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Impacts and Distribution of Premiums from Temporal Social Networks across Generations*

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Impacts and distribution of premiums from temporal social networks across generations*

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Abstract

Social networks certainly play an important role in labor market outcomes. In particular, the structures affect inter-group inequality via referral hiring. Through the network effects, while workers surely get premiums from the group to which they belong, they may get premiums or penalties from other groups than their own. Young workers do not obtain sufficient network premiums since referrals cannot be used well due to the higher unemployment rates of their friends. As time goes by, the network structure of each generation of course changes. In other words, not only premiums from their own network group but also those from the other network groups, or the spillovers from other generations, change over time. However, these changes in intra- and inter-group network effects have been rather overlooked so far. In this paper, we compute the network premiums for each generation in a search and matching model, and clarify which generation benefits the most from time-varying networks called temporal networks. New connections are generated proportional to the number of friends of each worker over time, while the existing connections are broken at a constant rate. Under this setting, workers get premiums or penalties depending on their network structures. On average, workers receive premiums from the overall network effects although they incur penalties from their network structures in wage and unemployment rates.

JEL Classification: E24, J31, J64;
Keywords: Referral hiring; temporal network; network structure; intergenerational inequality.

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1 Introduction

Referral hiring plays a non-negligible role in labor market outcomes, especially in determining inequality among workers. Many workers ask their friends for vacant positions. Thus, those who have many friends have high job-finding probabilities and accordingly strong outside options in wage bargaining leading to high wages, whereas those who have a few friends do not (Fontaine, 2008; Ioannides and Soetevent, 2006). Workers benefit from referral hiring through their social networks in which workers and friendships are nodes and edges, respectively.

Social networks surely change over time, and those time-varying networks are called *temporal networks* (Holme and Saramäki, 2019). This idea applies to a worker network since new friendships are formed and existing ones are broken up. If a worker is friendly, and has many friends, he is easy to make new friends, otherwise, he is difficult to make new ones, which is measured by degree correlations of *assortativity* (Jackson and Rogers, 2007; Newman, 2002, 2003). If this is the fact for all workers, then social networks are structured in such a way that a small number of people become to have a large number of friends as time goes by. This leads to structural differences in the social networks across generations, say, the young and old generations, meaning that workers of different generations (or ages) can have different premiums from their social networks.

The labor market with referral hiring using social networks has been analyzed since seminal works by Rees (1966), Granovetter (1973), and Boorman (1975) because referral hiring is one of the typical methods used by workers (Holzer, 1988; Pellizzari, 2010).[1] An analytical framework is established by Calvó-Armengol and Zenou (2005) and Fontaine (2007) in which referral hiring is incorporated as social networks into a search and matching model proposed by Diamond (1981), Mortensen and Pissarides (1994), and Pissarides (2000).

In search and matching models, social networks are often assumed to have a simple structure (e.g. a regular network in which all nodes have the identical number of edges) for analytical tractability (e.g. Calvó-Armengol and Zenou, 2005; Fontaine, 2007; Galenianos, 2014). However, it is well known that the network structure itself, e.g. how edges connect nodes in a network, and how workers differ in the number of friends, has a significant impact on the unemployment probability of a worker, as Calvó-Armengol and Jackson (2004) and Tassier and Menczer (2008) have shown using small networks.

To clarify the characteristic of a network structure in search and matching models, Ioannides and Soetevent (2006) uses the idea of *degree distribution*. The degree of a node in a network is the number of connections each node has to other nodes, and hence the degree distribution is defined as the probability distribution of the degrees over the network.[2] The degree distribution in a social network corresponds to the distribution of the number of friends that each worker has. With degree distributions, differences in social network structures lead to differences in congestion for sharing information about vacant job positions (Ogisu, 2022). Since social networks in the previous models are static, we develop a theoretical model with temporal networks in which network structures can be different among generations.

In this paper, we attempt to capture generation-dependent premiums (or penalties) caused by temporal social networks. We employ a search and matching model with overlapping generations, called life-cycle search models (e.g. Chéron et al., 2013; Fujimoto, 2013), and incorporate temporal networks into it. First, we construct the model with sim-

[1] Comprehensive surveys are provided by Ioannides and Loury (2004); Topa (2011).

[2] See Barabási (2013) and Newman (2018) for basics about degrees and degree distributions.

ple social networks, random regular networks, and analytically demonstrate the existence of equilibrium. Second, we extend the model to include temporal networks generated by a *fitness model* (e.g. De Masi et al., 2006; Kobayashi et al., 2019). In a fitness model, each node has an activity parameter, and the probability of creating an edge between two nodes depends on the parameters. To describe degree correlations in a social network, we set the activity parameters proportional to the number of friends the worker has.

The main results in the paper are summarized as follows. First, workers can receive positive premiums from their social networks. Judging from the Japanese data, the premiums in wage and unemployment rates are up to about 1.6%pt (measured by the average wage) and 0.2%pt, respectively, although the premiums are partly offset by structures of social networks up to 0.3%pt and 0.03%pt, respectively.

Second, young workers cannot benefit from their own social networks well even if they have relatively good social network structures. This is because the probability that their friends are also unemployed is high, and hence young workers are difficult to obtain valuable information from their friends.

Third, an increase in the network connectivity of some generations (e.g. average number of friends) can lead to a decline in the unemployment rates not only in that generation but also in the other generations. When the network connectivity in some generation increases, it improves the matching probability via referrals in that generation, which in turn brings about an increase in the number of new entry firms. If the number exceeds additional matches made by workers in the generation in which the change takes place at first, the vacancy rate increases in the network as a whole, leading to improved matching in the labor market. As a result, the other generations can also obtain additional matches. If, however, the number of entry firms is not sufficient, then the opposite case is also possible that the vacancy rate decreases and the unemployment rates of the other generations rise. The results of our numerical calculations confirm that an increase in network connectivity of the middle aged group (40 y.o.) raises the unemployment rate in the other generations.

This paper contributes to the existing literature threefold. The first is to present a theoretical analysis to explore the effects of referral hiring using social networks (e.g. Alaverdyan and Zaharieva, 2022; Cahuc and Fontaine, 2009; Galenianos, 2013; Galeotti and Merlino, 2014; Horvath and Zhang, 2018; Zaharieva, 2018). In the literature, some works have investigated efficiency losses (e.g. Cahuc and Fontaine, 2009; Galenianos, 2014; Horváth, 2014), while some examine the inequality caused by referral hiring (e.g. Calvó-Armengol and Jackson, 2004, 2007; Galenianos, 2021; Horváth, 2014; Igarashi, 2016; Stupnytska and Zaharieva, 2015; Zaharieva, 2013). Our analysis is in the line with the latter. It is particularly different from the former in using a life-cycle model to introduce generational differences among worker networks to capture generation-dependent premiums.

The second is examining inequality across generations in a life-cycle search model (e.g. Chéron et al., 2011, 2013; Esteban-Pretel and Fujimoto, 2014; Fujimoto, 2013; Hahn, 2009). We explicitly introduce referrals with social networks into the model in a tractable manner.

The third is to focus on the role of temporal networks (e.g. Holme and Saramäki, 2019; Kobayashi and Takaguchi, 2018; Kobayashi et al., 2019). In our model, social networks are exogenously given based on a fitness model that is frequently used in recent studies in network science. Since little is known about the impact of temporal networks on the labor market, this paper is one of the important applications of social networks using a search and matching model.

The rest of the paper is constructed as follows. Sec. 2 describes the basic model with a life-cycle search and matching model. With assumptions for analytical tractability, Sec. 3

shows the existence of equilibrium and deals with comparative analysis showing that an expansion in the effect of network connectivity has spillover effects. With parameters calibrated to fit the Japanese data, Sec. 4 presents the algorithm generating temporal social networks and identifies premiums and/or penalties from social networks numerically. Sec. 5 summarizes the results and describes research avenues.

2 Model

Time is a discrete and infinite horizon, and the economy is in a steady state. Workers who are risk neutral enter the market for generation 0 at first, and leave the market when they reach the end of the last generation, $\bar{g}$. Thus, there are generations of workers g ranging from 0 to $\bar{g}$ in the economy. Assume that workers in each generation make up a social network with a sufficiently large number of workers, N, which is constant and the same for all generations. For simplicity, there is no friendship between social networks in different generations. A worker who is unemployed searches for a job and produces home production $b > 0$, or one who is matched with a job supplies a unit of labor and receives wage w_g.

There is an infinite number of jobs which is ready to enter the market. If a job is vacant, the job is posted in the labor market with cost $c > 0$. If a worker arrives at the job, the job becomes a matched job and produces y_g unit of general good and pays wage w_g to the matched worker of generation g. Firms are destructed with a constant probability of $\delta \in (0, 1)$ for each end of the time period. Variables c, y_g, δ are exogenous parameters, although w_g is determined endogenously at the equilibrium.

Let p_g be the (aggregate) job arrival rate for a worker in generation g. Following a life-cycle search model (e.g. Esteban-Pretel and Fujimoto, 2014; Fujimoto, 2013), value functions are given by, for generation $g \in \{0, 1, \cdots, \bar{g} - 1\}$,

$$W_g = w_g + \beta[(1 - \delta)W_{g+1} + \delta U_{g+1}], \tag{1}$$

$$U_g = b + \beta[p_g W_{g+1} + (1 - p_g)U_{g+1}], \tag{2}$$

$$J_g = y_g - w_g + \beta[(1 - \delta)J_{g+1} + \delta V], \tag{3}$$

for the oldest generation $\bar{g}$,

$$W_{\bar{g}} = w_{\bar{g}}, \tag{4}$$

$$U_{\bar{g}} = b, \tag{5}$$

$$J_{\bar{g}} = y_{\bar{g}} - w_{\bar{g}} + \beta V, \tag{6}$$

where $\beta \in (0, 1)$ is the discount factor, and W_g, U_g, J_g represent the value functions of a worker of generation g who has the matched job, a worker of generation g who does not have a matched job, and a job which is matched to a worker of generation g, respectively.

V denotes the value function of a vacancy job, not matched with any workers, and it is given by

$$V = -c + \beta \left[q \sum_{g=0}^{\bar{g}} J_g \lambda_g + (1 - q)V \right], \tag{7}$$

where $q \equiv \sum_g p_g u_g / v(\bar{g} + 1)$ is the average arrival rate of a worker in generation g to the job, and $\lambda_g \equiv p_g u_g / \sum_{g'} p_{g'} u_{g'}$ is the probability that the matched worker is in generation g.

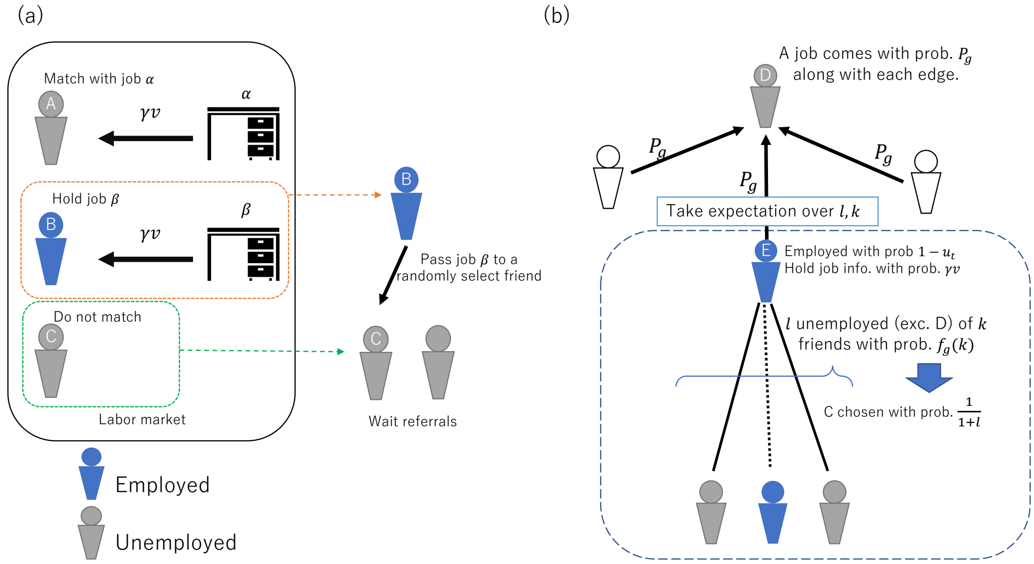

Fig. 1. Schematic of matching procedures. (a) A job arrives at a worker with probability γv. When Worker B is employed and gets a vacancy, Worker B passes it to one of his friends who is unemployed. If Worker B has two friends, a vacancy is passed to Worker C with probability $1/2$. (b) Referral procedure. Worker D has 3 edges in this case and receives a vacancy from each friend with probability P_g. If Worker E has k friends and l of k friends are unemployed excluding worker D, E chooses D with probability $1/(l+1)$. We sum up all possible cases with the probability taking k, l to get the expected probability that a vacancy goes from E to D, which is P_g.

The matching procedure primarily follows the setting in Calvó-Armengol and Zenou (2005). Vacancies are made available to workers through the market or their social networks. p^M and p_g^R denote the arrival rate via the market and the conditional probability of the job arrival rate by referral, respectively. The aggregate job arrival rate, p_g, is given by

$$p_g = p^M + (1 - p^M) p_g^R. \tag{8}$$

We note that the matching function induced by p_g is not a constant return scale in u_g and v since we employ the same matching procedure in Calvó-Armengol and Zenou (2005).

We assume that the matching probability in the market is such that the linear function of vacancy rate v, meaning that

$$p^M = \gamma v, \tag{9}$$

where $\gamma > 0$ is a parameter.

In the search and matching model with referral hiring, it is assumed that vacancy information goes not only to unemployed workers, but also to employed workers (Fig. 1a). Every worker can hold only one vacancy information and if multiple vacancies are made available to a worker, one is chosen by the worker at random and the others vanish. If an employed worker receives a vacancy, he passes it to one of his unemployed friends chosen at random. When unemployed workers cannot get job information in the labor market, they wait for their friends to pass on information regarding a job position. The referral procedure is described in Fig. 1b. We first define P_g as the probability that an unemployed worker receives a vacancy from an arbitrary one of his friends. Starting from an unemployed worker (Worker D in Fig. 1b), one of his friends (Worker E) is employed with probability $1-u_g$.[3] Worker E receives information about a vacancy in the labor market with probability γv. If Worker E has a vacancy, she sends it to one of her friends with probability $\psi \in [0,1]$. When Worker E has l unemployed of k friends except for Worker D, Worker E passes Worker D a vacancy with probability $1/(l+1)$. Furthermore, combinations of selecting l unemployed workers from k friends are given by a binomial distribution with probability u_g. Lastly, we get

$$\begin{aligned}
P_g &= \psi(1-u_g)\gamma v \sum_k \sum_{l=0}^{k-1} \binom{k-1}{l} \frac{1}{l+1} u_g^l (1-u_g)^{k-l-1} f_g(k) \\
&= \psi(1-u_g)\gamma v \sum_k \frac{1-(1-u_g)^k}{k u_g} f_g(k),
\end{aligned}$$

where $f_g(k)$ is the probability that a worker of generation g has k friends.[4] Thus, at least one vacancy information is made available to a worker having k friends with probability $1-(1-P_g)^k$.

The average arrival rate via referral for generation g is given by

$$p_g^R = \sum_k \left[1-(1-P_g)^k\right] f_g(k), \quad g \in \{0, 1 \cdots, \bar{g}-1\}, \tag{10}$$

and $p_{\bar{g}}^R = 0$. This is basically the same formulation as in Calvó-Armengol and Zenou (2005). Finally, if we know the degree distribution $f_g(k)$ for all g and assume $u_0 = 1$, the equilibrium can be defined.

Definition 1. *An equilibrium is defined such that:*

1. *The unemployment rate for each generation is determined by the following rules:*

$$\begin{aligned}
f_g(k)\ \forall g &: \ \textit{given}, \\
u_0 &= 1, \\
u_{g+1} &= (1-p_g)u_g + \delta(1-u_g) \quad \forall g \in \{0, \cdots, \bar{g}-1\},
\end{aligned} \tag{11}$$

   where $p_{\bar{g}} = 0$.

2. *Wage is determined by Nash bargaining with worker's bargaining power* $\eta \in (0,1)$, *which implies*

$$w_g = \arg\max_w \ (W_g - U_g)^\eta (J_g - V)^{1-\eta}. \tag{12}$$

3. *Free entry condition is satisfied:*

$$V = 0. \tag{13}$$

Let the match surplus be $S_g \equiv W_g - U_g + J_g - V$. From the Nash bargaining problem of (12), we obtain,

$$\begin{aligned}
W_g - U_g &= \eta S_g, \\
J_g - V &= (1-\eta) S_g.
\end{aligned}$$

[3] We consider u_g as the probability that a worker chosen from the worker pool of generation g is unemployed. That is one of the approximations by mean-field.

[4] In this formulation, we implicitly assume that social networks have a locally tree-like structure in which the local cycle by three nodes can be ignored(e.g. Molloy and Reed, 1995; Newman, 2018).

Combining them with Eq.(4)-(6) and free entry condition (13), we obtain,

$$W_{\bar{g}} = w_{\bar{g}} = \eta y_{\bar{g}} + (1-\eta)b, \tag{14}$$
$$J_{\bar{g}} = (1-\eta)(y_{\bar{g}} - b). \tag{15}$$

The match surplus is determined such that

$$S_g = y_g - b + \beta[1-\delta - p_g\eta]S_{g+1}, \quad \forall g \in \{0,\cdots,T-1\}, \tag{16}$$
$$S_{\bar{g}} = y_{\bar{g}} - b. \tag{17}$$

At the equilibrium, by Eq.(7) and free entry condition (13),

$$\frac{c(\bar{g}+1)}{\beta(1-\eta)} = G(v), \tag{18}$$

where $G(v) \equiv \sum_{g=0}^{\bar{g}} u_g p_g S_g / v$.

3 Analytical framework in regular networks

For the first step, we solve the model analytically with some assumptions that simplify the analysis. In this section, we suppose that $y_g = y > b \ \forall g$, and each network structure is a random regular network with degree $\bar{k}_g$. In that case, the degree distribution for generation g is given by

$$f_g(k) = \begin{cases} 1 & \text{if } k = \bar{k}_g, \\ 0 & \text{otherwise.} \end{cases} \tag{19}$$

3.1 Equilibrium

With the assumption of random regular networks, P_g and p_g^R can be rewritten as

$$P_g = \psi(1-u_g)\gamma v \frac{1-(1-u_g)^{\bar{k}_g}}{\bar{k}_g u_g},$$
$$p_g^R = 1-(1-P_g)^k, \quad \text{for } g \in \{0,\cdots,\bar{g}-1\}.$$

In this simple case, we derive the next proposition.

Proposition 1. *Assume that social networks are regular networks for every generation, i.e., degree distributions are given by* (19)*. Then an equilibrium exists if parameters satisfy the condition*

$$\frac{\gamma\beta(1-\eta)(y-b)}{c(\bar{g}+1)} \frac{1-[\beta(1-\delta-\eta)]^{\bar{g}-g+1}}{1-\beta(1-\delta-\eta)} < 1 < \frac{\gamma\beta(1-\eta)(y-b)}{c(\bar{g}+1)} \sum_{g=0}^{\bar{g}} \left[\frac{1-[\beta(1-\delta)]^{\bar{g}-g+1}}{1-\beta(1-\delta)} \right]. \tag{20}$$

Proof. See Appendix A. □

There are two remarks for condition (20). First, the existence of the equilibrium is irrelevant to the network parameter $\bar{k}_g$, which is the same as in Calvó-Armengol and Zenou (2005). The condition only needs information at the end points of v, meaning that $v = 0, 1/\gamma$. For the existence of the equilibrium, the key is not social networks, and this is held in the model including the generational structure. At these two end points, the

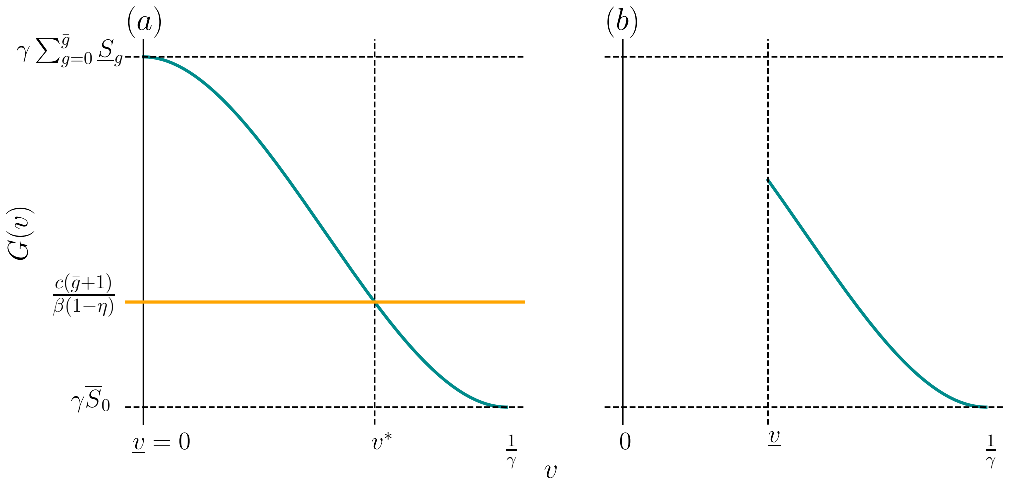

Fig. 2. Graphical image of the uniqueness of the equilibrium. (a) When $\underline{v} = 0$, the slope of $G(v)$ is negative for $v \in (0, 1/\gamma)$, and the unique equilibrium exists if Eq. (20). (b) If $\underline{v} > 0$, the slope of $G(v)$ in $v \in (\underline{v}, 1/\gamma)$ is negative but ambiguous in $v \in (0, \underline{v}]$. When $v < \underline{v}$, the uniqueness of the equilibrium is not assured.

job arrival rate in the labor market to 0 and to 1, respectively, meaning that no one can find jobs and anybody can find jobs in the labor market. Therefore, nobody can rely on referrals in these polarized cases.

Second, the job opening cost must be in the moderate value range. If job opening cost c is relatively high, the first inequality is held. However, if c is too high, the system violates the second inequality. As a result, the job opening cost needs to remain at a moderate level for the equilibrium to exist. If the job opening cost is too low, infinite jobs keep entering the market since jobs expect to make profits, while if it is too high, jobs leave the market which is no longer attractive.

At extreme points of v, $G(v)$ approaches two real values, denoting $\gamma \overline{S}_0$ and $\gamma \sum_{g=0}^{\bar{g}} \underline{S}_g$, although its slope between these points is ambiguous. However, we can prove that the slope is negative above a certain level of v, which is summarized in the next lemma.

Lemma 1. *For any parameter sets, there exists $\underline{v} < 1/\gamma$ such that for $v \in (\underline{v}, 1/\gamma)$,*

$$\frac{\partial G(v)}{\partial v} < 0. \tag{21}$$

Proof. See Appendix B. □

Moreover, following discussions in this section, we make the next assumption.

Assumption 1. *The parameter sets are chosen such that $\underline{v} = 0$.*

We note here that Assumption 1 is held under the parameters calibrated in Sec. 4. Combining Lemma 1 and Assumption 1 leads to the next proposition.

Proposition 2. *Assume $\underline{v} = 0$. Then Eq.* (20) *is the necessary and sufficient condition for the existence of the unique equilibrium.*

Proof. By Assumption 1 and Lemma 1, $G(v)$ is monotonically decreased in v and

$$\lim_{v\to 0} G(v) = \gamma(y-b)\sum_{g=0}^{\bar{g}} \frac{1-[\beta(1-\delta)]^{\bar{g}+1}}{1-\beta(1-\delta)}, \tag{22}$$

$$\lim_{v\to \frac{1}{\gamma}} G(v) = \gamma(y-b)\frac{1-[\beta(1-\delta-\eta)]^{\bar{g}+1}}{1-\beta(1-\delta-\eta)}. \tag{23}$$

Therefore, when

$$\lim_{v\to \frac{1}{\gamma}} G(v) < \frac{c(\bar{g}+1)}{\beta(1-\eta)} < \lim_{v\to 0} G(v),$$

the unique equilibrium exists and this condition is identical to that in Eq. (20) (also see Fig. 2). □

3.2 Spillover effects from network connectivity

In this model, the relationship between the number of friends ($\bar{k}_g$) and matching probability via referral (p_g^R) draws an inverse-U shape (see Proposition 1 (i) in Calvó-Armengol and Zenou, 2005). The crucial difference from Calvó-Armengol and Zenou (2005) is that network expansion of a generation affects the job arrival rate via referral not only of the generation, but also of the other generations. We define $\hat{k}_g$ as the $\bar{k}_g$ at which p_g^R is maximized. Then, at $\bar{k}_g = \hat{k}_g$, p_g is also maximized. For simplicity, we tentatively treat $\bar{k}_g$ as a real number, as Calvó-Armengol and Zenou (2005) do. We obtain the following result.

Proposition 3. *Given v;*

(i) for $\tau \in \{g+1, g+2\cdots, \bar{g}\}$,

$$\bar{k}_g \gtreqless \hat{k}_g \Longrightarrow \frac{\partial u_\tau}{\partial \bar{k}_g} \gtreqless 0, \tag{24}$$

(ii) for $\tau \in \{g, g+1, \cdots, \bar{g}\}$,

$$\bar{k}_g \gtreqless \hat{k}_g \quad \Longrightarrow \quad \frac{\partial p_\tau}{\partial \bar{k}_g} \lesseqgtr 0, \tag{25}$$

(iii) for $\tau \in \{0, 1, \cdots, \bar{g}\}$,

$$\bar{k}_g \gtreqless \hat{k}_g \Longrightarrow \frac{\partial S_\tau}{\partial \bar{k}_g} \gtreqless 0. \tag{26}$$

Proof. See Appendix C. □

Regarding Proposition 3, an increment in $\bar{k}_g$ means that all workers gain additional friendships when they become generation (or age) g. Therefore, if the increment in $\bar{k}_g < \hat{k}_g$ improves the job arrival rate for workers in generation g, it improves the unemployment rate and the job arrival rate of the generation in the following periods. For $\bar{k}_g > \hat{k}_g$, however, p_g decreases as $\bar{k}_g$ increases, resulting in *congestion effects* stated in Calvó-Armengol and Zenou (2005). When the network connectivity is too high, the probability of an unemployed worker getting multiple vacancies is also high, which is congestion in sharing information.

With Proposition 3, the equilibrium effects can be obtained. The keys are shifts in $G(v)$ and Beveridge curves. First, based on Eq. (24)-Eq. (26), an increment (or decline) in $\bar{k}_g$ causes a shift in $G(v)$ and moves the equilibrium vacancy rate also since

$$\frac{\partial G(v)}{\partial \bar{k}_g} = \sum_{\tau=0}^{g-1} \frac{u_\tau p_\tau}{v} \frac{\partial S_\tau}{\partial \bar{k}_g} + \frac{u_g}{v}\left(\frac{\partial p_g}{\partial \bar{k}_g} S_g + p_g \frac{\partial S_g}{\partial \bar{k}_g}\right) + \sum_{\tau=g+1}^{\bar{g}} \frac{1}{v}\left(\frac{\partial u_\tau}{\partial \bar{k}_g} p_\tau S_\tau + u_\tau \frac{\partial p_\tau}{\partial \bar{k}_g} S_\tau + u_\tau p_\tau \frac{\partial S_\tau}{\partial \bar{k}_g}\right). \tag{27}$$

The sign of Eq. (27) is ambiguous, and consequently, it is impossible to state the direction of the change in the new equilibrium vacancy rate. If the equilibrium vacancy rate increases, which means that additional jobs enter the market because of an improvement in worker arrival rate, the labor market becomes loose (and vice versa).

Second, for generation $\tau > g$, the Beveridge curves shift downward due to an increase in $\bar{k}_g$ (Eq. 24) while for other generations, they remain constant. Therefore, if the vacancy rate increases by a change in $\hat{k}_g$, unemployment rates for all generations decrease. By contrast, if the vacancy rate decreases due to the change, many generations incur negative impacts on unemployment rates except for the generations for which the Beveridge curves shift enough to offset the decrease in unemployment rates.

To summarize, we see lead to three possible effects on u_τ for each $\tau \in \{0, \cdots, \bar{g}\}$ at the equilibrium via the change in the increment in $\bar{k}_g$ which we term *spillover effects*. Spillover effects can be decomposed into direct and indirect effects. An increment in $\bar{k}_g$ *directly* decreases in unemployment rates through shifts in Beveridge curves for some generations (direct effects) while it *indirectly* changes in unemployment rates for all generations through a change in vacancy rate (indirect effects). Direct effects surely decrease unemployment rates although indirect effects can increase those. As a result, the overall spillover effect for each generation is determined by the relative size of the two effects.

Fig. 3 illustrates the effects from the increment in friendships in generation g from $\bar{k}_g = \kappa$ to $\bar{k}_g = \kappa'$. Suppose that the equilibrium vacancy rate v^* is achieved with $\bar{k}_\tau = \kappa\ \forall\tau$ at first and $\kappa < \kappa' < \hat{k}_\tau$. If the increment in $\bar{k}_g$ leads to a shift of $G(v)$ to the right (Fig. 3a), the vacancy rate moves from v^* to v', which is indirect effects. Due to the increase in the vacancy rate, the unemployment rate for *every* generation decreases. In this case, the intuition of the spillover effect is very simple. When workers in a generation gain new friendships, the referral probability in the generation increases. This improves the job-worker matching probability and encourages jobs to enter the market. As a result, the aggregate matching probability is improved and the other generations also find it easy to find jobs. The benefits from the increase in $\bar{k}_g$ are larger in older generations since the Beveridge curves of generations $\tau > g$ shift down (direct spillover effects) although those of the other generations do not (remember Proposition 3 (i)).[5]

Figs. 3b and 3c reveal that $G(v)$ shifts to the left. In this case, the increment in $\bar{k}_g$ rises unemployment rates for all generations along with the prior Beveridge curves. However, this change shifts the Beveridge curves downward for generations above g, leading to a decrease in unemployment rates for these generations. For a generation older than g, if the latter effect exceeds the former, in other words, the direct spillover effect is larger than the indirect spillover effect, the generation can achieve a lower unemployment rate than before (Fig. 3b), otherwise, the unemployment rate rises (Fig. 3c). For both cases of

[5]The slopes of the Beveridge curves are negative except for generation 0. See discussions in Appendix B.

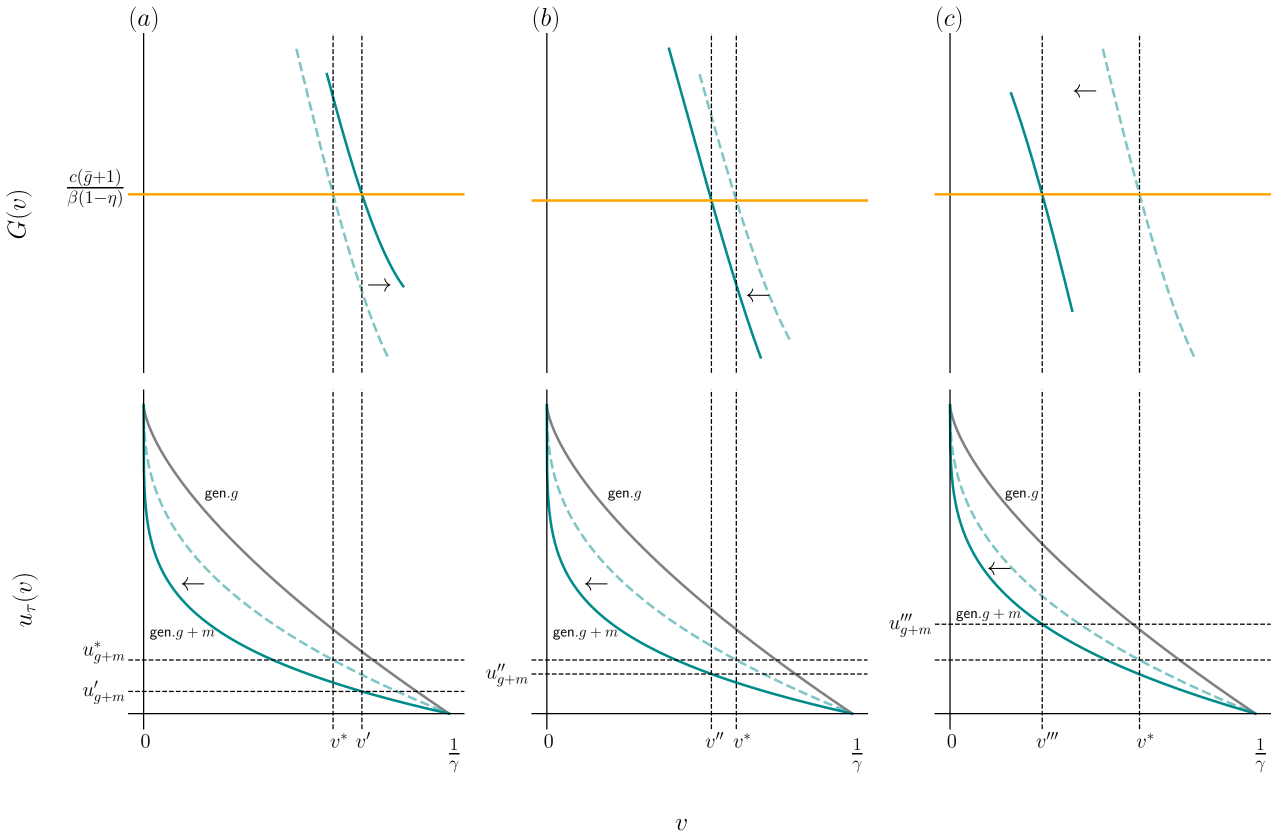

Fig. 3. Three possible spillover effects from increment $\bar{k}_g$. $m > 0$ is assumed and Beveridge curves only for generation g and $g + m$ are shown and the others are omitted. (a) $G(v)$ shift to the right, leading to lower unemployment rates for all generations. (b) $G(v)$ shift to the right, leading to lower and higher unemployment rates for generation $\tau > g$ and $g \leq \tau$, respectively. (c) $G(v)$ shift to the right, leading to higher unemployment rates for all generations.

Fig. 3b and 3c, the young generations incur negative spillover effects in total, since they do not benefit from direct spillover effects.

In this section, we find that the increase in the number of friends in a social network has spillover effects with a simple network structure, a random regular network, for social networks. Although the simple network structure simplifies the analysis, it cannot capture the effects of a network structure. It is known that the network structure can affect the unemployment rates and wages for social groups (e.g. Ogisu, 2022; Tassier and Menczer, 2008). In the next section, we incorporate a more complex network structure than a random regular network to analyze network structural effects on intergenerational inequality.

4 Numerical analysis with temporal networks

Social networks change when new friendships are created and existing ones are broken up. If the change in social networks is governed by a specific process, each generation has a specific feature depending on the time in the social network. The class of time-dependent networks is called *temporal networks* (Holme and Saramäki, 2019). We suppose that the change in social networks over time follows a *fitness model* (e.g. Kobayashi et al., 2019), and calculate the quantum of benefits that workers of a generation get, incremental to those of the other generations, from their network structure.

4.1 Social network formation and degree distributions

Suppose that when entering the labor market (generation 0), a social network is created such that the probability that each pair of workers has a friendship is given by $\phi \in (0, 1)$, which is known as an Erdős-Rényi network. Then, $f_0(k)$ is given by binomial distribution such that

$$f_0(k) = \binom{N-1}{k} \phi^k (1-\phi)^{N-k-1}. \tag{28}$$

Once a social network is created, the network changes following a fitness model.

In a fitness model, each worker has the activity parameter, and a pair of two workers will connect by an edge with the probability depending on these activity parameters. If it is allowed for the activity parameter to change, and it depends on the number of existing friendships of each worker over time, the procedure in which workers make up friendships is similar to preferential attachment processes (Barabási and Albert, 1999; Barrat and Pastor-Satorras, 2005). Let a_{ig} be activity parameter of worker i in generation g and assume that

$$a_{ig} = \frac{k_{ig} + 1}{\langle k_g \rangle + 1}, \tag{29}$$

where $\langle k_g \rangle$ is the average number of friends of generation g.

Let $\mathcal{U}_{ig}$ be the set of workers who are not friends of i in generation g. The probability that worker i will connect with worker $j \in \mathcal{U}_{ig}$ when they move to generation $g+1$ is

$$Pr_g(i, j) = \lambda a_{ig} a_{jg}, \tag{30}$$

where $\lambda > 0$ is a parameter. While new friendships are created in the network, the existing ones are broken up with probability $s \in (0, 1)$.

Based on Eqs. (28), (**??**), and (30), we obtain $f_g(k)$ for generation $g > 0$ by the following simulation.

1. Generate Erdős-Rényi network with ϕ and set iterator $l = 0$.
2. Let $\mathcal{A}$ and $\mathcal{B}$ be the set of worker pairs having and not having a friendship, respectively.
3. Break up a friendship for each pair in $\mathcal{A}$ with probability s.
4. Create a new edge for each pair in $\mathcal{B}$ with probability $Pr_g(i, j)$.
5. Increment l by one and if $l < g$, go back 2. Else, the simulation is finished, and degree distribution $f_g(k)$ is computed.

We calibrate parameter set $\{s, \lambda, \phi\}$ to fit the data in the next subsection.[6]

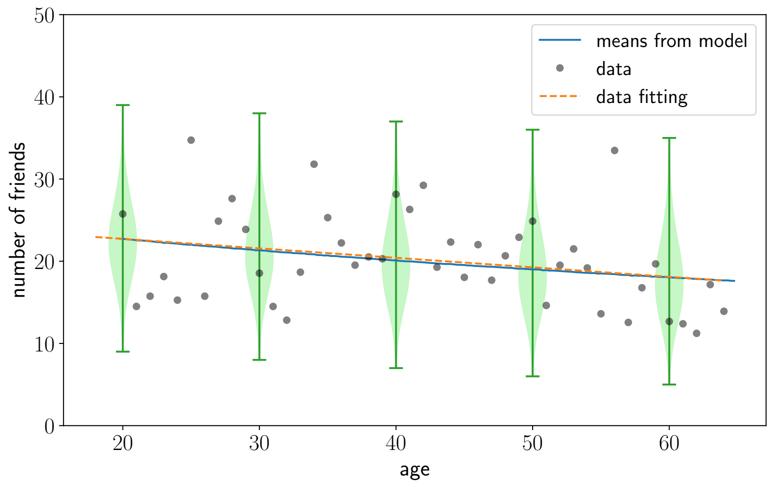

Fig. 4. Average number of friendships. Green plots describe the volume of the degree distributions at ages 20, 30, 40, 50, and 60 produced by the fitness model.

4.2 Parameterization

To obtain the numerical result, we calibrate parameters to fit Japanese data. One period is a quarter, and workers work from the 1st quarter of 20 to the 4th quarter of 64 years, while they begin to search for jobs when they are in the last quarter of 19 years which corresponds to generation 0. There are 1000 workers for each generation, meaning that $N = 1000$. We set discount factor β to 0.99. Home production b is 0.336 given by 60% of average wage which is targeted at 0.56 (mentioned later). We give output y_g by linear function of generation g;

$$y_g = \frac{2(1-b)}{\bar{g}} g + b \ \forall g,$$

which assures us that $y_g \geq b \ \forall g$, the average output over generations $(\sum_{g=0}^{\bar{g}} y_g/(\bar{g}+1))$ equals to one, and workers' productivity grows by age. The assumption of productivity growth is adopted by Esteban-Pretel and Fujimoto (2014), in which worker productivity grows stochastically for each individual. In our formulation, workers' productivity grows deterministically for simplicity. Other parameters are calibrated to fit targets.

Calibration is divided into two steps. In the first step, parameter set $\{s, \lambda, \phi\}$ is calibrated. We compute the average number of friends by age based on *International Social Survey Programme (ISSP): Social Networks and Social Resources*, 2017.[7] In Fig. 4, black dots indicate the averaged number of friends in each age and the orange broken line is the first order fitting by OLS. To fit the data, the parameter set is provided such that $\{s, \lambda, \phi\} = \{3.21 \times 10^{-5}, 0.0014, 0.023\}$, and in Fig. 4, the blue solid line is the average number of friends calculated by the model. Also, we show the volume of the degree distributions for ages 20, 30, 40, 50, and 60 based on the model by the green plots. With the calibrated parameters, the model can roughly capture the average and dispersion of the number of friends.

In the second step, based on the simulated degree distribution, the remaining parameters are calibrated to fit 2017 Japanese data. The calibrated parameters are job opening

[6] We actually use the distribution $f_g(k)$ averaged over 100 trials for each generation g to exclude stochastic fluctuation. In other words, letting $\tilde{f}_g^i(k)$ be the realization value in trial i, $f_g(k) = \sum_{g=1}^{100} \tilde{f}_g^i(k)/100$.

[7] See calculation procedure in Appendix D.

Tab. 1. Parameters for numerical calculations.

Exogenous parameters		
	period	quarter
	working age	20-64
N	number of workers in a generation	1000
y	output	$2(1-b)g/\bar{g}+b$
b	home production	0.336 (60% of avg. wage)
β	discount factor	0.99
Calibrated parameters		
λ	scale of friendship making probability	3.21×10^{-5}
s	probability of breaking a friendship	0.0014
ϕ	probability of friendships made at entering the market	0.023
c	vacancy opening cost	6.310
η	worker's bargaining power	0.024
δ	job destruction rate	0.022
ψ	efficiency in referral hiring	0.355
γ	efficiency in the labor market	15.37

cost c, worker's bargaining power η, job destruction rate δ, efficiency in referral ψ, and efficiency in the labor market γ. We set the following five targets:

1. From *Survey on Employment Trends*, the ratio of referral matches to all matches is 0.213;
$$\frac{\sum_{g=1}^{179}(1-v)p_g^R u_g}{\sum_{g=1}^{179} p_g u_g} = 0.213.$$

2. From *Penn World Table*, the ratio of workers' compensation to GDP is 0.56;
$$\sum_{g=1}^{180} \frac{(1-u_g)w_g}{180} = 0.56,$$

3. and the labor market tightness, which is the ratio of the unemployment rate to the vacancy rate, is 1.5;
$$\frac{v}{u} = 1.5,$$
where $u \equiv \sum_{g=1}^{180} u_g/(180)$.

4. From *Labour Force Survey*, the unemployment rate among 20-24 year olds is 0.047 given by;
$$\sum_{g=1}^{20} \frac{u_g}{20} = 0.047,$$

5. and the average unemployment rate among 20-64 year olds is 0.03;
$$\sum_{g=1}^{180} \frac{u_g}{180} = 0.03.$$

Each of these targets determines $c = 6.310$, $\eta = 0.024$, $\delta = 0.022$, $\psi = 0.355$, $\gamma = 15.37$ respectively. The resulting parameters are summarized in Tab. 1.

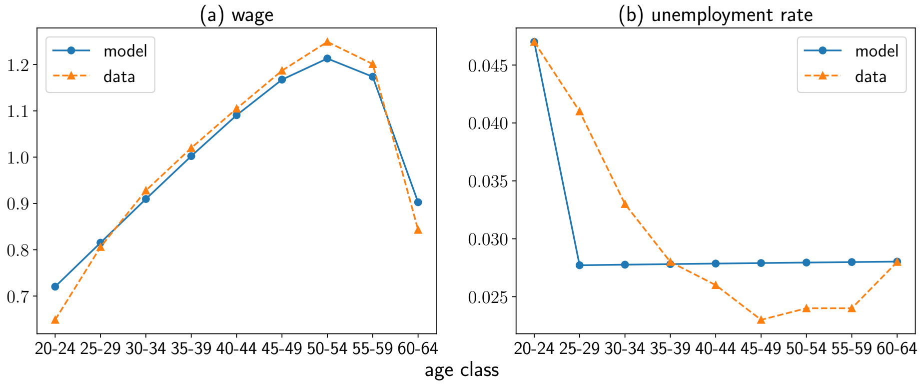

Fig. 5. Wages and unemployment rates at the equilibrium. Wages are normalized such that the average is equal to one in (a).

4.3 Numerical solution and social network premium

The results in Fig. 5 show the average by five-year age group, in which blue and orange lines are the values by the model and by data, respectively. In Fig. 5a, we normalize the average wage to be one to compare results from the model and data. The model mainly succeeds in capturing age-dependent tendencies in the context of wages and unemployment rates. Then, how much does each generation benefit from the social network in the equilibrium?

Workers can receive additional wages from their social network since they can lower unemployment probabilities through referral hiring. We define the social network premium as the difference in the equilibrium values achieved with and without referrals. Here we calculate the average wage and unemployment premium for each generation.

Workers in a social group benefit not only from the mean number of friends in the network, but also from the structure of the network (Ogisu, 2022; Tassier and Menczer, 2008). We can also compute the equilibrium values if the structure of networks does not change over time, and use it for calculating how much workers benefit from network *structures*.

Let $w_{g,tem}$, $w_{g,stat}$, and $w_{g,no}$ be an equilibrium wage of generation g when all workers can use referrals and social networks change over time, when all workers can use referrals but social networks do not change over time, and when all workers cannot use referrals, respectively. We denote a social network premium to the wage of generation g by ρ_g^w, and define it as

$$\rho_g^w \equiv w_{g,tem} - w_{g,no}. \tag{31}$$

It can be decomposed such that

$$\rho_g^w = \underbrace{w_{g,tem} - w_{g,stat}}_{\text{structural effect}} + \underbrace{w_{g,stat} - w_{g,no}}_{\text{common effect}}. \tag{32}$$

When social network structures do not change, all generations have the same network structure and degree distribution following Eq. (28). Thus, the additional premium (or penalty) from time-dependent structures of social networks can be captured by the difference between $w_{g,tem}$ and $w_{g,stat}$, and we call it structural effect and the rest common effect.

In the same manner, we also define social network premium to the unemployment rate of generation g as

$$\rho_g^u \equiv u_{g,no} - u_{g,tem},$$

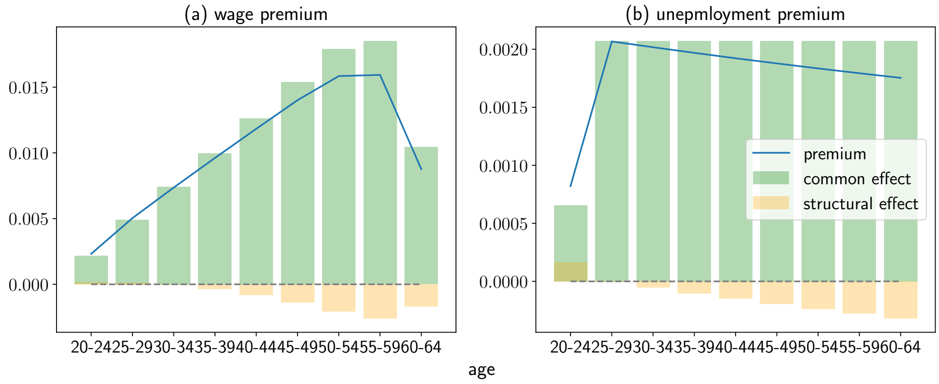

Fig. 6. Premiums from social networks. Social network premiums are positive while parts of the premiums are counteracted by structural effects for most generations.

and decompose such that

$$\rho_g^u = \underbrace{u_{g,no} - u_{g,stat}}_{\text{common effect}} + \underbrace{u_{u,stat} - u_{u,tem}}_{\text{structural effect}}. \tag{33}$$

Note that the sign of ρ_g^u is reversed in the definition of ρ_g^w since the premium in unemployment rates is captured as the decrease.

The calculated premiums averaged by the five-year age group are shown in Fig. 6. All generations receive premiums from their social networks and they are up to 1.6%pt at age class 55-59 in wages and up to 0.2%pt at the age class 25-29 in unemployment rates. The premium is the lowest in the youngest group. In turn, the youngest group can get a relatively large structural effect, meaning that workers in the group have good network structures in referral hiring. This can be interpreted as young workers being unable to use referrals frequently even if they have social networks since most friends are unemployed. Thus, when workers use referrals, most job slots are filled by middle- or old-age workers.

The relationship between wage and unemployment premiums is not monotonic. In particular, the wage premium of age class 60-64 decreases from that of the 55-59 class although the unemployment premium does not decrease as much. This is because workers in age 60-64 have little room for bargaining compared to young ones since they will retire soon. Workers belonging to the last quarter of age 64, for example, cannot get additional premiums whenever the change of friendships happens in the other generation because they receive $w_{\bar{g}}$ or b and exit the market. In other words, their incomes are fixed by the terminal conditions.

While there are positive premiums overall, structural effects of premiums decrease as age classes move up, which is caused by the congestion within the network for each generation. The penalties from the network structures are up to 0.3%pt at age class 55-59 in wages and up to 0.03%pt at age class 60-64 in unemployment rates. Based on Eq. (30), new friendships are created proportional to the number of friends of workers; in other words, a worker with more friends can get new friends with a higher probability than a worker with fewer friends. By this mechanism, workers in the social network are divided into two groups as time goes on: some workers who are popular and lots of workers who do not have many friends. A worker group with such a network structure is at a disadvantage in the labor market (Ogisu, 2022) since vacancy information concentrates in the hands of some popular workers, and as a result, older generations incur penalties from their network structure more than younger ones.

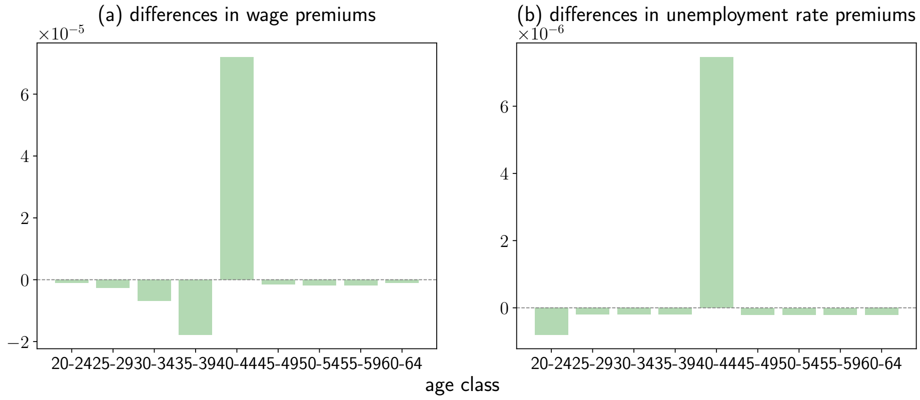

Fig. 7. Spillover effects by increment in the number of friends at the 1st quarter of age 40. Bars show changes in the structural effects by definition. The class including age 40 obviously benefits although the others do not. In the classes more than age 45, the effects are relatively small since they are partly offset by the shift of Beveridge curves.

4.4 An example of spillover effects

In this subsection, we provide an example of spillover effects by an increment in the number of friends of a generation. We choose the 1st quarter of age 40 ($g = 81$) for the generations in which degree distribution will change. To describe the increment in the number of friends, the mass of the k is moved to the $k + 2$ for all k in the degree distribution. This shift in the degree distribution changes the average number of friends from 20.1 to 22.1, which is an increase of about 10% only for the generation.[8]

We compute the new equilibrium and record differences in old and new ones for each age class, shown in Fig. 7. All of these changes in premiums are explained by the change in structural effect by definition. Although the networks are complex compared to the case in Sec. 3, the basic mechanism is generally the same. The age class 40-44 years, including the generation in which the degree distribution shifts right, obviously obtains the additional premiums to unemployment rates, meaning that the average unemployment rate decrease. However, premiums decrease for the older age classes compared with age 45, therefore, these unemployment rates at the new equilibrium increase. These changes are consistent with the case in Fig. 3b and 3c. Equilibrium wages decrease except for the age class 40-44 while the absolute values of the changes are smaller in the older ages than the younger ages since shifts of the Beveridge curves in the older generations can mitigate the decrease in these unemployment rates.

In this section, we demonstrate that workers benefit from their social networks while the benefit can be partly counteracted by the congestion led by these network structures. In the calibrated case suited to the Japanese context, spillover effects can be negative because of a decrease in the vacancy rate.

5 Conclusion

In this paper, we demonstrate the generation-dependent premiums from their social networks in a search and matching model with overlapping generations. These premiums are shown to be small for young workers compared to middle- and old-aged ones. A crucial reason is that young workers cannot utilize referrals sufficiently even if they have many

[8] We calculate two other examples in Appendix E; increase in connectivity of young and old generation.

friends since many of them are also unemployed. This implies that benefits from social networks vary in response to the unemployment dynamics. We note here that premiums are partially offset by congestion caused by network structures for most generations.

A change in the connectivity of a network affects not only the generation of which the degree distribution changes, but also the other generations through the changes in the vacancy rate in equilibrium. When the vacancy rate increases in response to a change in the degree distribution, then the unemployment rates of the other generations decrease. Even if, however, the vacancy rate decreases, it is possible that the unemployment rates of older generations become lower than before, or the increments in unemployment rates are partially offset due to the shift of the Beveridge curves.

Our model can be extended in various directions. Among them, the following three are important and hence worth investigating. First, an alternative model could be presented to be used for a realistic description of social networks. We have used a fitness model with simple rules to describe the dynamics in social networks. However, other specifications than this one are also possible. We use only the average number of friendships from data for the calibration target. If a data set of detailed social networks in the real economy becomes available, the model specification will be modified in a way to capture the actual characteristics of networks (e.g. clustering coefficients, diameters).

Second, social networks could be endogenized. In the model presented in this paper, workers do not invest in keeping or making friendships. If workers expect to receive referrals from their friends, they have incentives to make friends (e.g. Galenianos, 2021; Galeotti and Merlino, 2014; Horvath and Zhang, 2018). Including the endogenized network, the dynamics of variables in equilibrium (e.g. unemployment rates, job arrival rates) are determined as a result of the interaction between the social network dynamics and workers' decisions, which allows us to analyze premiums from a temporal network in a realistic sense.

Third, on-the-job search could be included. Referrals can work significantly to step workers' careers in reality. One of the natural extensions is to make job productivity stochastic. As a result, the referral process becomes complicated because all workers wait for good job information. Since a job passed down from a friend is not better than the friend's, if workers hope to get better-paid jobs, they need to wait for such a job in the labor market which is a tiny probability, or to form friendships with workers who earn much. Thus, in a model with on-the-job search, positions on the social networks, i.e. who you know, will become more crucial to determine premiums than in the model presented in this paper.

An important remark exists in interpreting the results put forward in the paper. To introduce network structure into the model, we employ degree distributions for the information on the network structures, similar to Ioannides and Soetevent (2006) and Ogisu (2022). This formulation is unable to capture the micro characteristics of the network structures like clustering coefficients. Thus, social networks in this paper cannot be distinguished from *configuration models*. In a configuration model, workers' degree distribution is prespecified, while the connection between workers is given at uniformly random, and hence clustering coefficients are equal to zero with a sufficiently large number of nodes (Catanzaro et al., 2005; Newman, 2018), although in an assortative mixing model, it is possible that clustering coefficients are positive (Jackson and Rogers, 2007). The impact of micro characteristics of network structure is one of the most important issues to be examined in the future.

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Appendix

A Proof of Proposition 1

We respectively rewrite S_g, p_g to

$$\begin{aligned}
S_g &= (y-b)\left(1+\prod_{\tau=g}^{g}\Delta_\tau+\prod_{\tau=g}^{g+1}\Delta_\tau+\cdots+\prod_{\tau=g}^{\bar{g}-1}\Delta_\tau\right)\\
&= (y-b)\left(1+\sum_{m=g}^{\bar{g}-1}\prod_{\tau=g}^{m}\Delta_\tau\right),\\
P_g &= \psi\gamma v(1-u_g)\frac{1-(1-u_g)^{\bar{k}_g}}{\bar{k}_g u_g}\\
&= \frac{\psi\gamma v(1-u_g)}{\bar{k}_g}\sum_{\kappa=0}^{\bar{k}_g-1}(1-u_g)^\kappa,\\
p_g &= \gamma v+(1-\gamma v)[1-(1-P_g)^{\bar{k}_g}]\\
&= \gamma v\,(1-P_g)^{\bar{k}_g}+1-(1-P_g)^{\bar{k}_g}\\
&= \gamma v\,(1-P_g)^{\bar{k}_g}+P_g\sum_{\kappa=0}^{\bar{k}_g-1}(1-P_g)^\kappa,
\end{aligned}$$

where $\Delta_g \equiv \beta(1-\delta-p_g\eta)$. Therefore, we can calculate

$$\frac{p_g}{v}=\gamma(1-P_g)^{\bar{k}_g}+\frac{\psi\gamma(1-u_g)}{\bar{k}_g}\left(\sum_{\kappa=0}^{\bar{k}_g-1}(1-u_g)^\kappa\right)\left(\sum_{\kappa=0}^{\bar{k}_g-1}(1-P_g)^\kappa\right). \tag{34}$$

When v approaches zero, for all $g \in \{0,1,\cdots\bar{g}\}$

$$\begin{aligned}
&\lim_{v\to 0}P_g=0,\quad \lim_{v\to 0}p_g=0,\quad \lim_{v\to 0}u_g=1,\\
&\lim_{v\to 0}S_g=(y-b)\left[\frac{1-[\beta(1-\delta)]^{\bar{g}-g+1}}{1-\beta(1-\delta)}\right]\equiv \underline{S}_g,\\
&\lim_{v\to 0}\frac{p_g}{v}=\gamma.
\end{aligned}$$

Thus,

$$\lim_{v\to 0}\sum_{g=0}^{\bar{g}}u_g\frac{p_g}{v}S_g=\gamma\sum_{g=0}^{\bar{g}}\underline{S}_g. \tag{35}$$

As $v\to 1/\gamma$, for all $g\in\{1,2,\cdots,\bar{g}\}$

$$\begin{aligned}
&\lim_{v\to \frac{1}{\gamma}}P_g=\psi,\quad \lim_{v\to \frac{1}{\gamma}}p_g=1,\quad \lim_{v\to \frac{1}{\gamma}}u_g=0,\\
&\lim_{v\to \frac{1}{\gamma}}S_g=(y-b)\frac{1-[\beta(1-\delta-\eta)]^{\bar{g}-g+1}}{1-\beta(1-\delta-\eta)}\equiv \overline{S}_g,\\
&\lim_{v\to \frac{1}{\gamma}}\frac{p_g}{v}=\gamma,
\end{aligned}$$

and for $g = 0$ with $u_0 = 1$,

$$\lim_{v\to\frac{1}{\gamma}} P_0 = 0, \quad \lim_{v\to\frac{1}{\gamma}} p_0 = 1,$$
$$\lim_{v\to\frac{1}{\gamma}} S_0 = (y-b)\frac{1-[\beta(1-\delta-\eta)]^{\bar{g}+1}}{1-\beta(1-\delta-\eta)} \equiv \overline{S}_0,$$
$$\lim_{v\to\frac{1}{\gamma}} \frac{p_0}{v} = \gamma.$$

Thereby,

$$\lim_{v\to\frac{1}{\gamma}} \sum_{g=0}^{\bar{k}_g-1} u_g \frac{p_g}{v} S_g = \gamma\overline{S}_0. \tag{36}$$

As a result,

$$\gamma\overline{S}_0 < \frac{c(\bar{g}+1)}{\beta(1-\eta)} < \gamma\sum_{g=0}^{\bar{g}} \underline{S}_g$$
$$\iff \quad \frac{\gamma\beta(1-\eta)(y-b)}{c(\bar{g}+1)}\frac{1-[\beta(1-\delta-\eta)]^{\bar{g}-g+1}}{1-\beta(1-\delta-\eta)} < 1 < \frac{\gamma\beta(1-\eta)(y-b)}{c(\bar{g}+1)}\sum_{g=0}^{\bar{g}}\left[\frac{1-[\beta(1-\delta)]^{\bar{g}-g+1}}{1-\beta(1-\delta)}\right]$$

is the sufficient condition for the existence of the equilibrium.

B Proof of Lemma 1

$$\frac{\partial P_g}{\partial u_g} \gtreqless 0,$$
$$\iff (\bar{k}_g u_g + 1)(1-u_g)^{\bar{k}_g} \gtreqless 1. \tag{37}$$

LHS of (37) decreases in u_g and approaches one when $u_g \to 0$. Therefore,

$$\frac{\partial P_g}{\partial u_g} \leq 0.$$

And we can get

$$\frac{\partial p_g^R}{\partial u_g} = \bar{k}_g(1-P_g)^{\bar{k}_g-1}\frac{\partial P_g}{\partial u_g} \leq 0 \quad \forall u_g \in (0,1],$$
$$\frac{\partial p_g}{\partial u_g} \leq 0.$$

By definition,

$$P_0 = 0, \quad p_0 = \gamma v,$$

and

$$\begin{aligned}
\frac{\partial P_g}{\partial v} &= \frac{\psi\gamma}{\bar{k}_g u_g^2}\left[\left(u_g - v\frac{\partial u_g}{\partial v}\right)\left(1-u_g-(1-u_g)^{\bar{k}_g+1}\right) + vu_g\left((\bar{k}_g+1)(1-u_g)^{\bar{k}_g}-1\right)\frac{\partial u_g}{\partial v}\right] \\
&= \frac{\psi\gamma}{\bar{k}_g u_g^2}\left[u_g\left(1-u_g-(1-u_g)^{\bar{k}_g+1}\right) + v\frac{\partial u_g}{\partial v}\left(u_g(\bar{k}_g+1)(1-u_g)^{\bar{k}_g} + (1-u_g)^{\bar{k}_g+1}-1\right)\right] \\
&= \frac{P_g}{v} + \frac{\psi\gamma v}{\bar{k}_g u_g^2}\left[(\bar{k}_g+1)u_g(1-u_g)^{\bar{k}_g} + (1-u_g)^{\bar{k}_g+1}-1\right]\frac{\partial u_g}{\partial v} \\
&= \frac{P_g}{v} + \frac{\psi\gamma v}{\bar{k}_g u_g^2}\left[(\bar{k}_g u_g+1)(1-u_g)^{\bar{k}_g}-1\right]\frac{\partial u_g}{\partial v},
\end{aligned}$$

where $(\bar{k}_g u_g + 1)(1-u_g)^{\bar{k}_g} - 1 < 0 \ \forall u_g \in (0,1]$ and

$$\begin{aligned}
\frac{\partial p_g}{\partial v} &= \left[\gamma(1-P_g) + \bar{k}_g(1-\gamma v)\frac{\partial P_g}{\partial v}\right](1-P_g)^{\bar{k}_g-1} \\
&= \gamma(1-P_g)^{\bar{k}_g} + \bar{k}_g(1-\gamma v)(1-P_g)^{\bar{k}_g-1}\frac{\partial P_g}{\partial v} \\
&= \gamma(1-P_g)^{\bar{k}_g} + \bar{k}_g(1-\gamma v)(1-P_g)^{\bar{k}_g-1}\left[\frac{P_g}{v} + \frac{\psi\gamma v}{\bar{k}_g u_g^2}\left[(\bar{k}_g u_g+1)(1-u_g)^{\bar{k}_g}-1\right]\frac{\partial u_g}{\partial v}\right] \\
&= \gamma(1-P_g)^{\bar{k}_g} + \bar{k}_g(1-\gamma v)(1-P_g)^{\bar{k}_g-1}\frac{P_g}{v} \\
&\qquad + (1-\gamma v)(1-P_g)^{\bar{k}_g-1}\frac{\psi\gamma v}{u_g^2}\left[(\bar{k}_g u_g+1)(1-u_g)^{\bar{k}_g}-1\right]\frac{\partial u_g}{\partial v} \\
&= \left[\gamma(1-P_g) + \bar{k}_g(1-\gamma v)\frac{P_g}{v}\right](1-P_g)^{\bar{k}_g-1} \\
&\qquad + (1-\gamma v)(1-P_g)^{\bar{k}_g-1}\frac{\psi\gamma v}{u_g^2}\left[(\bar{k}_g u_g+1)(1-u_g)^{\bar{k}_g}-1\right]\frac{\partial u_g}{\partial v} \\
&= \left[\gamma(1-P_g) + \bar{k}_g(1-\gamma v)\frac{P_g}{v}\right](1-P_g)^{\bar{k}_g-1} \\
&\qquad + (1-\gamma v)(1-P_g)^{\bar{k}_g-1}\frac{\psi\gamma v}{u_g^2}\left[(\bar{k}_g u_g+1)(1-u_g)^{\bar{k}_g}-1\right]\frac{\partial u_g}{\partial v} \\
&= \left[\gamma(1-P_g) + (1-\gamma v)\psi\gamma\frac{1-u_g-(1-u_g)^{\bar{k}_g+1}}{u_g}\right](1-P_g)^{\bar{k}_g-1} \\
&\quad + (1-\gamma v)(1-P_g)^{\bar{k}_g-1}\frac{\psi\gamma v}{u_g^2}\left[(\bar{k}_g u_g+1)(1-u_g)^{\bar{k}_g}-1\right]\frac{\partial u_g}{\partial v},
\end{aligned}$$

and by the law of motion,

$$\begin{aligned}
\frac{\partial u_g}{\partial v} &= -\frac{\partial p_{g-1}}{\partial v}u_{g-1} + (1-\delta-p_{g-1})\frac{\partial u_{g-1}}{\partial v} \\
&= -u_{g-1}\left[\gamma(1-P_{g-1}) + \bar{k}_g(1-\gamma v)\frac{P_{g-1}}{v}\right](1-P_{g-1})^{\bar{k}_g-1} \\
&\qquad - u_{g-1}(1-\gamma v)(1-P_{g-1})^{\bar{k}_g-1}\frac{\psi\gamma v}{u_{g-1}^2}\left[(\bar{k}_g u_{g-1}+1)(1-u_{g-1})^{\bar{k}_g}-1\right]\frac{\partial u_{g-1}}{\partial v} \\
&\qquad\qquad + (1-\delta-p_{g-1})\frac{\partial u_{g-1}}{\partial v}
\end{aligned}$$

$\lim_{v\to 0}\frac{\partial u_2}{v} = -\gamma - \gamma(1-\delta)$

thus,

$$\frac{\partial u_0}{\partial v} = 0, \quad \frac{\partial u_1}{\partial v} = -\gamma < 0,$$

and for $g = 2$,

$$\frac{\partial u_2}{\partial v} < 0,$$

since

$$\frac{\partial P_1}{\partial v} > 0, \quad \frac{\partial p_1}{\partial v} > 0.$$

And after that, similarly, for all $g \geq 2$,

$$\frac{\partial u_g}{\partial v} < 0, \quad \frac{\partial P_g}{\partial v} > 0, \quad \frac{\partial p_g}{\partial v} > 0.$$

Consequently,

$$\begin{aligned}
\frac{\partial S_g}{\partial v} &= -\beta\eta(y-b)\left[\frac{\partial p_g}{\partial v}\frac{1}{\Delta_g}\left\{\prod_{\tau=g}^{g}\Delta_\tau + \prod_{\tau=g}^{g+1}\Delta_\tau + \cdots + \prod_{\tau=g}^{\bar{g}-1}\Delta_\tau\right\}\right. \\
&\quad + \frac{\partial p_{g+1}}{\partial v}\frac{1}{\Delta_{g+1}}\left\{\prod_{\tau=g}^{g+1}\Delta_{g+1} + \cdots + \prod_{\tau=g}^{\bar{g}-1}\Delta_\tau\right\} \\
&\quad \vdots \\
&\quad + \frac{\partial p_{\bar{g}-2}}{\partial v}\frac{1}{\Delta_{\bar{g}-2}}\left\{\prod_{\tau=g}^{\bar{g}-2}\Delta_\tau + \prod_{\tau=g}^{\bar{g}-1}\Delta_\tau\right\} \\
&\quad \left. + \frac{\partial p_{\bar{g}-1}}{\partial v}\frac{1}{\Delta_{\bar{g}-1}}\left\{\prod_{\tau=g}^{\bar{g}-1}\Delta_{\bar{g}-1}\right\}\right] < 0 \quad \forall g \in \{0, 1, \cdots, \bar{g}-1\}, \\
\frac{\partial S_{\bar{g}}}{\partial v} &= 0.
\end{aligned}$$

Differentiating $G(v)$ by v,

$$\begin{aligned}
&\frac{\partial (u_g(p_g/v)S_g)}{\partial v} \\
&= \frac{\partial u_g}{\partial v}\frac{p_g}{v}S_g + u_g\frac{\partial (p_g/v)}{\partial v}S_g + u_g\frac{p_g}{v}\frac{\partial S_g}{\partial v} \\
&= \frac{\partial u_g}{\partial v}\frac{p_g}{v}S_g + u_g\frac{p_g}{v}\frac{\partial S_g}{\partial v} \\
&+ u_g S_g\left[-\gamma\bar{k}_g(1-P_g)^{\bar{k}_g-1}\frac{\partial P_g}{\partial v} + \frac{\psi\gamma}{\bar{k}_g u_g}\frac{\partial u_g}{\partial v}\left((\bar{k}_g+1)(1-u_g)^{\bar{k}_g}-1\right)\sum_{\kappa=0}^{\bar{k}_g-1}(1-P_g)^\kappa\right. \\
&- \frac{\psi\gamma}{\bar{k}_g u_g}\left(1-u_g-(1-u_g)^{\bar{k}_g+1}\right)\sum_{\kappa=1}^{\bar{k}_g-1}\kappa(1-P_g)^{\kappa-1}\frac{\partial P_g}{\partial v} \\
&\left. - \frac{\psi\gamma}{\bar{k}_g u_g^2}\frac{\partial u_g}{\partial v}\left(1-u_g-(1-u_g)^{\bar{k}_g+1}\right)\sum_{\kappa=0}^{\bar{k}_g-1}(1-P_g)^\kappa\right] \\
&= \frac{\partial u_g}{\partial v}\frac{p_g}{v}S_g + u_g\frac{p_g}{v}\frac{\partial S_g}{\partial v} \\
&+ u_g S_g\left[-\gamma\bar{k}_g(1-P_g)^{\bar{k}_g-1}\frac{\partial P_g}{\partial v} - \frac{\psi\gamma}{\bar{k}_g u_g}\left(1-u_g-(1-u_g)^{\bar{k}_g+1}\right)\sum_{\kappa=1}^{\bar{k}_g-1}\kappa(1-P_g)^{\kappa-1}\frac{\partial P_g}{\partial v}\right] \\
&+ S_g\frac{\psi\gamma}{\bar{k}_g}\frac{\partial u_g}{\partial v}\left((\bar{k}_g+1)(1-u_g)^{\bar{k}_g}-1\right)\sum_{\kappa=0}^{\bar{k}_g-1}(1-P_g)^\kappa \\
&- S_g\frac{\psi\gamma}{\bar{k}_g u_g}\frac{\partial u_g}{\partial v}\left(1-u_g-(1-u_g)^{\bar{k}_g+1}\right)\sum_{\kappa=0}^{\bar{k}_g-1}(1-P_g)^\kappa
\end{aligned}$$

$$
\begin{aligned}
&= u_g \frac{p_g}{v}\frac{\partial S_g}{\partial v} \\
&- u_g S_g \left[\gamma \bar{k}_g (1-P_g)^{\bar{k}_g - 1}\frac{\partial P_g}{\partial v} + \frac{\psi\gamma}{\bar{k}_g u_g}\left(1 - u_g - (1-u_g)^{\bar{k}_g+1}\right)\sum_{\kappa=1}^{\bar{k}_g-1}\kappa(1-P_g)^{\kappa-1}\frac{\partial P_g}{\partial v}\right] \\
&+ S_g \frac{\partial u_g}{\partial v}\left[\gamma(1-P_g)^{\bar{k}_g} + \frac{\psi\gamma}{\bar{k}_g}\left((\bar{k}_g+1)(1-u_g)^{\bar{k}_g} - 1\right)\sum_{\kappa=0}^{\bar{k}_g-1}(1-P_g)^{\kappa}\right] \\
&= u_g \frac{p_g}{v}\frac{\partial S_g}{\partial v} \\
&- u_g S_g \gamma \left[\bar{k}_g (1-P_g)^{\bar{k}_g - 1} + \frac{\psi}{\bar{k}_g u_g}\left(1 - u_g - (1-u_g)^{\bar{k}_g+1}\right)\sum_{\kappa=1}^{\bar{k}_g-1}\kappa(1-P_g)^{\kappa-1}\right]\frac{\partial P_g}{\partial v} \\
&+ S_g \frac{\partial u_g}{\partial v}\gamma\left[(1-P_g)^{\bar{k}_g} + \frac{\psi}{\bar{k}_g}\left((\bar{k}_g+1)(1-u_g)^{\bar{k}_g} - 1\right)\sum_{\kappa=0}^{\bar{k}_g-1}(1-P_g)^{\kappa}\right],
\end{aligned}
$$

where only the last term can be positive. When the bracket in the last term becomes positive, all terms in the equation become negative since $\partial u_g/\partial v < 0$. For $g = 0$, the bracket in the last term is

$$
1 - \psi > 0,
$$

and for $g \in \{1, 2, \cdots, \bar{g}\}$, it becomes such that

$$
\lim_{v \to \frac{1}{\gamma}}\left[(1-P_g)^{\bar{k}_g} + \frac{\psi}{\bar{k}_g}\left((\bar{k}_g+1)(1-u_g)^{\bar{k}_g} - 1\right)\sum_{\kappa=0}^{\bar{k}_g-1}(1-P_g)^{\kappa}\right] = 1 > 0,
$$

Thus, for each $g \in \{0, 1, \cdots \bar{g}\}$ there exists $\underline{v}_g \in (0, 1/\gamma)$ such that for $v \in (\underline{v}_g, 1/\gamma)$

$$
\frac{\partial u_g(p_g/v)S_g}{\partial v} < 0.
$$

Therefore, let

$$
\underline{v} \equiv \max\{v_g\}_{g=0}^{\bar{g}},
$$

then for $\underline{v} < v$, $\partial u_g(p_g/v)S_g/\partial v < 0$ for all g, which is sufficient for $\partial G(v)/\partial v < 0$.

C Proof of Proposition 3

For $\bar{k}_g > 0$,

$$
\begin{aligned}
\frac{\partial P_g}{\partial \bar{k}_g} &= \frac{\gamma\psi v(1-u_g)}{u_g \bar{k}_g}\left[\left\{1 - \bar{k}_g \log(1-u_g)\right\}(1-u_g)^{\bar{k}_g} - 1\right] < 0, \\
\frac{\partial p_g}{\partial \bar{k}_g} &= -(1-\gamma v)(1-P_g)^{\bar{k}_g}\left(\log(1-P_g) - \frac{\bar{k}_g}{1-P_g}\frac{\partial P_g}{\partial \bar{k}_g}\right) \\
&= (1-\gamma v)(1-P_g)^{\bar{k}_g}\left(\frac{\bar{k}_g}{1-P_g}\frac{\partial P_g}{\partial \bar{k}_g} - \log(1-P_g)\right) \\
&= (1-\gamma v)(1-P_g)^{\bar{k}_g}\left(\frac{\gamma\psi v(1-u_g)}{(1-P_g)u_g}\left[\left\{1 - \bar{k}_g \log(1-u_g)\right\}(1-u_g)^{\bar{k}_g} - 1\right] - \log(1-P_g)\right).
\end{aligned}
$$

$\log(1-P_g)$ increases monotonically in $\bar{k}_g$,

$$\lim_{\bar{k}_g \to 0} \log(1-P_g) = \log\left(1 + \frac{\gamma\psi v(1-u_g)}{u_g}\log(1-u_g)\right) < 0,$$
$$\lim_{\bar{k}_g \to \infty} \log(1-P_g) = 0,$$
$$\lim_{\bar{k}_g \to 0} \frac{\bar{k}_g}{1-P_g}\frac{\partial P_g}{\partial \bar{k}_g} = 0.$$

Therefore, there is $\bar{k}_g = \hat{k}_g$ such that $\partial p_g/\partial \bar{k}_g = 0$, at which p_g is maximized, and

$$\frac{\partial p_g}{\partial \bar{k}_g} \gtreqless 0 \Longleftrightarrow \bar{k}_g \lesseqgtr \hat{k}_g$$

Intergenerational propagations are such that

$$\begin{aligned}
\frac{\partial P_g}{\partial k_{g-1}} &= \frac{\partial P_g}{\partial u_g}\frac{\partial u_g}{\partial k_{g-1}} \\
&= -u_{g-1}\frac{\partial P_g}{\partial u_g}\frac{\partial p_{g-1}}{\partial \bar{k}_{g-1}} \\
\therefore \frac{\partial P_g}{\partial k_{g-1}} &\gtreqless 0 \Longleftrightarrow \bar{k}_{g-1} \lesseqgtr \hat{k}_{g-1}
\end{aligned}$$

and

$$\frac{\partial p_g}{\partial \bar{k}_{g-1}} = \bar{k}_g(1-\gamma v)(1-P_g)^{\bar{k}_g - 1}\frac{\partial P_g}{\partial \bar{k}_{g-1}}.$$

Similarly,

$$\begin{aligned}
\frac{\partial P_{g+m}}{\partial \bar{k}_g} &= \frac{\partial P_{g+m}}{\partial u_{g+m}}\frac{\partial u_{g+m}}{\partial \bar{k}_g}, \\
\frac{\partial p_{g+m}}{\partial \bar{k}_g} &= \bar{k}_{g+m}(1-\gamma v)(1-P_g)^{\bar{k}_{g+m}-1}\frac{\partial P_{g+m}}{\partial \bar{k}_g} \\
&= \bar{k}_{g+m}(1-\gamma v)(1-P_g)^{\bar{k}_{g+m}-1}\frac{\partial P_{g+m}}{\partial u_{g+m}}\frac{\partial u_{g+m}}{\partial \bar{k}_g} \\
\frac{\partial u_{g+m}}{\partial \bar{k}_g} &= -\frac{\partial p_{g+m-1}}{\partial \bar{k}_g}u_{g+m-1} + (1-\delta-p_{g+m-1})\left[-\frac{\partial p_{g+m-2}}{\partial \bar{k}_g}u_{g+m-2} + (1-\delta-p_{g+m-2})\frac{\partial u_{g+m-2}}{\partial \bar{k}_g}\right] \\
&= -\frac{\partial p_{g+m-1}}{\partial \bar{k}_g}u_{g+m-1} - (1-\delta-p_{g+m-1})\frac{\partial p_{g+m-2}}{\partial \bar{k}_g}u_{g+m-2} \\
&\quad + (1-\delta-p_{g+m-1})(1-\delta-p_{g+m-2})\frac{\partial u_{g+m-2}}{\partial \bar{k}_g} \\
&= -\frac{\partial p_{g+m-1}}{\partial \bar{k}_g}u_{g+m-1} - \frac{\partial p_{g+m-2}}{\partial \bar{k}_g}u_{g+m-2}\prod_{\tau=g+m-1}^{g+m-1}(1-\delta-p_\tau) - \cdots - \frac{\partial p_g}{\partial \bar{k}_g}u_g\prod_{\tau=g+1}^{g+m-1}(1-\delta-p_\tau) \\
&= -\left[\frac{\partial p_{g+m-1}}{\partial \bar{k}_g}u_{g+m-1} + \sum_{r=g+1}^{g+m-1}\frac{\partial p_{r-1}}{\partial \bar{k}_g}u_{r-1}\prod_{\tau=r}^{g+m-1}(1-\delta-p_\tau)\right]
\end{aligned}$$

And for $m \geq 0$,

$$
\begin{aligned}
\frac{\partial S_{g+m}}{\partial \bar{k}_g} &= -\beta\eta(y-b)\left[\frac{\partial p_{g+m}}{\partial \bar{k}_g}\frac{1}{\Delta_{g+m}}\left\{\Delta_{g+m} + \Delta_{g+m}\Delta_{g+m+1} + \cdots + \Delta_{g+m}\cdots\Delta_{\bar{g}-1}\right\}\right.\\
&\quad + \frac{\partial p_{g+m+1}}{\partial \bar{k}_g}\frac{1}{\Delta_{g+m+1}}\left\{\Delta_{g+m}\Delta_{g+m+1} + \cdots + \Delta_{g+m}\cdots\Delta_{\bar{g}-1}\right\}\\
&\quad \vdots\\
&\quad + \frac{\partial p_{\bar{g}-2}}{\partial \bar{k}_g}\frac{1}{\Delta_{\bar{g}-2}}\left\{\Delta_{g+m}\cdots\Delta_{\bar{g}-2} + \Delta_{g+m}\cdots\Delta_{\bar{g}-1}\right\}\\
&\quad \left. + \frac{\partial p_{\bar{g}-1}}{\partial \bar{k}_g}\frac{1}{\Delta_{\bar{g}-1}}\left\{\Delta_{g+m}\cdots\Delta_{\bar{g}-1}\right\}\right]\\
&= -\beta\eta(y-b)\left[\frac{\partial p_{g+m}}{\partial \bar{k}_g}\frac{1}{\Delta_{g+m}}\left\{\prod_{\tau=g+m}^{g+m}\Delta_\tau + \prod_{\tau=g+m}^{g+m+1}\Delta_\tau + \cdots + \prod_{\tau=g+m}^{\bar{g}-1}\Delta_\tau\right\}\right.\\
&\quad + \frac{\partial p_{g+m+1}}{\partial \bar{k}_g}\frac{1}{\Delta_{g+m+1}}\left\{\prod_{\tau=g+m}^{g+m+1}\Delta_\tau + \cdots + \prod_{\tau=g+m}^{\bar{g}-1}\Delta_\tau\right\}\\
&\quad \vdots\\
&\quad + \frac{\partial p_{\bar{g}-2}}{\partial \bar{k}_g}\frac{1}{\Delta_{\bar{g}-2}}\left\{\prod_{\tau=g+m}^{\bar{g}-2}\Delta_\tau + \prod_{\tau=g+m}^{\bar{g}-1}\Delta_\tau\right\}\\
&\quad \left. + \frac{\partial p_{\bar{g}-1}}{\partial \bar{k}_g}\frac{1}{\Delta_{\bar{g}-1}}\left\{\prod_{\tau=g+m}^{\bar{g}-1}\Delta_\tau\right\}\right]\\
&= -\beta\eta(y-b)\left[\frac{1}{\Delta_{g+m}}\frac{\partial p_{g+m}}{\partial \bar{k}_g}\prod_{\tau=g+m}^{g+m}\Delta_\tau\right.\\
&\quad + \left(\frac{1}{\Delta_{g+m}}\frac{\partial p_{g+m}}{\partial \bar{k}_g} + \frac{1}{\Delta_{g+m+1}}\frac{\partial p_{g+m+1}}{\partial \bar{k}_g}\right)\prod_{\tau=g+m}^{g+m+1}\Delta_\tau\\
&\quad + \left(\frac{1}{\Delta_{g+m}}\frac{\partial p_{g+m}}{\partial \bar{k}_g} + \frac{1}{\Delta_{g+m+1}}\frac{\partial p_{g+m+1}}{\partial \bar{k}_g} + \frac{1}{\Delta_{g+m+2}}\frac{\partial p_{g+m+2}}{\partial \bar{k}_g}\right)\prod_{\tau=g+m}^{g+m+2}\Delta_\tau\\
&\quad \vdots\\
&\quad \left. + \left(\sum_{\tau=g+m}^{\bar{g}-1}\frac{1}{\Delta_\tau}\frac{\partial p_\tau}{\partial \bar{k}_g}\right)\prod_{\tau=g+m}^{\bar{g}-1}\Delta_\tau\right].
\end{aligned}
$$

Thus, since $\partial P_g/\partial u_g < 0$,

$$
\begin{aligned}
\bar{k}_g &\lesseqqgtr \hat{k}_g\\
\Longrightarrow \frac{\partial p_g}{\partial \bar{k}_g} &\gtreqqless 0\\
\Longrightarrow \frac{\partial u_{g+1}}{\partial \bar{k}_g} &\lesseqqgtr 0\\
\Longrightarrow \frac{\partial p_{g+1}}{\partial \bar{k}_g} &\gtreqqless 0
\end{aligned}
$$

$$\vdots$$

$$\implies \frac{\partial u_{g+m}}{\partial \bar{k}_g} \lesseqgtr 0$$

$$\implies \frac{\partial p_{g+m}}{\partial \bar{k}_g} \gtreqless 0$$

$$\implies \frac{\partial S_{g+m}}{\partial \bar{k}_g} \lesseqgtr 0.$$

D Construction of the number of friendships

We use *International Social Survey Programme (ISSP): Social Networks and Social Resources*.[9] In the questionnaire, a respondent chooses how many people she has contact with on a typical weekday from options that include "0-4," "5-9," "10-19," "20-49," "50-99," "100 or more," "Can't choose," and "No answer." We exclude samples answering the last two, and set the middle value of each range to a representative value(e.g. 2 for range "0-4"). We set it 100 if the answer is "100 or more." Lastly, we take the average of the value by each age classes and show it in Fig. 4.

E Spillover effects from young and old age

We check here the spillover effects from other generations than middle age not discussed in Sec. 4.4. Fig. 8 and 9 are respectively spillover effects by increasing in connectivity in the social network of 1st quarter of age 20 and of 3rd quarter of age 64. In both cases, the network connectivity increase by about 10% by the same procedure in Sec. 4.4.

In Fig. 8, young workers incur relatively large penalties despite the increase in connectivity. When the network connectivity expands in the young generation, Beveridge curves of middle and old generations also shift. Thus, the probability of referrals increases for all generations. It leads to additional job-worker matches in many generations, and to a decrease in the equilibrium vacancy rate. However, young generations have still high unemployment rates and cannot use referrals well. As a result, the benefits do not go to young generations, but to middle and old generations.

In Fig. 9, by contrast, all generations can benefit from the increase in network connectivity. In this case, the Beveridge curve of the 3rd quarter of age 64 shifts lower although these curves of other generations do not. A part of new entrant jobs allocates to the generation in which network connectivity increase by referrals while the other part goes to the labor market. It makes the tightness of the labor market loose and improves the job arrival rate in the labor market. Consequently, all generations benefit from the change.

[9] `https://search.gesis.org/research_data/ZA6980`, (access in July 2022)

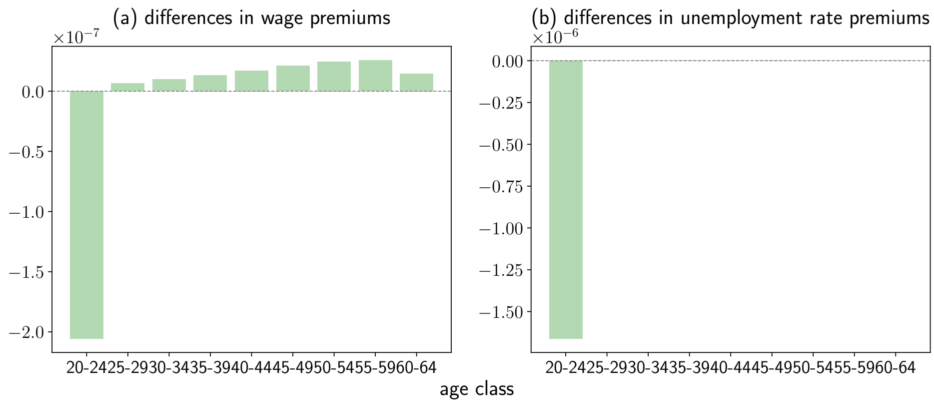

Fig. 8. Spillover effects by increment in the number of friends at the 1st quarter of age 20. Bars show changes in the structural effects.

Fig. 9. Spillover effects by increment in the number of friends at the 3rd quarter of age 64. Bars show changes in the structural effects.