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Delegating Decisions to Independent Committees*

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Abstract

This paper analyzes the delegation of binary decisions to a committee of homogeneous agents. The principal determines the committee size and a reward scheme contingent on the revealed state and the committee's choice. Agents can acquire private information at a cost but lack intrinsic motives to make correct decisions. The main results are as follows: For any committee size and any prior distribution of the state, the reward scheme that minimizes the cost of making agents acquire information induces the committee to make decisions by majority rule. If the principal is ex-ante indifferent between the two alternatives, the optimal reward scheme for the principal induces the committee to use the majority rule and the optimal committee size is inversely U-shaped regarding information quality.

JEL classification: D71, D82, D86

Keywords: Moral Hazard, Free-Rider Problem, Majority Rule, Committee Size, Information Acquisition, Monetary Transfers.

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1 Introduction

Decisions are often delegated to expert committees. For instance, shareholders entrust a company's management to a board of directors, and governments establish advisory bodies to make policy decisions. In these situations, committee designers jointly determine the committee size and the rewards for the experts. How many experts should the committee designer hire? What rewards should the committee designer offer the experts?

To address these questions, I have constructed a model in which the principal, who delegates a binary decision to a committee of homogeneous agents, can choose the committee size and a reward scheme. Each agent can receive a binary signal regarding the unknown state at a cost. Their payoffs are the expected reward minus the cost of information acquisition.[1] The unknown state is revealed after the decision is made. In addition, the principal cannot receive contractible messages from the agents due to communication costs.[2] Thus, the rewards depend only on the realized state and the committee's choice, not on the individual members' messages. It is assumed that the principal must offer an identical reward scheme to all committee members. The committee, like a mediator in Myerson (1986), selects the best alternative for all members based on their information. Such an alternative exists because all members are offered the same reward scheme. The strategy of the committee is regarded as the collective choice rule. The principal controls the committee's collective choice rule through monetary transfers.

The principal faces a trade-off between the benefit from the decision and the cost of making agents acquire information when choosing the committee size. As illustrated by Condorcet's jury theorem (Condorcet, 1785), increasing the number of agents leads to more precise decision-making. However, if the number of agents increases, each agent is less

[1] Committee members do not have intrinsic motives to make correct decisions. These agents are called *non-ethical* by Strulovici (2020).

[2] In many situations where decisions are delegated to committees, sending and receiving verifiable messages is extremely costly. For example, shareholders do not communicate with individual directors. Moreover, directors' compensation depends only on the company's benefit, which is determined by the collective choice of the directors and the state of the world in my model. Mookherjee and Tsumagari (2014) explicitly incorporate communication costs into mechanism design problems.

likely to play a pivotal role in the collective choice. Thus, increasing the number of agents discourages members from gathering information. Therefore, committee designers should take this trade-off into account when choosing the committee size.

The principal may face this trade-off not only when choosing the committee size but also when setting a reward scheme, that is, controlling the collective choice rule. This is because the reward scheme that minimizes the cost of making agents acquire information does not necessarily induce the committee to use the collective choice rule that maximizes the benefit from the decision. Committee designers should consider this trade-off when setting a reward scheme.

The main contribution of this paper is its elucidation of the relationship between collective choice rules and agency costs. Many studies on strategic voting assume that voters share a common purpose with social planners. Hence, they need not motivate voters to acquire information. In the literature on delegating decisions to experts, collective choice rules tend to be fixed. Consequently, the relationship between voting rules and agency costs has not been adequately studied.

It is shown that, for any committee size and any prior distribution of the state, the reward scheme that minimizes the cost of making agents acquire information induces the committee to make decisions by majority rule. Under the assumption that the principal is ex-ante indifferent between the two alternatives, the optimal reward scheme for the principal induces the committee to use the majority rule. Under this assumption, I examine the relationship between the optimal committee size and information quality. If only one agent's information quality improves, he is encouraged to acquire information but others are tempted to be free-riders. Thus, the effect of information quality on the optimal committee size is unclear. It turns out that the optimal committee size is inversely U-shaped regarding information quality. In other words, the free-rider effect becomes dominant as information quality increases.

Given the committee size and a reward scheme, my model is similar to that of strategic voting. While conventional voting models assume that voters have the same preferences as social planners, mine allows the principal to control the agents' preferences through monetary transfers. Austen-Smith and Banks (1996) and Persico (2004) focus on threshold voting rules. Austen-Smith and Banks (1996) imply that the strategy profile in which all voters report their truthful information is a Nash equilibrium in the threshold voting rule that selects the best alternative based on the vote profile. In my model, as the committee selects the best alternative for agents based on their information profile, agents report their information honestly. Persico (2004) clarifies that the incentive for information acquisition is influenced by the probability that a vote of an agent makes a difference in the voting outcome, i.e., the probability that he is pivotal in the collective choice. Gerardi and Yariv (2008) examine a voting model that includes stochastic voting rules. They point out that social planners could increase the incentive for information acquisition by committing to ex-post inefficient voting rules. In my model, the collective choice rule is selected by the committee. Thus, the principal cannot improve her payoff by employing ex-post inefficient rules.

My study is related to the literature on delegating decisions with transfers. While the number of experts was fixed in most earlier studies, my model allows the principal to select it. Zermeno (2011), Carroll (2019), and Clark and Reggiani (2021) analyze the delegation of decision-making to one expert in an environment where rewards can be contingent on the realized state. Gromb and Martimort (2007) compare the delegation to one expert who can obtain information twice with that to two experts who can obtain information once. They conclude that if experts do not collude, the delegation to two experts is more profitable for the principal due to the peer-monitoring effect. Azrieli (2021) shows that the principal can motivate multiple agents to acquire information using peer monitoring even if the state of the world is never revealed. In my model, the principal does not use peer monitoring because she cannot receive verifiable messages from the agents.

Feddersen and Pesendorfer (1998), Gershkov and Szentes (2009), Gershkov, Moldovanu and Shi (2017), etc. consider optimal voting rules from different perspectives. Mukhopadhaya (2003), Koriyama and Szentes (2009), Gersbach, Mamageishvili and Tejada (2022), etc. investigate the optimal committee size in different settings.

The rest of this paper is organized as follows. Section 2 constructs the model. Section 3 formulates the principal's optimization problem. Section 4 examines the optimal reward scheme for a fixed committee size and the relationship between the collective choice rules and agency costs. Section 5 characterizes the optimal committee size and develops comparative statics under the assumption that the principal is ex-ante indifferent between the two alternatives. Section 6 concludes and suggests directions for future research. All proofs are presented in Appendix A. Appendix B discusses the equilibrium selection.

2 The Model

The principal must choose between two alternatives, I and N. The state of the world $\theta \in \{H, L\}$ is distributed according to the common prior, $\Pr(H)$ and $\Pr(L)$. If the choice is I and the state is H (resp. L), she earns the benefit of $b_H > 0$ (resp. $b_L < 0$). If the choice is N, the benefit is zero regardless of the state. The state of the world is revealed after the decision is made.

The principal has the option of delegating her decision to a committee of agents. She can arbitrarily choose the size of the committee and transfer money to each agent according to the realized state and the committee's choice. More specifically, she chooses the number of agents n and a reward scheme $\boldsymbol{w} = (w_H^I, w_L^I, w_H^N, w_L^N)$, where w_θ^j denotes the amount of transfer to an agent when the realized state is $\theta \in \{H, L\}$ and the committee's choice is $j \in \{I, N\}$. The principal must offer the same reward scheme to all committee members. Agents are protected by limited liability, i.e., $\boldsymbol{w} \geq 0$. The principal is risk-neutral and her payoff is the expected benefit from the decision minus the expected transfers. I refer to a pair of a committee size and a reward scheme as a contract. A contract is said to be optimal if its committee size is the smallest among contracts that maximize the principal's payoff.

The principal obtains a payoff of $\max\{\Pr(H)b_H + \Pr(L)b_L, 0\}$ by setting the committee size to zero, i.e., no making committee. Hence, if the optimal committee size is greater than zero, she can obtain a payoff greater than $\max\{\Pr(H)b_H + \Pr(L)b_L, 0\}$.

By incurring information cost $c > 0$, each agent $i \in \{1, 2, \ldots, n\}$ can receive one conditionally independent private signal $s_i \in \{h, \ell\}$, which is informative:

$$\Pr(s_i = h \mid H) = \Pr(s_i = \ell \mid L) = q \in \left(\frac{1}{2}, 1\right).$$

Let $t_i \in \{\emptyset, h, \ell\}$ denote the type of agent i, where $\emptyset$ denotes the type of an uninformed agent. Agents are homogeneous, that is, the cost and quality of information are identical across agents. They are risk-neutral, and their payoffs are the expected reward minus the information cost if they acquire information. After observing the contract, the agents simultaneously decide whether to acquire information and report their types to the committee.

The committee selects the best alternative for the agents based on their type profile. This alternative exists because all committee members are offered the same reward scheme. I refer to the committee's strategy as the collective choice rule $f : \{\emptyset, h, \ell\}^n \to [0, 1]$, which maps the type profile $\boldsymbol{t} \in \{\emptyset, h, \ell\}^n$ to the probability that the committee selects I. For simplicity, the committee is assumed to select N if it is indifferent between the two alternatives. In short, a collective choice rule f satisfies the following condition:

$$\Pr(H \mid \boldsymbol{t})w_H^I + \Pr(L \mid \boldsymbol{t})w_L^I > \Pr(H \mid \boldsymbol{t})w_H^N + \Pr(L \mid \boldsymbol{t})w_L^N \iff f(\boldsymbol{t}) = 1,$$

$$\Pr(H \mid \boldsymbol{t})w_H^I + \Pr(L \mid \boldsymbol{t})w_L^I \leq \Pr(H \mid \boldsymbol{t})w_H^N + \Pr(L \mid \boldsymbol{t})w_L^N \iff f(\boldsymbol{t}) = 0.$$

Note that a contract uniquely determines the collective choice rule.

The game proceeds as follows.

Stage 1. The principal chooses the committee size and a reward scheme.

Stage 2. All the agents simultaneously decide whether to acquire information.

Stage 3. The agents report their types to the committee.

Stage 4. The committee selects the best alternative based on the agents' type profile.

Stage 5. The state of the world is revealed, and the principal pays the reward.

Perfect Bayesian equilibrium is adopted as the solution concept. In Stage 3, all committee members report their truthful types. This is because, given the equilibrium strategy of the committee, truth-telling is a best response for a pivotal agent if the others report their truthful types. The following assumption regarding the equilibrium selection is made.

Assumption 1. If multiple equilibria exist given a contract, the equilibrium in which the number of informed agents is maximized is realized.

This assumption seems convenient for the principal. However, given the optimal contract, agents also gain the highest payoff on the path where all of them gather information. This result is proven in Appendix B.

3 Optimization Problem

In this section, the principal's optimization problem is formulated. First, it is shown that all committee members acquire information given the optimal contract.

Lemma 1. *Suppose that the optimal committee size is greater than zero. Given the optimal contract, all committee members acquire information.*

Suppose that uninformed agents exist on the equilibrium path where the number of informed agents is maximized. Assumption 1 ensures that this equilibrium is realized. By removing uninformed agents from the committee, the principal saves on transfers without changing the probability of making correct decisions. Therefore, all committee members acquire information given the optimal contract.

If there exists an integer R such that

$$f(t_1 = \cdots = t_k = h, t_{k+1} = \cdots = t_n = \ell) = 0 \quad \text{for all } k \leq R,$$
$$f(t_1 = \cdots = t_k = h, t_{k+1} = \cdots = t_n = \ell) = 1 \quad \text{for all } k \geq R+1,$$

then R is said to be the (voting) threshold. Since the agents are homogeneous, the threshold R is invariant to the permutation of the types. Note that the threshold is defined only on the path where all committee members acquire information. On other paths, the committee does not necessarily make decisions using the threshold R. From Lemma 1 and Assumption 1, the path where all agents acquire information is realized in the optimal contract. Therefore, the threshold determines the committee's choice given the optimal contract. Let X_n denote the number of h signals in n informed agents. The committee makes decisions using the threshold R if and only if the contract satisfies the following inequalities:

$$\begin{aligned} &\Pr(H \mid X_n = R) w_H^N + \Pr(L \mid X_n = R) w_L^N \\ \geq &\Pr(H \mid X_n = R) w_H^I + \Pr(L \mid X_n = R) w_L^I, \end{aligned} \tag{1}$$

$$\begin{aligned} &\Pr(H \mid X_n = R+1) w_H^I + \Pr(L \mid X_n = R+1) w_L^I \\ \geq &\Pr(H \mid X_n = R+1) w_H^N + \Pr(L \mid X_n = R+1) w_L^N. \end{aligned} \tag{2}$$

An agent acquires information if the profit from information acquisition exceeds the information cost c. For contracts that induce the committee to use the threshold R, the profit from information acquisition when the others are informed is expressed as follows:

$$\begin{aligned} &\Pr(X_{n-1} = R) \left[\Pr(H \mid X_{n-1} = R) \left(q w_H^I + (1-q) w_H^N\right) + \Pr(L \mid X_{n-1} = R) \left(q w_L^N + (1-q) w_L^I\right)\right] \\ &- \max\left\{\Pr(X_{n-1} = R) \left[\Pr(H \mid X_{n-1} = R) w_H^I + \Pr(L \mid X_{n-1} = R) w_L^I\right],\right. \\ &\qquad \left.\Pr(X_{n-1} = R) \left[\Pr(H \mid X_{n-1} = R) w_H^N + \Pr(L \mid X_{n-1} = R) w_L^N\right]\right\}. \end{aligned}$$

The profit from information acquisition is derived by focusing on pivotal events. From Assumption 1, all the agents gather information if the following condition is satisfied:

$$\Pr(X_{n-1}=R)\left[\Pr(L\mid X_{n-1}=R)q\left(w_L^N-w_L^I\right)-\Pr(H\mid X_{n-1}=R)(1-q)\left(w_H^I-w_H^N\right)\right]\geq c, \tag{IAC_I}$$

$$\Pr(X_{n-1}=R)\left[\Pr(H\mid X_{n-1}=R)q\left(w_H^I-w_H^N\right)-\Pr(L\mid X_{n-1}=R)(1-q)\left(w_L^N-w_L^I\right)\right]\geq c. \tag{IAC_N}$$

This condition is said to be the information acquisition constraint (IAC). Since $c>0$, if (IAC_I) (resp. (IAC_N)) holds, then so does (1) (resp. (2)). Hence, in effect, the principal selects a threshold as well as a contract within the information acquisition constraint.

Let π denote the principal's payoff. The optimization problem is formulated as follows.

$$\max_{n,R,\boldsymbol{w}} \quad \pi(n,R,\boldsymbol{w}) \qquad \text{subject to} \quad (IAC)\ \&\ \boldsymbol{w}\geq 0.$$

4 Optimal Reward Scheme

This section examines the optimal reward scheme for a fixed committee size. The committee size is fixed in this section.

First, I derive the reward scheme that minimizes the cost of making agents acquire information among reward schemes that induce the committee to use a fixed voting threshold R. Since $w_H^I-w_H^N$ and $w_L^N-w_L^I$ determine the profit from information acquisition, the optimal reward scheme satisfies $w_L^I=w_H^N=0$ by limited liability. Since (IAC_N) becomes easier to satisfy as w_L^N decreases, the optimal reward scheme satisfies (IAC_I) with equality. Similarly, it also satisfies (IAC_N) with equality. The optimal rewards are derived by solving this system of equations.

$$w_H^I(n,R) = \frac{1}{\Pr(H)\Pr(X_{n-1} = R \mid H)}\frac{c}{2q-1}, \quad w_L^N(n,R) = \frac{1}{\Pr(L)\Pr(X_{n-1} = R \mid L)}\frac{c}{2q-1}.$$

$\Pr(X_{n-1} = R \mid H)$ and $\Pr(X_{n-1} = R \mid L)$ represent the probabilities of being pivotal. Therefore, these rewards decrease as they increase.

Next, I derive the reward scheme that minimizes the cost of making agents acquire information. Let $W(n,R)$ denote the expected transfer to an agent given $w_H^I(n,R)$ and $w_L^N(n,R)$.

$$W(n,R) \equiv w_H^I(n,R)\Pr(H)\sum_{k=R+1}^{n}\Pr(X_n = k \mid H) + w_L^N(n,R)\Pr(L)\sum_{k=0}^{R}\Pr(X_n = k \mid L).$$

Since $w_H^I(n,R)\Pr(H)$ (resp. $w_L^N(n,R)\Pr(L)$) does not depend on $\Pr(H)$ (resp. $\Pr(L)$), $W(n,R)$ is independent of the prior distribution of the state.

Since $w_H^I(n,R) = w_L^N(n,n-1-R)$ and $\sum_{k=R+1}^{n}\Pr(X_n = k \mid H) = \sum_{k=0}^{n-1-R}\Pr(X_n = k \mid L)$,

$$W(n,R) = w_L^N(n,n-1-R)\Pr(L)\sum_{k=0}^{n-1-R}\Pr(X_n = k \mid L) + w_L^N(n,R)\Pr(L)\sum_{k=0}^{R}\Pr(X_n = k \mid L).$$

Thus, $W(n,R)$ is symmetric about the simple majority rule, i.e., $W(n,R) = W(n,n-1-R)$. Note that the cost of making agents acquire information $C(n,R)$ equals to $nW(n,R)$.

Proposition 1. *The cost of making agents acquire information $C(n,R)$ decreases as the voting threshold R approaches the simple majority $R = \frac{n-1}{2}$.*

Note that, when n is even, $C(n,R)$ is minimized at $R = \frac{n}{2} - 1$ and $R = \frac{n}{2}$.

Observe that

$$W(n,R) = \left(\frac{\sum_{k=R+1}^{n}\Pr(X_n = k \mid H)}{\binom{n-1}{R}q^R(1-q)^{n-1-R}} + \frac{\sum_{k=0}^{R}\Pr(X_n = k \mid L)}{\binom{n-1}{R}q^{n-1-R}(1-q)^R}\right)\cdot\frac{c}{2q-1}.$$

The cost of making agents acquire information has three elements that depend on threshold R: (i) The probability of paying rewards in state H (resp. L), i.e., $\sum_{k=R+1}^{n} \Pr(X_n = k \mid H)$ (resp. $\sum_{k=0}^{R} \Pr(X_n = k \mid L)$), (ii) The probability of a pivotal event in state H (resp. L), i.e., $q^R(1-q)^{n-1-R}$ (resp. $q^{n-1-R}(1-q)^R$), and (iii) The number of pivotal events, i.e., $\binom{n-1}{R}$. When the principal control the collective choice rule, she cannot reduce the probability of paying rewards in both states. In addition, she cannot increase the probability of a pivotal event in both states. On the other hand, the number of pivotal events is common to both states and increases as the threshold approaches the simple majority. The main reason why the reward scheme that minimizes the cost of making agents acquire information induces the committee to make decisions by majority rule is that it maximizes the number of pivotal events. Since the unanimity rule minimizes the number of pivotal events, the reward scheme that maximizes the cost of making agents acquire information induces the committee to use the unanimity rule. Thus, we can regard the majority rule as a voting rule that minimizes agency costs and the unanimity rule as one that maximizes agency costs. Proposition 1 holds for any prior distribution of the state because $W(n, R)$ does not depend on it.

Let $B(n, R)$ denote the expected benefit from the decision.

$$B(n, R) \equiv \sum_{k=R+1}^{n} \Pr(X_n = k) \left[\Pr(H \mid X_n = k) b_H + \Pr(L \mid X_n = k) b_L\right].$$

The benefit-maximizing threshold R satisfies the following two inequalities:

$$\Pr(H \mid X_n = R + 1) b_H + \Pr(L \mid X_n = R + 1) b_L \geq 0,$$

$$\Pr(H \mid X_n = R) b_H + \Pr(L \mid X_n = R) b_L \leq 0.$$

Without information asymmetry, the principal would have compelled the committee to use the benefit-maximizing threshold. Hence, the benefit-maximizing threshold voting rule should be interpreted as the first-best rule. The optimal threshold is determined by both the benefit from the decision and the cost of making the agents acquire information. Proposition 2 follows from Proposition 1.

Proposition 2. *The collective choice rule that the optimal reward scheme induces the committee to use (i.e., the second-best rule) is closer to the majority rule than the benefit-maximizing threshold voting rule (i.e., the first-best rule).*

If the principal is ex-ante indifferent between the two alternatives, the benefit-maximizing rule is the majority rule. Hence, the following corollary is established.

Corollary 1. *Suppose that the principal is ex-ante indifferent between the two alternatives. Then the optimal reward scheme induces the committee to use the majority rule.*

5 Optimal Committee Size

This section assumes that the principal is ex-ante indifferent between the two alternatives, $\Pr(H)b_H + \Pr(L)b_L = 0$. First, the optimal committee size is characterized. Second, comparative statics on it is conducted.

Since the benefit from the decision is bounded and the cost of making agents acquire information diverges as the committee size grows to infinity, the optimal committee size is finite. When the optimal committee size is greater than zero, the following lemma holds.

Lemma 2. *Suppose that the principal is ex-ante indifferent between the two alternatives and that the optimal committee size is greater than zero. Then the optimal committee size is odd.*

The intuition of Lemma 2 is as follows. Suppose that n is even. Since the principal is ex-ante indifferent between the two alternatives, she is ex-post indifferent when $\frac{n}{2}$ agents receive signal h and the others receive signal ℓ. Thus, the benefit from the decision does not change between majority decisions by n agents and $n-1$ agents. However, the cost of making them acquire information in n-sized committees is more expensive than that in $(n-1)$-sized committees. Therefore, the optimal committee size is odd.

From Lemma 2 and Corollary 1, we can restrict the committee size to be odd and the second-best rule to be the simple majority rule. Define

$$\bar{\pi}(n) \equiv \pi\left(n, \frac{n-1}{2}, w_H^I(n, \frac{n-1}{2}), 0, 0, w_L^N(n, \frac{n-1}{2})\right).$$

Lemma 3. *Suppose that the principal is ex-ante indifferent between the two alternatives and that the optimal committee size is greater than zero. Then the optimal committee size is the smallest odd number n that satisfies $\bar{\pi}(n+2) - \bar{\pi}(n) \leq 0$.*

Lemma 3 follows from the concavity of $\bar{\pi}(n)$ with odd number n. Since the marginal probability of correct decisions monotonically decreases regarding committee size, so does the marginal benefit from the decision. As the probability of being pivotal monotonically decreases regarding committee size, the marginal cost of making agents acquire information monotonically increases regarding committee size. Consequently, the principal's payoff is concave regarding committee size.

Next, comparative statics on the optimal committee size is developed. Under the assumption that $\Pr(H)b_H + \Pr(L)b_L = 0$, rises in b_H cause falls in b_L. Hence, a rise in b_H increases the variance in benefits from the decision. A rise in b_H does not reduce the optimal committee size for the following reasons. The variance in benefits influences the principal's payoff only through the benefit from the decision. When b_H rises, the principal has a greater incentive to make the correct decision. Therefore, the principal is encouraged to enlarge the committee size.

A rise in the information cost c does not expand the optimal committee size for the following reasons. The information cost affects the principal's payoff only through the cost of making agents acquire information. A rise in c increases the expected transfer to an agent. Therefore, the principal is encouraged to reduce the committee size.

The effect of information quality q on the optimal committee size is unclear. Information quality influences both the benefit from the decision and the cost of making agents acquire

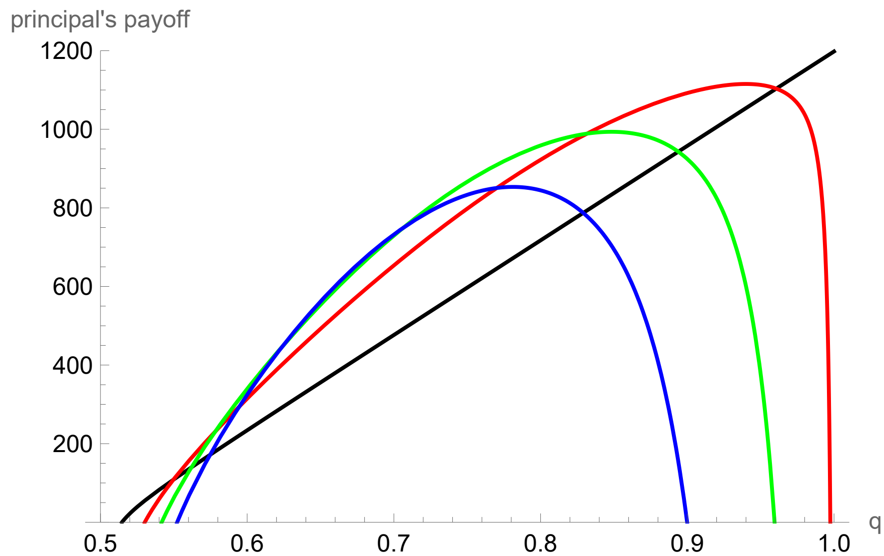

Figure 1: $\bar{\pi}(1)$:black, $\bar{\pi}(3)$:red, $\bar{\pi}(5)$:green, $\bar{\pi}(7)$:blue. Parameter values: $\Pr(H) = \Pr(L) = \frac{1}{2}$, $b_H = -b_L = 2400$, and $c = 1$

information. Improvements in information quality have a positive effect on the benefit side. It is unclear how information quality affects the cost side. This is because its improvement induces agents to not only exert effort but also be free-riders. It is shown that the optimal committee size is inversely U-shaped regarding information quality. In other words, the free-rider effect becomes dominant as information quality increases.

Proposition 3. *Suppose that the principal is ex-ante indifferent between the two alternatives. Then the optimal committee size is inversely U-shaped regarding information quality.*

Figure 1 illustrates that the optimal committee size is inversely U-shaped regarding information quality under specific parameters. Note that the black line $\bar{\pi}(1)$ is monotone increasing in information quality. This is because the free-rider problem does not exist in one-sized committees.

6 Concluding Remarks

This paper analyzes the optimal committee design where the committee designer chooses the committee size and indirectly controls the committee's collective choice rule through monetary transfers.

A significant contribution of this paper is that it provides new insights into voting rules from the perspective of agency costs. Since the cost of making agents acquire information decreases as the voting threshold approaches the simple majority, this paper concludes that the majority rule is the best, while the unanimity rule is the worst concerning agency costs.

Finally, I present two extensions of the model as remaining work for future research. The first considers the circumstances where agents' messages are contractible. The principal can reduce agency costs by making contracts contingent on them. However, such contracts may be vulnerable to collusion among agents. It might be worth examining collusion-proof contracts in such environments. The second examines committees consisting of heterogeneous experts. It would be intriguing to incorporate heterogeneity into information quality. Competent agents increase the precision of decision-making. In contrast, they tempt others to be free-riders. Designing a committee that considers this trade-off would be interesting.

A Appendix: Proofs

Proof of Lemma 1. Let $(n, \boldsymbol{w})$ denote the optimal contract and f_n denote the collective choice rule induced by the optimal contract $(n, \boldsymbol{w})$. Since the optimal committee size is greater than zero, $n \geq 1$. Suppose that $m \geq 1$ uninformed agents exist in equilibrium where the number of informed agents is maximized. Consider a new contract in which the committee size is $n-m$, and the reward scheme is $\boldsymbol{w}$. Since $\Pr(H \mid \emptyset_1, \ldots, \emptyset_m, t_{m+1}, \ldots, t_n) = \Pr(H \mid t_{m+1}, \ldots, t_n)$, the collective choice rule induced by the new contract is f_{n-m} such that $f_{n-m}(t_{m+1}, \ldots, t_n) = f_n(\emptyset_1, \ldots, \emptyset_m, t_{m+1}, \ldots, t_n)$. Thus, the strategy profile in which all $n - m$ agents acquire information is an equilibrium in the new contract. Moreover, it is realized from Assumption 1. The committee makes the same decisions between in the new contract and in the optimal one. Since the optimal committee size is greater than zero, the optimal contract makes some agents acquire information. Thus, $w_\theta^j > 0$ for some (θ, j).

Hence, the principal can save on the transfers of uninformed agents in the new contract. This contradicts the fact that $(n, \boldsymbol{w})$ is optimal. Therefore, $m = 0$. $\square$

Proof of Proposition 1. Since $W(n,R) = W(n, n-1-R)$, it is enough to show that $W(n,R) - W(n,R-1) < 0$ for $R < \frac{n}{2}$.

Since $w_H^I(n,R)$ and $w_L^N(n,R)$ satisfy (IAC_I) with equality,

$$\begin{aligned} &W(n,R) - c \\ =& \Pr(H) w_H^I(n,R) \sum_{k=R}^{n-1} \Pr(X_{n-1} = k \mid H) + \Pr(L) w_L^N(n,R) \sum_{k=0}^{R-1} \Pr(X_{n-1} = k \mid L). \end{aligned}$$

Since $w_H^I(n,R-1)$ and $w_L^N(n,R-1)$ satisfy (IAC_N) with equality,

$$\begin{aligned} &W(n,R-1) - c \\ =& \Pr(H) w_H^I(n,R-1) \sum_{k=R}^{n-1} \Pr(X_{n-1} = k \mid H) + \Pr(L) w_L^N(n,R-1) \sum_{k=0}^{R-1} \Pr(X_{n-1} = k \mid L). \end{aligned}$$

Thus,

$$\begin{aligned} W(n,R) - W(n,R-1) =& - \Pr(H) \left[w_H^I(n,R-1) - w_H^I(n,R) \right] \sum_{k=R}^{n-1} \Pr(X_{n-1} = k \mid H) \\ &+ \Pr(L) \left[w_L^N(n,R) - w_L^N(n,R-1) \right] \sum_{k=0}^{R-1} \Pr(X_{n-1} = k \mid L). \end{aligned}$$

Since $\sum_{k=R}^{n-1} \Pr(X_{n-1} = k \mid H) = \sum_{k=0}^{n-1-R} \Pr(X_{n-1} = k \mid L)$, $\sum_{k=R}^{n-1} \Pr(X_{n-1} = k \mid H) > \sum_{k=0}^{R-1} \Pr(X_{n-1} = k \mid L)$ for any $R < \frac{n}{2}$. Hence, the proof is completed by showing that

$$\Pr(H) \left[w_H^I(n,R-1) - w_H^I(n,R) \right] > \Pr(L) \left[w_L^N(n,R) - w_L^N(n,R-1) \right] (> 0) \text{ for } R < \frac{n}{2}.$$

Since $\left(\frac{q}{1-q}\right)^x$ is convex in x and $\binom{n-1}{R} > \binom{n-1}{R-1}$ if $R < \frac{n}{2}$,

$$\Pr(H)w_H^I(n, R-1) + \Pr(L)w_L^N(n, R-1) = \frac{c}{2q-1}\frac{1}{\binom{n-1}{R-1}q^{n-1}}\left[\left(\frac{q}{1-q}\right)^{n-R} + \left(\frac{q}{1-q}\right)^{R-1}\right]$$
$$> \frac{c}{2q-1}\frac{1}{\binom{n-1}{R}q^{n-1}}\left[\left(\frac{q}{1-q}\right)^{n-R-1} + \left(\frac{q}{1-q}\right)^{R}\right] = \Pr(H)w_H^I(n, R) + \Pr(L)w_L^N(n, R).$$

Therefore, for any $R < \frac{n}{2}$,

$$\Pr(H)\left[w_H^I(n, R-1) - w_H^I(n, R)\right] > \Pr(L)\left[w_L^N(n, R) - w_L^N(n, R-1)\right] (> 0)$$

Consequently, $W(n, R) - W(n, R-1) < 0$ for any $R < \frac{n}{2}$. $\square$

Proof of Lemma 2. Since $\Pr(H)b_H + \Pr(L)b_L = 0$, if the committee size n is even, then the benefit-maximizing thresholds are $R = \frac{n}{2}$ and $R = \frac{n}{2} - 1$. If the committee size n is odd, the benefit-maximizing threshold is $R = \frac{n-1}{2}$. Define $\bar{B}(n)$ as follows.

$$\bar{B}(n) \equiv \begin{cases} B(n, \frac{n-1}{2}) & \text{if } n \text{ is odd} \\ B(n, \frac{n}{2}) & \text{if } n \text{ is even} \end{cases}$$

From Proposition 1, if the committee size n is even, then the cost-minimizing thresholds are $R = \frac{n}{2}$ and $R = \frac{n}{2} - 1$. If the committee size n is odd, the cost-minimizing threshold is $R = \frac{n-1}{2}$. Define $\bar{C}(n)$ as follows.

$$\bar{C}(n) \equiv \begin{cases} C(n, \frac{n-1}{2}) & \text{if } n \text{ is odd} \\ C(n, \frac{n}{2}) & \text{if } n \text{ is even} \end{cases}$$

Define $\bar{\pi}(n) \equiv \bar{B}(n) - \bar{C}(n)$ for all n.

I prove that $\bar{\pi}(n) > \bar{\pi}(n+1)$ for any odd number n by showing that $\bar{B}(n) = \bar{B}(n+1)$ and

$\bar{C}(n) < \bar{C}(n+1)$. First, it is proven that $\bar{B}(n) = \bar{B}(n+1)$. Since $\Pr(H)b_H + \Pr(L)b_L = 0$,

$$\bar{B}(n+1) = \Pr(H)b_H \sum_{k=0}^{\frac{n+1}{2}} \binom{n+1}{k} \left(q^{n+1-k}(1-q)^k - q^k(1-q)^{n+1-k}\right).$$

Noting that $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ for any $1 \leq k \leq n-1$, and that $\binom{n+1}{0} = \binom{n}{0}$,

$$\begin{aligned}\bar{B}(n+1) =& \Pr(H)b_H \left(\sum_{k=0}^{\frac{n+1}{2}} \binom{n}{k} \left(q^{n+1-k}(1-q)^k - q^k(1-q)^{n+1-k}\right) \right. \\ &\left. + \sum_{k=1}^{\frac{n+1}{2}} \binom{n}{k-1} \left(q^{n+1-k}(1-q)^k - q^k(1-q)^{n+1-k}\right) \right) \\ =& \Pr(H)b_H \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \left(q^{n-k}(1-q)^k - q^k(1-q)^{n-k}\right) = \bar{B}(n).\end{aligned}$$

The second equality follows since

$$\sum_{k=1}^{\frac{n+1}{2}} \binom{n}{k-1} \left(q^{n+1-k}(1-q)^k - q^k(1-q)^{n+1-k}\right) = \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} \left(q^{n-k}(1-q)^{k+1} - q^{k+1}(1-q)^{n-k}\right).$$

Next, I prove that $\bar{C}(n+1) > \bar{C}(n)$ for any odd number n. It is enough to show that $W(n+1, \frac{n+1}{2}) > W(n, \frac{n-1}{2})$. Since the optimal reward scheme satisfies (IAC_N) with equality,

$$\begin{aligned}W(n+1, \frac{n+1}{2}) =& w_H(n+1, \frac{n+1}{2}) \Pr(H) \sum_{k=\frac{n+1}{2}}^{n} \Pr(X_n = k \mid H) \\ &+ w_L(n+1, \frac{n+1}{2}) \Pr(L) \sum_{k=0}^{\frac{n-1}{2}} \Pr(X_n = k \mid L) + c.\end{aligned}$$

Since $c > 0$, $w_H^I(n+1, \frac{n+1}{2}) > w_H^I(n, \frac{n-1}{2})$ and $w_L^N(n+1, \frac{n+1}{2}) > w_L^N(n, \frac{n-1}{2})$,

$$W(n+1, \frac{n+1}{2}) > W(n, \frac{n-1}{2}).$$

$\square$

Proof of Lemma 3. It is sufficient to show that $\bar{\pi}(n)$ is concave in odd number n. I prove that $[\bar{B}(n{+}4){-}\bar{B}(n{+}2)]{-}[\bar{B}(n{+}2){-}\bar{B}(n)] < 0$ and $[\bar{C}(n{+}4){-}\bar{C}(n{+}2)]{-}[\bar{C}(n{+}2){-}\bar{C}(n)] > 0$ for any odd number n. For any odd number n, let me define

$$F(n) \equiv \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} q^{n-k}(1-q)^k \quad \text{and} \quad G(n) \equiv \binom{n-1}{\frac{n-1}{2}}(2q-1)q^{\frac{n-1}{2}}(1-q)^{\frac{n-1}{2}}.$$

Then, for any odd number n.

$$\bar{B}(n) = \Pr(H)b_H\left(2F(n)-1\right) \quad \text{and} \quad \bar{C}(n) = \frac{2ncF(n)}{G(n)}.$$

Since $q(1-q) < \frac{1}{4}$ for $q \in \left(\frac{1}{2}, 1\right)$,

$$\binom{n+1}{\frac{n+1}{2}}q(1-q) = \binom{n-1}{\frac{n-1}{2}}\frac{4n}{n+1}q(1-q) < \binom{n-1}{\frac{n-1}{2}}.$$

Therefore, $G(n)$ monotonically decreases with odd number n.

First, I prove $[\bar{B}(n+4) - \bar{B}(n+2)] - [\bar{B}(n+2) - \bar{B}(n)] < 0$. It is enough to show that $[F(n+4) - F(n+2)] - [F(n+2) - F(n)] < 0$. Since $\binom{n+2}{k} = \binom{n+1}{k} + \binom{n+1}{k-1}$ for any $1 \leq k \leq n$ and $\binom{n+2}{0} = \binom{n+1}{0}$,

$$\begin{aligned}
F(n+2) =& \sum_{k=0}^{\frac{n+1}{2}} \binom{n+1}{k} q^{n+2-k}(1-q)^k + \sum_{k=1}^{\frac{n+1}{2}} \binom{n+1}{k-1} q^{n+2-k}(1-q)^k \\
=& q\sum_{k=0}^{\frac{n+1}{2}} \binom{n+1}{k} q^{n+1-k}(1-q)^k + (1-q)\sum_{k=0}^{\frac{n-1}{2}} \binom{n+1}{k} q^{n+1-k}(1-q)^k \\
=& q\binom{n+1}{\frac{n+1}{2}} q^{\frac{n+1}{2}}(1-q)^{\frac{n+1}{2}} + \sum_{k=0}^{\frac{n-1}{2}} \binom{n+1}{k} q^{n+1-k}(1-q)^k.
\end{aligned}$$

Since $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$ for any $1 \le k \le n-1$ and $\binom{n+1}{0} = \binom{n}{0}$,

$$
\begin{aligned}
& F(n+2) \\
=& q\binom{n+1}{\frac{n+1}{2}} q^{\frac{n+1}{2}}(1-q)^{\frac{n+1}{2}} + \sum_{k=0}^{\frac{n-1}{2}} \binom{n}{k} q^{n+1-k}(1-q)^k + \sum_{k=1}^{\frac{n-1}{2}} \binom{n}{k-1} q^{n+1-k}(1-q)^k \\
=& q\binom{n+1}{\frac{n+1}{2}} q^{\frac{n+1}{2}}(1-q)^{\frac{n+1}{2}} + qF(n) + (1-q)F(n) - \binom{n}{\frac{n-1}{2}} q^{\frac{n+1}{2}}(1-q)^{\frac{n+1}{2}} \\
=& F(n) + \frac{1}{2}G(n+2).
\end{aligned}
$$

The final equality follows from $\binom{n}{\frac{n-1}{2}} = \frac{1}{2}\binom{n+1}{\frac{n+1}{2}}$. Hence,

$$
[F(n+4) - F(n+2)] - [F(n+2) - F(n)] = \frac{1}{2}\left[G(n+4) - G(n+2)\right] < 0.
$$

The final inequality is derived from the monotonically decreasing property of $G(n)$ in n. Thus, $[\bar{B}(n+4) - \bar{B}(n+2)] - [\bar{B}(n+2) - \bar{B}(n)] < 0$.

Second, it is shown that $[\bar{C}(n+4) - \bar{C}(n+2)] - [\bar{C}(n+2) - \bar{C}(n)] > 0$. Since $F(n+2) - F(n) = \frac{1}{2}G(n+2)$,

$$
\begin{aligned}
& [\bar{C}(n+4) - \bar{C}(n+2)] - [\bar{C}(n+2) - \bar{C}(n)] \\
=& 2c\left(\frac{(n+4)F(n+2)}{G(n+4)} + \frac{n+4}{2} - \frac{2(n+2)F(n+2)}{G(n+2)} + \frac{nF(n+2)}{G(n)} - \frac{n}{2}\right).
\end{aligned}
$$

Hence, it is enough to show that

$$
(n+4) - 2q(1-q)(n+2)\frac{\binom{n+3}{\frac{n+3}{2}}}{\binom{n+1}{\frac{n+1}{2}}} + q^2(1-q)^2 n\frac{\binom{n+3}{\frac{n+3}{2}}}{\binom{n-1}{\frac{n-1}{2}}} \ge 0. \tag{3}
$$

Note that

$$\frac{\binom{n-1}{\frac{n-1}{2}}}{\binom{n+1}{\frac{n+1}{2}}}\frac{n+2}{n} = \frac{(n+1)(n+2)}{4n^2} > \frac{1}{4}.$$

Since $0 < q(1-q) < \frac{1}{4}$ for $q \in (\frac{1}{2}, 1)$, the left-hand side of (3) is monotone increasing in $q \in (\frac{1}{2}, 1)$. Note that

$$\binom{n+3}{\frac{n+3}{2}} = \frac{4(n+2)}{n+3}\binom{n+1}{\frac{n+1}{2}} = \frac{16(n+2)n}{(n+3)(n+1)}\binom{n-1}{\frac{n-1}{2}}.$$

By substituting $q = \frac{1}{2}$ on the left-hand side of (3) and multiplying both sides by $(n+3)(n+1)$, we obtain the following inequality.

$$(n+4)(n+3)(n+1) - 2(n+2)^2(n+1) + (n+2)n^2 = 3n+4 > 0.$$

Therefore, $[\bar{C}(n+4) - \bar{C}(n+2)] - [\bar{C}(n+2) - \bar{C}(n)] > 0$. $\square$

Proof of Proposition 3. Summarizing the results so far, we obtain the following equations.

$$2[F(n+2) - F(n)] = G(n+2).$$

$$\bar{\pi}(n) = \Pr(H)b_H(2F(n) - 1) - \frac{2ncF(n)}{G(n)}.$$

$$\bar{\pi}(n+2) - \bar{\pi}(n) = \Pr(H)b_H G(n+2) - (n+2)c - 2cF(n)\left(\frac{n+2}{G(n+2)} - \frac{n}{G(n)}\right).$$

To prove Proposition 3, I derive the following Lemma.

Lemma A.1. *Suppose that the principal is ex-ante indifferent between the two alternatives. If $\bar{\pi}(1) \leq 0$, then $\bar{\pi}(n) < 0$ for any odd number $n \geq 3$.*

Proof. It is sufficient to show that for any odd number n, $\bar{\pi}(n+2) \geq 0 \implies \bar{\pi}(n) > 0$. If

$\bar{\pi}(n+2) \geq 0$,

$$\Pr(H)b_H \geq \frac{2(n+2)cF(n+2)}{G(n+2)[2F(n+2)-1]} > \frac{2(n+2)cF(n)}{G(n)[2F(n)+G(n+2)-1]}.$$

Hence,

$$\begin{aligned}\bar{\pi}(n) =& \Pr(H)b_H(2F(n)-1) - \frac{2ncF(n)}{G(n)} \\ >& \frac{2cF(n)}{G(n)}\left(\frac{(n+2)(2F(n)-1)}{2F(n)+G(n+2)-1} - n\right) = \frac{2cF(n)\,[2(2F(n)-1)-nG(n)]}{G(n)[2F(n)+G(n+2)-1]}.\end{aligned}$$

Note that $2F(n)-1>0$. $2(2F(n)-1)-nG(n)>0$ is shown by induction. When $n=1$,

$$2(2F(1)-1)-G(1) = 2(2q-1)-(2q-1) = (2q-1) > 0.$$

Suppose that $2(2F(n)-1)-nG(n)>0$. Then,

$$2(2F(n+2)-1)-(n+2)G(n+2) = 2(2F(n)-1)-nG(n+2) > n[G(n)-G(n+2)] > 0.$$

$\square$

From Lemma A.1, we can derive the condition that the optimal committee size is greater than zero. The principal obtains a payoff of $\max\{\Pr(H)b_H + \Pr(L)b_L, 0\} = 0$ without making committees. Therefore, from Lemma A.1, the optimal committee size is greater than zero if and only if $\bar{\pi}(1) > 0$.

$$\bar{\pi}(1) = \Pr(H)b_H(2q-1) - \frac{c}{2q-1}2q > 0 \iff c < \frac{(2q-1)^2}{2q}\Pr(H)b_H.$$

From Lemma A.1, when q increases from $\frac{1}{2}$, $\bar{\pi}(1;q)$ becomes positive first among $\bar{\pi}(n;q)$. Since $\bar{\pi}(n;q)$ is concave regarding n, $\bar{\pi}(n+2;q)-\bar{\pi}(n;q)$ monotonically decreases in n. Thus, the optimal committee size can be changed to the adjacent odd numbers. Lemma A.2 implies that once the optimal committee size shrinks, it never expands. Furthermore, for any odd number n that is greater than one, $\bar{\pi}(n;q)$ diverges to negative infinity as $q \to 1$. Therefore, the optimal committee size is inversely U-shaped regarding information quality. $\square$

Lemma A.2. *If* $\frac{\partial}{\partial q}\left[\bar{\pi}(n+2;q) - \bar{\pi}(n;q)\right] < 0$ *for some* $q^* \in \left(\frac{1}{2}, 1\right)$*, then* $\frac{\partial}{\partial q}\left[\bar{\pi}(n+2;q) - \bar{\pi}(n;q)\right] < 0$ *for any* $q \in (q^*, 1)$

Proof. Note that

$$\bar{\pi}(n+2;q) - \bar{\pi}(n;q) = \Pr(H)b_H G(n+2;q) - (n+2)c - \frac{2cF(n;q)}{G(n+2;q)}\left(n+2-\frac{4n^2}{n+1}q(1-q)\right).$$

It is follows that

$$\begin{aligned}
&\frac{\partial}{\partial q}\left[\bar{\pi}(n+2;q) - \bar{\pi}(n;q)\right] \\
=&\Pr(H)b_H \cdot \frac{\partial}{\partial q}G(n+2;q) - 2c \cdot \frac{\partial}{\partial q}\left(\frac{F(n;q)}{G(n+2;q)}\right)\left(n+2-\frac{4n^2}{n+1}q(1-q)\right) \\
&-\frac{2cF(n;q)}{G(n+2;q)}\frac{4n^2}{n+1}(2q-1).
\end{aligned}$$

If $\frac{\partial}{\partial q}G(n+2;q) \leq 0$, then $\frac{\partial}{\partial q}\left(\frac{F(n;q)}{G(n+2;q)}\right) > 0$ because $\frac{\partial}{\partial q}F(n;q) > 0$. Thus, if $\frac{\partial}{\partial q}G(n+2;q) \leq 0$, then $\frac{\partial}{\partial q}\left[\bar{\pi}(n+2;q) - \bar{\pi}(n;q)\right] < 0$.

Consider the case where $\frac{\partial}{\partial q}G(n+2;q) > 0$. The proof is completed by showing that

$$\begin{aligned}
&\frac{\partial^2}{\partial q^2}\left[\bar{\pi}(n+2;q) - \bar{\pi}(n;q)\right] < 0 \\
\iff &\Pr(H)b_H \frac{\partial^2}{\partial q^2}G(n+2;q) - 2c \cdot \frac{\partial^2}{\partial q^2}\left(\frac{F(n;q)}{G(n+2;q)}\right)\left(n+2-\frac{4n^2}{n+1}q(1-q)\right) \\
&-2c(2q-1)\frac{4n^2}{n+1}\frac{\partial}{\partial q}\left(\frac{F(n;q)}{G(n+2;q)}\right) - \frac{2cF(n;q)}{G(n+2;q)}\frac{8n^2}{n+1} < 0.
\end{aligned}$$

First, it is shown that $\frac{\partial^2}{\partial q^2}G(n+2;q) < 0$.

$$\frac{\partial}{\partial q}G(n+2;q) = \left(q-q^2\right)^{\frac{n-1}{2}}\left[\frac{1}{2} - \frac{n+2}{2}(2q-1)^2\right]$$

$\frac{\partial}{\partial q}G(n+2;q) > 0$ implies that $\frac{1}{2} - \frac{n+2}{2}(2q-1)^2 > 0$. Hence,

$$\begin{aligned}&\frac{\partial^2}{\partial q^2}G(n+2;q)\\ =&-\frac{n-1}{2}(2q-1)\left(q-q^2\right)^{\frac{n-3}{2}}\left[\frac{1}{2}-\frac{n+2}{2}(2q-1)^2\right]-2(n+2)(2q-1)\left(q-q^2\right)^{\frac{n-1}{2}} < 0.\end{aligned}$$

Second, I show that

$$\begin{aligned}&\frac{\partial^2}{\partial q^2}\left(\frac{F(n;q)}{G(n+2;q)}\right)\left(n+2-\frac{4n^2}{n+1}q(1-q)\right)+(2q-1)\frac{4n^2}{n+1}\frac{\partial}{\partial q}\left(\frac{F(n;q)}{G(n+2;q)}\right)\\ &+\frac{F(n;q)}{G(n+2;q)}\frac{8n^2}{n+1} > 0.\end{aligned}$$

Note that

$$\begin{aligned}&\binom{n+1}{\frac{n+1}{2}}\frac{\partial}{\partial q}\left(\frac{F(n;q)}{G(n+2;q)}\right) = \frac{\partial}{\partial q}\left(\frac{1}{(2q-1)(1-q)}\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}\left(\frac{q}{1-q}\right)^{\frac{n-1}{2}-k}\right)\\ =&\frac{4q-3}{(2q-1)^2(1-q)^2}\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}\left(\frac{q}{1-q}\right)^{\frac{n-1}{2}-k}+\frac{1}{(2q-1)(1-q)^2q}\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}\left(\frac{n-1}{2}-k\right)\left(\frac{q}{1-q}\right)^{\frac{n-1}{2}-k}.\end{aligned}$$

From some computations,

$$\begin{aligned}&\binom{n+1}{\frac{n+1}{2}}\frac{\partial^2}{\partial q^2}\left(\frac{F(n;q)}{G(n+2;q)}\right)\\ =&\frac{1}{(2q-1)(1-q)^3q^2}\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}\left(\frac{q}{1-q}\right)^{\frac{n-1}{2}-k}\left[\left(\frac{n-1}{2}-k\right)^2\right.\\ &\left.-(2q-1)\left(\frac{n-1}{2}-k\right)+\frac{4(2q-1)(1-q)q^2+2(4q-3)^2q^2}{(2q-1)^2}\right].\end{aligned}$$

Since $0 < 2q-1 < 1$,

$$\left(\frac{n-1}{2}-k\right)^2-(2q-1)\left(\frac{n-1}{2}-k\right)+\frac{4(2q-1)(1-q)q^2+2(4q-3)^2q^2}{(2q-1)^2}$$

is minimized at $\frac{n-1}{2} - k = 0$. Therefore, $\frac{\partial^2}{\partial q^2}\left(\frac{F(n;q)}{G(n+2;q)}\right)$ is positive. If

$$(2q-1)\frac{4n^2}{n+1}\frac{\partial}{\partial q}\left(\frac{F(n;q)}{G(n+2;q)}\right) + \frac{F(n;q)}{G(n+2;q)}\frac{8n^2}{n+1} \geq 0,$$

the proof is completed. Consider the case where it is negative. Since $(n+2)(n+1) > n^2$, it is sufficient to show that

$$\frac{\partial^2}{\partial q^2}\left(\frac{F(n;q)}{G(n+2;q)}\right)(1-4q(1-q)) + 4(2q-1)\frac{\partial}{\partial q}\left(\frac{F(n;q)}{G(n+2;q)}\right) + \frac{8F(n;q)}{G(n+2;q)} > 0.$$

Note that

$$\begin{aligned}
&(2q-1)\frac{\partial}{\partial q}\left(\frac{F(n;q)}{G(n+2;q)}\right) + 2\frac{F(n;q)}{G(n+2;q)} \\
=&\frac{1}{(1-q)^2}\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}\left(\frac{q}{1-q}\right)^{\frac{n-1}{2}-k} + \frac{1}{(1-q)^2 q}\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}\left(\frac{n-1}{2}-k\right)\left(\frac{q}{1-q}\right)^{\frac{n-1}{2}-k}.
\end{aligned}$$

It follows that

$$\begin{aligned}
&(2q-1)^2\frac{\partial^2}{\partial q^2}\left(\frac{F(n;q)}{G(n+2;q)}\right) + 4(2q-1)\frac{\partial}{\partial q}\left(\frac{F(n;q)}{G(n+2;q)}\right) + \frac{8F(n;q)}{G(n+2;q)} \\
=&\frac{2q-1}{(1-q)^3 q^2}\sum_{k=0}^{\frac{n-1}{2}}\binom{n}{k}\left(\frac{q}{1-q}\right)^{\frac{n-1}{2}-k}\left[\left(\frac{n-1}{2}-k\right)^2\right. \\
&\left. - \frac{8q^2-8q+1}{2q-1}\left(\frac{n-1}{2}-k\right) + \frac{q^2((4q-3)^2+1)}{(2q-1)^2}\right].
\end{aligned}$$

Since

$$\frac{\partial}{\partial q}\left(\frac{8q^2-8q+1}{2q-1}\right) = \frac{-16(q-1)^2+6}{(2q-1)^2} > 0 \quad \text{for } q \in (\frac{1}{2}, 1),$$

$\frac{8q^2-8q+1}{2q-1} < 1$ for $q \in (\frac{1}{2}, 1)$. Thus,

$$\left(\frac{n-1}{2}-k\right)^2 - \frac{8q^2-8q+1}{2q-1}\left(\frac{n-1}{2}-k\right) + \frac{q^2((4q-3)^2+1)}{(2q-1)^2}$$

is minimized at $\frac{n-1}{2} - k = 0$. Therefore, it is positive. □

B Appendix: Equilibrium Selection

Appendix B proves that, in the optimal contract, the intended equilibrium in Assumption 1 Pareto-dominates all the other strategy profiles. Note that this section does not assume that the principal is ex-ante indifferent between the two alternatives.

First, I explain why multiple equilibria exist given the optimal contract. Suppose that the optimal committee size is n and the optimal reward scheme induces the committee to use the voting threshold R. Since the optimal reward scheme satisfies the two inequalities of the information acquisition constraint with equality, the committee is indifferent between the two alternatives when R agents report signal h, $n-1-R$ agents report signal ℓ, and one declares $\emptyset$. Being indifferent between the two alternatives for some information profile means that the agent's payoff takes the same value regardless of the chosen alternative on pivotal events. Thus, if the number of informed agents is $n-1$, an informed agent deviates from information acquisition.

Proposition B.1 justifies Assumption 1.

Proposition B.1. *Given the optimal contract, agents gain the highest payoff on the path where all committee members acquire information.*

From Lemma 1, in the optimal contract, the principal achieves the maximum payoff on the path where all agents acquire information. From Proposition B.1, all agents gain the highest payoff on that path. Thus, the intended equilibrium in Assumption 1 Pareto-dominates all the other strategy profiles. Consequently, Proposition B.1 ensures the plausibility of Assumption 1.

Subsequently, I prepare for the proof of Proposition B.1. Define

$$D(k; m, w_H^I, w_L^N) \equiv \Pr(H \mid X_m = k) w_H^I - \Pr(L \mid X_m = k) w_L^N.$$

Let $R^*_m(w^I_H, w^N_L)$ denote an integer k that satisfies $D(k; m, w^I_H, w^N_L) \leq 0$ and $D(k+1; m, w^I_H, w^N_L) > 0$. Given the contract $(n, (w^I_H, 0, 0, w^N_L))$, the collective choice rule is characterized by $\{R^*_m(w^I_H, w^N_L)\}_{m=0,1,\ldots,n}$. Since $w^I_H(n, R)$ and $w^N_L(n, R)$ induce the committee to use threshold R, $R^*_n(w^I_H(n, R), w^N_L(n, R)) = R$.

Similar to the result of Persico (2004), the following lemma holds.

Lemma B.1. *Suppose* $0 \leq R^*_m(w^I_H, w^N_L) \leq m - 1$. *Then* $R^*_{m-2}(w^I_H, w^N_L) = R^*_m(w^I_H, w^N_L) - 1$.

Proof.

$$\begin{aligned} D(k; m, w^I_H, w^N_L) &= \Pr(H)\binom{m}{k} q^k (1-q)^{m-k} w^I_H - \Pr(L)\binom{m}{k} q^{m-k}(1-q)^k w^N_L \\ &= \binom{m}{k} q^{m-k}(1-q)^k \left[\Pr(H)\left(\frac{1-q}{q}\right)^{m-2k} w^I_H - \Pr(L) w^N_L\right] \end{aligned}$$

Since $m - 2k = (m-2) - 2(k-1)$, the sign of $D(k; m, w^I_H, w^N_L)$ coincides with that of $D(k-1; m-2, w^I_H, w^N_L)$. Therefore, $D(R^*_m(w^I_H, w^N_L) - 1; m-2, w^I_H, w^N_L) \leq 0$ and $D(R^*_m(w^I_H, w^N_L); m, w^I_H, w^N_L) > 0$, that is, $R^*_{m-2}(w^I_H, w^N_L) = R^*_m(w^I_H, w^N_L) - 1$. □

Proof of Proposition B.1. Take $w^I_H(n, R)$ and $w^N_L(n, R)$ as given. Let $W(m, R^*_m)$ denote the expected transfer to one agent when m informed and $n - m$ uninformed agents report their truthful types.

$$W(m, R^*_m) \equiv \sum_{k=R^*_m+1}^{m} \Pr(H)\Pr(X_m = k \mid H) w^I_H(n, R) + \sum_{k=0}^{R^*_m} \Pr(L)\Pr(X_m = k \mid L) w^N_L(n, R).$$

The proof is completed by showing that, for any $m < n$, $W(n, R^*_n) - W(m, R^*_m) \geq c$. Since the optimal reward scheme satisfies the two inequalities of the information acquisition constraint with equality, $R^*_{n-1}(w^I_H(n, R), w^N_L(n, R)) = R$. From Lemma B.1,

$$W(n-2k+1, R^*_{n-2k+1}) - W(n-2k, R^*_{n-2k}) = W(n-2k+1, R-k+1) - W(n-2k, R-k).$$

Focusing on pivotal events,

$$
\begin{aligned}
& W(n-2k+1, R^*_{n-2k+1}) - W(n-2k, R^*_{n-2k}) \\
= & \Pr(X_{n-2k} = R-k) \left[\Pr(H \mid X_{n-2k} = R-k) q w^I_H(n,R) + \Pr(L \mid X_{n-2k} = R-k) q w^N_L(n,R)\right] \\
& - \Pr(X_{n-2k} = R-k) \Pr(H \mid X_{n-2k} = R-k) w^I_H(n,R) \\
= & - \Pr(H) \binom{n-2k}{R-k+1} q^{R-k+1} (1-q)^{n-k-R} w^I_H(n,R) \\
& + \Pr(L) \binom{n-2k}{R-k+1} q^{n-k-R} (1-q)^{R-k+1} w^N_L(n,R) = 0.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
& W(n-2k, R^*_{n-2k}) - W(n-2k-1, R^*_{n-2k-1}) \\
= & \Pr(H) \binom{n-2k-1}{R-k} q^{R-k+1} (1-q)^{n-k-R-1} w^I_H(n,R) \\
& - \Pr(L) \binom{n-2k-1}{R-k} q^{n-k-R-1} (1-q)^{R-k+1} w^N_L(n,R) \\
= & \frac{\binom{n-2k-1}{R-k}}{\binom{n-1}{R}} \frac{1}{q^k (1-q)^k} c > 0.
\end{aligned}
$$

From the information acquisition constraint, $W(n, R^*_n) - W(n-1, R^*_{n-1}) = c$. Therefore, for any $m < n$,

$$
W(n, R^*_n) - W(m, R^*_m) = \sum_{k=0}^{n-m-1} \left[W(n-k, R^*_{n-k}) - W(n-k-1, R^*_{n-k-1})\right] \geq c
$$

□

References

Austen-Smith, David and Jeffrey S Banks (1996) "Information aggregation, rationality, and the Condorcet jury theorem," *American Political Science Review*, 90 (1), 34–45.

Azrieli, Yaron (2021) “Monitoring experts,” *Theoretical Economics*, 16 (4), 1313–1350.

Carroll, Gabriel (2019) “Robust incentives for information acquisition,” *Journal of Economic Theory*, 181, 382–420.

Clark, Aubrey and Giovanni Reggiani (2021) “Contracts for acquiring information,” *arXiv preprint arXiv:2103.03911.*

Condorcet, Marquis de (1785) *Essai sur l’application de l’analyse à la probabilité des décisions rendues à la pluralité des voix*, Paris: Imprimerie Royale.

Feddersen, Timothy and Wolfgang Pesendorfer (1998) “Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting,” *American Political science review*, 92 (1), 23–35.

Gerardi, Dino and Leeat Yariv (2008) “Information acquisition in committees,” *Games and Economic Behavior*, 62 (2), 436–459.

Gersbach, Hans, Akaki Mamageishvili, and Oriol Tejada (2022) “Appointed learning for the common good: Optimal committee size and monetary transfers,” *Games and Economic Behavior*, 136, 153–176.

Gershkov, Alex, Benny Moldovanu, and Xianwen Shi (2017) “Optimal voting rules,” *The Review of Economic Studies*, 84 (2), 688–717.

Gershkov, Alex and Balázs Szentes (2009) “Optimal voting schemes with costly information acquisition,” *Journal of Economic Theory*, 144 (1), 36–68.

Gromb, Denis and David Martimort (2007) “Collusion and the organization of delegated expertise,” *Journal of Economic Theory*, 137 (1), 271–299.

Koriyama, Yukio and Balázs Szentes (2009) “A resurrection of the Condorcet Jury Theorem,” *Theoretical Economics*, 4 (2), 227–252.

Mookherjee, Dilip and Masatoshi Tsumagari (2014) "Mechanism design with communication constraints," *Journal of Political Economy*, 122 (5), 1094–1129.

Mukhopadhaya, Kaushik (2003) "Jury size and the free rider problem," *Journal of Law, Economics, and Organization*, 19 (1), 24–44.

Myerson, Roger B (1986) "Multistage games with communication," *Econometrica: Journal of the Econometric Society*, 323–358.

Persico, Nicola (2004) "Committee design with endogenous information," *The Review of Economic Studies*, 71 (1), 165–191.

Strulovici, Bruno (2020) "Can society function without ethical agents? an informational perspective," *arXiv preprint arXiv:2003.05441*.

Zermeno, Luis (2011) "A principal-expert model and the value of menus," *unpublished paper, Massachusetts Institute of Technology*, 4.