hold, hence R is not *assistance-proof*.

Case 2. Consider the case where $a_2 \geq \frac{1}{2}$. Let $\succ$ be a ranking profile such that

$$\begin{aligned}
&p_1(\succ, 2) = 1, p_1(\succ, 3) = 2, p_1(\succ, 1) = 3,\\
&p_2(\succ, 2) = 1, p_2(\succ, 3) = 2, p_2(\succ, 1) = 3, \text{ and}\\
&p_3(\succ, 3) = 1, p_3(\succ, 1) = 2, p_3(\succ, 2) = 3.
\end{aligned}$$

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$\begin{aligned}
&p_3(\succ', 2) = 2, p_3(\succ', 1) = 3, \text{ and}\\
&p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M\backslash\{(1, 3), (2, 3)\}.
\end{aligned}$$

score\event	1	2	3
$a_1 = 1$	2	2	3
a_2	3	3	$\cancel{1}$ 2
$a_3 = 0$	1	1	$\cancel{2}$ 1

Then, the scores of each competitor are as follows:

$$\begin{aligned}
&score(\succ, 1) = a_2, score(\succ', 1) = 0,\\
&score(\succ, 2) = 2, score(\succ', 2) = 2 + a_2, \text{ and}\\
&score(\succ, 3) = score(\succ', 3) = 1 + 2a_2.
\end{aligned}$$

Thus, we have

$$\forall l \in N \backslash \{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, if $a_2 = \frac{1}{2}$, then $3R(\succ)2$ holds but $3R(\succ')2$ does not hold. Otherwise, $2R(\succ')3$ holds but $2R(\succ)3$ does not hold. Hence R is not *assistance-proof*.

Next, we show that the statement holds if $n \geq 4$. Take any scoring rule R.

Case 1. Consider the case where R is neither null nor dichotomous. Then R has at least three distinct scores. We show that R is not *assistance-proof*. Without loss of generality, assume that $a_n = 0$ and the third-lowest score is 1. Let a^s denote the second-lowest score. Let t_1 and t_2 be the largest integers satisfying $a_{t_1} = 1$ and $a_{t_2} = a^s$, respectively.

Case 1-1. Consider the case where $|\{q \in \{1, ..., n\} | a_q = 0\}| = 1$. Let $\succ$ be a ranking profile such that

$$\begin{aligned}
&p_1(\succ, 2) = n-3, p_1(\succ, 4) = n-2, p_1(\succ, 3) = n-1, p_1(\succ, 1) = n,\\
&p_2(\succ, 3) = n-3, p_2(\succ, 2) = n-2, p_2(\succ, 4) = n-1, p_2(\succ, 1) = n,\\
&p_3(\succ, 4) = n-3, p_3(\succ, 3) = n-2, p_3(\succ, 1) = n-1, p_3(\succ, 2) = n, \text{ and}\\
&p_1(\succ, i) = p_2(\succ, i) = p_3(\succ, i) \text{ for all } i \in N \backslash \{1, 2, 3, 4\}.
\end{aligned}$$

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$\begin{aligned}
&p_3(\succ', 2) = n-1, p_3(\succ', 1) = n, \text{ and}\\
&p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M \backslash \{(1, 3), (2, 3)\}.
\end{aligned}$$

score\event	1	2	3
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	2	3	4
$\vdots$	4	2	3
$a_{n-1} = a^s$	3	4	$\cancel{1}$ 2
$a_n = 0$	1	1	$\cancel{2}$ 1

Then, the scores of each competitor are as follows:

$$\begin{aligned}
&score(\succ, 1) = a^s, score(\succ', 1) = 0,\\
&score(\succ, 2) \geq 2a^s, score(\succ', 2) \geq 3a^s,\\
&score(\succ, 3) = score(\succ', 3) \geq 3a^s,\\
&score(\succ, 4) = score(\succ', 4) \geq 3a^s, \text{ and}\\
&score(\succ, i) = score(\succ', i) \geq 3a^s \text{ for all } i \in N\backslash\{1, 2, 3, 4\}.
\end{aligned}$$

Thus, we have

$$\forall l \in N\backslash\{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, it follows that

$$\begin{aligned}
&score(\succ, 3) - score(\succ, 2) = a^s > 0, \text{ and}\\
&score(\succ', 3) = score(\succ', 2).
\end{aligned}$$

Therefore, $2R(\succ')3$ holds but $2R(\succ)3$ does not hold, and hence R is not *assistance-proof*.

Case 1-2. Consider the case where $|\{q \in \{1, ..., n\}|a_q = 0\}| > 1$ and $|\{q \in \{1, ..., n\}|a_q = 1\}| = 1$. Let $\succ$ be a ranking profile such that

$$\begin{aligned}
&p_1(\succ, 1) = t_1, p_1(\succ, 2) = t_1 + 1, p_1(\succ, 4) = t_1 + 2, p_1(\succ, 3) = t_1 + 3,\\
&p_2(\succ, 1) = t_1, p_2(\succ, 3) = t_1 + 1, p_2(\succ, 2) = t_1 + 2, p_2(\succ, 4) = t_1 + 3,\\
&p_3(\succ, 1) = t_1, p_3(\succ, 4) = t_1 + 1, p_3(\succ, 3) = t_1 + 2, p_3(\succ, 2) = t_1 + 3, \text{ and}\\
&p_1(\succ, i) = p_2(\succ, i) = p_3(\succ, i) \text{ for all } i \in N\backslash\{1, 2, 3, 4\}.
\end{aligned}$$

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$\begin{aligned}
&p_1(\succ', 2) = t_1, p_1(\succ', 1) = t_1 + 1, \text{ and}\\
&p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M\backslash\{(1, 1), (2, 1)\}.
\end{aligned}$$

score\event	1	2	3
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_{t_1} = 1$	$\cancel{1}$ 2	1	1
$a_{t_1+1} = a^s$	$\cancel{2}$ 1	3	4
$\vdots$	4	2	3
$\vdots$	3	4	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Then, the scores of each competitor are as follows:

$score(\succ, 1) = 3, score(\succ', 1) = 2 + a^s,$
$score(\succ, 2) \le 3a^s, score(\succ', 2) \le 1 + 2a^s,$
$score(\succ, 3) = score(\succ', 3) \le 3a^s,$
$score(\succ, 4) = score(\succ', 4) \le 3a^s,$
$score(\succ, i) = score(\succ', i) \le 3a^s$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_1(\succ, j)} < 1\},$ and
$score(\succ, i) = score(\succ', i) > 3$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_1(\succ, j)} > 1\}.$

Thus, we have

$$\forall l \in N\backslash\{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, it follows that

$$\begin{aligned} &score(\succ, 3) = score(\succ, 2), \text{ and} \\ &score(\succ', 2) - score(\succ', 3) = 1 - a^s > 0. \end{aligned}$$

Therefore, $3R(\succ)2$ holds but $3R(\succ')2$ does not hold, and hence R is not *assistance-proof.*

Case 1-3. Consider the case where $1 < |\{q \in \{1, ..., n\} | a_q = 1\}| \le |\{q \in \{1, ..., n\} | a_q = 0\}|$. Let $\succ$ be a ranking profile such that

$p_1(\succ, 4) = t_1 - 1, p_1(\succ, 1) = t_1, p_1(\succ, 2) = t_1 + 1, p_1(\succ, 3) = t_2 + 1,$
$p_2(\succ, 3) = t_1 - 1, p_2(\succ, 1) = t_1, p_2(\succ, 4) = t_1 + 1, p_2(\succ, 2) = t_2 + 1,$
$p_3(\succ, 2) = t_1 - 1, p_3(\succ, 1) = t_1, p_3(\succ, 3) = t_1 + 1, p_3(\succ, 4) = t_2 + 1,$
$a_{p_2(\succ, i)} = a_{p_3(\succ, i)} = 0$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_1(\succ, j)} = 1\},$
$a_{p_1(\succ, i)} = 0, a_{p_3(\succ, i)} = 1$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_2(\succ, j)} = 1\},$ and
$p_1(\succ, i) = p_2(\succ, i) = p_3(\succ, i)$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_1(\succ, j)} \ne 1 \text{ and } a_{p_2(\succ, j)} \ne 1\}.$

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$\begin{aligned} &p_1(\succ', 2) = t_1, p_1(\succ', 1) = t_1 + 1, \text{ and} \\ &p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M\backslash\{(1, 1), (2, 1)\}. \end{aligned}$$

score\event	1	2	3
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_{t_1-1} = 1$	4	3	2
$a_{t_1} = 1$	~~1~~ 2	1	1
$a_{t_1+1} = a^s$	~~2~~ 1	4	3
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_{t_2+1} = 0$	3	2	4
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Then, the scores of each competitor are as follows:

$score(\succ, 1) = 3, score(\succ', 1) = 2 + a^s$,

$score(\succ, 2) = 1 + a^s, score(\succ', 2) = 2$,

$score(\succ, 3) = 1 + a^s, score(\succ', 3) = 1 + a^s$,

$score(\succ, 4) = 1 + a^s, score(\succ', 4) = 1 + a^s$,

$score(\succ, i) = score(\succ', i) = 1$ for all $i \in \{j \in N\backslash\{1,2,3,4\} | a_{p_1(\succ,j)} = 1\}$,

$score(\succ, i) = score(\succ', i) = 2$ for all $i \in \{j \in N\backslash\{1,2,3,4\} | a_{p_1(\succ,j)} = 0$ and $a_{p_2(\succ,j)} = 1\}$,

$score(\succ, i) = score(\succ', i) = 0$ for all $i \in \{j \in N\backslash\{1,2,3,4\} | a_{p_1(\succ,j)} = 0$ and $a_{p_2(\succ,j)} = 0\}$,

$score(\succ, i) = score(\succ', i) > 3$ for all $i \in \{j \in N\backslash\{1,2,3,4\} | a_{p_1(\succ,j)} > 1\}$, and

$score(\succ, i) = score(\succ', i) = 3a^s$ for all $i \in \{j \in N\backslash\{1,2,3,4\} | a_{p_1(\succ,j)} = a^s\}$.

Thus, we have

$$\forall l \in N\backslash\{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, it follows that $3R(\succ)2$ holds but $3R(\succ')2$ does not hold, and hence R is not *assistance-proof*.

Case 1-4. Consider the case where $1 < |\{q \in \{1, ..., n\} | a_q = 0\}| < |\{q \in \{1, ..., n\} | a_q = 1\}|$. Let $\succ$ be a ranking profile such that

$p_1(\succ, 2) = t_1 - 1, p_1(\succ, 4) = t_1, p_1(\succ, 3) = t_2, p_1(\succ, 1) = t_2 + 1$,

$p_2(\succ, 3) = t_1 - 1, p_2(\succ, 2) = t_1, p_2(\succ, 4) = t_2, p_2(\succ, 1) = t_2 + 1$,

$p_3(\succ, 4) = t_1 - 1, p_3(\succ, 3) = t_1, p_3(\succ, 1) = t_2, p_3(\succ, 2) = t_2 + 1$,

$a_{p_2(\succ,i)} = a_{p_3(\succ,i)} = 1$ for all $i \in \{j \in N\backslash\{1,2,3,4\} | a_{p_1(\succ,j)} = 0\}$,

$a_{p_1(\succ,i)} = 1, a_{p_3(\succ,i)} = 0$ for all $i \in \{j \in N\backslash\{1,2,3,4\} | a_{p_2(\succ,j)} = 0\}$, and

$p_1(\succ, i) = p_2(\succ, i) = p_3(\succ, i)$ for all $i \in \{j \in N\backslash\{1,2,3,4\} | a_{p_1(\succ,j)} \neq 1$ and $a_{p_2(\succ,j)} \neq 1\}$.

In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$p_3(\succ', 2) = t_2, p_3(\succ', 1) = t_2 + 1, \text{ and}$$
$$p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M \backslash \{(1, 3), (2, 3)\}.$$

score\event	1	2	3
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_{t_1-1} = 1$	2	3	4
$a_{t_1} = 1$	4	2	3
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_{t_2} = a^s$	3	4	$\cancel{1}$ 2
$a_{t_2+1} = 0$	1	1	$\cancel{2}$ 1
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Then, the scores of each competitor are as follows:

$score(\succ, 1) = a^s, score(\succ', 1) = 0,$

$score(\succ, 2) = 2, score(\succ', 2) = 2 + a^s,$

$score(\succ, 3) = 2 + a^s, score(\succ', 3) = 2 + a^s,$

$score(\succ, 4) = 2 + a^s, score(\succ', 4) = 2 + a^s,$

$score(\succ, i) = score(\succ', i) = 2$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_1(\succ, j)} = 0\},$

$score(\succ, i) = score(\succ', i) = 1$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_1(\succ, j)} = 1$ and $a_{p_2(\succ, j)} = 0\},$

$score(\succ, i) = score(\succ', i) = 3$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_1(\succ, j)} = 1$ and $a_{p_2(\succ, j)} = 1\},$

$score(\succ, i) = score(\succ', i) > 3$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_1(\succ, j)} > 1\},$ and

$score(\succ, i) = score(\succ', i) = 3a^s$ for all $i \in \{j \in N\backslash\{1, 2, 3, 4\} | a_{p_1(\succ, j)} = a^s\}.$

Thus, we have

$$\forall l \in N\backslash\{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, it follows that $2R(\succ')3$ holds but $2R(\succ)3$ does not hold, and hence R is not *assistance-proof*.

Next, we show that dichotomous rule is *assistance-proof* if and only if $\theta \in (\frac{n+1}{3}, \frac{2n-1}{3})$. Take any dichotomous rule R. Without loss of generality, assume that $a_{\text{high}} = 1$ and $a_{\text{low}} = 0$. Let t denote the largest integer satisfying $a_t = 1$.

Case 2-1. Consider the case where $\theta \in (\frac{n+1}{3}, \frac{2n-1}{3})$. We show that R is *assistance-proof*. Suppose a competitor i whose score is 3 can assist anyone. Then there does not exist any competitor whose score is 2 or 3 and there exists some competitor whose score is 0. However, such a profile does not exist, since the total score of all competitors except i is

$$3\theta - 3 > n - 2.$$

Suppose a competitor i whose score is 1 can assist anyone. Then there does not exist any competitor whose score is 1 or 0 and there exists some competitor whose score is 2. However, such a profile does not exist, since the total score of all competitors except i is

$$3\theta - 3 < 2(n - 2).$$

Evidently, any competitor whose score is 2 or 0 cannot assist anyone. Thus, R is *assistance-proof*.

Case 2-2. Consider the case with $\theta \leq \frac{n+1}{3}$. Let $\succ$ be a ranking profile such that

$$\begin{aligned}
&p_1(\succ, 1) = t, p_1(\succ, 2) = t + 1,\\
&p_2(\succ, 1) = t, p_2(\succ, 2) = t + 1,\\
&p_3(\succ, 1) = t, p_3(\succ, 2) = t + 1,\\
&a_{p_2(\succ,i)} = a_{p_3(\succ,i)} = 0 \text{ for all } i \in \{j \in N\backslash\{1,2\} | a_{p_1(\succ,j)} = 1\},\\
&a_{p_3(\succ,i)} = a_{p_1(\succ,i)} = 0 \text{ for all } i \in \{j \in N\backslash\{1,2\} | a_{p_2(\succ,j)} = 1\}, \text{ and}\\
&a_{p_1(\succ,i)} = a_{p_2(\succ,i)} = 0 \text{ for all } i \in \{j \in N\backslash\{1,2\} | a_{p_3(\succ,j)} = 1\}.
\end{aligned}$$

Note that, such a profile exists since $3\theta - 3 \leq n - 2$. In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$$\begin{aligned}
&p_1(\succ', 2) = t, p_1(\succ', 1) = t + 1, \text{ and}\\
&p_h(\succ, i) = p_h(\succ', i) \text{ for all } (i, h) \in N \times M\backslash\{(1,1),(2,1)\}.
\end{aligned}$$

score\event	1	2	3
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_t = 1$	$\cancel{1}\ 2$	1	1
$a_{t+1} = 0$	$\cancel{2}\ 1$	2	2
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Then, the scores of each competitor are as follows:

$score(\succ, 1) = 3, score(\succ', 1) = 2,$

$score(\succ, 2) = 0, score(\succ', 2) = 1,$

$score(\succ, i) = score(\succ', i) = 1$

for all $i \in \{j \in N\backslash\{1,2\} | a_{p_1(\succ,j)} = 1 \text{ or } a_{p_2(\succ,j)} = 1 \text{ or } a_{p_3(\succ,j)} = 1\}$, and

$score(\succ, i) = score(\succ', i) = 0$

for all $i \in \{j \in N\backslash\{1,2\} | a_{p_1(\succ,j)} = 0 \text{ and } a_{p_2(\succ,j)} = 0 \text{ and } a_{p_3(\succ,j)} = 0\}$.

Thus, we have

$$\forall l \in N\backslash\{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, there exists a competitor $i \in N\backslash\{1,2\}$ such that $score(\succ, i) = score(\succ', i) = 0$ or $score(\succ, i) = score(\succ', i) = 1$. Thus R is not *assistance-proof*.

Case 2-3. Consider the case where $\theta \geq \frac{2n-1}{3}$. Let $\succ$ be a ranking profile such that

$p_1(\succ, 2) = t, p_1(\succ, 1) = t + 1,$

$p_2(\succ, 2) = t, p_2(\succ, 1) = t + 1,$

$p_3(\succ, 1) = t, p_3(\succ, 2) = t + 1,$

$a_{p_2(\succ,i)} = a_{p_3(\succ,i)} = 1$ for all $i \in \{j \in N\backslash\{1,2\} | a_{p_1(\succ,j)} = 0\}$,

$a_{p_3(\succ,i)} = a_{p_1(\succ,i)} = 1$ for all $i \in \{j \in N\backslash\{1,2\} | a_{p_2(\succ,j)} = 0\}$, and

$a_{p_1(\succ,i)} = a_{p_2(\succ,i)} = 1$ for all $i \in \{j \in N\backslash\{1,2\} | a_{p_3(\succ,j)} = 0\}$.

Note that, such a profile exists since $3\theta - 3 \geq 2(n-2)$. In addition, consider $\succ' \in \mathscr{L}(\succ, 1)$ such that

$p_3(\succ', 2) = t, p_3(\succ', 1) = t + 1,$ and

$p_h(\succ, i) = p_h(\succ', i)$ for all $(i, h) \in N \times M\backslash\{(1,3),(2,3)\}$.

score\event	1	2	3
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$a_t = 1$	2	2	$\cancel{1}\ 2$
$a_{t+1} = 0$	1	1	$\cancel{2}\ 1$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Then, the scores of each competitor are as follows:

$score(\succ, 1) = 1, score(\succ', 1) = 0,$

$score(\succ, 2) = 2, score(\succ', 2) = 3,$

$score(\succ, i) = score(\succ', i) = 2$

for all $i \in \{j \in N \backslash \{1,2\} | a_{p_1(\succ,j)} = 0$ or $a_{p_2(\succ,j)} = 0$ or $a_{p_3(\succ,j)} = 0\}$, and

$score(\succ, i) = score(\succ', i) = 3$

for all $i \in \{j \in N \backslash \{1,2\} | a_{p_1(\succ,j)} = 1$ and $a_{p_2(\succ,j)} = 1$ and $a_{p_3(\succ,j)} = 1\}$.

Thus, we have

$$\forall l \in N \backslash \{1\}, 1R(\succ)l \Longrightarrow 1R(\succ')l \text{ and } 1P(\succ)l \Longrightarrow 1P(\succ')l.$$

However, there exists a competitor $i \in N \backslash \{1, 2\}$ such that $score(\succ, i) = score(\succ', i) = 2$ or $score(\succ, i) = score(\succ', i) = 3$. Thus R is not *assistance-proof.* □

Proof of Theorem 5:

Take any generalized scoring rule R. If R is null, then R is *assistance-proof.* If R is nonnull and $w_1 = w_2$, then by Theorem 2, R is *assistance-proof* if and only if R is dichotomous rule. Suppose R is nonnull and $w_1 \neq w_2$. Without loss of generality, assume that $a_1 = 1, a_3 = 0,$ and $w_1 > w_2 = 1$.

score\weight	$w_1 > 1$	$w_2 = 1$
$a_1 = 1$	(w_1)	(1)
a_2	$(a_2 w_1)$	(a_2)
$a_3 = 0$	(0)	(0)

We show that R is *assistance-proof* if and only if R is dictatorial. First, we derive the necessary and sufficient conditions for R to be dictatorial.

Lemma 1. *R is dictatorial if and only if $a_2 w_1 > 1$ and $w_1 > a_2 w_1 + 1$.*

Proof. First, we show the "if" part. Suppose $a_2 w_1 > 1$ and $w_1 > a_2 w_1 + 1$. We show that for all $i, j \in N$ and $\succ \in \mathscr{P}^m$, $i \succ_1 j \Longrightarrow iP(\succ)j$. Take any $i, j \in N$ and $\succ \in \mathscr{P}^m$. Suppose $i \succ_1 j$.

Case 1-1. Consider the case with $p_1(\succ, i) = 1$. Then, i's score is not less than w_1 and j's score is not more than $a_2 w_1 + 1$. Now $w_1 > a_2 w_1 + 1$, so we have $iP(\succ)j$.

Case 1-2. Consider the case with $p_1(\succ, i) = 2$. Then, i's score is not less than $a_2 w_1$ and j's score is not more than 1. Now $a_2 w_1 > 1$, so we have $iP(\succ)j$.

Second, we show the "only if" part by showing the contraposition. Suppose either $a_2 w_1 \leq 1$ or $w_1 \leq a_2 w_1 + 1$.

Case 2-1. Consider the case with $a_2 w_1 \leq 1$. Take $\succ \in \mathscr{P}^m$ such that $3 \succ_1 1 \succ_1 2$ and $2 \succ_2 3 \succ_2 1$.

score\weight	$w_1 > 1$	$w_2 = 1$
$a_1 = 1$	3	2
a_2	1	3
$a_3 = 0$	2	1

Then we have $3R(\succ)2R(\succ)1$. Thus R is not dictatorial.

Case 2-2. Consider the case with $w_1 \leq a_2 w_1 + 1$. Take $\succ \in \mathscr{P}^m$ such that $2 \succ_1 1 \succ_1 3$ and $1 \succ_2 3 \succ_2 2$.

score\weight	$w_1 > 1$	$w_2 = 1$
$a_1 = 1$	2	1
a_2	1	3
$a_3 = 0$	3	2

Then we have $1R(\succ)2R(\succ)3$. Thus R is not dictatorial. □

By showing the next lemma, we obtain the proof of the theorem.

Lemma 2. *R is assistance-proof if and only if $a_2 w_1 > 1$ and $w_1 > a_2 w_1 + 1$.*

Proof. By lemma 1, the "if" part is obvious. We show the "only if" part by showing contraposition. Suppose either $a_2 w_1 \leq 1$ or $w_1 \leq a_2 w_1 + 1$.

Case 1. Consider the case where $a_2 w_1 \leq 1$. Take $\succ \in \mathscr{P}^m$ such that $3 \succ_1 1 \succ_1 2$ and $3 \succ_2 2 \succ_2 1$.

score\weight	$w_1 > 1$	$w_2 = 1$
$a_1 = 1$	3	3
a_2	1	2
$a_3 = 0$	2	1

In addition, take $\succ' \in \mathscr{L}(\succ, 3)$ such that $3 \succ'_1 1 \succ'_1 2$ and $2 \succ'_2 3 \succ'_2 1$.

score\weight	$w_1 > 1$	$w_2 = 1$
$a_1 = 1$	3	$\cancel{3}$ 2
a_2	1	$\cancel{2}$ 3
$a_3 = 0$	2	1

If $a_2 = 0$, we have $3P(\succ)1R(\succ)2$ and $3P(\succ')2P(\succ')1$. Otherwise, we have $3P(\succ)1P(\succ)2$ and $3P(\succ')2R(\succ')1$. Therefore, R is not *assistance-proof*.

Case 2. Consider the case where $w_1 \leq a_2 w_1 + 1$. Take $\succ \in \mathscr{P}^m$ such that $2 \succ_1 1 \succ_1 3$ and $1 \succ_2 3 \succ_2 2$.

score\weight	$w_1 > 1$	$w_2 = 1$
$a_1 = 1$	2	1
a_2	1	3
$a_3 = 0$	3	2

In addition, take $\succ' \in \mathscr{L}(\succ, 3)$ such that $2 \succ'_1 1 \succ'_1 3$ and $1 \succ'_2 2 \succ'_2 3$.

score\weight	$w_1 > 1$	$w_2 = 1$
$a_1 = 1$	2	1
a_2	1	$\cancel{3}$ 2
$a_3 = 0$	3	$\cancel{2}$ 3

If $a_2 = 1$, then we have $1P(\succ)2P(\succ)3$ and $2R(\succ')1P(\succ')3$. Otherwise, we have $1R(\succ)2P(\succ)3$ and $2P(\succ')1P(\succ')3$. Therefore, R is not *assistance-proof*. This completes the proof of the lemma and the theorem. $\square$

References

[1] Arrow, K.J. (1951), *Social Choice and Individual Values*, Wiley, New York.

[2] Baigent, N. (1987), Twitching weak dictators, *Journal of Economics*, 47, 407–411.

[3] Campbell, D.E., Kelly, J.S. (2000), Information and preference aggregation, *Social Choice and Welfare*, 17, 3–24.

[4] Campbell, D.E., Kelly, J.S. (2007), Social welfare functions that satisfy Pareto, anonymity, and neutrality, but not independence of irrelevant alternatives, *Social Choice and Welfare*, 29, 69–82.

[5] Cato, S. (2014), Independence and irrelevant alternatives revisited, *Theory and Decision*, 76, 511–527.

[6] Duggan, M., Levitt, S. D. (2002), Winning isn't everything: Corruption in sumo wrestling, *American Economic Review*, 92, 1594–1605.

[7] Dutta, B., Jackson, M. O., Le Breton, M. (2001), Strategic candidacy and voting procedures, *Econometrica*, 69, 1013–1037.

[8] Eso, P., Schummer, J., (2004), Bribing and signaling in second price auctions, *Games and Economic behavior*, 47, 299–324.

[9] Fleurbaey, M., Suzumura, K., Tadenuma, K. (2005), Arrovian aggregation in economic environments: How much should we know about indifference surfaces? *Journal of Economic Theory*, 124, 22–24.

[10] Holliday, W.H., Kelley, M. (2020), A note on Murakami's theorems and incomplete social choice without the Pareto principle, *Social Choice and Welfare*, 55, 243–253.

[11] Malawski, M., Zhou, L. (1994), A note on social choice without the Pareto principle, *Social Choice and Welfare*, 11, 103–107.

[12] Murakami, Y. (1968), *Logic and Social Choice*, Dover, New York.

[13] Preston, I., Szymanski, S. (2003), Cheating in contests, *Oxford Review of Economic Policy*, 19, 612–624.

[14] Sakai, T., Shimoji, M. (2006), Dichotomous preferences and the possibility of Arrovian social choice, *Social Choice and Welfare*, 26, 435–445.

[15] Sato, S. (2013), A sufficient condition for the equivalence of strategy-proofness and nonmanipulability by preferences adjacent to the sincere one, *Journal of Economic Theory*, 148, 259–278.

[16] Satterthwaite, M., Sonnenschein, H. (1981), Strategy-proof allocation mechanisms at differentiable points, *The Review of Economic Studies*, 48, 587–597.

[17] Schummer, J., (2000a), Eliciting preferences to assign positions and compensation, *Games and Economic Behavior*, 30, 293–318.

[18] Schummer, J., (2000b), Manipulation through bribes, *Journal of Economic Theory*, 91, 180–198.

[19] Thomson, W. (2016), Non-bossiness, *Social Choice and Welfare*, 47, 665–696.

[20] Wilson, R.B. (1972), Social choice theory without the Pareto principle, *Journal of Economic Thoery*, 5, 478–486.