A Spatial Panel Data Analysis of Fertility Rates: Unraveling Two Myths*

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A Spatial Panel Data Analysis of Fertility Rates: Unraveling Two Myths

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Abstract

This paper analyzes the relationship between total fertility rates and other socio-economic variables. The main results are against two myths: rural areas have higher total fertility rates than urban ones, and economic inequality harms total fertility rates. The key is a spatial panel data analysis with the lag between inception and birth. Here, spatial panel data analysis improves the efficiency of explanatory variables, and the lag produces a different result from former papers.

1 Introduction

In 2019, only 864 thousand babies were born in Japan, with a 126 million population, the number is the lowest in the statistics, and the percentage of the total population is only 0.69%. Several academics and journalists cite the chronically low birth rates as one of Japan’s crucial challenges. Because East Asian countries have similar problems, and even the United States now records the lowest birth rates in its history, the mechanism between the total fertility rates (TFRs) and other socio-economic variables are supposed to be vital to find the solution and evaluate the policy on low total fertility rates.

Numerous people suggest that income inequality and educational costs have adverse effects on TFRs. One may question if TFRs relate to the Gini index and the admission rates to colleges and the age cohort, and one may also be interested in the time series structure of TFRs. Moreover, several think rural areas have higher TFRs than cities, and economic inequality affects TFRs negatively.

Several papers suggest that the factors affect TFRs, such as female employment, male and female wage, wealth transfer between the generations, parental care costs, the number of children in one household. Moreover, in Japan, the policies to promote parental care should be evaluated. However, researchers have not estimated the relationship between the fertility rates, the Gini index, and the admission rates after high school.

The spatial panel data analyses show that the increase of the Gini index has adverse effects on TFRs. Also, the relationship between the population density and TFRs is the inverted U-shape. Furthermore, real expenditure and TFRs have an inverted J-shape relationship. Then, the impulse response of the panel vector autoregression (Sigmund & Ferstl 2019[22]) indicates that even a one-shot policy on TFRs has favorable effects for about 20 years and adverse economic shocks have opposite effects. This paper’s methods also validate the Easterlin hypothesis on fertility (Macunovich & Easterlin 2018[15]). This paper shows that the negative relationship between TFRs and the population density is inverted-U-shaped, using the simple unbalanced panel data with large cross-sectional dimensions and the balanced

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panel data with smaller cross-sectional dimensions denser time-series ones. However, this analysis faults that one does not consider the lag between inception and birth. With this lag, there is no statistically significant relationship between population density and TFRs. The relationship between economic inequality and TFRs is statistically false with this lag. The bivariate nonparametric regression and time series of Moran I and Theil indices indicate the negative relationship between TFRs and colleges' admission rates. However, the panel non-Granger causality test and the spatial panel regression do not show a clear relationship between TFRs and the admission rates.

2 The Literature and Data Characteristics

2.1 The Literature

2.1.1 Economic and Analytical Studies on Total Fertility Rates

Several social scientists work on human fertility. However, here we focus on socio-economic approaches.

Becker (2009)\[^4\] considers the interaction between the quantity and quality of children to explain why education per child tends to be lower in families having more children.

Willis (1973)\[^23\] points out that the interaction model captures an essential empirical regularity in the cross-section relationship between fertility and measures of husband's income and wife's education that has become apparent in the emergence of a U-shaped relationship between fertility and income. The predictions of the theoretical model of fertility demand developed in his paper fit well with this paper's estimation result.

Prettner et al. (2013)\[^19\] theoretically and empirically show that if individuals put more weight on education, they reduce fertility, increase educational investments, and hold consumption, savings, and health investments constant.

Lutz et al. (2006)\[^14\] empirically find a consistent and significant negative relationship between human fertility and population density, using fixed-effects models on the time series of 145 countries and controlling for critical social and economic variables such as GDP per capita, infant mortality, female labor force participation, and female literacy. In the empirical study, Kulu (2013)\[^12\] shows that the desired family size in small towns and rural areas is larger than that in urban areas.

Myrskylä et al. (2009)\[^17\] find that TFRs in highly developed countries have inverted J-shape curves over the Human Development Index (HDI) except Japan, Canada, and South Korea. The HDI is a summary measure of average achievement in key dimensions of human development: a long and healthy life, being knowledgeable, and having a decent living (United Nations Development Programme's Human Development Report\[^20\]). However, their finding is not robust to the redefined HDP or the decomposition of the HDI into its subindices of education, the standard of living, and health (Harttgen & Vollmer 2014\[^9\]).

Aldieri & Vinci (2012)\[^1\] estimate the correlation between the female’s education and the number of children in Italy and use the partner’s education to take into account the family dimension. They use a zero-inflated Poisson regression and observe a negative correlation between the number of children born and the educational level.

2.1.2 Sociological and Demographic Studies on Total Fertility Rates

As Pampel & Peters (1995)\[^18\] review the Easterlin hypothesis, Easterlin presented his basic argument that swings in relative cohort size and resulting levels of relative income among cohorts of child-bearing age produced the baby boom and then the baby bust. Initially, his discussion is concerned with describing demographic cycles (Easterlin 1968\[^7\]). Moreover, the linkage between higher birth rates and
adverse social-economic effects arises because large cohorts face crowding problems in three major social institutions such as the family, education, and labor markets (Macunovich & Easterlin 2018[15]).

The second demographic transition (SDT) explains the inverted J-shape curve in sociology and demography. Zaidi & Morgan (2017)[24] summarize the SDT as three arguments: the shift from king-child to king-couple, the Maslowian drift, the rise of individualism, and pushback against economic explanations.

2.1.3 Studies on Total Fertility Rates in Japan

Gini Index

Few researchers have the Gini index as the explanatory variable for total fertility rates. The panel data in this study shed light on the Gini index’s effects on TFRs.

Educational level

Some papers regress the female educational levels to TFRs.

2.2 Data

Total Fertility Rates \((TFR)\) and the population of each cohort of every prefecture \((p0to4, p5to9, p10to14, p15to19, p20to24, p25to29, p30to34, p35to39, p40to44, p45to49, p50to54, p55to59, p60to64, p65to69, p70to74, p75to79 and pover80)\) are from the National Institute of Population and Social Security Research (Japan) and its former organization. Municipal data on \(TFR\) is available online from the Statistical Bureau of Japan.

Figure. 1 depicts the movements of total fertility rates of each prefecture in Japan.

Data on the Gini index \((gini)\) and mean real expenditure \(real\_exp\_mean\) are estimated originally from the microdata courtesy of the Statistic Bureau, Ministry of Internal Affairs and Communications.

3 Results of the Econometric Approaches

3.1 Nonparametric Analysis and the Panel Non-Granger Causality

At first, one can nonparametrically regress the bivariate relationship between the total fertility rates \((TFRs)\) and other socio-economic variables. Figure 2 shows the relationship between \(TFR\) and \(pop\_density\) and one between \(TFR\) and \(ad\_rate\) appear inverse correlations.

The relationship between \(TFR\) and \(pop\_density\) and one between \(TFR\) and \(real\_exp\_mean\_100\) have the panel Granger causality in both directions. These tests are based on Dumitrescu & Hurlin (2012)[6]. They note that the null of Homogeneous Non-Causality does not provide any guidance concerning the number or the identity of the particular panel units for which the null of non-causality is rejected. In other words, only non-Granger causality is meaningful. However, \(gini\) and \(ad\_rate\) have the panel Granger causality from \(TFR\), and \(TFR\) has the panel non-Granger causality from \(gini\) and \(ad\_rate\). This \(gini\) and \(ad\_rate\) do not determine \(TFR\) in the non-Granger causality standpoint. Then, the panel non-Granger Causality test assumes that the individuals of the panel have no cross-sectional dependence.

3.2 The Relationship Between the Total Fertility Rates in the Municipal Level and the Population Density

Because municipal data on \(TFR\) have relatively large cross-sections (at least two thousand) and seven terms, panel data analysis is available. The estimation is based on the fixed-effects model, and the Least
Figure 1: Total Fertility Rates in Prefectures
Figure 2: Nonparametric Regression Results
### Table 1: Estimation Results: The Large Cross-Sectional Panel ($n = 3755$, $T = 1 − 7$, and $N = 19235$)

<table>
<thead>
<tr>
<th></th>
<th>Fixed Effects</th>
<th>Random Effects</th>
<th>Pooling</th>
</tr>
</thead>
<tbody>
<tr>
<td>log((TFR))</td>
<td>0.2332224</td>
<td>0.36260635</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0154558)</td>
<td>(0.01481855)</td>
<td></td>
</tr>
<tr>
<td><strong>Intercept</strong></td>
<td>***</td>
<td>***</td>
<td></td>
</tr>
<tr>
<td>log((pop_density))</td>
<td>0.6627606</td>
<td>0.1294545</td>
<td>0.0831563</td>
</tr>
<tr>
<td></td>
<td>(0.0654288)</td>
<td>(0.0057179)</td>
<td>(0.00562807)</td>
</tr>
<tr>
<td></td>
<td>***</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>(log((pop_density)))^2</td>
<td>-0.0504881</td>
<td>-0.0140688</td>
<td>-0.01043975</td>
</tr>
<tr>
<td></td>
<td>(0.0056033)</td>
<td>(0.0005074)</td>
<td>(0.00051242)</td>
</tr>
<tr>
<td></td>
<td>***</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>Adj.R^2</td>
<td>-0.11254</td>
<td>0.16</td>
<td>0.19968</td>
</tr>
</tbody>
</table>

Double clustered standard deviations are in parentheses.

Square Dummy Variable as explanatory variables are few, and the omitted variables bias is severe. The Hausman test between the fixed effects and the random effects supports this reasoning.

\[
\exp(0.6627606/2(0.0504881)) = 708.7712
\]  
(1)

This subsection’s relationship between the \(TFR\) and \(pop\_density\) is spurious because it is natural that we consider the lag between inception and birth.

### 3.3 The Spatial Autocorrelation and the Inequality Index

Moran’s I in the spatial autocorrelation of cross-sectional spatial data has an analogy to Durbin-Watson in the serial autocorrelation of time-series data (Li et al., 2007[13]). Then, the higher Moran’s I is, the higher the degree of spatial autocorrelation is. The spatial dependence (global spatial autocorrelation) measure of Moran’s I is represented by equation 2 (the formulation follows Rey 2004[21]):

\[
Moran_I = \frac{n}{s} \sum_i \sum_j w_{ij} z_i z_j \\
(2)
\]

where \(n\) is the number of prefectures (i.e.\(n = 47\)), \(z_i\) and \(z_j\) are the deviation of each variable from the mean of each prefecture in each year (the prefecture \(i, j\) and the year \(t\)), \(w_{ij}\) are the elements of weight matrix \(W (n \times n)\) and it is equal to 1 if \(i\) and \(j\) are neighbors and 0 if they are not; \(s\) is the sum of all elements of \(W\) (spatial weights). One note that \(n\), \(s\) and \(w_{ij}\) are invariant over time.

Figure 3 plots Moran’s I of each variable. The critical value, which rejects at the significant level 0.05 null hypothesis that there is no spatial correlation, is about 0.14 in the case of this dataset and is added as The Critical Value(horizontal lines) in line plots of Figure 3.

According Rey (2004)[21], Figure 4 plots Moran’s I and Theil index of each variable. The Theil index of inter-prefectural inequalities is presented as follows(the formulation follows Cowell 2011[5]):

\[
Theil_t = \frac{1}{n} \sum_{i=1}^{n} \frac{y_{it}}{\mu_t} \log\left(\frac{y_{it}}{\mu_t}\right)
\]  
(4)
Figure 3: Moran’s I of Each Variable in This Study
where \( y_{it} \) is the value of each variable in each prefecture and each year, \( \mu_t \) is the mean over Japan in each year and \( n \) is the number of prefectures (i.e. \( n = 47 \)). Rey (2004)[21] empirically shows higher Moran’s I’s tend to be higher Theil indices. Theil indices have the drawback that they are affected by the sample size. With higher inequality of TFR, ad_rate has lower inequality over 36 years in this paper’s dataset. Figure 4 indicates that inequality of TFR and that of ad_rate have a negative relationship, and one must consider geographical inequality to analyze the relationship of both. Next, we focus on spatial panel regression as the model of spatial autocorrelation.

### 3.4 The Spatial Panel Data Analysis

Hsiao (2014)[10] suggests that panel data analysis could control the impact of omitted variables (or individual or time heterogeneity) and generate more accurate predictions for individual outcomes than the cross-sectional analysis and time-series analysis.

One can apply the spatial panel data analysis to the analysis of Total Fertility Rates (TFRs) to gain more efficiency than the static panel analysis such as the Least Squares Dummy Variable (LSDV) or the dynamic panel analysis such as the dynamic panel Generalized Method of Moments (GMM) (Arellano & Bond, 1991[2]).

One can conduct the Maximum Likelihood (ML) (Baltagi et al. 2007[3]) and the Generalized Moments (GM) estimation (Kapoor et al. 2007[11]) to estimate the spatial panel. In this paper, the fixed effects spatial error model is estimated because the fixed effects model is generally more appropriate than the random effects model since adjacent spatial units’ space-time data are located in unbroken study areas (Elhorst 2014[8]). Furthermore, as the spatial error model’s parameters are considered marginal effects themselves (Elhorst 2014[8]), these parameters are easily interpretable. Then, other models using the spatial weight matrix are similar to the spatial error model. This paper’s notations are based on Millo and Piras (2012)[16], and we use their package. This paper utilizes the spatial error model to control spatial autocorrelation. In the spatial error model, spatial error terms and the spatial lag terms are considered the spatial autocorrelation terms in the error terms.

The fixed effects spatial error panel data model estimated by the ML and the GM in this paper is as follows:

\[
y = (\iota_T \otimes I_N) \mu + X \beta + \varepsilon
\]

\[
\varepsilon = \rho (I_T \otimes W_N) \varepsilon + \nu
\]

\[
\nu \sim IID(0, \sigma^2_\nu)
\]

\[
y_t = (\iota_T \otimes W_N) \mu + X_{t-1} \beta + u_t
\]

\[
u_t \sim IID(0, \sigma^2_\nu)
\]

\[
t = 1, \ldots, T
\]

\[
y_t = (\iota_T \otimes W_N) \mu + X_{t-1} \beta + u_t
\]

\[
u_t \sim IID(0, \sigma^2_\nu)
\]

\[
t = 1, \ldots, T
\]
Figure 4: Moran's I and Theil Index of Each Variable in This Study
Table 2: Estimation Results: Spatial Panel Maximum Likelihood (SPML), Spatial Panel Generalized Moments (SPGM), Least Squares Dummy Variable (LSDV) and Dynamic Panel Generalized Methods of Moments (Arellano & Bond 1991[2]) (DPGMM(AB)) ($n = 47$, $T = 35$, and $N = 1645$)

<table>
<thead>
<tr>
<th></th>
<th>SPML</th>
<th>SPGM</th>
<th>LSDV</th>
<th>DPGMM(AB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log($TFR$)</td>
<td>0.674254</td>
<td>0.54946607</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>(0.022147)***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma^2_\nu$</td>
<td></td>
<td>0.00082763</td>
<td></td>
<td>0.353606 (0.037013)***</td>
</tr>
<tr>
<td>$\text{lag}$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>log(lag(pop_density, 1))</td>
<td>-0.0163771 (0.0174095)</td>
<td>-0.017743 (0.015476)</td>
<td>0.0776786 (0.2496147)</td>
<td>0.485169 (0.324223)***</td>
</tr>
<tr>
<td>(log(lag(pop_density, 1)))$^2$</td>
<td>0.0016538</td>
<td>0.0018175</td>
<td>0.0080262</td>
<td>-0.030917</td>
</tr>
<tr>
<td>log(lag(real_exp_mean, 1))</td>
<td>1.7326863 (0.0685412)***</td>
<td>1.7248 (0.068263)***</td>
<td>2.0155213 (0.0796065)***</td>
<td>2.337559</td>
</tr>
<tr>
<td>(log(lag(real_exp_mean, 1)))$^2$</td>
<td>0.2270749 (0.0016538)***</td>
<td>0.24826 (0.0018175)***</td>
<td>0.3227035 (0.0080262)***</td>
<td>0.20214 (0.0016538)***</td>
</tr>
<tr>
<td>log(lag(gini, 1))</td>
<td>0.0118918</td>
<td>0.014496</td>
<td>-0.0124552</td>
<td>0.011498</td>
</tr>
<tr>
<td>log(lag(ad_rate, 1))</td>
<td>(0.0137283)***</td>
<td>(0.013994)***</td>
<td>(0.0159547)***</td>
<td>(0.01756)***</td>
</tr>
<tr>
<td>log(lag(p0to4, 1))</td>
<td>(0.0315136)***</td>
<td>(0.031383)***</td>
<td>(0.0550940)***</td>
<td>(0.050675)***</td>
</tr>
<tr>
<td>log(lag(p5to9, 1))</td>
<td>0.2359812 (0.0399109)***</td>
<td>0.21131 (0.040257)***</td>
<td>0.0257211 (0.0621721)***</td>
<td>0.10345 (0.051732)***</td>
</tr>
<tr>
<td>log(lag(p10to14, 1))</td>
<td>0.0467749 (0.0313585)***</td>
<td>0.041251 (0.031855)***</td>
<td>0.022933 (0.0479956)***</td>
<td>-0.09145 (0.054953)***</td>
</tr>
<tr>
<td>log(lag(p15to19, 1))</td>
<td>(0.0205411)***</td>
<td>(0.020838)***</td>
<td>(0.0307829)***</td>
<td>(0.031187)***</td>
</tr>
<tr>
<td>log(lag(p20to24, 1))</td>
<td>0.0138215 (0.0138215)***</td>
<td>0.013685 (0.013685)***</td>
<td>0.0236034 (0.0236034)***</td>
<td>(0.021896)***</td>
</tr>
<tr>
<td>log(lag(p25to29, 1))</td>
<td>(0.0209416)***</td>
<td>(0.021170)***</td>
<td>(0.0330438)***</td>
<td>(0.034114)***</td>
</tr>
</tbody>
</table>
Table 2 does not indicate that \( \log(\text{pop\_density}) \) and \( \log(\text{gini}) \) in the model with the lag have the statistical significance in the regression to \( \log(\text{TFR}) \). This result is consistent with the fact that \( \text{TFR} \) has panel non-Granger causality from \( \text{gini} \) in Section 3.1. In short, \( \text{gini} \) has no impact on \( \text{TFR} \) in the viewpoint of panel data. Moreover, \( \text{pop\_density} \)'s effect on \( \text{TFR} \) remains unclear.

## 4 Conclusion

Several people think rural areas have higher \( \text{TFR} \) than cities, and economic inequality negatively correlates with \( \text{TFR} \). However, the negative relationship between \( \text{TFR} \) and \( \text{pop\_density} \) and the positive relationship between \( \text{TFR} \) and \( \text{gini} \) is false if one considers the lag between inception and birth. The spatial panel data analysis validates the Easterlin hypothesis. \( \text{ad\_rate} \) has adverse effects on \( \text{TFR} \) in the
spatial panel data model. However, the panel non-Granger causality shows \( ad rate \) does not determine \( TFR \) in the non-Granger causality standpoint.

**Compliance with Ethical Standards**

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**Conflict of interest**

The author has no conflicts of interest to declare.

**Ethical approval**

This article does not contain any studies with human participants performed by any of the authors.

**References**


