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Abstract

This article analytically investigates the mechanism behind the biases in price elasticities of demand in applying static demand models under dynamic demand, which has been pointed out by the previous empirical studies. There are three sources of biases: disregard of state variables (affecting short-run elasticity), inconsistent utility parameter estimates, and changing expectations of consumers (affecting long-run elasticity). Disregard of state variables, such as durable product holdings, leads to overestimate of short-run own elasticities. Especially when the focus is on the large conditional choice probability products, the first and the third sources of biases might induce large biases in price elasticities.

Keywords: Dynamic demand; Static demand model; Estimation bias; Price elasticity of demand; Dynamic discrete choice.

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1 Introduction

Many markets are characterized by dynamic demand structures, wherein future demand has connections with current demand. Examples of goods with dynamic demand include durable goods, storable goods, goods with switching costs, and experience goods. Given the significant presence of these goods in many industries, empirical analysis of these goods is essential for a deeper understanding of the economy.

So far, to analyze these goods, a large number of studies have applied static demand models, in which the connections between current and future demand are not explicitly specified. For example, Berry et al. (1995) (henceforth BLP) applied a static demand model to automobiles, which is one of the typical examples of durable goods. Many of the studies derived implications on competition, industrial, trade, and environmental policies. Nevertheless, previous empirical studies specifying dynamic demand models have found that applying static demand models yields biased estimates of price elasticities of demand, as summarized in Table 1. For example, Hendel and Nevo (2006), studying storable goods, showed that static demand estimates, which neglect dynamics, overestimate long-run own price elasticities by 30%, and underestimate long-run cross price elasticities by up to a factor of 5. Gowrisankaran and Rysman (2012), studying new durable goods, showed that applying a BLP style static demand model yields price elasticities sufficiently close to zero.

The purpose of this study is to investigate why applying static demand models yields biased price elasticities under dynamic demand. Even though numerous empirical studies have found the biases in applying static models, they are limited to empirical contexts. It is not clear whether these results, such as the sign and magnitude of the biases, are the same in other markets. Furthermore, some of the studies showed the opposite results. For instance, Chen et al. (2008), analyzing the automobile market, showed that the own price elasticity of demand computed from a static model is larger than the short-run own price elasticity of demand computed from a dynamic model. In contrast, Schiraldi (2011), also studying the automobile market, showed that the own price elasticity computed from a static model is smaller than the short-run own price elasticity of demand computed from a dynamic model. In this article, I develop a model of dynamic demand and show how biases arise and in what cases the biases are small or large.

The understanding of the mechanism is important for the appreciation of static demand models. Even though the estimation and simulation of dynamic demand models are getting easier due to the accumulation of knowledge and advancement of computational power, analysis with dynamic demand models is time-consuming and requires greater effort. In addition, for researchers interested in the supply side behavior, the introduction of dynamic demand structures might not be attractive, since they need to consider firms' dynamic pricing setting behaviors to make the models consistent with the theory[1]. When the dynamics in both the demand and supply sides are introduced, the problem gets more complicated and computationally burdensome. In that sense, static demand models would survive even in the future. If we can expect that the biases in applying static demand models are small for the goods of interest, it can justify the studies that use static demand models. If not, researchers should be careful about the use of static demand models.

In this study, I classify the biases into three types: disregard of state variables, inconsistent utility parameter estimates, and changing expectations of consumers. The first source of bias, namely, the disregard of state variables, causes biased estimates of short-run price elasticity of demand. Here, short-run price elasticity is defined as the elasticity of the current period's demand in response to the current period's temporary price change, given fixed consumers' future expectations. The examples of state variables are durable goods holdings. Especially when using aggregate data, researchers may not have data on the consumers' inventory[2], and existence of consumer inventory is totally ignored in the standard static demand models. In these static models, it is implicitly assumed that even the consumers already owning durable goods are highly likely to consider additional purchases, which might lead to biased price elasticities. I show that applying a static model ignoring state variables leads to the overestimation of short-run own elasticity[3]. Further, short-run cross price elasticities are

[1] Under dynamic demand, current price change affects consumers' expectations on future outcomes, and indirectly affects current demand. Furthermore, the current price change affects future demand through the change in current demand. Hence, firms would consider these dynamic elements when setting product prices.

[2] In the case of storable goods, individual- level data (scanner data) is typically available, and many previous studies have utilized them. Nevertheless, data on consumer inventory is not available, and it has been recognized as the one obstacle for studying the goods.

[3] By simple calculation, we can easily construct an example where short-run own elasticity is overestimated by over 300%.

underestimated for durable goods with unit inventory, and storable goods with the same package sizes. Note that one remedy for the problem is the introduction of random coefficients, especially the one on the constant term, to approximate the distribution of consumers' state variables. Nevertheless, there is no guarantee that such "reduced-form" approach works well.

The second source of bias, namely, inconsistent utility parameter estimates, arises due to the failure to account for consumers' future expectations and unobserved state variables[4]. Utility parameters are inconsistently estimated under static models without controlling for these components, because product characteristics might be correlated with consumers' future expectations or the ratio of the consumers in each unobserved state variable. So far, several articles have proposed remedies for static demand models, such as the introduction of time and brand dummies. In this article, we discuss under what conditions these remedies yield consistent parameter estimates or mitigate the problems associated with the use of the static demand models.

The third source of bias, namely, the changing expectations of consumers, causes biased estimates of long-run price elasticity of demand. Here, long-run price elasticity is defined as the elasticity of the current period's demand change in response to the permanent or long-term price change, allowing the changing expectations of consumers. The difference between the short-run and the long-run elasticity is the changing expectation of consumers. In the case of short-run elasticity, price change is limited to only the current period, and we consider the setting where consumers' expectations do not change. However, in the case of long-run elasticity, price changes not only in the current period, but also in the future period. Consumers change expectations on future market conditions, and consequently change current demand[5]. Even though the direct counterpart of long-run elasticity does not exist in static demand models, the elasticity is essential for a more precise evaluation of the consumers' responses to prices. I show that the sign and the size of the difference between the short-run elasticity and long-run elasticity are determined by the index representing the connection between current and future demand under a perfect foresight setting. Using the statement, we can easily guess that long-run own elasticity is smaller than short-run own elasticity, and long-run cross price elasticity is larger than short-run cross elasticity for durable and storable goods[6].

With these classifications and statements, we can explain the mechanism behind the results of the previous empirical articles discussing the biases that result from the application of static demand models summarized in Table 1. Furthermore, the insight presented here can be applied to other settings not explored in the previous literature. Note that I also show that the first and the third sources of biases are large when the focus is on the product with large CCPs. Hence, researchers should be careful about the use of static demand models especially when the focus is on the products with large market shares.

The main contribution of this article is twofold. First, this article provides insight into the biases associated with the first source of bias: disregard of state variables. Previous literature has paid much attention to the changing expectations of consumers and inconsistent utility parameter estimates, but not disregarding of state variables. Consequently, the literature could not fully explain why some studies have shown contradicting results. This article contributes to the literature by stressing the importance of the biases due to ignoring consumers' state variables, such as consumers' durable product holdings.

Second, this article provides insight on when the biases in applying static demand models are small or large. Even though introducing dynamic demand models would fully solve the problems, sometimes it is not attractive, since it makes the supply-side model more complex and sometimes not manageable. I show the conditions when applying static demand models yield small biases in price elasticities of demand. When the biases are negligible, applying static demand models causes only minor problems.

The rest of this article is organized as follows. In Section 2, I describe the relationship between this study and the previous studies. In Section 3, we develop a dynamic demand model and discuss the three sources of the biases. In Section 4, we extend the discussions in Section 3, and show the magnitude of the biases and remedies for static demand models. Further, we extend the base model and discuss storable goods. In Section 5 we discuss how the results in this article provide insight into the previous studies' findings summarized in

[4]For instance, the existence of product stock corresponds to the consumers' unobserved state variables in the case of durable goods.

[5]For instance, consumers would postpone purchasing durable goods if the price reduction of the product is permanent, rather than temporary.

[6]Still, the result is the opposite for goods with switching costs.

Table 1. Finally, Section 6 concludes. All the proof of the statements are shown in Appendix A. In Appendix B, I show the results of Monte Carlo experiments to demonstrate the effectiveness of introducing time dummies for utility parameter estimates. In Appendix C, I show the sources of the information on the literature on the biases in static demand models represented in Table 1 and Section 5.

2 Literature

This study relates and contributes to several strands of literature.

2.1 Literature on dynamic demand models

First, this study closely relates to the recent empirical literature on dynamic demand models[7]. Many of them specified Dynamic Discrete Choice (DDC) models as in Rust (1987), estimated consumers' utility parameters and derived implications. As summarized in Table 1, many articles have shown that applying static demand models yields biased price elasticities of demand. This study attempts to explain the mechanism behind the biases by developing a general model.

Gowrisankaran and Rysman (2020) also develop a general model of dynamic demand, and the framework in this article is similar to theirs. Their focus is surveying previous studies and clarifying the implicit assumptions imposed in previous empirical studies, which alleviate the computational burden of solving dynamic demand models. In contrast to their article, we focus on the difference between the dynamic and static demand models.

Fukasawa (2022) also investigates dynamic demand using a unified framework similar to the current article, but the focus is different. The existing article focuses on the supply side, and investigates the specific cases in which a firm's dynamic pricing behavior can be approximated as static under dynamic demand. On the other hand, the current article focuses on the demand side, and investigates whether applying a static demand model yields a good approximation of the true dynamic demand model. So, these articles are complementary. It is interesting that Fukasawa (2022) shows a firm's dynamic pricing behavior under dynamic demand can be approximated as static when CCPs of the firm's product are small for all consumer types and state variables. The condition is the same as the condition for small biases in price elasticities shown in the current article. If the condition holds and utility parameters are consistently estimated, we can argue that applying the "static model," namely, the combination of the static demand-side model and the static supply-side model, has valid implications on firms' markups and the effects of short-run policy interventions[8].

2.2 Literature applying static demand models

Second, this article contributes to the large number of studies applying static demand models to goods with dynamic demand. For example, since the advent of the Berry et al. (1995)'s BLP method, many studies have applied static demand models to products with dynamic demand, such as durable goods. To justify the use of static demand models, several articles have discussed some remedies. For instance, Goldberg and Verboven (2001), studying automobiles, argued that adding time and brand dummies mitigates the problem. Nevertheless, these discussions have been limited to informal ones. In this study, I formally show the extent to which these remedies alleviate the issues.

2.3 Literature on dynamic discrete choice models

Third, this article contributes to the recent literature on Dynamic Discrete Choice (DDC) models, which is the basis of dynamic demand models. So far, the main focus of the literature has been on the identification of structural parameters, including the discount factor (e.g. Magnac and Thesmar, 2002; Abbring and Daljord, 2020) and more straightforward estimation methods of DDC models (e.g. Hotz and Miller, 1993; Kalouptsidi

[7]For a survey of the literature, see Aguirregabiria and Nevo (2013), Gowrisankaran and Rysman (2020), and Gandhi and Nevo (2021).

[8]Note that static models cannot deal with long-run policy interventions that largely affect consumers' expectations.

Articles	Category	Market	Logit spec.	Ind. data	Hetero.	Est. of price coef.	Own elas.	Cross elas.
Chen et al. (2008)*	Durables	Automobile	Yes	No	No	S(2.26)<D(2.31)	DS*(4.19)<S(4.79)	-
Prince (2008)	Durables	PC	Yes	Yes	No	S(-0.145)<M(0.293)<D(0.305)	DL(2.17)<M(2.63)<DS(2.74)	-
Gordon (2009)	Durables	PC processor	Yes	No	Yes	M(0.849)<D(1.803)	M(3.68,3.40)<DL(5.70,5.186)	M(1.63,1.59) < DL(2.70,2.187)
Schiraldi (2011)*	Durables	Automobile	Yes	No	Yes	S(9.32)<D(52.37)	S(2.39)<DS(8.81) **	S(0.0225)<DS(0.2564)
Gowrisankaran and Rysman (2012)*	Durables	Digital camera	Yes	No	Yes	S(0.099)<D(3.43)	S(0)<DL(2.41)<DS(2.59)	-
Lou et al. (2012)	Durables	Digital camera	Yes	No	Yes	S(2.610)<D(4.402)	S(1.103∼1.256)<DL(3.387∼3.425)	S(0.0035∼0.0223)<DL(0.047∼0.061)
Erdem et al. (2003)	Storables	Ketchup	Yes	Yes	Yes	-	DL(3.6)<DS(4.9)	DS(0.2)<DL(0.75∼1)
Sun et al. (2003)	Storables	Ketchup	Yes	Yes	Yes	D(0.44)<S(0.47)	DS<S (bias:3.9%)	-
Hendel and Nevo (2006)*	Storables	Detergent	Yes	Yes	Yes	D<S (bias:15%)	DL<S(bias:30%)	S<DL (bias:∼500%)
Hendel and Nevo (2013)	Storables	Soda	No	No	**	-	DL(2.16,2.78)<S(2.46,2.94)	-
Wang (2015)	Storables	Soda	Yes	Yes	Yes	-	DL<S(bias:60.8%)	-
Perrone (2017)	Storables	Food product	No	Yes	**	-	DL<DS (ratio:1.899∼3.406)	-
Li (2021)	Storables	(Theory)	-	-	**	-	**	**
Ho (2015)	Switching cost	Bank deposits	Yes	No	No	S(100)<D(192)	S(0.05∼0.26) < DS(0.11∼0.43) <DL(0.19∼0.77)	S(0.02)<DS(0.05)<DL(0.05∼0.08)
Shcherbakov (2016)	Switching cost	TV service	Yes	No	**	D(0.078)<S(0.114)	DS(0.95,1.13) < DL(1.14,2.32) **	-
Yeo and Miller (2018)	Switching cost	Health insurance	Yes	No	**	D(0.029)<M(0.038)<S(0.045)	DS(1.033)<M(1.368)<DL(1.606) **	M(0.0093)<DL(0.0135) **
Hartmann (2006)	Consumption capital	Golf	Yes	Yes	**	S(0.048)<D(0.074)	DL(2.7789)<DS(3.0550)	-
Osborne (2011)	Switching cost&Learning	Detergent	Yes	Yes	Yes	**	**	**
Seiler (2013)	Storables & CS	Detergent	Yes	Yes	Yes	**	**	-
Pires (2016)	Storables & CS	Detergent	Yes	Yes	Yes	S(No CS;0.747)<D(1.265)	**	**

Table 1: Literature on the biases in static demand models

Notes:
For additional information, see Section 5 and Appendix C of the current article.
"Est. of Price coef." shows the estimated mean coefficient of price in the consumers' utility function.
"Logit spec." indicates whether each study assumed logit distribution in the model.
"Ind. data" indicates whether each study used individual data.
"Hetero." indicates whether each study introduced consumer heterogeneity (random coefficients) in the dynamic and static demand models.
"S" and "D" in the column of "Est. of price coef." denote the estimates of price coefficients applying static models and dynamic models. "M" denotes the estimates applying myopic models (dynamic models with consumers' discount factor set to zero).
"S", "DS" "DL", "M" in the columns of "Own elas." and "Cross elas." denote the estimates of price elasticities computed from static demand models, short-run elasticity computed from dynamic models, long-run elasticity computed from dynamic models, and elasticity computed from myopic models (dynamic models with consumers' discount factor set to zero).
"CS" denotes the consideration set.
"-" indicates that a result was not provided in the articles.
"*" indicates that the results of the paper is discussed in the main part of this paper.
"**" indicates that an additional explanation is presented in Appendix C of the current article.

et al., 2021b). In contrast to the previous studies, the main purpose of the current article is neither to propose a new identification strategy nor a new estimation method of DDC models. Instead, we discuss the situations in which static models yield large biases and when applying dynamic models is essential, which have not been discussed in the previous literature. In that sense, this study is complementary to the literature on DDC models[9].

3 Model

In general, applying static demand models yields biased price elasticities of demand when the true demand structure is dynamic. To show the essence of the problem, we develop a standard dynamic demand model in Section 3.1. Then, using the model, we discuss the three sources of the biases: disregard of state variables, inconsistent utility parameter estimates, and changing consumer expectations. In this section, we mainly discuss with durable goods in mind. Nevertheless, we can use the general model for other types of goods with dynamic demand, such as goods with switching costs. Besides, since we consider the model without the choice of consumption level, we cannot directly apply the results to storable goods. Nevertheless, we can extend the model and use some of the results to storable goods, as discussed in Section 4.3.

3.1 Setup

3.1.1 State variables

First, let $x_t \in X_t$ be a consumer's individual-level state variables at the beginning of time t. X_t denotes the set of individual-level states. For instance, x_t indicates whether the consumer already owns durable products, in the case of durable goods. In this study, we assume that X_t is a discrete set, for notational simplicity. Nevertheless, we can easily extend this to the case where X_t is a continuous set. Let $Pr_{lt}(x_t)$ be the ratio of type l consumers at state x_t among type l consumers at time t. Note that $\sum_{x_t \in X_t} Pr_{lt}(x_t) = 1$ holds by definition.

3.1.2 Choices

Let $\mathcal{J}_t$ denotes the set of products available at time t. "0" represents the outside option, namely, the option of not buying any product. Further, let $a_t \in \mathcal{J}_t \cup \{0\}$ be the choice of a consumer at time t. $a_t = j$ means that the consumer purchases product j, and $a_t = 0$ means that they do not purchase any product.

3.1.3 Utility function

In this study, we assume each consumer purchases at most one product in each period. Let the expected discounted utility of type l consumer i whose state is x_t and choice is a_t given product prices $p_t \equiv (p_{jt})_{j \in \mathcal{J}_t}$ and continuation values $g_t \equiv (g_{ljt}(x_t))_{l, x_t, j \in \mathcal{J}_t \cup \{0\}}$ be $v_{ilt}(x_t, p_t, g_t, a_t)$. Here, p_{jt} denotes product j's price at time t. Type l consumer i maximizes utility $v_{ilt}(x_t, p_t, g_t, a_t)$ with respect to $a_t \in \mathcal{J}_t \cup \{0\}$.

Utility $v_{ilt}(x_t, p_t, g_t, a_t)$ is in the following form:

$$v_{ilt}(x_t, p_t, g_t, a_t) = \begin{cases} -\alpha_l p_{jt} + f_{ljt} + \phi_{ljt}(x_t) + \beta_C g_{ljt}(x_t) + \epsilon_{ijt} & \text{if } a_t = j \\ f_{l0t}(x_t) + \beta_C g_{l0t}(x_t) + \epsilon_{i0t} & \text{if } a_t = 0 \end{cases} \quad (1)$$

where f_{ljt} denotes the flow utility type l consumers gain when buying product j. $f_{l0t}(x_t)$ denotes the flow utility type l consumers at state x_t gain when not buying anything. For instance, it represents the utility from continuing using previous durable product x_t in the case of durable goods. $\phi_{ljt}(x_t)$ denotes the flow utility for consumers at state x_t gains when purchasing product j, which is not captured in f_{ljt}. For instance,

[9] In the recent literature on DDC models, Kalouptsidi et al. (2021a) compare "static" and dynamic models to investigate the identification of counterfactuals. In their discussion, it is assumed that the static model is the right specification when using the static model. On the other hand, in the current article, we consider the setting where the dynamic model is the correct specification when applying the static model. Hence, the meaning of the static model in the current article is different from the one in Kalouptsidi et al. (2021a).

it represents the resell value of old durable products x_t. $(\epsilon_{ijt})_{j\in\mathcal{J}_t\cup\{0\}}$ denotes the individual-level random preference shock, and we assume that they follow i.i.d. mean-zero type-I extreme value distribution. β_C represents the consumers' discount factor, and E_t represents the expectation operator given the information available at time t. α_l represents type l consumers' marginal utility of money, and we assume $\alpha_l > 0$ holds for all l. Besides, continuation value $g_{ljt}(x_t)$ is in the following form:

$$g_{ljt}(x_t) = E_t V^C_{lt+1}(x_{t+1}, p_{t+1}|x_t, p_t, a_t = j) \tag{2}$$

Note that g_t depends on the expected path of future product prices $\{p_{t+\tau}\}_{\tau\geq 1}$.

$V^C_{lt}(x_t, p_t, g_t)$ is the value function of type l consumers given states x_t at time t given product prices p_t and continuation values g_t, and defined as follows:

$$V^C_{lt}(x_t, p_t, g_t) \equiv E_\epsilon\left[\max_{a_t\in\mathcal{J}_t\cup\{0\}} v_{ilt}(x_t, p_t, g_t, a_t)\right]$$

where E_ϵ denotes the expectation operator with respect to random i.i.d. shocks $\{\epsilon_{ijt}\}_{j\in\mathcal{J}_t\cup\{0\}}$. Under the assumption that ϵ_{ijt} follows i.i.d. type-I extreme value distribution, the following formula holds:

$$V^C_{lt}(x_t, p_t, g_t) = \log\left(\sum_{j\in\mathcal{J}_t}\exp\left(-\alpha_l p_{jt} + f_{ljt} + \phi_{ljt}(x_t) + \beta_C g_{ljt}(x_t)\right) + \exp\left(f_{l0t}(x_t) + \beta_C g_{l0t}(x_t)\right)\right) \tag{3}$$

3.1.4 Choice probability

The CCP that type l consumer buys product j at time t conditional on being at states x_t is:

$$\begin{aligned} s^{(ccp)}_{ljt}(x_t, p_t, g_t) &= Pr\left(v_{ilt}(x_t, p_t, g_t, a_t = j) > v_{ilt}(x_t, p_t, g_t, a_t = k)\ \forall k \in \mathcal{J}_t \cup \{0\} - \{j\}\right) \\ &= \frac{\exp\left(-\alpha_l p_{jt} + f_{ljt} + \phi_{ljt}(x_t) + \beta_C g_{ljt}(x_t)\right)}{\exp\left(V^C_{lt}(x_t, p_t, g_t)\right)} \end{aligned} \tag{4}$$

The CCP that type l consumer does not buy any product at time t conditional on being at states x_t is:

$$s^{(ccp)}_{l0t}(x_t, p_t, g_t) = \frac{\exp\left(f_{l0t}(x_t) + \beta_C g_{l0t}(x_t)\right)}{\exp\left(V^C_{lt}(x_t, p_t, g_t)\right)} \tag{5}$$

The probability that type l consumer buys product j at time t is:

$$s_{ljt}(p_t, g_t) = \sum_{x_t\in X_t} s^{(ccp)}_{ljt}(x_t, p_t, g_t)\cdot Pr_{lt}(x_t) \tag{6}$$

The probability that type l consumer does not buy any product at time t is:

$$s_{l0t}(p_t, g_t) = \sum_{x_t\in X_t} s^{(ccp)}_{l0t}(x_t, p_t, g_t)\cdot Pr_{lt}(x_t) \tag{7}$$

The market share of product j at time t, namely, the fraction of consumers purchasing product j at time t is:

$$s_{jt}(p_t, g_t) = \int s_{ljt}(p_t, g_t)dP(l) \tag{8}$$

where $dP(l)$ denotes the measure of type l consumers.

The fraction of consumers not purchasing any product at time t is:

$$s_{0t}(p_t, g_t) = \int s_{l0t}(p_t, g_t)dP(l) \tag{9}$$

3.1.5 State transition

The transition probability of consumer-level state variables x_t is given by $\psi(x_{t+1}|x_t, a_t)$. It depends on the previous period's states x_t and choices a_t. For instance, in the case of durable goods which depreciate over time, x_{t+1}, product holding at time $t+1$, depends on the previous state x_t and the product choice a_t at time t. The transition process depends on the depreciation rate of the durable products.

Note that $Pr_{lt}(x_t)$ satisfies the following state transition formula:

$$Pr_{lt+1}(x_{t+1}) = \sum_{x_t \in X_t} Pr_{lt}(x_t) \cdot \sum_{j \in \mathcal{J}_t \cup \{0\}} s_{ljt}^{(ccp)}(x_t, p_t, g_t) \cdot \psi(x_{t+1}|x_t, a_t = j) \tag{10}$$

The dynamic demand system is composed of equations (2)-(10).

3.1.6 Price elasticity of demand

In this study, short-run price elasticity is defined as the elasticity of the current period's demand in response to the current period's temporary price change, given fixed consumers' future expectations. Long-run price elasticity is defined as the elasticity of the current period's demand change in response to the permanent or long-term price change, allowing changing expectations of consumers[10][11].

Under the specifications above, short-run own price elasticity of product j at time t given product prices $p_t^0 = (p_{jt}^0)_{j \in \mathcal{J}_t}$ and continuation values g_t^0 is:

$$\begin{aligned}
\eta_{jt}^{(short)}(p_t^0, g_t^0) &\equiv -\frac{\partial s_{jt}(p_t^0, g_t^0)}{\partial p_{jt}} \frac{p_{jt}^0}{s_{jt}^0} \\
&= -\left[\int \sum_{x_t \in X_t} Pr_{lt}(x_t) \frac{\partial s_{ljt}^{(ccp)}(x_t, p_t^0, g_t^0)}{\partial p_{jt}} dP(l)\right] \frac{p_{jt}^0}{s_{jt}^0} \\
&= \left[\int \alpha_l \sum_{x_t \in X_t} Pr_{lt}(x_t) s_{ljt}^{(ccp)}(x_t, p_t^0, g_t^0)(1 - s_{ljt}^{(ccp)}(x_t, p_t^0, g_t^0)) dP(l)\right] \frac{p_{jt}^0}{s_{jt}^0}
\end{aligned} \tag{11}$$

Here, we define the term $s_{jt}^0 \equiv s_{jt}(p_t^0, g_t^0)$.

Short-run cross price elasticity of product k with respect to product $j(\neq k)$ at time t given product prices p_t^0 and continuation values g_t^0 is:

$$\begin{aligned}
\eta_{jkt}^{(short)}(p_t^0, g_t^0) &\equiv \frac{\partial s_{kt}(p_t^0, g_t^0)}{\partial p_{jt}} \frac{p_{jt}^0}{s_{kt}^0} \\
&= \left[\int \sum_{x_t \in X_t} Pr_{lt}(x_t) \frac{\partial s_{lkt}^{(ccp)}(x_t, p_t^0, g_t^0)}{\partial p_{jt}} dP(l)\right] \frac{p_{jt}^0}{s_{kt}^0} \\
&= \left[\int \alpha_l \sum_{x_t \in X_t} Pr_{lt}(x_t) s_{ljt}^{(ccp)}(x_t, p_t^0, g_t^0) s_{lkt}^{(ccp)}(x_t, p_t^0, g_t^0) dP(l)\right] \frac{p_{jt}^0}{s_{kt}^0}
\end{aligned} \tag{12}$$

3.2 Disregard of state variables and Short-run price elasticity

In the remaining part of this section, we discuss the three sources of biases in applying static demand models under dynamic demand: disregard of state variables, inconsistent utility parameter estimates, and changing expectations of consumers.

[10] Note that some of the studies have used different terms for these elasticities. In this study, I use the definitions here.

[11] In this study, we assume that the price changes are not expected by the consumers before the current period. This type of specification is used by most of the studies in Table 1, except for Hartmann (2006).

In this subsection, we focus on the bias associated with the disregard of state variables. Here, to clarify the point, we consider the case where no persistent consumer heterogeneity exists in the dynamic model.

First, we can derive a static representation of the dynamic demand model with no random coefficients:[12]:

$$s_{jt}(p_t, g_t) = \frac{\exp(-\alpha p_{jt} + f_{jt} + \widehat{c_{jt}}(p_t, g_t))}{1 + \sum_{k \in \mathcal{J}_t} \exp(-\alpha p_{kt} + f_{kt} + \widehat{c_{kt}}(p_t, g_t))} \tag{13}$$

$$s_{0t}(p_t, g_t) = \frac{1}{1 + \sum_{k \in \mathcal{J}_t} \exp(-\alpha p_{kt} + f_{kt} + \widehat{c_{kt}}(p_t, g_t))} \tag{14}$$

where

$$\widehat{c_{jt}}(p_t, g_t) \equiv \log \left(\frac{\sum_{x_t \in X_t} \frac{\exp(\phi_{jt}(x_t) + \beta_C g_{jt}(x_t))}{\exp\left(V_t^C(x_t, p_t, g_t)\right)} \cdot Pr_t(x_t)}{\sum_{x_t \in X_t} \frac{\exp(f_{0t}(x_t) + \beta_C g_{0t}(x_t))}{\exp\left(V_t^C(x_t, p_t, g_t)\right)} \cdot Pr_t(x_t)} \right) \tag{15}$$

Next, suppose that the value of $\widehat{c_{jt}}$ is available at product prices p_t^0 and continuation values g_t^0. Then, we can construct the following static demand model with market share $\widehat{s_{jt}}$[13]:

$$\widehat{s_{jt}}(p_t; p_t^0, g_t^0) = \frac{\exp(-\alpha p_{jt} + f_{jt} + \widehat{c_{jt}}(p_t^0, g_t^0))}{1 + \sum_{k \in \mathcal{J}_t} \exp(-\alpha p_{kt} + f_{kt} + \widehat{c_{kt}}(p_t^0, g_t^0))} \tag{16}$$

$$\widehat{s_{0t}}(p_t; p_t^0, g_t^0) = \frac{1}{1 + \sum_{k \in \mathcal{J}_t} \exp(-\alpha p_{kt} + f_{kt} + \widehat{c_{kt}}(p_t^0, g_t^0))} \tag{17}$$

Note that $\widehat{s_{jt}}(p_t = p_t^0; p_t^0, g_t^0) = s_{jt}(p_t^0, g_t^0) = s_{jt}^0$ holds by construction.

Under the static model, own price elasticity of product j at time t is:

$$\begin{aligned}
\widehat{\eta_{jt}}(p_t^0, g_t^0) &\equiv -\frac{\partial \widehat{s_{jt}}(p_t = p_t^0; p_t^0, g_t^0)}{\partial p_{jt}} \frac{p_{jt}^0}{\widehat{s_{jt}}(p_t^0; p_t^0, g_t^0)} \\
&= \alpha \widehat{s_{jt}}(p_t^0; p_t^0, g_t^0)(1 - \widehat{s_{jt}}(p_t^0; p_t^0, g_t^0)) \frac{p_{jt}^0}{\widehat{s_{jt}}(p_t^0; p_t^0, g_t^0)} \\
&= \alpha s_{jt}^0 (1 - s_{jt}^0) \cdot \frac{p_{jt}^0}{s_{jt}^0}
\end{aligned} \tag{18}$$

Similarly, cross price elasticity of product k with respect to product $j (\neq k)$ at time t is:

$$\begin{aligned}
\widehat{\eta_{jkt}}(p_t^0, g_t^0) &\equiv \frac{\partial \widetilde{s_{kt}}(p_t = p_t^0; p_t^0, g_t^0)}{\partial p_{jt}} \frac{p_{jt}^0}{\widehat{s_{kt}}(p_t^0; p_t^0, g_t^0)} \\
&= \alpha \widehat{s_{jt}}(p_t^0; p_t^0, g_t^0) \widehat{s_{kt}}(p_t^0; p_t^0, g_t^0) \frac{p_{jt}^0}{\widehat{s_{kt}}(p_t^0; p_t^0, g_t^0)} \\
&= \alpha s_{jt}^0 s_{kt}^0 \cdot \frac{p_{jt}^0}{s_{kt}^0}
\end{aligned} \tag{19}$$

Then, static elasticities $\widehat{\eta_{jt}}(p_t^0, g_t^0), \widehat{\eta_{jkt}}(p_t^0, g_t^0)$ and dynamic short-run elasticities $\eta_{jt}^{(short)}(p_t^0, g_t^0), \eta_{jkt}^{(short)}(p_t^0, g_t^0)$ satisfy the following formulas:

[12] Since only one consumer type exists, we omit l.

[13] As we will discuss in the next subsection, we can recover the values of $f_{kt} + \widehat{c_{kt}}$ from the observed market share data with product prices p_t^0.

Lemma 1. *The following equations hold:*

$$\begin{aligned} \widehat{\eta_{jt}}(p_t^0, g_t^0) - \eta_{jt}^{(short)}(p_t^0, g_t^0) &= \alpha Var_{jt}(p_t^0, g_t^0) \cdot \frac{p_{jt}^0}{s_{jt}^0} \\ \widehat{\eta_{jkt}}(p_t^0, g_t^0) - \eta_{jkt}^{(short)}(p_t^0, g_t^0) &= -\alpha Cov_{jkt}(p_t^0, g_t^0) \cdot \frac{p_{jt}^0}{s_{kt}^0} \end{aligned}$$

Here, we define the following terms:

$$\begin{aligned} E_{x_t} s_{jt}^{(ccp)}(x_t, p_t, g_t) &\equiv \sum_{\widetilde{x_t} \in X_t} Pr_t(\widetilde{x_t}) s_{jt}^{(ccp)}(\widetilde{x_t}, p_t, g_t) \\ Var_{jt}(p_t, g_t) &\equiv \sum_{\widetilde{x_t} \in X_t} Pr_t(\widetilde{x_t}) \left[s_{jt}^{(ccp)}(\widetilde{x_t}, p_t, g_t) - E_{x_t} s_{jt}^{(ccp)}(x_t, p_t, g_t) \right]^2 \\ Cov_{jkt}(p_t, g_t) &\equiv \sum_{\widetilde{x_t} \in X_t} Pr_t(\widetilde{x_t}) \left(s_{jt}^{(ccp)}(\widetilde{x_t}, p_t, g_t) - E_{x_t} s_{jt}^{(ccp)}(x_t, p_t, g_t) \right) \left(s_{kt}^{(ccp)}(\widetilde{x_t}, p_t, g_t) - E_{x_t} s_{kt}^{(ccp)}(x_t, p_t, g_t) \right) \end{aligned}$$

Intuitively, $E_{x_t} s_{jt}^{(ccp)}(x_t, p_t, g_t), Var_{jt}(p_t, g_t), Cov_{jkt}(p_t, g_t)$ represent the mean, variance, and covariance of CCPs given product prices p_t and continuation values g_t. Note that $Cov_{jjt} = Var_{jt}$ holds, and the diagonal components of the matrix $((Cov_{jkt})_{j,k})$ correspond to Var_{jt}.

Using the lemma, we can easily obtain the following statement on the short-tun own price elasticities:

Proposition 1. $\widehat{\eta_{jt}}(p_t^0, g_t^0) \geq \eta_{jt}^{(short)}(p_t^0, g_t^0)$

Furthermore, equality holds only when $s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) = s_{jt}(p_t^0, g_t^0)\ \forall x_t \in X_t$.

The proposition implies that short-run price elasticity is overestimated when applying the static demand model.

The next proposition is on cross price elasticities:

Proposition 2. *The following inequalities hold:*

$$\widehat{\eta_{jkt}}(p_t^0, g_t^0) \begin{Bmatrix} > \\ = \\ < \end{Bmatrix} \eta_{jkt}^{(short)}(p_t^0, g_t^0) \quad \text{if } Cov_{jkt}(p_t^0, g_t^0) \begin{Bmatrix} < \\ = \\ > \end{Bmatrix} 0$$

Since we cannot determine the sign of Cov_{jkt} in general, the relative size of the two cross elasticities is unclear from this proposition. Nevertheless, we can derive a stronger result for durable goods with unit stock where consumers own at most one durable product:

Corollary 1. *Suppose that* $\phi_{jt}(x_t) = \phi_t(x_t)\ \forall j \in \mathcal{J}_t$. *Then, for durable goods with unit stock (inventory),* $Cov_{jkt}(p_t^0, g_t^0) \geq 0$ *and* $\widehat{\eta_{jkt}}(p_t^0, g_t^0) \leq \eta_{jkt}^{(short)}(p_t^0, g_t^0)$.

The problem of the static model is that the values of $\widehat{c_{jt}}$ is treated to be fixed even when product prices p_t change. In reality, $\widehat{c_{jt}}$ depends on product prices since $V_t^C(x_t, p_t, g_t)$ is a function of product prices, and the static model cannot capture such an effect. Consequently, elasticities based on the static model and the short-run elasticities based on the dynamic model do not necessarily coincide. Of course, we can allow changing values of $\widehat{c_{jt}}$ if we can compute the derivatives of $\widehat{c_{jt}}$ with respect to product prices. Nevertheless, to do so, we need the knowledge of the distribution of consumers' state variables ($Pr_t(x_t)$), and the process is generally omitted in the standard static models.

Note that when $Pr_t(x_t) = 1$ (only one state variable; no role of state variables) and $\beta_C = 0$ (myopic consumers), $\widehat{c_{jt}} = \phi_{jt} - f_{0t}$ holds[14]. Then, $\widehat{c_{jt}}$ does not depend on product prices p_t, and no problem happens even when applying the static demand model.

[14] Since only one state variable exists, we omit x_t of $\phi_t(x_t)$ and $f_{0t}(x_t)$.

3.3 Inconsistent utility parameter estimates

The discussion above implies that static models yield overestimated short-run price own elasticities, and underestimated short-run cross elasticities for durable goods with unit stock. However, the discussion above hinges on the condition that the static dynamic models share the same price coefficient α. Nevertheless, there is no guarantee that the estimated α based on the static model coincides with α in the dynamic model. Next, we discuss the second source of bias: inconsistent utility parameter estimates.

Here, we consider the estimation process using aggregate data. As in the previous subsection, we consider the static model without random coefficients. Let $f_{jt} = X_{jt}\theta + c_0 + \xi_{jt}$, where X_{jt}, ξ_{jt} denote product j's observed and unobserved characteristics. Let Z_{jt} be instrumental variables satisfying $E[\xi_{jt}|Z_{jt}] = 0$. Additionally, we impose the following condition as in the static BLP model:

$$S_{jt} \quad = \quad s_{jt} \quad j \in \mathcal{J}_t \cup \{0\}$$

where S_{jt} denotes the market share data of product j at time t.

Then, by (13) and (14) we obtain the following linear equation:

$$\log S_{jt} - \log S_{0t} \quad = \quad -\alpha p_{jt} + X_{jt}\theta + \widehat{c_0} + \zeta_{jt}$$

where $\zeta_{jt} \equiv \xi_{jt} + \widehat{c_{jt}} - E[\widehat{c_{jt}}]$ and $\widehat{c_0} = c_0 + E[\widehat{c_{jt}}]$.

In general, estimating the linear equation above by treating ζ_{jt} as the error term and applying GMM does not yield consistent estimates of α and θ. Since $\widehat{c_{jt}}$ is a function of the value function $V_t^C(x_t, p_t, g_t)$ as shown in (15), it depends on the current product characteristics and prices. Consequently, $E[\widehat{c_{jt}}|Z_{jt}] \neq 0$, and it implies $E[\zeta_{jt}|Z_{jt}] \neq 0$. Besides, correlation between instrumental variables Z_{jt} and consumers' expectations $g_{jt}(x_t) = E_t V_{t+1}^C(x_t, a_t = j)$, or the correlations between IVs Z_{jt} and $Pr_t(x_t)$ also lead to inconsistent utility parameter estimates.

Note that when $Pr_t(x_t) = 1$ (only one state variable; no role of state variables) and $\beta_C = 0$ (myopic consumers), $\widehat{c_{jt}} = \phi_{jt} - f_{0t}$ holds[15]. Further, if $\phi_{jt} = \phi_t$ for all $j \in \mathcal{J}_t$ and if ϕ_t and f_{0t} do not change over time, we can treat $\widehat{c_{jt}}$ as a constant term that does not depend on product j, and we can consistently estimate α and θ. If neither of the conditions above fails, applying the static model leads to the inconsistent estimates of utility parameters.

3.4 Changing expectations of consumers and Long-run price elasticity

Next, we consider the third source of bias: changing expectations of consumers, which affects long-run price elasticities. In the static model, there is no counterpart of long-run price elasticities, and we compare long-run price elasticity based on the dynamic model and short-run price elasticities based on the dynamic model.

To clarify the point, we assume that consumers have perfect foresight on the future price path[16]. We consider the case where the price of product j is expected to be raised from time t to time $t+T$. We assume that the price change is not expected by the consumers before time t. We further assume that the increments of the price increases are the same for all the periods. To derive statements, we define:

$$\lambda_{ljt}(x_t, \{p_{t+\tau}\}_{\tau \geq 0}) \equiv \sum_{\tau=1}^{T} \beta_C^{\tau-1} \left[Pr(l \ choose \ j \ at \ t+\tau | x_t, a_t = j, \{p_{t+\tau}\}_{\tau \geq 0}) - Pr(l \ choose \ j \ at \ t+\tau | x_t, \{p_{t+\tau}\}_{\tau \geq 0}) \right]$$

[15]Since only one state variable exists, we omit x_t of $\phi_t(x_t)$ and $f_{0t}(x_t)$.

[16]Note that many of the previous studies have not used the perfect foresight specification, and specified alternative expectation formulation (e.g., Expectation with stochastic fluctuations of prices as in Erdem et al., 2003; Expectations with Inclusive Value Sufficiency as in Hendel and Nevo, 2006, Gowrisankaran and Rysman, 2012). We can derive similar results under the alternative expectation formation specifications. Nevertheless, the results under perfect foresight are the clearest ones. Even though minor differences exist between different specifications, the essence of the claims would not be lost even when applying perfect foresight specification.

$$\lambda_{ljkt}(x_t, \{p_{t+\tau}\}_{\tau\geq 0}) \equiv \sum_{\tau=1}^{T} \beta_C^{\tau-1} \left[Pr(l\ choose\ j\ at\ t+\tau | x_t, a_t = k, \{p_{t+\tau}\}_{\tau\geq 0}) - Pr(l\ choose\ j\ at\ t+\tau | x_t, \{p_{t+\tau}\}_{\tau\geq 0}) \right]$$

Here, $Pr(l\ choose\ j\ at\ t+\tau | x_t, a_t = k, \{p_{t+\tau}\}_{\tau\geq 0})$ denotes the probability that type l consumers with state x_t and choice k at time t choose product j at time $t+\tau$ given future product prices $\{p_{t+\tau}\}_{\tau\geq 0}$. $Pr(l\ choose\ j\ at\ t+\tau | x_t, \{p_{t+\tau}\}_{\tau\geq 0})$ is defined in a similar way. Intuitively, $\lambda_{ljkt}(x_t) > 0$ implies that type l consumers choosing product k at time t are more likely to choose product j in the future periods.

Let $\eta_{jt}^{(long)}(\{p_{t+\tau}^0\}_{\tau\geq 0})$ be the long-run own price elasticity of product j at time t, and $\eta_{jkt}^{(long)}$ be the long-run cross elasticity of product k with respect to product j at time t given the future price path $\{p_{t+\tau}\}_{\tau\geq 0}$. We further assume that continuation value g_t^0 is consistent with the future price path $\{p_{t+\tau}^0\}_{\tau\geq 0}$. Then, short-run and long-run price elasticities satisfy the following lemma:

Lemma 2. *The following equations hold:*

$$\begin{aligned}
\eta_{jt}^{(long)}(\{p_{t+\tau}^0\}_{\tau\geq 0}) - \eta_{jt}^{(short)}(p_t^0, g_t^0) &= \frac{p_{jt}^0}{s_{jt}^0} \beta_C \int \alpha_l \sum_{x_t \in X_t} Pr_{lt}(x_t) s_{ljt}^{(ccp)}(x_t, p_t^0) \lambda_{ljt}(x_t, \{p_{t+\tau}^0\}_{\tau\geq 0}) dP(l) \\
\eta_{jkt}^{(long)}(\{p_{t+\tau}^0\}_{\tau\geq 0}) - \eta_{jkt}^{(short)}(p_t^0, g_t^0) &= -\frac{p_{jt}^0}{s_{kt}^0} \beta_C \int \alpha_l \sum_{x_t \in X_t} Pr_{lt}(x_t) s_{lkt}^{(ccp)}(x_t, p_t^0) \lambda_{ljkt}(x_t, \{p_{t+\tau}^0\}_{\tau\geq 0}) dP(l)
\end{aligned}$$

Then, we can easily obtain the following proposition:

Proposition 3. *The following inequalities hold:*

$$\eta_{jt}^{(long)}(\{p_{t+\tau}^0\}_{\tau\geq 0}) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} \eta_{jt}^{(short)}(p_t^0, g_t^0) \text{ if } \lambda_{ljt}(x_t, \{p_{t+\tau}^0\}_{\tau\geq 0}) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} 0\ \forall l, x_t \in X_t$$

$$\eta_{jkt}^{(long)}(\{p_{t+\tau}^0\}_{\tau\geq 0}) \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} \eta_{jkt}^{(short)}(p_t^0, g_t^0) \text{ if } \lambda_{ljkt}(x_t, \{p_{t+\tau}^0\}_{\tau\geq 0}) \left\{ \begin{array}{c} < \\ = \\ > \end{array} \right\} 0\ \forall l, x_t \in X_t$$

In the case of durable goods, generally the current demand for a product implies less future demand for any product due to the durability of products ($\lambda_{ljt}(x_t) < 0, \lambda_{ljkt}(x_t) < 0\ j, k \in \mathcal{J}_t$). Then, Proposition 3 implies that long-run own price elasticity is smaller than the short-run own price elasticity, and long-run cross elasticity is larger than the short-run cross price elasticity[17]. Note that the two elasticities coincide when the consumers are myopic ($\beta_C = 0$).

4 Extensions

In this section, we extend the results in Section 3 for a deeper appreciation of the static demand model. In Section 4.1, we consider the magnitude of the biases in price elasticities. In Section 4.2, we discuss some remedies for static demand models. In Section 4.3, we extend the model to storable goods, which is also an important product with dynamic demand.

[17] In the case of goods with switching costs, in general the current demand for a product implies more future demand for the product due to the existence of switching costs, but less demand for the other products ($\lambda_{ljt}(x_t) > 0, \lambda_{ljkt}(x_t) < 0\ j, k \in \mathcal{J}_t, j \neq k$). Then, Proposition 3 implies that long-run own price elasticity is larger than the short-run own price elasticity, and long-run cross elasticity is larger than the short-run cross price elasticity.

4.1 Magnitude of the biases in price elasticities

4.1.1 Short-run price elasticity

We can derive the upper bound of the biases in short-run price elasticities associated with the disregard of state variables discussed in Section 3.2. The following proposition shows the statement:

Proposition 4. *The following inequalities hold:*

$$\begin{aligned} 0 \leq \widehat{\eta_{jt}}(p_t^0, g_t^0) - \eta_{jt}^{(short)}(p_t^0, g_t^0) &\leq \alpha p_{jt}^0 \left(\max_{x_t} s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) \right) \\ \left| \widehat{\eta_{jkt}}(p_t^0, g_t^0) - \eta_{jkt}^{(short)}(p_t^0, g_t^0) \right| &\leq \alpha p_{jt}^0 \left(\max_{x_t} s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) \right) \end{aligned}$$

The inequalities indicate that the biases in short-run elasticities are small when CCPs $s_{jt}^{(ccp)}(x_t, p_t^0)$ are sufficiently small for all the state variables x_t.

To understand why the size of the CCPs affects the magnitude of the biases in short-run price elasticities, consider the simplified setting where $\phi_{jt}(x_t) = 0$ and $\beta_C = 0$. Then, the utility function based on the dynamic model can be reformulated as:

$$v_{it}(x_t, p_t, a_t) = \begin{cases} -\alpha p_{jt} + f_{jt} + \epsilon_{ijt} & \text{if } a_t = j \\ f_{0t}(x_t) + \epsilon_{i0t} & \text{if } a_t = 0 \end{cases}$$

Further, we assume that only one product j exists in the market. Then, by defining the term $\Delta v_{it}(x_t) \equiv v_{it}(x_t, p_t, a_t = j) - v_{it}(x_t, p_t, a_t = 0)$, the market share s_{jt} based on the static model can be expressed as:

$$\begin{aligned} s_{jt} &= \sum_{x_t \in X_t} Pr(v_{it}(x_t, p_t, a_t = j) > v_{it}(x_t, p_t, a_t = 0)) \cdot Pr_t(x_t) \\ &= \sum_{x_t \in X_t} Pr(\Delta v_{it}(x_t) > 0) \cdot Pr_t(x_t) \end{aligned}$$

Next, by (16) and (17), we can specify the "static" utility function based on the static model:

$$\widehat{v_{it}}(p_t, a_t) = \begin{cases} -\alpha p_{jt} + f_{jt} + \widehat{c_{jt}} + \epsilon_{ijt} & \text{if } a_t = j \\ \epsilon_{i0t} & \text{if } a_t = 0 \end{cases}$$

By defining the term $\Delta \widehat{v_{it}} \equiv \widehat{v_{it}}(p_t, a_t = j) - \widehat{v_{it}}(p_t, a_t = 0)$, static market share $\widehat{s_{jt}}$ can be expressed as:

$$\begin{aligned} \widehat{s_{jt}} &= Pr(\widehat{v_{it}}(p_t, a_t = j) > \widehat{v_{it}}(p_t, a_t = 0)) \\ &= Pr(\Delta \widehat{v_{it}} > 0) \end{aligned}$$

Consequently, market shares and price elasticities are determined by the distributions of $\Delta v_{it}(x_t)$ (in the case of the dynamic model) and $\Delta \widehat{v_{it}}$ (in the case of the static model).

The real line in Figure 1 shows the shape of the density function of $\Delta v_{it}(x_t)$. Here, we assume that $\{\epsilon_{ijt}\}_{j \in \mathcal{J}_t}$ follows i.i.d. type-I extreme value distribution. Since there are multiple types of consumers with different x_t, multiple peaks exist. In contrast, in the static demand model, we abstract away the existence of consumer-level state variables and fit a single distribution of $\Delta \widehat{v_{it}}$ as in the dashed line in Figure 1. The two distributions take different shapes, and leading to biased estimates of price elasticities.

Generally, consumers purchasing the small CCP product is located in the right tail of the distribution. In the tail of the distribution, we can find minor differences between the real line and the dashed line. Consequently, the difference between static and dynamic models is small when focusing on the small CCP product.

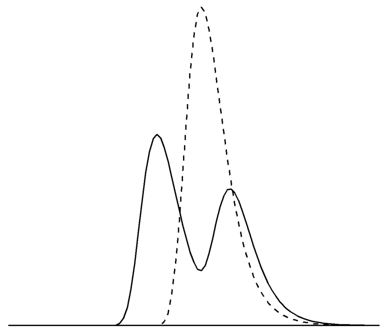

Figure 1: Distribution of the utilities $\Delta v_{it}(x_t)$ and $\Delta \widehat{v_{it}}$

Notes:
The real line shows the density function of $\Delta v_{it}(x_t)$ based on the dynamic model accounting for the existence of state variables x_t. The dashed line shows the density function of $\Delta \widehat{v_{it}}$ based on the "static" model.

Example. Durable goods with exogenous replacement timing

To understand the results more clearly, consider the example of durable goods with exogenous replacement timing, where consumers consider purchases only when they do not have any product. Formally, consider the following setting: $s_{jt}^{(ccp)}(x_t \neq 0, p_t^0, g_t^0) \approx 0$. Here, $x_t = 0$ denotes the state where the consumer does not possess any product at the beginning of time t.

Then, since $s_{jt}(p_t, g_t) = Pr_t(x_t = 0) \cdot s_{jt}^{(ccp)}(x_t = 0, p_t, g_t)$ holds, by (11), short-run own price elasticity of demand based on the dynamic model is:

$$\eta_{jt}^{(short)}(p_t^0, g_t^0) \equiv \alpha p_{jt}^0 (1 - s_{jt}^{(ccp)}(x_t = 0, p_t^0, g_t^0))$$

By (18), the own price elasticity of demand computed from the static model is:

$$\widehat{\eta_{jt}}(p_t^0, g_t^0) \equiv \alpha p_{jt}^0 (1 - s_{jt}(p_t^0, g_t^0))$$

Using these simple formulas, we can easily compute the biases in short-run price elasticities when applying the static demand model. Table 2 shows examples.

$Pr_t(x_t = 0)$	$s_{jt}^{(ccp)}(x_t = 0)$	s_{jt}	$\eta_{jt}^{(short)}$	$\widehat{\eta_{jt}}$	Bias(%)
0.2	0.8	0.16	2	8.4	320%
0.2	0.5	0.1	5	9	80%
0.2	0.1	0.02	9	9.8	9%
0.2	0.01	0.002	9.9	9.98	1%
0.9	0.8	0.72	2	2.8	40%
0.9	0.5	0.45	5	5.5	10%
0.9	0.1	0.09	9	9.1	1%
0.9	0.01	0.009	9.9	9.91	0%

Table 2: Bias in short-run own price elasticity of demand (Durable goods with exogenous replacement timing)

Notes:
$Pr_t(x_t = 0)$ denotes the fraction of no-stock consumers. $\eta_{jt}^{(short)}$ denotes the short-run own price elasticity derived from the dynamic demand model. $\widehat{\eta_{jt}}$ denotes the price elasticity of demand derived from the static demand model. In the table, the terms p_t^0 and g_t^0 are omitted to simplify the notation.The biases are calculated as $\frac{\widehat{\eta_{jt}} - \eta_{jt}^{(short)}}{\eta_{jt}^{(short)}} \times 100$.
$\alpha = 0.1$, $p_{jt}^0 = \$100$.

As this table shows, the bias is large when the fraction of consumers currently possessing products is high ($Pr_t(x_t = 0)$ is small), and consumer's CCP of the product ($s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)$) is high. For instance, when $Pr_t(x_t = 0) = 0.2$ and $s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) = 0.8$, the bias is 320%. This large bias comes from the implicit assumption in the static model that even the consumers already possessing any product will consider a purchase as if they do not own anything. In the true dynamic demand model, consumers already possessing products will not buy. Note that the bias is small if the value of $s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)$ is small, even when $Pr_t(x_t = 0)$ is high. In that sense, the product's CCPs and market shares are also important for assessing the biases when applying static demand models.

4.1.2 Long-run price elasticity

The next proposition shows the upper bound of the biases in long-run price elasticities, which are discussed in Section 3.4:

Proposition 5. *The following inequalities hold:*

$$\begin{aligned}
\left|\eta_{jt}^{(long)}(\{p_{t+\tau}^0\}_{\tau\geq 0}) - \eta_{jt}^{(short)}(p_t^0, g_t^0)\right| &\leq \kappa_{jt}\left(\max_{l, x_{t+\tau}, \tau\geq 1} \alpha_l s_{ljt+\tau}^{(ccp)}(x_{t+\tau}, p_{t+\tau}^0, g_{t+\tau}^0)\right) \\
\left|\eta_{jkt}^{(long)}(\{p_{t+\tau}^0\}_{\tau\geq 0}) - \eta_{t}^{(short)}(p_t^0, g_t^0)\right| &\leq \kappa_{jt}\left(\max_{l, x_{t+\tau}, \tau\geq 1} \alpha_l s_{ljt+\tau}^{(ccp)}(x_{t+\tau}, p_{t+\tau}^0, g_{t+\tau}^0)\right)
\end{aligned}$$

where $\kappa_{jt} \equiv 2\beta_C \frac{1-\beta_C^T}{1-\beta_C} p_{jt}^0$

The inequalities indicate that the biases in long-run elasticities are small when future CCPs are sufficiently small for all the state variables and consumer types.

In general, consumers expect that the probability they will purchase the product in the future is small if the CCP of a product is small. Then, they are less likely to be affected by the future price change of the small CCP product.

4.2 Remedies for static demand models

In general, applying static models yields biased price elasticities, unless consumers are myopic ($\beta_C = 0$) and only one state variable exists. Nevertheless, sometimes it is possible to mitigate the problems. In this subsection, we discuss some remedies for static demand models. In Section 4.2.1, we discuss the remedies for the problems associated with the disregard of state variables. In Section 4.2.2, we consider the remedies for utility parameter estimates. Even though these remedies do not work in all cases, it is worth considering based on each empirical context.

4.2.1 Disregard of state variables

In Section 3.2, we have not introduced any random coefficients in the static model. Nevertheless, by introducing random coefficients, especially the random coefficient on the constant term, we might be able to mitigate the bias.

Here, to make the point clear we consider the case where no persistent heterogeneity exists in the dynamic model as in Section 3.2. Note that a similar argument holds even in the case where persistent consumer heterogeneity exists in the dynamic model. By introducing random coefficients in the static model, we can derive a static representation of the dynamic model in an alternative way. The static model is composed of market shares $\widetilde{s_{jt}}$, type specific choice probabilities $\widetilde{s_{\tilde{l}jt}}(p_t, g_t)$, additional terms $\widetilde{c_{\tilde{l}jt}}$ and a mapping σ from x_t to $\widetilde{l}$ such that:

$$
\begin{aligned}
s_{\widetilde{l}jt}(p_t,g_t) &= \frac{\exp(-\alpha p_{jt}+f_{jt}+c_{\widetilde{l}jt}(g_t))}{1+\sum_{k\in\mathcal{J}_t}\exp(-\alpha p_{kt}+f_{kt}+c_{\widetilde{l}kt}(g_t))} \\
\widetilde{s_{jt}}(p_t,g_t) &= \int s_{\widetilde{l}jt}(p_t,g_t)dP(\widetilde{l}) \\
P(\widetilde{l}=\sigma(x_t)) &= Pr_t(x_t) \\
c_{\widetilde{l}jt}(g_t) &\equiv \phi_{jt}(x_t)-f_{0t}(x_t)+\beta_C g_{jt}(x_t)-\beta_C g_{0t}(x_t) \text{ for } \widetilde{l}=\sigma(x_t)
\end{aligned}
$$

where P denotes the density of type $\widetilde{l}$ consumers. Then, we can easily show that the static and dynamic models are related in the following ways:

$$
\begin{aligned}
s_{\widetilde{l}jt}(p_t,g_t) &= s_{jt}^{(ccp)}(x_t,p_t,g_t) \quad \text{for } \widetilde{l}=x_t \\
\widetilde{s_{jt}}(p_t,g_t) &= s_{jt}(p_t,g_t)
\end{aligned}
$$

Then, own price elasticity of product j is:

$$
\widetilde{\eta_{jt}}(p^0,g_t^0)=-\frac{\partial\widetilde{s_{jt}}(p_t^0,g_t^0)}{\partial p_{jt}}\frac{p_{jt}^0}{\widetilde{s_{jt}}(p_t^0,g_t^0)}=-\left[\int\frac{\partial s_{\widetilde{l}jt}(p_t^0,g_t^0)}{\partial p_{jt}}dP(\widetilde{l})\right]\frac{p_{jt}^0}{s_{jt}^0}=\widetilde{s_{ljt}}(p_t^0,g_t^0)(1-\widetilde{s_{ljt}}(p_t^0,g_t^0))\frac{p_{jt}^0}{s_{jt}^0}=\eta_{jt}^{(short)}(p_t^0,g_t^0)
$$

Cross price elasticity of product k with respect to product j is:

$$
\widetilde{\eta_{jkt}}(p_t^0,g_t^0)=\frac{\partial\widetilde{s_{kt}}(p_t^0,g_t^0)}{\partial p_{jt}}\frac{p_{jt}^0}{\widetilde{s_{kt}}(p_t^0,g_t^0)}=-\left[\int\frac{\partial s_{\widetilde{l}kt}(p_t^0,g_t^0)}{\partial p_{jt}}dP(\widetilde{l})\right]\frac{p_{jt}^0}{s_{kt}^0}=\widetilde{s_{ljt}}(p_t^0,g_t^0)\widetilde{s_{lkt}}(p_t^0,g_t^0)\frac{p_{jt}^0}{s_{kt}(p_t^0,g_t^0)}=\eta_{jkt}^{(short)}(p_t^0,g_t^0)
$$

They imply that we can obtain consistent estimates of the short-run price elasticities, if we can fit the static model so that $s_{\widetilde{l}jt}(p_t^0,g_t^0)\approx s_{jt}^{(ccp)}(x_t,p_t^0,g_t^0)$ for $\widetilde{l}=\sigma(x_t)$ and α is consistently estimated.

To fit the static model so that $s_{\widetilde{l}jt}(p_t^0,g_t^0)\approx s_{jt}^{(ccp)}(x_t,p_t^0,g_t^0)$ for $\widetilde{l}=\sigma(x_t)$, we should well approximate the distribution of the term $c_{\widetilde{l}jt}(g_t)=\phi_{jt}(x_t)-f_{0t}(x_t)+\beta_C g_{ljt}(x_t)-\beta_C g_{l0t}(x_t)$ by random coefficients. For example, in order to approximate the distribution of the term $f_{0t}(x_t)$, which does not depend on product characteristics, introducing a random coefficient on the constant term is the most straightforward way. Note that such a "reduced-form" approach might not work in all cases. The distribution of $f_{0t}(x_t)$ changes over time based on the values of $Pr_t(x_t)$, and it is not clear to what extent this type of strategy works well.

4.2.2 Inconsistent utility parameter estimates

The case of static models without random coefficients We continue the discussion in Section 3.3: We abstract away the existence of persistent consumer heterogeneity in the dynamic model, and we consider the case where random coefficients are not introduced in the static model. The problem is that the term $\widehat{c_{jt}}(p_t,g_t)=\log\left(\frac{\sum_{x_t\in X_t}\frac{\exp\left(\phi_{jt}(x_t)+\beta_C g_{jt}(x_t)\right)}{\exp\left(V_t^C(x_t,p_t,g_t)\right)}\cdot Pr_t(x_t)}{\sum_{x_t\in X_t}\frac{\exp(f_{0t}(x_t)+\beta_C g_{0t}(x_t))}{\exp\left(V_t^C(x_t,p_t,g_t)\right)}\cdot Pr_t(x_t)}\right)$ is not controlled in the estimation process:

$$
\log S_{jt}-\log S_{0t}=-\alpha p_{jt}+X_{jt}\theta+\widehat{c_{jt}}(p_t,g_t)+c_0+\xi_{jt}
$$

If we can well approximate the term $\widehat{c_{jt}}$ with the variables other than p_{jt} and X_{jt}, we can obtain consistent estimates of utility parameter estimates α and θ.

One strategy for approximating $c_0 + \widehat{c_{jt}}$ is the introduction of time / group dummies[18][19]. Suppose that the set of products $\mathcal{J}_t$ can be divided into mutually exclusive groups $\mathcal{J}_{gt}$ ($g = 1, \cdots, G$), and products in the same group share the same values of $c_0 + \widehat{c_{jt}}$ ($c_0 + \widehat{c_{jt}} = \widehat{c_{gt}}\ \forall j \in \mathcal{J}_{gt}$). Then, we can consistently estimate α and θ by treating $\widehat{c_{gt}}$ as fixed effect dummies:

$$\log S_{jt} - \log S_{0t} = -\alpha p_{jt} + X_{jt}\theta + \widehat{c_{gt}} + \xi_{jt}$$

Note that products in the same group share the same values of $\widehat{c_{jt}}$ only when products in the same group share the same values of $g_{jt}(x_t)$ (continuation value given purchasing product j) and $\phi_{jt}(x_t)$ (flow utility from purchasing product j given state x_t other than $-\alpha p_{jt} + f_{jt}$)[20] .

The introduction of time and group dummies are informally proposed in Goldberg and Verboven (2001). In Goldberg and Verboven (2001), automobile brand is the group. If it is plausible to assume that consumers perceive that future utility from purchasing the same brand automobiles are the same, the strategy works well and we can obtain the consistent estimate of α and θ.

Note that when the market environment is stable over time, the values of $Pr_t(x_t)$ and $\beta_C g_{jt}(x_t)$ are mostly stable, and $\widehat{c_{jt}}$ also take mostly stable values over time. In this case, we can treat the term $\widehat{c_{jt}}$ as a constant term, and we can obtain mostly precise estimate of the utility parameters unless cross-sectional correlations between $\beta_C g_{jt}(x_t)$ and p_{jt} or Z_{jt} exists.

The case of static models with random coefficients We continue the discussion in Section 4.2.1, and we abstract away the existence of persistent consumer heterogeneity in the dynamic model. Note that a similar argument holds even when persistent consumer heterogeneity exists in the dynamic model.

If we can well approximate the term $\widetilde{c_{ljt}}(g_t) = \phi_{jt}(x_t) - f_{0t}(x_t) + \beta_C g_{jt}(x_t) - \beta_C g_{0t}(x_t)$ with random coefficients and the variables other than p_{jt} and X_{jt}, we can obtain consistent estimates of utility parameters α and θ. As in the case of static models without random coefficients, introducing time / group dummies would mitigate the problems in some cases, because it approximates the values of continuation values $g_{jt}(x_t)$ to some extent. Nevertheless, the value also depends on the state variables x_t, and the interactions with random coefficients would be necessary. There is no guarantee that the estimated utility parameters are consistent, when introducing random coefficients in the static models.

4.3 Storable goods

We can extend the discussion in Section 3 to storable goods, by introducing the choice of consumption level other than the product choice, which is also the essential component in the storable goods models.

Here, we consider the model where consumers solve the dynamic optimization problem with inventory represented by the following Bellman equation[21]:

$$\begin{aligned}\widetilde{V_{lt}^{C}}(x_t, p_t) &= \int \max_{C_{lt}, \{d_{iljt}\}_{j \in \mathcal{J}_t}} \Bigg[U(C_{lt}) - F(x_t + q_t - C_{lt}) + \sum_{j \in \mathcal{J}_t} d_{iljt} \left(-\alpha p_{jt} + f_{ljt} + \epsilon_{ijt}\right) \\ &\quad + \beta_C E_t \widetilde{V_{lt+1}^{C}}(x_t + q_t - C_{lt}, p_{t+1} | x_t, p_t) \Bigg] p(\epsilon) d\epsilon\end{aligned}$$

Here, x_t denotes the inventory of the consumer at time t. d_{iljt} denotes the number of purchases of product j. For instance, $d_{iljt} = 1$ implies that the consumer purchases a product j at time t. C_{lt} denotes the amount of

[18]The effectiveness of the introduction of time / group dummies is quantitatively shown in Appendix B by conducting Monte Carlo simulation.

[19]Besides, Lou et al. (2012) proposed to approximate the term associated with the consumers' expectations by the age of the product. The strategy works well when the continuation value of each product $g_{jt}(x_t)$ is highly correlated with the age of the product.

[20]Gowrisankaran and Rysman (2020) formulated the assumptions as "Constant Continuation Value within Group (CCV)" Assumption and "Separability of Previous Purchases in Flow Utility SPP-F)" Assumption. They argued that they play essential roles in reducing the number of state variables when solving dynamic demand models.

[21]The model is mainly based on Hendel and Nevo (2006).

consumption, and $U(C_{lt})$ represents the utility from the consumption of storable goods. Here, we assume that only the quantity of consumption matters (brand does not matter in the consumption stage). $q_t = \sum_{j\in\mathcal{J}_t} q_{jt}d_{iljt}$ denotes the purchased quantity at time t, where q_{jt} denotes the package size of product j. $x_t + q_t - C_{lt}$ represents the quantity of the goods for storage, and $F(x_t + q_t - C_{lt})$ represents the storage cost. We assume that $x_{t+1} = x_t + q_t - C_{lt}$ holds.

In the following discussion, we assume that consumers purchase at most one product ($\sum_{j\in\mathcal{J}_t} d_{iljt} \leq 1$) in each period. Then, the optimal consumption level given purchasing product j at time t is:

$$C^*_{lt}|_j = \arg\max_{C_{lt}} U(C_{lt}) - F(x_t + q_{jt} - C_{lt}) + \beta_C E_t \widetilde{V^C_{lt+1}}(x_t + q_{jt} - C_{lt}, p_{t+1}|x_t, p_t)$$

The optimal consumption level given purchasing nothing at time t is:

$$C^*_{lt}|_0 = \arg\max_{C_{lt}} U(C_{lt}) - F(x_t - C_{lt}) + \beta_C E_t \widetilde{V^C_{lt+1}}(x_t - C_{lt}, p_{t+1}|x_t, p_t)$$

Note that $C^*_{lt}|_{j\in\mathcal{J}_t\cup\{0\}}$ does not depend on the current product prices p_t, given the continuation values $E_t\widetilde{V^C_{lt+1}}(x_t + q_{jt} - C^*_{lt}|_{j\in\mathcal{J}_t\cup\{0\}}, p_{t+1}|x_t, p_t)$. In contrast, $C^*_{lt}|_{j\in\mathcal{J}_t\cup\{0\}}$ may depend on the future prices $\{p_{t+\tau}\}_{\tau\geq 1}$ through the terms on continuation values g_t.

Then, by defining $\phi_{ljt}(x_t) \equiv U(C^*_{lt}|_j) - F(x_t + q_{jt} - C^*_{lt}|_j)$ for $j \in \mathcal{J}_t$, $f_{l0t}(x_t) \equiv U(C^*_{lt}|_j) - F(x_t + q_{jt} - C^*_{lt}|_j)$, and $g_{ljt}(x_t) \equiv E_t\widetilde{V^C_{lt+1}}(x_t + q_{jt} - C^*_{lt}|_j, p_{t+1}|x_t, p_t)$, we can specify the utility from choosing the alternative $j \in \mathcal{J}_t \cup \{0\}$ as in (1):

$$v_{ilt}(x_t, p_t, g_t, a_t) = \begin{cases} -\alpha_l p_{jt} + f_{ljt} + \phi_{ljt}(x_t) + \beta_C g_{ljt}(x_t) + \epsilon_{ijt} & \text{if } a_t = j \\ f_{l0t}(x_t) + \beta_C g_{l0t}(x_t) + \epsilon_{i0t} & \text{if } a_t = 0 \end{cases}$$

4.3.1 Short-run price elasticity

Since $\phi_{jt}(x_t)$ and $f_{l0t}(x_t)$ does not depend on the current product prices $\{p_{kt}\}_{k\in\mathcal{J}_t}$, given the continuation values g_t, we can easily show that the same statements in Section 3.2 (Propositions 1 and 2) hold.

In terms of short-run cross price elasticities, the signs of the biases are not clear, as shown in Proposition 2. Nevertheless, for the "same package size products", short-run cross price elasticities are underestimated. The following corollary shows the statement:

Corollary 2. *For storable goods $j, k \in \mathcal{J}_t$ with $q_{jt} = q_{kt}$, $Cov_{jkt}(p^0_t, g^0_t) \geq 0$ and $\widehat{\eta_{jkt}}(p^0_t, g^0_t) \leq \eta^{(short)}_{jkt}(p^0_t, g^0_t)$.*

4.3.2 Long-run price elasticity

If the future price change does not affect the current consumption level, then the terms $\phi_{ljt}(x_t)$ and $f_{l0t}(x_t)$ do not depend on future prices, and we can easily show that the statements in Section 3.4 (Proposition 3) also holds even for storable goods. In the case of storable goods, in general the current demand for a product implies less future demand for any product ($\lambda_{ljt}(x_t) < 0, \lambda_{ljkt}(x_t) < 0\ j, k \in \mathcal{J}_t$). Then, Proposition 3 implies that long-run own price elasticity is smaller than the short-run own price elasticity, and long-run cross elasticity is larger than the short-run cross price elasticity.

Nevertheless, in reality, future price change may affect the current consumption level. For instance, consumers expecting higher future product prices may reduce the amount of consumption and increase inventory. Consequently, Proposition 3 cannot be directly applied to storable goods, and another source of bias, namely, changing consumption level in response to the future price change exists.

5 Applications of the results to empirical researches

In this section, we discuss how the results so far provide insights into the previous studies' findings on the biases in applying static demand models. Here, we focus on the results of four papers in Table 1.

5.1 Automobiles (Chen et al. (2008) and Schiraldi (2011))

Chen et al. (2008), analyzing the automobile market, showed that a static model overestimates short-run own price elasticity[22] by 14%. In contrast, Schiraldi (2011), also analyzing the automobile market, showed that a static model underestimates short-run own price elasticity by 73%. We can guess that the difference comes from the primary sources of the biases: Chen et al. (2008)'s result is mainly due to disregarding state variables, and Schiraldi (2011)'s result is primarily due to the inconsistent utility parameter estimates.

Chen et al. (2008) considered the market with only one homogeneous new car model and one homogeneous used car model. Moreover, the model did not incorporate persistent consumer heterogeneity. Since they introduced new and used car dummy variables in the estimation, the identification mainly comes from time-series variations. Since they implicitly considered the setting where the market is stationary, $Pr_t(x_t)$ and consumers' expectations $g_{jt}(x_t)$ are mostly stable over time. Even though they used cost-shifters as instruments that may be correlated with consumers' expectations over time, static estimates yielded only minor biases. In fact, the bias of the price coefficient was only 2%. Then, the effect of the bias due to the disregard of state variables dominated, and the static model overestimated the short-run price elasticity.

In contrast, Schiraldi (2011) considered the market with multiple products. The identification came from both time-series variations and cross-sectional variations. When focusing on cross-sectional variations, consumers' continuation values $g_{ljt}(x_t)$ are positively correlated with the product prices p_{jt}, since higher quality and expensive products would have higher remaining values in the next period. We can suspect that the positive correlation leads to a large positive correlation between product prices p_{jt} and $\widetilde{c_{ljt}}$, and consequently, an underestimation of the price coefficient[23]. Then, since the bias due to the underestimation of the price coefficient is so large (bias:82%), we can suspect that the effect of the bias dominated the bias due to the disregard of state variables[24], and the static model underestimated the short-run price elasticity.

.

5.2 New durable goods (Gowrisankaran and Rysman (2012))

Gowrisankaran and Rysman (2012) studied a new durable goods market with replacement demand. They showed that the estimated price coefficient was sufficiently close to zero when applying a static BLP model. In their model, we can expect that not only continuation values $g_{ljt}(x_t)$ are correlated with product prices p_{jt} as in Schiraldi (2011), but also $Pr_{lt}(x_t = 0)$ is correlated with product prices p_{jt}. Here, $x_t = 0$ denotes the state where consumers do not own any durable product. Especially in the latter periods of the diffusion process, the decreasing fraction of no-stock consumers $Pr_{lt}(x_t = 0)$ induces lower demand s_{jt}. Product prices p_{jt} decline over time at the same time, and a static estimate fits the model as if declining product prices induces lower demand[25].

5.3 Storable goods (Hendel and Nevo (2006))

Hendel and Nevo (2006) argued that applying a static model leads to the overestimation of long-run own price elasticities and underestimation of long-run cross price elasticities. The biases come from three sources: disregard of state variables (consumer inventory), inconsistent utility parameter estimates, and changing expectations of consumers. Disregard of state variables leads to the overestimation of short-run own price

[22]Note that Chen et al. (2008) considered the elasticity in which changing consumers' expectations in response to the current temporary price change is allowed. Hence, it also affected the estimated price elasticity based on the dynamic model. Besides, we compare the elasticities given fixed used car prices, even though the article also showed the results allowing changing used car prices.

[23]Schiraldi (2011) used instrumental variables, including lagged product prices, and assumed that error term e_{jt} satisfies the moment condition that $e_{jt} - \lambda e_{jt-1}$ is orthogonal to instrumental variables. Even with this identification strategy, the endogeneity problem might not be solved under the static specification.

[24]Note that Schiraldi (2011) did not introduce random coefficients in the static estimation, and the problem of the disregard of state variables might not be solved in the static specification. Nevertheless, the effect might be smaller than the bias due to the inconsistent utility parameter estimate.

[25]Note that Gowrisankaran and Rysman (2012) used standard variables that capture how crowded a model is in a characteristic space as instruments. For new durable goods, it is natural that these variables increase or decline over time. For instance, more and more products are introduced in the new durable goods market. Hence, $Pr_{lt}(x_t = 0)$ and instrumental variables would be correlated, and the use of instrumental variables might not solve the problem.

elasticities for all the products, and underestimation of short-run own price elasticities for the same package size products as discussed in Section 4.3. Changing consumers' expectations affect the current consumption level and product choice. If the current consumption level does not change in response to the future price change, it is plausible to assume that long-run price elasticities are higher than short-run price elasticities, as discussed in Section 4.3.

Regarding inconsistent utility parameter estimates, Hendel and Nevo (2006) empirically showed that applying a static model results in the overestimation of the price coefficient. To understand the result, consider a simplified model in which there are only two states: state with / without inventory ($x_t \neq 0$ and $x_t = 0$). Further, we abstract away consumer heterogeneity and assume that consumers do not buy anything at the with inventory state ($x_t \neq 0$). Then, $\widehat{c_{jt}}$ can be expressed as[26]:

$$\widehat{c_{jt}}(p_t, g_t) \quad = \quad \log \left(\frac{\frac{\exp(\phi_{jt}(x_t=0)+\beta_C g_{jt}(x_t))}{\exp(V_t^C(x_t=0,p_t,g_t))} Pr_t(x_t = 0)}{1 - \frac{\sum_{k \in \mathcal{J}_t} \exp(-\alpha p_{kt}+f_{kt}+\phi_{kt}(x_t)+\beta_C g_{kt}(x_t))}{\exp(V_t^C(x_t=0,p_t,g_t))} Pr_t(x_t = 0)} \right) \tag{20}$$

The equation implies $\widehat{c_{jt}}$ is an increasing function of $Pr_t(x_t = 0)$. Generally, consumers are more likely to buy storables under temporary price reductions and when they are at the no-inventory state. Then, it is plausible to assume that $Pr_t(x_t = 0)$ and product prices p_{jt} are negatively correlated ($Pr_t(x_t = 0)$ is high when product prices are low, and $Pr_t(x_t = 0)$ is low when the product prices are high). Then, negative correlation between $Pr_t(x_t = 0)$ and product prices p_{jt} induces negative correlation between product prices p_{jt} and $\widehat{c_{jt}}$. Of course, other effects, such as the correlation between p_{jt} and $g_{jt}(x_t)$ would exist. Nevertheless, these effects might be dominated by the effect of the negative correlation between $Pr_t(x_t = 0)$ and product prices p_{jt}[27].

6 Conclusion

In this article, I investigated the mechanisms behind the biases in applying static demand models when the true demand structure is dynamic. There are three sources of biases: disregard of state variables, inconsistent utility parameter estimates, and changing expectations of consumers. The bias due to the disregard of state variables, which has not been discussed so much in the previous literature, leads to the overestimation of short-run own price elasticities. The first and third sources of biases are small when the focus is on the small CCP products.

In this study, we assumed that idiosyncratic preference shock ϵ_{ijt} follows type-I extreme value distribution as in most literature. Nevertheless, there is no guarantee that the distributional assumption is correct. Considering the results in nonparametric settings might be a promising extension of this study.

Besides, we can find an analogy with the limited consideration set models (e.g., Abaluck and Adams-Prassl, 2021, Crawford et al., 2021). For example, in the default specific models, one of the limited consideration set models, only a fraction of consumers make purchase decisions. Similarly, in the extreme case of the durable goods model, only a fraction of consumers with no durable goods holdings make additional purchase decisions[28]. However, the fraction of consumers making decisions is generally not observed, and it causes problems in both cases. It is interesting to investigate how the identification strategies in the recent literature on consideration set models help mitigate the issues associated with the use of static demand models.

[26] See Appendix A for the proof.

[27] Note that Hendel and Nevo (2006) didn't introduce unobserved product characteristics ξ_{jt} and estimated the utility parameters by maximum likelihood estimation, unlike the discussion in Section 3 and 4. Nevertheless, a similar argument holds.

[28] CCPs of purchasing any product are zero for consumers with durable product holdings.

A Proof

A.1 Proof of the statements in Section 3.2

A.1.1 Proof of Lemma 1

Own price elasticity:

Proof. By (11) and (18),

$$\begin{aligned}
& \widehat{\eta_{jt}}(p_t^0, g_t^0) - \eta_{jt}^{(short)}(p_t^0, g_t^0) \\
= & \left[\alpha s_{jt}^0(1-s_{jt}^0) - \alpha \sum_{x_t \in X_t} Pr_t(x_t) \cdot s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)\left(1 - s_{jt}^{(ccp)}(x_t, p_t^0)\right)\right] \frac{p_{jt}^0}{s_{jt}^0} \\
= & \alpha \left[\left(-\left(\sum_{x_t \in X_t} Pr_t(x_t) s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)\right)^2 + \sum_{x_t \in X_t} Pr_t(x_t) s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)^2\right)\right] \frac{p_{jt}^0}{s_{jt}^0} \\
= & \alpha \sum_{x_t \in X_t} Pr_t(x_t) \left[s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) - \sum_{\widetilde{x}_t \in X_t} Pr_t(\widetilde{x}_t) s_{jt}^{(ccp)}(\widetilde{x}_t, p_t^0, g_t^0)\right]^2 \cdot \frac{p_{jt}^0}{s_{jt}^0} \\
= & \alpha Var_{jt}(p_t^0, g_t^0) \cdot \frac{p_{jt}^0}{s_{jt}^0}
\end{aligned}$$

□

Cross price elasticity:

Proof. By (12) and (19),

$$\begin{aligned}
& \widehat{\eta_{jkt}}(p_t^0, g_t^0) - \eta_{jkt}^{(short)}(p_t^0, g_t^0) \\
= & \left[\alpha s_{jt}^0 s_{kt}^0 - \alpha \sum_{x_t \in X_t} Pr_t(x_t) \cdot s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) s_{kt}^{(ccp)}(x_t, p_t^0, g_t^0)\right] \frac{p_{jt}^0}{s_{kt}^0} \\
= & -\alpha \left[\sum_{x_t \in X_t} Pr_t(x_t) \left(s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) - \sum_{\widetilde{x}_t \in X_t} Pr_t(\widetilde{x}_t) s_{jt}^{(ccp)}(\widetilde{x}_t, p_t^0, g_t^0)\right) \cdot \right. \\
& \left. \left(s_{kt}^{(ccp)}(x_t, p_t^0, g_t^0) - \sum_{\widetilde{x}_t \in X_t} Pr_t(\widetilde{x}_t) s_{kt}^{(ccp)}(\widetilde{x}_t, p_t^0, g_t^0)\right)\right] \frac{p_{jt}^0}{s_{kt}^0} \\
= & -\alpha Cov_{jkt}(p_t^0, g_t^0) \cdot \frac{p_{jt}^0}{s_{kt}^0}
\end{aligned}$$

□

A.1.2 Proof of Corollary 1

Proof. First, continuation value $g_{jt}(x_t)$ does not depend on the state variables x_t for durable goods with unit stock, because current product holdings do not affect future continuation values when they sell or throw away the old product and purchase a new product.

Next, we define $\overline{s_{jt}}(p_t, g_t) \equiv s_{jt}^{(ccp)}(x_t, p_t, g_t) / \left(\sum_{m \in \mathcal{J}_t} s_{mt}^{(ccp)}(x_t, p_t, g_t)\right)$, which represents the conditional choice probability of product j given purchasing any product. $\overline{s_{jt}}(p_t, g_t)$ does not depend on consumers' state variables x_t, because:

$$
\begin{aligned}
\overline{s_{jt}}(p_t, g_t) &\equiv \frac{s_{jt}^{(ccp)}(x_t, p_t, g_t)}{\sum_{m\in\mathcal{J}_t} s_{mt}^{(ccp)}(x_t, p_t, g_t)} \\
&= \frac{\frac{\exp(\phi_t(x_t)-\alpha p_{jt}+f_{jt}+\beta_C g_{jt}(x_t))}{\exp\left(V_t^C(x_t,p_t,g_t)\right)}}{\sum_{m\in\mathcal{J}_t}\frac{\exp(\phi_t(x_t)-\alpha p_{mt}+f_{mt}+\beta_C g_{mt}(x_t))}{\exp\left(V_t^C(x_t,p_t,g_t)\right)}} \\
&= \frac{\exp\left(-\alpha p_{jt} + f_{jt} + \beta_C g_{jt}(x_t)\right)}{\sum_{m\in\mathcal{J}_t}\exp\left(-\alpha p_{mt} + f_{mt} + \beta_C g_{mt}(x_t)\right)}
\end{aligned}
$$

. Then, we can derive:

$$
\begin{aligned}
Cov_{jkt}(p_t, g_t) &\equiv \sum_{\widetilde{x_t}\in X_t} Pr_t(\widetilde{x_t})\left(s_{jt}^{(ccp)}(\widetilde{x_t}, p_t, g_t) - E_{x_t}s_{jt}^{(ccp)}(x_t, p_t, g_t)\right)\left(s_{kt}^{(ccp)}(\widetilde{x_t}, p_t, g_t) - E_{x_t}s_{kt}^{(ccp)}(x_t, p_t, g_t)\right) \\
&= \sum_{\widetilde{x_t}\in X_t} Pr_t(\widetilde{x_t})\left(\left(1 - s_{0t}^{(ccp)}(\widetilde{x_t}, p_t, g_t)\right)\overline{s_{jt}}(p_t, g_t) - E_{x_t}\left(1 - s_{0t}^{(ccp)}(x_t, p_t, g_t)\right)\overline{s_{jt}}(p_t, g_t)\right)\cdot \\
&\quad \left(\left(1 - s_{0t}^{(ccp)}(\widetilde{x_t}, p_t, g_t)\right)\overline{s_{kt}}(p_t, g_t) - E_{x_t}\left(1 - s_{0t}^{(ccp)}(x_t, p_t, g_t)\right)\overline{s_{kt}}(p_t, g_t)\right) \\
&= \overline{s_{jt}}(p_t, g_t)\overline{s_{kt}}(p_t, g_t)\sum_{\widetilde{x_t}\in X_t} Pr_t(\widetilde{x_t})\left(s_{0t}^{(ccp)}(\widetilde{x_t}, p_t, g_t) - E_{x_t}s_{0t}^{(ccp)}(x_t, p_t, g_t)\right)^2 \\
&\geq 0
\end{aligned}
$$

$\square$

A.2 Proof of the statements in Section 3.4

A.2.1 Proof of Lemma 2

To prove Lemma 2, we define the term

$$
\widetilde{v_{ljt}}(x_t, p_t, g_t) \equiv \begin{cases} -\alpha_l p_{jt} + f_{ljt} + \phi_{ljt}(x_t) + \beta_C g_{ljt}(x_t) & \text{if } j \in \mathcal{J}_t \\ f_{l0t}(x_t) + \beta_C g_{l0t}(x_t) & \text{if } j = 0 \end{cases}
$$

. Using the term, $v_{ilt}(x_t, p_t, g_t, a_t = j) = \widetilde{v_{ljt}}(x_t, p_t, g_t) + \epsilon_{ijt}$ holds. For convenience, we further define $\mathcal{A}_t = \mathcal{J}_t \cup \{0\}$. Besides, we omit the terms $p_{t+\tau}^0$ and $g_{t+\tau}^0$ in the following discussion to simplify the notation. First, we prove the following lemmas:

Lemma 3. *Define the following term:*

$$
\zeta_{lt+\tau}(\widetilde{x_{t+1}}) \equiv \sum_{x_{t+\tau-1}}\sum_{h\in\mathcal{A}_{t+\tau-1}} Pr(l \text{ choose } j \text{ at } t+\tau | x_{t+\tau-1}, a_{t+\tau-1} = h)\frac{\partial V_{lt+1}^C(\widetilde{x_{t+1}})}{\partial \widetilde{v_{lht+\tau-1}}(x_{t+\tau-1})}
$$

for $\tau = 2, \cdots, T$. Then, the following formula holds:

$$
\zeta_{lt+\tau}(\widetilde{x_{t+1}}) = \beta_C^{\tau-2} Pr(l \text{ choose } j \text{ at } t+\tau | \widetilde{x_{t+1}})
$$

Proof. First,

$$
\begin{aligned}
& \zeta_{lt+\tau}(\widetilde{x_{t+1}}) \\
\equiv & \sum_{x_{t+\tau-1}} \sum_{h \in \mathcal{A}_{t+\tau-1}} Pr(l \ choose \ j \ at \ t+\tau | x_{t+\tau-1}, a_{t+\tau-1}=h) \frac{\partial V^C_{lt+1}(\widetilde{x_{t+1}})}{\partial \widetilde{v_{lht+\tau-1}}(x_{t+\tau-1})} \\
= & \sum_{x_{t+\tau-1}} \sum_{h \in \mathcal{A}_{t+\tau-1}} Pr(l \ choose \ j \ at \ t+\tau | x_{t+\tau-1}, a_{t+\tau-1}=h) \cdot \\
& \frac{\partial V^C_{lt+\tau-1}(x_{t+\tau-1})}{\partial \widetilde{v_{lht+\tau-1}}(x_{t+\tau-1})} \sum_{x_{t+\tau-2}} \sum_{k \in \mathcal{A}_{lt+\tau-2}(x_{t+\tau-2})} \frac{\partial \widetilde{v_{lkt+\tau-2}}(x_{t+\tau-2})}{\partial V^C_{lt+\tau-1}(x_{t+\tau-1})} \frac{\partial V^C_{lt+1}(\widetilde{x_{t+1}})}{\partial \widetilde{v_{lkt+\tau-2}}(x_{t+\tau-2})} \\
= & \sum_{x_{t+\tau-1}} \sum_{h \in \mathcal{A}_{t+\tau-1}} Pr(l \ choose \ j \ at \ t+\tau | x_{t+\tau-1}, a_{t+\tau-1}=h) \cdot \\
& s^{(ccp)}_{lht+\tau-1}(x_{t+\tau-1}) \sum_{x_{t+\tau-2}} \sum_{k \in \mathcal{A}_{t+\tau-2}} \beta_C \psi(x_{t+\tau-1} | x_{t+\tau-2}, a_{t+\tau-2}=k) \frac{\partial V^C_{lt+1}(x_{t+1})}{\partial \widetilde{v_{lkt+\tau-2}}(x_{t+\tau-2})}
\end{aligned}
$$

Since

$$
\begin{aligned}
& \sum_{x_{t+\tau-1}} \sum_{h \in \mathcal{A}_{t+\tau-1}} Pr(l \ choose \ j \ at \ t+\tau | x_{t+\tau-1}, a_{t+\tau-1}=h) s^{(ccp)}_{lht+\tau-1}(x_{t+\tau-1}) \psi(x_{t+\tau-1} | x_{t+\tau-2}, a_{t+\tau-2}=k) \\
= & \ Pr(l \ choose \ j \ at \ t+\tau | x_{t+\tau-2}, a_{t+\tau-2}=k)
\end{aligned}
$$

holds, we obtain:

$$
\begin{aligned}
& \zeta_{lt+\tau}(\widetilde{x_{t+1}}) \\
= & \ \beta_C \sum_{x_{t+\tau-2}} \sum_{k \in \mathcal{A}_{t+\tau-2}} Pr(l \ choose \ j \ at \ t+\tau-1 | x_{t+\tau-2}, a_{t+\tau-2}=k) \frac{\partial V^C_{lt+1}(\widetilde{x_{t+1}})}{\partial \widetilde{v_{lkt+\tau-2}}(x_{t+\tau-2})} \\
= & \ \beta_C \zeta_{lt+\tau-1}(\widetilde{x_{t+1}})
\end{aligned}
$$

By repeatedly applying the equation, we have:

$$
\zeta_{lt+\tau}(\widetilde{x_{t+1}}) = \beta_C^{\tau-2} \zeta_{lt+2}(\widetilde{x_{t+1}})
$$

Here,

$$
\begin{aligned}
& \zeta_{lt+2}(\widetilde{x_{t+1}}) \\
\equiv & \sum_{x_{t+1}} \sum_{k \in \mathcal{A}_{t+1}} Pr(l \ choose \ j \ at \ t+\tau | x_{t+1}, a_{t+1}=k) \frac{\partial V^C_{lt+1}(\widetilde{x_{t+1}})}{\partial \widetilde{v_{lkt+1}}(x_{t+1})} \\
= & \sum_{k \in \mathcal{A}_{t+1}} Pr(l \ choose \ j \ at \ t+\tau | \widetilde{x_{t+1}}, a_{t+1}=k) \frac{\partial V^C_{lt+1}(\widetilde{x_{t+1}})}{\partial \widetilde{v_{lkt+1}}(\widetilde{x_{t+1}})} \\
= & \sum_{k \in \mathcal{A}_{t+1}} Pr(l \ choose \ j \ at \ t+\tau | \widetilde{x_{t+1}}, a_{t+1}=k) s^{(ccp)}_{lkt+1}(\widetilde{x_{t+1}}) \\
= & \ Pr(l \ choose \ j \ at \ t+\tau | \widetilde{x_{t+1}})
\end{aligned}
$$

Hence, we obtain:

$$
\zeta_{lt+\tau}(\widetilde{x_{t+1}}) = \beta_C^{\tau-2} Pr(l \ choose \ j \ at \ t+\tau | \widetilde{x_{t+1}})
$$

$\square$

Lemma 4. *The following equation holds:*

$$\frac{\partial E_t V^C_{lt+1}(x_t, a_t = k)}{\partial p_{jt+\tau}} \quad = \quad -\alpha_l \beta_C^{\tau-1} Pr(l \ choose \ j \ at \ t+\tau | x_t, a_t = k)$$

Proof. Using Lemma 3,

$$\begin{aligned}
& \frac{\partial V^C_{lt+1}(\widetilde{x_{t+1}})}{\partial p_{jt+\tau}} \\
= \ & \sum_{x_{t+\tau}} \frac{\partial \widetilde{v_{ljt+\tau}}(x_{t+\tau})}{\partial p_{jt+\tau}} \frac{\partial V^C_{lt+\tau}(x_{t+\tau})}{\partial \widetilde{v_{ljt+\tau}}(x_{t+\tau})} \sum_{x_{t+\tau-1}} \sum_{h \in \mathcal{A}_{t+\tau-1}} \frac{\partial \widetilde{v_{lht+\tau-1}}(x_{t+\tau-1})}{\partial V^C_{lt+\tau}(x_{t+\tau})} \frac{\partial V^C_{lt+1}(\widetilde{x_{t+1}})}{\partial \widetilde{v_{lht+\tau-1}}(x_{t+\tau-1})} \\
= \ & \sum_{x_{t+\tau}} (-\alpha_l) s^{(ccp)}_{ljt+\tau}(x_{t+\tau}) \sum_{x_{t+\tau-1}} \sum_{h \in \mathcal{A}_{t+\tau-1}} \beta_C \psi(x_{t+\tau} | x_{t+\tau-1}, a_{t+\tau-1} = h) \frac{\partial V^C_{lt+1}(\widetilde{x_{t+1}})}{\partial \widetilde{v_{lht+\tau-1}}(x_{t+\tau-1})} \\
= \ & -\beta_C \alpha_l \cdot \sum_{x_{t+\tau-1}} \sum_{h \in \mathcal{A}_{t+\tau-1}} Pr(l \ choose \ j \ at \ t+\tau | x_{t+\tau-1}, a_{t+\tau-1} = h) \frac{\partial V^C_{lt+1}(\widetilde{x_{t+1}})}{\partial \widetilde{v_{lht+\tau-1}}(x_{t+\tau-1})} \\
= \ & -\beta_C \alpha_l \zeta_{lt+\tau}(\widetilde{x_{t+1}}) \\
= \ & -\beta_C^{\tau-1} \alpha_l Pr(l \ choose \ j \ at \ t+\tau | \widetilde{x_{t+1}})
\end{aligned}$$

Consequently,

$$\begin{aligned}
& \frac{\partial E_t V^C_{lt+1}(x_t, a_t = k)}{\partial p_{jt+\tau}} \\
= \ & \sum_{x_{t+1}} \frac{\partial V^C_{lt+1}(x_{t+1})}{\partial p_{jt+\tau}} \psi(x_{t+1} | x_t, a_t = k) \\
= \ & -\alpha_l \beta_C^{\tau-1} \sum_{x_{t+1}} Pr(l \ choose \ j \ at \ t+\tau | x_{t+1}) \psi(x_{t+1} | x_t, a_t = k) \\
= \ & -\alpha_l \beta_C^{\tau-1} Pr(l \ choose \ j \ at \ t+\tau | x_t, a_t = k)
\end{aligned}$$

□

Lemma 5. *The sfollowing equation holds:*

$$\sum_{m \in \mathcal{A}_t} \sum_{\tau=1}^{T} \frac{\partial \beta_C E_t V^C_{lt+1}(x_t, a_t = m)}{\partial p_{jt+\tau}} \frac{\partial s^{(ccp)}_{lkt}(x_t)}{\partial \widetilde{v_{lmt}}(x_t)} \quad = \quad -\alpha_l \beta_C s^{(ccp)}_{lkt}(x_t) \lambda_{ljkt}(x_t)$$

Proof. First,

$$\begin{aligned}
& \sum_{m \in \mathcal{A}_t} \sum_{\tau=1}^{T} \frac{\partial \beta_C E_t V^C_{lt+1}(x_t, a_t = m)}{\partial p_{jt+\tau}} \frac{\partial s^{(ccp)}_{lkt}(x_t)}{\partial \widetilde{v_{lmt}}(x_t)} \\
= \ & \sum_{\tau=1}^{T} \left[\frac{\partial \beta_C E_t V^C_{lt+1}(x_t, a_t = k)}{\partial p_{jt+\tau}} \frac{\partial s^{(ccp)}_{lkt}(x_t)}{\partial \widetilde{v_{lkt}}(x_t)} + \sum_{m \in \mathcal{A}_t - \{k\}} \frac{\partial \beta_C E_t V^C_{lt+1}(x_t, a_t = m)}{\partial p_{jt+\tau}} \frac{\partial s^{(ccp)}_{lkt}(x_t)}{\partial \widetilde{v_{lmt}}(x_t)} \right] \\
= \ & \sum_{\tau=1}^{T} \left[\frac{\partial \beta_C E_t V^C_{lt+1}(x_t, a_t = k)}{\partial p_{jt+\tau}} s^{(ccp)}_{lkt}(x_t) \left(1 - s^{(ccp)}_{lkt}(x_t)\right) - \sum_{m \in \mathcal{A}_t - \{k\}} \frac{\partial \beta_C E_t V^C_{lt+1}(x_t, a_t = m)}{\partial p_{jt+\tau}} s^{(ccp)}_{lmt}(x_t) s^{(ccp)}_{lkt}(x_t) \right] \\
= \ & \beta_C s^{(ccp)}_{lkt}(x_t) \sum_{\tau=1}^{T} \left[\frac{\partial E_t V^C_{lt+1}(x_t, a_t = k)}{\partial p_{jt+\tau}} - \sum_{m \in \mathcal{A}_t} \frac{\partial E_t V^C_{lt+1}(x_t, a_t = m)}{\partial p_{jt+\tau}} s^{(ccp)}_{lmt}(x_t) \right]
\end{aligned}$$

By Lemma 4,

$$
\begin{aligned}
& \frac{\partial E_t V_{lt+1}^C(x_t, a_t = k)}{\partial p_{jt+\tau}} - \sum_{m \in \mathcal{A}_t} \frac{\partial E_t V_{lt+1}^C(x_t, a_t = m)}{\partial p_{jt+\tau}} s_{lmt}^{(ccp)}(x_t) \\
= \quad & -\alpha_l \beta_C^{\tau-1} Pr(l \ choose \ j \ at \ t+\tau | x_t, a_t = k) + \alpha_l \beta_C^{\tau-1} \sum_{m \in \mathcal{A}_t} Pr(l \ choose \ j \ at \ t+\tau | x_t, a_t = m) \cdot s_{lmt}^{(ccp)}(x_t) \\
= \quad & -\alpha_l \beta_C^{\tau-1} \left[Pr(l \ choose \ j \ at \ t+\tau | x_t, a_t = k) - Pr(l \ choose \ j \ at \ t+\tau | x_t) \right]
\end{aligned}
$$

Then, we obtain:

$$
\begin{aligned}
& \sum_{m \in \mathcal{A}_t} \sum_{\tau=1}^{T} \frac{\partial \beta_C E_t V_{lt+1}^C(x_t, a_t = m)}{\partial p_{jt+\tau}} \frac{\partial s_{lkt}^{(ccp)}(x_t)}{\partial \widetilde{v_{lmt}}(x_t)} \\
= \quad & -\alpha_l \beta_C s_{lkt}^{(ccp)}(x_t) \sum_{\tau=1}^{T} \beta_C^{\tau-1} \left[Pr(l \ choose \ j \ at \ t+\tau | x_t, a_t = k) - Pr(l \ choose \ j \ at \ t+\tau | x_t) \right] \\
= \quad & -\alpha_l \beta_C s_{lkt}^{(ccp)}(x_t) \lambda_{ljkt}(x_t)
\end{aligned}
$$

□

Proof of Lemma 2

Own price elasticity:

Proof. It follows that:

$$
\begin{aligned}
& \eta_{jt}^{(long)} - \eta_{jt}^{(short)} \\
= \quad & -\frac{p_{jt}^0}{s_{jt}^0} \int \left(\sum_{x_t \in X_t} Pr_{lt}(x_t) \sum_{m \in \mathcal{A}_t} \sum_{\tau=1}^{T} \frac{\partial \beta_C E_t V_{lt+1}^C(x_t, a_t = m)}{\partial p_{jt+\tau}} \frac{\partial s_{ljt}^{(ccp)}(x_t)}{\partial \beta_C E_t V_{lt+1}^C(x_t, a_t = m)} \right) dP(l) \\
= \quad & -\frac{p_{jt}^0}{s_{jt}^0} \int \left(\sum_{x_t \in X_t} Pr_{lt}(x_t) \sum_{m \in \mathcal{A}_t} \sum_{\tau=1}^{T} \frac{\partial \beta_C E_t V_{lt+1}^C(x_t, a_t = m)}{\partial p_{jt+\tau}} \frac{\partial s_{ljt}^{(ccp)}(x_t)}{\partial \widetilde{v_{lmt}}(x_t)} \right) dP(l) \\
= \quad & -\frac{p_{jt}^0}{s_{jt}^0} \int \left(\sum_{x_t \in X_t} Pr_{lt}(x_t) \left(-\alpha_l \beta_C s_{ljt}^{(ccp)}(x_t) \lambda_{ljt}(x_t) \right) \right) dP(l) \quad (\because \text{Lemma 5}) \\
= \quad & \beta_C \frac{p_{jt}^0}{s_{jt}^0} \int \alpha_l \left(\sum_{x_t \in X_t} Pr_{lt}(x_t) s_{ljt}^{(ccp)}(x_t) \lambda_{ljt}(x_t) \right) dP(l)
\end{aligned}
$$

Hence, the statement holds. □

Cross price elasticity:

Proof. It follows that:

$$
\begin{aligned}
&\eta_{jkt}^{(long)} - \eta_{jkt}^{(short)} \\
= &\frac{p_{jt}^0}{s_{kt}^0} \int \left(\sum_{x_t \in X_t} Pr_{lt}(x_t) \sum_{m \in \mathcal{A}_t} \sum_{\tau=1}^{T} \frac{\partial \beta_C E_t V_{lt+1}^C(x_t, a_t = m)}{\partial p_{jt+\tau}} \frac{\partial s_{lkt}^{(ccp)}(x_t)}{\partial \beta_C E_t V_{lt+1}^C(x_t, a_t = m)} \right) dP(l) \\
= &\frac{p_{jt}^0}{s_{kt}^0} \int \left(\sum_{x_t \in X_t} Pr_{lt}(x_t) \sum_{m \in \mathcal{A}_t} \sum_{\tau=1}^{T} \frac{\partial \beta_C E_t V_{lt+1}^C(x_t, a_t = m)}{\partial p_{jt+\tau}} \frac{\partial s_{lkt}^{(ccp)}(x_t)}{\partial \widetilde{v_{lmt}}(x_t)} \right) dP(l) \\
= &\frac{p_{jt}^0}{s_{kt}^0} \int \left(\sum_{x_t \in X_t} Pr_{lt}(x_t) \left(-\alpha_l \beta_C s_{lkt}^{(ccp)}(x_t) \lambda_{ljkt}(x_t) \right) \right) dP(l) \quad (\because \text{Lemma 5}) \\
= &-\beta_C \frac{p_{jt}^0}{s_{kt}^0} \int \alpha_l \left(\sum_{x_t \in X_t} Pr_{lt}(x_t) s_{lkt}^{(ccp)}(x_t) \lambda_{ljkt}(x_t) \right) dP(l)
\end{aligned}
$$

Hence, the statement holds. □

A.3 Proof of the statements in Section 4.1

A.3.1 Proof of Proposition 4

Own price elasticity:

Proof. It follows that:

$$
\begin{aligned}
&\widehat{\eta_{jt}}(p_t^0, g_t^0) - \eta_{jt}^{(short)}(p_t^0, g_t^0) \\
= &\left[\alpha \left(- \left(\sum_{x_t \in X_t} Pr_t(x_t) s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) \right)^2 + \sum_{x_t \in X_t} Pr_t(x_t) s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)^2 \right) \right] \frac{p_{jt}^0}{s_{jt}^0} \\
\leq &\frac{\sum_{x_t \in X_t} Pr_t(x_t) s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) \cdot \alpha s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)}{\sum_{x_t \in X_t} Pr_t(x_t) s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)} p_{jt}^0 \\
\leq &\frac{\left(\max_{x_t} \alpha s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) \right) \cdot \sum_{x_t \in X_t} Pr_t(x_t) s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)}{\sum_{x_t \in X_t} Pr_t(x_t) s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0)} p_{jt}^0 \\
= &\alpha p_{jt}^0 \left(\max_{x_t} s_{jt}^{(ccp)}(x_t, p_t^0, g_t^0) \right)
\end{aligned}
$$

□

Cross price elasticity:

Proof. When $\widehat{\eta_{jkt}}(p_t^0, g_t^0) - \eta_{jkt}^{(short)}(p_t^0, g_t^0) \geq 0$ holds,

$$
\begin{aligned}
\left|\widehat{\eta_{jkt}}(p_t^0,g_t^0)-\eta_{jkt}^{(short)}(p_t^0,g_t^0)\right| &= \widehat{\eta_{jkt}}(p_t^0,g_t^0)-\eta_{jkt}^{(short)}(p_t^0,g_t^0)\\
&= \left[\alpha s_{jt}^0 s_{kt}^0-\alpha\sum_{x_t\in X_t}Pr_t(x_t)\cdot s_{jt}^{(ccp)}(x_t,p_t^0,g_t^0)s_{kt}^{(ccp)}(x_t,p_t^0,g_t^0)\right]\frac{p_{jt}^0}{s_{kt}^0}\\
&\leq \frac{\alpha s_{jt}^0 s_{kt}^0}{s_{kt}^0}p_{jt}^0\\
&\leq \left(\alpha\sum_{x_t\in X_t}Pr_t(x_t)s_{jt}^{(ccp)}(x_t,p_t^0,g_t^0)\right)p_{jt}^0\\
&\leq \alpha\max_{x_t}s_{jt}^{(ccp)}(x_t,p_t^0)p_{jt}^0\quad\left(\because\sum_{x_t\in X_t}Pr_t(x_t)=1\right)
\end{aligned}
$$

On the other hand, when $\widehat{\eta_{jkt}}(p_t^0,g_t^0)-\eta_{jkt}^{(short)}(p_t^0,g_t^0)\leq 0$ holds,

$$
\begin{aligned}
\left|\widehat{\eta_{jkt}}(p_t^0,g_t^0)-\eta_{jkt}^{(short)}(p_t^0,g_t^0)\right| &= -\widehat{\eta_{jkt}}(p_t^0,g_t^0)+\eta_{jkt}^{(short)}(p_t^0,g_t^0)\\
&= \left[-s_{jt}(p_t^0,g_t^0)s_{kt}(p_t^0,g_t^0)+\alpha\sum_{x_t\in X_t}Pr_t(x_t)\cdot s_{jt}^{(ccp)}(x_t,p_t^0,g_t^0)s_{kt}^{(ccp)}(x_t,p_t^0,g_t^0)\right]\frac{p_{jt}^0}{s_{kt}^0}\\
&\leq \frac{\alpha\sum_{x_t\in X_t}Pr_t(x_t)\cdot s_{jt}^{(ccp)}(x_t,p_t^0,g_t^0)s_{kt}^{(ccp)}(x_t,p_t^0,g_t^0)}{s_{kt}^0}p_{jt}^0\\
&\leq \frac{\left(\max_{x_t}\alpha s_{jt}^{(ccp)}(x_t,p_t^0,g_t^0)\right)\cdot\sum_{x_t\in X_t}Pr_t(x_t)s_{kt}^{(ccp)}(x_t,p_t^0,g_t^0)}{s_{kt}^0}p_{jt}^0\\
&\leq \alpha\max_{x_t}s_{jt}^{(ccp)}(x_t,p_t^0,g_t^0)p_{jt}^0
\end{aligned}
$$

Hence, the statement holds. □

A.3.2 Proof of Proposition 5

Cross price elasticity:

Proof. In the following, we omit $p_{t+\tau}^0$ and $g_{t+\tau}^0$ to make the notation simpler. It follows that:

$$
\begin{aligned}
& \left|\eta_{jkt}^{(long)} - \eta_{jkt}^{(short)}\right| \\
= \; & \frac{p_{jt}^0}{s_{kt}^0}\beta_C \int \alpha_l \sum_{x_t \in X_t} Pr_{lt}(x_t) s_{lkt}^{(ccp)}(x_t) \left|\lambda_{ljkt}(x_t)\right| dP(l) \\
\leq \; & \beta_C p_{jt}^0 \frac{\int \sum_{x_t \in X_t} Pr_{lt}(x_t) s_{lkt}^{(ccp)}(x_t) dP(l)}{s_{kt}^0} \left(\max_{l,x_t} \alpha_l \left|\lambda_{ljkt}(x_t)\right|\right) \\
= \; & \beta_C p_{jt}^0 \left(\max_{l,x_t} \alpha_l \left|\lambda_{ljkt}(x_t)\right|\right) \\
= \; & \beta_C p_{jt}^0 \left(\max_{l,x_t} \alpha_l \left|\sum_{\tau=1}^{T} \beta_C^{\tau-1} \left[Pr(l\ choose\ j\ at\ t+\tau | x_t, a_t = j) - Pr(l\ choose\ j\ at\ t+\tau | x_t)\right]\right|\right) \\
= \; & \beta_C p_{jt}^0 \left(\max_{l,x_t} \alpha_l \left|\sum_{\tau=1}^{T} \beta_C^{\tau-1} \sum_{x_{t+\tau}} \left[s_{ljt+\tau}^{(ccp)}(x_{t+\tau}) Pr(x_{t+\tau}|x_t, a_t = j) - s_{ljt+\tau}^{(ccp)}(x_{t+\tau}) Pr(x_{t+\tau}|x_t)\right]\right|\right) \\
= \; & \beta_C p_{jt}^0 \left(\max_{l,x_t} \alpha_l \left|\sum_{\tau=1}^{T} \beta_C^{\tau-1} \sum_{x_{t+\tau}} s_{ljt+\tau}^{(ccp)}(x_{t+\tau}) \left[Pr(x_{t+\tau}|x_t, a_t = j) - Pr(x_{t+\tau}|x_t)\right]\right|\right) \\
\leq \; & \beta_C p_{jt}^0 \left(\max_{l} \alpha_l \cdot 2 \sum_{\tau=1}^{T} \beta_C^{\tau-1} \max_{x_{t+\tau}} s_{ljt+\tau}^{(ccp)}(x_{t+\tau})\right) \;\; \left(\because \sum_{x_{t+\tau}} Pr(x_{t+\tau}|x_t, a_t = j) = \sum_{x_{t+\tau}} Pr(x_{t+\tau}|x_t) = 1\right) \\
= \; & 2\beta_C \frac{1-\beta_C^T}{1-\beta_C} p_{jt}^0 \cdot \max_{x_{t+\tau}} s_{ljt+\tau}^{(ccp)}(x_{t+\tau})
\end{aligned}
$$

□

Own price elasticity:

Proof. We can prove the statement as in the case of cross price elasticity. □

A.4 Proof of the statements in Section 4.3

A.4.1 Proof of Corollary 2

Proof. Let $\mathcal{J}_{gt}$ be the set of products in the same package size, and let $j, k \in \mathcal{J}_{gt}$. Then, $\phi_{jt}(x_t) = \phi_{gt}(x_t)$ holds for all $j \in \mathcal{J}_{gt}$, because $C_t^*|_j = \arg\max_{C_t} U(C_t) - F(x_t + q_{jt} - C_t) + \beta_C E_t V_{lt+1}^C(x_t + q_{jt} - C_t, p_{t+1}|x_t, g_t)$ takes common values for all $j \in \mathcal{J}_{gt}$ (same package size (q_{jt}) products), and $\phi_{ljt}(x_t) = U(C_t^*|_j) - F(x_t + q_{jt} - C_t^*|_j)$ holds by construction. Moreover, since $x_{t+1} = x_t + q_t - C_t$, continuation values $g_{jt}(x_t)$ take common values for all $j \in \mathcal{J}_{gt}$.

Then, $s_{j|g,t}(p_t, g_t) \equiv s_{jt}^{(ccp)}(x_t, p_t, g_t) / \left(\sum_{m \in \mathcal{J}_{gt}} s_{mt}^{(ccp)}(x_t, p_t, g_t)\right)$, the probability that a consumer purchases product j conditional on purchasing products in $\mathcal{J}_{gt}$, does not depend on state variables x_t, because:

$$
\begin{aligned}
s_{j|g,t}(p_t, g_t) \; & \equiv \; s_{jt}^{(ccp)}(x_t, p_t, g_t) / \left(\sum_{m \in \mathcal{J}_{gt}} s_{mt}^{(ccp)}(x_t, p_t, g_t)\right) \\
& = \; \frac{\frac{\exp(\phi_{jt}(x_t) - \alpha p_{jt} + f_{jt} + \beta_C g_{jt}(x_t))}{\exp\left(V_t^C(x_t, p_t, g_t)\right)}}{\sum_{m \in \mathcal{J}_{gt}} \frac{\exp(\phi_{mt}(x_t) - \alpha p_{mt} + f_{mt} + \beta_C g_{mt}(x_t))}{\exp\left(V_t^C(x_t, p_t, g_t)\right)}} \\
& = \; \frac{\exp(-\alpha p_{jt} + f_{jt})}{\sum_{m \in \mathcal{J}_{gt}} \exp\left(-\alpha p_{mt} + f_{mt}\right)}
\end{aligned}
$$

Hence, we obtain:

$$
\begin{aligned}
Cov_{jkt}(p_t,g_t) &\equiv \sum_{\widetilde{x_t}\in X_t} Pr_t(\widetilde{x_t})\left(s_{jt}^{(ccp)}(\widetilde{x_t},p_t,g_t)-E_{x_t}s_{jt}^{(ccp)}(x_t,p_t,g_t)\right)\left(s_{kt}^{(ccp)}(\widetilde{x_t},p_t,g_t)-E_{x_t}s_{kt}^{(ccp)}(x_t,p_t,g_t)\right)\\
&= \sum_{\widetilde{x_t}\in X_t} Pr_t(\widetilde{x_t})\left(\left(\sum_{m\in\mathcal{J}_{gt}} s_{mt}^{(ccp)}(\widetilde{x_t},p_t,g_t)\right)s_{j|g,t}(p_t,g_t)-E_{x_t}\left(\sum_{m\in\mathcal{J}_{gt}} s_{mt}^{(ccp)}(x_t,p_t,g_t)\right)s_{j|g,t}(p_t,g_t)\right)\cdot\\
&\quad\left(\left(\sum_{m\in\mathcal{J}_{gt}} s_{mt}^{(ccp)}(\widetilde{x_t},p_t,g_t)\right)s_{k|g,t}(p_t,g_t)-E_{x_t}\left(\sum_{m\in\mathcal{J}_{gt}} s_{mt}^{(ccp)}(x_t,p_t,g_t)\right)s_{k|g,t}(p_t,g_t)\right)\\
&= s_{j|g,t}(p_t,g_t)s_{k|g,t}(p_t,g_t)\sum_{\widetilde{x_t}\in X_t} Pr_t(\widetilde{x_t})\left(\sum_{m\in\mathcal{J}_{gt}} s_{mt}^{(ccp)}(\widetilde{x_t},p_t,g_t)-E_{x_t}\sum_{m\in\mathcal{J}_{gt}} s_{mt}^{(ccp)}(x_t,p_t,g_t)\right)^2\\
&\geq 0
\end{aligned}
$$

□

A.5 Proof of the statements in Section 5

A.5.1 Proof of equation (20)

Proof. Since we consider the case where consumers do not purchase under the state $x_t \neq 0$, let $\phi_{jt}(x_t \neq 0) = -\infty$ so that $s_{jt}^{(ccp)}(x_t \neq 0, p_t, g_t) = 0$. Then, $\frac{\exp(\phi_{jt}(x_t\neq 0)+\beta_C g_{jt}(x_t\neq 0))}{\exp(V_t^C(x_t\neq 0,p_t,g_t))} = 0$ and $\frac{\exp(f_{0t}(x_t\neq 0)+\beta_C g_{0t}(x_t\neq 0))}{\exp(V_t^C(x_t\neq 0,p_t,g_t))} = 1$. Hence,

$$
\begin{aligned}
\widehat{c_{jt}} &= \log\left(\frac{\frac{\exp(\phi_{jt}(x_t=0)+\beta_C g_{jt}(x_t=0))}{\exp(V_t^C(x_t=0,p_t,g_t))}Pr_t(x_t=0)+\frac{\exp(\phi_{jt}(x_t\neq 0)+\beta_C g_{jt}(x_t\neq 0))}{\exp(V_t^C(x_t\neq 0,p_t,g_t))}(1-Pr_t(x_t=0))}{\frac{\exp(f_{0t}(x_t=0)+\beta_C g_{0t}(x_t=0))}{\exp(V_t^C(x_t=0,p_t,g_t))}Pr_t(x_t=0)+\frac{\exp(f_{0t}(x_t\neq 0)+\beta_C g_{0t}(x_t\neq 0))}{\exp(V_t^C(x_t\neq 0,p_t,g_t))}(1-Pr_t(x_t=0))}\right)\\
&= \log\left(\frac{\frac{\exp(\phi_{jt}(x_t=0)+\beta_C g_{jt}(x_t=0))}{\exp(V_t^C(x_t=0,p_t,g_t))}Pr_t(x_t=0)}{\frac{\exp(f_{0t}(x_t=0)+\beta_C g_{0t}(x_t=0))}{\exp(V_t^C(x_t=0,p_t,g_t))}Pr_t(x_t=0)+(1-Pr_t(x_t=0))}\right)\\
&= \log\left(\frac{\frac{\exp(\phi_{jt}(x_t=0)+\beta_C g_{jt}(x_t=0))}{\exp(V_t^C(x_t=0,p_t,g_t))}Pr_t(x_t=0)}{1-\frac{\sum_{k\in\mathcal{J}_t}\exp(-\alpha p_{kt}+f_{kt}+\phi_{kt}(x_t=0)+\beta_C g_{kt}(x_t=0))}{\exp(V_t^C(x_t=0,p_t,g_t))}Pr_t(x_t=0)}\right)
\end{aligned}
$$

□

B Monte Carlo Simulation

To show the effectiveness of introducing fixed effect terms for utility parameter estimates discussed in Section 3.3, we conduct a Monte Carlo Experiment. We consider the market of durable goods with exogenous replacement timing, where consumers consider purchases only when they do not have any product. In the model, all the products share the same continuation values, and the introduction of time dummies yields consistent estimates of utility parameters under the nonexistence of persistent consumer heterogeneity. We try the static estimation as in Berry et al. (1995) with and without fixed effect terms and compare the estimation results. In addition, we estimate the parameters under two market environments: a stationary environment and a nonstationary environment. Here, "stationary" implies that product holdings and consumer expectations are mostly stable over time. I show the case where the biases in parameter estimates are not large if the market is stationary, even when not introducing fixed effect terms.

B.1 Specifications of the Monte Carlo experiment

The Monte Carlo experiments are conducted in the following specifications and procedures.

Specifications

The synthetic data generated in this study are similar to those in Dubé et al. (2012) and Sun and Ishihara (2019)[29], except for the introduction of the replacement demand.

Let x_t be consumer's individual-level state variables. We consider the case where consumers consider purchases only when they do not own any product ($x_t = 0$). Then, type l consumer i's present value discounted sum of utility at time t is specified as:

$$\begin{aligned} v_{iljt}(x_t = 0) &= \theta_l^0 + \sum_{m=1}^{2} \theta_l \chi_{jt}^{(m)} - \alpha_l p_{jt} + \xi_{jt} + E_t \beta_C^L V_{lt+L}^C (x_{t+L} = 0) + \epsilon_{ijt} \\ v_{il0t}(x_t = 0) &= \beta_C E_t V_{lt+1}^C (x_{t+1} = 0) + \epsilon_{i0t} \end{aligned}$$

for $j = 1, ...J$, and $t = 1, ..., T$. Here, $\chi_{jt}, p_{jt}, \xi_{jt}$ denote the product j's observed characteristics, price, and unobserved characteristics. L denotes the lifetime of products.

We assume that all the products follow the same lifetime distribution. Products follow geometric depreciation with a depreciation rate ρ. Then, $Pr(x_t)$ follows the following process:

$$Pr_{lt+1}(x_{t+1} = 0) = \left(1 - Pr_{lt}(x_t = 0) s_{l0t}^{(cd)}(x_t = 0)\right) \rho + Pr_{lt}(x_t = 0) s_{l0t}^{(cd)}(x_t = 0)$$

We consider the case where only one market exists, and we let the values of T and J be $T = 100$ and $J = 50$. In addition, we assume that none of the consumers possess the products at the beginning of $t = 1$ ($Pr_{lt=1}(x_{t=1} = 0) = 1$).

Observed product characteristics $\chi_{jt} = \left(\chi_{jt}^{(1)}, \chi_{jt}^{(2)}\right)'$ follows log-normal distribution:

$$\chi_{jt} \equiv \begin{pmatrix} x_{jt}^1 \\ x_{jt}^2 \end{pmatrix} \sim LN\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0.5^2 & \\ & 0.5^2 \end{pmatrix} \right)$$

Unobserved product characteristic ξ_{jt} follows the i.i.d. normal $N(0, 0.5^2)$.

Product price is generated in the following way:

$$p_{jt} = \gamma_z^0 + \gamma_x \chi_{jt} + \gamma_z z_{jt} + \gamma_w w_{jt} + \gamma_\xi \xi_{jt} - \gamma_p \frac{\sum_{k \neq j} X_{kt}}{J-1} + u_{jt}^p$$

where $u_{jt}^p \sim N(0, 0.01^2)$. We consider the environment where product prices decline over time, but finally converge to stable levels.

w_{jt} and z_{jt} represents cost shifter, and w_{jt} follows standard normal distribution $N(0, 1^2)$. z_{jt} follows $AR(1)$ process $z_{jt} = \lambda_0^z + \lambda_1^z z_{jt-1} + u_{jt}^z$, where $u_t^z \sim N(0, 0.01^2)$.

The values of the parameters are set as follows:

$z_{j0} = 6$, $[\lambda_0^z, \lambda_1^z] = [0.1, 0.9]$, $[\gamma_z^0, \gamma_{x_1}, \gamma_{x_2}, \gamma_z, \gamma_w, \gamma_\xi] = [1, 0.2, 0.2, 1, 0.2, 0.7]$, $\gamma_p = [0.05, 0.05]$

The values of the product depreciation rate and consumers' discount factor are set to 0.03 and 0.99.

For estimation, we use the weight matrix $W = (Z'Z)^{-1}$, where Z is the matrix of instrumental variables. As instrumental variables, we use polynomial expansions of $[\chi_{jt}, z_{jt}, w_{jt}, \sum_{k \neq j} \chi_{kt}]$. There are 35 instrumental variables.

[29] These studies considered the alternative estimation procedures other than the method relying on the nested fixed-point algorithm.

Procedure

I repeated the following process:

1. Generate price and product characteristics data (p_{jt}, χ_{jt})
2. Solve $V_{lt}^{C}(x_t = 0)$ and $s_{ljt}^{(cd)}(x_{tr} = 0)$

   To solve these values, we assume the following AR(1) transition process of inclusive value $V_{lt}^{C}(x_t = 0)$:[30]

$$V_{lt+1}^{C}(x_{t+1} = 0) = \gamma_0^{\delta} + \gamma_1^{\delta} V_{lt}^{C}(x_t = 0) + u_{lt}^{\delta} \quad (E u_{lt}^{\delta} = 0)$$

3. Calculate the path of stock $(Pr_{lt}(x_t = 0))$ and market share (s_{jt})
4. Using the generated data $(p_{jt}, \chi_{jt}, s_{jt})$, estimate parameters by assuming a static demand model and applying the BLP method. Static BLP estimation was implemented by using PyBLP (Conlon and Gortmaker, 2020).

B.2 Results under the dynamic model without persistent consumer heterogeneity

First, I show the results under the dynamic model without persistent consumer heterogeneity. I generate simulated data under the following parameter setting: $[\theta_l^0, \theta_l^1, \theta_l^2, \alpha_l] = [2, 1.5, 1, 2]\ \forall l$.

Before looking at the results of parameter estimates, we look at the environment of the market generated in the simulation.

Figure 2 shows the fraction of consumers who do not possess any product at the beginning of each period. As time passes, more and more consumers possess the products in the earlier periods. On the other hand, in the later periods, the fraction is mostly stable over time. In the later periods, most of the demand comes from the replacement demand, rather than new purchases. Hence, the demand structure is largely different between the two ranges of periods.

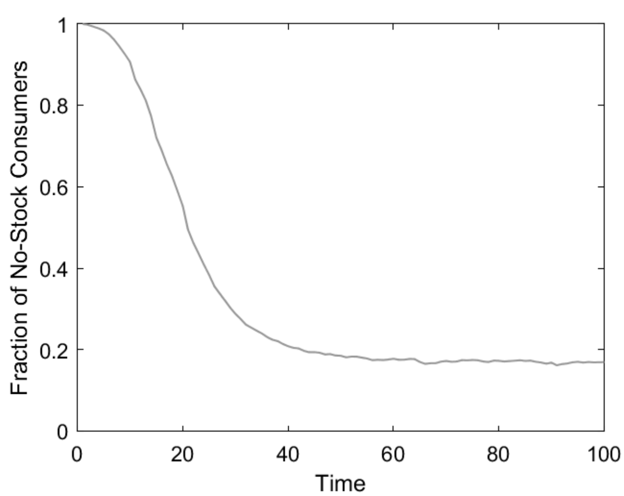

Figure 2: Fraction of no-stock consumers (No persistent consumer heterogeneity)

Notes:
Fraction of no-stock consumers is defined as $Pr_{lt}(x_t = 0)$.
Averages of 20 simulated data are shown.

Figure 3 shows the time trend of average price($\frac{1}{J}\sum_j p_{jt}$) and demand($\sum_j s_{jt}$). First, the price declines gradually, and finally it reaches a stable level. On the other hand, demand increases from time 1 to time 20,

[30]In this study, inclusive value δ_{lt} is defined as follows: $\delta_{lt} = E_\epsilon \left[\max_{a_t \in \mathcal{J} \cup \{0\}} v_{ilt}(x_t, a_t)\right]$. On the other hand, most of the literature has used the alternative specification of inclusive value equivalent to $\delta_{lt} = E_\epsilon \left[\max_{a_t \in \mathcal{J}} v_{ilt}(x_t, a_t)\right]$. When using the latter specification, we need to specify the distribution of u_{lt}^{δ} and calculate expectations based on the distribution of u_{lt}^{δ}. In this case, the computation gets more complicated. Hence, I use the former specification.

and declines from time 20 to time 50. From time 50, the demand is at a stable level, even though it fluctuates over time. Then, we can observe negative correlations between price and demand in time 1–20 and positive correlations between price and demand in time 20–50. The positive correlation may seem strange, but it is mainly caused by the declining fraction of no-stock consumers as shown in Figure 2.

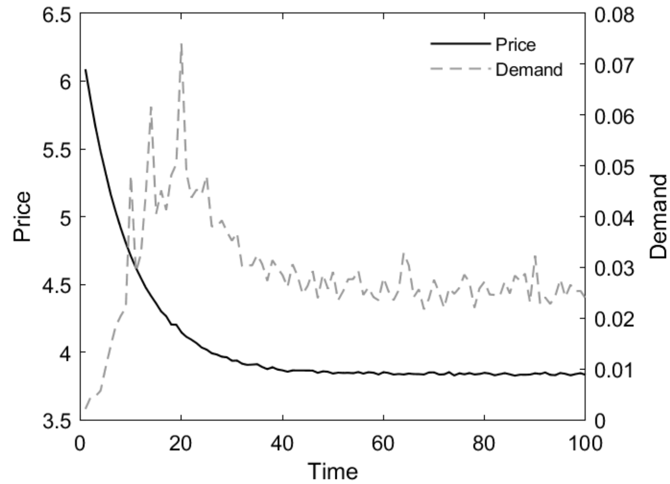

Figure 3: Price and demand (No persistent consumer heterogeneity)

Notes:
Price is defined as $\frac{1}{J}\sum_j p_{jt}$. Demand is defined as $\sum_j s_{jt}$.
Averages of 20 simulated data are shown.

The estimation results are shown in Table 3. Since the environment is largely different between the earlier periods and the later periods, I divide the data into two parts (t =1–50 and t =51–100), and estimate parameters in each case.

The first and second columns of Table 3 show the estimation results using the data of the nonstationary market (t =1–50). In the case of the nonstationary market, estimated parameters are largely different from the true values when fixed effect terms are not introduced. For instance, the average estimated value of α is 0.885, though the true value is 2.0. On the other hand, when fixed effect terms are introduced, estimated values are closer to the true values.

Similarly, the third and fourth columns of Table 3 show the estimation results using the data of the stationary market (t =51–100). In this case, both specifications yield parameter estimates close to the true values. From time 51 to time 100, the market is in a stationary environment: consumers' expectations and product stock are mostly stable over time. Hence, the role of controlling the fixed effect terms is limited. Even when estimating parameters with the static model without controlling fixed effect terms, the biases are relatively small[31].

Parameters	True value	Nonstationary (t =1–50)		Stationary (t =51–100)	
		without FE	with FE	without FE	with FE
θ^1	1.5	1.268	1.490	1.485	1.495
		(0.030)	(0.009)	(0.013)	(0.01)
θ^2	1.0	0.780	0.993	0.988	0.996
		(0.023)	(0.009)	(0.012)	(0.008)
α	2.0	0.885	1.970	1.957	1.978
		(0.061)	(0.027)	(0.037)	(0.026)

Table 3: Results of the Monte Carlo Experiment (No persistent consumer heterogeneity)

Notes:
Based on 20 simulated data and 5 initial parameter values.
Root mean squared errors are shown inside parenthesis.

[31] Similar insight was also discussed in Melnikov (2000).

As discussed in Section 3.2, even when utility parameters are consistently estimated by static specification under the dynamic model without persistent consumer heterogeneity, estimated short-run price elasticities are biased. Nevertheless, even though the true model does not include any random coefficients, the introduction of random coefficients may mitigate the biases in price elasticities by providing more flexible demand structures. To validate the conjecture, I estimate the utility parameters and price elasticities under the two specifications: without/with random coefficients.

Table 4 shows the parameter estimates. Columns (1) and (3) are the estimation results without introducing random coefficients. These results are the same as the ones in Table 3. Columns (2) and (4) are the estimation results introducing random coefficients. As in Table 3, I divide the samples into two parts: nonstationary(t =1–50) and stationary(t =51–100) markets. In all the specifications time fixed effects are introduced. Though the RMSEs get large, estimated parameters are not very close to the true values.

Parameters	True value	Nonstationary (t =1–50)		Stationary (t =51–100)	
		(1)	(2)	(3)	(4)
mean of θ_l^1	1.5	1.490	1.480	1.495	1.492
		(0.009)	(0.019)	(0.010)	(0.016)
mean of θ_l^2	1.0	0.993	0.984	0.996	0.993
		(0.009)	(0.015)	(0.008)	(0.014)
mean of α_l	2.0	1.970	3.391	1.978	3.789
		(0.027)	(1.516)	(0.026)	(1.582)
s.d. of θ_l^1	0	-	0.069	-	0.038
			(0.076)		(0.061)
s.d. of θ_l^2	0	-	0.053	-	0.030
			(0.059)		(0.055)
s.d. of α_l	0	-	0.838	-	1.077
			(0.864)		(0.881)
with FE		Yes	Yes	Yes	Yes

Table 4: Results of Monte Carlo experiment (Parameter estimates; without persistent consumer heterogeneity)
Notes:
Based on 20 simulated data and 5 initial parameter values.
Root mean squared errors (RMSEs) are shown inside parenthesis.

Next, we stack all the samples of 20 simulated data used in the estimation, and compute elasticities. Here, we do not distinguish between nonstationary and stationary markets. Table 5 shows the summary statistics of the short-run own price elasticities computed from the true dynamic model. Figure 4 compares the values of own price elasticities computed from the static demand model and the ones computed from the true dynamic demand model. Panel (a) shows the results under the specification without random coefficients, which correspond to columns (1) and (3) in Table 4. Panel (b) shows the results under the specification with random coefficients, which correspond to columns (2) and (4) in Table 4. To understand when the bias is large, I present the scatter diagrams between products' CCPs and biases in elasticities measured by percentage in Figure 4.

As Figure 4(a) shows, the biases are tremendously large especially for large CCP products: For instance, the bias of the product whose CCP value is roughly 0.9 is over 1,000%[32]. In contrast, the biases are small for small CCP products. As shown in Figure 4(b), the introduction of random coefficients partially mitigate the biases. Nevertheless, negligible magnitudes of the biases remain.

Note that in columns (2) and (4), I did not introduce the random coefficient of the constant term. Even though the introduction might further reduce the biases, many of the estimations did not converge when introducing the random coefficient of the constant term.

[32] As shown in Table 5, short-run price elasticities are not necessarily close to zero. Hence, we cannot attribute the large biases to small values of true price elasticities.

Obs	Median	Mean	s.d.	Min	Max
100000	7.87	8.11	1.09	0.61	14.44

Table 5: Summary statistics of short-run own price elasticities (Without persistent consumer heterogeneity)

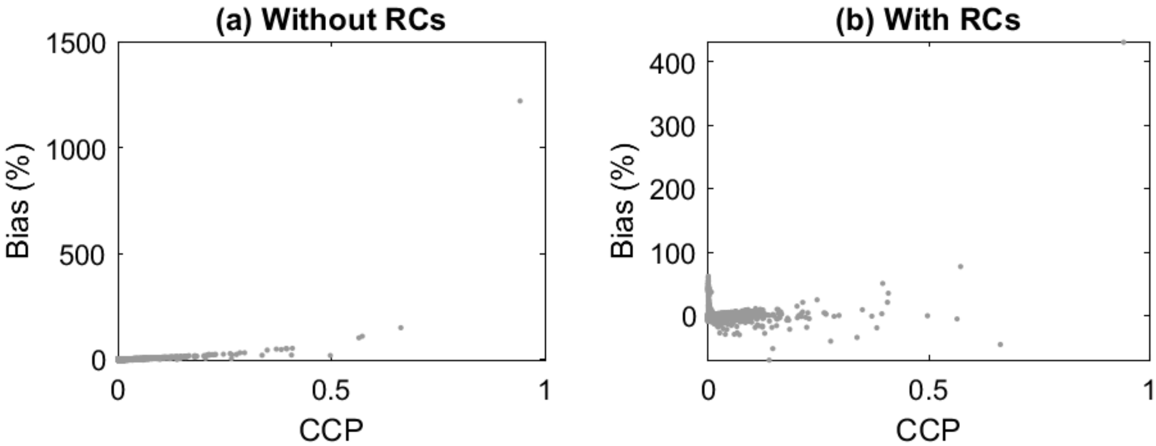

Figure 4: Biases in short-run own price elasticities (Without persistent consumer heterogeneity)

Notes:
Bias is calculated by: $\frac{\text{(static estimate)} - \text{(true value)}}{\text{(true value)}} \times 100$.
CCP is defined as $s^{(cd)}_{ljt}(x_t = 0)$.

B.3 Results under the model with persistent consumer heterogeneity

Next, I show the results under the dynamic model with persistent consumer heterogeneity. I generate simulated data under the following parameter settings:

$$\begin{pmatrix} \theta_l^0 \\ \theta_l^1 \\ \theta_l^2 \\ \alpha_l \end{pmatrix} \sim N\left(\begin{pmatrix} 2 \\ 1.5 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 & & & \\ & 0.5^2 & & \\ & & 0.5^2 & \\ & & & 0.25^2 \end{pmatrix} \right)$$

Figures 5 and 6 show the trend of the main variables of the simulated data.

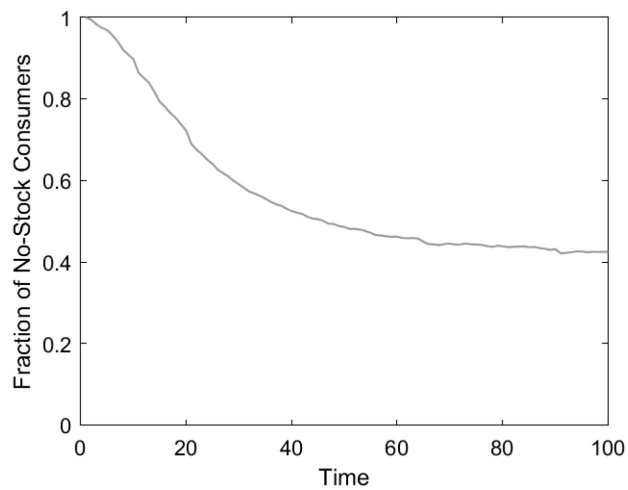

Figure 5: Fraction of no-stock consumers (With persistent consumer heterogeneity)

Notes:
Fraction of no-stock consumers is defined as $Pr_t \equiv \int Pr_{lt}(x_t = 0)dP(l)$.
Averages of 20 simulated data are shown.

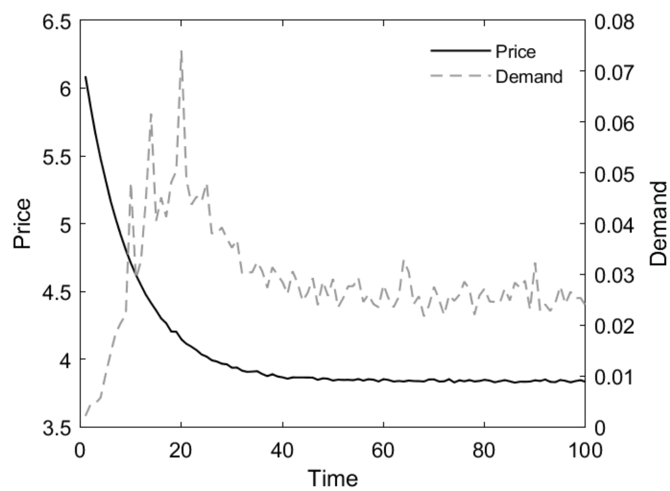

Figure 6: Price and demand (With persistent consumer heterogeneity)

Notes:
Price is defined as $\frac{1}{J}\sum_j p_{jt}$. Demand is defined as $\sum_j s_{jt}$.
Averages of 20 simulated data are shown.

Table 6 shows the estimation results. As in Table 3, I estimate the parameters with and without time fixed effect terms. As in the case of the model without persistent consumer heterogeneity, the estimations without fixed effect terms lead to large biases in parameter estimates when the environment of the market is nonstationary. We can also observe that even when introducing fixed effect terms, estimated coefficients are not very close to the true values, even though the RMSEs are large.

Parameters	True value	Nonstationary (t =1–50) without FE	with FE	Stationary (t =51–100) without FE	with FE
mean of $\theta_l^{x_1}$	1.5	1.137	1.348	1.291	1.291
		(0.080)	(0.041)	(0.030)	(0.033)
mean of $\theta_l^{x_2}$	1.0	0.686	0.864	0.826	0.830
		(0.051)	(0.032)	(0.036)	(0.035)
mean of α_l^p	2.0	1.094	2.491	2.937	3.188
		(0.798)	(0.238)	(0.959)	(1.152)
s.d. of $\theta_l^{x_1}$	0.5	0.388	0.426	0.440	0.446
		(0.079)	(0.052)	(0.025)	(0.028)
s.d. of $\theta_l^{x_2}$	0.5	0.419	0.464	0.457	0.459
		(0.054)	(0.031)	(0.032)	(0.031)
s.d. of α_l^p	0.25	0.138	0.441	0.675	0.802
		(0.448)	(0.110)	(0.512)	(0.619)

Table 6: Results of Monte Carlo simulation (Parameter estimates; with persistent consumer heterogeneity)
Notes:
Based on 20 simulated data and 5 initial parameter values.
Root mean squared errors (RMSEs) are shown inside parenthesis.

Next, as in the case without persistent consumer heterogeneity presented in the previous subsection, we stack all the samples of generated data used in the estimation, and compute elasticities. Table 7 shows the summary statistics of short-run own price elasticities computed from the true dynamic model. Figure 7 compares the values of own price elasticities computed from the static demand model and the ones computed from the true dynamic demand model. Here, random coefficients and fixed effect terms are introduced in the static estimates. As in the cases of the model without persistent consumer heterogeneity, the biases are large especially for large CCP products.

Obs	Median	Mean	s.d.	Min	Max
100000	7.52	7.64	0.76	2.37	11.69

Table 7: Summary statistics of short-run own price elasticities (With persistent consumer heterogeneity)

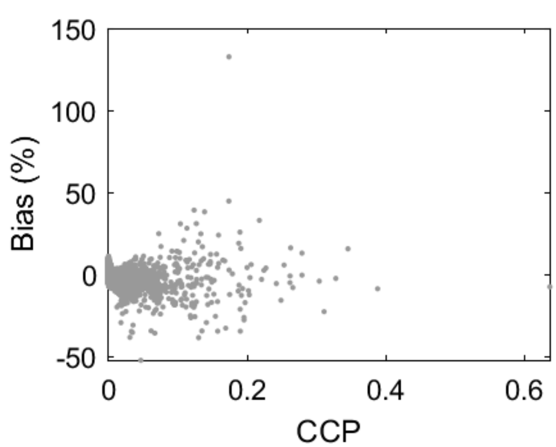

Figure 7: Biases in short-run price elasticities (With persistent consumer heterogeneity)
Notes:
Bias is calculated by: $\frac{\text{(static estimate)}-\text{(true value)}}{\text{(true value)}} \times 100$.
CCP is defined as $\int s_{ljt}^{(cd)}(x_t = 0)dP(l)$.

Overall, the biases in own price elasticities may be large when the products' CCPs are large. Caution is required when we use static demand models especially when the CCPs or the market shares of the products we focus on are large.

C Literature on the biases in static demand models

In this section, I show the source of the information of each article's estimates summarized in Table 1. In addition, for some articles I provide additional interpretation of the results based on the discussion in this article.

C.1 Durable goods

Chen et al. (2008)

Chen et al. (2008) specified both dynamic demand-side model and dynamic supply-side model, and generated data of automobile market given calibrated parameter values. Besides, both new and used goods exist in the market, and no persistent consumer heterogeneity exists.

- Price coefficient: Table 3 ($N = 3$, δ=0.11, $\rho = 0.10$, β_1=0.96, $\beta_2 = 0.96$)

  $\widehat{\gamma}$ represents the static estimates of the price coefficient (marginal utility of money), and γ represents the true marginal utility of money based on the dynamic model.

- Own elasticity: Table 3 ($N = 3$, δ=0.11, $\rho = 0.10$, β_1=0.96, $\beta_2 = 0.96$)

  $\widehat{\eta}$ represents the static estimate of the price elasticity (of the new product). e represents the true price elasticity of the new product based on the dynamic model. Note that η considers the effect of temporary price change on the current demand, yet it includes the changing future expectations of consumers after the temporary price change. In contrast to e, η abstracts away the changing price of used car prices under market clearing conditions of the secondary market.

Prince (2008)

- Price coefficient: Table V ("1999 Full Data Estimate") (dynamic), Table VII ("1999 Myopic Model Estimate") (myopic) and Table VII ("No Stock Model #1 Estimate") (static)

  We look at the row of "Marginal utility of Money" in the tables. Note that the "Myopic" model corresponds to the case where state variables (product holdings) are correctly specified, but setting the consumers' discount factor to zero. "No Stock Model #1" corresponds to the case where state variables are not specified (researchers assume that consumers do not possess anything). This model corresponds to the "static model", in our terminology.

- Own elasticity: Table IX (1999)

Gordon (2009)

- Price coefficient: Table 2

  Estimates of Price coefficients under "Myopic"(myopic) model and "Two Segment"(dynamic; Segment 1) are shown in Table 1 of the current article.

- Own elasticity: Table 5

  Estimates of Intel and AMD's own elasticities of demand under dynamic and static models are presented.

- Cross elasticity: Table 5

  Estimates of Intel and AMD's cross elasticities of demand under dynamic and static models are presented.

Schiraldi (2011)

Schiraldi (2011) introduced the existence of used goods in the model.

- Price coefficient: Table 4 ("Dynamic Model with Micromoments" / "Static Model")
- Own elasticity: Table 5 ("Dynamic Model with Transaction Costs and Micromoments, Short run", "Static Model"; Small Car)
- Cross elasticity: Table 6 ("Full dynamic model (Short-run)", "Static Model"; Small Car, New)

  Average price elasticities weighted by market shares are shown in the article. Note that Schiraldi (2011) also considered "long-run price elasticity" ("Dynamic Model with Transaction Costs and Micromoments, Long run" in Table 5 (own elasticity) and "Full dynamic model (Long run)" in Table 6 (cross elasticity)). Nevertheless, the elasticity is different from ours, in that Schiraldi (2011) allowed changing used car prices. For example, the author mentioned in footnote 28 that "I increase the price of a new FIAT compact in 2000, the 1-year-old FIAT compact in 2001, the 2-year-old Fiat compact in 2002, and so on" for computing long-run elasticities. Under the specification, "price changes" not only affect consumers' expectations on future new car prices but also resell values of the cars already owned by consumers. In contrast to the discussion of the current article, Schiraldi (2011) showed the case where long-run cross price elasticity is smaller than short-run cross elasticity (0.1558 (long-run, New, Small car) / 0.2564 (short-run, New, Small car)) in Table 6[33].

Gowrisankaran and Rysman (2012)

- Price coefficient: Table 1 ("Dynamic Model with Micro Moment", "Static Model")

  Mean coefficients of "Log price" are reported in Table 1 of the current article.
- Own elasticity: Figure 13

  Figure 13 shows the own price elasticity for the product "Sony DCRTRV250", which had the largest market share in the median period, and the values of the elasticities are mentioned on page 1206 (2.59 for long-run elasticity and 2.41 for short-run elasticity). Note that industrywide elasticities are reported to be 2.55 (long-run) and 1.23 (short-run), which is mentioned on page 1206 and plotted in Figure 12. Besides, the price elasticity derived from the static model are based on the information in footnote 29 of the article.

Lou et al. (2012)

- Price coefficient: Table 3 ("BLP model"(static), "GR model"(dynamic))
- Own and cross elasticity: Table 5 ("BLP model"(static), "GR model"(dynamic))

  In addition to the static and dynamic models, Lou et al. (2012) compared "BLPWP" model proposed in the article.

C.2 Storable goods

Erdem et al. (2003)

- Own elasticity:

  Own elasticities of quantity demanded with respect to Heinz's price cut are shown. The values of 3.6(long-run) and 4.9(short-run) are shown in Table 11 (short-run; "Heinz", "Fixed") and Table 12 (long-run; "Heinz", "Permanent 10% drop in mean offer price of Heinz" "Purchase quantity", "Heinz").

[33] For larger cars, he showed that long-run cross price elasticity is larger than short-run cross elasticity (0.0797 (long-run, New) / 0.0691 (short-run, New))

- Cross elasticity:

  Cross elasticities of several products are reported. The values of 0.2(short-run) and 0.75∼1(long-run) are shown in Table 11 (short-run; "Hunts / Del Monte / Store Brand", "Fixed") and Table 12 (long-run; "Hunts / Del Monte / Store Brand", "Permanent 10% drop in mean offer price of Heinz, ", "Purchase quantity", "Heinz").

In Erdem et al. (2003), "short-run price elasticity" is defined as the current demand change in response to the current price change allowing for changing future expectations of consumers. In contrast, in the current article, "short-run price elasticity" is defined as the current demand change in response to the current price change given fixed consumers' future expectations.

Sun et al. (2003)

- Price coefficient: ("Logit with No-Purchase Alternative"(static) and "Dynamic Structural Model"(dynamic))

  "Logit with No-Purchase Alternative" model corresponds to the static model in our setting. Note that "Logit" model in Table 4 does not include an outside option (buy nothing) as a choice, and it does not coincide with the "static model" in our setting.

- Own elasticity: Table 5 ("Logit with No-Purchase Alternative"(static) and "Dynamic Structural Model"(dynamic))

  Based on the results in Table 5, under "Logit with No-Purchase Alternative" (static) specification, "short-run switching elasticity" is 0.346, and its ratio to "short-run promotion effect" is 77%. Then, the short-run promotion effect is $\frac{0.346}{0.77} = 0.449$ in this case.

  Similarly, under "Dynamic Structural Model" (dynamic) specification, "short-run switching elasticity" is 0.242, and its ratio to "short-run promotion effect" is 56%. Then, the short-run promotion effect is $\frac{0.242}{0.56} = 0.432$ in this case.

  Hence, the ratio of static estimates of price elasticity to short-run price elasticity based on the dynamic model is $\frac{0.449}{0.432} = 1.039$.

Hendel and Nevo (2006)

Hendel and Nevo (2006) allowed household level heterogeneity, but they did not introduce persistent unobserved consumer heterogeneity.

- Price coefficient

  On page 1665, the authors mentioned that "The price coefficient estimated in the static model is roughly 15 percent higher than that estimated in the first stage of the dynamic model".

- Own elasticity

  The information of "overestimate own-price elasticities by 30 percent" is based on the description of the results by the authors in the abstract of the article. The result is based on Table VIII.

- Cross elasticity

  In the abstract of this article, the authors mention that "underestimate cross-price elasticities by up to a factor of 5". The result is based on Table VIII.

Hendel and Nevo (2013)

Hendel and Nevo (2013) considered the model and estimation procedures where utility parameters are not explicitly estimated. The estimation procedure of price elasticity is largely different from the articles applying standard discrete choice models.

- Own elasticity

  The authors mentioned on page 2741 of the article that "The own-price elasticity implied by the estimates from the dynamic model, evaluated at the quantity-weighted price, is 2.16 for Coke and 2.78 for Pepsi. The elasticities implied by the static estimates are 2.46 and 2.94, respectively. As expected, neglecting dynamics in the estimation overstates own-price elasticities".

Wang (2015)

- Own elasticity

  The author mentioned in the abstract and introduction of the article that "static analyses overestimate the long-run own-price elasticity of regular soda by 60.8%".

Perrone (2017)

Perrone (2017) considered the model where only one product exists. The estimation procedure of price elasticity is largely different from the articles applying standard discrete choice models.

- Own elasticity: Table 3 ($\epsilon^{sr0}/\epsilon^{lr0}$)

Li (2021)

Li (2021) theoretically investigated the biases in price elasticities under the existence of consumer stockpiling. Nevertheless, he compared "elasticity of demand for immediate consumption" and "elasticity of demand for sum of immediate consumption and stockpiling". Even though the latter elasticity corresponds to either "short-run elasticity" or "long-run elasticity" discussed in the current article, the former elasticity is different from neither "short-run elasticity" nor "long-run elasticity" in our terminology.

C.3 Goods with switching costs

Ho (2015)

- Price Coefficient: Table 2 ("Dynamic-1" (dynamic) and "Myopia" (static))
- Own Elasticity

  Inequality on the absolute values of the three elasticities (Static<Short<Long) is obtained by comparing the results in Table 4 and Table 6 of the article.

- Cross Elasticity

  Inequality on the absolute values of the three elasticities (Static<Short<Long) is obtained by comparing the results in Table 4 and Table 6 of the article.

Shcherbakov (2016)

Shcherbakov (2016) did not introduce persistent consumer heterogeneity in the baseline model. The results in Table 1 are based on the specification. Note that he also showed the results with persistent consumer heterogeneity as robustness checks.

- Price Coefficient: Table 3 ("Static" and "Dynamic(1)")
- Own Elasticity: Table 4 ("Dynamic short run" and "Dynamic long run")

  Price elasticities of cable and satellite are shown. Note that Shcherbakov (2016) also showed static estimates of own price elasticities for cable and satellite. Nevertheless, since the results (inequalities with other elasticities) are largely different between the two alternatives, I did not show the result in Table 1 of the current article.

Yeo and Miller (2018)

Yeo and Miller (2018) introduced random coefficients for static models, but they did not introduce them for a dynamic model.

- Price Coefficient: Table 6 ((1); dynamic) and Table 7 ((1); static)
- Own Elasticity: Table 9 Panel A ("Myopic"(myopic), "Dyn-1"(short-run), "Dyn-3"(long-run), $t = 2006$)

  Median of the own elasticities are shown in Table 1 of the current article.
- Cross Elasticity: Table 9 Panel C ("Myopic"(myopic), "Forward-looking"(long-run), $t = 2006$)

  The means of the cross elasticities are shown in Table 1 of the current article.

Note that static estimates of own and cross elasticities are shown in Table 9 of the article. Nevertheless, since (inequalities with other elasticities) are largely different between the $t = 2006$ and $t \geq 2007$, I did not show the result in Table 1 of the current article.

C.4 Others

Hartmann (2006)

Even though Hartmann (2006) focused on consumption capital, the basic structure of the model is close to the one for durable goods.

- Price Coefficient: Table 4 ("Static logit Random coefficients" and "Dynamic logit Random coefficients")
- Own Elasticity:

  The value of 2.7789 (long-run elasticity) is mentioned on page 345. The value of 3.0550 (short-run elasticity) appears in Table 5 of the article.

Osborne (2011)

Osborne (2011) compared the results under the full model (with learning and switching costs) and the partial models (either with learning or with switching costs). This type of comparison is not intuitive in our model.

Seiler (2013)

Seiler (2013) compared the results under the full model (with inventory and search cost) and the partial model (with inventory but without search cost). This type of comparison is not intuitive in our model.

Pires (2016)

Pires (2016) compared four types of models: model (with/without) inventory and (with/without) consideration set. Parameter estimates are shown in Table 5 (model with inventory and consideration set) and Table 7 (other models).

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