Discussion Paper Series
RIEB
Kobe University

DP2022-28

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Revised November 21, 2022

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November 2022

Abstract

We study how tax policy affects economic growth through entrepreneurs' choice of commercialization mode. Introducing both *heterogeneous* quality jumps and a *leapfrog* versus *sell* choice into the quality-ladders model, we show that entrepreneurs use high-quality innovations to leapfrog incumbent firms and become new market leaders, but sell low quality ones to incumbents. Tax incentives that promote leapfrogging slow the rate of innovation. A numerical analysis concludes that subsidies to product design improve welfare. Corporate taxes, capital gains taxes, and subsidies to market entry all harm welfare.

Key Words: Innovation-based growth, heterogenous quality improvements, innovation sales, corporate tax, capital gains tax, market entry subsidy, product design subsidy

***JEL* Classifications**: O31; O33; O43

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Acknowledgements: The authors are grateful for helpful comments from participants of workshops at China Agricultural University, Dongnan University, Kobe University and Nankai University. They also acknowledge financial support from JSPS through Grants-in-Aid for Scientific Research: #19K01638, #19K01606 and #19H01484. All remaining errors are their own.

1 Introduction

In the innovation sector, "the-winner-taking-all" is a common phenomenon. In other words, many innovations are not directly commercialized as final products sold to consumers, simply because they are too small and trivial. Instead, they are sold to the large incumbent firms which become the winners, while the small innovators exit with the money, and possibly start new rounds of innovation. Sufficiently large innovations, however, enable innovators to leapfrog incumbents, forcing them to exit instead. Indeed, there are abundant stories and stylized facts documenting the entrepreneur's leapfrog/sell choice, especially in the high-tech industries.[1]

A key question then arises regarding which mode of commercialization best promotes economic growth and welfare. And, more practically, how policy makers should shape fiscal incentives for entrepreneurs investing in research and development (R&D)? In this paper, we tackle these issues by investigating the entrepreneur's choice between using a new innovation to start a new firm, thereby displacing (i.e., leapfrogging) the incumbent market leader, or selling the new innovation to the incumbent firm.

Specifically, we set up a framework for innovation-based endogenous growth in which entrepreneurs invest in the development of new product designs that generate *heterogenous* quality improvements over existing product lines. In each industry, we characterize an equilibrium pattern of entrepreneurial behavior, whereby, once the quality of a new design is known, the entrepreneur makes a leapfrog versus sell decision. High-quality designs are used to leapfrog incumbent leaders, while low-quality designs are sold to the incumbents.

The long-run evolution of product quality in the average industry can be characterized with two variables: a threshold design quality for which entrepreneurs are indifferent between leapfrogging vs. selling, and the product quality of the incumbent leader. These variables are intrinsically linked as a lower threshold expands the share of designs used to leapfrog, reducing the frequency

[1] Large firms such as Google, Facebook and Microsoft buy thousands of applications to improve on their own platforms. The relationship between Google and Yahoo is especially interesting. In 1998, Google's Larry Page and Sergey Brin reportedly approached Yahoo to sell their PageRank system for as little as $1 million, because they wanted to focus on their studies at Stanford University, but Yahoo did not value the system much and declined.

with which incumbents update their product lines by purchasing new designs.

The interaction between the entrepreneur's leapfrog/sell choice and the rate of innovation depends on how the labor and investment markets adjust. In the labor market, an increase in the threshold quality lowers the average cost of introducing a new product as incumbents are more productive at commercializing new designs; and in the investment market, the increases in the threshold quality and average incumbent quality improve the threshold profits from leapfrogging over selling, raising the return to investment in product development. Together, these channels generate a positive relationship between the threshold quality required for leapfrogging and the innovation rate.

We then use the framework to study a number of policy initiatives: corporate taxes on operating profits, capital gains taxes on the sale of new designs, subsidies to entrepreneurs developing new product designs, and subsidies to entrepreneurs entering production with new designs. The direct effects of these policies are standard, with lower taxation and higher subsidy rates promoting investment in product development. However, because entrepreneurs face a choice between leapfrogging vs. selling, the overall policy impacts on the rate of innovation are not obvious. Indeed, we find that when firms receive tax deductions for R&D costs, the relationship between tax policy and investment in innovation depends on the balance between opposing profit and tax exemption effects. Surprisingly, tax incentives that increase the likelihood of leapfrogging rather than selling, tend to slow the rate of innovation, through an indirect effect–the positive link between the threshold quality required to leapfrog and the innovation rate. Because either the direct or the indirect channel may dominate when their directions do not align, the overall impact of each policy on the rate of innovation is generally ambiguous. For a benchmark parameter set, however, numerical analysis suggests that subsidies to product design increase investment in innovation, raising the rate of economic growth and improving welfare. Corporate taxes on operating profits, capital gains taxes on the sale of product designs, and subsidies to market entry slow the rate of innovation, thereby reducing the rate of economic growth and harming welfare.

In the literature, Akcigit et al. (2016) study issues similar to ours, but using a Lancaster varieties' approach, where they show that incumbent firms keep or buy innovations of varieties close to their existing business but sell off faraway ones. Akcigit and Kerr (2018) examine heterogenous quality improvements, but they assume incumbents invest in internal innovations to improve their existing products, while new entrants invest in external innovations to acquire new product lines. Cunningham et al. (2020) investigate "killer acquistions" in the pharmaceutical industry and find that acquired drug projects are less likely to be developed when they overlap with the acquirer's existing product portfolio, especially when the acquirer's market power is large because of weak competition or distant patent expiration. In contrast, the decision to sell or leapfrog in our model depends on the endogenous significance of the innovation rather than on exogenous technical closeness and overlap.

Also, in Klette and Kortum (2004), innovation is concentrated in incumbents and more productive incumbents innovate more, whereas in Dinopoulos and Unel (2011), the variety expansion approach is used to show that high-quality firms select to export while low-quality ones sell domestically. In earlier models of endogenous growth, such as Segerstrom et al. (1990), Grossman and Helpman (1991) and Agion and Howitt (1992), quality improvements are constant. While recent studies have extended the quality-ladders framework to consider heterogenous quality improvements (Minniti et al., 2013; Acemoglu and Cao, 2015; Chu et al., 2017; Parello, 2018; Iwaisako and Ohki, 2019), they do not consider the entrepreneur's option to sell to an incumbent firm.

Finally, our framework is related to a literature that adopts a game-theoretic approach to study how acquisition and licensing between startups and incumbent firms in the commercialization of new technologies affects investment in R&D (Gans and Stern, 2000; Henkel et al., 2015). In particular, Haufler et al. (2014) investigate how tax incentives affect the entrepreneur's choice between market entry and selling to an incumbent, and show that entrepreneurs may choose R&D projects with too little risks when entering the market, and hence tax incentives aimed at promoting market entry may lead to welfare losses. In contrast, our macroeconomic model explores the links between the entrepreneur's choice of commercialization mode, economic growth, and welfare, within a dynamic and quantitative rather than game-theoretic framework.

The paper proceeds as follows. In the next section, we introduce our framework in which entrepreneurs choose between leapfrogging and selling based on heterogenous quality improvements. Then, Section 3 provides a characterization of the long-run equilibrium. In Section 4, we investigate how changes in economic policy affect the entrepreneur's leapfrog/sell choice, economic growth and social welfare. Section 5 concludes. All mathematical derivations are relegated to the appendices.

2 Model

We analyze the choice between leapfrogging and selling in a quality ladders framework of innovation-based endogenous growth (e.g., Grossman and Helpman, 1991). A unit mass of industries employs labor in the production of final goods for household consumption. Entry into a representative industry requires a new product design that introduces a quality improvement to the existing product line. New designs are created in a competitive innovation sector where entrepreneurs develop designs with heterogeneous quality improvements. Depending on the size of the quality improvement, entrepreneurs choose between either using the designs to leapfrog incumbent market leaders to become leaders themselves (*leapfrogging*), or selling the designs to incumbent leaders (*selling*). Labor is the sole factor of production.

2.1 Household Preferences

The demand side of the economy consists of a fixed population of dynastic households that maximize utility over an infinite time horizon. The intertemporal preferences of a household are described by

$$U = \int_0^\infty e^{-\rho t} \ln D(t) dt, \tag{1}$$

where ρ is the subjective discount rate. The instantaneous utility derived from consumption follows a quality-augmented index with unitary elasticity of substitution across the products produced within a unit mass of industries indexed by $i \in [0,1]$:

$$\ln D(t) = \int_0^1 \ln \left(\sum_{j(i)} \prod_{h=1}^{j(i)} \lambda(h) q(i,j;t) \right) di, \tag{2}$$

where $q(i,j)$ is the quantity demanded of product vintage j, with $j(i)$ indicating the number of quality innovations (product generations) that have been introduced to date in industry i. The product quality of the jth generation in industry i is captured by $\prod_{h=1}^{j(i)} \lambda(h)$, with $\lambda(h) > 1$ measuring the heterogeneous quality improvement associated with each generation.

With each household supplying one unit of labor inelastically, intertemporal optimization requires that the representative household select an optimal path for expenditure with the objective of maximizing lifetime utility (1) subject to the flow budget constraint:

$$\dot{A}(t) = r(t)A(t) + w(t) - E(t) - T, \tag{3}$$

where A is household asset wealth, r is the interest rate, w is the wage rate, E is household expenditure, T is a lump-sum tax on household income, and a dot over a variable denotes time differentiation. The optimal expenditure path is described by the Euler condition:

$$\frac{\dot{E}(t)}{E(t)} = r(t) - \rho. \tag{4}$$

Henceforth, setting household expenditure as the model numeraire, $E = 1$, we have $r = \rho$ at all moments in time. Time notation is suppressed hereafter for simplification.

Given the unitary elasticity of substitution across industries, at each moment in time the household sets an even allocation of expenditure across the unit mass of product lines. The household then purchases only the product with the lowest quality-adjusted price $p_x(i,j)/\prod_{h=1}^{j(i)} \lambda(h)$ from each product line, with $p_x(i,j)$ being the product price, and sets its demand for all remaining products to zero. Defining the state-of-the-art as the product with the lowest quality-adjusted price and denoting it by $j(i) = J(i)$ in industry i, the total demand for product $j(i)$ is

$$x(i,j) = \begin{cases} q(i,j)L = \frac{L}{p_x(i,j)} & for \quad j(i) = J(i), \\ 0 & otherwise, \end{cases} \tag{5}$$

where we have aggregated across the total population of households L and used $E = 1$. Whenever the consumer is indifferent between two products with the same quality-adjusted price, we assume that he/she always buys the higher quality (newer generation) product.

2.2 Production

The production sector features firms that compete via Bertrand pricing strategies. Each firm holds a patent protecting its right as the sole producer of a given generation of the industry's product line. All firms have access to the same production technology

$$x(i) = L_X(i), \tag{6}$$

where $x(i)$ and $L_X(i)$ are firm-level output and labor employment in production respectively. The common marginal cost of production across firms is the wage rate, w.

The optimal pricing strategy of the industry leader, who with a patent for the state-of-the-art product design of the highest quality level, is to set the quality-adjusted price just equal to the closest rival firm's marginal cost, with the aim of forcing the rival out of the market. Given the production technology (6), this limit price is $p_x(i) = w\lambda(i)$, where $\lambda(i)$ is the difference in quality between the industry leader over the closest rival firm in industry i.

Following Grossman and Helpman (1991), we rewrite $\delta(i) \equiv 1/\lambda(i) \in (0,1)$ as the *inverse-quality-gap* between the state-of-the-art product and the closest vintage product. In other words, the lower the value of δ, the greater the difference in quality between the products of the market leader and the closest rival firm. Applying the limit pricing rule, we can write the gross profit of the market leader as

$$\pi(i) = p_x(i)x(i) - wL_X(i) = (1 - \delta(i))\, L, \tag{7}$$

where demand condition (5) has been used. An increase in the quality improvement λ (i.e., a decrease in δ) clearly raises the profit of the market leader by expanding the price-cost markup.

2.3 Innovation

New product designs are created by entrepreneurs working independently in the competitive innovation sector. Each new product design adds a *heterogeneous* quality improvement to the current state-of-the-art in its respective industry, while adopting all of the quality improvements that have been introduced to date. At the completion of the design process, the entrepreneur applies for a patent to protect the design from imitation. Before issuing a patent, however, the patent office reviews the application to ensure that the design represents a sufficiently large improvement over the current state-of-the-art in the industry. If awarded, the patent protects the product design indefinitely. Entrepreneurs that have successfully obtained a patent choose whether to sell the patented design to the incumbent market leader, or even better, to use the design to leapfrog the incumbent firm to become the new market leader. Further, because leapfrogging requires an additional fixed cost for the entrepreneur, the sell/leapfrog choice is intrinsically linked with the size of the quality improvement generated by the new design.

More formally, an entrepreneur in industry i employs $\iota(i)\alpha aL$ units of labor at each moment in time to generate an instantaneous probability $\iota(i)$ of creating a new product design, where $a > 0$ is a constant parameter, $\alpha \in (0,1]$ is the share of product development in innovation costs, and L captures a dilution effect in which the product development cost increases in market size.[2] The inverse-quality gap (δ) is randomly drawn from a continuous, strictly increasing probability distribution function $F(\delta)$, with density $f(\delta)$ and support $[0,1]$. The strength of patent regulation is captured by $\delta^* \in (0,1)$, with a patent application rejected for $\delta > \delta^*$, when the patent examiner judges that the quality improvement associated with the product design is not sufficient to be

[2]See Laincz and Peretto (2006) for a discussion of the dilution effect.

granted a patent. Therefore, an entrepreneur employing $\iota(i)\alpha aL$ units of labor in industry i invents a patentable product design with an instantaneous probability of $\iota(i)F(\delta^*)dt$.

Once a patent has been successfully obtained, the entrepreneur chooses between selling and leapfrogging. If the entrepreneur decides to leapfrog the incumbent, an additional $(1-\alpha)aL$ units of labor are required to market the product. Alternatively, if the entrepreneur decides to sell the design, the incumbent firm employs an additional $(1-\xi)(1-\alpha)aL$ units of labor to market the product, where $\xi \in (0,1)$ regulates the degree to which the incumbent is more proficient at commercializing new product designs, given its existing production capacity and supply networks.

In addition to the patent regulation (δ^*) introduced above, the government implements the following policies: (i) a corporate tax rate of $\tau_\pi \in (0,1)$ on per-period operating profits; (ii) a capital gains tax of $\tau_V \in (0,1)$ on the sale of designs; (iii) a subsidy of $s_E \in (0,1)$ for initial product design; and (iv) a subsidy of $s_M \in (0,1)$ for market entry through leapfrogging. All R&D investment costs are fully deductible from taxable income. We assume that firms survive in the market long enough to use the full tax deduction by carrying forward investment costs. In the sections below, we characterize each of the entrepreneur's choices and derive a threshold inverse-quality level that determines whether an entrepreneur chooses selling or leapfrogging. We then consider how policy changes affect the entrepreneur's choice and study the implications for growth and welfare.

2.4 Leapfrogging the Incumbent

When a new product design is used to leapfrog the incumbent, creating a new market leader, the design generates an after-tax profit stream with a present value equal to

$$V_M(i) = \int_0^\infty e^{-\int_0^t (\rho + \iota(i,t')F(\delta^*))dt'} \pi_M(i)dt, \tag{8}$$

where the subscript M denotes variables associated with leapfrogging, and the gross operating profit is $\pi_M(i) = (1-\delta(i))L$. The discount factor is $\rho + \iota(i,t')F(\delta^*)$ with $\iota(i,t')F(\delta^*)$ capturing the risk of potentially losing the market to a rival firm entering with a patentable superior product design at a future date t'.

Accounting for the additional labor cost incurred when marketing the product, the profit with a new design of inverse-quality $\delta(i)$ is

$$\Pi_M(i) = \begin{cases} (1-\tau_\pi)V_M(i) - C_M waL, & for \quad \delta(i) \leq \delta_0, \\ 0, & for \quad \delta(i) > \delta_0, \end{cases} \tag{9}$$

where we summarize the cost parameters associated with leapfrogging using $C_M \equiv (1-\tau_\pi)((1-s_E)\alpha + (1-s_M)(1-\alpha))$, with $s_E \in (0,1)$ and $s_M \in (0,1)$ denoting the government subsidies to initial product design and market entry through leapfrogging. The profit associated with leapfrogging is illustrated by the $\Pi_M(i)$ curve in Figure 1. Importantly, the leapfrogging value of a design is increasing in product quality (i.e., $\partial V_M(i)/\partial\delta(i) < 0$). As such, the entrepreneur can only use the design to leapfrog the incumbent if $\delta \leq \delta_0$, where we define δ_0 as the threshold inverse-quality gap at which $\Pi_M(i) = 0$, signaling that the entrepreneur is indifferent between entering or not entering the market through leapfrogging. If $\delta(i) > \delta_0$, because leapfrogging is unprofitable, the entrepreneur will not enter the market, but may instead have an incentive to sell the design to the incumbent market leader, to which we turn next.

2.5 Selling to the Incumbent

When an entrepreneur sells a new product design to the incumbent market leader, the incumbent's product quality improves from $\lambda_I(i) \equiv 1/\delta_I(i)$ to $\lambda_I(i)\lambda(i) \equiv 1/(\delta_I(i)\delta(i))$, allowing the incumbent's product price to rise to $p_x(i) = w/(\delta_I(i)\delta(i))$ after the design has been adopted.

Prior to purchasing the design, the incumbent's gross value is

$$V_I(i) = \int_0^{\infty} e^{-\int_0^t (\rho + \iota(i,t')F(\delta^*))dt'} \pi_I(i)dt, \tag{10}$$

where the subscript I denotes variables associated with incumbent firms, and referencing (7) we have $\pi_I(i) = (1 - \delta_I(i))L$. But after purchasing and adopting the new design, the incumbent's gross value becomes

$$V_S(i) = \int_0^{\infty} e^{-\int_0^t (\rho + \iota(i,t')F(\delta^*))dt'} \pi_S(i)dt, \tag{11}$$

where the subscript S indicates variables associated with selling, and $\pi_S(i) = (1 - \delta_I(i)\delta(i))L$.

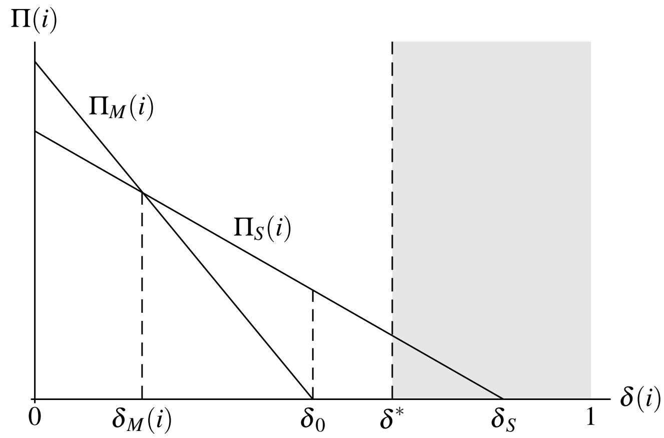
Also, the incumbent firm's per-period operating profit rises by $\pi_S(i) - \pi_I(i) = \delta_I(i)(1 - \delta(i))L > 0$, and firms with a higher $\delta_I(i)$ benefit more from the adoption of a new design purchase. Denoting the design price by $P_V(i)$, the change in the incumbent's net value after purchasing a new design is $\Pi_B(i) = (1 - \tau_\pi)(V_S(i) - V_I(i) - P_V(i) - (1 - \xi)(1 - \alpha)waL)$. We assume that the entrepreneur makes a take-it-or-leave-it offer that just sets the change in the net value of the incumbent to zero: $P_V(i) = V_S(i) - V_I(i) - (1 - \xi)(1 - \alpha)waL$.

Then entrepreneur's net profit from selling the design is

$$\Pi_S(i) = (1 - \tau_V)(V_S(i) - V_I(i)) - C_S waL, \tag{12}$$

where we summarize the cost parameters associated with selling using $C_S \equiv (1 - \tau_V)((1 - s_E)\alpha + (1 - \xi)(1 - \alpha))$. The net profit from selling is an increasing function of product quality (i.e., $\partial V_S(i)/\partial \delta(i) < 0$), and becomes negative if the quality of the design is too low ($\delta(i) > \delta_S$), as illustrated by the $\Pi_S(i)$ curve in Figure 1. Importantly, low incumbent product quality (i.e., a higher $\delta_I(i)$) is associated with a higher design price, and greater net profit from selling (i.e., $\partial(V_S(i) - V_I(i))/\partial \delta_I(i) > 0$). Thus, a rise (fall) in δ_I induces an upward (downward) shift in the $\Pi_S(i)$ curve.

Figure 1: Quality Thresholds



2.6 Leapfrogging vs. Selling

Having separately described the options of the entrepreneur to either leapfrog or sell to the incumbent, we now consider the entrepreneur's equilibrium choice. To facilitate the analysis, we first define a threshold inverse-quality gap $\delta(i) = \delta_M(i)$ that equates the profits associated with leapfrogging and selling ($\Pi_M(\delta_M) = \Pi_S(\delta_M)$), as shown by the intersection of the $\Pi_M(i)$ and $\Pi_S(i)$ curves in Figure 1. We find that $\Pi_M(i) > \Pi_S(i)$, for $\delta(i) < \delta_M(i)$, and the entrepreneur leapfrogs. Alternatively, $\Pi_M(i) < \Pi_S(i)$ for $\delta(i) > \delta_M(i)$, and the entrepreneur sells the design. Thus, referencing the effects of changes in $\delta_I(i)$ on the profit from selling $\Pi_S(i)$, we straightforwardly obtain

Proposition 1 *There are three cases for the entrepreneur's leapfrogging vs. selling choice:*
(i) For $\delta_I(i) > (1-\tau_\pi)/(1-\tau_V)$, we have $\delta_M(i) = 0$, and the entrepreneur sells the product design to the incumbent firm for all inverse-quality gaps $\delta(i) \in (0, \delta^*)$.
(ii) For $(1-\tau_\pi)/(1-\tau_V) > \delta_I(i) > ((1-s_E)\alpha + (1-\xi)(1-\alpha))/((1-s_E)\alpha + (1-s_M)(1-\alpha))$, we have $\delta_M(i) \in (0, \delta^*)$, and the entrepreneur leapfrogs the incumbent for $\delta(i) \in (0, \delta_M(i))$, but sells the design for $\delta(i) \in (\delta_M(i), \delta^*)$.
(iii) For $((1-s_E)\alpha + (1-\xi)(1-\alpha))/((1-s_E)\alpha + (1-s_M)(1-\alpha)) > \delta_I(i)$, we have $\delta_M(i) = \delta^*$, and the entrepreneur leapfrogs the incumbent for all inverse-quality gaps $\delta(i) \in (0, \delta^*)$.
Proof: See Appendix A.

The entrepreneur's leapfrog/sell choice is closely linked with the inverse-quality gap of the incumbent through the effect of changes in $\delta_I(i)$ on the net profit from selling. If the incumbent's product design is low-quality, $\delta_I(i) > (1-\tau_\pi)/(1-\tau_V)$, the entrepreneur always sells its design to the incumbent, because the net profit is greater from selling than from leapfrogging for all values of $\delta(i) \in (0, \delta^*)$. Alternatively, if the incumbent's design is high-quality, $((1-s_E)\alpha + (1-\xi)(1-\alpha))/((1-s_E)\alpha + (1-s_M)(1-\alpha)) > \delta_I(i)$, the entrepreneur leapfrogs because the net profit is higher from leapfrogging than from selling.

In the intermediate case presented in Figure 1, where $(1-\tau_\pi)/(1-\tau_V) > \delta_I(i) > ((1-s_E)\alpha + (1-\xi)(1-\alpha))/((1-s_E)\alpha + (1-s_M)(1-\alpha))$, the entrepreneur's leapfrog/sell choice depends on the quality of its new product design. Leapfrogging arises when $\delta(i) \in (0, \delta_M(i))$, but selling occurs when $\delta(i) \in (\delta_M(i), \delta^*)$. We take the time derivative of $\Pi_M(\delta_M) = \Pi_S(\delta_M)$ to derive an investment condition that describes the leapfrog/sell decision:

$$\rho + \iota F(\delta^*) = \frac{(1-\tau_\pi)\pi_M(\delta_M) - (1-\tau_V)(\pi_S(\delta_M) - \pi_I)}{(C_M - C_S)waL} + \frac{\dot{w}}{w}. \tag{13}$$

This no-arbitrage condition equates the threshold relative return to leapfrogging over selling with the risk-adjusted interest rate for the overall economy. Consistent with Proposition 1, the existence of $\delta_M(i) \in (0,1)$ requires $1-\tau_\pi > (1-\tau_V)\delta_I$, since no level of design quality generates a positive return to leapfrogging if $(1-\tau_\pi)\pi_M(\delta_M) - (1-\tau_V)(\pi_S(\delta_M) - \pi_I) = ((1-\tau_\pi) - (1-\tau_V)\delta_I)\pi_M(\delta) < 0$.

2.7 Free Entry

Expected profits drive the entry and exit of entrepreneurs in innovation. Referencing Figure 1, the expected profit associated with a new product design in industry i is

$$\Pi_E(i) = \int_0^{\delta_M(i)} \Pi_M(\delta) dF(\delta) + \int_{\delta_M(i)}^{\delta^*} \Pi_S(\delta) dF(\delta) - \int_{\delta^*}^{1} (1-s_E)\alpha waL dF(\delta), \tag{14}$$

where the first term on the RHS is the expected profit from leapfrogging, the second term is that from selling the design, and the third term is the expected loss from market exit when the quality of the design is not sufficient to obtain a patent.

Free entry and exit into R&D drives the expected profit of entrepreneurs to zero, $\Pi_E(i) = 0$. Setting (14) to zero and taking the time derivative of the result yields the following no-arbitrage condition for investment in the development of a new product design in industry i:

$$\rho + \iota F(\delta^*) = \frac{\pi_E(\delta_M, \delta_I)}{C_E(\delta_M) w a L} + \frac{\dot{w}}{w}, \tag{15}$$

where we have used (8), (9), (10), (11), and (12). The expected per-period profit, net of taxes and conditional on the design being patentable, is $\pi_E(\delta_M, \delta_I) \equiv (1-\tau_\pi) \int_0^{\delta_M} \pi_M dF(\delta) + (1-\tau_V) \int_{\delta_M}^{\delta^*} (\pi_S - \pi_I) dF(\delta)$, and the expected cost associated with product development is described by $C_E(\delta_M) \equiv (1-s_E)\alpha(1-F(\delta^*)) + C_M F(\delta_M) + C_S(F(\delta^*) - F(\delta_M))$. As usual, the asset condition (15) requires that the expected after-tax return to investment equal the risk-free interest rate plus a risk premium to account for the threat of future market entry. The return consists of the expected dividends from leapfrogging and selling, in addition to expected capital gains.[3]

2.8 Evolution of Incumbent Product Quality

We now describe the evolution of the quality gap between the incumbent market leader and its nearest rival firm. The dynamics of δ_I are driven by two mechanisms. First, as the entrant replaces the incumbent when $\delta < \delta_M(i)$, the expected change in the inverse-quality gap that results from leapfrogging is $\iota(i) \int_0^{\delta_M(i)} (\delta - \delta_I(i)) dF(\delta)$; Second, if the product design is sold to the incumbent under $\delta > \delta_M(i)$, the expected change in the inverse-quality gap is $\iota(i)\delta_I(i) \int_{\delta_M(i)}^{\delta^*} (\delta - 1) dF(\delta)$. Combining the above delivers the expected change in the inverse-

[3]In our analysis, capital gains taxes are not applied to adjustments in stock prices ($\dot{w}/w$) in secondary markets. This assumption will not influence our policy analysis as we find that stock prices are constant in the steady state of the model. See also Peretto (2011) for a study of the effects of capital gains taxation on stock price fluctuations in an endogenous growth framework.

quality gap,

$$\dot{\delta}_I(i) = \left(\int_0^{\delta_M(i)} \delta dF(\delta) + \delta_I(i) \int_{\delta_M(i)}^{\delta^*} \delta dF(\delta) - \delta_I(i) \int_0^{\delta^*} dF(\delta) \right) \iota(i), \tag{16}$$

with the first term indicating the effect of leapfrogging, the second term the effect of selling, and the third term the exit of the industry leader.

2.9 Labor Market Equilibrium

At each moment in time, households inelastically supply a total of L units of labor to production and innovation. First, combining the demand condition (5) and the limit price $p_x = w/\delta_I$, employment in production is $L_X = \delta_I L/w$ for the average industry, where $\delta_I = \int_0^1 \delta_I(i)di$ is the average inverse-quality gap associated with the average incumbent firm.

Second, in the average industry, entrepreneurs employ $\iota \alpha a L$ units of labor in product development and $\iota(1-\alpha)aLF(\delta_M)$ units of labor in market entry, conditional on leapfrogging, where δ_M denotes the leapfrog/sell quality threshold associated with the average industry. Meanwhile, incumbent firms employ $\iota(1-\alpha)(1-\xi)aL(F(\delta^*)-F(\delta_M))aL$ units of labor in market entry, conditional on purchasing new product designs. Combining these yields the average labor demand from R&D: $L_R = \iota R(\delta_M)aL$, where $R(\delta_M) \equiv \alpha + (1-\alpha)(F(\delta_M) + (1-\xi)(F(\delta^*) - F(\delta_M)))$ describes the productivity of labor in innovation.

Summing up the labor demands from production and R&D, we derive the average arrival rate for new product designs that clears the labor market ($L = L_X + L_R$):

$$\iota_L = \frac{1-\delta_I/w}{R(\delta_M)a}. \tag{17}$$

Thus, the average innovation rate is decreasing in the fixed cost of innovation (a) and the share of product development in innovation costs (α), but increasing in the strength of patent regulation (δ^*) and incumbent productivity in market entry (ξ).

2.10 Government

We set a relatively passive role for the government in adjusting the lump-sum tax on household income (T), to balance the fiscal budget at all moments in time; that is,

$$\iota s_E \alpha waL + \iota s_M (1-\alpha) waLF(\delta_M) = \tau_\pi \pi_I(\delta_I) + \iota \tau_V \int_{\delta_M}^{\delta^*} P_V(\delta) dF(\delta) + TL. \qquad (18)$$

The LHS represents the cost of subsidies, and the RHS consists of the expected tax revenue. We assume that the government budget is supported fully by tax revenue, and that the government is unable to issue bonds; i.e., budget deficits are not allowed.

3 Long-Run Equilibrium

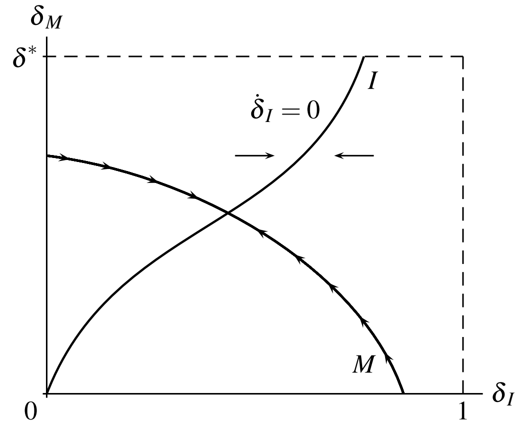
The long-run equilibrium of the model features a constant allocation of labor between innovation and production, presupposing constant values for average incumbent quality, the innovation rate, and the wage rate. Interestingly, the model can be reduced to two autonomous dynamic systems, with the first governing the dynamics of product quality, and the second the dynamics of innovation and wages.[4] In the next subsection, we reduce the model to a steady-state system with two conditions that implicitly determine the average incumbent inverse-quality gap (δ_I) and the threshold inverse-quality gap associated with the average industry (δ_M). Then, in the following subsection, the steady-state innovation rate (ι) and wage rate (w) are resolved through equilibrium in the investment and labor markets.

3.1 Steady-State Innovation for Sale and Incumbent Firm Quality

Turning to the product quality gaps, the first condition captures combinations of the average incumbent inverse-quality gap and the threshold inverse-quality gap where the free-entry condition

[4] Alternatively, the dynamics of the model can be studied within a single dynamic system that describes the evolution of the average incumbent inverse-quality gap (δ_I), the threshold inverse-quality gap (δ_M), and the wage rate (w). The consideration of two independent dynamic systems, however, allows for a more intuitive explanation of the comparative static results associated with the policy analysis presented in Section 4.

Figure 2: Steady-state Product Quality Gaps



This stylized figure can be reproduced numerically setting $f(\delta) = k\delta^{k-1}$ with the following parameter values: a = 1.0625, α = 0.671, ξ = 0.95, ρ = 0.02, k = 7.3, τ_π = 0.21, τ_V = 0.20, s_E = 0.05, s_M = 0.1, and δ^* = 0.895. This parameter set generates δ_I = 0.758, δ_M = 0.877, δ_0 = 0.908, and δ_S = 0.917.

for investment in a new product design and the threshold investment condition for leapfrogging are simultaneously satisfied. Combining (13) and (15) gives the following investment condition:

$$\frac{\pi_E(\delta_M, \delta_I)}{C_E(\delta_M)} = \frac{(1-\tau_\pi)\pi_M(\delta_M) - (1-\tau_V)(\pi_S(\delta_M, \delta_I) - \pi_I(\delta_I))}{C_M - C_S}. \tag{19}$$

The LHS describes the return to product development as the ratio of the expected profit stream from a new product design to the expected cost of product development. The RHS shows the relative return to leapfrogging over selling at the threshold inverse-quality gap (δ_M), as the ratio of the per-period profit differential to the cost differential associated with the entrepreneur's leapfrog/sell choice. The M-curve depicts (19) in Figure 2, and sets δ_M to equate the return to product development with the relative return to leapfrogging over selling, for a given average incumbent inverse-quality gap (δ_I). Thus, we find that equilibrium in the investment market requires that the economy lie on the M-curve at all moments in time. Note that (19) is satisfied independently of the innovation rate (ι) and the wage rate (w).

The second condition is derived by setting the dynamics of incumbent quality (16) to zero for the average industry ($\dot{\delta}_I = 0$), and reorganizing the result:

$$\int_0^{\delta_M} \pi_M(\delta) dF(\delta) + \int_{\delta_M}^{\delta^*} \pi_S(\delta, \delta_I) dF(\delta) = \int_0^{\delta^*} \pi_I(\delta_I) dF(\delta). \tag{20}$$

This expression implicitly determines the steady-state inverse-quality gap for the average incumbent firm (δ_I). In the long-run equilibrium, incumbent operating profit converges to the expected operating profit associated with a patentable new product design. The steady-state locus captures the positive relationship between δ_M and δ_I, as depicted by the I-curve in Figure 2. Intuitively, a decrease in the share of product designs sold to incumbent firms (i.e., a rise in δ_M) reduces the frequency with which incumbent products are updated, leading to a fall in average incumbent product quality (i.e., a rise in δ_I).

The system includes one differential equation (20) and one side condition (19). By investigating the local dynamics around the steady state described by the intersection of the M-curve and I-curve, we have:

Lemma 1 *The steady-state equilibrium with positive shares of product designs for both selling and leapfrogging is stable.*

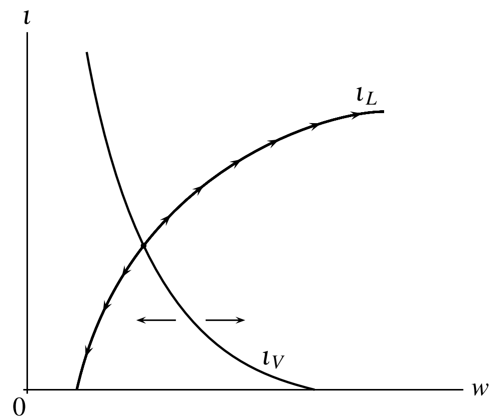
Proof: See Appendix B.

The dynamic system can be reduced to a single differential equation for the average incumbent quality gap ($\dot{\delta}_I$). As shown in Figure 2, for values to the left (right) of the I-curve, δ_I rises (falls). Accordingly, the slope of the I-curve must be strictly greater than that of the M-curve, to ensure that the economy converges along the M-curve to the long-run equilibrium. In Appendix B, we show that the slope of the M-curve is strictly negative, and that the necessary slope ranking is therefore always satisfied.

3.2 Long-Run Innovation

With the average incumbent quality gap and the threshold quality gap pinned down by (19) and (20), the long-run rate of innovation is determined together with the wage rate through

Figure 3: Long-run Innovation Rate



This stylized figure can be reproduced numerically setting $f(\delta) = k\delta^{k-1}$ with the following parameter values: a = 1.0625, α = 0.671, ξ = 0.95, ρ = 0.02, k = 7.3, τ_π = 0.21, τ_V = 0.20, s_E = 0.05, s_M = 0.1, and δ^* = 0.895. This parameter set generates δ_I = 0.758, δ_M = 0.877, ι = 0.181, and w = 1.022.

equilibrium in the labor and investment markets. On the one hand, the labor market equilibrium is described by (17) with a strictly positive relationship between ι and w, as illustrated by the ι_L-curve in Figure 3. On the other hand, the investment market equilibrium is captured by the no-arbitrage condition in new product designs (15), which can be rewritten in its steady-state form using (19):

$$\iota_V = \frac{(1-\tau_\pi-(1-\tau_V)\delta_I)\pi_M(\delta_M)}{waLF(\delta^*)(C_M-C_S)} - \frac{\rho}{F(\delta^*)}, \tag{21}$$

where we have set $\dot{w} = 0$. This expression captures the strictly negative relationship between the innovation rate and the wage rate in the investment market, as depicted in Figure 3 by the ι_V-curve.

The local dynamics of the system around the steady state (i.e., the intersection of the ι_L-curve and ι_V-curve) are consistent with the standard quality ladders model of innovation-based growth (e.g., Grossman and Helpman, 1991). In Appendix B, we reduce the system to a single differential equation in the wage rate ($\dot{w}$). And, as shown in Figure 3, for values to the left (right) of the ι_V-curve the wage rate is falling (rising). Therefore, at each moment in time, the economy must jump immediately to the steady-state equilibrium for the investment and labor markets, given the current values for the average incumbent inverse-quality gap (δ_I) and the threshold inverse-quality gap (δ_M). The resulting innovation rate is

$$\iota = \frac{(1-\tau_\pi-(1-\tau_V)\delta_I)\pi_M(\delta_M)-\delta_I aL\rho(C_M-C_S)}{(1-\tau_\pi-(1-\tau_V)\delta_I)\pi_M(\delta_M)R(\delta_M)a+\delta_I aLF(\delta^*)(C_M-C_S)}. \tag{22}$$

Clearly, $(1-\tau_\pi-(1-\tau_V)\delta_I)\pi_M(\delta_M) > \delta_I aL\rho(C_M-C_S)$ is required for $\iota \geq 0$. Hereafter, we assume that the long-run equilibrium satisfies the necessary conditions for a positive rate of innovation.

In preparation for the policy analysis of Section 4, we consider how the innovation rate (22) adjusts with changes in the threshold quality gap (δ_M) associated with the leapfrog/sell decision and the average incumbent quality gap (δ_I). The following lemma outlines the relationship that arises between δ_M, δ_I, and ι in the labor and investment markets:

Lemma 1 *Decreases in the average threshold quality gap for leapfrogging (i.e., a rise in δ_M) and in the average incumbent quality gap (i.e., a rise in δ_I) lower the rate of innovation (ι).*

First, in the labor market (17), the average amout of labor required to introduce a new product design $R(\delta_M)aL$ increases with a rise in the share of leapfrogging designs, as incumbent firms require less labor in the marketing of new products. In addition, a fall in the average quality gap of incumbent firms lowers the average product price $p_x = w/\delta_I$, inducing an expansion in production as household demand rises. These two effects result in a lower level of employment in innovation that exhibits as a downward shift in the ι_L-curve in Figure 3.

Second, in the investment market (21), increases in δ_M and δ_I both reduce the threshold profit from leapfrogging over selling $(1-\tau_\pi-(1-\tau_V)\delta_I)\pi_M(\delta_M)$, lowering the threshold relative return to leapfrogging. This in turn decreases investment in innovation, described by a downward shift in the ι_V-curve in Figure 3. From the labor market and investment market channels, decreases in the average threshold quality gap and in the average incumbent quality gap lead to a slower rate of innovation.

3.3 Social Welfare

Due to the random quality improvements of new product designs in each industry and the choice between leapfrogging and selling, a general analysis of welfare is not tractable. As an alternative, we follow Minniti et al. (2013) in assuming that the random quality improvements are governed by a Pareto distribution with a probability density function of $f^{-1}(\lambda) = k/\lambda^{k+1}$, where $k > 1$ is a shape parameter. Then, the random inverse-quality gaps are generated by $f(\delta) = k\delta^{k-1}$.

As households only consume the state-of-the-art product in each industry, instantaneous utility (2) reduces to $\log D(t) = \log(x(t)/L) + I(t)\mathbb{E}[\log\lambda]$, with the first term describing the utility stemming from the average quantity of consumption ($x(t) = \int_0^1 x(t,i)di$), and the second term capturing the utility derived from the average quality of products consumed. Importantly, $I(t) = \int_0^t \iota(t')dt' = \iota t$ measures the expected number of new product designs introduced before time t, under a constant steady-state rate of innovation, and $\mathbb{E}[\log\lambda] = (1 - k\log\delta^*)\delta^{*k}/k$ describes the average size of the quality improvements associated with new product designs.

With constant steady-state employment in production and innovation, the time derivative of instantaneous utility yields the long-run rate of growth as $g \equiv \iota\mathbb{E}[\log\lambda]$, where we have used the fact that only designs satisfying patent regulation update product quality (i.e., $\lambda \geq \lambda^* \equiv 1/\delta^*$). Combining the growth rate with lifetime utility (1), we can calculate steady-state welfare as

$$U = \frac{\log(1 - \iota R(\delta_M)a)}{\rho} + \frac{\iota(1 - k\log\delta^*)\delta^{*k}}{k\rho^2}, \tag{23}$$

where $R(\delta_M) \equiv \alpha + (1-\alpha)\xi\delta_M^k + (1-\alpha)(1-\xi)\delta^{*k}$. The first term on the RHS shows the welfare derived from the steady-state level of consumption, and the second term describes that derived from future growth in product quality. Thus, we find that there are three possible channels through which economic policy may affect household welfare. First, an increase in the frequency of leapfrogging (i.e., a rise in δ_M) lowers welfare by increasing the labor required for market entry, thus contracting the production of goods for consumption. Second, an increase in the innovation rate similarly has a negative welfare effect as it pulls labor away from production.

Third, an increase in the rate of innovation improves welfare by accelerating growth in product quality. As such, the net welfare changes depend on both the type of policy and its magnitude.

4 Policy Analysis

This section studies the effects of economic policy on the entrepreneur's leapfrog/sell choice, the rate of innovation, and social welfare. We focus on several alternative policy instruments that are available to the government: the corporate tax rate (τ_π), the capital gains tax (τ_V), the product design subsidy (s_E), and the leapfrog subsidy (s_M).

Our policy analysis includes numerical evaluations of steady-state welfare. As the numerical results are sensitive to changes in parameter values, we select the benchmark parameters based on three documented facts. First, Serrano (2010) reports that 13.5% of patents are sold (transferred) at least once in the United States. Second, empirical evidence suggests that the average price-cost markup lies between 1.2 and 1.6 in the United States (De Loecker et al., 2020). Third, we target at a rate of economic growth of 2% as the benchmark for our policy analysis to approximate the average rate of growth observed historically for the United States (Jones, 2005). We also use a value of $\rho = 0.02$ for the intertemporal discount rate following Jones et al. (1993). Given these motivations, we set the corporate tax rate to $\tau_\pi = 0.21$ and the capital gains tax rate to $\tau_V = 0.20$ to match with the current rates in the United States.[5] The product design subsidy and the leapfrog subsidy are respectively set to $s_E = 0.05$ and $s_M = 0.1$. The share of R&D costs incurred before patent application is $\alpha = 0.671$, the relative productivity of incumbent firms in marketing new product designs is $\xi = 0.95$, and the scale parameter for innovation costs is $a = 1.0625$. Finally, we let $k = 7.3$ and $\delta^* = 0.895$ for the shape parameter of the density function and the strength of patent regulation. This benchmark parameter set produces the following equilibrium values: $\delta_I = 0.758$, $\delta_M = 0.877$, $\iota = 0.184$, and $w = 1.022$, $x/L = 0.899$, and $U = 42.45$. The price-cost markup is therefore $p_x/w = 1/\delta_I = 1.31$. The rate of innovation is substantially lower than

[5] The US 2017 Tax Cuts and Jobs Act revised the definition of capital assets to exclude patents, thereby raising the tax rate imposed on primary patent sales from the capital gains tax rate of 20% to the ordinary income tax rate of 37% for individuals in the highest income tax bracket.

the rate of economic growth, as in the real world only 44.5% of new product designs have a sufficient quality level to obtain patent protection.[6], and 13.5% of patented product designs are sold to incumbent firms.

4.1 Corporate Taxes

First, we consider the effects of changes in the corporate tax rate (τ_π). The relationships between the corporate tax rate, the leapfrog/sell decision (δ_M), and average incumbent quality (δ_I) can be uncovered by taking the partial derivative of the investment condition (19) with respect to τ_π, revealing two opposing effects between leapfrogging and selling:

$$-\frac{(1-\tau_V)}{(1-\tau_\pi)}\left(\frac{\delta_I \int_{\delta_M}^{\delta^*} \pi_M dF(\delta)}{C_E}+\frac{\delta_I \pi_M(\delta_M)}{C_M-C_S}\right)+\frac{((1-s_E)\alpha(1-F(\delta^*))+C_S F(\delta^*))C_M \pi_E}{(1-\tau_\pi)(C_M-C_S)C_E^2}.$$

The first term is the negative profit effect of higher corporate taxation, showing that an increase in τ_π reduces the incentive for entrepreneurs to leapfrog, pushing the M-curve downwards in Figure 2. The second term is the positive tax exemption effect, stemming from an increase in the tax allowance for R&D expenditures associated with leapfrogging. A rise in the tax exemption tends to shift the M-curve upwards, as entrepreneurs have more incentive to leapfrog.

If the profit effect dominates, the share of designs used to leapfrog falls, (i.e., δ_M falls), and average incumbent quality rises as the product designs of market leaders are updated more frequently (i.e., δ_I rises). Otherwise, if the tax exemption effect dominates, the design quality required to induce leapfrogging decreases (i.e., δ_M rises), leading to lower average incumbent quality (i.e., δ_I rises). These results and their implications for the long-run rate of innovation are summarized as:

Proposition 2 *(i). If the positive tax exemption effect dominates, an increase in the corporate tax rate (τ_π) lowers the share of product designs sold to incumbent firms (i.e, a rise in δ_M), while*

[6]Carley et al. (2015) estimate that only 55.8% of applications filed with the US Patent and Trademark Office were granted patents between 1996 and 2013. The lower share of new designs awarded patents in our analysis can be rationalized by considering that many inventors recognize that their product designs do not include innovations that are insufficient to be awarded a patent and therefore do not initiate a patent application.

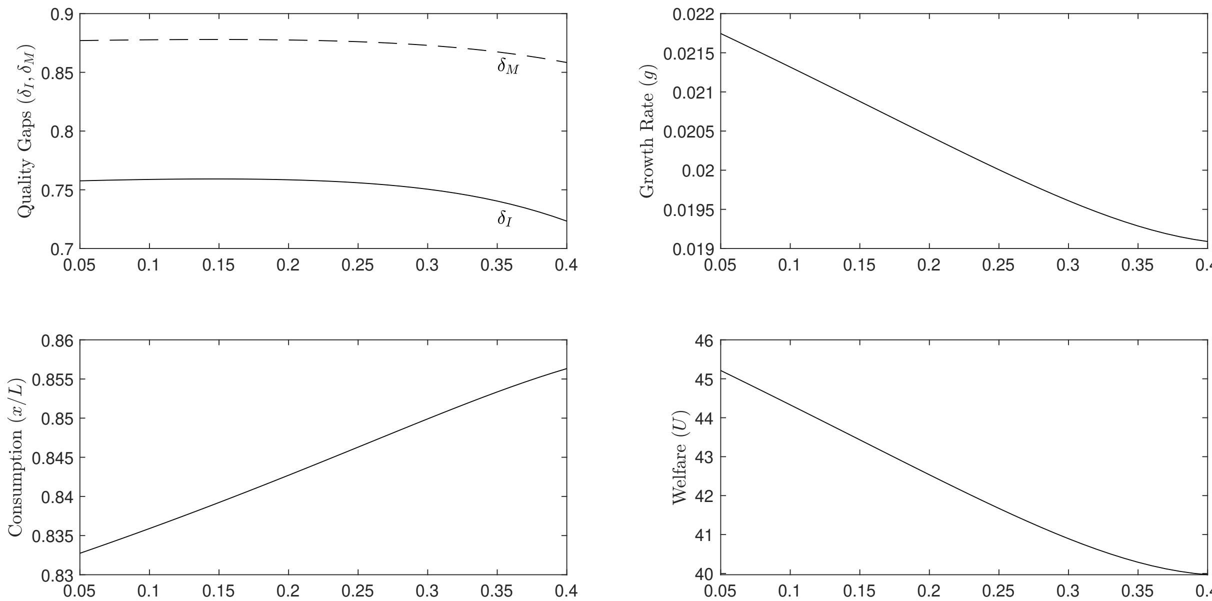
reducing the average product quality of market leaders (i.e., a rise in δ_I). The rate of innovation (ι) falls. (ii). If the negative profit effect dominates, however, an increase in τ_π lowers δ_M and δ_I. The effect on the innovation rate is ambiguous.

Proof: See Appendices C and D.

An increase in the corporate tax rate (τ_π) affects the innovation rate through adjustments in the labor and investment markets. First, referring to (21), we find that the negative profit and positive tax exemption effects have a direct impact on the threshold return to leapfrogging over selling in the investment market. However, the direct negative profit effect always dominates the direct tax exemption effect, slowing the rate of innovation as the ι_V-curve shifts downwards in Figure 3 (i.e., $\partial\iota/\partial\tau_\pi < 0$). Next, following the results of Lemma 1, the increase in τ_π also affects innovation indirectly through the adjustments in the labor and innovation markets that coincide with changes in the average threshold inverse-quality gap (δ_M) and the average incumbent inverse-quality gap (δ_I). On the one hand, if the positive tax exemption effect dominates, δ_M and δ_I increase, generating downward shifts in the ι_L-curve and the ι_V-curve (i.e., $\partial\iota/\partial\delta_M \cdot \partial\delta_M/\partial\tau_\pi < 0$ and $\partial\iota/\partial\delta_I \cdot \partial\delta_I/\partial\tau_\pi < 0$). In this case, the directions of the direct and indirect effects align, and an increase in the corporate tax rate slows the rate of innovation. On the other hand, if the negative profit effect dominates, δ_M and δ_I decrease, causing the ι_L-curve and ι_V-curve to shift upwards (i.e., $\partial\iota/\partial\delta_M \cdot \partial\delta_M/\partial\tau_\pi > 0$ and $\partial\iota/\partial\delta_I \cdot \partial\delta_I/\partial\tau_\pi > 0$). Then, in this case, the balance between the direct and indirect effects of a change in τ_π determines the relationship between the corporate tax rate and the rate of innovation.

Figure 4 presents a numerical analysis of the effects of changes in the corporate tax rate on the share of designs sold to incumbent firms (δ_M), average incumbent firm quality (δ_I), the rate of economic growth (g), household consumption (x/L), and welfare (U) over the policy range $\tau_\pi \in (0, 0.4)$. The plots suggest that the tax exemption effect dominates when the corporate tax rate is low, with an increase in τ_π reducing the share of designs sold to incumbents, and lowering average incumbent product quality, while the profit effect dominates when the corporate tax rate is high, with an increase in τ_π lowering δ_M and δ_I. Turning to innovation, we find that under

Figure 4: Corporate Tax



These figures are produced using $f(\delta) = k\delta^{k-1}$ with the parameter values a = 1.0625, α = 0.671, ξ = 0.95, ρ = 0.02, k = 7.3, s_E = 0.05, s_M = 0.1, τ_V = 0.2, and δ^* = 0.895. This benchmark parameter set yields δ_I = 0.758, δ_M = 0.877, w = 1.022, ι = 0.184, g = 0.02, x/L = 0.899, and U = 42.45.

the assumed parameter set the direct effect of changes in the corporate tax rate dominates across the policy range, and an increase in τ_π reduces employment in innovation and slows the rate of economic growth. Adjustments in the price-cost markup depend on the balance of the profit and tax exemption effects, but a decrease in the wage rate is the key factor behind a fall in the product price of the average industry ($p_x = w/\delta_I$) that generates an expansion in household consumption (x/L). Ultimately, household welfare follows the rate of economic growth, with rising corporate taxes reducing steady-state utility (U).

4.2 Capital Gains Taxes

Second, we investigate the effects of changes in the capital gains tax (τ_V). Partially differentiating the investment condition (19) yields:

$$\left(\frac{\delta_I \int_{\delta_M}^{\delta^*} \pi_M dF(\delta)}{C_E} + \frac{\delta_I \pi_M(\delta_M)}{C_M - C_S}\right) - \frac{((1-s_E)\alpha(1-F(\delta^*)) + C_M F(\delta^*))C_S \pi_E}{(1-\tau_V)(C_M - C_S)C_E^2}.$$

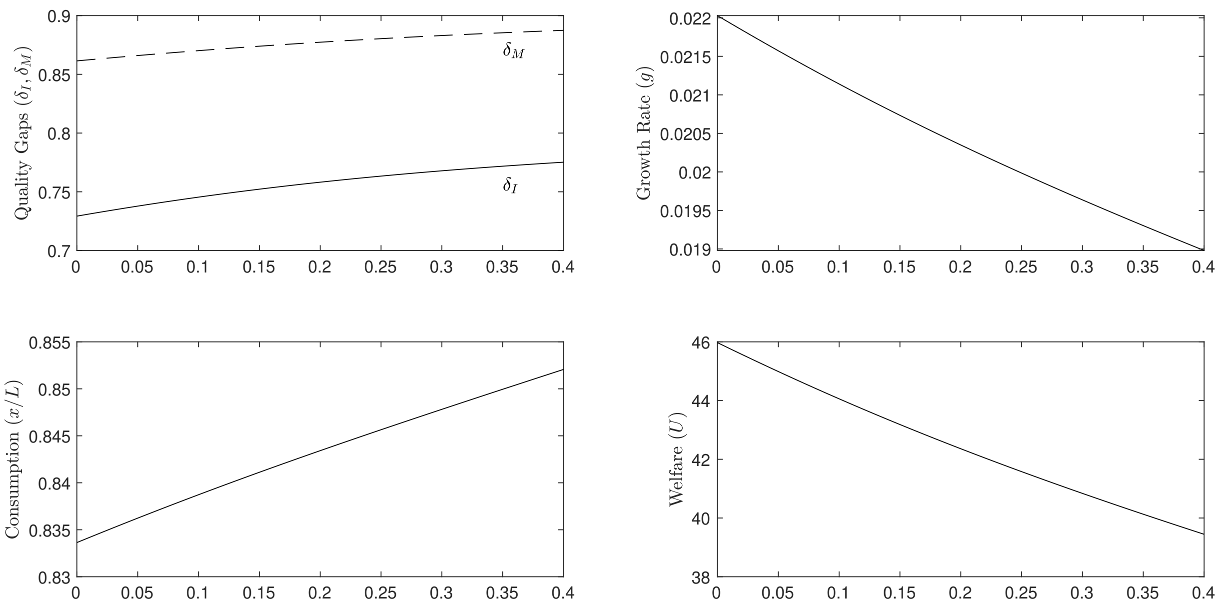
The first term is the positive profit effect, indicating that an increase in τ_V makes entrepreneurs more likely to leapfrog incumbents, causing an upward shift in the M-curve. The second term is the negative tax exemption effect, showing that an increase in the capital gains tax associated with selling product designs tends to shift the M-curve downwards.

When the positive profit effect dominates, the share of designs for leapfrogging increases (i.e., δ_M rises), and average incumbent product quality falls, as their product designs are updated less frequently (i.e., δ_I rises). Alternatively, when the negative tax exemption effect dominates, the design quality required to leapfrog rises (i.e., δ_M falls), leading to more frequent updating of incumbent designs, and higher average incumbent quality (i.e., δ_I falls). We summarize these results and their implications for the rate of innovation as:

Proposition 3 *(i). When the positive tax effect dominates, an increase in the capital gains tax rate (τ_V) decreases the share of product designs sold to ithe ncumbent firms (i.e., a rise in δ_M), while lowering the average product quality of market leaders (i.e., a rise in δ_I). The rate of innovation (ι) slows down. (ii). When the negative tax exemption effect dominates, an increase in τ_V lowers δ_M and δ_I. The effect on the innovation rate (ι) is ambiguous.*
Proof: See Appendices C and D.

An increase in the capital gains tax (τ_V) also affects the rate of innovation (ι) via adjustments in the labor and investment markets. First, the direct effects of raising τ_V are a negative effect on the profit from selling designs and a positive effect on the tax allowances for R&D expenditures associated with selling. The direct negative profit effect dominates, causing a downward shift in the ι_V-curve in Figure 3 (i.e., $\partial\iota/\partial\tau_V < 0$). Recalling Lemma 1, the increase in τ_V also affects innovation indirectly through adjustments in the labor and investment markets that coincide with changes in the threshold inverse-quality gap (δ_M) and the average incumbent inverse-quality gap (δ_I). If the positive profit effect dominates, δ_M and δ_I increase, causing the ι_L-curve and the ι_V-curve to shift downwards (i.e., $\partial\iota/\partial\delta_M \cdot \partial\delta_M/\partial\tau_V < 0$ and $\partial\iota/\partial\delta_I \cdot \partial\delta_I/\partial\tau_V < 0$). Hence, because the directions of the direct and indirect effects match, increasing the capital gains tax slows the rate of innovation. Alternatively, if the negative tax exemption effect dominates, δ_M

Figure 5: Capital Gains Tax



These figures are produced using $f(\delta) = k\delta^{k-1}$ with the parameter values a = 1.0625, α = 0.671, ξ = 0.95, ρ = 0.02, k = 7.3, s_E = 0.05, s_M = 0.1, τ_π = 0.21, and δ^* = 0.895. This benchmark parameter set yields δ_I = 0.758, δ_M = 0.877, w = 1.022, ι = 0.184, g = 0.02, x/L = 0.899, and U = 42.45.

and δ_I decrease, shifting the ι_L-curve and the ι_V-curve upwards (i.e., $\partial\iota/\partial\delta_M \cdot \partial\delta_M/\partial\tau_V > 0$ and $\partial\iota/\partial\delta_I \cdot \partial\delta_I/\partial\tau_V > 0$). The balance of the direct and indirect effects then determines the relationship between the capital gains tax and the innovation rate.

Figure 5 plots numerical results for the effects of a change in the capital gains tax rate (τ_V) on the leapfrog/sell choice (δ_M), the average incumbent inverse-quality gap (δ_I), the rate of growth (g), household consumption (x/L), and welfare (U) over the policy range $\tau_V \in (0, 0.4)$. From the plots, we infer that the positive profit effect dominates, with an increase in the capital gains tax reducing the share of designs sold to incumbents and lowering the average product quality of market leaders, given the assumed parameter set. The indirect effects align with the negative direct effect of capital gains taxation on employment in innovation, resulting in a slower rate of growth. Despite a rising wage rate, a fall in the price-cost markup ensures that the product price of the average industry falls ($p_x = w/\delta_I$), allowing for an expansion in household consumption (x/L). Adjustments in the rate of economic growth drive the direction of changes in household utility, however, with an increase in the capital gains tax rate hurting welfare (U).

4.3 Product Design Subsidy

Next, we study the effects of changes in the subsidy to product design (s_E). Partially differentiating (19) with respect to s_E yields

$$\frac{\alpha(1-F(\delta^*)+(1-\tau_\pi)F(\delta_M)+(1-\tau_V)(F(\delta^*)-F(\delta_M)))\pi_E}{C_E^2}+\frac{\alpha(\tau_\pi-\tau_V)\pi_E}{(C_M-C_S)C_E}.$$

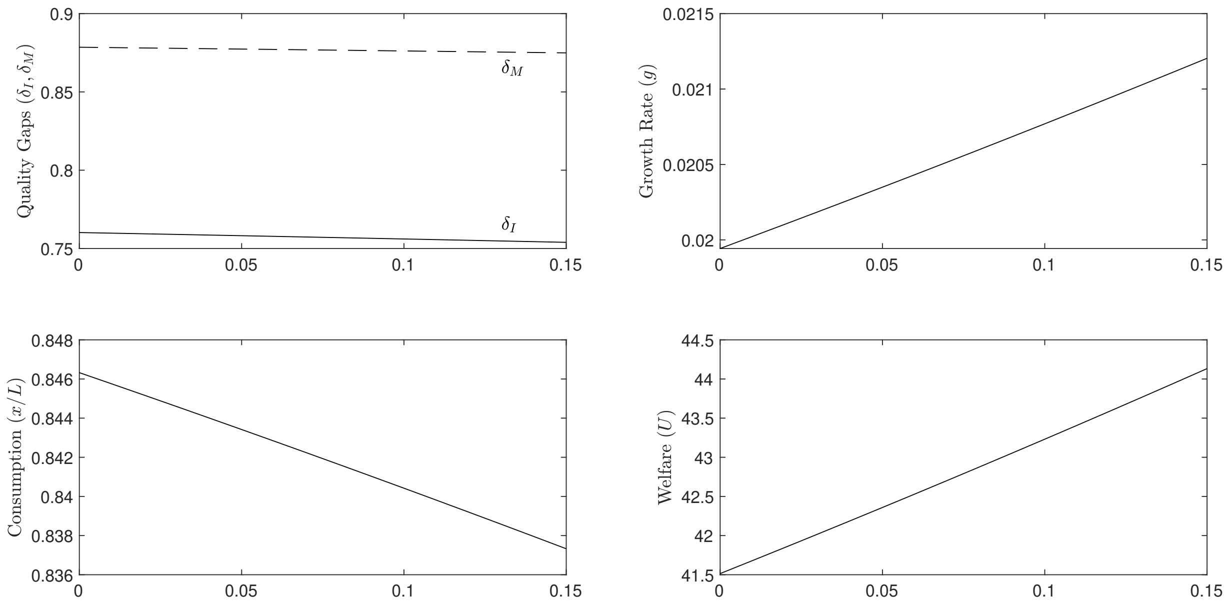
The first term is the positive effect of the product design subsidy on the expected return to product development that results from a reduction in the expected cost (C_E). This effect shifts the M-curve downwards in Figure 2. The second term captures the effect of the subsidy on the cost of leapfrogging relative to that of selling ($C_M - C_S$), the sign of which depends on the balance of the corporate and capital gains tax rates. This effect shifts the M-curve downwards for $\tau_\pi > \tau_V$, and upwards for $\tau_\pi < \tau_V$. Overall, we find an increase in s_E shifts the M-curve downwards, causing the share of product designs sold to incumbents to rise (i.e., δ_M falls). The higher frequency with which incumbent product designs are updated then results in higher average incumbent quality (i.e., δ_I falls). We summarize these results as:

Proposition 4 *An increase in the subsidy (s_E) to product design (i.e., initial R&D) increases the share of product designs sold to incumbent firms (i.e., δ_M falls), while raising the average product quality of market leaders (i.e., δ_I falls). The effect of a change in the subsidy to product design on the innovation rate (ι) is generally ambiguous.*

Proof: See Appendices C and D.

Changes in the product design subsidy (s_E) affect the innovation rate (ι) directly through the investment market. When $\tau_\pi > \tau_V$, an increase in s_E lowers the cost of selling more than that of leapfrogging, shifting the ι_V-curve upwards in Figure 3. Alternatively, when $\tau_\pi < \tau_V$, there is a greater decrease in the cost of selling, and the ι_V-curve shifts downwards. From Lemma 1, the design subsidy also has indirect effects on the innovation rate, with decreases in δ_M and δ_I

Figure 6: Product Design Subsidy



These figures are produced using $f(\delta) = k\delta^{k-1}$ with the parameter values a = 1.0625, α = 0.671, ξ = 0.95, ρ = 0.02, k = 7.3, s_M = 0.1, τ_π = 0.21, τ_V = 0.2, and δ^* = 0.895. This benchmark parameter set yields δ_I = 0.758, δ_M = 0.877, w = 1.022, ι = 0.184, g = 0.02, x/L = 0.899, and U = 42.45.

shifting both the ι_L-curve and the ι_V-curve upwards in Figure 3. (i.e., $\partial\iota/\partial\delta_M \cdot \partial\delta_M/\partial s_E > 0$ and $\partial\iota/\partial\delta_I \cdot \partial\delta_I/\partial s_E > 0$). Summarizing, the net effect of the design subsidy on innovation is determined by whether the direct effect or indirect effect dominates.

In Figure 6, we present a numerical analysis of the effects of the design subsidy (s_E) on the leapfrog/sell choice (δ_M), the average incumbent inverse-quality gap (δ_I), the rate of growth (g), household consumption (x/L), and welfare (U) over the range $s_E \in (0, 0.15)$. Essentially, the theoretical results of Proposition 4 are reproduced numerically, with an increase in s_E expanding the share of designs sold to incumbents, and raising the average quality of market leaders. For the benchmark parameter set, the positive indirect effects of investment and labor market adjustments dominate, ensuring that the design subsidy encourages investment in innovation, and causing an increase in the growth rate. A rise in the price-cost markup of the average industry and a higher wage rate raise product price ($p_x = w/\delta_I$), leading to a contraction in household consumption (x/L). Overall, we find that introducing a subsidy to product design improves household welfare

(U) as the positive effect of faster growth dominates the negative effect of lower consumption for the benchmark parameter set.

4.4 Leapfrog Subsidy

Finally, we consider the effects of changes in the subsidy to market entry through leapfrogging (s_M). An increase in s_M reduces the cost to entrepreneurs of marketing a new product design ($\partial C_M/\partial s_M = -(1-\tau_\pi)(1-\alpha) < 0$), raising the threshold return to leapfrogging and shifting the M-curve upwards in Figure 2. Consequently, a lower design quality is required to entice entrepreneurs to leapfrog (i.e., δ_M rises). And, as the share of designs sold to incumbents contracts, average incumbent quality falls (i.e., δ_I rises). These results and the subsequent effect on the innovation rate are summarized as:

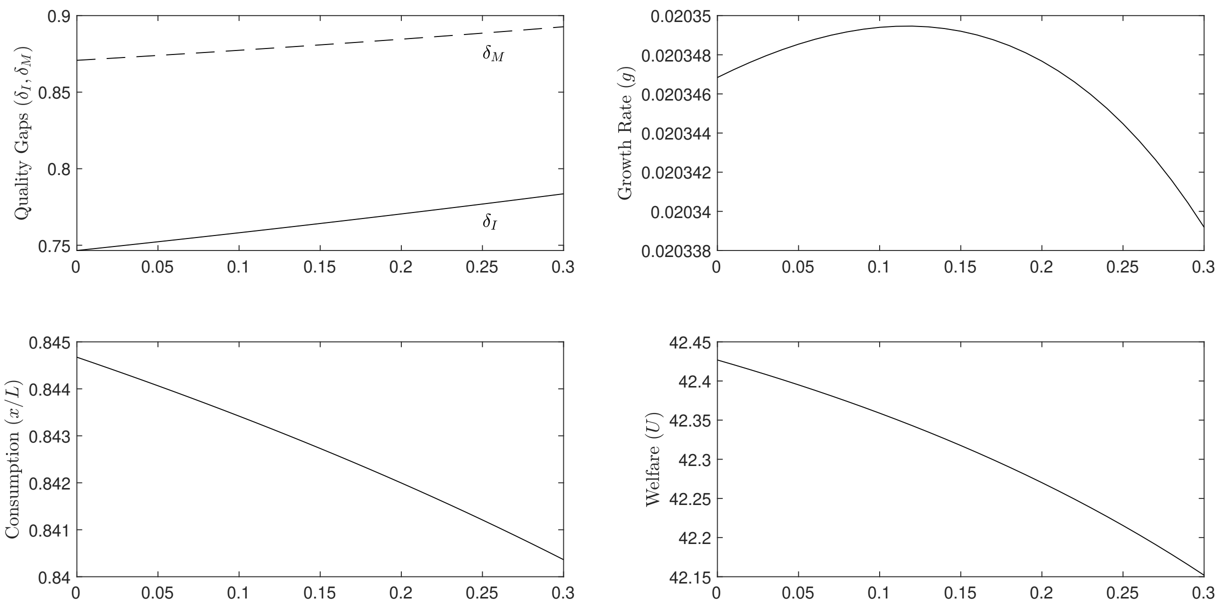
Proposition 5 *An increase in the subsidy (s_M) to market entry through leapfrogging decreases the share of product designs sold to incumbent firms (i.e., δ_M rises), while lowering the average product quality of market leaders (i.e., δ_I rises). The effect on the innovation rate (ι) is generally ambiguous.*

Proof: See Appendices C and D.

The direct effect of the leapfrog subsidy (s_M) on the innovation rate (ι) is an increase in the threshold return to leapfrogging over selling that shifts the ι_V-curve upwards in Figure 3 (i.e., $\partial\iota/\partial s_M > 0$), as the R&D costs associated with leapfrogging fall. From Lemma 1, however, the market entry subsidy also has indirect effects on the labor and investment markets, with the increases in δ_M and δ_I shifting both the ι_L-curve and the ι_V-curve downwards (i.e., $\partial\iota/\partial\delta_M \cdot \partial\delta_M/\partial s_M < 0$ and $\partial\iota/\partial\delta_I \cdot \partial\delta_I/\partial s_M < 0$). Accordingly, the relationship between the leapfrog subsidy and the innovation rate depends on whether the direct effect or the indirect effect dominates.

Figure 7 provides a numerical analysis of the effects of changes in the subsidy to market entry (s) on the leapfrog/sell choice (δ_M), the average incumbent inverse-quality gap (δ_I), the

Figure 7: Leapfrog Subsidy



These figures are produced using $f(\delta) = k\delta^{k-1}$ with the parameter values a = 1.0625, α = 0.671, ξ = 0.95, ρ = 0.02, k = 7.3, s_E = 0.05, τ_π = 0.21, τ_V = 0.2, and δ^* = 0.895. This benchmark parameter set yields δ_I = 0.758, δ_M = 0.877, w = 1.022, ι = 0.184, g = 0.02, x/L = 0.899, and U = 42.45.

growth rate (g), household consumption (x/L), and welfare (U) over the range $s_M \in (0, 0.3)$. The plots demonstrate the theoretical results of Proposition 5, with an increase in the leapfrog subsidy reducing the share of designs sold to incumbents, and lowering the average product quality of market leaders. Although the adjustments in economic growth are small, under the assumed parameter set, our numerical analysis suggests that initially the positive direct effect dominates, hastening innovation and accelerating growth. Beyond a certain level, however, the negative indirect effects of investment and labor market adjustments dominate, and further increases in the leapfrog subsidy slow innovation, thereby reducing the growth rate. While the rise in δ_I reduces the price-cost markup in the average industry, the wage rate increases leading to a higher product price ($p_x = w/\delta_I$). As a result, household consumption (x/L) contracts. Overall, we find that introducing a leapfrog subsidy hurts household welfare (U) for the parameter set that we have considered here.

5 Conclusion

This paper studies how tax policy affects economic growth through its influence on entrepreneurs' choice of commercialization mode for new innovations. In particular, we introduce a model of innovation-based economic growth in which entrepreneurs invest in new product designs that improve the quality of vintage product lines. The size of quality improvements are randomly drawn, generating *heterogeneous* values for product designs. After the quality of a design is revealed, an entrepreneur decides whether to use the design to leapfrog the incumbent firm and become the market leader or sell the design to the incumbent. We show that high-quality designs are used to leapfrog and low-quality designs are sold.

We use the framework to study how tax incentives affect the entrepreneur's leapfrog/sell choice, and consider the implications for economic growth and social welfare. Characterizing an average industry using incumbent product quality and the threshold design quality required to leapfrog, we find that tax incentives influence the rate of innovation through two channels. The first is the standard direct channel where tax incentives reduce expected R&D costs and promote investment in innovation. The second indirect channel operates through the positive relationship that arises between the threshold design quality required for leapfrogging and the innovation rate. Tax incentives that increase the likelihood of leapfrogging rather than selling, tend to slow the rate of innovation. Because either the direct or the indirect channel may dominate when the directions of the channels do not align, the overall impact of tax policy on the rate of innovation is generally ambiguous. Further, numerical analysis suggests that although subsidies to product design raise the growth rate and improve welfare, corporate taxes on operating profits, capital gains taxes on the sale of product designs and subsidies to market entry all slow the rate of economic growth and harm welfare.

Appendix A

This appendix derives the existence conditions for $\delta_M(i) \in (0, \delta^*)$. First, we obtain a condition for the required horizontal intercept ranking $\delta_S > \delta_0$. Referencing (9) and (12), and reorganizing $\Pi_M(\delta_0) = 0$ and $\Pi_S(\delta_S) = 0$ yields $waL = (1-\tau_\pi)V_M(\delta_0)/C_M = (1-\tau_V)(V_M(\delta_s) - V_I)/C_S$, which implies $(1-\tau_\pi)(1-\delta_0)/C_M = (1-\tau_V)(1-\delta_S)\delta_I/C_S$. Rearranging gives

$$\delta_S - \delta_0 = (1-\delta_0)\left(1 - \frac{(1-s_E)\alpha + (1-\xi)(1-\alpha)}{((1-s_E)\alpha + (1-s_M)(1-\alpha))\delta_I(i)}\right),$$

and thus $\delta_I > ((1-s_E)\alpha + (1-\xi)(1-\alpha))/((1-s_E)\alpha + (1-s_M)(1-\alpha))$ is required for $\delta_S > \delta_0$. Second, we derive a condition for the required slope ranking $\partial\Pi_M(i)/\partial\delta(i) < \partial\Pi_S(i)/\partial\delta(i)$. Taking the derivatives of (9) and (12) yields

$$\frac{d\Pi_M(\delta(i))}{d\delta(i)} - \frac{d\Pi_S(\delta(i))}{d\delta(i)} = -\int_0^\infty e^{-\int_0^t (\rho + \iota(i,t'))dt'}((1-\tau_\pi) - (1-\tau_V)\delta_I(i))Ldt < 0.$$

Therefore, $(1-\tau_\pi)/(1-\tau_V) > \delta_I(i)$ is required for $d\Pi_M(i)/d\delta(i) - d\Pi_S(i)/d\delta(i) < 0$. Combining the above results, the threshold quality level $\delta_M(i) \in (0, \delta^*)$ exists for $(1-\tau_\pi)/(1-\tau_V) > \delta_I(i) > ((1-s_E)\alpha + (1-\xi)(1-\alpha))/((1-s_E)\alpha + (1-s_M)(1-\alpha))$. Finally, noting that at $\delta(i) = 1$, we have $V_M(i) = 0$ and $V_S(i) - V_I(i) = 0$. Thus, we require $C_M > C_S$ for $\delta_S > \delta_0$, or $(1-\tau_\pi)/(1-\tau_V) > ((1-s_E)\alpha + (1-\xi)(1-\alpha))/((1-s_E)\alpha + (1-s_M)(1-\alpha))$. This completes the proof of Proposition 1.

Appendix B: Stability of Long-Run Equilibrium

We first consider the stability of the steady state illustrated in Figure 2. A Taylor expansion of

$$\dot{\delta}_I = \left(\int_0^{\delta_M} \delta dF(\delta) + \delta_I \int_{\delta_M}^{\delta^*} \delta dF(\delta) - \delta_I \int_0^{\delta^*} dF(\delta)\right)\iota,$$

evaluating around (19) and $\dot{\delta}_I = 0$ yield

$$\frac{\partial \dot{\delta}_I}{\partial \delta_I} = -(1-\delta_I)\delta_M f(\delta_M)\iota \left(\left.\frac{d\delta_M}{d\delta_I}\right|_{I-curve} - \left.\frac{d\delta_M}{d\delta_I}\right|_{M-curve} \right) < 0,$$

where

$$\begin{aligned}
\left.\frac{d\delta_M}{d\delta_I}\right|_I &= \frac{\int_0^{\delta_M} \delta dF(\delta)/\delta_I}{(1-\delta_I)\delta_M f(\delta_M)} > 0, \\
\left.\frac{d\delta_M}{d\delta_I}\right|_M &= -\frac{(1-\tau_V)(C_M-C_S)}{((1-\tau_\pi)-(1-\tau_V)\delta_I)L} \left(\frac{\int_{\delta_M}^{\delta^*} \pi_M(\delta)dF(\delta)}{C_E(\delta_M)} + \frac{\pi_M(\delta_M)}{C_M-C_S} \right) < 0,
\end{aligned}$$

and the expected entry cost is denoted by $C_E(\delta_M) \equiv (1-s_E)\alpha(1-F(\delta^*)) + (C_M-C_S)F(\delta_M) + C_S F(\delta^*)$. As δ_I is a state variable, stability requires $\partial\dot{\delta}_I/\partial\delta_I < 0$.

Second, we examine the stability of the steady state depicted in Figure 3. Using (15) and (17), the system can be reduced to a single differential equation describing the dynamics of the wage rate:

$$\dot{w} = w\rho + \frac{(w-\delta_I)F(\delta^*)}{C_E(\delta_M)a} - \frac{(1-\tau_\pi)\pi_M(\delta_M) - (1-\tau_V)(\pi_S(\delta_M)-\pi_I)}{(C_M-C_S)aL}.$$

Setting w as a control variable, a Taylor expansion delivers $\partial\dot{w}/\partial w = \rho + F(\delta^*)/(C_E(\delta_M)a) > 0$, indicating that the system is saddle-path stable, with the wage rate jumping immediately and permanently to its steady-state level.

Appendix C: Comparative Statics for Product Quality

Appendix C calculates the effects of policy changes on the entrepreneur's leapfrog/sell choice and incumbent quality. To facilitate the analysis, we rewrite (19) and (20) in implicit form:

$$\begin{aligned}
\Omega_1 &= \frac{\pi_E(\delta_M,\delta_I)}{C_E(\delta_M)} - \frac{((1-\tau_\pi)-(1-\tau_V)\delta_I)\pi_M(\delta_M)}{C_M-C_S}, \\
\Omega_2 &= \int_0^{\delta_M} \delta dF(\delta) + \delta_I \int_{\delta_M}^{\delta^*} \delta dF(\delta) - \delta_I \int_0^{\delta^*} dF(\delta),
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial \Omega_1}{\partial \delta_M} &= \frac{((1-\tau_\pi)-(1-\tau_V)\delta_I)L}{C_M - C_S} > 0, \\
\frac{\partial \Omega_1}{\partial \delta_I} &= \frac{(1-\tau_V)\int_{\delta_M}^{\delta^*} \pi_M(\delta)dF(\delta)}{C_E(\delta_M)} + \frac{(1-\tau_V)\pi_M(\delta_M)}{C_M - C_S} > 0, \\
\frac{\partial \Omega_2}{\partial \delta_M} &= (1-\delta_I)\delta_M f(\delta_M) > 0, \\
\frac{\partial \Omega_2}{\partial \delta_I} &= -\frac{1}{\delta_I}\int_0^{\delta_M} \delta dF(\delta) < 0,
\end{aligned}$$

and thus we have $|J_1| = (\partial\Omega_1/\partial\delta_M)(\partial\Omega_2/\partial\delta_I) - (\partial\Omega_1/\partial\delta_I)(\partial\Omega_2/\partial\delta_M) < 0$.

Corporate Tax: Taking the total derivatives of Ω_1 and Ω_2 with respect to τ_π yields

$$\frac{d\delta_M}{d\tau_\pi} = -\frac{\partial\Omega_1}{\partial\tau_\pi}\frac{\partial\Omega_2}{\partial\delta_I}\frac{1}{|J_1|} \gtreqless 0, \qquad \frac{d\delta_I}{d\tau_\pi} = \frac{\partial\Omega_1}{\partial\tau_\pi}\frac{\partial\Omega_2}{\partial\delta_M}\frac{1}{|J_1|} \gtreqless 0,$$

where $\partial\Omega_2/\partial\delta_M > 0$, $\partial\Omega_2/\partial\delta_I < 0$, $|J_1| < 0$, and

$$\frac{\partial\Omega_1}{\partial\tau_\pi} = \frac{(1-\tau_V)}{(1-\tau_\pi)}\left(\frac{\delta_I\int_{\delta_M}^{\delta^*}\pi_M dF(\delta)}{C_E} + \frac{\delta_I\pi_M(\delta_M)}{C_M - C_S}\right) - \frac{((1-s_E)\alpha(1-F(\delta^*)) + C_S F(\delta^*))C_M\pi_E}{(1-\tau_\pi)(C_M - C_S)C_E^2} \gtreqless 0.$$

Capital Gains Tax: Taking the total derivatives of Ω_1 and Ω_2 with respect to τ_V gives

$$\frac{d\delta_M}{d\tau_V} = -\frac{\partial\Omega_1}{\partial\tau_V}\frac{\partial\Omega_2}{\partial\delta_I}\frac{1}{|J_1|} \gtreqless 0, \qquad \frac{d\delta_I}{d\tau_V} = \frac{\partial\Omega_1}{\partial\tau_V}\frac{\partial\Omega_2}{\partial\delta_M}\frac{1}{|J_1|} \gtreqless 0,$$

where $\partial\Omega_2/\partial\delta_M > 0$, $\partial\Omega_2/\partial\delta_I < 0$, $|J_1| < 0$, and

$$\frac{\partial\Omega_1}{\partial\tau_V} = -\frac{\delta_I\int_{\delta_M}^{\delta^*}\pi_M dF(\delta)}{C_E} - \frac{\delta_I\pi_M(\delta_M)}{C_M - C_S} + \frac{((1-s_E)\alpha(1-F(\delta^*)) + C_M F(\delta^*))C_S\pi_E}{(1-\tau_V)(C_M - C_S)C_E^2} \gtreqless 0.$$

Product Design Subsidy: Taking the total derivatives of Ω_1 and Ω_2 with respect to s_E generates

$$\frac{d\delta_M}{ds_E} = -\frac{\partial\Omega_1}{\partial s_E}\frac{\partial\Omega_2}{\partial\delta_I}\frac{1}{|J_1|} < 0, \qquad \frac{d\delta_I}{ds_E} = \frac{\partial\Omega_1}{\partial s_E}\frac{\partial\Omega_2}{\partial\delta_M}\frac{1}{|J_1|} < 0,$$

where $\partial\Omega_2/\partial\delta_M > 0$, $\partial\Omega_2/\partial\delta_I < 0$, $|J_1| < 0$, and

$$\frac{\partial\Omega_1}{\partial s_E} = \frac{((1-\tau_\pi)(1-s_M)(1-\tau_V F(\delta^*)) - (1-\tau_V)(1-\xi)(1-\tau_\pi F(\delta^*)))\alpha(1-\alpha)\pi_E}{(C_M - C_S)C_E^2} > 0.$$

To show $\partial\Omega/\partial s_E > 0$, we compare the minimum corporate tax rate that satisfies $(1-\tau_\pi)(1-s_M)(1-\tau_V F(\delta^*)) \geq (1-\tau_V)(1-\xi)(1-\tau_\pi F(\delta^*))$; that is,

$$\tau_{\pi 1} \equiv 1 - \frac{(1-\tau_V)(1-\xi)(1-F(\delta^*))}{(1-s_M)(1-F(\delta^*)) + (\xi - s_M)(1-\tau_V)F(\delta^*)},$$

with the minimum corporate tax rate required to ensure the existence of an interior equilibrium for both leapfrogging and selling,

$$\tau_{\pi 2} \equiv 1 - \frac{(1-\tau_V)((1-s_E)\alpha + (1-\xi)(1-\alpha))}{(1-s_E)\alpha + (1-s_M)(1-\alpha)},$$

as outlined in case (ii) of Proposition 1. Comparing these minimum tax rates yields

$$\tau_{\pi 1} - \tau_{\pi 2} = \frac{(1-\tau_\pi)(1-\tau_V)(\xi - s_M)((1-s_E)\alpha(1-F(\delta^*) + C_S F(\delta^*))}{C_M((1-s_M)(1-F(\delta^*)) + (\xi - s_M)(1-\tau_V)F(\delta^*))} > 0,$$

where $\xi > s_M$ is also required for the existence of an interior equilibrium. Thus, as $\tau_{\pi 1} > \tau_{\pi 2}$, we find that when the corporate tax rate is sufficiently low, $\partial\Omega/\partial s_E > 0$ holds.

Leapfrog Subsidy: Taking the total derivatives of Ω_1 and Ω_2 with respect to s_M leads to

$$\frac{d\delta_M}{ds_M} = -\frac{\partial\Omega_1}{\partial s_M}\frac{\partial\Omega_2}{\partial\delta_I}\frac{1}{|J_1|} > 0, \qquad \frac{d\delta_I}{ds_M} = \frac{\partial\Omega_1}{\partial s_M}\frac{\partial\Omega_2}{\partial\delta_M}\frac{1}{|J_1|} > 0,$$

where $\partial\Omega_2/\partial\delta_M > 0$, $\partial\Omega_2/\partial\delta_I < 0$, $|J_1| < 0$, and

$$\frac{\partial\Omega_1}{\partial s_M} = -\frac{(1-\tau_\pi)(1-\alpha)((1-s_E)\alpha(1-F(\delta^*)) + C_S F(\delta^*))\pi_E}{(C_M - C_S)C_E^2} < 0.$$

Appendix D: Comparative Statics for Innovation

This appendix derives the policy effects on the long-run innovation rate, using the following implicit functions to describe equilibrium in the labor and investment markets:

$$
\begin{aligned}
\iota_L &= \frac{1}{R(\delta_M)a} - \frac{\delta_I}{R(\delta_M)wa}, \\
\iota_V &= \frac{(1-\tau_\pi-(1-\tau_V)\delta_I)\pi_M(\delta_M)}{waLF(\delta^*)(C_M-C_S)} - \frac{\rho}{F(\delta^*)},
\end{aligned}
$$

where we have referenced (17) and (21), and we define $|J_2| = \pi_E(\delta_M,\delta_I)/(C_E(\delta_M)aLF(\delta^*)) + \delta_I/(R(\delta_M)a) > 0$.

Corporate Tax: Taking the total derivatives of ι_L and ι_V with respect to τ_π, we have

$$
\frac{d\iota}{d\tau_\pi} = \left(\frac{\pi_E}{C_E aLF(\delta^*)}\frac{\partial\iota_L}{\partial\tau_\pi} + \frac{\delta_I}{Ra}\frac{\partial\iota_V}{\partial\tau_\pi}\right)\frac{1}{|J_2|} \gtrless 0,
$$

where $\partial\delta_M/\partial\tau_\pi \gtrless 0$, $\partial\delta_I/\partial\tau_\pi \gtrless 0$, $|J_2| = \rho/(wF(\delta^*)) + 1/(w(\alpha+\beta F(\delta_M))) > 0$, and

$$
\begin{aligned}
\frac{\partial\iota_L}{\partial\tau_\pi} &= -\frac{(1-\alpha)\xi f(\delta_M)}{R^2 a}\frac{d\delta_M}{d\tau_\pi} - \frac{1}{wRa}\frac{d\delta_I}{d\tau_\pi} \gtrless 0, \\
\frac{\partial\iota_V}{\partial\tau_\pi} &= -\frac{((1-\tau_V)\delta_I C_M-(1-\tau_\pi)C_S)\pi_M(\delta_M)}{(1-\tau_\pi)(C_M-C_S)^2 waLF(\delta^*)} \\
&\quad -\frac{(1-\tau_\pi-(1-\tau_V)\delta_I)}{(C_M-C_S)waF(\delta^*)}\frac{d\delta_M}{d\tau_\pi} - \frac{(1-\tau_V)\pi_M(\delta_M)}{(C_M-C_S)waLF(\delta^*)}\frac{d\delta_I}{d\tau_\pi} \gtrless 0.
\end{aligned}
$$

Capital Gains Tax: Taking the total derivatives of ι_L and ι_V with respect to τ_V gives

$$
\frac{d\iota}{d\tau_V} = \left(\frac{\pi_E}{C_E aLF(\delta^*)}\frac{\partial\iota_L}{\partial\tau_V} + \frac{\delta_I}{Ra}\frac{\partial\iota_V}{\partial\tau_V}\right)\frac{1}{|J_2|} \gtrless 0,
$$

where $\partial\delta_M/\partial\tau_V \gtrless 0$, $\partial\delta_I/\partial\tau_V \gtrless 0$, $|J_2| = \rho/(wF(\delta^*)) + 1/(w(\alpha+\beta F(\delta_M))) > 0$, and

$$\begin{aligned}
\frac{\partial\iota_L}{\partial\tau_V} &= -\frac{(1-\alpha)\xi f(\delta_M)}{R^2 a}\frac{d\delta_M}{d\tau_V} - \frac{1}{wRa}\frac{d\delta_I}{d\tau_V} \gtrless 0,\\
\frac{\partial\iota_V}{\partial\tau_V} &= -\frac{((1-\tau_V)\delta_I C_M - (1-\tau_\pi)C_S)\pi_M(\delta_M)}{(1-\tau_V)(C_M-C_S)^2 waLF(\delta^*)}\\
&\quad -\frac{(1-\tau_\pi-(1-\tau_V)\delta_I)}{(C_M-C_S)waF(\delta^*)}\frac{d\delta_M}{d\tau_V} - \frac{(1-\tau_V)\pi_M(\delta_M)}{(C_M-C_S)waLF(\delta^*)}\frac{d\delta_I}{d\tau_V} \gtrless 0.
\end{aligned}$$

Product Design Subsidy: Taking the total derivatives of ι_L and ι_V with respect to s_E yields

$$\frac{d\iota}{ds_E} = \left(\frac{\pi_E}{C_E aLF(\delta^*)}\frac{\partial\iota_L}{\partial s_E} + \frac{\delta_I}{Ra}\frac{\partial\iota_V}{\partial s_E}\right)\frac{1}{|J_2|} \gtrless 0,$$

where $\partial\delta_M/\partial s_E < 0$, $\partial\delta_I/\partial s_E < 0$, $|J_2| > 0$, and

$$\begin{aligned}
\frac{\partial\iota_L}{\partial s_E} &= -\frac{(1-\alpha)\xi f(\delta_M)}{R^2 a}\frac{d\delta_M}{ds_E} - \frac{1}{wRa}\frac{d\delta_I}{ds_E} > 0,\\
\frac{\partial\iota_V}{\partial s_E} &= \frac{(1-\tau_V)\int_{\delta_M}^{\delta^*}\pi_M dF(\delta)}{C_E waLF(\delta^*)}\frac{d\delta_I}{ds_E} + \frac{\alpha(\tau_\pi-\tau_V)}{C_M-C_S} \gtrless 0,
\end{aligned}$$

where we have used $(\partial\Omega_1/\partial\delta_M)(d\delta_M/ds_E) + (\partial\Omega_1/\partial\delta_I)(d\delta_I/ds_E) = -\partial\Omega_1/\partial s_E$.

Leapfrog Subsidy: Taking the total derivatives of ι_L and ι_V with respect to s_M yields

$$\frac{d\iota}{ds_M} = \left(\frac{\pi_E}{C_E aLF(\delta^*)}\frac{\partial\iota_L}{\partial s_M} + \frac{\delta_I}{Ra}\frac{\partial\iota_V}{\partial s_M}\right)\frac{1}{|J_2|} \gtrless 0,$$

where $\partial\delta_M/\partial s_M > 0$, $\partial\delta_I/\partial s_M > 0$, $|J_2| > 0$, and

$$\begin{aligned}
\frac{\partial\iota_L}{\partial s_M} &= -\frac{(1-\alpha)\xi f(\delta_M)}{R^2 a}\frac{d\delta_M}{ds_M} - \frac{1}{wRa}\frac{d\delta_I}{ds_M} < 0,\\
\frac{\partial\iota_V}{\partial s_M} &= \frac{(1-\tau_V)\int_{\delta_M}^{\delta^*}\pi_M dF(\delta)}{C_E waLF(\delta^*)}\frac{d\delta_I}{ds_M} + \frac{(1-\tau_\pi)(1-\alpha)F(\delta_M)\pi_E}{C_E^2 waLF(\delta^*)} > 0,
\end{aligned}$$

where we have used $(\partial\Omega_1/\partial\delta_M)(d\delta_M/ds_M) + (\partial\Omega_1/\partial\delta_I)(d\delta_I/ds_M) = -\partial\Omega_1/\partial s_M$.

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