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Abstract

This paper examines the possibility of two types of rational asset price bubbles in a small open economy. Our model is a simple representative-agent model similar to the Lucas "tree" model except that the economy may be partially or completely open. There are goods, stock asset, pure bubbly asset, and loan markets, and we consider all possible cases depending on whether each of the markets is open or closed. We show that capital inflows give rise to asset bubbles. Moreover, two types of bubbles arise simultaneously in the economy if the goods market and loan markets are open.

Keywords: stock market bubbles, pure bubbles, small open economy.

JEL classification numbers: E21, E44

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1 Introduction

It is widely held that asset bubbles are often fueled, or at least accompanied, by capital inflows. Examples of such episodes include the bubbles in emerging market economies in the 1980s and 1990s, the Japanese asset bubble in the late 1980s, and the US housing bubble in the 2000s (see, e.g., Caballero and Krishnamurthy, 2006; Hattori et al., 2009; Calvo, 2011).

Perhaps the most standard asset pricing model in macroeconomics is the Lucas (1978) "tree" model as described in Sargent (1987), which is a representative agent economy where bubbles are typically ruled out. In fact, in deterministic representative agent economies, bubbles are simply impossible (Kamihigashi, 2001), and even in the stochastic case, bubbles are impossible except under rather pathological specifications (Kamihigashi, 1998; Montrucchio and Privileggi, 2001).

Regarding overlapping generations models, it has been known since Samuelson (1958) that fiat money can be valued in these models, and it has been common to interpret fiat money as a bubble since Tirole (1985). On the other hand, it has been known since Wilson (1981) that even in general overlapping generations models, bubbles, including fiat money, are impossible if the value of aggregate wealth is finite. This is also true for general overlapping generations models with incomplete markets (Santos and Woodford, 1997). For example, the value of aggregate wealth is finite if there is a traded stock whose dividend stream is larger than a fixed fraction of the aggregate endowment stream. Therefore, if one is to analyze a bubble on a "Lucas tree," which pays the per capital endowment of the economy as a dividend in each period, then some deviation from standard assumptions is necessary, whether the underlying economy has overlapping generations or not.

A common approach in the current literature on bubbles is to introduce an additional financial friction. This approach is used to study bubbles on intrinsically useless assets in overlapping generations models (Marin and Ventura, 2011; Farhi and Tirole, 2011; Clan-Chamosset-Yvrard and Seegmuller, 2012) as well as models with infinite lived agents (Hirano and Yanagawa, 2011; Aoki and Nikolov, 2015; Kunieda and Shibata, 2016). There are also some known cases in which bubbles on intrinsically useful assets are possible. For example, Kocherlakota (2008) shows that such bubbles are possible under an exogenous solvency constraint in a model with a finite number of infinite lived agents. Olivier (2000) considers bubbles on newly created assets (or firms) in an overlapping generations model. In variants of the Lucas tree mode, Kamihigashi (2008a, 2008b) shows that bubbles on Lucas trees are possible if utility directly depends on wealth.[1]

This paper complements the existing literature by taking an entirely different approach to bubbles. We consider two types of bubbles simultaneously. We introduce intrinsically useless assets into the Lucas tree model. In this study we call the bubble on the intrinsically useless asset *pure bubble* and bubble on Lucas tree *stock market bubble*. In the closed economy both bubbles are ruled out, as mentioned above. Then we assume that it is a small open economy, and consider the possibility of two types of bubbles. To our knowledge, this simple extension of the Lucas tree model with pure bubbly assets has never been analyzed in the context of bubbles.

This paper is not the first to study bubbles in an open economy. In fact, there are

[1]See also Miao and Wang (2018). For other studies on bubbles, we refer the reader to the surveys by Brunnermeier (2008) and Iraola and Santos (2008).

well-known studies on bubbles and capital flows in open economies that use overlapping generations models (Caballero and Krishnamurthy, 2006; Ventura, 2011; Clan-Chamosset-Yvrard and Kamihigashi, 2017; Ikeda and Phan, 2019;). However, these studies are not intended to explain why bubbles (or valued fiat money) are more likely to arise in open economies, since bubbles are possible even in closed economies with overlapping generations. In addition, these studies except Clan-Chamosset-Yvrard and Kamihigashi (2017) assume that bubble assets are never traded internationally, thus ruling out fully open economies. By contrast, this paper offers a simple explanation of why two types of bubbles assets easily arise in fully or partially open economies.

Like the Lucas tree model as described in Sargent (1987), our model is an exchange economy where there are homogenous goods and homogeneous stock assets, or "trees." The additional features that we introduce is that agents are allowed to buy pure bubbly asset in a competitive asset market and borrow and lend in a competitive loan market. Thus there are four markets: goods, stock assets, pure bubbly assets, and loans. We first show that both bubbles are impossible if the goods market is closed or if two types of assets and loan markets are closed. Therefore, for bubbles to ever arise, at least the goods market must be open, and at least one of the assets and loan markets must be open. These necessary conditions turn out to be also sufficient: we establish that both bubbles are possible if the goods market and one of the assets and loan markets are open. For example, stock market bubbles are possible if the goods and stock asset markets are open; this result can be interpreted as saying that foreign investors can drive up domestic asset prices. A less intuitive result is that two types of bubbles are possible simultaneously even if stock and pure bubbly asset markets are closed, provided that the goods and loan markets are open; i.e., even if foreign investors have no access to the domestic asset markets, bubbles can be fueled by capital inflows through external debt.

As indicated above, our model is completely standard except that it is open; thus the driving force behind bubbles in our model is the openness of the economy. In other words, capital inflows give rise to otherwise nonexistent bubbles. Furthermore, in contrast to most studies on bubbles, stock market and pure bubbles in our model are explosive; i.e., they grow unboundedly. Most studies on bubbles focus on the steady state of bubbles. On one hand, this explosive feature may not be consistent with general equilibrium unless the implicit world economy grows fast enough. On the other hand, in the real world, bubbles become a serious issue only when they appear to be explosive; thus the explosive feature may capture one of the defining characteristics of bubbles in reality. In any case, our results suggest that explosive bubbles easily arise in a country or region that is perceived as "small." It seems likely that our model is the simplest optimizing framework that can be used to explain explosive bubbles. Recently, Kikuchi and Thepmongkol (2020) investigate explosive bubbles on capital assets in the small open economy populated by overlapping generations. In contrast to them, we employ infinitely-lived agents model and consider simultaneously stock and pure bubbly asset on an explosive path.

The rest of the paper is organized as follows. In Section 2, we lay out the general framework common to all cases considered in this paper. We also develop some preliminary results and formally define bubbles. In Section 3, we show that bubbles are impossible if the goods market is closed or if both the asset and loan markets are closed. In Section 4, we show that bubbles are possible if the goods market and one of the assets and loan markets are open. In Section 5 we conclude the paper. All omitted proofs appear in the appendix.

2 The General Framework

Consider an economy with homogenous consumption goods and many homogeneous agents who derive utility from consumption. There are stock and intrinsically useless assets in the economy. We call the bubble on the intrinsically useless asset *pure bubble* and the bubble on the stock *stock market bubble*. Agents can borrow and lend in a competitive loan market. The economy can be completely closed, partially open, or completely open. This section lays out the general framework that is common to all possible cases, which are to be analyzed in subsequent sections.

Throughout the paper, we follow the convention that an equality or inequality dependent on t is understood to hold for all $t \in \mathbb{Z}_+$ unless otherwise indicated.

2.1 Agents' Maximization Problem

Each agent faces the following maximization problem:

$$\max_{\{c_t, s_t, m_t, d_t, w_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{2.1}$$

$$\text{s.t.} \quad c_t + p_t s_t + p_t^m m_t - d_t = w_t, \tag{2.2}$$

$$w_{t+1} = (p_{t+1} + \delta) s_t + p_{t+1}^m m_t - R_{t+1} d_t, \tag{2.3}$$

$$c_t \geq 0, \tag{2.4}$$

$$s_t, m_t, \geq 0 \tag{2.5}$$

$$w_{t+1} \geq 0, \tag{2.6}$$

$$w_0 = (p_0 + \delta) s_{-1} + p_0^m m_{-1} + R_0 d_{-1} > 0, \tag{2.7}$$

where $\beta \in (0, 1)$ is the discount factor, c_t is consumption in period t, w_t is wealth at the beginning of period t, p_t is the price of the stock in period t, s_t is shares of stock held at the end of period t, p_t^m is the price of the intrinsically useless asset in period t, m_t is the level of the intrinsically useless asset held at the end of period t, d_t is debt, $R_{t+1} > 0$ is the gross interest rate between periods t and $t + 1$, and $\delta > 0$ is the dividend per share, which is assumed to be constant over time. (2.5) denotes short-sales constraints. (2.6) is a non-negative constraint on wealth. With the short sale constraints, we interpret (2.6) as a borrowing constraint on d_t

$$(p_{t+1} + \delta) s_t + p_{t+1}^m m_t \geq R_{t+1} d_t. \tag{2.8}$$

Remark 2.1. We also consider cases in which in addition to the above constraints, the following no-Ponzi-game condition is required to be fulfilled:

$$\lim_{T \uparrow \infty} \prod_{t=1}^{T} \frac{1}{R_t} d_t \leq 0. \tag{2.9}$$

We assume that the utility function $u : \mathbb{R}_+ \to [-\infty, \infty)$ is continuously differentiable on $\mathbb{R}_{++}$, continuous, concave, strictly increasing, satisfying the Inada condition

$$\lim_{c \downarrow 0} u'(c) = \infty. \tag{2.10}$$

We say that a set of sequences $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is a *feasible path* (*from* w_0) if it satisfies (2.2)-(2.5). An *optimal path* (*from* w_0) is a feasible path that solves the maximization problem (2.1)-(2.7).[2]

2.2 Market Clearing Conditions

We normalize both the population of agents and the supply of assets to unity. If the economy is completely closed, the following four market clearing conditions are required to hold in equilibrium:

$$c_t = \delta, \tag{2.11}$$
$$s_t = 1, \tag{2.12}$$
$$m_t = 1 \tag{2.13}$$
$$d_t = 0. \tag{2.14}$$

(2.11) is the goods market clearing condition. As in the Lucas tree model, we assume for simplicity that the dividends on the stocks constitute the entire supply of goods. (2.12) is the stock asset market clearing condition. (2.13) is the pure bubbly asset market clearing condition. (2.14) is the loan market clearing condition, which states that there is no borrowing and lending in equilibrium since agents are homogeneous.

If the economy is partially open, then one, two, or three of the market clearing conditions above are required to hold. For example, if the goods market is closed and the asset and loan markets are open, then only the goods market clearing condition (2.11) is required to hold. If the economy is completely open, none of the market clearing conditions is required to hold. In this case, the model reduces to an optimization problem without equilibrium conditions. Unless otherwise indicated, we assume the following in (2.7) for all cases considered in this paper:

$$s_{-1} = 1, \quad m_{-1} = 1, \quad d_{-1} = 0. \tag{2.15}$$
$$w_0 = p_0 + \delta + p_0^m. \tag{2.16}$$

Formally, the closed economy consists of agents' maximization problem (2.1)–(2.7) and the four market clearing conditions (2.11)–(2.14). A partially open economy consists of agents' maximization problem (2.1)–(2.7) and a subset of the market clearing conditions (2.11)–(2.14). An *equilibrium* of a closed or partially open economy is defined as a set of sequences $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ such that given strictly positive sequences $\{p_t, p_t^m, R_{t+1}\}$, the path $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is optimal, and the corresponding subset of the market clearing conditions (2.11)–(2.14) hold. The completely open economy consists only of agents' maximization problem (2.1)–(2.7). An *equilibrium* of the completely open economy is defined as

[2] Strictly speaking, since we allow u to be unbounded, the maximand in (2.1) need not be well defined for all feasible paths. Thus, to be perfectly precise, we use weak maximality as our optimality criterion

a set of sequences $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ such that given strictly positive sequences $\{p_t, p_t^m, R_{t+1}\}$, the path $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is optimal.

We analyze possibility of bubbles in all possible cases, where one, two, three, or all of the market clearing conditions are required to hold in equilibrium.

For the rest of the paper we focus on interior equilibria where the short-sales constraints (2.5) are never binding.

2.3 Properties of Optimal paths

Here we derive some useful properties of optimal paths without considering market clearing conditions. The following proposition summarizes some of the most basic properties.

Proposition 2.1. *A feasible path $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is optimal if and only if*

$$c_t, w_{t+1} > 0, \tag{2.17}$$
$$p_t s_t + p_t^m m_t - d_t > 0, \tag{2.18}$$
$$u'(c_t) = \beta u'(c_{t+1}) R_{t+1}, \tag{2.19}$$
$$u'(c_t) p_t = \beta u'(c_{t+1})(p_{t+1} + \delta), \tag{2.20}$$
$$u'(c_t) p_t^m = \beta u'(c_{t+1}) p_{t+1}^m, \tag{2.21}$$
$$\lim_{t\uparrow\infty} \beta^t u'(c_t)(p_t s_t + p_t^m m_t - d_t) = 0. \tag{2.22}$$

Proof. See Appendix A. □

With $s_t > 0$ and $m_t > 0$, inequalities (2.17) and (2.3) mean that none of the inequality constraints is binding for an optimal path. Equations (2.19), (2.20)), and (2.21) are the Euler equations associated with d_t, s_t, and m_t, respectively. Equation (2.22) is a transversality condition.

From Euler equations (2.19), (2.20), and (2.21) we obtain the following no-arbitrage condition

$$R_{t+1} = \frac{p_{t+1} + \delta}{p_t} = \frac{p_{t+1}^m}{p_t^m}, \tag{2.23}$$

i.e., lending at the ongoing interest rate and investing in assets are perfect substitutes from agents' perspective. (2.3) and (2.23) yields

$$w_{t+1} = R_{t+1}(p_t s_t + p_t^m m_t - d_t), \tag{2.24}$$

and then the budget constraint (2.2) and (2.3) reduce to

$$c_t + \frac{w_{t+1}}{R_{t+1}} = w_t. \tag{2.25}$$

To state our next result, we define $\pi_t^0 = 1$, and

$$\pi_t^j = \prod_{i=1}^{j} \frac{1}{R_{t+i}} \tag{2.26}$$

for $j \in \mathbb{N}$. Then, by (2.19)

$$\pi_t^j = \beta^j \frac{u'(c_{t+j})}{u'(c_t)}. \tag{2.27}$$

Note that

$$\pi_t^i \pi_{t+i}^j = \pi_t^{i+j}, \quad \pi_{t+1}^j = R_{t+1} \pi_t^{j+1}. \tag{2.28}$$

Lemma 2.1. *Any optimal path* $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ *satisfies*

$$\sum_{t=0}^{\infty} \pi_0^t c_t = w_0. \tag{2.29}$$

Proof. See Appendix B. □

Recall that in this model the only source of income is initial wealth w_0. Lemma 2.1 says that initial wealth must be exhausted on consumption; i.e., the present discounted value of the optimal consumption stream is equal to initial wealth.

2.4 Bubbles

We now define bubbles in a standard way. Note from the Euler equation (2.20) and (2.27)that

$$p_t = \beta \frac{u'(c_{t+1})}{u'(c_t)} (p_{t+1} + \delta) \tag{2.30}$$

$$= \pi_t^1 \delta + \pi_t^1 p_{t+1}. \tag{2.31}$$

By successive forward substitution and (2.28)

$$p_t = \pi_t^1 \delta + \pi_t^2 \delta + \pi_t^2 p_{t+2} \tag{2.32}$$

$$= \pi_t^1 \delta + \pi_t^2 \delta + \pi_t^3 \delta + \pi_t^3 p_{t+3} \tag{2.33}$$

$$\vdots \tag{2.34}$$

$$= \sum_{j=1}^{J} \pi_t^j \delta + \pi_t^J p_{t+J} \tag{2.35}$$

$$= \sum_{j=1}^{\infty} \pi_t^j \delta + \lim_{j \uparrow \infty} \pi_t^J p_{t+J}. \tag{2.36}$$

The above limit exists since the sum in (2.35) is increasing in J and thus the second term in (2.35) is decreasing in J. Define

$$p_t^* = \sum_{j=1}^{\infty} \pi_t^j \delta = \sum_{j=1}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} \delta, \tag{2.37}$$

$$b_t = \lim_{J \uparrow \infty} \pi_t^J p_{t+J} = \lim_{J \uparrow \infty} \beta^J \frac{u'(c_{t+J})}{u'(c_t)} p_{t+J}. \tag{2.38}$$

Note that p_t^* is the present discounted value of the dividend stream. We call p_t^* the *fundamental value* of an asset, and b_t the *stock market bubble*. Now p_t is decomposed as

$$p_t = p_t^* + b_t. \tag{2.39}$$

Note from that (2.28) and (2.38) that

$$b_{t+1} = \lim_{J\uparrow\infty} \pi_{t+1}^J p_{t+1+J} \tag{2.40}$$

$$= R_{t+1} \lim_{J\uparrow\infty} \pi_t^{J+1} p_{t+1+J} = R_{t+1} b_t. \tag{2.41}$$

It follows that

$$b_0 > 0 \iff \forall t \in \mathbb{Z}_+, b_t > 0. \tag{2.42}$$

A stock market bubble can start only in period 0, and once it starts, it remains positive in all periods.

From (2.23) pure bubbles satisfy

$$p_{t+1}^m = R_{t+1} p_t^m \tag{2.43}$$

Thus, we have

$$p_0^m > 0 \iff \forall t \in \mathbb{Z}_+, p_t^m > 0. \tag{2.44}$$

3 Partially Open Economies without Bubbles

In this section, we take an equilibrium $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ of a closed or partially open economy as given; we specify the required set of market clearing conditions in the statements of our results.

Proposition 3.1. *If the goods market is closed, i.e., if* (2.11) *holds, then* $b_0 = p_0^m = 0$.

Proof. Recall from (2.16) and (2.29) that

$$\sum_{t=0}^{\infty} \pi_0^t c_t = w_0 = p_0 + \delta + p_0^m. \tag{3.1}$$

This together with the goods market clearing condition (2.11) yields

$$p_0 + p_0^m = \sum_{t=0}^{\infty} \pi_0^t \delta - \delta = \sum_{t=1}^{\infty} \pi_0^t \delta = p_0^*, \tag{3.2}$$

i.e., $b_0 + p_0^m = 0$. Thus, we have $b_0 = p_0^m = 0$ □

This result shows that as long as the goods market is closed, stock market and pure bubbles are impossible regardless of the openness of the remaining three markets. To understand this clearly, note from (2.16) and Lemma 2.1 that if there are two types bubbles, agents must consume more than $p_0^* + \delta$ in equilibrium. But since the goods market is closed, their consumption is fixed in equilibrium, and its present discounted value is exactly equal to $p_0^* + \delta$. Therefore, there is no room for both bubbles as long as the goods market is closed.

Proposition 3.2. *If the assets and loan markets are closed, i.e., if* (2.12)*,* (2.13)*, and* (2.14) *hold, then* $b_0 = 0$ *and* p_0^m*.*

Proof. (2.2) and (2.3) yields

$$c_t + p_s s_t - d_t = (p_t + \delta) + p_{t-1}^m - R_t d_{t-1}. \tag{3.3}$$

By using (2.12), (2.13), and (2.14), we have $c_t = \delta$. Then, it is easily to see $b_0 = p_0^m = 0$, as in the proof of Proposition 3.1. □

This result shows that as long as the assets and loan markets are closed, bubbles are impossible regardless of the openness of the goods market. The easiest way to understand this result is to notice that even if the goods market is open, it is totally useless unless another market is open, since otherwise there is no way for agents to increase consumption. Hence, as long as the asset s and loan markets are closed, the goods market is effectively closed, and there cannot be any bubbles for the reason described above.

4 Open Economies with Bubbles

Propositions 3.1 and 3.2 show that for bubbles to arise, the goods market and at least one of the assets and loan markets must be open. In this section, we consider partially or fully open economies in which this minimum requirement is met.

Since the returns on assets and loans are identical by (2.23), and since at least one of the assets and loan markets is open, the common rate of return must be equal to the rate of return prevailing in the world market, denoted R, which we assume is equal to $1/\beta$:

$$R_{t+1} = R = 1/\beta. \tag{4.1}$$

This is a common assumption for models of small open economies, implying that there is a steady state. Condition (4.1) makes it possible to obtain a closed form solution, but it is not crucial to our basic results on the existence of bubbles.

An *equilibrium* $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ of a partially or fully open economy is defined here as in the previous section with R_{t+1} given by (4.1).

Note from (2.19), (2.27), and (4.1) that $\pi_t^j = \beta^j$. Thus by (2.37), p_t^* is constant:

$$p_t^* = \frac{\beta}{1-\beta}\delta \equiv p^*. \tag{4.2}$$

Lemma 4.1. *Any* equilibrium $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ *satisfies the following:*

$$p_0 = p^* + b_0, \tag{4.3}$$

$$p_{t+1} = Rp_t - \delta, \tag{4.4}$$

$$p_t = p^* + R^t b_0, \tag{4.5}$$

$$p_{t+1}^m = Rp_t^m, \tag{4.6}$$

$$c_t = \delta + (1-\beta)(b_0 + p_0^m), \tag{4.7}$$

$$w_t = \frac{\delta}{1-\beta} + b_0 + p_0^m. \tag{4.8}$$

Proof. See Appendix C. □

4.1 Explosive Bubbles and Capital Inflow

Proposition 4.1. *Suppose that the pure bubbly asset and loan market are closed and the goods and asset markets are open, i.e., of the four market clearing conditions* (2.11)–(2.14)*, only* (2.13) *($m_t = 1$) and* (2.14) *($d_t = 0$) are required to hold. Then for any $b_0 \geq 0$, there exists a unique equilibrium $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ satisfying* (4.3)*. This equilibrium satisfies* (4.4)–(4.8)*, and*

$$s_t = \frac{p^* + \beta b_0}{p^* + R^t b_0} - \frac{(R^t - \beta)p_0^m}{p^* + R^t b_0}, \tag{4.9}$$

$$p_0^m = 0. \tag{4.10}$$

Proof. See Appendix D. □

In the setting of Proposition 4.1, by (4.9) and (4.10), if $b_0 > 0$

$$s_t < 1, \quad \lim_{t \uparrow \infty} s_t = 0. \tag{4.11}$$

Hence stock asset ownership gradually shifts from domestic agents to foreign investors. In the limit, domestic agents hold no asset. If $b_0 = 0$, (4.9) implies that $s_t = 1$ and the equilibrium can attain the same allocation as a closed economy even if stock asset market is opened.

Proposition 4.2. *Suppose that the stock asset and loan market are closed and the goods and pure bubbly asset markets are open, i.e., of the four market clearing conditions* (2.11)–(2.14)*, only* (2.12) *($s_t = 1$) and* (2.14) *($d_t = 0$) are required to hold. Then for any $p_0^m \geq 0$, there exists a unique equilibrium $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ satisfying* (4.3)–(4.8)*,*

$$m_t = \frac{\beta p_0^m}{R^t p_0^m} - \frac{(R^t - \beta)b_0}{R^t p_0^m} \tag{4.12}$$

$$b_0 = 0. \tag{4.13}$$

Proof. See Appendix E. □

As in the above argument, if $p_0^m > 0$

$$m_t < 1, \quad \lim_{t \uparrow \infty} m_t = 0. \tag{4.14}$$

Hence pure bubbly asset ownership gradually shifts from domestic agents to foreign investors. In the limit, domestic agents hold no pure bubbly assets.

We define aggregate bubbles as

$$B_t = b_t + p_t^m. \tag{4.15}$$

It satisfies

$$B_{t+1} = R^t B_0, \tag{4.16}$$

where $B_0 = b_0 + p_0^m$.

The following proposition shows that both stock market and pure bubbles arise in the economy.

Proposition 4.3. *Suppose that the stock and pure bubbly assets market are closed and the goods and loan are open, i.e., of the four market clearing conditions* (2.11)–(2.14)*, only* (2.12) *(*$s_t = 1$*) and* (2.13) *(*$m_t = 1$*) are required to hold. Then for any* $B_0 \geq 0$*, there exists a unique equilibrium* $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ *satisfying* (4.3)–(4.8)*,*

$$d_t = (R^t - \beta)B_0. \tag{4.17}$$

Proof. See Appendix F. □

Proposition 4.3 shows that there is a continuum of equilibria indexed by the initial size of the aggregate bubble $B_0 \geq 0$. Given B_t, the values of b_t and p^m are indeterminate. Note that wealth, consumption, and debt all depend positively on B_0. Therefore, bubbles are welfare improving in this model.

Proposition 4.4. *Suppose that the only loan market is closed and the goods, i.e.,* (2.14) *(*$d_t = 0$*) is required to hold. Let* B_0 *be given. Then there exists an equilibrium* $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ *satisfying* (4.3)–(4.8)*. Furthermore, for any* $\{\overline{s}_t\}$ *in* $\mathbb{R}$*, there exists a unique equilibrium* $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ *satisfying* (4.3) *such that* $\{m_t\} = \{\overline{m}_t\}$*, where the sequence* m_t *is determined by*

$$p_t s_t + p_t^m m_t = p^* + \beta B_0, \tag{4.18}$$

where given $B_0 = b_0 + p_0^m \geq 0$ *the values of* b_0 *and* p_0^m *are indeterminate. Likewise, for any sequence* $\{\overline{m}_t\}$ *in* $\mathbb{R}$*, there exists a unique equilibrium* $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ *satisfying*(4.3) *such that* $\{s_t\} = \{\overline{s}_t\}$*, where the sequence* $\{s_t\}$ *is determined by* (4.18)*.*

Proof. See Appendix G. □

Proposition 4.4 shows that there is a continuum of equilibria in the partially open economy in where the stock and pure bubbly assets market are open. The characteristic of the equilibria are identical to those of the partially open economies with bubbles studied above, except that $\{s_t\}$ or $\{m_t\}$ are indeterminate even when the initial size of the bubble B_0 is given.

4.2 Capital Flight and Endogenous Bubble Crashes.

Consider the partially open economy in which the loan market and pure bubbly asset market (or stock asset market) are closed; in this case, it does not matter whether the no-Ponzi-game condition is imposed or not. We suppose that the economy initially follows an equilibrium with a positive stock market bubble (or pure bubble). In period $t > 0$, however, all foreign investors suddenly withdraw from the domestic market, so that the asset market clearing condition is required to be satisfied:

$$s_t = 1 \quad or \quad m_t = 1. \tag{4.19}$$

Then, by Proposition 3.2, stock market bubbles (or pure bubbles) are never valued, that is, bubbles collapse. This can be due to a domestic policy change. For example, the government might suddenly restrict asset ownership to domestic agents. It can also be due to a dramatic change in the global financial landscape, which might in turn be caused by a global financial crisis. For example, foreign investors may be in sudden need of cash, and they need to cash in their asset holdings.

5 Conclusion

This study constructed a model of two types of rational bubbles in a small open economy. We showed that with capital inflow stock market and pure bubbles are possible. The driving force behind bubbles in our model is the openness of the economy. In addition, both bubbles are explosive.

The model has some limitations. We employed an endowment economy model and ignored production. Bubbles enhance production and economic growth. Thus, it is important to examine how two types of bubble affect production and economic growth. We considered the deterministic bubbles. It is important extension to consider the effects of a collapse of bubbles and how interaction between two types of stochastic bubble affect macroeconomy. We leave these issues as future work.

Appendix

A Proof of Proposition 2.1

If: Suppose a feasible path $\{c_t^*, s_t^*, m_t^*, d_t^*, w_{t+1}^*\}$ satisfies (2.17)–(2.22). From (2.19)–(2.21), we have

$$R_{t+1} = \frac{p_{t+1} + \delta}{p_t} = \frac{p_{t+1}^m}{p_t^m}. \tag{A.1}$$

Let $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ be an arbitrary feasible path. Then, (2.2), (2.3), and (A.1) imply that for any feasible path the followings hold

$$p_t s_t + p_t^m m_t - d_t = \frac{w_{t+1}}{R_{t+1}}, \tag{A.2}$$

$$c_t = w_t - (p_t s_t + p_t^m m_t - d_t) = w_t - \frac{w_{t+1}}{R_{t+1}}. \tag{A.3}$$

By concavity, we have

$$\begin{aligned}
&\sum_{t=0}^{T} \beta^t u(c_t) - \sum_{t=0}^{T} \beta^t u(c_t^*) && \text{(A.4)}\\
&\leq \sum_{t=0}^{T} \beta^t u'(c_t^*)(c_t - c_t^*) && \text{(A.5)}\\
&= \sum_{t=0}^{T} \beta^t u'(c_t^*) \left[w_t - w_t^* - (w_{t+1} - w_{t+1}^*)/R_{t+1}\right] && \text{(A.6)}\\
&= \beta^T u'(c_T^*)(w_{T+1}^* - w_{T+1})/R_{t+1} && \text{(A.7)}\\
&\leq \beta^T u'(c_T^*) w_{T+1}^*/R_{t+1} && \text{(A.8)}\\
&= \beta^T u'(c_T^*)(p_t s_t^* + p_t^m m_t^* - d_t^*) \to 0, && \text{(A.9)}
\end{aligned}$$

where (A.6) holds by (A.3). (A.7) holds by the Euler equation (2.19). The inequality in (A.8) holds by $w_{t+1} > 0$ in (2.18). The equality in (A.9) holds by (A.2). The convergence holds by the transversality condition (2.22). The optimality of $\{c_t^*, s_t^*, m_t^*, d_t^*, w_{t+1}^*\}$ now follows.

Only if: Let an optimal path be given. We verify (2.17)-(2.22). We start by claiming $c_t > 0$ for all $t \in \mathbb{Z}_+$. To see this, first note that since $R_{t+1} > 0$ for all $t \in \mathbb{Z}_+$, it is feasible to choose $c_t > 0$ for all $t \in \mathbb{Z}_+$. Thus, if u is unbounded below, then it is never optimal to choose $c_t = 0$. If u is bounded below, then Inada condition (2.10) implies that $c_t > 0$ for a given $t \in \mathbb{N}$. whenever $c_{t-1} > 0$ or $c_{t+1} > 0$, since $R_t, R_{t+1} > 0$. This in turn implies $c_t > 0$ for all $t \in \mathbb{Z}_+$.

Next we show $w_{t+1} > 0$ for all $t \in \mathbb{Z}_+$. To see this, suppose $w_t = 0$ for some $t \in \mathbb{N}$. We have $0 < c_t = -p_t s_t - p_t^m m_t + d_t$. By choosing $\tilde{s}_t = 2s_t$, $\tilde{m}_t = 2m_t$, and $\tilde{d}_t = 2d_t$ instead of s_t, m_t, and d_t, one can obtain $\tilde{c}_t = 2c_t > c_t$, $\tilde{w}_{t+1} = 2w_{t+1} \geq w_{t+1}$, and $\tilde{c}_{t+1} \geq c_{t+1}$ instead of c_t, w_{t+1}, and c_{t+1} without affecting the rest of the path. This contradicts the optimality of the original path, establishing $w_{t+1} > 0$ for all $t \in \mathbb{Z}_+$.

To see (2.18), note that as in the above argument, if $p_t s_t + p_t^m m_t - d_t \leq 0$, one could double w_{t+1} and increase c_{t+1} by doubling s_t, m_t, and d_t without affecting the rest of the path. Thus (2.18) follows. Since no inequality constraint is binding, the Euler equation (2.19), (2.20) and (2.21) hold. The transversality condition (2.22) follows from Kamihigashi (2001).

B Proof of Lemma 2.1

We start by establishing two simple lemmas:

Lemma A1. *Any feasible path $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ satsifies*

$$\sum_{t=0}^{\infty} \pi_0^t c_t \leq w_0. \tag{B.1}$$

Proof. Multiplying (2.25) by π_0^t and summing over from $t = 0$ to $t = T$, we get

$$w_0 = \sum_{t=0}^{T} \pi_0^t c_t + \pi_0^T w_{T+1}/R_{T+1} \geq \sum_{t=0}^{T} \pi_0^t c_t. \tag{B.2}$$

Letting $T \uparrow \infty$ yields (B.1). □

Lemma A2. *For any nonnegative sequence $\{c_t\}$ satisfying* (2.29)*, there exist sequences $\{s_t\}, \{m_t\}, \{d_t\}$, and $\{w_{t+1}\}$ such that the path $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is feasible.*

Proof. We construct such sequences. Let

$$s_t = \frac{w_{t+1}}{2(p_{t+1} + \delta)}, \quad m_t = \frac{w_{t+1}}{2p_{t+1}^m}, \quad d_t = 0, \quad w_t = \sum_{i=0}^{\infty} \pi_t^i c_{t+i} \geq 0. \tag{B.3}$$

Clearly, (2.3)–(2.7) hold. To verify (2.2), note that (B.3) that

$$
\begin{aligned}
c_t + p_t s_t + p^m m_t - d_t &= c_t + \frac{w_{t+1}}{2(p_{t+1}+\delta)/p_t} + \frac{w_{t+1}}{2p^m_{t+1}/p^m_t} && \text{(B.4)}\\
&= c_t + \frac{w_{t+1}}{R_{t+1}} && \text{(B.5)}\\
&= c_t + \sum_{i=0}^{\infty} \pi^i_{t+1} c_{t+1+i}/R_{t+1} && \text{(B.6)}\\
&= c_t + \sum_{i=0}^{\infty} \pi^{i+1}_t c_{t+1+i} && \text{(B.7)}\\
&= \sum_{j=0}^{\infty} \pi^j_t c_{t+j} && \text{(B.8)}
\end{aligned}
$$

where (B.5) holds by (2.23). (B.7) holds by (2.28). Now (2.2) follows, so the path $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is feasible. □

To complete the proof of Lemma 2.1, suppose there is an optimal path $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ that does not satisfy (2.29). Then by Lemma A1, we must have

$$\sigma \equiv \sum_{t=0}^{\infty} \pi^t_0 c_t < w_0. \tag{B.9}$$

Let

$$\tilde{c}_t = c_t w_0/\sigma > c_t. \tag{B.10}$$

Then, we have

$$\sum_{t=0}^{\infty} \pi^t_0 \tilde{c}_t = \sum_{t=0}^{\infty} \pi^t_0 c_t w_0/\sigma = w_0. \tag{B.11}$$

By Lemma A2, there is a feasible path $\{c'_t, s'_t, m'_t, d'_t, w'_{t+1}\}$ such that $\{c'_t\} = \{\tilde{c}_t\}$, contradicting the optimality of the original path by (B.11).

C Proof of Lemma 4.1

By (4.1) an the Euler equation (2.19),

$$u'(c_t) = u'(c_{t+1}). \tag{C.1}$$

Thus there is $c > 0$ such that $c_t = c$.

By the Euler equation (2.20), $c_t = c$, and (4.1), we have $p_t = (p_{t+1}+\delta)/R$. Solving for p_{t+1} yields (4.4). With $b_{t+1} = Rb_t$ (see (2.41)), (4.1), and (4.4), we have (4.3) and (4.5). (2.43) and (4.1) yields (4.6) Note from (2.19), (2.27), and (4.1) that $\pi^j_t = \beta^j$. Lemma 2.1, $c_t = c$, (2.16), and (4.2) yields (4.7). Note from (2.29), $c_t = c$, and $\pi^j_t = \beta^j$ that $c/(1-\beta) = w_0$ holds. Since (2.29) holds with period 0 shifted forward to period t, we have $c/(1-\beta) = w_t = w_0$. By (2.16), and (4.2), (4.8) is obtained.

D Proof of Proposition 4.1

Let $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ be an equilibrium satisfying (4.3). Then it also satisfies (4.4)–(4.8) by Lemma 4.1. Note from $c_t/(1-\beta) = w_t = w_0$ and (2.2) that

$$p_t s_t + p_t^m m_t - d_t = \beta w_0 \tag{D.1}$$

Substituting the market clearing conditions ($m_t = 1$ and $d_t = 0$) into the above equation yields

$$\begin{aligned} s_t &= \frac{\beta w_0 - p_t^m}{p_t} && \text{(D.2)}\\ &= \frac{\beta(p_0 + \delta + p_0^m) - p_t^m}{p^* + R^t b_0} && \text{(D.3)}\\ &= \frac{p^* + \beta b_0 - (p_t^m - \beta p_0^m)}{p^* + R^t b_0} && \text{(D.4)}\\ &= \frac{p^* + \beta b_0}{p^* + R^t b_0} - \frac{(R^t - \beta)p_0^m}{p^* + R^t b_0}. && \text{(D.5)} \end{aligned}$$

The second line uses (2.16) and (4.5). The third line uses (4.2) and (4.3). The last line uses (4.6).

Next, we show that $p_0^m = 0$. By (D.1), $m_t = 1$, and $d_t = 0$ we have

$$p_t s_t + p_t^m = \beta w_0, \tag{D.6}$$

which implies $p_t^m < \beta w_0$. We rearrange this inequality as

$$(R^t - \beta)p_0^m < \beta(p_0 + \delta), \tag{D.7}$$

where we use (2.16) and (4.6). The inequality means that for sufficiently large T, agents can not buy pure bubbly assets even if they use all of assets they hold. Thus, we must have p_0^m.

Now we show that uniqueness and existence of the equilibrium. Uniqueness follows since (4.4)–(4.8), (4.9), and (4.10) uniquely determine an equilibrium satisfying (2.13), (2.14), and (4.3). To establish existence, let $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ be given by (4.3)–(4.8), (4.9), and (4.10). It satisfies (2.13) and (2.14) by construction. It is routine to verify the feasibility of $\{c_t, s_t, m_t, d_t, w_{t+1}\}$. As for optimality, it is easy to see that (2.17)–(2.21) hold. The transversality condition (2.22) also holds since $p_t s_t + p_t^m m_t - d_t$ is constant by (D.1). Thus by Proposition 2.1, $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is optimal. It follows that $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ is an equilibrium.

E Proof of Proposition 4.2

First, suppose that $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ be an equilibrium satisfying (2.12) and (2.14). Then, it also satisfy (4.3)–(4.8) by Lemma 4.1. By the stock asset and loan market clearing conditions (2.12) and (2.14) and (D.1),

$$m_t = \frac{\beta w_0 - p_t}{p_t^m} \tag{E.1}$$

$$= \frac{\beta(p_0 + \delta + p_0^m) - p_t}{R^t p_0^m} \tag{E.2}$$

$$= \frac{\beta p_0^m}{R^t p_0^m} - \frac{\beta(p^* + b_0 + \delta) - (p^* + R^t b_0)}{R^t p_0^m} \tag{E.3}$$

$$= \frac{\beta p_0^m}{R^t p_0^m} - \frac{(R^t - \beta) b_0}{R^t p_0^m}, \tag{E.4}$$

where the second line uses (2.16) and (4.6). The third line uses (4.3) and (4.5). The last line uses (4.2). Now (4.12) follows.

Next, we show that (4.13) ($b_0 = 0$). As in the derivation of (D.7), we have

$$p^* + (R^t - \beta) b_0 < \beta(p^* + \delta + p_0^m). \tag{E.5}$$

Since the stock market bubble b_t exceeds the total wealth of agents for a large T, $b_0 = 0$ must hold in the equilibrium with $s_t = 1$.

Uniqueness follows since (4.3)–(4.8), (4.12), and (4.13) uniquely determine an equilibrium satisfying (2.12) and (2.14) for any $p_0^m \geq 0$. To establish existence, let $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ be given by (4.3)–(4.8), (4.12), and (4.13). It satisfies (2.12) and (2.14) by construction. It is easy to verify that $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is feasible and optimal, as in the proof of Proposition 4.1. It follows that $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ is an equilibrium.

F Proof of Proposition 4.3

Let $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ be an equilibrium satisfying (2.12) and (2.13). Then, it also satisfy (4.3)–(4.8) by Lemma 4.1. By the stock and pure bubbly assets market clearing conditions (2.12) and (2.13) and (D.1),

$$d_t = p_t + p_t^m - \beta w_0 \tag{F.1}$$

$$= p^* + B_t - \beta(p^* + \delta + B_0) \tag{F.2}$$

$$= (R^t - \beta) B_0. \tag{F.3}$$

where the second line uses (2.16), (4.3), (4.5), and (4.15). The last line uses (4.16).

Uniqueness follows since (4.3)–(4.8), and (4.17) uniquely determine an equilibrium satisfying (2.12) and (2.13) for any $B_0 \geq 0$. To establish existence, let $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ be given by (4.3)–(4.8), and (4.17). It satisfies (2.12) and (2.13) by construction. It is easy to verify that $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is feasible and optimal, as in the proof of Proposition 4.1. It follows that $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ is an equilibrium.

G Proof of Proposition 4.4

Let $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ be an equilibrium satisfying (4.3). Then it also satisfies (4.4)–(4.8) by Lemma 4.1. (2.14) and (D.1) yields

$$p_t s_t + p_t^m m_t = \beta(p^* + b_0 + \delta + p^*)$$
$$= p^* + \beta B_0.$$

Thus, (4.18) is obtained.

For a given sequence $\{s_t\}$ in $\mathbb{R}$, (4.4)–(4.8) and (4.18) uniquely determine an equilibrium satisfying (4.3). Likewise, for a given $\{m_t\}$,(4.4)–(4.8) and (4.18) uniquely determine an equilibrium satisfying (4.3).

Finally, let $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ satisfy (4.3)–(4.8), and (4.18) . Then it is easy to verify that $\{c_t, s_t, m_t, d_t, w_{t+1}\}$ is feasible and optimal, as in the proof Proposition 4.1. Since there is no market clearing condition here, it follows that $\{c_t, s_t, m_t, d_t, w_{t+1}, p_t, p_t^m, R_{t+1}\}$ is an equilibrium.

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