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Abstract

We propose a new behavioral asset pricing model that can flexibly express the time-variability or state-dependence of aggregate risk attitude. We suppose that individual preferences are heterogeneous, and each preference changes depending on the state of the economy. While most previous studies, including Chan and Kogan (2002) which is the basis of our model, imply that aggregate risk aversion is counter-cyclical by assuming either heterogeneity or changeability of preferences, considering both of these can result in aggregate risk aversion being procyclical in a particular economic situation. The status quo hypothesis we propose suggests that the aggregate risk attitude is procyclical during recessions and counter-cyclical during booms and depressions. This stems from the fact that in our setting only risk-tolerant individuals face losses large enough to be loss-averse in order to maintain the status quo. We analyze the weekly returns of 10 major stock markets, including Canada, China, Eurozone, France, Germany, Hong Kong, India, Japan, the UK and the US, and provide some evidences to support our status quo hypothesis in Western stock markets, where stock markets are relatively mature. We conclude that it is essential at least in such matured stock markets to allow both heterogeneity and changeability of preferences in order to obtain an accurate aggregate risk attitude.

Keywords: risk attitude, heterogeneous preferences, status quo, loss-aversion, prospect theory
JEL classification: G12, G15, G40

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1. Introduction

An aggregate risk attitude is time-varying. A well-known study by Campbell and Cochrane (1999) showed that the relative risk aversion of a representative individual fluctuates with changes in wealth.

There are three major factors that cause the aggregate risk attitude to fluctuate over time. First, the preference parameters of individuals in the economy change such as Guiso et al. (2018) and Schildberg-Hörisch (2018). These studies focus on changes in individual risk attitudes rather than changes in an aggregate risk attitude. Second, the risk aversion and other preference parameters of a representative individual change such as Campbell and Cochrane (1999) and Barberis et al. (2001).[2] This means that the preferences of individuals in the economy are homogeneous. Third, the preference parameters of individuals in the economy does not change, but those of aggregate individual change such as Wang (1996), Chan and Kogan (2002), and Verdelhan (2010). The individual preferences are heterogeneous. Although these three factors rely on essentially different models and theories, they draw common conclusions. That is, the aggregate risk attitude is counter-cyclical.

The counter-cyclicality of aggregate risk attitude is extremely intuitive. Put briefly, people become risk averse when the economy goes bad. This corresponds to the claim of the first group. If this change is generally homogeneous for all individuals, it corresponds to the claim of the second group. On the other hand, Chan and Kogan (2002), in the third group, argue that the economic downturn increases the relative wealth of relatively risk-averse individuals, resulting in an increase in wealth-weighted aggregate risk aversion. This is because relatively risk-averse individuals are less willing to hold risk assets than relatively risk-tolerant individuals and therefore do not significantly impair wealth in the event of an economic downturn.

Few models of heterogeneous individuals that change their preferences have been considered. The reason is probably that the consequences of these explanations are the same. If the economic downturn makes individuals risk-averse, and the wealth of relatively risk-averse individuals increases, aggregate risk aversion should be even more risk-averse. It remains important to take into account both changes in individual preferences and their heterogeneity, nevertheless. This is because there are three possible hypotheses in which the aggregate risk attitude becomes more risk-tolerant as the economy deteriorates.

The first hypothesis is that in the event of an economic downturn, the preference of a sufficient number of individuals to affect the economy as a whole becomes more risk-tolerant. Some studies have shown that individuals who experience catastrophic shocks such as the Great East Japan Earthquake and the Corona disaster (Hanaoka et al., 2018, and Tsutsui and Tsutsui-Kimura, 2020, respectively) are rather risk tolerant. The reason for this phenomenon may be interpreted as "habituation" to repeated stress according to Tsutsui and Tsutsui-Kimura. Despite these extremely interesting results, most studies, including Guiso et al. (2018) and Schildberg-Hörisch (2018), suggest that catastrophic shocks such as the financial crisis make people more risk-averse. At least in the analysis of financial markets, this hypothesis seems to be somewhat

[2] Studies of similar settings are Menzly et al. (2004) and Santos and Veronesi (2010).

inadequate. In any case, when this hypothesis is correct at the aggregate level, the worse the economy, the lower the aggregate risk attitude.

The second hypothesis is that when the economy deteriorates, individuals become risk-loving. Prospect theory provided by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) suggests that individuals facing loss become risk-loving and loss-averse. When considering a CRRA (constant relative risk aversion) type utility function, if the relative risk aversion is negative, the utility function means risk lover. Their weighted aggregate risk attitude may decrease in a recession where many individuals face losses. A general equilibrium involving individuals with convex utility functions, however, generally does not hold. Araujo et al. (2017) and Araujo et al. (2018) have shown the existence of such a general equilibrium, but some conditions are required; for example, the condition that there must be enough risk-averse individuals compared to risk-loving individuals.

The third is the *status quo hypothesis*, and we pay the most attention to this hypothesis. This means that when the economy goes bad, individuals become more loss-averse. As in the second hypothesis, a recession means that many individuals face losses. People should be more loss-averse according to prospect theory. Loss avoidance is expressed by the slope of the value function (utility function) when facing loss becomes steeper than when facing gain. Interpreted in the framework of the expected utility theory, this means that the marginal utility of consumption increases, or the marginal negative utility of reduction in consumption increases. This implies that individuals who are forced to reduce their consumption in the face of loss cannot reduce their consumption as the original marginal utility suggests in order to maintain their status quo. In other words, it can be said that it expresses the *status quo propensity*.[3] This may represent the cost stickiness in households and businesses.

This status quo hypothesis can be used to link the economic downturn with a reduction in aggregate risk aversion, while using the general equilibrium theory without breaking it. Before explaining this mechanism, we explain the Chan and Kogan (2002) model with heterogeneous individuals who do not change their preference parameters. During a recession, optimal consumption generally decreases regardless of individual risk attitudes. Relatively risk-tolerant individuals hold more risk assets than relatively risk-averse individuals. They are likely to suffer significant losses first when the economy enters a recession. Consequently, relatively risk-tolerant individuals have a greater reduction in optimal consumption than relatively risk-averse individuals, and their aggregate consumption share (the ratio of their consumption to aggregate consumption) is lower. The influence of relatively risk-tolerant individuals is reduced, and the aggregate risk attitude become larger, or more risk-averse.

On the other hand, when relatively risk-tolerant individuals facing losses switch to loss-averse preferences, their optimal consumption does not decrease as much as the traditional model. In a recession,

[3] The expression *status quo bias* may be more common. Our status quo hypothesis represents a situation in which consumption cannot be reduced even if it is desired. This is not just due to psychological bias. Structural problems that lead to cost stickiness in households and businesses are also relevant. Therefore, it may be inappropriate to call it *bias*. We should collectively call these the *status quo propensity*.

relatively risk-averse individuals also reduce consumption. The aggregate consumption share of relatively risk-tolerant individuals may increase compared to that of relatively risk-averse individuals. The influence of relatively risk-tolerant individuals becomes rather large, and the deterioration of the economy leads to a decrease in the aggregate risk aversion.

The status quo hypothesis is thought to show interesting properties during further economic downturns such as the global financial crisis. In a minor recession, only relatively risk-tolerant individuals with large risk assets face significant losses. That is, individual loss-aversions are heterogeneous. As the economic downturn becomes more severe, relatively risk-averse individuals, however, also face significant losses. In other words, individual loss-aversions become homogeneous. The aggregate consumption share of relatively risk-averse individuals will rise, and the aggregate risk aversion will rise again.

From the above viewpoints, the aggregate risk aversion is expected to be procyclical in recessions, counter-cyclical in depressions, and counter-cyclical in booms. In order to capture more flexible movements, the aggregate risk aversion expressed as a function of the state variable of the economy is approximated by a polynomial. The GARCH-M (generalized autoregressive conditional heteroskedasticity in mean) model provided by Engle, Lilien and Robins (1987) is used to analyze the aggregate risk aversion. Since the conventional GARCH-M model does not allow time-variability of aggregate risk aversion, the constant aggregate risk aversion is replaced by the time-varying aggregate risk aversion as a function of the state variable.

We use the weighted average of historical returns in the stock market as the proxy for state variable. Chan and Kogan (2002) used “relative log-consumption” as a variable to capture the state of the economy. This is defined as the logarithm of the ratio of current consumption to the consumption benchmark, which is the weighted sum of past consumptions. This can be rewritten as a weighted average of past consumption or endowment growth rates. Data representing consumption or endowment, such as household consumption, CPI, or GDP, are less frequent than stock return, however. Furthermore, these data are lagging indices related to the business cycle, and it must be said that consumption as data is a little slow to show the business sentiment that investors feel. In fact, the stock market index will be the most important factor for investors to judge the future growth potential of consumption, productivity and endowment in the economy. Therefore, in this paper, as a variable to capture the state of the economy, we use a variable in which the aggregate consumption or endowment growth rate in the “relative log-consumption” is replaced by the stock market return.

We analyze weekly returns from July 2005 to June 2020 in the 10 major stock markets, including Canada, China, Eurozone, France, Germany, Hong Kong, India, Japan, the UK and the US. The time-varying (or state-dependent) model is supported in all Western markets, but not in Asian markets. As expected, in the Western markets, the aggregate risk attitude tends to decrease during the economic downturn. It is suggested that loss aversion, or status quo hypothesis, regarding consumption plays an important role during the recession. This does not necessarily deny the possibility that people will become relatively risk-averse during the recession. At the market level as a whole, the effect of the shift to more loss averse preference is greater than that. In addition, the aggregate risk attitude tends to rise again during extreme economic downturns such as the 2008 global financial crisis and the 2020 COVID-19 crisis. This result strongly supports the status quo hypothesis because the other two hypotheses cannot explain this phenomenon. In the boom period, as shown by the conventional models, a counter-cyclical relationship is confirmed between the state of the economy and the aggregate risk aversion.

We have shown that loss aversion, or status quo hypothesis, affects the market as a whole by considering both heterogeneity and changeability of individual preferences. Moreover, our empirical model uses only market returns and interest rates and can minimize the problems of variable selection arbitrariness and overfitting. Loss-aversion or status quo propensity is universal properties for humankind, and it is thought that these properties will be maintained in the mature stock market.

Section 2 introduces a Chan and Kogan (2002) model with heterogeneous individuals who do not change their preferences and provide some useful interpretations for the following sections. In Section 3, the Chan and Kogan model is extended so that the preference changes depending on the state of the economy. Section 4 provides some evidences in support of our loss avoidance hypothesis and Section 5 concludes.

2. Heterogeneous individuals and time-varying risk attitude

This section explains the time-variability of the (aggregate) risk attitude according to Chan and Kogan (2002). Their model is a continuous-time consumption-based asset pricing model with *catching up with the Joneses* preferences, or external habit formation. In their setting, while all individuals have constant relative risk aversions (hereafter, CRRA(s)), each CRRA coefficient is allowed to be different. Such heterogeneous risk attitude model implies that the aggregate risk aversion in such an economy is counter-cyclical. Next, we discuss the characteristics of the *relative log consumption,* or *state variable*, which plays an important role in their model.

2.1. Heterogeneous constant risk aversions

Chan and Kogan (2002) constructed a new continuous-time consumption-based asset pricing model that relaxes the assumptions of representative individuals in the conventional model. When an individual has the same CRRA-type utility function, the representative individual also has a CRRA-type utility function. This is a typical setting. This is not always the case if individuals can have different CRRA-type utility function. Wang (1996) is considering an exchange economy in which there are two individuals with different risk aversions. In their model, as wealth increases, a single type of individual dominates the market. Chan and Kogan used catching up with Joneses preference (Abel, 1990; Abel, 1999) to prevent the asymptotic cross-sectional distribution of wealth from degenerating. In the Chan and Kogan model, an individual with a constant relative risk aversion $\gamma^{ind} \in (1, \infty)$ has the following utility function.[4]

$$u(C_t, X_t; \gamma^{ind}) = \frac{1}{1-\gamma^{ind}} \left(\frac{C_t}{X_t}\right)^{1-\gamma^{ind}}, \tag{1}$$

where C_t is an individual consumption and X_t is an external benchmark which represents the standard of living in the economy. This reflects that the marginal utility of consumption is increasing in the consumption level of other individuals. In other words, individuals consume more during boom times when the standard of living rises. The dynamics of $x_t = \ln X_t$ are defined as follows using the past values of logarithmic aggregate endowment $y_t = \ln Y_t$:

$$x_t = x_0 e^{-\kappa t} + \kappa \int_0^t e^{-\kappa(t-s)} y_s \mathrm{d}s, \qquad \kappa > 0. \tag{2}$$

The parameter κ governs the degree of history dependence in X_t.

Considering the budget constraint up to the aggregate endowment Y_t, the consumption of an individual with a risk aversion γ^{ind} is a function of Y_t and X_t, that is, $C_t = C_t(Y_t, X_t; \gamma^{ind})$. The budget constraint at time t is expressed as $\int_1^\infty C_t(Y_t, X_t; \gamma^{ind}) \mathrm{d}\gamma^{ind} \le Y_t$. Let $f(\gamma^{ind})$ be the social weight attached by the central planner, or the distribution of individuals, that is $\int_1^\infty f(\gamma^{ind}) \mathrm{d}\gamma^{ind} = 1$. In an exchange economy, no intertemporal transfer of wealth is possible at the aggregate level, and the optimization problem can be separated at each time $t \in [0, \infty)$. The utility function of representative (or aggregate) individual at time t is

$$\begin{aligned} U(Y_t, X_t) = \sup_{\{c_t(Y_t, X_t; \gamma^{ind}); \gamma^{ind} > 1\}} & \int_1^\infty f(\gamma^{ind}) \frac{1}{1-\gamma^{ind}} \left(\frac{C_t(Y_t, X_t; \gamma^{ind})}{X_t}\right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind} \\ \text{s.t.} \quad & \int_1^\infty C_t(Y_t, X_t; \gamma^{ind}) \mathrm{d}\gamma^{ind} \le Y_t. \end{aligned} \tag{3}$$

We summarize the main implications of their model.[5] First, the aggregate relative risk aversion is expressed as follows.

$$\gamma(Y_t, X_t) \equiv \frac{-Y_t U_{YY}(Y_t, X_t)}{U_Y(Y_t, X_t)} = -h'(z_t) > 1, \tag{4}$$

where $z_t \equiv y_t - x_t = \ln(Y_t / X_t)$ is *relative log-consumption* according to Chan and Kogan (2002) because it measures aggregate consumption relative to the standard of living, and the function $h(z_t)$ is logarithmic Lagrange multiplier and implicitly defined as $\int_1^\infty f(\gamma^{ind})^{1/\gamma^{ind}} \exp\{h(z_t) - z_t\} \mathrm{d}\gamma^{ind} = 1$. Since z_t is convenient for describing the state of the economy, we call it simply *state variable*. Eq. (4) implies the aggregate risk aversion is the function of the state variable:

$$\gamma(Y_t, X_t) = \gamma(z_t). \tag{5}$$

Therefore, the aggregate risk aversion becomes time-varying although the risk aversion of each individual

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[4] The constraint $\gamma^{ind} > 1$ is necessary for the complementarity between individual consumption and the consumption benchmark described below.

[5] See Chan and Kogan (2002) and Munk (2013) for detail.

is constant.

Second, the first derivative of the aggregate risk aversion satisfies the following properties.

$$\gamma'(z_t) < 0. \tag{6}$$

This is derived from the property that $h(z_t)$ is a decreasing and convex function. This property implies that the aggregate risk aversion is counter-cyclical.

Third, the optimal aggregate consumption share $c_t^* \equiv C_t^*/Y_t$ is

$$c_t^*(z_t;\gamma^{ind}) = f(\gamma^{ind})^{1/\gamma^{ind}} \exp\left\{-\frac{1}{\gamma^{ind}}h(z_t) - z_t\right\} \quad \text{where} \quad \int_1^\infty c_t^*(z_t;\gamma^{ind})\mathrm{d}\gamma^{ind} = 1. \tag{7}$$

Differentiating with respect to z_t,

$$\frac{\mathrm{d}c_t^*(z_t;\gamma^{ind})}{\mathrm{d}z_t} = \frac{1}{\gamma^{ind}}(\gamma(z_t) - \gamma^{ind})c_t^*(z_t;\gamma^{ind}). \tag{8}$$

It is implied that when the state variable increases, individuals with risk aversion γ^{ind} lower (higher) than aggregate risk aversion $\gamma(z_t)$ increase (decrease) their optimal aggregate consumption share. Relatively risk-tolerant individuals have a greater share of risky asset in their wealth than relatively risk-averse individuals regardless of the state of the economy. As the state of the economy improves, their wealth share may increase by growth of the risky asset, and they can consume more. In contrast, when the state of the economy deteriorates, they face great losses. Therefore, they are forced to reduce their aggregate consumption share.

2.2. *Useful interpretations*

We provide three useful interpretations of the Chan and Kogan (2002) model in this subsection. First, we provide the property that the state variable is (approximately) a function of the history of the aggregate endowment growth. Using $\mathrm{d}x_t = \kappa(y_t - x_t)\mathrm{d}t$ according to Chan and Kogan, the current state variable, z_t, can be expressed as a function of the state variable Δt period ago, $z_{t-\Delta t}$, and the (log) aggregate endowment growth from Δt period ago to the present, $y_t - y_{t-\Delta t}$:

$$z_t = y_t - y_{t-\Delta t} + (1-\kappa\Delta t)z_{t-\Delta t}. \tag{9}$$

If t is large enough, the effect of z_0 can be ignored. Eq. (9) implies that z_t is a function of the history of the (log) aggregate endowment growth $y_t - y_{t-\Delta t}$ for $t \in (\Delta t, t]$. For sufficiently large t, the state variable can be approximated as follows.

$$z_t \approx z(\psi_t;\kappa), \tag{10}$$

where ψ_t is the history of the (log) aggregate endowment growth.

Second, we provide the property about reference period. Using eq. (2) and the fact that $\kappa\int_0^t e^{-\kappa(t-s)}\mathrm{d}s = 1 - e^{-\kappa t}$, the state variable, $z_t = y_t - x_t$, can be rewritten as follows.

$$z_t = \int_0^t e^{-\kappa(t-s)}(y_t - y_s)\mathrm{d}s + e^{-\kappa t}(y_t - x_0). \tag{11}$$

If t is sufficiently large compared to a unit of period, the second term can be ignored. In this case, z_t can be interpreted as a weighted average of $y_t - y_s$, which means the (log) endowment growth from $t - s$ periods ago. The coefficient $e^{-\kappa(t-s)}$ implies the influence of individuals who refer to $t - s$ periods ago. Assuming that the period referred to by the individual is not changed, while the individual who refers to $t - s$ periods ago appears once in the market, the individual who refers to one period ago will appear in the market $t - s$ times. Thus, the distribution of such individuals is expressed as follows.

$$g(s;\kappa) = (t-s)\kappa^2 e^{-\kappa(t-s)}. \tag{12}$$

The peak of the distribution is equal to κ^{-1}. For example, given κ =0.02, it means that 50 periods ago is the most referenced.

Third, we provide an intuitive interpretation of the negative state variable. Let $\mathcal{T}_t^+$ be the set of time s satisfied that $y_t - y_s \geq 0$ for all $s \in \mathcal{T}_t^+$. Similarly, let $\mathcal{T}_t^-$ be the set of time s satisfied that $y_t - y_s < 0$ for all $s \in \mathcal{T}_s^+$. Then, the state variable can be rewritten as follows.

$$z_t = \int_{\mathcal{T}_t^+} e^{-\kappa(t-s)} |y_t - y_s| \mathrm{d}s - \int_{\mathcal{T}_t^-} e^{-\kappa(t-s)} |y_t - y_s| \mathrm{d}s \tag{13}$$

The first term is the weighted average of the influence of the individual facing the gain, $e^{-\kappa(t-s)}$ for $s \in \mathcal{T}_t^+$, by the magnitude of each positive growth, $|y_t - y_s|$ for $s \in \mathcal{T}_t^+$. Similarly, the second term is the weighted average of the influence of the individual facing the loss by the magnitude of each negative growth. Consequently, the negative state variable means that the growth-weighted aggregate influence of an individual facing a gain outweighs that of an individual facing a loss.

3. State-dependent preferences of heterogeneous individuals, and status quo hypothesis

This section extends the Chan and Kogan (2002) model to allow individual preferences to change depending on the state of the economy. As shown in eq. (13), it is expected that the smaller the state variable z_t, the greater the influence of individuals facing losses.[6] Individuals facing loss are considered to be loss-averse as shown by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). Alternatively, feared individuals may be risk-averse as Guiso et al. (2018) suggest, and individuals facing significant losses may be risk-averse. First, we consider the case where only the individual risk aversions change. This is expressed by switching the distribution of social weights depending on the state of the economy. Next, we examine the situation in which the individual loss aversions change. It can be shown that an increase in loss aversion for individuals with a certain risk aversion is virtually equivalent to an increase in their social weight. We call it *status quo hypothesis*.

3.1. Switching risk aversions

Switching risk aversions means that a given distribution of social weights changes. For example, we consider the case where relatively risk-tolerant individuals change their risk aversion to be more risk-averse.

[6] Barberis et al. (2001) use similar variables to describe the situation in which an individual faces loss.

At this time, the social weight assigned to relatively risk-tolerant individuals will decrease, and the social weight assigned to relatively risk-averse individuals will increase.

To investigate this effect, we make the distribution of social weights in the Chan and Kogan model dependent on the latent state k:

$$\begin{aligned} U(Y_t, X_t; k) = \sup_{\{c_t(Y_t, X_t; \gamma^{ind}); \gamma^{ind}>1\}} \int_1^\infty f_k(\gamma^{ind}) \frac{1}{1-\gamma^{ind}} \left(\frac{C_t(Y_t, X_t; \gamma^{ind})}{X_t} \right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind}, \\ \text{s.t.} \int_1^\infty C_t(Y_t, X_t; \gamma^{ind}) \mathrm{d}\gamma^{ind} \le Y_t. \end{aligned} \tag{14}$$

Given various $f_k(\gamma^{ind})$, the optimal allocation of consumption in a sufficiently large number of Pareto equilibria is potentially derived. Let S_t be the latent variable that captures the latent state. $S_t = k$ determines the structure of $f_k(\gamma^{ind})$, and the Pareto equilibrium under its Pareto weight is adopted. It is thought that the latent state changes when the number of individuals facing loss increases. Therefore, we can naturally assume that the latent variable S_t depends on the state variable z_t, that is, $S_t \equiv S(z_t)$. This procedure provides the distribution of social weights pseudo time-variability. The aggregate risk aversion γ is still a function of z_t because the pseudo time-variability is captured by z_t.

The optimal aggregate consumption shares of individuals with γ^{ind} in latent state k are given below.

$$\begin{aligned} c^*_{kt}(z_t; \gamma^{ind}) = \left(f_k(\gamma^{ind}) \right)^{1/\gamma^{ind}} \exp\left\{ -\frac{1}{\gamma^{ind}} h_k(z_t) - z_t \right\}, \\ \text{where} \int_1^\infty c^*_{kt}(z_t; \gamma^{ind}) \mathrm{d}\gamma^{ind} = 1. \end{aligned} \tag{15}$$

Eq. (15) holds for any $\gamma^{ind} \in (1, \infty)$. Let γ_i^{ind} and γ_j^{ind} in $(1, \infty)$ be individual risk attitudes where $\gamma_i^{ind} > \gamma_j^{ind}$, that is, individuals with γ_i^{ind} are more risk-averse than individuals with γ_j^{ind}. Eq. (15) can be rewritten as follows.

$$\frac{f_k(\gamma_i^{ind})}{f_k(\gamma_j^{ind})} = \frac{c^*_{kt}(z_t; \gamma_i^{ind})^{\gamma_i^{ind}}}{c^*_{kt}(z_t; \gamma_j^{ind})^{\gamma_j^{ind}}} \exp\{(\gamma_i^{ind} - \gamma_j^{ind}) z_t\}. \tag{16}$$

An increase in the social weight ratio $f_k(\gamma_i^{ind})/f_k(\gamma_j^{ind})$ means an increase in $c^*_{kt}(z_t; \gamma_i^{ind})$, a decrease in $c^*_{kt}(z_t; \gamma_j^{ind})$, or both. This means an increase in the influence of γ_i^{ind} (relatively risk-averse individuals), a decrease in the influence of γ_j^{ind} (relatively risk-tolerant individuals), or both. This is because an increase in the optimal aggregate consumption share of individuals with γ^{ind} means an increase in the influence of γ^{ind} on the aggregate risk aversion (see Appendix for detail). In any case, as the social weight of relatively risk-averse individuals increases, the aggregate risk attitude increase.[7]

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[7] As shown by Schildberg-Hörisch (2018) and Guiso et al. (2018), if the economic downturn makes each individual's risk aversion more risk-averse, then the aggregate risk attitude becomes counter-cyclical. This makes the counter-cyclicality suggested by the conventional Chan and Kogan model more prominent.

3.2. *Status quo hypothesis: switching loss aversions*

Next, we consider the case where individuals with a certain risk aversion shifts to be loss averse. We provide the *status quo hypothesis* by considering this case. This case holds almost the same argument as the case of switching a given distribution of social weights substantially. This is because an increase in the parameter representing loss aversion in individuals with a certain risk aversion γ^{ind} has the same effect as an increase in its social weight.

To investigate this effect, eq. (14) is rewritten as follows.

$$\begin{gathered}\hat{U}(Y_t, X_t; k) = \sup_{\{c_t(Y_t, X_t; \gamma^{ind}); \gamma^{ind} > 1\}} \int_1^\infty f_k(\gamma^{ind}) \frac{\lambda_k(\gamma^{ind})}{1-\gamma^{ind}} \left(\frac{C_t(Y_t, X_t; \gamma^{ind})}{X_t} \right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind}, \\ \text{s.t.} \int_1^\infty C_t(Y_t, X_t; \gamma^{ind}) \mathrm{d}\gamma^{ind} \leq Y_t. \end{gathered} \tag{17}$$

where $\lambda_k(\gamma^{ind})$ is the parameter of loss-aversion for individuals with γ^{ind} in the latent state k.[8] An increase in loss aversion implies that individuals facing loss and wanting to reduce their consumption cannot reduce their consumption as they wish. This slowdown in consumption decline may be due to the status quo propensity.[9] This may represent the cost stickiness in households and businesses.

The optimal aggregate consumption shares of individuals with γ^{ind} in latent state k are given below.

$$\begin{gathered}\hat{c}^*_{kt}(z_t; \gamma^{ind}) = \left(\lambda_k(\gamma^{ind}) f_k(\gamma^{ind}) \right)^{1/\gamma^{ind}} \exp\left\{ -\frac{1}{\gamma^{ind}} \hat{h}_k(z_t) - z_t \right\}, \\ \text{where} \int_1^\infty \hat{c}^*_{kt}(z_t; \gamma^{ind}) \mathrm{d}\gamma^{ind} = 1. \end{gathered} \tag{18}$$

In the same way as eq. (16), we have

$$\frac{\lambda_k(\gamma_i^{ind}) f_k(\gamma_i^{ind})}{\lambda_k(\gamma_j^{ind}) f_k(\gamma_j^{ind})} = \frac{c^*_{kt}(z_t; \gamma_i^{ind})^{\gamma_i^{ind}}}{c^*_{kt}(z_t; \gamma_j^{ind})^{\gamma_j^{ind}}} \exp\{(\gamma_i^{ind} - \gamma_j^{ind}) z_t\}. \tag{19}$$

When $f_k(\gamma_i^{ind}) / \left(\lambda_k(\gamma_j^{ind}) f_k(\gamma_j^{ind}) \right)$ is fixed, an increase in the loss aversion $\lambda_k(\gamma_i^{ind})$ means an increase in $c^*_{kt}(z_t; \gamma_i^{ind})$, a decrease in $c^*_{kt}(z_t; \gamma_j^{ind})$, or both. As with social weights, in any case, as the loss-aversion of relatively risk-averse individuals increases, the aggregate risk attitude increase. Inversely, as the loss-aversion of relatively risk-tolerant individuals increases, the aggregate risk attitude decrease. If

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[8] The loss avoidance parameter λ does not necessarily have to be the same for individuals with the same γ^{ind}. For example, if individuals with $\gamma^{ind} = 2$ and $\lambda = 3$ and individuals with $\gamma^{ind} = 2$ and $\lambda = 1$ divide the social weight belonging to individuals with $\gamma^{ind} = 2$ in half, these can be regarded as a single group with $\gamma^{ind} = 2$ and $\lambda = 1.5$.

[9] At a glance, the slowdown in consumption decline may also be explained by switching to be more risk-averse. This is because C/X is generally a very small value and eq. (1) implies $u_C = X^{-1}(C/X)^{-\gamma^{ind}}$, and consequently, the more risk averse, the greater the marginal utility of consumption. The large γ^{ind}, however, makes u_C extremely volatile because exponential terms are included in $u_{CC} = -\gamma^{ind} X^{-2}(C/X)^{-\gamma^{ind}-1}$ and $u_{CX} = (\gamma^{ind} - 1) X^{-2}(C/X)^{-\gamma^{ind}}$. This property is not suitable for expressing the rigidity of the consumption level. On the other hand, the switch to loss-averse preference changes u_{CC} and u_{CX} only in a multiplier, which prevents u_C from becoming extremely volatile. Studies such as Guiso et al. (2018) that suggest that people become more risk-averse during an economic downturn may not be able to distinguish between switching to be loss-averse and risk-averse.

relatively risk-tolerant individuals become loss-averse due to the economic downturn, the influence of relatively risk-tolerant individuals on the aggregate risk aversion will increase. That is, the aggregate risk aversion decreases in the economic downturn, or is procyclical. Note that when all individuals become homogeneously loss-averse, the optimal aggregate consumption share of each individual does not change.

The relationship between the aggregate risk attitude and the state of the economy is summarized below as the status quo hypothesis. Relatively risk-tolerant individuals are more likely to face losses than relatively risk-averse individuals during recession because the former allocate more to risk assets than the latter. The aggregate risk aversion will be procyclical when the effect of such heterogeneous loss-aversion outweighs the effect that lead to counter-cyclicality such as increasing the social weight of relatively risk-averse individuals (e.g. Guiso et al., 2018) and increasing the influence of relatively risk-averse individuals on the aggregate risk aversion (e.g. Chan and Kogan, 2002). A serious economic downturn such as 2008 global financial crisis and 2020 COVID-19 crisis is expected to make aggregate risk aversion counter-cyclical again because it homogenizes the loss-aversion of each individual.

4. Empirical evidences

This section uses data to analyze how the aggregate risk attitude fluctuate in response to changes in the state of the economy. First, we construct empirical models with time-varying (state-dependent) aggregate risk attitude. Second, we explain our data for market returns in some main market and provide simple analysis. Third, we provide evidence that the aggregate risk attitudes in stock markets of Western countries are time-varying, and the status quo hypothesis is supported for the time-variability.

4.1. Empirical model

The aggregate risk attitude can be expressed as a function of the state of the economy, which represents the business-cycle of the economy. Most conventional models imply that the aggregate risk aversion is monotonic decreasing function of state variable. In contrast, our status quo hypothesis implies that the effect of state variable on aggregate risk attitudes depends on the level of state variable (or business sentiment). In other word, the function $\gamma(z)$ may have one or more inflection points. For this reason, we make some polynomial approximations to capture the various relationships between the state variable and the aggregate risk aversion:

$$\gamma(z) = \begin{cases} \delta_0 + \delta_1(1+z) \\ \delta_0 + \delta_1(1+z) + \delta_2(1+z)^2 \\ \delta_0 + \delta_1(1+z) + \delta_2(1+z)^2 + \delta_3(1+z)^3 \\ \delta_0 + \delta_1(1+z) + \delta_2(1+z)^2 + \delta_3(1+z)^3 + \delta_4(1+z)^4 \end{cases} \tag{20}$$

The first line in eq. (20), linear model, corresponds to the Campbell and Cochrane (1999) model, Chan and Kogan (2002) model, and other main conventional models if $\delta_1 < 0$. The second to fourth lines, quadratic to quartic models, approximate the situation where the inflection point exists. Our status quo hypothesis suggests procyclicality during the recession and counter-cyclicality during the boom and depression. Therefore, the status quo hypothesis is supported by the cubic function model. Note that these parameters, δs, are not constrained to flexibly capture market movements.

Moreover, an appropriate κ is required to specify the state variable since $z_t \approx z(\psi_t;\kappa)$ from eq. (10). We consider 9 patterns of κ; 0.02, 0.025, 0.0325, 0.0375, 0.045, 0.055, 0.075, 0.115 and 0.25. From eq. (12), these mean that the peaks in the distribution of past time referred to in the weekly data are 1 year, 9 months, 7 months, 6 months, 5 months, 4 months, 3 months, 2 months, and 1 month, respectively (see for details in Section 2.2). As a result, the number of models is 4 × 9 = 36. Let the model with the smallest AIC (Akaike's information criterion) be the time-varying or state-dependent model. We also consider a constant model, which has a constant aggregate risk aversion, as a benchmark model. That is, $\gamma(z) = \delta_0$.

We use the weighted average of historical returns in the stock market as the proxy for state variable. Chan and Kogan (2002) used "relative log-consumption" as a variable to capture the state of the economy. Data representing consumption or endowment, such as household consumption, CPI, or GDP, are less frequent than stock return, however. Furthermore, these data are lagging indices related to the business cycle, and it must be said that consumption as data is a little slow to show the business sentiment that investors feel. In fact, the stock market index will be the most important factor for investors to judge the future growth potential of consumption, productivity and endowment in the economy. Therefore, as a variable to capture the state of the economy, we use a variable in which the aggregate consumption or endowment growth rate in the relative log-consumption is replaced by the stock market return.

We use GARCH-in-mean (generalized autoregressive conditional heteroskedasticity in mean) model proposed by Engle, Lilien and Robins (1987) in order to get the aggregate risk attitude of the economy:

$$r_{Mt} - r_{ft} = \gamma(z_{t-1})\sigma^2_{t|t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}\big(0, \sigma^2_{t|t-1}\big). \tag{21}$$

where r_{Mt} and r_{ft} are the stock market return and risk-free rate at time t, respectively. We assume that the dynamics of the conditional variance follows the GARCH (1,1) process:

$$\sigma^2_{t|t-1} = \alpha_0 + \alpha_1 e^2_{t-1} + \beta\sigma^2_{t-1|t-2}, \tag{22}$$

where e_{t-1} is the residual at time $t-1$, and α_0, α_1 and β are constant. The specification corresponds to the discrete-time capital asset pricing model with the time-varying state-dependent aggregate risk aversion. [10]

Ultimately, the only variables needed in our model are market returns and risk-free rates. Such simplicity can suppress the problems of variable selection arbitrariness and overfitting.

[10] Chan and Kogan (2002) imply that the Sharpe ratio of the single risky asset is expressed as the product of the state-dependent aggregate risk attitude and the volatility of the aggregate endowment growth rate. Since the volatility of the aggregate endowment growth rate in the Chan and Kogan model is treated as a constant, it is necessary to verify the state dependence of the Sharpe ratio rather than the state dependence of the aggregate risk attitude in order to accurately verify the Chan and Kogan model. As mentioned above, Statistical data on consumption and endowment do not necessarily represent the intrinsic aggregate endowment growth rate, and there is no guarantee that its volatility is constant. We use market returns as a proxy for aggregate endowment growth, and finally, we obtain eq. (21).

4.2. *Data and summary*

The data required in this paper are (log) market returns and (log) risk-free rates. We analyze the 10 major stock markets to investigate whether the mechanism for time-variability in the aggregate risk attitude varies from country to country; Canada, China, Eurozone, France, Germany, Hong Kong, India, Japan, the United Kingdom, and the United States. The index of each market is used for the weekly market return; TSX, SSE, STOXX 600, CAC 40, DAX, HSI, SENSEX, TOPIX, FTSE 100, and S&P 500, respectively. The risk-free rate of each market is used for the 3 months interbank rate of each market or currency except for Canada and China. The risk-free rate of Canada is used for Canadian 3 months government bond yield, and that of China is used for Chinese 3 months interbank repo rate. The length of the sample period is 783 weeks from July 1, 2005 to June 26, 2020 in order to have the same period in each market.[11] We need, however, longer sample to eliminate the arbitrariness of the initial value for the state variable z because the state variable constructed by historical stock market returns in discrete-time framework can be rewritten as $z_t = r_{Mt} + (1-\kappa)z_{t-1} = (1-\kappa)^t z_0 + \sum_{s=0}^{t-1}(1-\kappa)^s r_{M,t-s}$ from eq. (9). We set the initial value of $z_0 = y_0 - x_0$ to zero and burn-in-period to 100 weeks (about 2 years). Therefore, the precise length of market returns is 883 weeks.[12]

Table 1 shows the summary statistics of logarithmic returns on 10 market indices. The skewness of the market index is negative in all markets. The kurtosis of the market index is well above 3 in all markets. All distributions of market indices are distorted in the negative direction and are fat-tail. This is as previously pointed out, and these characteristic does not change in emerging markets such as China and India. The rightmost column shows the correlation with the US market. It can be seen that the stock markets in Western countries including Canada, Eurozone, France, Germany, and the UK are highly correlated with the US. On the other hand, the stock markets of Asian countries including China, Hong Kong, India and Japan have extremely low correlation with the US.

[Table 1]

4.3. *Results*

We analyze the structure of state-dependent aggregate risk attitude specified by equations (20) and (21) with conditional volatility specified by eq. (22). Table 2 shows the results of constant and state-dependent aggregate risk attitude model with conditional volatility. The constant aggregate risk attitude model in the Canadian stock market provides the estimate of constant term δ_0 is 2.589. Its standard error is 1.312, and the parameter is statistically significant at 5% level. The AIC (Akaike's information criterion) of the constant model is 3,225.055. On the other hand, the state-dependent aggregate risk attitude model with cubic function and $\kappa = 0.0375$ in the Canadian stock market has the smallest AIC. $\kappa = 0.0375$

[11] It also aims to eliminate the period when the exchange rates of each country are unstable due to the introduction of the euro.

[12] All these data are taken from Bloomberg Professional.

implies the peak of individual reference time point is 6 months ago (see also 2.2 in this paper for details). All estimates of coefficients, δ_0, δ_1, δ_2 and δ_3, are statistically significant at 5% level. The AIC of state-dependent model is 3,222.594 and smaller than that of constant model. In addition, likelihood ratio test (LR test in Table 2) shows that the null hypothesis that the constant model is correct is rejected at the 5% level. To sum up, the aggregate risk attitude in the Canadian market is state-dependent.

In the same way, it can be seen that stock markets in Western countries, including Canada, Eurozone, France, Germany, the UK and the US have a representative individual with state-dependent aggregate risk attitude. Inversely, representative individuals in Asian stock markets, including China, Hong Kong, India and Japan have constant aggregate risk attitudes. In the Chinese market, the state-dependent model has some insignificant parameters in spite of being selected by the likelihood ratio test and AIC. In the Indian market, the state-dependent model is not selected by the likelihood ratio test and AIC although all parameters are significant. The constant model is generally supported in the Asian stock markets. At least, polynomial approximations from linear to quartic functions cannot capture their state-dependence or time-variability.

[Table 2]

Figure 1 shows the structure of aggregate risk attitude with conditional volatility. The horizontal and vertical axes represent the state variable and the aggregate risk attitude, respectively. Western stock markets have similar cubic structure. On the other hand, the structures of aggregate risk attitudes in Asian stock markets are very different from those in Western stock markets and are different from each other. We will consider only the Western market from now on since the state-dependent model with polynomial approximations is not supported in the Asian stock market.[13]

Cubic curves in these markets commonly have the first inflection point or local minimum point around -0.25, and the second inflection point or local maximum point near 0. The decreasing structure in the range over 0, the movement between the upper left and the lower right, corresponds to the counter-cyclicality of aggregate risk attitude at times when most individuals face gain, that is, booms. It can be seen that the conventional model such as Campbell and Cochrane (1999) and Chan and Kogan (2002) holds in such a stable economic environment.

The aggregate risk attitude, however, become increasing function of state variable in the range -0.25 to 0. This structure corresponds to the procyclicality of aggregate risk attitude at times when risk-taking individuals face loss, that is, recessions. This implies two hypotheses as shown in Section 3. The first hypothesis is that the social weight of relatively risk-tolerant individuals increases due to the economic

[13] We also analyze the Sharpe ratio by a similar way, in order to match the formation of Chan and Kogan (2002) model (not listed in table). That is, we use following model instead of eq. (21).

$$\mu_t - r_{ft} = \gamma(z_{t-1})\sigma\sigma_{t|t-1} + \epsilon_t, \qquad \epsilon_t \sim \mathcal{N}\left(0, \sigma^2_{t|t-1}\right) \tag{21$'$}$$

where σ is the constant volatility of consumption (or endowment) growth. Results in the US stock market and German stock market also support our status quo hypothesis while results in the other markets reject our status quo hypothesis. It may be necessary to consider the time-variability of the consumption volatility, the dependence of the market on the US market, and the effects of exchange rates.

downturn. The second hypothesis, or the status quo hypothesis, is that the relatively risk-tolerant individuals will switch to more loss-averse preferences in order to maintain their status quo as the economy deteriorates.[14]

The decreasing structure outside the range below -0.25 corresponds to the counter-cyclicality of aggregate risk attitude at times when most individuals face loss, that is, depressions. The first hypothesis cannot explain this counter-cyclicality. This is because if an individual facing a loss becomes risk-tolerant, the aggregate risk attitude should still be procyclical during depressions. On the other hand, the second status quo hypothesis can explain this counter-cyclicality because the depression provides almost homogeneous switching to loss aversions. In other words, the depression causes loss not only to individuals who take risks but also to individuals who do not take risks, and eliminates distortions based on differences in preferences. Consequently, the relationship between aggregate risk attitude and state variable becomes the same negative relationship as during the boom period.

[Figure 1]

Finally, the relationship between inflection points and the state variable z is described. Figure 2 show the time-series of z and inflection points. Areas filled in purple (blue) mean that the state of the economy is below the first (second) inflection point. The first inflection point is around -0.25 and the second inflection point is around 0 as can be seen from Figure 1. The purple areas generally correspond to the 2008 global financial crisis and the 2020 COVID-19 crisis. During such a serious crisis, almost all individuals may switch to loss-averse preferences in order to maintain their status quo. The blue areas are observed in 2011 and 2016-2017, except around those two crises. The former may respond to the Greek and Spanish crises, and the latter may respond to the Brazilian economic crisis. It can be said that shocks that are not due to their own country cannot reduce the wealth of relatively risk-averse individuals who do not take risks, except for two global crises.

[Figure 2]

5. Conclusion

We improved Chan and Kogan (2002) model and constructed a new behavioral asset pricing model with heterogeneous individuals whose preferences change depending on the state of the economy. Based on this model, we proposed a status quo hypothesis. In our status quo hypothesis, individuals facing loss switch to loss-averse preferences to maintain status quo. In the boom period, most individuals face gains, so the aggregate risk attitude is countercyclical as in the conventional model. In a minor recession, the effect of switching to loss averse preference on aggregate risk attitudes is heterogeneous, as only risk-taking individuals face significant losses. As a result, the aggregate risk attitude becomes procyclical. On the other

[14] A hypothesis in which the proportion of risk-loving individuals increases can also be considered. In this case, the general equilibrium theory may not hold. This hypothesis is beyond the scope of this paper. Recent studies have also developed a general equilibrium model of the economy, including risk-loving individuals (Araujo et al., 2017; Araujo et al., 2018).

hand, in a major recession, or depression, almost all individuals face large losses. The effect of loss aversion on aggregate risk attitudes is nearly homogeneous. Consequently, the aggregate risk attitude becomes countercyclical again.

Next, the structure of aggregate risk attitude was approximated by a polynomial to take into account the possibility of multiple inflection points. As a result, the structures of aggregate risk attitudes in the stock markets of Western countries including Canada, Eurozone, France, Germany, the UK and the US could be well approximated by a cubic function. This cubic function implies the counter-cyclicality of these aggregate risk attitudes during booms and depressions and the procyclicality during recessions. This result supports our status quo hypothesis. We conclude that it is essential to allow both heterogeneity and changeability of preferences in order to determine an accurate aggregate risk attitude.

Appendix

We provide a formal proof that an increase in the optimal aggregate consumption share of individuals with γ^{ind} means an increase in the influence of γ^{ind} on the aggregate risk aversion, while this is intuitively clear.

Chan and Kogan (2002) proved that the aggregate risk aversion is expressed as follows.

$$\gamma(z_t) = \left(\int_1^\infty \frac{1}{\gamma^{ind}} c_t^*(z_t;\gamma^{ind})\mathrm{d}\gamma^{ind}\right)^{-1} \quad \text{s.t.} \quad \int_1^\infty c_t^*(z_t;\gamma^{ind})\mathrm{d}\gamma^{ind} = 1. \tag{A1}$$

We focus on a specific range $(\underline{\gamma}^{ind}, \bar{\gamma}^{ind})$ and define $\hat{c}_t^*$, γ_1^{ind} and γ_2^{ind} as follows.

$$\hat{c}_t^* = \int_{\underline{\gamma}^{ind}}^{\bar{\gamma}^{ind}} c_t^*(z_t;\gamma^{ind})\mathrm{d}\gamma^{ind} \tag{A2}$$

$$\gamma_1^{ind} = \frac{\hat{c}_t^*}{\int_{\underline{\gamma}^{ind}}^{\bar{\gamma}^{ind}} \frac{1}{\gamma^{ind}} c_t^*(z_t;\gamma^{ind})\mathrm{d}\gamma^{ind}}, \tag{A3}$$

$$\gamma_2^{ind} = \frac{1-\hat{c}_t^*}{\int_{\gamma^{ind}\in(1,\infty)\setminus(\underline{\gamma}^{ind},\bar{\gamma}^{ind})} \frac{1}{\gamma^{ind}} c_t^*(z_t;\gamma^{ind})\mathrm{d}\gamma^{ind}} \tag{A4}$$

$\hat{c}_t^*$ means the aggregate consumption share of the individuals with $\gamma^{ind} \in (\underline{\gamma}^{ind}, \bar{\gamma}^{ind})$. γ_1^{ind} means the aggregate risk aversion of only these individuals. γ_2^{ind} means the aggregate risk aversion except for these individuals. For example, if $\gamma_1^{ind} < \gamma_2^{ind}$, the individuals with $\gamma^{ind} \in (\underline{\gamma}^{ind}, \bar{\gamma}^{ind})$, or simply γ_1^{ind}, mean relatively risk-tolerant individual than representative (aggregate) individual since $\gamma_2^{ind} \approx \gamma$ for small $\hat{c}_t^*$. Eq. (A1) can be rewritten as follows.

$$\gamma(z_t, \hat{c}_t^*) = \left(\frac{1}{\gamma_1^{ind}}\hat{c}_t^* + \frac{1}{\gamma_2^{ind}}(1-\hat{c}_t^*)\right)^{-1}. \tag{A5}$$

Consequently,

$$\frac{\partial\gamma(z_t,\hat{c}_t^*)}{\partial\hat{c}_t^*} = -\gamma^2\left(\frac{1}{\gamma_1^{ind}} - \frac{1}{\gamma_2^{ind}}\right) = -\frac{\gamma^2}{\gamma_1^{ind}\gamma_2^{ind}}(\gamma_2^{ind} - \gamma_1^{ind}). \tag{A6}$$

If $\gamma_1^{ind} < \gamma_2^{ind}$, an increase in the optimal aggregate consumption share of individuals with $\gamma^{ind} \in (\underline{\gamma}^{ind}, \bar{\gamma}^{ind})$, who are relatively risk-tolerant individuals, decrease the aggregate risk aversion. This implies that the representative (aggregate) individual become more risk-tolerant. Similarly, increasing the optimal aggregate consumption share of relatively risk-averse individuals makes representative individual more risk-averse.

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Table 1
Summary statistics

	Mean	Std. Dev	Skew	Kurt	Min	Med	Max	Corr
Canada	0.053	2.425	-1.514	14.405	-17.542	0.293	12.817	0.822
China	0.127	3.411	-0.359	5.512	-14.898	0.237	13.945	0.191
Eurozone	0.033	2.717	-1.790	16.755	-24.252	0.314	12.437	0.834
France	0.020	3.069	-1.435	12.989	-25.050	0.268	12.432	0.812
Germany	0.124	3.150	-1.249	12.513	-24.347	0.441	14.942	0.813
Hong Kong	0.070	2.967	-0.333	6.075	-17.815	0.327	11.719	0.604
India	0.203	2.990	-0.475	6.948	-17.381	0.378	13.171	0.557
Japan	0.038	2.926	-1.044	9.202	-22.019	0.338	12.871	0.620
UK	0.025	2.546	-1.639	18.139	-23.632	0.200	12.584	0.829
US	0.118	2.534	-1.131	12.747	-20.084	0.253	11.424	1.000

Note: Results are based on market index returns [%, weekly] from 1 Jul 2003 to 26 Jun 2020 (783 weeks).
The statistics of “Corr” means the correlation with the US stock returns.

Table 2

GARCH estimation (1/2)

	Canada		China		Eurozone		France		Germany	
	constant	time-varying	constant	time-varying	constant	time-varying	constant	time-varying	constant	time-varying
α_0	0.277 ***	0.267 ***	0.245 ***	0.248 ***	0.490 ***	0.462 ***	0.528 ***	0.489 ***	1.122 ***	1.000 ***
	(0.066)	(0.063)	(0.094)	(0.095)	(0.130)	(0.124)	(0.164)	(0.153)	(0.313)	(0.269)
α_1	0.228 ***	0.229 ***	0.136 ***	0.135 ***	0.237 ***	0.232 ***	0.173 ***	0.163 ***	0.267 ***	0.255 ***
	(0.035)	(0.035)	(0.029)	(0.029)	(0.042)	(0.042)	(0.035)	(0.034)	(0.051)	(0.049)
β	0.726 ***	0.727 ***	0.845 ***	0.847 ***	0.712 ***	0.721 ***	0.778 ***	0.791 ***	0.634 ***	0.656 ***
	(0.035)	(0.035)	(0.030)	(0.030)	(0.046)	(0.045)	(0.043)	(0.041)	(0.065)	(0.058)
δ_0	2.589 **	899 **	-0.368 -	73 -	3.079 ***	505 **	2.441 **	736 **	3.411 ***	979 ***
	(1.312)	(364)	(0.979)	(49)	(1.118)	(235)	(1.098)	(298)	(0.999)	(307)
δ_1		-3,221 **		-163 -		-1,842 **		-2,605 **		-3,431 ***
		(1,284)		(100)		(843)		(1,048)		(1,064)
δ_2		3,776 **		88 *		2,200 **		3,026 **		3,944 ***
		(1,484)		(50)		(993)		(1,208)		(1,211)
δ_3		-1,449 **				-858 **		-1,153 **		-1,485 ***
		(564)				(384)		(458)		(453)
κ		0.0375		0.1150		0.0450		0.0450		0.0750
		(6 months)		(2 months)		(5 months)		(5 months)		(3 months)
AIC	3,225.055	3,222.594	3,961.982	3,960.718	3,558.762	3,557.803	3,794.387	3,793.196	3,824.198	3,820.795
LR test		8.4604 **		5.2637 *		6.9589 *		7.1914 *		9.4026 **

Note: Values in parentheses denote standard errors. ***,** and * denote statistical significance at 1, 5 and 10% levels, respectively.

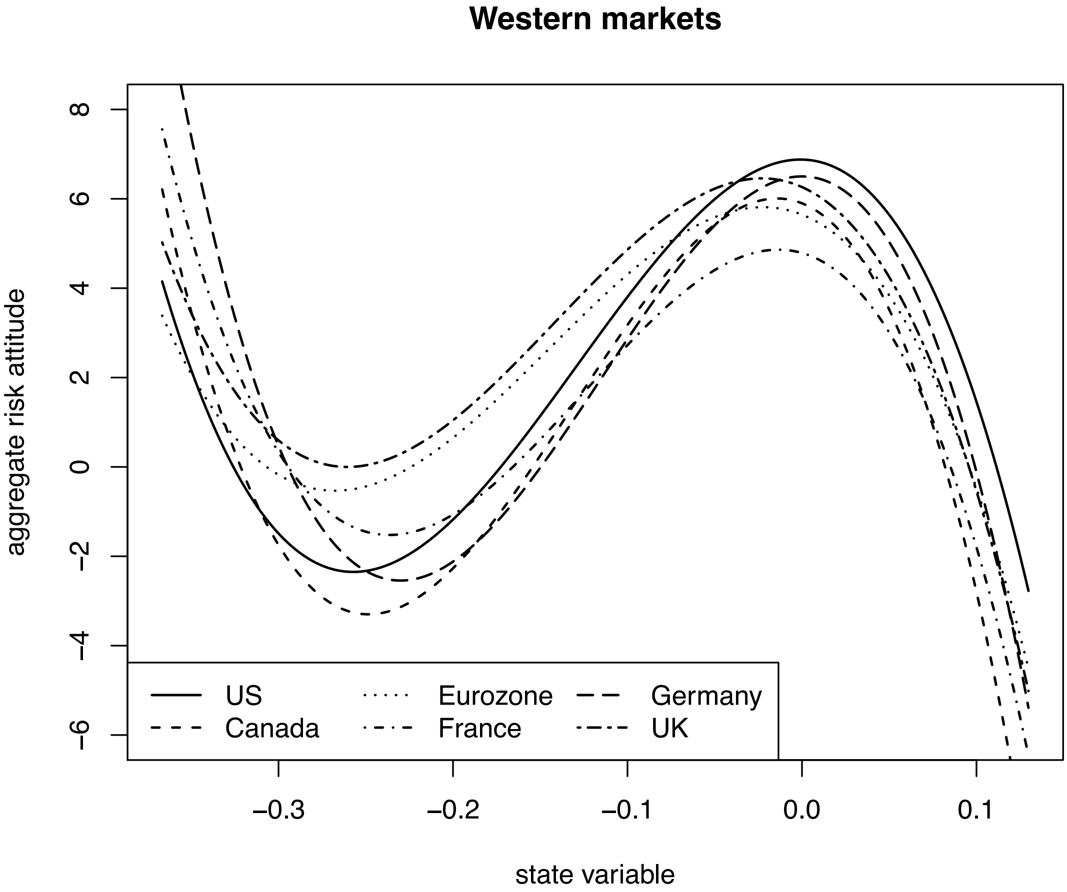
Table 2

GARCH estimation (2/2)

	Hong Kong		India		Japan		UK		US	
	constant	**time-varying**	**constant**	**time-varying**	**constant**	**time-varying**	**constant**	**time-varying**	**constant**	**time-varying**
α_0	0.227 **	0.227 **	0.311 ***	0.305 ***	1.544 ***	1.520 ***	0.429 ***	0.394 ***	0.445 ***	0.425 ***
	(0.097)	(0.097)	(0.101)	(0.102)	(0.542)	(0.525)	(0.117)	(0.114)	(0.107)	(0.101)
α_1	0.081 ***	0.080 ***	0.160 ***	0.162 ***	0.251 ***	0.251 ***	0.215 ***	0.200 ***	0.285 ***	0.273 ***
	(0.021)	(0.021)	(0.031)	(0.032)	(0.053)	(0.052)	(0.041)	(0.041)	(0.042)	(0.041)
β	0.891 ***	0.892 ***	0.804 ***	0.802 ***	0.573 ***	0.576 ***	0.731 ***	0.749 ***	0.648 ***	0.661 ***
	(0.028)	(0.027)	(0.037)	(0.038)	(0.098)	(0.095)	(0.046)	(0.047)	(0.045)	(0.043)
δ_0	1.769 -	6.403 -	1.965 *	186 **	2.618 **	-0.439 -	3.227 **	578 *	4.130 ***	680 **
	(1.197)	(6.262)	(1.133)	(86)	(1.145)	(5.639)	(1.265)	(323)	(1.151)	(271)
δ_1		-4.704 -		-667 **		3.185 -		-2,091 *		-2,443 **
		(6.267)		(299)		(5.735)		(1,149)		(973)
δ_2				764 **				2,484 *		2,867 **
				(334)				(1,336)		(1,141)
δ_3				-280 **				-965 *		-1,097 **
				(120)				(508)		(439)
κ		0.0200		0.0325		0.0200		0.0200		0.0375
		(1 year)		(7 months)		(1 year)		(1 year)		(6 months)
AIC	3,797.196	3,798.647	3,693.526	3,694.102	3,780.373	3,782.059	3,460.558	3,459.313	3,338.053	3,335.517
LR test		0.5491 -		5.4234 -		0.3139 -		7.2458 *		8.5357 **

Note: Values in parentheses denote standard errors. ***,** and * denote statistical significance at 1, 5 and 10% levels, respectively.

Figure 1
Structure of aggregate risk attitude



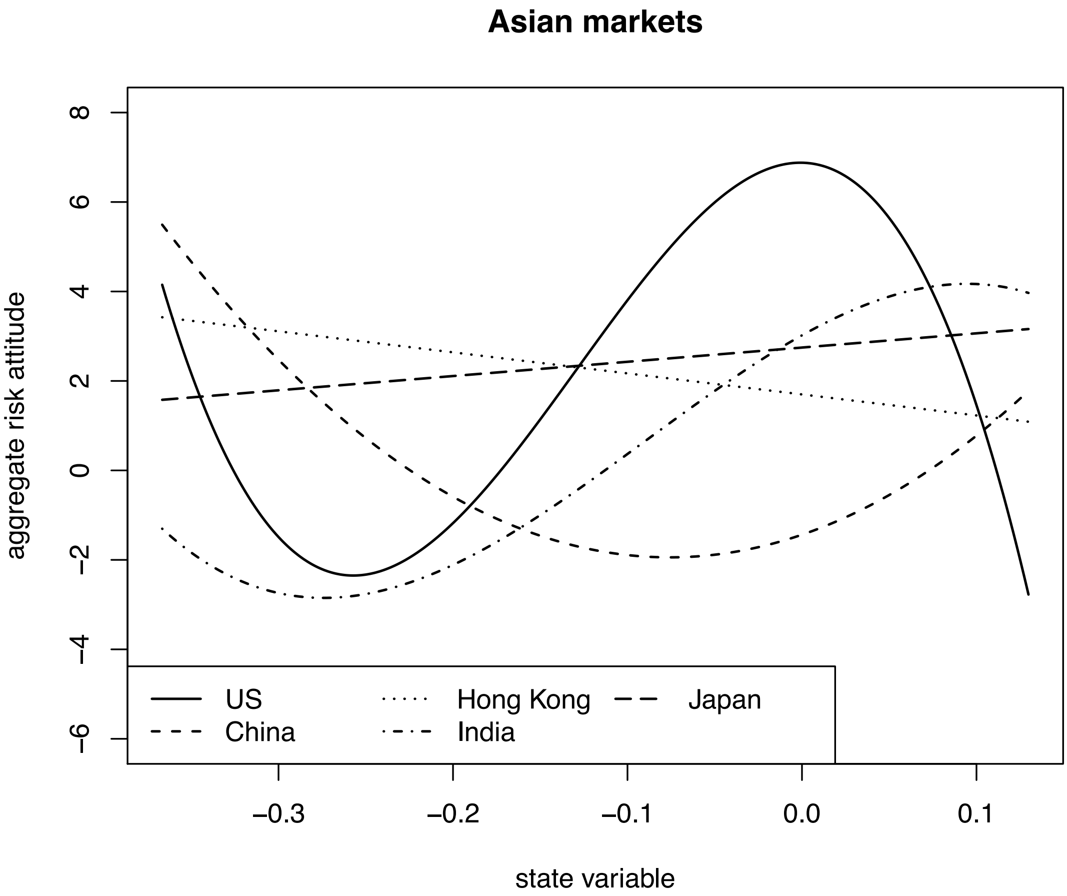


Figure 2
Time-series of state variable and inflection points