Discussion Paper Series

RIEB

Kobe University

DP2022-19

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Revised September 14, 2022

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September 14, 2022

Abstract

This study investigates in what cases a firm's dynamic price-setting behavior can be approximated as static under dynamic demand, by developing a dynamic discrete choice model. Under dynamic demand with random utility shock following Gumbel distribution, this study shows that an oligopolistic firm's optimal price-setting behavior is well approximated by the static one with no strategic consideration, when consumers' conditional choice probabilities (CCPs) of choosing the firm's product are small for all consumer types and state variables. If the condition does not hold, the firm's behavior might be far from static.

Keywords: Dynamic demand; Dynamic price-setting behavior; Static approximation; Monopolistic competition; Dynamic discrete choice.

*This work was supported by JSPS KAKENHI Grant Number JP21J21410.

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1 Introduction

Many industries exhibit dynamic demand structures, wherein future demand has connections with current demand. Examples of goods with dynamic demand include durable goods, storable goods, goods with switching costs, experience goods, and network goods. Under dynamic demand, firm's price-setting behavior should also be dynamic. First, current price indirectly affects current demand through changing consumers' expectations of future outcomes. Second, current price and future demands are related; thus, current price affects future demand. Consequently, firms set their product prices considering these dynamic effects.

This article investigates in what cases a firm's dynamic price-setting behavior can be approximated as static under dynamic demand. We should not miss the dynamic aspects of firms' price-setting behaviors when considering industries with dynamic demand if their magnitude is not negligible. Nevertheless, if the magnitude of the dynamic elements is small, ignoring them causes minor problems. By developing a unified theoretical model of dynamic discrete choice, This study analytically shows when the firm's dynamic price-setting behavior is close to or far from the static one.

So far, many studies have applied "static models," which this article defines as the combination of static demand models and firm's static price-setting models, to analyze many industries and derive implications. There are several advantages to static models. First, static demand structures are simple, and we do not need to solve dynamic models. Second, firms' optimality condition of price-setting behavior is also static and simple. There is no need to solve complicated dynamic optimization problems[1]; however, numerous recent empirical studies have found that applying static demand models to industries with dynamic demand yields biased results[2]. For example, price elasticities are incorrectly estimated when applying static demand model, as Hendel and Nevo (2006) and others have indicated. In recent years, the estimation and simulation of dynamic demand models have been getting easier due to the accumulation of knowledge and computational power. In that sense, developing dynamic demand model is becoming desirable to derive more precise implications. Nevertheless, once introducing dynamic demand, firms' price-setting behaviors should also be dynamic, as described earlier. Then, developing both dynamic demand-side and dynamic supply-side models is indispensable to make the model consistent. In this case, the model gets so complicated.

This article's main objective is to derive easier ways to analyze industries with dynamic demand. If the firm's dynamic price-setting behavior can be approximated as static, applying a static specification causes minor problems. It is relatively easier to solve the supply-side behavior when applying static conditions. We can then derive appropriate implications with less cost or effort, even though introducing dynamics on the demand side is necessary.

Under dynamic demand with random utility shock following Gumbel distribution, this study shows that a firm's dynamic price-setting behavior is close to static when conditional choice probabilities (CCPs) of the firm's product are small for all consumer types and state variables. It implies that the firm's behavior might be far from static if at least one consumer type or state variable exists in which the CCP takes a large value.

The existence of random utility shock following Gumbel distribution is prevalent in empirical studies. The introduction of random utility shock is needed to explain why consumers do not always purchase the same goods over time in the real world[3]. Random utility shock brings important implications for firm's behavior under dynamic demand[4]. Intuitively, under random utility shock, current buyers may not be future buyers because of the randomness of consumer behavior. Then, the connection between current and future periods is small, and the firm is less likely to consider future outcomes.

[1] Aguirregabiria et al. (2021) mentioned that computation of dynamic problems is still enormously difficult even now, especially when state space is large. Researchers are haunted by the problem of the curse of dimensionality.

[2] See Fukasawa (2022) for the literature review on the biases in applying static demand model to markets with dynamic demand.

[3] See the discussion in Belleflamme and Peitz (2015) for instance. Note that the introduction of random utility shock largely affects substitution patterns among products. Rysman and Ackerberg (2005) argued that the model with random utility shock might not be so attractive for evaluating the value of new goods, since consumer surplus grows without an upper bound as the number of products increases under random utility shock. This study does not delve into these arguments in detail in this article. Berry and Pakes (2007) proposed a pure characteristics model which does not introduce random utility shock terms. In recent years, Lu and Saito (2021) theoretically investigated to what extent the model with random utility shock can well approximate the pure characteristics model in the case of the mixed logit model.

[4] In the theoretical literature on durable goods, several articles, such as Biehl (2001), Johnson (2011), and Garrett (2016), have found that the introduction of random utility shock overturns the previous results.

Small CCPs for all consumer types and state variables require small persistent heterogeneity of consumers and small market share. To understand why small persistent heterogeneity of consumers is essential, consider the case where persistent consumer heterogeneity is large. A small fraction of consumers may strongly prefer the product; they would always purchase it if they have the opportunity, even under random utility shock. Then, current buyers would be future buyers, and the connection between the current and future periods would be large. In this case, the firm is more likely to consider future demand. Otherwise, when persistent heterogeneity is small, future periods are unimportant for the firm.

Small market share is realized if the number of firms is large, or the firm is fringe. As Anderson et al. (1992) and others described, when idiosyncratic utility shock follows Gumbel distribution, even small firms can set prices higher than the products' marginal costs in contrast to perfect competition or homogeneous goods with many firms. If the market share is small, the probability that the current buyer coincides with the future buyer is so small. Then, the connection between the current and the future periods is small, and the firm is less likely to consider the future periods.

Under the conditions mentioned above, we can also show that the firm behaves like a static monopolistic competitor, which has no impact on its competitors but is free to choose the price that maximizes its profits as a monopolist (Chamberlin, 1949). As discussed in Thisse and Ushchev (2018), the model of monopolistic competition makes it easier to analyze problems with general equilibrium effects, which are hard to deal with in oligopolistic models that allow for strategic interactions among firms. Hence, the condition of small CCPs for all consumer types and state variables presented in the current article is important for simplifying the analysis of industries with dynamic demand.

To validate the performance of the approximation when the conditions are satisfied, this study builds a numerical model of durable goods as a typical example of the goods with dynamic demand and run numerical experiments. After generating simulated data of durable goods market, this study conducts two types of experiments prevalent in empirical applications: recovering marginal cost based on the firm's static optimality condition and calculating markup, and calculating the cost pass-through rate based on the recovered marginal cost. Then, we compare the results based on the true dynamic optimality condition and those based on the static optimality condition. The simulation results show that a firm's dynamic pricing behavior is close to static if the conditions are satisfied but far from when they are violated.

While our focus is on the markets with dynamic demand, we can find an analogy with natural resources extraction. Consider the case of fishery, where the number of fishers is large and no coordination among them exists. If each fisher catches too many fish, the number of surviving fish shrinks, making it difficult to catch them in the next period. Nevertheless, each fisher behaves myopically: their current action has negligible effects on the number of remaining fish, and they do not have incentives to adjust their activities to maintain the resources in the future. In that sense, fisher's behavior is static. Section 6 discusses the counterparts of the random utility shock and small persistent heterogeneity under dynamic demand in the case of natural resources extraction.

This article's main contribution is the proof of the conditions under which a firm's pricing behavior is (approximately) static and the firm behaves like a monopolistic competitor under dynamic demand. The results make it easier to analyze industries with dynamic demand theoretically and empirically if the conditions are plausible to assume[5]; this is the first article of this sort.

The organization of the rest of this article is as follows. Section 2 describes the relationship between this study and the previous studies, Section 3 presents the theoretical model and the main results, and Section 4 validates the analytical results through numerical experiments. Section 5 considers the applications of the results to each type of goods with dynamic demand, and Section 6 describes the similarity with the natural resources extraction problem. Section 7 concludes and describes possible extensions.

2 Literature

This article is related to several strands of literature.

[5]As discussed in the next section, Carranza (2010) empirically applied a similar idea to analyze product innovation in the durable digital camera market, even though theoretical justification was not formally discussed.

First, this article builds on and contributes to the growing empirical literature on dynamic demand. Recent studies have shown that dynamic demand structure characterize many industries in the real world. Examples include automobiles (durable goods; Chen et al., 2013), electronic devices (durable goods; Goettler and Gordon, 2011, Gowrisankaran and Rysman, 2012), houses (durable goods; Bayer et al., 2016), detergents (storable goods; Hendel and Nevo, 2006), drugs (experience goods; Crawford and Shum, 2005), subscription TV services (goods with switching cost; Shcherbakov, 2016), and video games (network goods; Dubé et al., 2010; Lee, 2013). Building on the literature, Gowrisankaran and Rysman (2020) developed a framework for empirical models of dynamic demand. Developing the dynamic demand-side and dynamic supply-side models is ideal for analyzing these industries and deriving more precise policy implications; however, the number of studies is limited[6], probably because of the effort required to solve dynamic problems. This article contributes to the literature by showing when a firm's dynamic behavior is or not is not negligible and when ignoring firm's dynamic behavior may lead to biased results, using the unified analytical framework.

Carranza (2010) conducted an empirical study applying a similar idea to analyze product innovation in the digital camera market. Since digital cameras are durable goods, the author specified a dynamic demand model on the demand side. On the supply side, he argued that each firm's product-level decisions (pricing and innovation)[7] have negligible effect on market variables including consumers' expectations (inclusive values), since the number of products was large in the digital camera market. He justified this claim by showing numerical evidence that the derivatives of the consumers' inclusive values concerning individual prices were close to zero. Then, he used the firms' static price-setting model to analyze the market[8]. Even though the effect of the current price on future demand was not explicitly mentioned and no formal proof exists, the study applied an idea in line with this current article.

Fukasawa (2022) also investigated dynamic demand using a unified framework similar to this article, but the focus is different. The existing article focuses on the demand side, and investigates when applying static demand model yields biased results in key parameters (utility parameters and price elasticities of demand) when the true demand structure is dynamic. In Fukasawa (2022), supply-side behavior is not explicitly specified. In contrast, the current article focuses on the supply-side and investigates how firms behave given dynamic demand structure. Hence, these articles are complementary. It is interesting to note that Fukasawa (2022) showed that applying static demand models yields small biases in estimated price elasticity of a product when its CCPs are small for all consumer types and state variables, given consistent consumer utility parameter estimates and under the assumption that random utility shock follows Gumbel distribution. The conditions are the same as those in this article, we can argue that applying a "static model," namely, a static demand-side model and a static supply-side model, brings valid implications if the condition holds.

This study's main motivation, namely, considering easier ways to analyze industries with dynamic demand, is closely related to oblivious equilibrium (Weintraub et al., 2008; Benkard et al., 2015; Ifrach and Weintraub, 2017) and experience-based equilibrium (Fershtman and Pakes, 2012). Oblivious equilibrium and its variants mainly focus on a firm's investment decisions when it is small under static demand structures. Even though they are approximations of Markov perfect equilibrium, they perform well when certain conditions are satisfied. Experience-based equilibrium focuses on dynamic games with asymmetric information, and agents make their actions based on their experiences. As discussed in the article, the equilibrium can also be applied to the case with dynamic demand. They effectively reduce the number of state variables and mitigate the computational burden to solve the problems. The analysis in the current article does not provide any new equilibrium concepts but shows when firm's dynamic pricing behavior in solving a dynamic problem is close to the one based on the commonly used static solutions under dynamic demand. Hence, this study is complementary to these equilibrium concepts.

This article also contributes to the monopolistic competition literature. The idea of monopolistic competition has been widely applied in many fields, including industrial organization, trade, and macroeconomics. One significant reason for the prevalence of the idea of monopolistic competition would be theoretical simplicity which does not appear in the standard oligopolistic competition models with strategic interactions among firms.

[6]For instance, only a handful of articles (Esteban and Shum, 2007; Chen et al., 2008; Chen et al., 2013, Gillingham et al., 2019) have specified and investigated firms' dynamic behaviors in the case of oligopolistic durable goods market.

[7]Note that he considered the model where each firm produces multiple products.

[8]Note that he assumed the firms' innovation decisions are dynamic.

AsThisse and Ushchev (2018) discussed, it has facilitated the studies dealing with general equilibrium effects. Moreover, several studies, such as Perloff and Salop (1985) and Gabaix et al. (2016), have investigated whether the market environment of monopolistic competition can be represented as the limit case of the oligopolistic competition with a sufficiently large number of firms; however, these studies appear to have been confined to the case of static demand. Nevertheless, dynamic demand structure is prevalent in the real world, as described previously. This study contributes to the literature by presenting the condition under which the idea of monopolistic competition can be applied under discrete choice model with a dynamic demand structure, which is not negligible in many industries.

Finally, this article contributes to many theoretical studies investigating the supply-side behavior under dynamic demand. They have investigated the characteristics of firms' behaviors for each type of goods with dynamic demand: studies on durable goods since the work of Coase (1972), studies on switching costs since Klemperer (1987), and studies on network effects since Katz and Shapiro (1986), to name a few. This study's primary main focus is not on each goods' unique characteristics but on common characteristics: when a firm's pricing behavior is close to or far from static. This article develops a general model of the market with dynamic demand and provides insights from the viewpoints different from the previous studies.

3 Model

We build a model of dynamic demand with product differentiation. Time is discrete. Both consumers and firms are forward-looking, and formulate correct expectations. We consider the case where firms set their product prices in each period, and assume that consumers and firms behave according to Markov perfect equilibrium. Hence, consumers and firms optimize their behavior conditional only on the current state variables and private shocks. In Appendix C, we consider the case where firms can commit to future prices, but similar results hold.

3.1 Consumers

On the demand side, a continuum of consumers exists. Let M be the total number of consumers. There are L types of consumers ($l = 1, \cdots, L$) who have common (deterministic) flow utilities. Let π_l be the fraction of type l consumers. π_l satisfies $\sum_l \pi_l = 1$. We assume L is finite[9].

3.1.1 State variables

First, let $x_t \in X_t$ be consumer's individual-level state variables at the beginning of time t. X_t denotes the set of individual-level states. For instance, x_t indicates the age (time since the last purchase) of the products, in the case of durable goods. We assume that X_t is a discrete set. Besides, let B_t be the vector of aggregated-level state variables at time t. For instance, B_t includes $Pr_{lt}(x_t)$, where $Pr_{lt}(x_t)$ denotes the ratio of type l consumers at state x_t among type l consumers at time t. Note that $\sum_{x_t} Pr_{lt}(x_t) = 1$ holds.

3.1.2 Consideration set and choices

Let $\mathcal{A}_l(x_t)$ be the available alternatives (consideration set) of type l consumers at states x_t. $\mathcal{A}_l(x_t)$ satisfies $\mathcal{A}_l(x_t) \subseteq \mathcal{J} \cup \{0\}$, where $\mathcal{J}$ denotes the set of products[10]. We allow a limited consideration set. The consideration sets depend on consumer's individual states x_t.

Let $a_t \in \mathcal{A}_l(x_t)$ be the choice of type l consumer at time t. $a_t = j$ means that the consumer purchases product j, and $a_t = 0$ means that they do not purchase any product.

3.1.3 State transition

The transition probability of individual-level state variables x_t is given by $\psi(x_{t+1}|x_t, B_t, a_t)$, and it depends on the previous period's states and choices. We allow stochastic transitions of state variables. Conversely, we

[9]The assumption of finite L is imposed just for a technical reason.

[10]In this study, we abstract away the existence of secondary market (used goods market).

assume that aggregate state variables B_t follow a deterministic process[11]. B_{t+1} is a function of previous period's aggregate states B_t and consumers' CCPs $s_t^{(ccp)} \equiv \left(s_{ljt}^{(ccp)}(x_t, B_t, p_t)\right)_{x_t, l, j \in \mathcal{A}_l(x_t)}$, where $p_t = \{p_{kt}\}_{k \in \mathcal{J}}$ denote product prices. Since the vector of CCPs $s_t^{(ccp)}$ is a function of B_t, B_{t+1} is a function of B_t.

Note that $Pr_{lt}(x_t)$ satisfies the following state transition formula:

$$Pr_{lt+1}(x_{t+1}) = \sum_{x_t \in X_t} Pr_{lt}(x_t) \cdot \sum_{j \in \mathcal{A}_l(x_t)} s_{ljt}^{(ccp)}(x_t, B_t, p_t) \cdot \psi(x_{t+1}|x_t, B_t, a_t = j) \quad (1)$$

3.1.4 Utility function

This study assumes that each consumer purchases at most one product in each period. Let the expected discounted (decisive) utility of type l consumer i whose states are (x_t, B_t) and choice is a_t given product prices $p_t = \{p_{kt}\}_{k \in \mathcal{J}}$ be $v_{ilt}(x_t, B_t, a_t, p_t)$. Type l consumer i maximizes utility $v_{ilt}(x_t, B_t, a_t, p_t)$ regarding $a_t \in \mathcal{A}_l(x_t)$.

Utility v_{ilt} is in the following form:

$$v_{ilt}(x_t, B_t, a_t, p_t) = \begin{cases} f_{lj}(x_t, B_t, p_{jt}) + \beta_C E\left[V_l^C(x_{t+1}, B_{t+1}(B_t, s_t^{(ccp)}(B_t, p_t)))|x_t, a_t = j\right] + \epsilon_{iljt} & \text{if } a_t = j \\ f_{l0}(x_t, B_t) + \beta_C E\left[V_l^C(x_{t+1}, B_{t+1}(B_t, s_t^{(ccp)}(B_t, p_t)))|x_t, a_t = 0\right] + \epsilon_{il0t} & \text{if } a_t = 0 \end{cases} \quad (2)$$

where $f_{lj}(x_t, B_t, p_{jt})$ denotes the flow utility type l consumers at states (x_t, B_t) gain when buying product j whose price is p_{jt}, and $f_{l0}(x_t, B_t)$ denotes the flow utility type l consumers at states (x_t, B_t) gain when not buying anything. $(\epsilon_{iljt})_{j \in \mathcal{A}_l(x_t)}$ denotes the individual-level random preference shock, and we assume they follow i.i.d. distribution across (i, t)[12]. In the following, even though we do not restrict the functional form of the distribution of ϵ_{iljt} provided the assumptions mentioned later are satisfied, we keep Gumbel distribution in mind. β_C represents the consumers' discount factor, and E represents the expectation operator. Further, let $\widetilde{v_{ljt}}(x_t, B_t, p_t) = f_{lj}(x_t, B_t, p_{jt}) + \beta_C E\left[V_l^C(x_{t+1}, B_{t+1}(B_t, s_t^{(ccp)}(B_t, p_t)))|x_t, a_t = j\right]$.

Here, $V_l^C(x_t, B_t)$ is the value function of type l consumers given states (x_t, B_t), and satisfies the following equation under the presumption that firms and consumers behave according to Markov perfect equilibrium:

$$V_l^C(x_t, B_t) = E_\epsilon\left[\max_{a_t \in \mathcal{A}_l(x_t)} v_{ilt}(x_t, B_t, a_t, p^*(B_t))\right] \quad (3)$$

where E_ϵ denotes the expectation operator with respect to random i.i.d. shocks $\{\epsilon_{ilkt}\}_{k \in \mathcal{A}_l(x_t)}$.

3.1.5 Choice probability

The CCP that type l consumer buys product j at time t conditional on being at state (x_t, B_t) and product prices p_t is:

$$\begin{aligned} s_{ljt}^{(ccp)}(x_t, B_t, p_t) &= \begin{cases} Pr\left(v_{ilt}(x_t, B_t, a_t = j, p_t) > v_{ilt}(x_t, B_t, a_t = k, p_t)\ \forall k \in \mathcal{A}_l(x_t) - \{j\}\right) & \text{if } j \in \mathcal{A}_l(x_t) \\ 0 & \text{if } j \notin \mathcal{A}_l(x_t) \end{cases} \\ &= \begin{cases} Pr\left(\widetilde{v_{ljt}}(x_t, B_t, p_t) + \epsilon_{iljt} > \widetilde{v_{lkt}}(x_t, B_t, p_t) + \epsilon_{ilkt}\ \forall k \in \mathcal{A}_l(x_t) - \{j\}\right) & \text{if } j \in \mathcal{A}_l(x_t) \\ 0 & \text{if } j \notin \mathcal{A}_l(x_t) \end{cases} \end{aligned} \quad (4)$$

The probability that type l consumer buys product j at time t is:

$$s_{ljt}(B_t, p_t) = \sum_{x_t} Pr_{lt}(x_t) s_{ljt}^{(ccp)}(x_t, B_t, p_t) \quad (5)$$

[11]This is just for simplicity.

[12]Here, we allow the case where ϵ_{iljt} is not i.i.d across j, as in the case of generalized extreme value (GEV) distribution.

The probability that type l consumer does not buy any product at time t is:

$$s_{l0t}(B_t, p_t) = \sum_{x_t} Pr_{lt}(x_t) s_{l0t}^{(ccp)}(x_t, B_t, p_t) \quad (6)$$

The market share of product j at time t, namely, the fraction of consumers purchasing product j at time t given the state variables B_t and product prices p_t is:

$$s_{jt}(B_t, p_t) = \sum_{l} \pi_l s_{ljt}(B_t, p_t) \quad (7)$$

The fraction of consumers not purchasing any product at time t is:

$$s_{0t}(B_t, p_t) = \sum_{l} \pi_l s_{l0t}(B_t, p_t) \quad (8)$$

The dynamic demand system is composed of equations (2)-(8).

3.1.6 Examples

Example 1. Durable goods
Consider the case where consumers purchase durable goods that depreciate over time. Suppose that consumers do not make additional purchases when they already own a product. Let x_t be the age of the product consumer owns. If consumers do not own any product, $x_t = 0$. Then, x_{t+1} follows the following stochastic process:

In the case of durable goods, x_t indicates the age of the products or the dummy variables representing the holding of functioning products, for instance. Let $x_t = 0$ be the state where consumers do not possess any product at time t. In the case of durable goods, it is natural to assume that consumers do not consider replacement or additional purchases when they already own functioning products ($x_t \neq 0$)[13]. In this case, consideration set $\mathcal{A}_l(x_t)$ satisfies $\mathcal{A}_l(x_t \neq 0) = \{0\}$. Since durable goods depreciate, x_{t+1}, product holding at time $t+1$, depends on previous state x_t and product choice a_t. The transition process depends on the depreciation rate of the products.

Example 2. Goods with switching costs
When switching cost exists, consumer's flow utility $f_{lj}(x_t, B_t, p_{jt})$ depends on their past choices. Namely, $x_t = a_{t-1}$. Generally, $f_{lj}(x_t = a_{t-1} = j, B_t, p_{jt}) > f_{lj}(x_t = a_{t-1} \neq j, B_t, p_{jt})$ holds if switching costs exist.

Example 3. Experience goods
The quality of the products sold in the market is sometimes uncertain for consumers. Let $x_t^{(j)}$ be the knowledge level each consumer has on product j. Specifically, let $x_t^{(j)} = 0$ denote the state where consumers do not possess incomplete information and $x_t^{(j)} = 1$ denote the state where consumers possess complete information on product j. Then, $x_t^{(j)}$ affects consumer's flow decisive utility $f_{lj}(x_t, B_t, p_{jt})$: If the consumer has incomplete information about the product ($x_t^{(j)} = 0$) and expects the quality of the product to be low, $f_{lj}(x_t^{(j)} = 0, B_t, p_{jt}) < f_{lj}(x_t^{(j)} = 1, B_t, p_{jt})$ holds. Similarly, if the consumer has incomplete information about the products ($x_t^{(j)} = 0$) and expects the quality of the products to be high, $f_{lj}(x_t^{(j)} = 0, B_t, p_{jt}) > f_{lj}(x_t^{(j)} = 1, B_t, p_{jt})$ holds. The knowledge level $x_{t+1}^{(j)}$ depends on the past period's product choice a_t and the past period's knowledge level $x_t^{(j)}$.

Example 4. Durable goods with network effects
Many durable goods exhibit network effects, wherein the value of the products depends on the number of other consumers using the products. When the number of consumers who already own the products is large, consumers can gain high utility by purchasing the products. There are two channels: direct network effect and

[13] This kind of specification is also used and discussed in Chen (2016), studying durable network goods with switching costs.

indirect network effect. First, if the fraction of consumers already using format k, $B_t^{(k)}$, increases, the direct benefit from using the format increases, known as direct network effect. Second, higher $B_t^{(k)}$ increases the variety of complementary products, and the benefit from using the format increases, known as indirect network effects. Here, the number of consumers who already own the products is included as one of the variables in B_t. Hence, consumer's flow utility $f_{lj}(x_t, B_t, p_{jt})$ depends on B_t.

3.2 Firms

On the supply side, we assume that each firm produces only one product, just for simplicity. Firms follow Markov perfect equilibrium, and firms set their product prices given aggregate state variables B. We assume constant marginal costs, and assume that marginal costs are constant over time. We assume that no entry and exit exist in the market and consider the case where firms' strategic variables are only their products' prices.

Firm j's dynamic optimization problem is:

$$\max_{\{p_{jt+\tau}\}_{\tau\in\mathbb{Z}_+}} \sum_{\tau=0}^{\infty} \beta_F^\tau M s_{jt+\tau}(B_{t+\tau}, p_{jt+\tau}, p^*_{-jt+\tau}(B_{t+\tau}))(p_{jt+\tau} - mc_j) \tag{9}$$

where β_F and mc_j represent the firms' discount factor and product j's marginal cost. p^*_{-j} denotes the firms other than firm j's optimal prices.

Then, the Bellman equation that characterizes firm j's value function given that other firms and consumers behave according to Markov perfect equilibrium is:

$$V_j^F(B_t) \quad = \quad \max_{p_{jt}} \left[M s_{jt}(B_t, p_{jt}, p^*_{-j}(B_t))(p_{jt} - mc_j) + \beta_F V_j^F(B_{t+1}(B_t, s_t^{(ccp)}(p_{jt}, p^*_{-j}(B_t)))) \right] \tag{10}$$

The first order condition of the optimization problem is:

$$\begin{aligned} 0 \quad = \quad & \underbrace{M s_{jt}(B_t, p_{jt}, p^*_{-j}(B_t)) + M(p_{jt} - mc_j) \sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \frac{\partial f_{lj}(x_t, B_t, p_{jt})}{\partial p_{jt}} \frac{\partial s_{ljt}^{(ccp)}(x_t, B_t, p_{jt}, p^*_{-j}(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p_{jt}, p^*_{-j}(B_t))}}_{\text{static}} + \\ & \underbrace{M(p_{jt} - mc_j) \sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \sum_{k \in \mathcal{A}_l(x_t)} \frac{\partial \beta_C E\left[V_l^C(x_{t+1}, B_{t+1}(B_t, s_t^{(ccp)}(B_t, p_{jt}, p^*_{-j}(B_t))))|x_t, a_t = k\right]}{\partial p_{jt}} \frac{\partial s_{ljt}^{(ccp)}(x_t, B_t)}{\partial \widetilde{v_{lkt}}(x_t, B_t)}}_{\text{changing expectations of consumers}} + \\ & \underbrace{\beta_F \frac{\partial V_j^F(B_{t+1}(B_t, p_{jt}, p^*_{-j}(B_t)))}{\partial p_{jt}}}_{\text{forward-looking firms}} \end{aligned} \tag{11}$$

The first term represents the static effect of the current price change, namely, the direct effect of the current price change on the current profit by changing current flow utilities. The second term indicates the indirect effect of the current price change on the current profit through the change in consumers' expectations on future outcomes. The third term indicates the effect of current price change on future profits through the change in the current demand. The second and the third term appear in the case of dynamic demand.

Then, equilibrium price p_j^* given aggregate state variables B_t satisfies:

$$p_j^*(B_t) \quad = \quad mc_j - \frac{M s_{jt}(B_t, p^*(B_t)) \cdot (1 + \lambda_j^F(B_t))}{M \sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} \frac{\partial s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} \cdot (1 + \lambda_j^C(B_t))} \tag{12}$$

where

$$\lambda_j^C(B_t) \equiv \frac{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \sum_{k\in\mathcal{A}_l(x_t)} \frac{\partial \beta_C E\left[V_l^C(x_{t+1},B_{t+1}(B_t,s_t^{(ccp)}(B_t,p^*(B_t))))|x_t,a_t=k\right]}{\partial p_{jt}} \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))}}{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}} \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}}$$

$$\lambda_j^F(B_t) \equiv \frac{\beta_F \frac{\partial V_j^F(B_{t+1}(B_t,s_t^{(ccp)}(p_j^*(B_t),p_{-j}^*(B_t))))}{\partial p_{jt}}}{M s_{jt}(B_t,p^*(B_t))}$$

Intuitively, λ_j^C represents the importance of the effect of changing consumers' expectations in firm j's price-setting behavior relative to the static effect. λ_j^F represents the importance of the effect on the firm's future profits relative to the static effect.

By rearranging the equation, we obtain:

$$\begin{aligned}
\frac{p_j^*(B_t) - mc_j}{p_j^*(B_t)} &= \left(-\frac{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial \log p_{jt}} \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}}{s_{jt}(B_t,p^*(B_t))}\right)^{-1} \frac{1+\lambda_j^F(B_t)}{1+\lambda_j^C(B_t)} \\
&= \left(-\left.\frac{\partial \log s_{jt}(B_t,p_j^*(B_t),p_{-j}^*(B_t))}{\partial \log p_{jt}}\right|_{\text{static}}\right)^{-1} \frac{1+\lambda_j^F(B_t)}{1+\lambda_j^C(B_t)}
\end{aligned} \tag{13}$$

Three elements affect the markup: "static" price elasticity of demand defined by $\left.\frac{\partial \log s_{jt}(B_t,p_j^*(B_t),p_{-j}^*(B_t))}{\partial \log p_{jt}}\right|_{\text{static}} \equiv \frac{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial \log p_{jt}} \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}}{s_{jt}(B_t,p^*(B_t))}$, the index representing the effect of consumers' expectations, λ_j^C, and the index representing the effect of forward-looking behavior of the firm, λ_j^F.

The following subsections show that the second and the third terms in the right-hand side of (11) are negligible, namely, the values of λ_j^C and λ_j^F are sufficiently close to zero when CCPs of product (firm) j are small for all consumer types and state variables under the existence of random utility shock following Gumbel distribution.

To proceed with the discussion, we first presume the existence of Markov perfect equilibrium:

Assumption 1 (Existence of Markov perfect equilibrium)**.** *Markov perfect equilibrium exists.*

Next, we define the following terms in advance:

$$\begin{aligned}
\gamma_1 &\equiv \left\|(I - C(B_t))^{-1}\right\|_\infty \\
\gamma_2 &\equiv \max_{l,x_{t+1}} \sum_n \left|\frac{\partial V_l^C(x_{t+1},B_{t+1}(B_t))}{\partial B_{t+1}^{(n)}}\right| \\
\gamma_3 &\equiv \sum_l \sum_{x_t} \max_{k\in\mathcal{A}_l(x_t),n} \left|\frac{\partial B_{t+1}^{(n)}(B_t,s_t^{(ccp)})}{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right| \\
\gamma_4 &\equiv \max_{l,x_t} \sum_{k\in\mathcal{A}_l(x_t)} \frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))} \\
\gamma_5 &\equiv \max_{\tau\in\mathbb{N},m} \sum_n \left|\frac{\partial B_{t+\tau}^{(m)}(B_{t+1}(B_t))}{\partial B_{t+1}^{(n)}}\right|
\end{aligned}$$

Furthermore, we define the following terms for firm j:

$$
\begin{aligned}
b_{j1} &\equiv \max_{l,x_t} \left| \frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} \right|^{-1} \\
b_{j2} &\equiv \max_{\tau \in \mathbb{N}} \left| p_j^*(B_{t+\tau}(B_t)) - mc_j \right| \\
b_{j3} &\equiv \max_{l,\tau \in \mathbb{Z}_+} \sum_n \left| \frac{\partial \log s_{ljt+\tau+1}(B_{t+\tau+1}(B_t))}{\partial B_{t+\tau+1}^{(n)}} \right| \\
b_{j4} &\equiv \max_{\tau \in \mathbb{Z}_+} \sum_n \left| \frac{\partial p_j^*(B_{t+\tau+1}(B_t))}{\partial B_{t+\tau+1}^{(n)}} \right| \\
b_{j5} &\equiv \max_{l} \sum_{\tau=0}^{\infty} \beta_F^{\tau+1} \frac{s_{ljt+\tau+1}(B_{t+\tau+1}(B_t))}{s_{ljt}(B_t)} \\
\nu_j &\equiv \max_{l,x_t} \left| \frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} \frac{\partial s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} \right|
\end{aligned}
$$

where $B_{t+1}^{(n)}$ represents the n-th element of the vector of aggregate state variables B_{t+1}. $B_{t+1}(B_t, s_t^{(ccp)})$ implies that aggregate level state variables B_{t+1} depend on the previous period's aggregate state variables B_t and CCPs $s_t^{(ccp)}$. Regarding b_{j3} and b_{j5}, $s_{ljt+\tau+1}$ is a function of $B_{t+\tau+1}$ and $p^*(B_{t+\tau+1})$, and consequently we can presume that it is a function of only $B_{t+\tau+1}$. ν_j represents the direct responsiveness of current CCP $s_{ljt}^{(ccp)}(x_t, B_t, p_t)$ to changing current price p_{jt}. As discussed, current prices affect consumers' expectations concerning future outcomes and subsequently affect current CCPs $s_{ljt}^{(ccp)}(x_t, B_t, p_t)$. ν_j represents the current CCP's response to the current price change excluding these effects. $C(B_t) \equiv (c_{qr})_{qr}$ is the $N_{B_t} \times N_{B_t}$ dimensional matrix with

$$
c_{qr} \equiv \beta_C \sum_{x_{t+1}} \psi(x_{t+1} | \widetilde{x_t}, B_t, a_t = \widetilde{k}) \sum_n \frac{\partial V_{\widetilde{l}}^C(x_{t+1}, B_{t+1}(B_t))}{\partial B_{t+1}^{(n)}} \sum_{h \in \mathcal{A}_l(x_t)} \frac{\partial B_{t+1}^{(n)}(B_t, s_t^{(ccp)}(B_t, p^*(B_t)))}{\partial s_{lht}^{(ccp)}(x_t, B_t, p^*(B_t))} \frac{\partial s_{lht}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t, B_t, p^*(B_t))}
$$

for $q = G_{B_t}(\widetilde{l}, \widetilde{x_t}, \widetilde{k})$ and $r = G_{B_t}(l, x_t, k)$. $G_{B_t}(\cdot)$ is the one-to-one mapping from $\{(l, x_t, k) \mid l \in \{1, \cdots, L\}, x_t \in X_t, k \in \mathcal{A}_l(x_t)\}$ to $\{1, \cdots, N_{B_t}\}$. N_{B_t} represents the dimension of B_t. $\|A\|_\infty \equiv \max_i \sum_{j=1}^n |a_{ij}|$ is the infinity norm for a $n \times n$ dimensional matrix $A = (a_{ij})$. Note that $\gamma_1, \cdots, \gamma_5$ and $b_{j1} \cdots, b_{j5}$ depend on aggregate state variables B_t. Further, some of these terms are determined by the dynamic demand structure. For instance, the failure rates of products affect some of these values in the case of durable goods.

We impose the following regularity conditions.

Assumption 2 (Regularity Conditions). *1.* $\frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} < 0$ *holds for all* l, x_t, j *(Positive marginal utility of money)*

2. Distribution function of ϵ_{iljt} *is continuous, differentiable and has unbounded support.*

3. $(I - C(B_t))^{-1}$ *exists.*

4. The terms $b_{j1}, \cdots, b_{j5}$ *and* $\gamma_1, \cdots, \gamma_5$ *take finite values.*

The first condition is satisfied as long as the marginal utility of money is positive. The second condition is satisfied if ϵ_{iljt} follows Gumbel distribution. The third and fourth conditions would be satisfied if the equilibrium is well-behaved and ϵ_{iljt} follows Gumbel distribution, even when the number of firms is large. In Appendix B, we discuss the validity of Assumption 2 in detail.

3.3 Changing consumers' expectations

First, consider λ_j^C, the relative importance of changing consumers' expectations in firm j's price-setting behavior. The following proposition shows the upper bound of the absolute value of λ_j^C.

Proposition 1. *Under Assumptions 1 and 2,*

$$\left|\lambda_j^C(B_t)\right| \leq 2\beta_C\gamma_1\gamma_2\gamma_3 b_{j1}\nu_j$$

Proof. See Appendix A. □

3.4 Forward-looking behavior of firms

Next, consider λ_j^F, the relative importance of future profits for firm j in firm j's price-setting behavior. The following proposition shows the upper bound of the absolute value of λ_j^F.

Proposition 2. *Under Assumptions 1 and 2,*

$$\left|\lambda_j^F(B_t)\right| \leq 2\gamma_3(1+\beta_C\gamma_1\gamma_2\gamma_3\gamma_4)(b_{j2}b_{j3}+b_{j4})b_{j5}\gamma_5\nu_j$$

Proof. See Appendix A. □

3.5 Static approximation of firm's behavior under dynamic demand

To make the point clear, let $p_j^{(dynamic)}(B_t) \equiv p_j^*(B_t)$ be the solution of first order condition (11) derived from the dynamic optimization problem given aggregate state variables B_t.

Here, let $p_j^{(static)}(B_t)$ be the solution of the following "static" first order condition:

$$0 = Ms_{jt} + M(p_j^{(static)}(B_t) - mc_j)\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t)\frac{\partial f_{lj}(x_t, B_t, p_j^{(static)}(B_t))}{\partial p_{jt}}\frac{\partial s_{ljt}^{(ccp)}(x_t, B_t, p_j^{(static)}(B_t), p_{-j}^{(dynamic)}(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p_j^{(static)}(B_t), p_{-j}^{(dynamic)}(B_t))} \tag{14}$$

The solution of this condition satisfies the following equation:

$$\frac{p_j^{(static)}(B_t) - mc_j}{p_j^{(static)}(B_t)} = \left(-\left.\frac{\partial \log s_{jt}(B_t, p_j^{(static)}(B_t), p_{-j}^{(dynamic)}(B_t))}{\partial \log p_{jt}}\right|_{\text{static}}\right)^{-1} \tag{15}$$

Here, we impose the following additional assumption:

Assumption 3 (Finite semi-elasticity of CCP)**.** *The maximum value of semi-elasticity of CCPs* $b_{j6} \equiv \max_{x_t,l}\left|\frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}} \cdot \frac{\partial \log s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}\right|$ *takes a finite value.*

The assumption holds when ϵ_{iljt} follows Gumbel distribution with scale parameter 1, since

$$\begin{aligned}
&\left|\frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} \cdot \frac{\partial \log s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))}\right| \\
=& \left|\frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} \cdot \frac{\partial s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))}\frac{1}{s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))}\right| \\
\leq& \max_{x_t,l}\left|\frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}}\left[1 - s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))\right]\right| \left(\because \frac{\partial s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} = s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))\left(1 - s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))\right)\right) \\
\leq& \max_{l,x_t}\left|\frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}}\right|
\end{aligned}$$

Since

$$\begin{aligned}
\nu_j &\equiv \max_{l,x_t}\left|\frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}}\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}\right| \\
&\leq \underbrace{\max_{l,x_t}\left|\frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}}\cdot\frac{\partial \log s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}\right|}_{\text{Static semi-elasticity of CCP}}\cdot\underbrace{\left[\max_{l,x_t} s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))\right]}_{\text{CCP}}
\end{aligned}$$

, ν_j takes a small value if $\max_{l,x_t} s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))$ is sufficiently close to zero under Assumption 3. Hence, using Propositions 1 and 2, we obtain the following informal but intuitive claim:

Proposition 3. *Under Assumptions 1, 2, and 3, $s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) \approx 0\ \forall l,x_t$ implies $p_j^{(dynamic)}(B_t) \approx p_j^{(static)}(B_t)$.*

The claim shows that a firm's optimal price solving dynamic problem is close to the price solving static condition, if the CCPs of the firm's product are sufficiently small and close to zero for all consumer types and (individual level) state variables. The performance of the approximation depends on how large the values of $b_{j1},\cdots,b_{j6}$ and $\gamma_1,\cdots,\gamma_5$ are. Nevertheless, if these values are finite, optimal prices based on the static condition converge to the true prices based on the dynamic condition as the values of CCPs go to zero for all consumer types and state variables.

If we further assume that firms are symmetric, we can derive stronger results. Here, we impose the following assumption:

Assumption 4 (Symmetry of firms in terms of CCPs)**.** *$s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))$ take common values regardless of the product $j\in\mathcal{J}$ for all x_t and l.*

Let J be the number of the firms in the market. Since $s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) = \frac{1-s_{l0t}^{(ccp)}(x_t,B_t,p^*(B_t))}{J} \leq \frac{1}{J} \to 0\ (J\to\infty)$ under Assumption 4, we obtain the following statement:

Corollary 1. *Suppose that Assumptions 1 and 4 hold. Furthermore, suppose that Assumptions 2 and 3 hold for all $j\in\mathcal{J}$. Then, $\lim_{J\to\infty}\left|p_j^{(dynamic)}(B_t)-p_j^{(static)}(B_t)\right| = 0$ holds.*

Note that Proposition 3 includes the case of asymmetric firms, such as the case where a dominant firm exists in the market yet firm j is fringe.

3.6 Meaning of $\max_{l,x_t} s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) \approx 0$

Here, we interpret the meaning of $\max_{l,x_t} s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) \approx 0$.

To clarify, suppose that random utility shock ϵ_{iljt} follows Gumbel distribution with scale parameter σ_ϵ. Then, $s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))$ is in the following form:

$$s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) = \frac{\exp\left(\frac{\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}{\sigma_\epsilon}\right)}{\sum_{k\in\mathcal{A}_l(x_t)}\exp\left(\frac{\widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))}{\sigma_\epsilon}\right)}$$

To understand when the condition does not or does hold, consider the case where the scale parameter of Gumbel distribution σ_ϵ is so small. If $\sigma_\epsilon\to 0$, it converges to the case where no random utility shock exists. If we consider the case where a consumer type l exists such that $\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t)) > \widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))$ for all $k\in\mathcal{A}_l(x_t)-\{j\}$, we can show that $s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) \to 1\ (\sigma_\epsilon\to 0)$ holds[14]. Then, the condition $\max_{l,x_t} s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) \approx 0$ does not hold. Therefore, the existence of random utility shock is necessary to

[14]In contrast, $s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t)) \to 0\ (\sigma_\epsilon\to 0)$ holds for $k\in\mathcal{A}_l(x_t)-\{j\}$.

derive the condition. Intuitively, current buyers may not be future buyers under random utility shock. Then, the connection between current and future periods is small, and firm is less likely to consider future outcomes.

Next, consider the case where some consumers strongly prefer the product[15]. Then, $s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))$ take large values far from zero for part of l, and $\max_{l,x_t} s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t)) \approx 0$ does not hold. In that sense, small persistent heterogeneity of consumers is required to satisfy the condition. Intuitively, if consumers' persistent heterogeneity is large and part of the consumers strongly prefer the product, they would always purchase the product given the opportunity, even under random utility shock. Then, current buyers would be the future buyers, and the connection between current and future periods would be large. In this case, the firm is more likely to consider future periods. Otherwise, when persistent heterogeneity is small, future periods are unimportant for the firm.

Finally, small market share of the firm (product) is also necessary. It holds when the value of each product is similar but the number of firms is large, or when the product's value is much lower than the other products (fringe firm case). Under product differentiation and the existence of utility shock following Gumbel distribution, even small firms can gain positive profits, in contrast to the case of perfect competition or homogeneous goods with many firms. If the market share is small, the probability that current buyers coincide with the future buyers is small. Then, the connection between current and future periods is small, and the firm is less likely to consider the effects future period.

3.7 Static monopolistic competition under dynamic demand

If the condition $s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t)) \approx 0\ \forall x_t, l$ holds, we can further argue that firm j behaves approximately like a (static) monopolistic competitor, which has no impact on its competitors, but is free to choose the price that maximizes its profits as a monopolist (Chamberlin, 1949). To see this, we assume that ϵ_{iljt} follows the Gumbel distribution with scale parameter 1. Note that rescaling both $f_{lj}(x_t, B_t, p_{jt})$ and ϵ_{iljt} to set the standard deviation of random utility shock ϵ_{iljt} to 1 does not make any difference in CCPs and the equilibrium. Let $b_{j7} \equiv \max_{l,x_t} \left| \frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} \right|$. Then, (static and dynamic version of) cross price elasticities of product $k \neq j$ concerning product j, $\frac{\partial \log s_{kt}}{\partial p_{jt}}$ and $\frac{\partial \log s_{kt+\tau}}{\partial p_{jt}} (\tau \in \mathbb{Z}_+)$, satisfy the following inequalities shown in the proposition:

Proposition 4. *If ϵ_{iljt} follows the Gumbel distribution with scale parameter 1 and Assumptions 1, 2, and 3 hold, following inequalities hold:*

$$\begin{aligned} \left| \frac{\partial \log s_{kt}(B_t, p^*(B_t))}{\partial p_{jt}} \right| &\leq [b_{j7} + 2\beta_C \gamma_1 \gamma_2 \gamma_3 b_{j6}] \cdot \left[\max_{l,x_t} s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t)) \right] \ (k \neq j) \\ \left| \frac{\partial \log s_{kt+\tau}(B_{t+\tau}(B_{t+1}(B_t, s_t^{(ccp)}(B_t, s_t^{(ccp)}(B_t, p^*(B_t))))))}{\partial p_{jt}} \right| &\leq 2\gamma_3(1 + \beta_C \gamma_1 \gamma_2 \gamma_3 \gamma_4) b_{j3} \gamma_5 b_{j6} \left[\max_{l,x_t} s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t)) \right] \ (k \in \mathcal{J}, \tau \in \mathbb{N}) \end{aligned}$$

Proof. See Appendix A. □

Hence, if $s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t)) \approx 0\ \forall x_t, l$ holds, the effect of firm j's price on competitors' demand (current demand/future demand) is negligible. Consequently, the role of firm j's strategic incentive considering the effects on competitors is limited, and firm j behaves approximately like a static monopolistic competitor under the assumptions.

Note that our model is not restricted to a case where all the firms behave approximately as monopolistic competitors, like the standard monopolistic competition model (Dixit and Stiglitz, 1977). Our model encompasses the case where a handful of large firms with the ability to manipulate the market coexist with small firms having small market share as in Shimomura and Thisse (2012). Proposition 4 shows that if the CCPs are small for all consumer types and state variables, the small firm has negligible effect on the market even under dynamic demand.

So far, we have mainly considered the case where the random utility shock follows the Gumbel distribution. As Anderson et al. (1992) discussed, even small firms can set prices higher than the products' marginal costs in a

[15] See the numerical model (the case of $f_{sd} = 3$) in Section 4 as an example.

static setting under Gumbel distribution, which contrasts with the case of perfect competition or homogeneous goods with many firms. Conversely, if the random utility shock follows a uniform or normal distribution, the equilibrium prices firms set converge to their marginal costs as the number of firms increases in static symmetric-firm models (Perloff and Salop, 1985; Gabaix et al., 2016). In Appendix C, we consider whether this section's results hold under distributions other than Gumbel.

4 Numerical experiments

The analytical results show that a firm's price-setting behavior can be approximated as static under dynamic demand with random utility shock following Gumbel distribution if the CCPs of the firm's product are small for all consumer types and state variables. Nevertheless, the extent to which the approximation works well is unclear from the analytical results. This section numerically shows the extent to which prices solving the static optimality condition well approximate the dynamic behavior of firms, by building a numerical model of durable goods[16], one of the typical examples of goods with dynamic demand.

In the numerical experiments, we conduct two types of experiments, which are common in empirical applications:

1. Computing markup rates from data:

   We first numerically solve the dynamic equilibrium (based on the dynamic optimality condition (dynamic optimality condition (13)) and generate the simulated data, including equilbrium product prices. Then, we recover firm $j = 1$'s marginal cost based on the following static optimality condition from the observed equilibrium prices $p^{(data)}(B_t)$, calculate markups, and compare the results with the ones based on the dynamic optimality condition (13):

$$\frac{p_j^{(data)}(B_t) - mc_j}{p_j^{(data)}(B_t)} = \left(-\left.\frac{\partial \log s_{jt}(B_t, p_j^{(data)}(B_t), p_{-j}^{(data)}(B_t))}{\partial \log p_{jt}}\right|_{\text{static}}\right)^{-1}$$

   Note that we can easily guess that the static solution is sufficiently close to the dynamic solution when the maximum of the CCPs of firm $j = 1$ is sufficiently close to zero, as in the discussion in Section 3.5.

2. Computing cost path-through rate:

   We compute two types of equilibria. The first one is the equilibrium where all the firms follow the following static optimality condition:

$$\frac{p_j^{(static)}(B_t) - mc_j}{p_j^{(static)}(B_t)} = \left(-\left.\frac{\partial \log s_{jt}(B_t, p_j^{(static)}(B_t), p_{-j}^{(static)}(B_t))}{\partial \log p_{jt}}\right|_{\text{static}}\right)^{-1}$$

   The second one is the equilibrium where all the firms follow the dynamic optimality condition (13).

   We repeat solving the two equilibria for baseline levels of marginal costs and counterfactual levels of marginal costs (10% increase from the baseline levels). Then, we compute the cost pass-through rates for two types of equilibria, and compare the outcomes.

   Even in this case, we can easily guess that the static solution is sufficiently close to the dynamic solution when the maximum of all the firms' CCPs are sufficiently close to zero, as in the discussion in Section 3.5.

[16] We can think of many kinds of electronic devices, such as light bulbs, washing machines, PCs, as the examples of the products represented in the numerical model. Note that we do not introduce the existence of used goods market, which is not negligible in the automobile industry.

4.1 Model settings

We employ the following model settings.

State variables

Let x_t be the "age" of the product, namely, the passed time since the last purchase. $x_t = 0$ denotes the state where consumers do not possess any product at the beginning of time t. We assume that products always fail when their ages reach x_{max}. Let B_{lt} be the fraction of type l consumers not owning any product at the beginning of time t. Namely, $B_{lt} = Pr_{lt}(x_t = 0)$ holds.

Consideration set

We assume that consumers do not consider replacement or additional purchases when they already own functioning products ($x_t \neq 0$). Consideration set $\mathcal{A}_l(x_t)$ is specified in the following form:

$$\mathcal{A}_l(x_t) = \begin{cases} \mathcal{J} \cup \{0\} & \text{if } x_t = 0 \\ \{0\} & \text{if } x_t \neq 0 \end{cases}$$

Consumers' flow utility

Consumers' flow utility is specified as follows:

$$\begin{aligned} f_{lj}(x_t = 0, B_t, p_{jt}) &= \widetilde{f_{lj}} + f_0 - \alpha p_{jt} \; j \in \mathcal{J} \\ f_{l0}(x_t, B_t) &= \begin{cases} 0 & \text{if } x_t = 0 \\ f_0 & \text{if } x_t \neq 0 \end{cases} \end{aligned}$$

$\widetilde{f_{lj}}$ indicates the flow utility of consumer l from purchasing product j at time t. f_0 indicates the flow utility of consumers using the products. Furthermore, we assume that ϵ_{ijt} follows Gumbel distribution with scale parameter σ_ϵ. Then, $\frac{\alpha}{\sigma_\epsilon}$ represents the consumers' marginal utility of money.

Here, we consider the existence of two types of consumers $l = 1$ and $l = 2$. $\widetilde{f_{lj}}$ is specified as follows:

$$\widetilde{f_{lj}} = \begin{cases} \overline{f} + f_{sd} & j = 1, l = 1 \\ \overline{f} - f_{sd} & j = 1, l = 2 \\ \overline{f} & j \neq 1, l = 1, 2 \end{cases}$$

Type $l = 1$ consumers strongly prefer product $j = 1$ more than type $l = 2$ consumers when $f_{sd} > 0$. The valuation of products other than product $j = 1$ are the same for all the consumers. We can interpret f_{sd} as the intensity of consumers' persistent heterogeneity.

State transition

We assume that each product owned by consumers fails with probability $1 - \phi$, if the product's age is under x_{max}; here, ϕ denotes the survival rate of the products. Furthermore, we presume that the products always fail if their age reaches x_{max}. Then, $Pr_{lt+1}(x_{t+1} = 0)$ satisfies the following equation:

$$Pr_{lt+1}(x_{t+1} = 0) = (1 - \widetilde{s_{l0t}}(x_t = 0, B_t, p_t))(1 - \phi) + \widetilde{s_{l0t}}(x_t = 0, B_t, p_t)$$

where $\widetilde{s_{l0t}}(x_t = 0, B_t, p_t)$ represents the fraction of consumers not possessing products after purchase decisions, and the term is defined as:

$$\widetilde{s_{l0t}}(x_t = 0, B_t, p_t) = Pr_{lt}(x_t = 0) \cdot s_{l0t}^{(ccp)}(x_t = 0, B_t, p_t)$$

Firms

We consider the case where there are $|\mathcal{J}| = J$ firms in the market, and each firm produces only one product. The demand structure of firms $j \neq 1$ is symmetric, as represented in the forms of $f_{lj}(x_t = 0, B_t, p_{jt})$; however, firm $j = 1$ is not symmetric when $f_{sd} > 0$.

Furthermore, we assume that marginal costs are constant across firms: $mc_j = mc\ \forall j \in \mathcal{J}$.

Parameter settings

In the baseline case we employ the following parameter settings: $\alpha = 2, \overline{\overline{f}} = 5, f_0 = 0.4, \sigma_\epsilon = 1, \beta_C = \beta_F = 0.9, \phi = 0.9, \pi_1 = 0.01, mc = 2, x_{max} = 100$. Here, we define the term $\overline{\overline{f}} \equiv \overline{f} + \frac{1-\beta_C^{x_{max}+1}}{1-\beta_C} f_0$, and set the parameter value of $\overline{\overline{f}}$, since the value of $\overline{f}$ and f_0 do not affect the consumers' discounted sum of flow utility given $\overline{\overline{f}}$ and f_{sd}[17]. Note that $\phi = 0.9$ implies that products fail with probability 0.1 in each period. $\pi_1 = 0.01$ implies that fraction of type $l = 1$ consumers is only 1%. Even though the weight among all the consumers is small, they significantly impact firm $j = 1$'s behavior when persistent heterogeneity of consumers is large, as shown later.

For details of the experiments and the algorithm for solving the equilibrium, see Appendix D.

4.2 Results

The experiment results are shown in Figures 1-5. In the experiments, we change the number of firms and the scale of consumers' persistent heterogeneity (f_{sd}) to see how the results change. We look at the cases where $Pr_{lt}(x_t = 0) = 1$ holds for all l, namely, cases where no consumers own the goods at the beginning of time t.

4.2.1 Experiment 1: Computing markup rates from data

First, Figure 1 shows each type of consumers' CCPs of the product (firm) $j = 1$ in the simulated data. Other than CCPs, market share, which is calculated as their weighted sum is also plotted. The left panel shows the case where no persistent consumer heterogeneity exists ($f_{sd} = 0$). Since no persistent consumer heterogeneity exists, the two types of CCPs coincide, and market shares also coincide. As the number of firms increases, CCPs and the market share decline. Conversely, the right panel shows the case where persistent consumer heterogeneity exists ($f_{sd} = 3$). Type $l = 1$ consumers strongly prefer product $j = 1$, and their CCPs of product $j = 1$ is over 20%, even when the number of firms exceeds 10. In contrast, type $l = 2$ consumers' valuation of product (firm) $j = 1$ is low, and CCPs are close to zero, regardless of the number of firms (products). Since the ratio of type $l = 1$ consumers is only 1%, the market share are very close to the CCP of type $l = 2$ consumers, who comprise 99% of consumers.

[17]Discounted sum of flow utility from purchasing product j at time t excluding the expected future utility after the the product failure is $\widetilde{f_{lj}} + \sum_{\tau=0}^{x_{max}} \beta_C^\tau f_0 = \widetilde{f_{lj}} + \frac{1-\beta_C^{x_{max}+1}}{1-\beta_C} f_0$.

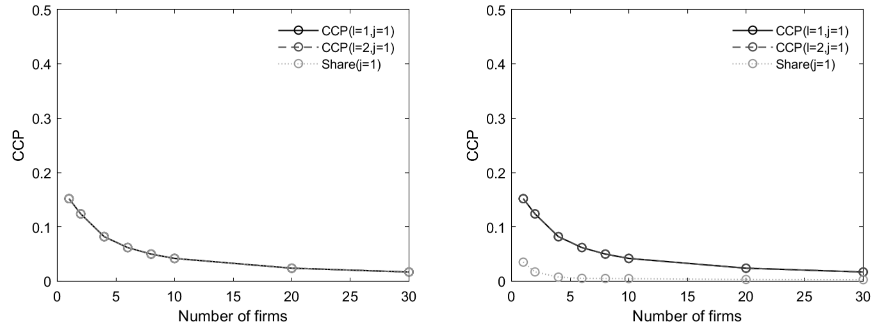

Figure 1: Number of firms and CCPs

Notes.
The figures show the values of $s_{ljt}^{(ccp)}(x_t = 0, B_t, p^*(B_t))$ ($j = 1$; CCPs) and $s_{jt}(B_t, p^*(B_t))$ ($j = 1$; market share) in each setting.
Left: Without persistent consumer heterogeneity ($f_{sd} = 0$)
Right: With persistent consumer heterogeneity ($f_{sd} = 3$)
"Share" is defined as the weighted sum of each type of consumers' CCP.

Next, we look at how the the solutions derived from the static optimality condition well approximate the true solutions based on the dynamic optimality condition. Figure 2 shows the results on firms' markups. When no persistent consumer heterogeneity exists ($f_{sd} = 0$) and only one firm ($j = 1$) exists, the static condition yields significant biases: firm $j = 1$'s markup is roughly 1.25 even though the true value is roughly 1.5, as shown in the left panel of Figure 2. Nevertheless, as the number of firms increases, the markups derived from the static condition converge to the markup based on the actual dynamic optimality condition. When the number of firms is 30, the two values are almost the identical. Note that since the random utility shock following Gumbel distribution exists, markup does not converge to 1 when the number of firms increases, unlike the case of the homogeneous good.

In contrast, the result differs significantly when persistent consumer heterogeneity exists ($f_{sd} = 3$). As shown in the right panel, markups based on static condition are largely different from the dynamic solution, even when the number of firms increases[18].

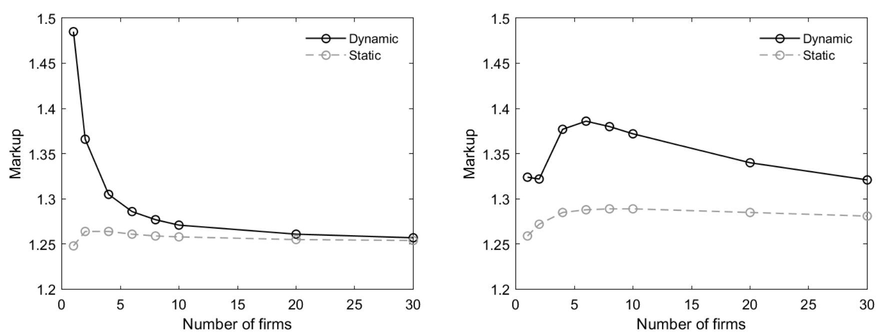

Figure 2: Number of firms and markups

Notes.
The figures show the values of product (firm) $j = 1$'s markup, defined as $\frac{p_{jt}}{mc_j}$ ($j = 1$), in each setting.
Left: Without persistent consumer heterogeneity ($f_{sd} = 0$)
Right: With persistent consumer heterogeneity ($f_{sd} = 3$)

Figure 3 shows the values of λ_j^C and λ_j^F. When consumers' persistent heterogeneity exists ($f_{sd} = 3$), λ_j^F does not converge to zero. Conversely, when consumers' persistent heterogeneity does not exist ($f_{sd} = 0$), the value of λ_j^F converges to zero as the number of firms increases. Note that the values of $|\lambda_j^C|$ are close to zero in

[18]The right panel of Figure 2 shows the case where the firm's equilibrium price solving the dynamic optimality condition given $J = 1$ (monopolistic case) is lower than the equilibrium price given $J \geq 3$ (oligopolistic case). Gul (1987) and Ausubel and Deneckere (1987) theoretically showed that durable goods producers could set higher prices in an oligopolistic environment than the cases in a monopolistic environment. The results in Figure 2 reflect of this phenomenon.

Figure 3. Hence, in our setting, when considering firms' dynamic decisions, consumers' changing expectations is less important than forward-looking behavior of firms.

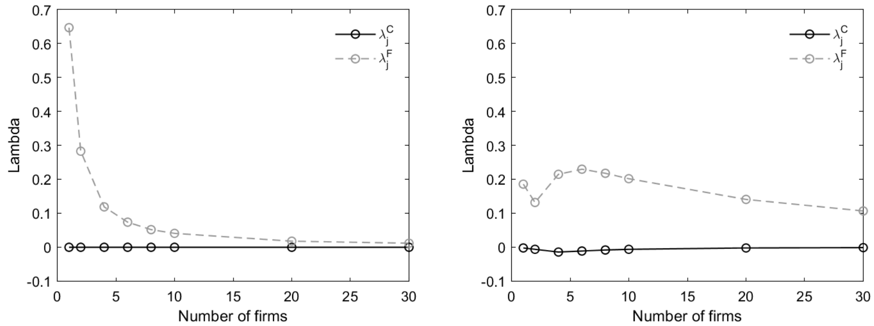

Figure 3: Number of firms and λ_j^C, λ_j^F

Notes.
The figures show the values of λ_j^C and λ_j^F ($j = 1$) in each setting.
Left: Without persistent consumer heterogeneity ($f_{sd} = 0$)
Right: With persistent consumer heterogeneity ($f_{sd} = 3$)

4.2.2 Experiment 2: Computing cost path-through rate

First, Figure 4 shows the maximum value of the all the firms' CCPs under the dynamic equilibrium and baseline level of marginal costs. We can see that the maximum of CCPs gets smaller as the number of firms increases in the no persistent consumer heterogeneity case (left figure), but it does not in the case with persistent consumer heterogeneity (right figure).

Next, Figure 5 shows the pass-through rate of firm (product) $j = 1$. "Static" represents the pass-through rate derived from the model based on the static optimality conditions. As in the case of markups, we can observe significant biases when no consumer heterogeneity exists ($f_{sd} = 0$) and the number of firms is 1. Nevertheless, the pass-through rate based on the static condition converges to the dynamic counterpart, as the number of firms increases. In contrast, when persistent heterogeneity exists ($f_{sd} = 3$), it does not necessarily converge to the dynamic solution, even though the difference is not large.

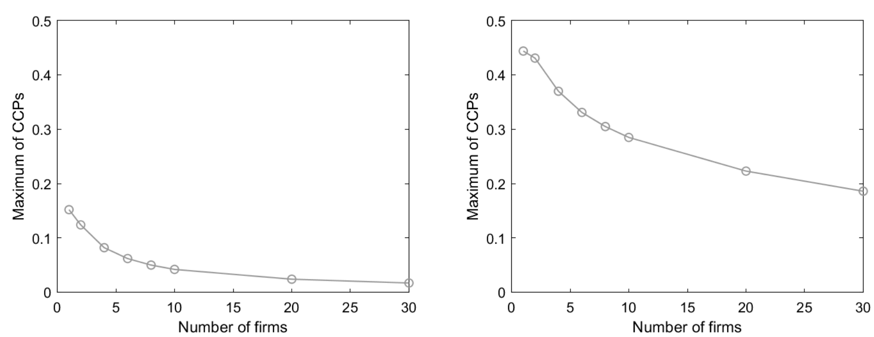

Figure 4: Number of firms and the maximum of CCPs

Notes.
The figures show the values of $\max_{l,j} s_{ljt}^{(ccp)}(x_t = 0, B_t, p^{(dynamic)}(B_t))$ under the baseline level of marginal costs.
Left: Without persistent consumer heterogeneity ($f_{sd} = 0$)
Right: With persistent consumer heterogeneity ($f_{sd} = 3$)

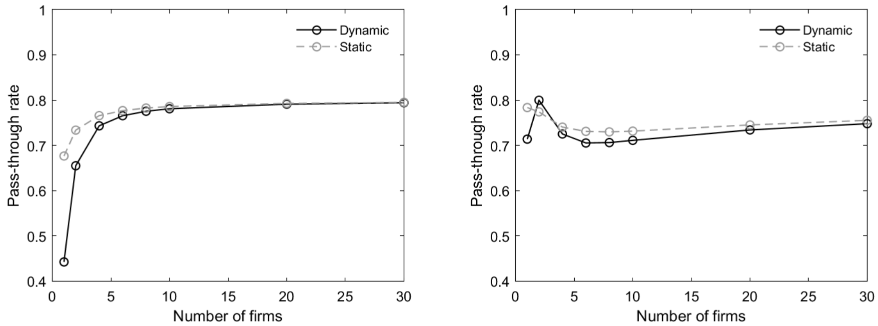

Figure 5: Number of firms and cost pass-through rates (10% increase in marginal costs)

Notes.
The figures show the values of cost pass-through rate of product firm $j = 1$ in response to 10% increase in all the firms' marginal costs in each setting. Pass-through rate is defined as $\frac{p'_{jt}-p_{jt}}{mc'_j-mc_j}$, where mc'_j and p'_{jt} denote the marginal cost and the equilibrium price after the 10% cost increase. We consider the case where all the firms' marginal costs increase permanently by 10%.
Left: Without persistent consumer heterogeneity ($f_{sd} = 0$)
Right: With persistent consumer heterogeneity ($f_{sd} = 3$)

Overall, the results of the numerical experiments are consistent with the analytical results, and provide several implications. First, we can well-approximate the markup rate of a firm based on the static optimality condition using the observed data, when the firm's CCP is small for all consumer types and state variables. Second, if all the firms' CCPs are small, we can compute the static equilibrium which is sufficiently close to the dynamic equilibrium, and we can well-approximate the cost pass-through rate using the static equilibrium model. Third, the static condition does not necessarily yield a good approximation if the persistent consumer heterogeneity is large, because small CCPs conditions do not necessarily hold.

5 Applications

This section presents examples of how the main results presented in Section 3 can be applied or provide insight into each type of goods with dynamic demand.

Example 1 (Continued)**.** Durable goods
In the case of durable goods, when firms can commit to future prices, greater current demand implies smaller future demand due to the durability of goods. Hence, firms have incentives to set higher prices considering their future profits ($p_j^{(dynamic)}(B_t) > p_j^{(static)}(B_t)$). If firms cannot commit, consumers expect future price declines and postpone their purchases. Then, firms have incentives to lower current prices. In extreme cases, $p_j^{(dynamic)}(B_t) \approx mc_j$ holds, which is known as Coase's conjecture (Coase, 1972, Stokey, 1981).

This study shows that firm's forward-looking behavior and the effect of consumers' expectations is negligible when $\max_{l,x_t} s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t)) \approx 0$ holds. Hence, ignoring firm's dynamic price-setting behavior causes minor problems if the industry of interest satisfies the condition.

Example 2 (Continued)**.** Goods with switching cost
Recall that state variable x_t is the past choice of the consumer a_{t-1}, namely, $x_t = a_{t-1}$. Consumers opt to choose the previously purchased goods, and CCPs $s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))$ are large and far from zero when $x_t = a_{t-1} = j$. Then, based on this study's results, firms' decisions are far from static. This is consistent with the previous studies stressing the importance of firms' dynamic price-setting behavior (investing/harvesting).

Example 3 (Continued)**.** Experience goods
In the case of experience goods, recall that consumers do not have complete information on the quality of products and learn them through purchasing. Let $x_t^{(j)}$ be the knowledge level of consumers on product j, Let $x_t^{(k)} = 0$ and $x_t^{(k)} = 1$ denote the state where consumers possess incomplete information and the state where consumers possess complete information on product j. Let $x_t^{(j)}$ depend on whether a consumer has already

purchased product j. Here, consider the case where consumers' flow decisive utility of product j is smaller than the true value ($f_{lj}(x_t^{(j)}=1,B_t,p_{jt}) > f_{lj}(x_t^{(j)}=0,B_t,p_{jt})$) when the consumer has incomplete information on the product ($x_t^{(j)}=0$). If the CCPs of choosing product j are larger than zero given complete knowledge of the product($x_t^{(j)}=1$), the main results of this article show that firm j's behavior cannot be approximated as static. This holds even when the CCPs of choosing product j is sufficiently close to zero given incomplete knowledge of the product($x_t^{(j)}=0$).

Example 4 (Continued)**.** Durable goods with network effects
Consider the case where several types of formats exist in the products[19]. Let $\mathcal{J}^{(k)}$ be the set of format k products (firms). Furthermore, let $x_t^{(k)}$ be a dummy variable indicating whether the consumer owns format k products, and let $B_t^{(k)}$ be the fraction of consumers already using format k products. If network effects exist, flow utility $f_{lj}(x_t,B_t,p_{jt})$ $j\in\mathcal{J}^{(k)}$ depends on $B_t^{(k)}$. Firms generally have incentives to offer penetration pricing (Katz and Shapiro, 1986), namely, setting lower prices at early stages of the diffusion when network effects exist. Here, consider a firm with a small market share among firms producing format k products, implying that the firm's CCPs are small. The main result of this article shows that $p_j^{(dynamic)}(B_t) \approx p_{jt}^{(static)}(B_t)$ holds when $\max_{l,x_t} s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) \approx 0$. Hence, the firm does not make much future profit and has little incentive to offer penetration pricing.

6 Comparison with natural resources extraction

Even though our focus has been on markets with dynamic demand, we find an analogy with natural resources extraction. Consider the case of fisheries, which is a typical example of natural resources extraction[20]. If each fisher catches too many fish, the number of surviving fish shrinks, and it is challenging for fishers to catch them in the next period. Nevertheless, each fisher behaves myopically if the area hosts a large number of fishers in the area (open access) with no coordination among them; their current action has negligible effects on the remaining number of fish, and they do not have incentives to adjust their activities to maintain the resources in the future. In that sense, fishers' behavior is very close to static. The problem of social inefficiency is known as "Tragedy of the commons".

Conversely, if one fisher controls the area (closed access), the fisher's problem is dynamic. Consider the case where fish is cultured by one fisher. If catching too many fish in the current period, the number of the remaining fish gets smaller, and future profits decrease. Then, the fisher would consider the remaining future number of fish and dynamically optimize the number of fish caught.

Figure 6 illustrates the comparison between natural resources extraction (fishery) and the durable goods market as an example of dynamic demand. Consumers in the durable goods market correspond to fish. Firms in the durable goods market correspond to fishers. Consumers are "caught" by firms, and current "capture" affects future "capture". Consumers' preference corresponds to the location of the fish.

In the case of durable goods, if consumers' preferences change stochastically (random utility shock exists), the firm can catch other consumers that approach the firm in the next period, even if the firm caught a large number of nearby consumers in the current period. Similarly, in the case of fisheries, if the area is open access, and fish can move freely, each fisher can catch other fish entering the area near the fisher in the next period, even when catching numerous fish near the fisher in the current period (shown in the left panel of Figure 6). In contrast, in the durable goods case, if consumers' preference is constant over time (no random utility shock exists), the number of consumers near the firm would be smaller in the next period if the firm catches the consumers near the firm in the current period. Analogously, in the fishery case, if the area is closed access and fish cannot move freely, the number of fish in the controlled area would be smaller in the next period if the firm catches numerous fish in the area in the current period (shown in the right panel of Figure 6). In both cases, catching many consumers/fish is not profitable in the long run, and the firm/fisher has incentives to optimize their action dynamically.

[19]Consider the case of format war between VHS and Beta, for instance.
[20]See Conrad (2010) and others.

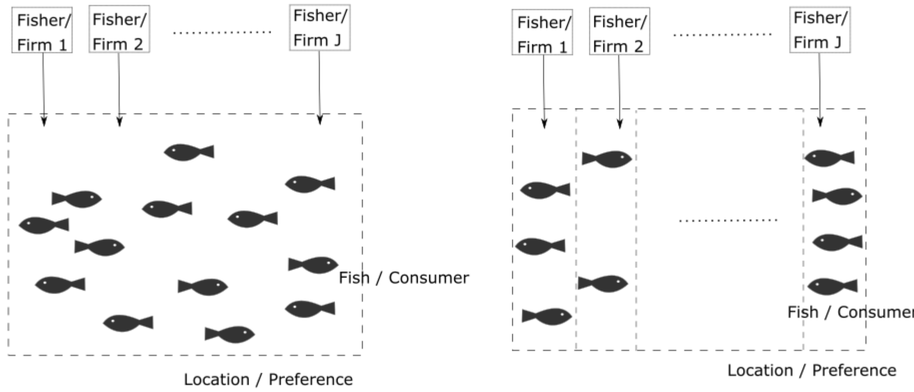

Figure 6: Analogy with natural resources extraction (The case of fishery)

Notes.
Left: Open access (fishery) / Without persistent consumer heterogeneity (durable goods) case
Right: Closed access (fishery) / With persistent consumer heterogeneity (durable goods) case

7 Conclusions

This study developed a unified theoretical framework and investigated in what cases a firm's dynamic price-setting behavior can be approximated as static under dynamic demand. The results show that a firm's dynamic price-setting behavior is close to the static one under dynamic demand with random utility shock following Gumbel distribution, when consumers' CCPs of choosing the firm's product are small for all consumer types and state variables. Small persistent heterogeneity and small market share of the firm are necessary to satisfy the conditions. The results in our study would justify the simple static specifications and further facilitate the studies on the industries with dynamic demand, if the conditions are plausible to assume.

In contrast, based on this article's discussion, we should note that applying static supply-side model is inadequate when the firm of interest has a large market share, or large persistent consumer heterogeneity exists. In some cases, such as the case of merger policies, we are interested in the behavior of large firms. Hence, applying static supply-side model may yield misleading policy implications. Further consideration of more accessible methods to analyze industries with large firms is necessary for further development.

Although we have not introduced firms' dynamic investment decisions, oblivious equilibrium and variants (Weintraub et al., 2008; Benkard et al., 2015; and Ifrach and Weintraub, 2017) have considered situations where small firms keep track of their own state variables when making decisions and strategic interactions are negligible under static demand structures. They are motivated by the need to analyze industries with many firms, where fully solving a Markov perfect equilibrium is sometimes computationally demanding. Their ideas and motivations are close to the current article, and investigating the properties of the combinations of the two ideas is an interesting extension.

Finally, even though this article focuses on the discrete choice models, previous studies have investigated the connections between the discrete choice models and the representative consumer models (e.g. CES utility function) under static framework (Anderson et al., 1992; Thisse and Ushchev, 2018). Investigating firms' behavior under representative consumer models with dynamic demand is an interesting avenue for future research.

A Proof

To prove Propositions 1, 2, and 4, we show the following lemmas in advance.

Lemma 1. *Following equations hold:*

$$\sum_{k\in\mathcal{A}_l(x_t)}\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\right| = 2\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\ \forall x_t,B_t,p_t,j\in\mathcal{A}_l(x_t)$$

$$\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\right| \leq \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\ \forall x_t,B_t,p_t,j\in\mathcal{A}_l(x_t)$$

Proof. Since $\sum_{k\in\mathcal{A}_{lt}(x_t)} s_{lkt}^{(ccp)}(x_t,B_t,p_t)=1$, we have:

$$\begin{aligned}
\sum_{k\in\mathcal{A}_l(x_t)-\{j\}}\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\right| &= -\sum_{k\in\mathcal{A}_l(x_t)-\{j\}}\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\ \left(\because \frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}<0\ \forall k\neq j\right)\\
&= -\frac{\partial\left(1-s_{ljt}^{(ccp)}(x_t,B_t,p_t)\right)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\\
&= \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}
\end{aligned}$$

Then, for $k\in\mathcal{A}_l(x_t)$, we obtain the following equations:

$$\begin{aligned}
\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\right| &\leq \sum_{k\in\mathcal{A}_l(x_t)-\{j\}}\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\right|\\
&= \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}
\end{aligned}$$

$$\begin{aligned}
\sum_{k\in\mathcal{A}_l(x_t)}\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\right| &= \left|\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\right| + \sum_{k\in\mathcal{A}_l(x_t)-\{j\}}\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\right|\\
&= \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)} + \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\\
&= 2\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}
\end{aligned}$$

□

Lemma 2. $\sum_{k\in\mathcal{A}_l(x_t)}\left|\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{lkt}}(x_t,B_t,p_t)}\right| = 2\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}\ \forall x_t,B_t,p_t,j\in\mathcal{A}_l(x_t)$

Proof. It is sufficient to show that $\sum_{k\in\mathcal{A}_l(x_t)-\{j\}}\left|\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{lkt}}(x_t,B_t,p_t)}\right| = \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{ljt}}(x_t,B_t,p_t)}$.

First,

$$\begin{aligned}
s_{ljt}^{(ccp)}(x_t,B_t,p_t) &\equiv Pr(\widetilde{v_{ljt}}(x_t,B_t,p_t)+\epsilon_{iljt} > \widetilde{v_{lkt}}(x_t,B_t,p_t)+\epsilon_{ilkt}\ \forall k\in\mathcal{A}_l(x_t)-\{j\})\\
&= Pr(\widetilde{v_{ljt}}(x_t,B_t,p_t)-\widetilde{v_{lkt}}(x_t,B_t,p_t) > \epsilon_{ilkt}-\epsilon_{iljt}\ \forall k\in\mathcal{A}_l(x_t)-\{j\})\\
&= Pr(z_{lt}^{(j,k)}(x_t,B_t,p_t) > \widetilde{\epsilon_{ilt}}^{(k,j)}\ \forall k\in\mathcal{A}_l(x_t)-\{j\})
\end{aligned}$$

Here, we define $z_{lt}^{(j,k)}(x_t,B_t,p_t)\equiv\widetilde{v_{ljt}}(x_t,B_t,p_t)-\widetilde{v_{lkt}}(x_t,B_t,p_t)$ and $\widetilde{\epsilon_{ilt}}^{(k,j)}\equiv\epsilon_{ilkt}-\epsilon_{iljt}$. Then,

$$\begin{aligned}
&\sum_{k\in\mathcal{A}_l(x_t)-\{j\}}\left|\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{lkt}}(x_t,B_t,p_t)}\right|\\
&= -\sum_{k\in\mathcal{A}_l(x_t)-\{j\}}\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{lkt}}(x_t,B_t,p_t)}\quad\left(\because \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial\widetilde{v_{lkt}}(x_t,B_t,p_t)}<0\ \forall k\neq j\right)\\
&= -\left[\sum_{k\in\mathcal{A}_l(x_t)-\{j\}}\frac{\partial z_{lt}^{(j,k)}(x_t,B_t,p_t)}{\partial\widetilde{v_{lkt}}(x_t,B_t,p_t)}\frac{\partial Pr(z_{lt}^{(j,k)}(x_t,B_t,p_t)>\widetilde{\epsilon_{ilt}}^{(k,j)}\ \forall k\in\mathcal{A}_l(x_t)-\{j\})}{\partial z_{lt}^{(j,k)}(x_t,B_t,p_t)}\right]\\
&= \sum_{k\in\mathcal{A}_l(x_t)-\{j\}}\frac{\partial Pr(z_{lt}^{(j,k)}(x_t,B_t,p_t)>\widetilde{\epsilon_{ilt}}^{(k,j)}\ \forall k\in\mathcal{A}_l(x_t)-\{j\})}{\partial z_{lt}^{(j,k)}(x_t,B_t,p_t)}\ \left(\because \frac{\partial z_{lt}^{(j,k)}(x_t,B_t,p_t)}{\partial\widetilde{v_{lkt}}(x_t,B_t,p_t)}=-1\ \forall k\in\mathcal{A}_l(x_t)-\{j\}\right)
\end{aligned}$$

In contrast,

$$\frac{\partial s_{ljt}^{(ccp)}(x_t, B_t, p_t)}{\partial \widetilde{v_{ljt}}(x_t, B_t, p_t)} = \sum_{k \in \mathcal{A}_l(x_t)-\{j\}} \frac{\partial z_{lt}^{(j,k)}(x_t, B_t, p_t)}{\partial \widetilde{v_{ljt}}(x_t, B_t, p_t)} \frac{\partial Pr(z_{lt}^{(j,k)}(x_t, B_t, p_t) > \widetilde{\epsilon_{ilt}}^{(k,j)} \ \forall k \in \mathcal{A}_l(x_t)-\{j\})}{\partial z_{lt}^{(j,k)}(x_t, B_t, p_t)}$$

Hence, $\sum_{k \in \mathcal{A}_l(x_t)-\{j\}} \left| \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial \widetilde{v_{lkt}}(x_t,B_t,p_t)} \right| = \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p_t)}{\partial \widetilde{v_{ljt}}(x_t,B_t,p_t)}$ holds.

□

Lemma 3. $\max_{l,x_t,k \in \mathcal{A}_l(x_t)} \left| \frac{\partial E\left[V_l^C(x_{t+1},B_{t+1}(B_t,s_t^{(ccp)}(B_t,p^*(B_t))))|x_t,a_t=k\right]}{\partial p_{jt}} \right| \leq \gamma_1\gamma_2\gamma_3\nu_j$

Proof. First, $\frac{\partial E\left[V_l^C(x_{t+1},B_{t+1}(B_t,s_t^{(ccp)}(B_t,p^*(B_t))))|x_t,a_t=k\right]}{\partial p_{jt}}$ satisfies the following equation:

$$\begin{aligned}
&\frac{\partial E\left[V_l^C(x_{t+1}, B_{t+1}(B_t, s_t^{(ccp)}(B_t, p^*(B_t))))|x_t, a_t = k\right]}{\partial p_{jt}} \\
= &\sum_{x_{t+1}} \psi(x_{t+1}|\widetilde{x}_t, B_t, a_t = \widetilde{k}) \sum_n \frac{\partial V_{\widetilde{l}t+1}^C(x_{t+1}, B_{t+1})}{\partial B_{t+1}^{(n)}} \sum_l \sum_{x_t} \sum_{k \in \mathcal{A}_l(x_t)} \frac{\partial B_{t+1}^{(n)}(B_t, s_t^{(ccp)})}{\partial s_{lkt}^{(ccp)}(x_t, B_t, p^*(B_t))} \frac{\partial s_{lkt}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} \frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} + \\
&\sum_{x_{t+1}} \psi(x_{t+1}|\widetilde{x}_t, B_t, a_t = \widetilde{k}) \sum_n \frac{\partial V_{\widetilde{l}t+1}^C(x_{t+1}, B_{t+1})}{\partial B_{t+1}^{(n)}} \sum_l \sum_{x_t} \sum_{k,h \in \mathcal{A}_l(x_t)} \frac{\partial B_{t+1}^{(n)}(B_t, s_t^{(ccp)})}{\partial s_{lht}^{(ccp)}(x_t, B_t, p^*(B_t))} \frac{\partial s_{lht}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t, B_t, p^*(B_t))} \cdot \\
&\frac{\partial \beta_C E\left[V_l^C(X_{t+1}, B_{t+1}(B_t, s_t^{(ccp)}(B_t, p^*(B_t))))|x_t, a_t = k\right]}{\partial p_{jt}}
\end{aligned}$$

Here, we define the following $N_{B_t} \times N_{B_t}$ dimensional matrix $C(B_t) = (c_{qr})$ where

$$c_{qr} \equiv \beta_C \sum_{x_{t+1}} \psi(x_{t+1}|\widetilde{x}_t, B_t, a_t = \widetilde{k}) \sum_n \frac{\partial V_{\widetilde{l}t+1}^C(x_{t+1}, B_{t+1})}{\partial B_{t+1}^{(n)}} \sum_{h \in \mathcal{A}_l(x_t)} \frac{\partial B_{t+1}^{(n)}(B_t, s_t^{(ccp)})}{\partial s_{lht}^{(ccp)}(x_t, B_t, p^*(B_t))} \frac{\partial s_{lht}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t, B_t, p^*(B_t))}$$

for $q = G_{B_t}(\widetilde{l}, \widetilde{x}_t, \widetilde{k})$ and $r = G_{B_t}(l, x_t, k)$. $G_{B_t}(\cdot)$ is the mapping from $\{(l, x_t, k) \ | l \in \{1, \cdots, L\}, x_t \in X_t, k \in \mathcal{A}_l(x_t)\}$ to $\{1, \cdots, N_{B_t}\}$.

Let $w(B_t) = (w_r(B_t))$ be N_{B_t} dimensional vector where

$$w_r(B_t) = \sum_{x_{t+1}} \psi(x_{t+1}|\widetilde{x}_t, B_t, a_t = \widetilde{k}) \sum_n \frac{\partial V_{\widetilde{l}t+1}^C(x_{t+1}, B_{t+1})}{\partial B_{t+1}^{(n)}} \frac{\partial B_{t+1}^{(n)}(B_t, s_t^{(ccp)})}{\partial s_{lkt}^{(ccp)}(x_t, B_t, p^*(B_t))} \frac{\partial s_{lkt}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} \frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}}$$

for $r = G_{B_t}(l, x_t, k)$.

Let $y(B_t) \equiv (y_r(B_t))$ be N_{B_t} dimensional vector where

$$y_r(B_t) \equiv \frac{\partial E\left[V_l^C(x_{t+1}, B_{t+1}(B_t, s_t^{(ccp)}(B_t, p^*(B_t))))|x_t, a_t = k\right]}{\partial p_{jt}}$$

for $r = G_{B_t}(l, x_t, a_{it} = k)$.

Since $y(B_t) = w(B_t) + C(B_t)y(B_t)$ holds, we have:

$$y(B_t) = (I - C(B_t))^{-1} w(B_t)$$

where I denotes the identity matrix.

Hence, using the formula in linear algebra, we have:

$$\|y(B_t)\|_\infty \leq \left\|(I - C(B_t))^{-1}\right\|_\infty \|w(B_t)\|_\infty$$

where $\|\mathbf{a}\|_\infty \equiv \max_i a_i$ is the infinity norm for a vector $\mathbf{a} = (a_i)$ and $\|A\|_\infty \equiv \max_i \sum_{j=1}^n |a_{ij}|$ is the infinity norm for a $n \times n$ dimensional matrix $A = (a_{ij})$.

Here, $\|w(B_t)\|_\infty$ satisfies:

$$
\begin{aligned}
& \|w(B_t)\|_\infty \\
= \quad & \max_{l,x_t,k\in\mathcal{A}_l(x_t)} \left| \sum_{x_{t+1}} \psi(x_{t+1}|\widetilde{x_t}, B_t, a_t=\widetilde{k}) \sum_n \frac{\partial V^C_{lt+1}(x_{t+1}, B_{t+1})}{\partial B^{(n)}_{t+1}} \frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t)}{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))} \frac{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right| \\
\leq \quad & \left[\max_{l,x_{t+1}} \sum_n \left| \frac{\partial V_l(x_{t+1}, B_{t+1})}{\partial B^{(n)}_{t+1}} \right| \right] \cdot \\
& \max_{l,x_t,k\in\mathcal{A}_l(x_t),n} \left| \sum_{x_{t+1}} \psi(x_{t+1}|\widetilde{x_t}, B_t, a_t=\widetilde{k}) \cdot \left[\left| \frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t)}{\partial s^{(ccp)}_{lkt}(x_t, B_t)} \right| \left| \frac{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} \right| \left| \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right| \right] \right| \\
\leq \quad & \gamma_2 \cdot \max_{l,x_t,k\in\mathcal{A}_l(x_t),n} \left| \frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t)}{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))} \right| \left| \frac{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} \right| \left| \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right| \\
\leq \quad & \gamma_2 \cdot \max_{l,x_t,k\in\mathcal{A}_l(x_t),n} \left| \frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t)}{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))} \right| \cdot \max_{l,x_t} \left[\left| \frac{\partial s^{(ccp)}_{ljt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} \right| \left| \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right| \right] \quad (\because \text{Lemma 1}) \\
\leq \quad & \gamma_2\gamma_3\nu_j
\end{aligned}
$$

Therefore, we obtain:

$$
\begin{aligned}
& \max_{l,x_t,k\in\mathcal{A}_l(x_t)} \left| \frac{\partial E\left[V^C_l(x_{t+1}, B_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t))))|x_t, a_t=k\right]}{\partial p_{jt}} \right| \\
= \quad & \|y(B_t)\|_\infty \\
\leq \quad & \left\|(I - C(B_t))^{-1}\right\|_\infty \cdot \|w(B_t)\|_\infty \\
= \quad & \left\|(I - C(B_t))^{-1}\right\|_\infty \cdot \gamma_2\gamma_3\nu_j \\
\leq \quad & \gamma_1\gamma_2\gamma_3\nu_j
\end{aligned}
$$

□

Proof of Proposition 1

Proof. It follows that:

$$
\begin{aligned}
& \left|\lambda^C_j(B_t)\right| \\
= \quad & \frac{\left| \sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \sum_{k\in\mathcal{A}_l(x_t)} \frac{\partial \beta_C E\left[V^C_l(x_{t+1}, B_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t))))|x_t, a_t=k\right]}{\partial p_{jt}} \frac{\partial s^{(ccp)}_{ljt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t, B_t, p^*(B_t))} \right|}{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \left| \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right| \frac{\partial s^{(ccp)}_{ljt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))}} \\
\leq \quad & \frac{\left[\max_{l,x_t,k\in\mathcal{A}_l(x_t)} \frac{\partial \beta_C E\left[V^C_l(x_{t+1}, B_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t))))|x_t, a_t=k\right]}{\partial p_{jt}} \right] \cdot \sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \sum_{k\in\mathcal{A}_l(x_t)} \left| \frac{\partial s^{(ccp)}_{ljt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t, B_t, p^*(B_t))} \right|}{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \left| \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right| \frac{\partial s^{(ccp)}_{ljt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))}} \\
\leq \quad & \beta_C \left[\max_{l,x_t,k\in\mathcal{A}_l(x_t)} \frac{\partial \beta_C E\left[V^C_l(x_{t+1}, B_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t))))|x_t, a_t=k\right]}{\partial p_{jt}} \right] \cdot \\
& \frac{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \cdot 2 \left| \frac{\partial s^{(ccp)}_{ljt}(x_t, B_t)}{\partial \widetilde{v_{ljt}}(x_t, B_t)} \right| \left| \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right| \left| \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right|^{-1}}{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \left| \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right| \frac{\partial s^{(ccp)}_{ljt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))}} \quad (\because \text{Lemma 2}) \\
\leq \quad & 2\beta_C \left[\max_{l,x_t,k\in\mathcal{A}_l(x_t)} \frac{\partial \beta_C E\left[V^C_l(x_{t+1}, B_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t))))|x_t, a_t=k\right]}{\partial p_{jt}} \right] \cdot \left[\max_{l,x_t} \left| \frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}} \right|^{-1} \right] \\
\leq \quad & 2\beta_C\gamma_1\gamma_2\gamma_3 b_{j1}\nu_j \ (\because \text{Lemma 3})
\end{aligned}
$$

□

Next, we prove Proposition 2. To prove it, we show the following lemmas in advance:

Lemma 4. *Following inequality holds:*

$$\frac{\sum_{\tau=0}^{\infty}\beta_F^{\tau+1}\sum_m\left|\frac{\partial\Pi^F_{jt+\tau+1}(B_{t+\tau+1})}{\partial B^{(m)}_{t+\tau+1}}\right|}{Ms_{jt}(B_t)} \quad \leq \quad (b_{j2}b_{j3}+b_{j4})$$

Proof. First, $\sum_m\left|\frac{\partial\Pi^F_{jt+\tau+1}(B_{t+\tau+1})}{\partial B^{(m)}_{t+\tau+1}}\right|$ satisfies the following inequality:

$$\begin{aligned}
& \sum_m\left|\frac{\partial\Pi^F_{jt+\tau+1}(B_{t+\tau+1})}{\partial B^{(m)}_{t+\tau+1}}\right| \\
= \quad & \sum_m\left|\frac{\partial\left[\left(p_j^*(B_{t+\tau+1})-mc_j\right)\cdot M\sum_l\pi_l s_{ljt+\tau+1}((B_{t+\tau+1},p^*(B_{t+\tau+1})))\right]}{\partial B^{(m)}_{t+\tau+1}}\right| \\
\leq \quad & \sum_m\left|\frac{\partial\left[M\sum_l\pi_l s_{ljt+\tau+1}\right]}{\partial B^{(m)}_{t+\tau+1}}\right|\cdot\max_{\tau\in\mathbb{Z}_+}|p_{jt+\tau+1}(B_{t+\tau+1})-mc_j|+ \\
& \left[M\sum_l\pi_l s_{ljt+\tau+1}\right]\cdot\sum_m\left|\frac{\partial p_{jt+\tau+1}(B_{t+\tau+1})}{\partial B^{(m)}_{t+\tau+1}}\right| \\
\leq \quad & M\sum_l\pi_l\left[\max_{l,}\sum_m\left|\frac{\partial\log s_{ljt+\tau+1}((B_{t+\tau+1},p^*(B_{t+\tau+1})))}{\partial B^{(m)}_{t+\tau+1}}\right|\right]s_{ljt+\tau+1}\cdot\max_{\tau\in\mathbb{Z}_+}\left|p_j^*(B_{t+\tau+1})-mc_j\right|+ \\
& \left[M\sum_l\pi_l s_{ljt+\tau+1}\right]\cdot\sum_m\left|\frac{\partial p_j^*(B_{t+\tau+1})}{\partial B^{(m)}_{t+\tau+1}}\right| \\
\leq \quad & \left[M\sum_l\pi_l s_{ljt+\tau+1}\right]\cdot(b_{j3}b_{j2}+b_{j4})
\end{aligned}$$

Therefore, we obtain:

$$\begin{aligned}
\frac{\sum_{\tau=0}^{\infty}\beta_F^{\tau+1}\sum_m\left|\frac{\partial\Pi^F_{jt+\tau+1}(B_{t+\tau+1})}{\partial B^{(m)}_{t+\tau+1}}\right|}{Ms_{jt}(B_t,p^*(B_t))} \quad & \leq \quad \frac{\sum_{\tau=0}^{\infty}\beta_F^{\tau+1}M\sum_l\pi_l s_{ljt+\tau+1}}{M\sum_l\pi_l s_{ljt}(B_t,p^*(B_t))}\cdot(b_{j2}b_{j3}+b_{j4}) \\
& \leq \quad \frac{\sum_l\pi_l s_{ljt}\cdot\left[\max_l\sum_{\tau=0}^{\infty}\beta_F^{\tau+1}\frac{s_{ljt+\tau+1}}{s_{ljt}}\right]}{\sum_l\pi_l s_{ljt}(B_t,p^*(B_t))}\cdot(b_{j2}b_{j3}+b_{j4}) \\
& = \quad (b_{j2}b_{j3}+b_{j4})b_{j5}
\end{aligned}$$

□

Lemma 5. $\max_n\left|\frac{\partial B^{(n)}_{t+1}(B_t,s_t^{(ccp)}(B_t,p^*(B_t)))}{\partial p_{jt}}\right|\leq 2\gamma_3(1+\beta_C\gamma_1\gamma_2\gamma_3\gamma_4)\nu_j$

Proof. It follows that:

$$
\begin{aligned}
&\max_n \left|\frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t)))}{\partial p_{jt}}\right| \\
\leq\ & \max_n \sum_l \sum_{x_t} \sum_{k\in\mathcal{A}_l(x_t)} \left|\frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}}\right| \left|\frac{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))}\right| \left|\frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t)))}{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}\right| + \\
& \max_n \sum_l \sum_{x_t} \sum_{k,h\in\mathcal{A}_l(x_t)} \left|\frac{\partial \beta_C E\left[V^C_l(x_{t+1}, B_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t))))|x_t, a_t = h\right]}{\partial p_{jt}}\right| \cdot \\
& \left|\frac{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{lht}}(x_t, B_t, p^*(B_t))}\right| \left|\frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t)}{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}\right| \\
\leq\ & \left[\sum_l \sum_{x_t} \max_{k\in\mathcal{A}_l(x_t), n} \left|\frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t)}{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}\right|\right] \cdot 2\max_{l,x_t}\left(\left|\frac{\partial f_{lj}(x_t, B_t, p^*_j(B_t))}{\partial p_{jt}}\right| \left|\frac{\partial s^{(ccp)}_{ljt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, , p^*(B_t))}\right|\right) + \\
& \beta_C \left[\max_{l, x_t, h\in\mathcal{A}_{lt}(x_t)} \left|\frac{\partial \beta_C E\left[V^C_l(x_{t+1}, B_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t))))|x_t, a_t = h\right]}{\partial p_{jt}}\right|\right] \cdot \\
& \left[\max_{l,x_t} \sum_{k,h\in\mathcal{A}_l(x_t)} \left|\frac{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{lht}}(x_t, B_t, p^*(B_t))}\right|\right] \left[\sum_l \sum_{x_t} \max_{k\in\mathcal{A}_l(x_t)} \left|\frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t)}{\partial s^{(ccp)}_{lkt}(x_t, B_t, p^*(B_t))}\right|\right] \ (\because \text{Lemma 1}) \\
\leq\ & 2\gamma_3\nu_j + \\
& \beta_C \cdot \gamma_1\gamma_2\gamma_3\nu_j \cdot \left[2\max_{l,x_t} \sum_{k\in\mathcal{A}_l(x_t)} \left|\frac{\partial s_{lkt}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t, B_t, p^*(B_t))}\right|\right] \gamma_3 \ (\because \text{Lemma 2}) \\
=\ & 2\gamma_3\nu_j + \beta_C\gamma_1\gamma_2\gamma_3\nu_j \cdot 2\gamma_4\gamma_3 \\
=\ & 2(1 + \beta_C\gamma_1\gamma_2\gamma_3\gamma_4)\gamma_3\nu_j
\end{aligned}
$$

□

Proof of Proposition 2

Proof. First, $\left|\lambda^F_j(B_t)\right|$ satisfies the following inequality:

$$
\begin{aligned}
\left|\lambda^F_j(B_t)\right| &= \left|\frac{\beta_F \frac{\partial V^F_j(B_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t))))}{\partial p_{jt}}}{M s_{jt}}\right| \\
&= \left|\frac{\beta_F \sum_n \frac{\partial V^F_j(B_{t+1})}{\partial B^{(n)}_{t+1}} \frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t)))}{\partial p_{jt}}}{M s_{jt}}\right| \\
&\leq \left[\frac{\sum_n \left|\frac{\partial \beta_F V^F_j(B_{t+1})}{\partial B^{(n)}_{t+1}}\right|}{M s_{jt}}\right] \cdot \left[\max_n \left|\frac{\partial B^{(n)}_{t+1}(B_t, s^{(ccp)}_t(B_t, p^*(B_t)))}{\partial p_{jt}}\right|\right]
\end{aligned}
$$

Here, $\left|\frac{\sum_n \frac{\partial \beta_F V^F_j(B_{t+1})}{\partial B^{(n)}_{t+1}}}{M s_{jt}}\right|$ satisfies:

$$
\begin{aligned}
\left|\frac{\sum_n \frac{\partial \beta_F V^F_j(B_{t+1})}{\partial B^{(n)}_{t+1}}}{M s_{jt}}\right| &\leq \frac{1}{M s_{jt}} \sum_{\tau=0}^{\infty} \beta_F^{\tau+1} \sum_{n,m} \left|\frac{\partial B^{(m)}_{t+\tau+1}}{\partial B^{(n)}_{t+1}}\right| \left|\frac{\partial \Pi^F_{jt+\tau+1}(B_{t+\tau+1})}{\partial B^{(m)}_{t+\tau+1}}\right| \\
&\leq \left[\max_{\tau\in\mathbb{Z}_+, m} \sum_n \left|\frac{\partial B^{(m)}_{t+\tau+1}}{\partial B^{(n)}_{t+1}}\right|\right] \cdot \frac{\sum_{\tau=0}^{\infty} \beta_F^{\tau+1} \sum_m \left|\frac{\partial \Pi^F_{jt+\tau+1}(B_{t+\tau+1})}{\partial B^{(m)}_{t+\tau+1}}\right|}{M s_{jt}} \\
&\leq \gamma_5 \cdot (b_{j2}b_{j3} + b_{j4}) \ (\because \text{Lemma 4})
\end{aligned}
$$

Hence, by Lemma 5, we have:

$$\left|\lambda_j^F(B_t)\right| \quad \leq \quad 2\gamma_3(1+\beta_C\gamma_1\gamma_2\gamma_3\gamma_4)(b_{j2}b_{j3}+b_{j4})b_{j5}\gamma_5\nu_j$$

$\square$

Proof of Proposition 4

Proof. When ϵ_{iljt} follows Gumbel distribution with scale parameter 1, $\left|\frac{\partial \log s_{kt}(B_t,p^*(B_t))}{\partial p_{jt}}\right|$ satisfies:

$$\begin{aligned}
& \left|\frac{\partial \log s_{kt}(B_t,p^*(B_t))}{\partial p_{jt}}\right| \\
= & \left|\frac{\partial s_{kt}(B_t,p^*(B_t))}{\partial p_{jt}}\frac{1}{s_{kt}(B_t,p^*(B_t)}\right| \\
= & \frac{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t)\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial p_{jt}}\right|}{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))} \\
= & \frac{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial p_{jt}}\frac{1}{s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right|}{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))} \\
\leq & \max_{l,x_t}\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial p_{jt}}\frac{1}{s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right| \\
\leq & \max_{l,x_t}\left[\left|\frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}}\right|\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}\frac{1}{s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right|\right]+ \\
& \max_{l,x_t}\left[\sum_{h\in\mathcal{A}_l(x_t)}\left|\frac{\partial \beta_C E\left[V_l^C(x_{t+1},B_{t+1}(B_t,s_t^{(ccp)}(B_t,p^*(B_t))))|x_t,a_t=h\right]}{\partial p_{jt}}\right|\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{lht}}(x_t,B_t,p^*(B_t))}\frac{1}{s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right|\right] \\
\leq & \max_{l,x_t}\left|\frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}}\right|\cdot\max_{l,x_t}\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}\frac{1}{s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right|+ \\
& \left[\max_{l,x_t,h\in\mathcal{A}_l(x_t)}\left|\frac{\partial \beta_C E\left[V_l^C(x_{t+1},B_{t+1}(B_t,s_t^{(ccp)}(B_t,p^*(B_t))))|x_t,a_t=h\right]}{\partial p_{jt}}\right|\right]\cdot \\
& \left[\max_{l,x_t}\sum_{h\in\mathcal{A}_l(x_t)}\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{lht}}(x_t,B_t,p^*(B_t))}\frac{1}{s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right|\right] \\
= & b_{j7}\cdot\max_{l,x_t}\left|s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))\frac{1}{s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right|+ \\
& \beta_C\gamma_1\gamma_2\gamma_3\nu_j\cdot\left[\max_{l,x_t}2\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))}\frac{1}{s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right|\right] \quad (\because \text{Lemmas } 2,3) \\
= & b_{j7}\cdot\max_{l,x_t}s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))+2\beta_C\gamma_1\gamma_2\gamma_3\nu_j\left[\max_{l,x_t}s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))(1-s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t)))\right] \\
= & b_{j7}\cdot\max_{l,x_t}s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))+2\beta_C\gamma_1\gamma_2\gamma_3 b_{j6}\max_{l,x_t}s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) \ \left(\because s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))(1-s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t)))<1\right)
\end{aligned}$$

Furthermore, $\left|\frac{\partial \log s_{kt+\tau}}{\partial p_{jt}}\right|$ satisfies:

$$
\begin{aligned}
\left|\frac{\partial \log s_{kt+\tau}(B_{t+\tau}(B_{t+1}(B_t, s_t^{(ccp)}(B_t,p^*(B_t)))))}{\partial p_{jt}}\right| &\leq \left[\max_n \left|\frac{\partial B_{t+1}^{(n)}(B_t, s_t^{(ccp)}(B_t,p^*(B_t)))}{\partial p_{jt}}\right|\right]\left[\sum_{n,m}\frac{\partial B_{t+\tau}^{(m)}}{\partial B_{t+1}^{(n)}}\frac{\partial \log s_{kt+\tau}}{\partial B_{t+\tau}^{(m)}}\right] \\
&\leq \left[\max_n \left|\frac{\partial B_{t+1}^{(n)}(B_t, s_t^{(ccp)}(B_t,p^*(B_t)))}{\partial p_{jt}}\right|\right]\cdot\left[\max_m \sum_n \frac{\partial B_{t+\tau}^{(m)}}{\partial B_{t+1}^{(n)}}\right]\cdot\left[\sum_m \frac{\partial \log s_{kt+\tau}}{\partial B_{t+\tau}^{(m)}}\right] \\
&\leq 2\gamma_3(1+\beta_C\gamma_1\gamma_2\gamma_3\gamma_4)\nu_j\cdot\gamma_5\cdot\left[\sum_m \frac{\partial \log s_{kt+\tau}}{\partial B_{t+\tau}^{(m)}}\right] \quad (\because \text{Lemma 5}) \\
&\leq 2\gamma_3(1+\beta_C\gamma_1\gamma_2\gamma_3\gamma_4)\nu_j\cdot\gamma_5\cdot\frac{\sum_l \pi_l \sum_m \left|\frac{\partial s_{lkt+\tau}}{\partial B_{t+\tau}^{(m)}}\right|}{\sum_l \pi_l s_{lkt+\tau}} \\
&\leq 2\gamma_3(1+\beta_C\gamma_1\gamma_2\gamma_3\gamma_4)\nu_j\cdot\gamma_5\cdot\max_{l,\tau\in\mathbb{N}}\sum_m\left|\frac{\partial s_{lkt+\tau}}{\partial B_{t+\tau}^{(m)}}\frac{1}{s_{lkt+\tau}}\right| \\
&= 2\gamma_3(1+\beta_C\gamma_1\gamma_2\gamma_3\gamma_4)\nu_j\cdot\gamma_5\cdot b_{j3} \\
&\leq 2\gamma_3(1+\beta_C\gamma_1\gamma_2\gamma_3\gamma_4)b_{j3}\gamma_5 b_{j6}\cdot\left[\max_{l,x_t} s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))\right]
\end{aligned}
$$

□

B Further consideration of Assumption 2

In this section, we consider the validity of Assumption 2. we show that $(I-C(B_t))^{-1}$ would exist and the terms $\gamma_1,\cdots,\gamma_5$ and $b_{j1},\cdots,b_{j5}$ would take finite values in standard settings. The results hold even when $s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))$ is so small, or the number of firms is large, for instance.

The existence of $(I-C(B_t))^{-1}$

Even though we imposed the assumption that the inverse matrix of $I-C(B_t)$ exists, we can show the existence of the matrix if $2\beta_C\gamma_2\gamma_3\gamma_4<1$. First, $\sum_r |c_{qr}|$ satisfies the following inequality:

$$
\begin{aligned}
&\sum_r |c_{qr}| \\
= \; & \sum_l \sum_{x_t}\sum_{k\in\mathcal{A}_l(x_t)}\beta_C\sum_{x_{t+1}}\psi(x_{t+1}|\widetilde{x}_t,B_t,a_t=\widetilde{k})\sum_n\left|\frac{\partial V_{\widetilde{l}t+1}^C(x_{t+1},B_{t+1})}{\partial B_{t+1}^{(n)}}\right|\sum_{h\in\mathcal{A}_l(x_t)}\left|\frac{\partial s_{lht}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial\widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))}\right|\left|\frac{\partial B_{t+1}^{(n)}(B_t,s_t^{(ccp)})}{\partial s_{lht}^{(ccp)}(x_t,B_t,p^*(B_t))}\right| \\
\leq \; & \left[\max_{\widetilde{l},x_{t+1}}\sum_n\left|\frac{\partial V_{\widetilde{l}t+1}^C(x_{t+1},B_{t+1})}{\partial B_{t+1}^{(n)}}\right|\right]\cdot\sum_l\sum_{x_t}\sum_{k,h\in\mathcal{A}_l(x_t)}\beta_C\left[\sum_{x_{t+1}}\psi(x_{t+1}|\widetilde{x}_t,B_t,a_t=\widetilde{k})\right]\left|\frac{\partial s_{lht}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial\widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))}\right|\left|\frac{\partial B_{t+1}^{(n)}(B_t,s_t^{(ccp)})}{\partial s_{lht}^{(ccp)}(x_t,B_t,p^*(B_t))}\right| \\
\leq \; & \beta_C\gamma_2\cdot\left[\sum_l\sum_{x_t}\max_{h\in\mathcal{A}_l(x_t),n}\left|\frac{\partial B_{t+1}^{(n)}(B_t,s_t^{(ccp)})}{\partial s_{lht}^{(ccp)}(x_t,B_t,p^*(B_t))}\right|\right]\cdot\left[\max_{l,x_t}\sum_{k,h\in\mathcal{A}_l(x_t)}\left|\frac{\partial s_{lht}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial\widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))}\right|\right] \\
\leq \; & 2\beta_C\gamma_2\gamma_3\gamma_4\quad\left(\because\sum_{k,h\in\mathcal{A}_l(x_t)}\left|\frac{\partial s_{lht}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial\widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))}\right| = 2\sum_{k\in\mathcal{A}_l(x_t)}\left|\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial\widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))}\right| \text{ by Lemma 2}\right)
\end{aligned}
$$

Here, we define $(e_{qr})_{qr}\equiv E(B_t)\equiv I-C(B_t)$, for convenience. If we presume that $2\beta_C\gamma_2\gamma_3\gamma_4<1$ holds, $|c_{qq}|<2\beta_C\gamma_2\gamma_3\gamma_4<1$ and

$$
\begin{aligned}
|e_{qq}| - \sum_{r\in\{1,\cdots,N_{B_t}\}-\{q\}}|e_{qr}| &= |1-c_{qq}| - \sum_{r\in\{1,\cdots,N_{B_t}\}-\{q\}}|c_{qr}| \\
&= 1-c_{qq}-\sum_{r\in\{1,\cdots,N_{B_t}\}-\{q\}}|c_{qr}| \\
&\geq 1-\sum_{r\in\{1,\cdots,N_{B_t}\}}|c_{qr}| \;\; (\because -c_{qq}\geq -|c_{qq}|) \\
&\geq 1-2\beta_C\gamma_2\gamma_3\gamma_4
\end{aligned}
$$

Hence, if $2\beta_C\gamma_2\gamma_3\gamma_4 < 1$ holds, $|e_{qq}| - \sum_{r\in\{1,\cdots,N_{B_t}\}-\{q\}} |e_{qr}| > 0$ and $E(B_t) \equiv I - C(B_t)$ is a strictly diagonally dominant matrix. Then, by Levy-Desplanques theorem (see Corollary 5.6.17 of Horn and Johnson (2012)), $I - C(B_t)$ is nonsingular and its inverse matrix exists.

γ_1 (Infinity norm of the inverse matrix)

If $2\beta_C\gamma_2\gamma_3\gamma_4 < 1$ holds, by using Ahlberg-Nilson-Varah bound (Ahlberg and Nilson, 1963, Varah, 1975; see Theorem 1 of Varah, 1975), it follows that:

$$\begin{aligned}
\gamma_1 &\equiv \left\|(I - C(B_t))^{-1}\right\|_\infty \\
&= \left\|(E(B_t))^{-1}\right\|_\infty \\
&\leq \frac{1}{\min_q\left(|e_{qq}| - \sum_{r\in\{1,\cdots,N_{B_t}\}-\{q\}} |e_{qr}|\right)} \\
&\leq \frac{1}{1-2\beta_C\gamma_2\gamma_3\gamma_4} \quad \left(\because |e_{qq}| - \sum_{r\in\{1,\cdots,N_{B_t}\}-\{q\}} |e_{qr}| \geq 1 - 2\beta_C\gamma_2\gamma_3\gamma_4 \ \forall q\right)
\end{aligned}$$

γ_2 (Change in consumers' value function in response to the aggregate state change)

Define the term $\zeta_{lt+\tau}(x_{t+1}, B_{t+1}) \equiv \sum_{x_{t+\tau}} \sum_{h\in\mathcal{A}_l(x_{t+\tau})} \frac{\partial V_l^C(x_{t+1},B_{t+1})}{\partial \widetilde{v_{lht+\tau}}(x_{t+\tau},B_{t+\tau},p^*(B_{t+\tau}))}$. We use the following lemma.

Lemma 6. *If ϵ_{iljt} follows Gumbel distribution with scale parameter 1,*

$$\begin{aligned}
\zeta_{lt+\tau}(x_{t+1}, B_{t+1}) &= \beta_C \zeta_{lt+\tau-1}(x_{t+1}, B_{t+1}) \ \forall \tau \geq 2 \\
&= \beta_C^{\tau-1}
\end{aligned}$$

Proof. First, $\zeta_{lt+\tau}(x_{t+1}, B_{t+1})$ satisfies the following equation:

$$\begin{aligned}
&\zeta_{lt+\tau}(x_{t+1}, B_{t+1}) \\
= &\sum_{x_{t+\tau}} \sum_{h\in\mathcal{A}_l(x_{t+\tau})} \frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lht+\tau}}(x_{t+\tau}, B_{t+\tau}, p^*(B_{t+\tau}))} \\
= &\sum_{x_{t+\tau}} \sum_{h\in\mathcal{A}_l(x_{t+\tau})} \frac{\partial V_{lt+\tau}^C(x_{t+\tau}, B_{t+\tau})}{\partial \widetilde{v_{lht+\tau}}(x_{t+\tau}, B_{t+\tau}, p^*(B_{t+\tau}))} \cdot \\
&\sum_{x_{t+\tau-1}} \sum_{k\in\mathcal{A}_l(x_{t+\tau-1})} \frac{\partial \widetilde{v_{lkt+\tau-1}}(x_{t+\tau-1}, B_{t+\tau-1}, p^*(B_{t+\tau-1}))}{\partial V_{lt+\tau}^C(x_{t+\tau}, B_{t+\tau})} \frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lkt+\tau-1}}(x_{t+\tau-1}, B_{t+\tau-1}, p^*(B_{t+\tau-1}))}
\end{aligned}$$

If ϵ_{iljt} follows Gumbel distribution with scale parameter 1,

$$\begin{aligned}
&\sum_{h\in\mathcal{A}_l(x_{t+\tau})} \frac{\partial V_{lt+\tau}^C(x_{t+\tau}, B_{t+\tau})}{\partial \widetilde{v_{lht+\tau}}(x_{t+\tau}, B_{t+\tau}, p^*(B_{t+\tau}))} \\
= &\sum_{h\in\mathcal{A}_l(x_{t+\tau})} \frac{\partial \log\left(\sum_{\widetilde{h}\in\mathcal{A}_l(x_{t+\tau})} \exp\left(\widetilde{v_{l\widetilde{h}t+\tau}}(x_{t+\tau}, B_{t+\tau}, p^*(B_{t+\tau}))\right)\right)}{\partial \widetilde{v_{lht+\tau}}(x_{t+\tau}, B_{t+\tau}, p^*(B_{t+\tau}))} \\
= &\sum_{h\in\mathcal{A}_l(x_{t+\tau})} \frac{\exp\left(\widetilde{v_{lht+\tau}}(x_{t+\tau}, B_{t+\tau}, p^*(B_{t+\tau}))\right)}{\sum_{\widetilde{h}\in\mathcal{A}_l(x_{t+\tau})} \exp\left(\widetilde{v_{l\widetilde{h}t+\tau}}(x_{t+\tau}, B_{t+\tau}, p^*(B_{t+\tau}))\right)} \\
= &\ 1
\end{aligned}$$

Using these equations, we obtain:

$$
\begin{aligned}
& \zeta_{lt+\tau}(x_{t+1}, B_{t+1}) \\
\equiv \; & \sum_{x_{t+\tau}} \sum_{h\in\mathcal{A}_l(x_{t+\tau})} \frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lht+\tau}}(x_{t+\tau}, B_{t+\tau}, p^*(B_{t+\tau}))} \\
= \; & \sum_{x_{t+\tau}} 1 \cdot \sum_{x_{t+\tau-1}} \sum_{k\in\mathcal{A}_l(x_{t+\tau-1})} \frac{\partial \widetilde{v_{lkt+\tau-1}}(x_{t+\tau-1}, B_{t+\tau-1}, p^*(B_{t+\tau-1}))}{\partial V_{lt+\tau}^C(x_{t+\tau}, B_{t+\tau})} \frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lkt+\tau-1}}(x_{t+\tau-1}, B_{t+\tau-1}, p^*(B_{t+\tau-1}))} \\
= \; & \sum_{x_{t+\tau-1}} \sum_{k\in\mathcal{A}_l(x_{t+\tau-1})} \left[\sum_{x_{t+\tau}} \frac{\partial \widetilde{v_{lkt+\tau-1}}(x_{t+\tau-1}, B_{t+\tau-1}, p^*(B_{t+\tau-1}))}{\partial V_{lt+\tau}^C(x_{t+\tau}, B_{t+\tau})}\right] \cdot \frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lkt+\tau-1}}(x_{t+\tau-1}, B_{t+\tau-1}, p^*(B_{t+\tau-1}))} \\
= \; & \sum_{x_{t+\tau-1}} \sum_{k\in\mathcal{A}_l(x_{t+\tau-1})} \left[\sum_{x_{t+\tau}} \beta_C \psi(x_{t+\tau}|x_{t+\tau-1}, B_{t+\tau-1}, a_{t+\tau-1}=k)\right] \cdot \frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lkt+\tau-1}}(x_{t+\tau-1}, B_{t+\tau-1}, p^*(B_{t+\tau-1}))} \\
= \; & \beta_C \sum_{x_{t+\tau-1}} \sum_{k\in\mathcal{A}_l(x_{t+\tau-1})} \cdot \frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lkt+\tau-1}}(x_{t+\tau-1}, B_{t+\tau-1}, p^*(B_{t+\tau-1}))} \\
= \; & \beta_C \zeta_{lt+\tau-1}(x_{t+1}, B_{t+1})
\end{aligned}
$$

Furthermore,

$$
\begin{aligned}
& \zeta_{lt+\tau}(x_{t+1}, B_{t+1}) \\
= \; & \beta_C \zeta_{lt+\tau-1}(x_{t+1}, B_{t+1}) \\
= \; & \cdots \\
= \; & \beta_C^{\tau-2} \zeta_{lt+2}(x_{t+1}, B_{t+1}) \\
= \; & \beta_C^{\tau-2} \sum_{x_{t+2}} \sum_{m\in\mathcal{A}_l(x_{t+2})} \frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lmt+2}}(x_{t+2}, B_{t+2}, p^*(B_{t+2}))} \\
= \; & \beta_C^{\tau-2} \sum_{x_{t+2}} \sum_{m\in\mathcal{A}_l(x_{t+2})} \frac{\partial V_{lt+2}^C(x_{t+2}, B_{t+2})}{\partial \widetilde{v_{lmt+2}}(x_{t+2}, B_{t+2}, p^*(B_{t+2}))} \sum_{n\in\mathcal{A}_l(x_{t+1})} \frac{\partial \widetilde{v_{lnt+1}}(x_{t+1}, B_{t+1}, p^*(B_{t+1}))}{\partial V_{lt+2}^C(x_{t+2}, B_{t+2})} \frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lnt+1}}(x_{t+1}, B_{t+1}, p^*(B_{t+1}))} \\
= \; & \beta_C^{\tau-2} \sum_{x_{t+2}} \sum_{m\in\mathcal{A}_l(x_{t+2})} \frac{\exp\left(\widetilde{v_{lmt+2}}(x_{t+2}, B_{t+2}, p^*(B_{t+2}))\right)}{\sum_{\widetilde{m}\in\mathcal{A}_l(x_{t+2})} \exp\left(\widetilde{v_{l\widetilde{m}t+2}}(x_{t+2}, B_{t+2}, p^*(B_{t+2}))\right)} \cdot \\
& \sum_{n\in\mathcal{A}_l(x_{t+1})} \beta_C \psi(x_{t+2}|x_{t+1}, B_{t+1}, a_{t+1}=n) \frac{\exp\left(\widetilde{v_{lnt+1}}(x_{t+1}, B_{t+1}, p^*(B_{t+1}))\right)}{\sum_{\widetilde{n}\in\mathcal{A}_l(x_{t+1})} \exp\left(\widetilde{v_{l\widetilde{n}t+1}}(x_{t+1}, B_{t+1}, p^*(B_{t+1}))\right)} \\
= \; & \beta_C^{\tau-1}
\end{aligned}
$$

□

Using Lemma 6, we can derive the upper bound of γ_2:

$$
\begin{aligned}
\gamma_2 \; \equiv \; & \max_{l,x_{t+1}} \sum_n \left|\frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial B_{t+1}^{(n)}}\right| \\
\leq \; & \max_{l,x_{t+1}} \sum_n \sum_{\tau=1}^{\infty} \sum_{h\in\mathcal{J}} \left|\frac{\partial p_{ht+\tau}}{\partial B_{t+1}^{(n)}}\right| \sum_{x_{t+\tau}} \left|\frac{\partial f_{lh}(x_{t+\tau}, B_{t+\tau}, p_h^*(B_{t+\tau}))}{\partial p_{ht+\tau}}\right| \left|\frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lht+\tau}}(x_{t+\tau}, B_{t+\tau},, p^*(B_{t+\tau}))}\right| \\
\leq \; & \left[\max_{h\in\mathcal{J},\tau\in\mathbb{N}} \sum_n \left|\frac{\partial p_{ht+\tau}}{\partial B_{t+1}^{(n)}}\right|\right] \cdot \left[\max_{lx_{t+\tau},\tau\in\mathbb{N},,h\in\mathcal{A}_l(x_{t+\tau})} \left|\frac{\partial f_{lh}(x_{t+\tau}, B_{t+\tau}, p_h^*(B_{t+\tau}))}{\partial p_{ht+\tau}}\right|\right] \cdot \\
& \left[\max_{l,x_{t+1}} \sum_{\tau=1}^{\infty} \sum_{x_{t+\tau}} \sum_{h\in\mathcal{A}_l(x_{t+\tau})} \left|\frac{\partial V_l^C(x_{t+1}, B_{t+1})}{\partial \widetilde{v_{lht+\tau}}(x_{t+\tau}, B_{t+\tau}, p^*(B_{t+\tau}))}\right|\right] \\
\leq \; & \left[\max_{h\in\mathcal{J},\tau\in\mathbb{N}} \sum_n \left|\frac{\partial p_{ht+\tau}}{\partial B_{t+1}^{(n)}}\right|\right] \cdot \left[\max_{l,x_{t+\tau},\tau\in\mathbb{N},h\in\mathcal{A}_l(x_{t+\tau})} \left|\frac{\partial f_{lh}(x_{t+\tau}, B_{t+\tau}, p_h^*(B_{t+\tau}))}{\partial p_{ht+\tau}}\right|\right] \cdot \left[\max_{l,x_{t+1}} \sum_{\tau=1}^{\infty} \zeta_{lt+\tau}(x_{t+1}, B_{t+1})\right] \\
= \; & \left[\max_{h\in\mathcal{J},\tau\in\mathbb{N}} \sum_n \left|\frac{\partial p_{ht+\tau}}{\partial B_{t+1}^{(n)}}\right|\right] \cdot \left[\max_{l,x_{t+\tau},\tau\in\mathbb{N},h\in\mathcal{A}_l(x_{t+\tau})} \left|\frac{\partial f_{lh}(x_{t+\tau}, B_{t+\tau}, p_h^*(B_{t+\tau}))}{\partial p_{ht+\tau}}\right|\right] \cdot \sum_{\tau=1}^{\infty} \beta_C^{\tau-1} \quad (\because \text{Lemma 6}) \\
= \; & \frac{1}{1-\beta_C} \left[\max_{l,x_t,h\in\mathcal{J},\tau\in\mathbb{N},,k\in\mathcal{A}_l(x_t)} \left|\frac{\partial p_{ht+\tau}}{\partial B_{t+1}^{(n)}}\right|\right] \cdot \left[\max_{l,x_{t+\tau},\tau\in\mathbb{N},,h\in\mathcal{A}_l(x_{t+\tau})} \left|\frac{\partial f_{lh}(x_{t+\tau}, B_{t+\tau}, p_h^*(B_{t+\tau}))}{\partial p_{ht+\tau}}\right|\right]
\end{aligned}
$$

Moreover, in the case of perfectly durable goods with no persistent consumer heterogeneity and no network effects, the term $\max_{h\in\mathcal{J},\tau\in\mathbb{N}}\sum_n\left|\frac{\partial p_{ht+\tau}}{\partial B_{t+1}^{(n)}}\right|$ is equal to zero and $\gamma_2=0$ holds. For details, see the discussion on perfectly durable goods at the end of this section.

γ_3 **(Next period's aggregate state's response to the current CCP's change)**

The value is determined by the state transition process. If it is well-behaved, $\gamma_3\equiv\sum_l\sum_{x_t}\max_{k\in\mathcal{A}_l(x_t),n}\left|\frac{\partial B_{t+1}^{(n)}(B_t,s_t^{(ccp)})}{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right|$ would take a finite value.

γ_4 **(Sum of the current CCP change in response to the current utility change)**

Consider the case where ϵ_{iljt} follows Gumbel distribution with scale parameter 1. Then,

$$\begin{aligned}
\gamma_4 &\equiv \max_{l,x_t}\sum_{k\in\mathcal{A}_l(x_t)}\frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial\widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))}\\
&= \max_{l,x_t}\sum_{k\in\mathcal{A}_l(x_t)} s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))\left(1-s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))\right)\\
&= \max_{l,x_t}\left[\sum_{k\in\mathcal{A}_l(x_t)} s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))-\sum_{k\in\mathcal{A}_l(x_t)} s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))^2\right]\\
&< 1\quad\left(\because\sum_{k\in\mathcal{A}_l(x_t)} s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))=1,\ \sum_{k\in\mathcal{A}_l(x_t)} s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))^2>0\right)
\end{aligned}$$

γ_5 **(Future aggregate state response to the current aggregate state change)**

The value of $\gamma_5\equiv\max_{\tau\in\mathbb{Z}_+,m}\sum_n\left|\frac{\partial B_{t+\tau+1}^{(m)}}{\partial B_{t+1}^{(n)}}\right|$ is determined by the future state transition process and firms' equilibrium prices based on the aggregate state variables. If they are well-behaved, γ_5 would take a finite value. Moreover, in the case of perfectly durable goods with no persistent consumer heterogeneity and no network effects, $\gamma_5=1$ holds. See the discussion on perfectly durable goods at the end of this section for details.

b_{j1} **(Inverse of the marginal utility of money)**

If the utility function is quasi-linear and $\frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}}=-\alpha_l\ \forall x_t,l$ holds, $b_{j1}\equiv\max_{l,x_t}\left|\frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}}\right|^{-1}=\max_l\alpha_l^{-1}$ and b_{j1} takes a finite value as long as the values of α_l^{-1} are finite for all consumer types l.

b_{j2}**(Absolute value of the margin)**

If the equilibrium price is well-behaved, $b_{j2}\equiv\max_{\tau\in\mathbb{N}}\left|p_j^*(B_{t+\tau}(B_t))-mc_j\right|$ would be finite.

b_{j3} **(Semi-elasticity of the future demand with respect to the aggregate state)**

b_{j3} satisfies the following inequality:

$$
\begin{aligned}
& b_{j3} \\
\equiv\ & \max_{l,\tau\in\mathbb{Z}_+} \sum_n \left| \frac{\partial \log s_{ljt+\tau+1}(B_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right| \\
\leq\ & \max_{l,\tau\in\mathbb{Z}_+} \frac{\sum_n \left| \sum_{x_{t+\tau+1}} Pr_{lt+\tau+1}(x_{t+\tau+1}) \frac{\partial s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right|}{\sum_{x_{t+\tau+1}} Pr_{lt+\tau+1}(x_{t+\tau+1}) s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1})} + \\
& \max_{l,\tau\in\mathbb{Z}_+} \frac{\sum_n \left| \sum_{x_{t+\tau+1}} \frac{\partial Pr_{lt+\tau+1}(x_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1}) \right|}{\sum_{x_{t+\tau+1}} Pr_{lt+\tau+1}(x_{t+\tau+1}) s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1})} \\
=\ & \max_{l,\tau\in\mathbb{Z}_+} \frac{\sum_n \left| \sum_{x_{t+\tau+1}} Pr_{lt+\tau+1}(x_{t+\tau+1}) s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1}) \frac{\partial \log s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right|}{\sum_{x_{t+\tau+1}} Pr_{lt+\tau+1}(x_{t+\tau+1}) s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1})} + \\
& \max_{l,\tau\in\mathbb{Z}_+} \frac{\sum_n \left| \sum_{x_{t+\tau+1}} \frac{\partial \log Pr_{lt+\tau+1}(x_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} Pr_{lt+\tau+1}(x_{t+\tau+1}) s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1}) \right|}{\sum_{x_{t+\tau+1}} Pr_{lt+\tau+1}(x_{t+\tau+1}) s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1})} \\
\leq\ & \max_{l,\tau\in\mathbb{Z}_+, x_{t+\tau+1}} \sum_n \left| \frac{\partial \log s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right| + \max_{l,\tau\in\mathbb{Z}_+, x_{t+\tau+1}} \sum_n \left| \frac{\partial \log Pr_{lt+\tau+1}(x_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right|
\end{aligned}
$$

First, the term $\max_{l,\tau\in\mathbb{Z}_+, x_{t+\tau+1}} \sum_n \left| \frac{\partial \log s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right|$ would take a finite value in normal settings. In the case of perfectly durable goods with no persistent consumer heterogeneity and no network effects, the term $\max_{l,\tau\in\mathbb{Z}_+, x_{t+\tau+1}} \sum_n \left| \frac{\partial \log s^{(ccp)}_{ljt+\tau+1}(x_{t+\tau+1}, B_{t+\tau+1}(B_t))}{\partial B^{(n)}_{t+\tau+1}} \right|$ is equal to zero, as we discuss later.

Second, it is plausible to assume that the term $\max_{l,\tau\in\mathbb{Z}_+, x_{t+\tau+1}} \sum_n \left| \frac{\partial \log Pr_{lt+\tau+1}(x_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right|$ takes a finite value. When the goods is perfectly durable and ϵ_{iljt} follows Gumbel distribution, we can show that the term $\max_{l,\tau\in\mathbb{Z}_+, x_{t+\tau+1}} \sum_n \left| \frac{\partial \log Pr_{lt+\tau+1}(x_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right|$ takes a finite value. Suppose that $\exists l, x_{t+\tau+1}\ s.t.\ Pr_{lt+\tau+1}(x_{t+\tau+1}) = B^{(n)}_{t+\tau+1}$. Then,

$$
\begin{aligned}
\max_{l,\tau\in\mathbb{Z}_+, x_{t+\tau+1}} \sum_n \left| \frac{\partial \log Pr_{lt+\tau+1}(x_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right| &= \max_{l,\tau\in\mathbb{Z}_+, x_{t+\tau+1}} \sum_l \left| \frac{\partial \log Pr_{lt+\tau+1}(x_{t+\tau+1})}{\partial Pr_{lt+\tau+1}(x_{t+\tau+1})} \right| \\
&= \max_{l,\tau\in\mathbb{Z}_+, x_{t+\tau+1}} \sum_l \frac{1}{Pr_{lt+\tau+1}(x_{t+\tau+1})}
\end{aligned}
$$

Hence, if $\frac{1}{Pr_{lt+\tau+1}(x_{t+\tau+1})}$ takes a finite value ($Pr_{lt+\tau+1}(x_{t+\tau+1})$ is not sufficiently close to zero), it is plausible to assume that b_{j3} is finite.

b_{j4} (Future price change in response to the current aggregate state change)

If the equilibrium price is well-behaved, $b_{j4} \equiv \max_{\tau\in\mathbb{Z}_+} \sum_n \left| \frac{\partial p^*_j(B_{t+\tau+1})}{\partial B^{(n)}_{t+\tau+1}} \right|$ would be finite. Moreover, in the case of perfectly durable goods with no persistent consumer heterogeneity and no network effects, b_{j4} is equal to zero. For details, see the discussion on perfectly durable goods at the end of this section.

b_{j5} (Ratio of the future demand to the current demand)

$b_{j5} \equiv \max_l \sum_{\tau=0}^{\infty} \beta_F^{\tau+1} \frac{s_{ljt+\tau+1}(B_{t+\tau+1}(B_t))}{s_{ljt}(B_t)}$ takes a small value when the relative size of the future demand $s_{ljt+\tau}(B_{t+\tau+1}(B_t))$ compared to the current demand $s_{ljt}(B_t)$ is small. Note that b_{j5} may take a large or infinite value if the size of the future demand is much larger than the current demand. For example, if network effects exist, demand for the product may grow so rapidly. In this case, b_{j5} might be large. Intuitively, if the future demand is so large compared to the current demand, the firm would make much of the future demand. Consequently, the static approximation might not work well if b_{j5} is large.

The case of perfectly durable goods with no persistent consumer heterogeneity and no network effects

Here, we consider the case of perfectly durable goods, where consumers would not buy anymore after purchasing any product. We do not allow the existence of persistent consumer heterogeneity here and assume that only one consumer type exists. Let $x_t = 0$

be the state where consumers do not own any product. Consumers make purchase decisions only when they are at state $x_t = 0$. Furthermore, let $B_t = Pr_t(x_t = 0)$ be the fraction of consumers who do not own any product at the beginning of time t[21]. We consider an environment where marginal cost and flow utility do not change over time, for simplicity. Besides, we consider the case where the CCPs do not depend on B_t, and focus on the environment where no network effects exist.

Under the setting, we can show that the equilibrium prices do not depend on the aggregate states B_t. To prove the statement, guess the equilibrium price of firm $k \in \mathcal{J}$ in the following form:

$$p_k^*(B_t) = \theta_k \cdot mc_k$$

where θ_k is a parameter which does not depend on B_t. In addition, guess that consumers' value functions $V_t^C(x_t, B_t)$ do not depend on B_t. In this case, $E\left[V_{t+1}^C(x_{t+1}, B_{t+1}(B_t, s_t^{(ccp)}(B_t, p_t)))|x_t, a_t\right]$ do not depend on p_t. The transition process of B_t is in the following form:

$$B_{t+1} = s_{0t}^{(ccp)}(x_t = 0, p^*) \cdot B_t \tag{16}$$

Then, firm k's value function V_{kt+1}^F is in the following form:

$$\begin{aligned} V_k^F(B_{t+1}) &= \sum_{\tau=1}^{\infty} \beta_F^{\tau-1} M B_{t+\tau} \cdot s_{kt+\tau}^{(ccp)}(x_{t+\tau}, p^*) \\ &= M \sum_{\tau=1}^{\infty} \beta_F^{\tau-1} \left[\prod_{u=1}^{\tau-1} s_{0t+u}^{(ccp)}(x_{t+u} = 0, p^*)\right] B_{t+1} \cdot s_{kt+\tau}^{(ccp)}(x_{t+\tau} = 0, p^*) \end{aligned}$$

By (16), $\frac{\partial B_{t+1}}{\partial p_{kt}} = \frac{\partial s_{0t}^{(ccp)}(x_t=0, p_k^*, p_{-k}^*)}{\partial p_{kt}} \frac{\partial B_{t+1}(B_t, s_t^{(ccp)})}{\partial s_{0t}^{(ccp)}(x_t=0, p_k^*, p_{-k}^*)} = \frac{\partial s_{0t}^{(ccp)}(x_t=0, p_k^*, p_{-k}^*)}{\partial p_{kt}} B_t$ holds. Then, by (11), $p_k^* = \theta_k \cdot mc_k$ should satisfy the following first order condition:

$$\begin{aligned} 0 &= M B_t s_{kt}^{(ccp)}(x_t = 0, p_k^*, p_{-k}^*) + (p_k^* - mc_k) M B_t \frac{\partial f_k(x_t = 0, p_k^*)}{\partial p_{kt}} \frac{\partial s_{kt}^{(ccp)}(x_t = 0, p^*)}{\partial \widetilde{v_{kt}}(x_t = 0, p^*)} + \\ &\quad \beta_F \frac{\partial B_{t+1}(B_t, s_t^{(ccp)}(B_t, p^*))}{\partial p_{kt}} \frac{\partial V_j^F(B_{t+1})}{\partial B_{t+1}} \\ &= M B_t s_{kt}^{(ccp)}(x_t = 0, p^*) + M(\theta_k - 1) mc_k B_t \frac{\partial f_{kt}(x_t = 0, p_k^*)}{\partial p_{kt}} \frac{\partial s_{kt}^{(ccp)}(x_t = 0, p^*)}{\partial \widetilde{v_{kt}}(x_t = 0, p^*)} + \\ &\quad \beta_F \frac{\partial s_{0t}^{(ccp)}(x_t = 0, p^*)}{\partial p_{kt}} B_t \cdot M \sum_{\tau=1}^{\infty} \beta_F^{\tau-1} \left[\prod_{u=1}^{\tau-1} s_{0t+u}^{(ccp)}(x_{t+u} = 0, p^*)\right] \cdot s_{kt+\tau}^{(ccp)}(x_{t+\tau} = 0, p^*) \quad (\because (16)) \end{aligned}$$

Hence, we obtain:

$$\begin{aligned} 0 &= s_{kt}^{(ccp)}(x_t = 0, p^*) + (\theta_k - 1) mc_k \frac{\partial f_{kt}(x_t = 0, p_k^*)}{\partial p_{kt}} \frac{\partial s_{kt}^{(ccp)}(x_t = 0, p^*)}{\partial \widetilde{v_{kt}}(x_t = 0, p^*)} + \\ &\quad \beta_F \frac{\partial s_{0t}^{(ccp)}(x_t = 0, p^*)}{\partial p_{kt}} \cdot \sum_{\tau=1}^{\infty} \beta_F^{\tau-1} \left[\prod_{u=1}^{\tau-1} s_{0t+u}^{(ccp)}(x_{t+u} = 0, p^*)\right] \cdot s_{kt+\tau}^{(ccp)}(x_{t+\tau} = 0, p^*) \end{aligned}$$

Note that this equation does not depend on aggregate state variables B_t. Then, if the parameters $\{\theta_k\}_{k \in \mathcal{J}}$ satisfy the equation above, $p_k^*(B_t) = \theta_k \cdot mc_k$ is the equilibrium price.

Intuitively, under the condition that the goods are perfectly durable, no persistent heterogeneity exists, and no network effects exist, firms and consumers play the same games over time except for the potential number of consumers $M \cdot B_t$. Hence, the equilibrium prices do not depend on aggregate state variables B_t.

Furthermore, the result implies that current CCPs do not affect future prices and future CCPs. Hence, we can easily derive the following equations:

[21] We omit subscript l since only one consumer type exists.

$$\max_{h\in\mathcal{J},\tau\in\mathbb{N}}\sum_n\left|\frac{\partial p_{ht+\tau}}{\partial B_{t+1}^{(n)}}\right| = 0$$

$$\max_{l,\tau\in\mathbb{Z}_+,x_{t+\tau+1}}\sum_n\left|\frac{\partial\log s_{ljt+\tau+1}^{(ccp)}(x_{t+\tau+1},B_{t+\tau+1})}{\partial B_{t+\tau+1}^{(n)}}\right| = 0$$

$$(\gamma_5\equiv)\max_{\tau\in\mathbb{Z}_+,m}\sum_n\left|\frac{\partial B_{t+\tau+1}^{(m)}}{\partial B_{t+1}^{(n)}}\right| = 1$$

$$(b_{j4}\equiv)\max_{\tau\in\mathbb{Z}_+}\sum_n\left|\frac{\partial p_{jt+\tau+1}(B_{t+\tau+1})}{\partial B_{t+\tau+1}^{(n)}}\right| = 0$$

C Additional results

C.1 Equilibrium under firms' commitment ability

In the main part of this article, we considered the case where firms and consumers follow Markov perfect equilibrium. We can show that the main argument holds even when firms can commit to their future prices. If firms have commitment ability, firms would not change their future pricing even when future state variables change. Hence, the term corresponding to $b_{j4}\equiv\max_{\tau\in\mathbb{Z}_+}\sum_n\left|\frac{\partial p_{jt+\tau+1}(B_{t+\tau+1})}{\partial B_{t+\tau+1}^{(n)}}\right|$ disappears. This is the difference, and we can prove a statement similar to the statement in the main part of this article in a similar way.

C.2 Distribution of the random utility shock

In the main part of this article, we have mainly considered the case where the random utility shock ϵ_{iljt} follows Gumbel distribution. Then, what if it follows a distribution other than Gumbel? This subsection mainly considers the case where the random utility shock follows uniform distribution and looks at how the main results change under the condition.

For simplicity, we impose Assumption 4 and consider the symmetric firms' case as in Perloff and Salop (1985) and Gabaix et al. (2016). Let G and g be the distribution function and density function of random utility shock ϵ_{iljt}. To make the notations simpler, let $\widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))=v$ and $\widetilde{v_{l0t}}(x_t,B_t,p^*(B_t))=v_0$. First, we obtain the following statement[22]:

Lemma 7. *Under Assumption 4, the following equations hold:*

$$s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) = G(\theta)^{J-1}G(v-v_0+\theta)g(\theta)d\theta$$
$$\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))} = (J-1)\int G(\theta)^{J-2}G(v-v_0+\theta)g(\theta)^2d\theta+\int G(\theta)^{J-1}g(v-v_0+\theta)g(\theta)d\theta$$

Proof. First, by the symmetry assumption,

$$s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t)) = \int G(\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))-v+\theta)^{J-1}G(\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))-v_0+\theta)g(\theta)d\theta$$
$$\begin{aligned}\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))} &= (J-1)\int G(\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))-v+\theta)^{J-2}G(\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))-v_0+\theta)g(\theta)^2d\theta+\\ &\quad\int G(\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))-v+\theta)^{J-1}g(\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))-v_0+\theta)g(\theta)d\theta\end{aligned}$$

Since $\widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))=v$ holds by the symmetry assumption, we obtain the statements.

□

Next, let $\epsilon_{iljt}\sim_{i.i.d.}U(0,1)$. Then, G and g are in the following forms:

$$G(\theta) = \begin{cases}0 & \text{if }\theta<0\\ \theta & \text{if }0\leq\theta\leq1\\ 1 & \text{if }1<\theta\end{cases}$$
$$g(\theta) = \begin{cases}0 & \text{if }\theta<0,1<\theta\\ 1 & \text{if }0\leq\theta\leq1\end{cases}$$

[22] An analogous discussion exists in Perloff and Salop (1985) and Gabaix et al. (2016). The difference is the existence of the outside option. Under dynamic demand structures, the existence of the outside option is essential. For instance, in the case of perfectly durable goods, no outside option implies no demand in the next period, and demand dynamics plays minor role.

To exclude the case where all the consumers choose some products rather than the outside option (no purchase), we assume that $0 < v_0 - v < 1$ holds. By Lemma 7, it follows that:

$$\begin{aligned}
\frac{\partial s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} &= (J-1)\int_{v_0-v}^{1} \theta^{J-2}(v - v_0 + \theta)d\theta + \int_{v_0-v}^{1} \theta^{J-1}(v - v_0 + \theta)d\theta \\
&= (v - v_0) + \frac{J-1+v-v_0}{J} + \frac{1}{J+1} \\
&\quad - \left(-(v_0-v)^J + \frac{J-1+v-v_0}{J}(v_0-v\right))^J + \frac{1}{J+1}(v_0-v)^{J+1} \\
&\to 1 + v - v_0 \in (0,1)\ (J \to \infty)
\end{aligned} \tag{17}$$

Hence, unless $v_0 - v$ converges to 1 as J goes to infinity, $\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}$ converge to a positive value. Consequently, $\gamma_4 \equiv \max_{l,x_t} \sum_{k \in \mathcal{A}_l(x_t)} \frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))} \geq \max_{l,x_t} \sum_{k \in \mathcal{J}} \frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))} = \max_{l,x_t} J \frac{\partial s_{lkt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{lkt}}(x_t,B_t,p^*(B_t))} \to \infty\ (J \to \infty)$ holds, and Assumption 2 is violated. Moreover, since $s_{ljt}^{(ccp)}(x_t, B_t) \leq \frac{1}{J} \to 0\ (J \to \infty)$, $b_{j6} \equiv \max_{l,x_t} \frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}} \frac{\partial \log s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))} \leq \left[\max_{l,x_t} \frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}}\right] \cdot \left[\max_{l,x_t} \frac{\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}}{s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}\right] \to \infty\ (J \to \infty)$ holds, and Assumption 3 is not satisfied. Hence, the argument in the main part of this article fails under the uniform distribution.

Nevertheless, $p_{jt}^{(static)}$ and $p_{jt}^{(dynamic)}$ take closer values as $J \to \infty$ from different logic. As discussed in Section 3, $p_{jt}^{(dynamic)}$ and $p_{jt}^{(static)}$ satisfy the following equations:

$$\begin{aligned}
\frac{p_{jt}^{(dynamic)}(B_t) - mc_j}{p_{jt}^{(dynamic)}(B_t)} &= \left(-\left.\frac{\partial \log s_{jt}}{\partial \log p_{jt}}\right|_{\text{static}}\right)^{-1} \frac{1+\lambda_j^F(B_t)}{1+\lambda_j^C(B_t)} \\
\frac{p_{jt}^{(static)}(B_t) - mc_j}{p_{jt}^{(static)}(B_t)} &= \left(-\left.\frac{\partial \log s_{jt}}{\partial \log p_{jt}}\right|_{\text{static}}\right)^{-1}
\end{aligned}$$

If the marginal cost is positive and p_{jt} does not converge to zero, it follows that:

$$\begin{aligned}
\left.\frac{\partial \log s_{jt}}{\partial \log p_{jt}}\right|_{\text{static}} &= \frac{\sum_l \pi_l \sum_{x_t} Pr_{lt}(x_t) \frac{\partial f_{lj}(x_t,B_t,p_j^*(B_t))}{\partial p_{jt}} \frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}}{\sum_l \pi_l \sum_{x_t} s_{ljt}^{(ccp)}(x_t, B_t)} p_{jt} \\
&\geq \min_{l,x_t} \left| \frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} \frac{\partial \log s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t, B_t, p^*(B_t))} \right| p_{jt} \\
&= \min_{l,x_t} \left| \frac{\partial f_{lj}(x_t, B_t, p_j^*(B_t))}{\partial p_{jt}} \frac{\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}}{s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))} \right| p_{jt} \\
&\to \infty\ (J \to \infty) \quad \left(\because \frac{\frac{\partial s_{ljt}^{(ccp)}(x_t,B_t,p^*(B_t))}{\partial \widetilde{v_{ljt}}(x_t,B_t,p^*(B_t))}}{s_{ljt}^{(ccp)}(x_t, B_t, p^*(B_t))} \to \infty\ (J \to \infty)\right)
\end{aligned}$$

Then, both $p_j^{(dynamic)}$ and $p_j^{(static)}$ converge to mc_j as the number of firms goes to infinity, if λ_j^C does not converge to -1[23] and λ_j^F takes a finite value. Consequently, $p_j^{(static)}$ and $p_j^{(dynamic)}$ take sufficiently close values when the number of the firms is so large even under uniform distribution.

D Details of the numerical experiments

Algorithm

To run the numerical experiments, we need to solve the equilibrium. this study uses the following fixed point algorithm:

[23] In the case of perfectly durable goods with no persistent consumer heterogeneity and no network effects, $\gamma_2 = 0$ holds, as we discussed in Appendix C, and consequently $\lambda_j^C = 0$ holds.

1. Take grid points of state variables $B_t = (B_{lt})_l = (Pr_{lt}(x_t = 0))_l$. Set initial values of $V^{C(0)} \equiv \left\{V_l^{C(0)}(x_t, B_t)\right\}_{l,x_t,B_t}$ (consumers' value function), $V^{F(0)} \equiv \left\{V_j^{F(0)}(B_t)\right\}_{j,B_t}$ (firm's value function), $p^{(0)} \equiv \left\{p_j^{(0)}(B_t)\right\}_{j,B_t}$ (equilibrium price), $B_{t+1}^{(0)} \equiv \left\{B_{lt+1}^{(0)}(B_t)\right\}_{l,B_t}$ (aggregate level state variables in the next period), and $\frac{\partial B_{t+1}^{(0)}}{\partial p_t} \equiv \left\{\frac{\partial B_{lt+1}^{(0)}}{\partial p_{jt}}(B_t)\right\}_{j,l,x_t,B_t}$ (derivative of the aggregate state variables in the next period with respect to the current price).

2. Iterate the following process until the convergence of $V^{C(n)}, V^{F(n)}, p^{(n)}, B_{t+1}^{(n)}, \frac{\partial B_{t+1}^{(n)}}{\partial p_t}$ $(n = 1, 2, \cdots)$:

   (a) Given $V^{C(n)}, V^{F(n)}, p^{(n)}, B_{t+1}^{(n)}, \frac{\partial B_{t+1}^{(n)}}{\partial p_t}$, solve:

$$
\begin{aligned}
s_{ljt}^{(ccp)}(x_t = 0, B_t, p^{(n)}(B_t)) &= \frac{\exp\left(f_{lj}(x_t = 0, B_t, p_j^{(n)}(B_t)) + \beta_C E\left[\widehat{V_l^{C(n)}}(x_{t+1}, B_{t+1}^{(n)}(B_t))|x_t = 0, a_t = j\right]\right)}{\sum_{k\in\mathcal{J}\cup\{0\}} \exp\left(f_{lk}(x_t = 0, B_t, p_k^{(n)}(B_t)) + \beta_C E\left[\widehat{V_l^{C(n)}}(x_{t+1}, B_{t+1}^{(n)}(B_t))|x_t = 0, a_t = k\right]\right)} \\
s_{l0t}^{(ccp)}(x_t = 0, B_t, p^{(n)}(B_t)) &= \frac{\exp\left(f_{lj}(x_t = 0, B_t, p_j^{(n)}(B_t)) + \beta_C E\left[\widehat{V_l^{C(n)}}(x_{t+1}, B_{t+1}^{(n)}(B_t))|x_t = 0, a_t = 0\right]\right)}{\sum_{k\in\mathcal{J}\cup\{0\}} \exp\left(f_{lkt}(x_t = 0, B_t) + \beta_C E\left[\widehat{V_l^{C(n)}}(x_{t+1}, B_{t+1}^{(n)}(B_t))|x_t = 0, a_t = k\right]\right)} \\
s_{jt}(B_t, p^{(n)}(B_t)) &= \sum_l \pi_l Pr_{lt}(x_t = 0) \cdot s_{ljt}^{(ccp)}(x_t = 0, B_t, p^{(n)}(B_t))
\end{aligned}
$$

   where $\widehat{\cdot}$ denotes the interpolated value based on the values at grid points B_t.

   (b) Given $V^{C(n)}, V^{F(n)}, p^{(n)}, B_{t+1}^{(n)}, \frac{\partial B_{t+1}^{(n)}}{\partial p_t}$ and $\left\{s_{ljt}^{(ccp)}(x_t = 0, B_t, p^{(n)}(B_t))\right\}_{l,j,B_t}$, $\left\{s_{l0t}^{(ccp)}(x_t = 0, B_t, p^{(n)}(B_t))\right\}_{l,B_t}$, $\left\{s_{jt}(B_t, p^{(n)}(B_t))\right\}_{j,B_t}$, solve:

$$
\begin{aligned}
V_l^{C(n+1)}(x_t = 0, B_t) &= \log\left(\sum_{k\in\mathcal{J}\cup\{0\}} \exp\left(f_{lk}(x_t = 0, B_t, p_{kt}^{(n)}(B_t)) + \beta_C E\left[\widehat{V_l^{C(n)}}(x_{t+1}, B_{t+1}^{(n)}(B_t))|x_t = 0, a_t = k\right]\right)\right) \\
V_l^{C(n+1)}(1 \le x_t \le x_{max} - 1, B_t) &= f_0 + \beta_C\left[\phi V_l^{C(n)}(x_{t+1} = x_t + 1, B_{t+1}^{(n)}(B_t)) + (1-\phi)V_l^{C(n)}(x_{t+1} = 0, B_{t+1}^{(n)}(B_t))\right] \\
V_l^{C(n+1)}(x_t = x_{max}, B_t) &= f_0 + \beta_C V_l^{C(n)}(x_{t+1} = 0, B_{t+1}^{(n)}(B_t)) \\
V_j^{F(n+1)}(B_t) &= \left(p_j^{(n)}(B_t) - mc_j\right) M s_{jt}(B_t, p^{(n)}(B_t)) + \beta^F \widehat{V_j^{F(n)}}(B_{t+1}^{(n)}) \\
p_j^{(n+1)}(B_t) &= mc_j - \frac{M s_{jt} + \beta_F \frac{\partial \widehat{V_j^{F(n)}}(B_t, s_t^{(ccp)}(B_t, p^{(n)}(B_t)))}{\partial p_{jt}}}{M \frac{\partial s_{jt}(B_t, p^{(n)}(B_t))}{\partial p_{jt}}} \\
B_{lt+1}^{(n+1)}(B_t) &= B_{lt}^{(n)} s_{l0t}^{(ccp)}(x_t = 0, B_t, p^{(n)}(B_t)) + (1-\phi)\left(1 - B_{lt}^{(n)} s_{l0t}^{(ccp)}(x_t = 0, B_t, p^{(n)}(B_t))\right) \\
\frac{\partial B_{lt+1}^{(n+1)}}{\partial p_{jt}}(B_t) &= \sum_{\tilde{l}} \sum_{k\in\mathcal{J}\cup\{0\}} \frac{\partial s_{\tilde{l}kt}^{(ccp)}(x_t = 0, B_t, p^{(n)}(B_t))}{\partial p_{jt}} \frac{\partial B_{lt+1}^{(n)}(B_t, s_t^{(ccp)})}{\partial s_{\tilde{l}kt}^{(ccp)}(x_t = 0, B_t, p^{(n)}(B_t))}
\end{aligned}
$$

To speed up the convergence process, this study incorporates the spectral algorithm developed in La Cruz et al. (2006) in the iteration. To interpolate the values at the points other than the grid points, this study uses the collocation method, and Chebyshev polynomials are used as basis functions. Furthermore, to efficiently interpolate the values of $\widehat{V_l^{C(n)}}$ and $\widehat{V_j^{F(n)}}$, this study uses Smolyak method developed by Smolyak (1963) and improved by Judd et al. (2014).

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