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Bilateral Integration Measures and Risk Attitudes in Large Stock Markets

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Bilateral Integration Measures and Risk Attitudes in Large Stock Markets[1]

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Abstract

This paper examines whether developed markets are more internationally integrated than emerging markets. A new bivariate regime switching model is constructed in order to take into account both international integration regime and segmentation regime, capture the endogenous and interactive effects between large markets, and pay attention to the economic structure of the price of variance risk. We estimated such regime switching model for 24 large stock markets and the US market as a reference market. That is, the regime reflects whether each of the 24 markets is integrated with the US. As a result, the structures of representative investor's risk attitude, or that of the price of variance risk, in each of the following 15 markets are almost the same; Canada, France, Italy, Australia, Hong Kong, Netherlands, Spain, Sweden, Switzerland, Brazil, South Korea, Taiwan, Indonesia, Mexico and Saudi Arabia. In such markets, these "international integration measures" defined as the (smoothed) probability of international integration regime are on average high, declining before the 2008 global financial crisis, but rising again after the crisis. This means that non-home-biased strategies such as an international diversification have advantages over home-biased strategies such as a domestic concentration except just before the crisis. In addition, the difference between the international integration measures of developed and emerging markets included in these markets is extremely small. In other words, being an emerging market does not mean that the market is segmented. Summing up the above results, it can be concluded that the international diversification is strongly recommended in these markets regardless of country or period.

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1. Introduction

The issue of “market scope” is a crucial issue in financial economics. Investors who misunderstand the true market scope may endure suboptimal strategies for their stockholdings, even if they have the ability to execute optimal strategies. We need to know the true scope of stock markets in order to avoid such failures and construct a new model considering the following three points; researchers’ arbitrary assumptions about market scopes, interdependence between stock markets, and the time-variability of the price of variance risk.

First, we consider the arbitrariness of market scopes. Many of the models for analyzing stock prices, such as the standard CAPM (capital asset pricing model) or standard multi-factor models, target the domestic stock market and do not consider the effects of foreign stock markets. If we use these models, we are implicitly assuming that the stock market is independent of the foreign stock market. In other words, investors recognize an internationally segmented market of only one country. On the other hand, with the rapid globalization in recent years, more and more evidence of interaction between stock markets has been observed. Some researchers think that all stock markets should be considered a single, integrated market (e.g. international CAPM). That is, investors recognize an internationally integrated market including multiple countries. In any case, it is a problem to “arbitrarily” decide that the market is internationally segmented or integrated.

Studies on international comparison between stock markets often assume that developed markets are integrated with each other and emerging markets are at least partially segmented. De Jong and de Roon (2005) classified stocks in emerging stock markets based on the investable asset index and the non-investable asset index provided by IFC (International Finance Corporation). They defined the ratio of non-investable assets in the total of both assets as the degree of international segmentation. All stocks in developed stock markets are treated as investable assets in their setting so that these markets are perfectly integrated. Karolyi and Wu (2018) defined “globally accessible securities” as securities traded on the market with few barriers to foreign investors. These securities are chosen in the spirit of the “global, long-term view” in Sarkissian and Schill (2016). In their definition, major stock markets in the US and Europe are perfectly integrated with each other and other markets are at least partially segmented. These studies clearly show the interaction between stock markets and these are different from conventional studies on the only domestic stock market. However, these have similar problems to such conventional studies in that these “arbitrarily” assume market scopes. It is necessary to construct a model that estimates market scopes instead of assuming them in order to find the optimal strategy.

Bekaert and Harvey (1995) estimated the degree of market integration between global market and emerging markets using a two-regime switching model. In their model, the international CAPM is applied when the global market and the emerging market are integrated and the domestic CAPM is applied when these are segmented. One of the only minor drawbacks of their model is that they can only analyze emerging markets and not developed markets, that is, most of the world market capitalization.

Second, we discuss the interdependence between stock markets. Pukthuanthong and Roll (2009) and Akbari, Ng and Solnik (2020) quantified market integration in a simple and elegant way. They expressed market integration by using the coefficient of determination when each market return or part of it was regressed on one of the global factors. Such global factors are constructed from the value-weighted returns of major developed markets or all markets analyzed. These are strongly affected by large stock markets such as the G20 markets and cannot be treated as pure exogenous variables when analyzing such markets. This problem of simultaneity is also the reason why the Bekaert and Harvey (1995) model cannot be applied to the developed markets.

It is ideal to construct a multivariate time series model with the returns of all markets as the explained variables in order to consider the endogeneity of the global factor. Then, the correlations of the error terms will be set to non-zero values. However, it is very difficult to estimate a large multivariate time series model, and even if it can be estimated, there is concern that the estimation accuracy will deteriorate. Therefore, this paper builds a bivariate model with the US market as a reference market and analyzes whether the US integrates with any market. The basic structure of the model is a bivariate GARCH-in-Mean (generalized ARCH-in-Mean) model with two regimes modified from the Bekaert and Harvey (1995) model. The smoothed probability of the integration regime was interpreted as an "international integration measure". A high international integration measure means that non-home-biased strategies such as international diversification have advantages over home-biased strategies such as domestic concentration.

Third, we regard the time-variability of the price of variance risk. This paper stands in the position that market integration should be estimated without arbitrary assumptions. The specification for the two models that make up the regime switching model, the international CAPM and the domestic CAPM, is refined in order to make a more reasonable estimate. We focus on the "price of variance risk", which is an important parameter in CAPM and is defined as the ratio of the market's expected excess return to its variance. The price of variance risk is treated as a constant in the one-period CAPM, but not in the multi-period or conditional CAPM. The price of (conditional) variance risk is defined as the ratio of the market's conditional expected excess return to its conditional variance and is time-varying. Harvey (1991), Bekaert and Harvey (1995) and De Santis and Gerard (1997) show empirical results that the price of variance risk is time-varying. However, they do not show "how" and "why" the price of variance risk is time-varying. Their analysis did not provide a sufficient economic interpretation of its time-variation. Therefore, this paper uses the Chan and Kogan (2002) model (hereafter, CK model), which is one of the continuous-time habit-formation consumption-based asset pricing models, to specify the time-variation of the price of variance risk. They defined the "state of the economy" as a weighted average of aggregate consumption in the past and showed that the relative risk aversion of representative investors in that economy is counter-cyclical in their model. Converting the consumption-based asset pricing model to the conventional CAPM format by replacing the growth of the aggregate endowment with the market return, the relative risk aversion of a representative investor equals the price of variance risk. The state of the economy is defined as a weighted average of past market returns and can be rewritten as a weighted average of market returns from each past date to the current date. These weights of the latter can be interpreted as expressing the distribution of rebalancing cycles of investors participating in the market.

The relative risk aversion of the representative investor, or the price of variance risk, is expressed as a function of the state of the economy and is expected to be a monotonically decreasing function according to the CK model. This means the price of variance risk is countercyclical. It would be sufficient to express the price of variance risk by a linear function of the state of the economy if it is always a decreasing function. It may, however, not always hold. The model assumes that individuals may have different utility functions, but these are all risk averse. On the other hand, this assumption is extremely restrictive from the perspective of prospect theory by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). One of their most basic arguments is that people tend to be risk averse for bets that are prospected to exceed a certain reference point, but loss averse for bets that are prospected to be below that. To put it more simply, the value function associated with the utility function is concave for gains and convex for losses.

We extend the CK model from the viewpoint of prospect theory. First, we extend the model to the case of the loss averse utility function. As a result, we prove that the risk attitude of the representative investor, or the price of variance risk, is a monotonically increasing function of the state of the economy. This means the price of variance risk is procyclical. It is suggested that the relationship between the price of variance risk and the state of the economy is unclear in a market where risk-averse individuals and loss-averse individuals coexist. Second, we provide a rational interpretation of the state of the economy in line with prospect theory. As mentioned earlier, the state of the economy is expressed as a weighted average of market returns from each past date to the present date, and its weight can be interpreted as representing the distribution of investors' rebalancing cycles. The negative state of the economy means that more investors will recognize the loss than those who recognize the gain, and vice versa. Investors who recognize losses are loss averse, according to prospect theory. It is expected that the influence of the extended CK model with a loss-averse utility function will exceed that of the original model with a risk-averse utility function when the state of the economy is negative. The price of variance risk is expected to be procyclical or increasing. On the contrary, the price of variance risk is expected to be countercyclical or decreasing when the state of the economy is positive. When the state of the economy is near zero, it is expected that the influences of risk-averse individuals and loss-averse individuals will antagonize. The procyclicality or countercyclicality of the price of variance risk will diminish and its slope will be gradual. Therefore, a graph with the state of the economy on the horizontal axis and the price of variance risk on the vertical axis is expected to be a curve with the maximum value at the point where the state of the economy is zero. We use polynomial approximation to verify the structure of the price of variance risk. Specifically, we construct a model with a linear function, a quadratic function, a cubic function, and a quartic function, and selected the most appropriate model using the Akaike information criterion.

As a result, the structures of representative investor's risk attitude, or that of the price of variance risk, in each of the following 15 markets are almost the same; Canada, France, Italy, Australia, Hong Kong, Netherlands, Spain, Sweden, Switzerland, Brazil, South Korea, Taiwan, Indonesia, Mexico and Saudi Arabia. It is considered that the effect of the time-variation of the price of variance risk in each market on the time-variation of the international integration measure estimated in such a market is considered to be almost eliminated.

International integration measures in such markets are on average high (except Australia), declining before the 2008 global financial crisis (except Australia and Mexico), but rising again after the crisis (except Australia). This means that non-home-biased strategies have advantages over home-biased strategies except just before the crisis. In addition, the difference between the international integration measures of developed and emerging markets was extremely small. In other words, being an emerging market does not mean that the market is segmented. Summing up the above results, we conclude that the international diversification is strongly recommended in these 15 markets (except Australia) regardless of country or period. The contribution of this paper is to draw this conclusion on the globally influential stock markets, paying attention to the distribution of investors' risk attitudes and behavioral bias suggested by prospect theory, without making arbitrary assumptions about market integration.

In Section 2, we examine a model that simultaneously expresses market integration and segmentation based on the standard discrete-time CAPM, while relating with the continuous-time habit-formation consumption-based asset pricing model of Chan and Kogan (2002). In Section 3, we discuss the characteristics of market return data. We provide some evidence of market integration in section 4 and conclude in section 5.

2. Theoretical background and empirical model

This section discusses many theoretical arguments and constructs an empirical model, focusing on three important points in preventing misidentification of market scopes; researchers' arbitrary assumptions about market scopes, interdependence between stock markets, and the time-variability of the price of variance risk.

2.1. Arbitrariness of market scopes

Our aim is to identify the market scope. If the true market scope spans the stock markets of multiple countries, it is advisable to adopt the returns of such integrated markets as the pricing factor. The set of countries included in the integrated market W is represented by $\mathcal{W}$. Let r_{Wt} be the integrated market returns, r_{it} be the market returns for each country $i \in \mathcal{W}$, r_{ft} be the risk-free rate, and Ω_t be the information set at time t. A multi-period discrete CAPM suggests the following equation.

$$\mathrm{E}[r_{i,t+1}|\Omega_t] = r_{ft} + \lambda_{W,t+1|t}\,\mathrm{Cov}[r_{i,t+1}, r_{W,t+1}|\Omega_t]\,, \qquad \forall i \in \mathcal{W}. \tag{1}$$

The coefficient of the conditional covariance, $\lambda_{W,t+1|t}$, is represented by the conditional moments of the conditional pricing factor.

$$\lambda_{W,t+1|t} = \frac{\mathrm{E}[r_{W,t+1}|\Omega_t] - r_{ft}}{\mathrm{Var}[r_{W,t+1}|\Omega_t]}. \tag{2}$$

$\lambda_{W,t+1|t}$ is referred to as the price of variance risk for the integrated market W.

On the other hand, if the true market scope is limited to the domestic market, it is the market return of each country i that becomes the conditional pricing factor. Eq. (1) can be rewritten as follows.

$$\mathrm{E}[r_{i,t+1}|\Omega_t] = r_{ft} + \lambda_{i,t+1|t}\,\mathrm{Var}[r_{i,t+1}|\Omega_t]\,. \tag{3}$$

The coefficient of the conditional variance is

$$\lambda_{i,t+1|t} = \frac{\mathrm{E}[r_{i,t+1}|\Omega_t] - r_{ft}}{\mathrm{Var}[r_{i,t+1}|\Omega_t]}. \tag{4}$$

It is clear that $\lambda_{i,t+1|t}$ is the price of variance risk for the market i. Of course, eq. (3) and eq. (4) are the same equation.

It can be said that such a formulation arbitrarily assumes the market scope. Such arbitrariness can be eliminated by using the regime switching model. Two opposite regimes at the time t are represented by a binary variable ϕ_t. The conditional expectation of r_{it} can be decomposed as $\mathrm{E}[r_{i,t+1}|\Omega_t] = \Pr[\phi_{t+1}=1|\Omega_t]\,\mathrm{E}[r_{i,t+1}|\Omega_t,\phi_{t+1}=1] + \Pr[\phi_{t+1}=0|\Omega_t]\,\mathrm{E}[r_{i,t+1}|\Omega_t,\phi_{t+1}=0]$. The integration regime is represented by $\phi=1$, and it is assumed that eqs. (1) and (2) hold in this regime. Similarly, the segmentation regime is represented by $\phi=0$, and it is assumed that eqs. (3) and (4) hold in this regime. Then, the following equation holds.

$$\begin{gathered}\mathrm{E}[r_{i,t+1}|\Omega_t] = r_{ft} + \Pr[\phi_{t+1}=1|\Omega_t]\,\lambda_{W,t+1|t}\sigma^{(1)}_{iW,t+1|t} + \Pr[\phi_{t+1}=0|\Omega_t]\,\lambda_{i,t+1|t}\sigma^{(0)}_{ii,t+1|t} \\ \text{where } \sigma^{(k)}_{lm,t+1|t} = \mathrm{Cov}[r_{l,t+1}, r_{m,t+1}|\Omega_t,\phi_{t+1}=k]\,.\end{gathered} \tag{5}$$

2.2. Interdependence between stock markets

Set $\mathcal{W}$ should include all markets around the world. However, the verification of eq. (1) requires an "about 200-variate model" because the eq. (1) must hold for all $i \in \mathcal{W}$ in the integrated regime. One way to solve this problem is to build a bivariate model of r_i and r_W. Suppose that each return r_i affects all foreign returns $j \in \mathcal{W} \setminus \{i\}$ in the integration regime, but the effects will be negligible when aggregated into r_W. Under this supposition, the "200-variate model" can be reduced to a bivariate model. Thus, the integrated market return r_W is typically set to be exogenous to each market return r_i (e.g. Bekaert and Harvey, 1995). Pukthuanthong and Roll (2009) and Akbari, Ng and Solnik (2020) also set global factors as exogenous variables, and such settings are very common.

The integrated market return or global factor is, however, strongly affected by extremely large stock market returns such as the US and China. In addition, shocks in countries with relatively large economies, even though not as large as these two countries, can lead to global shocks; for example, Thailand, Indonesia and South Korea in the Asian financial crisis, and Russia in the subsequent Russian financial crisis. Shocks in major countries such as the G20 are likely to cause global shocks. The integrated market return r_W can no longer be said to be exogenous to such market returns.

Another problem is that the setting of the above integration regime is extremely strict. The above setting requires that the markets $i \in \mathcal{W}$ are all integrated. In other words, a regime in which $i \in \mathcal{W}$ and $j \in \mathcal{W} \setminus \{i\}$ are integrated but i and $k \in \mathcal{W} \setminus \{i, j\}$ are segmented is not an integration regime. The probability that such integration regime will be applied, $\Pr[\phi_{t+1} = 1|\Omega_t]$, is not expected to be large. The bivariate model of r_i and r_W may underestimate the integration probability when there is a market k that disturbs the integration of the world market.

We analyze bilateral integration to clarify the interdependence between stock markets. In other words, the set of bilateral integrated markets of i and j is $\mathcal{W}_{ij} = \{i, j\}$, and the regime with its integrated market return $r_{W_{ij}}$ as the pricing factor is interpreted as the integration regime. The important assumption here is that the effects from i to j (or j to i) are only direct effects. That is, it is assumed that the indirect effect from i to j, which is the sum of the effect from i to the third country k and the effect from k to j, is negligible. Effects from third countries to i and j are also typically ignored. However, the shock from the third countries may affect the correlation structure between i and j. Allowing time variability in the correlation between the two countries may capture shocks from third countries. Note that all investors evaluate the returns using the US dollar as the numéraire, and do not consider the exchange rate in this paper.

The bivariate regime switching model is constructed based on the above discussion. In the case of integration regime,

$$\begin{pmatrix} r_{i,t+1} \\ r_{j,t+1} \end{pmatrix} = \begin{pmatrix} r_{ft} + \lambda_{W_{ij},t+1|t}\sigma_{iW,t+1|t} \\ r_{ft} + \lambda_{W_{ij},t+1|t}\sigma_{jW,t+1|t} \end{pmatrix} + \begin{pmatrix} \epsilon^{(1)}_{i,t+1} \\ \epsilon^{(1)}_{j,t+1} \end{pmatrix}, \tag{6}$$

where the error term vector $\begin{pmatrix} \epsilon^{(1)}_{i,t+1} & \epsilon^{(1)}_{j,t+1} \end{pmatrix}^\top$ follows a bivariate normal distribution;

$$\begin{pmatrix} \epsilon^{(1)}_{i,t+1} \\ \epsilon^{(1)}_{j,t+1} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{ii,t+1|t} & \sigma_{ij,t+1|t} \\ \sigma_{ji,t+1|t} & \sigma_{jj,t+1|t} \end{pmatrix} \right). \tag{7}$$

In the case of segmentation regime,

$$\begin{pmatrix} r_{i,t+1} \\ r_{j,t+1} \end{pmatrix} = \begin{pmatrix} r_{ft} + \lambda_{i,t+1|t}\sigma_{ii,t+1|t} \\ r_{ft} + \lambda_{j,t+1|t}\sigma_{jj,t+1|t} \end{pmatrix} + \begin{pmatrix} \epsilon^{(0)}_{i,t+1} \\ \epsilon^{(0)}_{j,t+1} \end{pmatrix}, \tag{8}$$

where the error term vector $\begin{pmatrix} \epsilon^{(0)}_{i,t+1} & \epsilon^{(0)}_{j,t+1} \end{pmatrix}^\top$ follows a bivariate normal distribution;

$$\begin{pmatrix} \epsilon^{(0)}_{i,t+1} \\ \epsilon^{(0)}_{j,t+1} \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_{ii,t+1|t} & 0 \\ 0 & \sigma_{jj,t+1|t} \end{pmatrix} \right). \tag{9}$$

The error term vector at time $t+1$ is expressed as follows.

$$\begin{pmatrix} \epsilon_{i,t+1} \\ \epsilon_{j,t+1} \end{pmatrix} = \Pr[\phi_{ij,t+1} = 1|\psi_{ijt}] \begin{pmatrix} \epsilon^{(1)}_{i,t+1} \\ \epsilon^{(1)}_{j,t+1} \end{pmatrix} + \Pr[\phi_{ij,t+1} = 0|\psi_{ijt}] \begin{pmatrix} \epsilon^{(0)}_{i,t+1} \\ \epsilon^{(0)}_{j,t+1} \end{pmatrix} \tag{10}$$

where $\phi_{ij,t+1}$ equals 1 if market i and j are integrated and 0 otherwise, ψ_{ijt} is the history of r_i and r_j. Eqs. (6) - (10) imply that conditional variances do not depend on the regime and conditional covariances are zero in the segmentation regime because expected returns in such regime should be estimated independently, as if they were seemingly uncorrelated. The dynamics of these conditional covariance matrices are specified by GARCH (1, 1).[3] The elements of these matrices are specified by the constant conditional correlation model (CCC model) proposed by Bollerslev (1990) as follows.

$$\sigma_{ii,t+1|t} = \alpha_{i0} + \alpha_{i1}e^2_{it} + \beta_i\sigma_{ii,t|t-1}, \tag{11}$$

$$\sigma_{jj,t+1|t} = \alpha_{j0} + \alpha_{j1}e^2_{jt} + \beta_j\sigma_{jj,t|t-1}, \tag{12}$$

$$\sigma_{ij,t+1|t} = \sigma_{ji,t+1|t} = \rho_{ij}\sqrt{\sigma_{ii,t+1|t}\sigma_{jj,t+1|t}} \tag{13}$$

where e_{it} and e_{jt} are the residuals correspond to the error terms ϵ_{it} and ϵ_{jt}. The parameters of α_{i0} and α_{j0} are constant terms for the dynamics of the volatility, the parameters of α_{i1} and α_{j1} capture the effects of past residuals, and the parameters of β_i and β_j capture the dependence of the past volatility. Assuming that market integration does not affect the intrinsic volatility of stocks, these parameters are independent on regimes. Consequently, these are considered as common parameters between regimes.[4] The parameter ρ_{ij} is the constant correlation between ϵ_{it} and ϵ_{jt}. In this model, we assume that the correlation coefficient is not time-varying, since the change over time of the bilateral interaction effect should be captured in the dynamics of the switching variable ϕ_{ijt}. At this

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[3] The GARCH model with regime switching requires careful formulation when conditional volatility depends on an infinite past regime (Haas, Mittnik and Paolella, 2004). This problem does not occur in this paper because the conditional volatility does not depend on the regime.

[4] If we set these parameters are not common between regimes, the transition of the switching variable cannot be interpreted well. That is because we cannot tell whether the regime shift is due to a shift between the integration and segmentation regimes, or a shift between the large and small volatility regimes like the SWARCH (Switching ARCH) model by Hamilton and Susmel (1994).

time, the variance-covariance matrix of the error term vector is

$$\begin{pmatrix} \sigma_{ii,t+1|t} & \Pr[\phi_{ij,t+1}=1|\psi_{ijt}]\,\rho_{ij}\sqrt{\sigma_{ii,t+1|t}\sigma_{jj,t+1|t}} \\ \Pr[\phi_{ij,t+1}=1|\psi_{ijt}]\,\rho_{ij}\sqrt{\sigma_{ii,t+1|t}\sigma_{jj,t+1|t}} & \sigma_{jj,t+1|t} \end{pmatrix}.$$

Therefore, the time variability of $\Pr[\phi_{ij,t+1}=1|\psi_{ijt}]$ makes the bilateral correlation substantially time-varying. The covariance with the integrated market W_{ij} is specified as follows.

$$\begin{aligned} \sigma_{iW_{ij},t+1|t} &= w_t\sigma_{ii,t+1|t} + (1-w_t)\rho_{ij}\sqrt{\sigma_{ii,t+1|t}\sigma_{jj,t+1|t}}, \\ \sigma_{jW_{ij},t+1|t} &= w_t\rho_{ij}\sqrt{\sigma_{ii,t+1|t}\sigma_{jj,t+1|t}} + (1-w_t)\sigma_{jj,t+1|t}, \end{aligned} \tag{14}$$

where w_t is the value-weight of market i at the time t, that is, the ratio of market capitalization of market i to the market capitalization of market W_{ij}.

The dynamics of ϕ_{ijt} are characterized by the probability of staying integration regime, $p_{1|1}^{(i,j)} \equiv \Pr[\phi_{ij,t+1}=1|\phi_{ij,t}=1]$ $\forall t$, and that of staying segmentation regime, $p_{0|0}^{(i,j)} \equiv \Pr[\phi_{ij,t+1}=0|\phi_{ij,t}=0]$ for all t. The transition probability matrix P_{ij} is composed of them as follows.

$$P_{ij} = \begin{pmatrix} p_{1|1}^{(i,j)} & 1-p_{0|0}^{(i,j)} \\ 1-p_{1|1}^{(i,j)} & p_{0|0}^{(i,j)} \end{pmatrix} \tag{15}$$

See Hamilton (1988), Hamilton (1989), Hamilton (1990) and Kim and Nelson (1999) for detail.

2.3. Time-varying price of variance risk

The price of variance risk is defined as the ratio of the conditional expected excess return on the market portfolio to its conditional variance. The price of variance risk is obviously time-varying, but many empirical models for CAPM and multi-factor models ignore that time variability. Harvey (1991), Bekaert and Harvey (1995) and de Santis and Gerard (1997) provided a lot of evidences that the price of variance risk is time-varying. They have shown the fact that the price of variance risk is time-varying but have not analyzed its economic background.

For this purpose, this paper uses the Chan and Kogan (2002) model (CK model), which is one of the continuous-time habit-formation consumption-based asset pricing models. Furthermore, we generalize the CK model so that we can take into account the behavioral bias suggested by prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). As a result, it can be theoretically shown that the price of variance risk is countercyclical in a boom, and procyclical in a recession.

2.3.1. Chan and Kogan model

Chan and Kogan (2002) construct a habit-formation consumption-based asset pricing model in which the assumptions of a representative individual is relaxed. In their model, individuals have constant but different relative risk aversions. This model implies that the aggregate risk aversion in the economy, corresponding to the risk aversion of a representative individual, is time-varying and its time-variation is countercyclical.

They solve the following representative individual's utility function $U(Y_t, X_t)$ (see also Chan and Kogan (2002) or Munk (2013) for details).

$$U(Y_t, X_t) = \sup_{\{c_t(Y_t, X_t; \gamma^{ind})\}} \left[\int_1^\infty f(\gamma^{ind}) \frac{1}{1-\gamma^{ind}} \left(\frac{c_t(Y_t, X_t; \gamma^{ind})}{X_t} \right)^{1-\gamma^{ind}} d\gamma^{ind}, \quad \text{s. t.} \quad \int_1^\infty c_t(Y_t, X_t; \gamma^{ind}) d\gamma^{ind} \leq Y_t \right] \tag{16}$$

$\gamma^{ind} \in (1, \infty)$ is a constant relative risk aversion of individual where the subscript "ind" means "individual", and $f(\gamma^{ind})$ normalized as $\int_1^\infty f(\gamma^{ind}) d\gamma^{ind} = 1$ denote the weight of individuals with the relative risk aversion γ^{ind}. Y_t and X_t are an aggregate endowment and a benchmark for habit-formation at time t, respectively[5]. The individual consumption c_t is the function of them. The log-benchmark $x_t = \ln X_t$ is specified as follows.

$$x_t = \exp(-\kappa t)\, x_0 + \kappa \int_0^t \exp\bigl(-\kappa(t-s)\bigr)\, y_s ds,$$

$$\text{where} \quad y_t = \ln Y_t\,, \qquad \kappa \text{ is positive and small.}$$

The parameter κ can be interpreted as the degree of history dependence in X_t according to Chan and Kogan (2002).

Given the discrete time period $t-1$ is correspond to $[t-1, t]$, and considered $1-\kappa \approx \exp(-\kappa)$ for small κ, this equation can be approximated in discrete time as follows.

$$x_t \approx (1-\kappa)^t x_0 + \kappa \sum_{s=0}^{t-1} (1-\kappa)^{t-1-s} y_s.$$

Consequently,

$$\begin{aligned} x_t &\approx (1-\kappa) \left((1-\kappa)^{t-1} x_0 + \kappa \sum_{s=0}^{t-2} (1-\kappa)^{t-2-s} y_s \right) + \kappa y_{t-1} \\ &= (1-\kappa) x_{t-1} + \kappa y_{t-1}. \end{aligned} \tag{17}$$

This approximation provides the useful intuition that the log-benchmark x_t is a weighted average of its lagged value x_{t-1} and the lagged log-endowment y_{t-1}. In addition, they introduce the state of the economy as the difference between the log-endowment and the log-benchmark, $z_t \equiv y_t - x_t$. This can be rewritten by using eq. (17) as follows.

$$\begin{aligned} z_t &\approx y_t - \bigl((1-\kappa) x_{t-1} + \kappa y_{t-1}\bigr) \\ &= y_t - y_{t-1} + (1-\kappa)(y_{t-1} - x_{t-1}) \\ &= \ln Y_t / Y_{t-1} + (1-\kappa) z_{t-1} \end{aligned}$$

[5] Chan and Kogan (2002) assume that the dynamics of the aggregate endowment follow a geometric Brownian motion and the log-benchmark is expressed as a weighted geometric average of past realizations of log-consumptions.

$$\equiv r_{Yt} + (1-\kappa)z_{t-1} \tag{18}$$

where r_{Yt} is the growth rate of the aggregate endowment. This equation means the state of the economy can be specified as like AR (1) model with exogeneous variable. The initial value is $z_0 = \mu_Y/\kappa$ where μ_Y is unconditional mean of r_{Yt}. Substituting eq. (18) successively,

$$z_t \approx r_{Yt} + (1-\kappa)r_{Y,t-1} + (1-\kappa)^2 r_{Y,t-2} + \cdots + (1-\kappa)^{t-1} r_{Y,1} + (1-\kappa)^t \mu_Y/\kappa \tag{19}$$

Therefore, the state of the economy can be written by the function of the set $\psi_{Yt} = \{R_{Yt}, R_{Y,t-1}, R_{Y,t-2}, \dots, R_{Y0}\}$ which is the history of the change rate of the aggregate endowment. To explicitly show this property, it can be written as $z_t \equiv z_{Yt} = z(\psi_{Yt}; \mu_Y, \kappa)$. Eq. (19) can be rewritten as:

$$z_{Yt} \approx \kappa\left(r_{Y,[t-1,t]} + (1-\kappa)r_{Y,[t-2,t]} + (1-\kappa)^2 r_{Y,[t-3,t]} + \cdots\right) \tag{20}$$

where $r_{Y,[t-s,t]}$ is the growth of aggregate endowment from the time $t-s$ to t. In the case of $\kappa = 0.02$, the state of the economy can be rewritten as $z_{Yt} \approx 0.02 \times \left(r_{Y,[t-1,t]} + 0.980 \times r_{Y,[t-2,t]} + 0.960 \times r_{Y,[t-3,t]} + 0.941 \times r_{Y,[t-4,t]} + \cdots\right)$. The influence of investors with a rebalancing cycle of 4-periods is 94.1% of the investors with a rebalancing cycle of 1-period, even though they only appear in the market once every four periods. That is, the density of 4-period investors is $0.941 \times 4 \approx 3.8$ times that of 1-period investors. The distribution of investor rebalancing cycles is given below.

$$g(s;\kappa) = s\kappa^2(1-\kappa)^{s-1}, \qquad s \in \{1,2,\dots\} \tag{21}$$

where $\lim_{t\to\infty} \sum_{s=1}^{t} g(s;\kappa) = 1$ holds.

Figure 1 shows the distribution of investor rebalancing cycles under various κ. The density of investors with a rebalancing cycle of s-periods is $s(1-\kappa)^{s-1}$ times that of 1-period investors. If s that maximizes this is s^*, then $s^* = -1/\ln(1-\kappa) \approx 1/\kappa$. For example, $s^* \approx 50$ when $\kappa = 0.02$, $s^* \approx 4$ when $\kappa = 0.25$, and so on. Therefore, this assumption implies that the distribution has the highest number of investors with a 50-periods rebalancing cycle. In the discrete-time model, κ plays the role of parameters that specify the distribution of investor rebalancing cycles. At this time, $1/\kappa$ represents the mode of the distribution.

[Figure 1]

Chan and Kogan indicate that the utility function of an aggregate (representative) individual is specified as follows, under the optimal consumption allocation.

$$U(Y_t, X_t) = \int_1^\infty \frac{1}{1-\gamma^{ind}} f(\gamma^{ind})^{1/\gamma^{ind}} \exp\left(-\frac{1-\gamma^{ind}}{\gamma^{ind}} h(z_t)\right) d\gamma^{ind}, \tag{22}$$

where the function h is implicitly defined as $\int_1^\infty f(\gamma^{ind})^{1/\gamma^{ind}} \exp\left(-\frac{1}{\gamma} h(z_{Yt}) - z_{Yt}\right) d\gamma^{ind} = 1$. They have proved that the function h is convex and decreasing in z_t and the first derivative is satisfied as $-h'(z_{Yt}) = -Y_t U_{YY}(Y_t, X_t)/U_Y(Y_t, X_t) > 1$ where U_Y and U_{YY} are the first and second derivative of U with respect to Y, respectively. Therefore, $-h'(z_{Yt})$ is interpreted as the relative risk aversion of the aggregate individual. In order to emphasize this characteristic, it is expressed as $-h'(z_{Yt}) \equiv \gamma(z_{Yt})$ where the function $\gamma(.)$ means the aggregate relative risk aversion. It is decreasing in the state of the economy (because of the convexity of h) so that we can consider its fluctuations to be time-varying and counter-cyclical. Intuitively, individuals with low risk aversions hold a large fraction of their portfolio in risky assets such as stocks and a small fraction in non-risky assets such as treasury bills than individuals with high risk aversions. When the stock prices go up, the wealth of individuals with low risk aversions will grow more than that of individuals with high risk aversions. Consequently, the aggregate relative risk aversion will be low. Conversely, when the stock prices fall, the wealth of individuals with low risk aversions will shrink more than others so that the aggregate relative risk aversion will be high. Therefore, the relative risk aversion of the aggregate (representative) individual is time-varying and counter-cyclical.

In their model, the Sharpe ratio of a risky asset i is

$$\frac{r_{it} - r_{ft}}{\sigma_{it}} = -h'(z_{Yt})\sigma_{Yt} = \gamma\big(z(\psi_{Yt};\mu_Y,\kappa)\big)\sigma_{Yt},$$

where r_{ft}, r_{it}, σ_{it} and σ_{Yt} denote the instantaneous risk-free rate, the (expected) return of asset i,[6] its volatility (or its sensitivity to external shock) and the volatility of the aggregate endowment Y, respectively. $\gamma\big(z(\psi_{Yt};\mu_Y,\kappa)\big)$ is the relative risk aversion of an aggregate individual.

Their general model includes the continuous-time CAPM like Merton (1973) as a special case by applying the market capitalization M as a proxy of Y. At this time, the growth of aggregate endowment, r_{Yt}, is replaced by the stock market return r_{Mt}. Thus, the Sharpe ratio of the stock market return is $(r_{Mt} - r_{ft})/\sigma_{Mt} = \gamma\big(z(\psi_{Mt};\mu_M,\kappa)\big)\sigma_{Mt}$. Therefore,

$$\gamma\big(z(\psi_{Mt};\mu_M,\kappa)\big) = \frac{r_{Mt} - r_t^f}{\sigma_{Mt}^2}$$

This equation implies that the ratio of expected excess market return to its volatility equals to the relative risk aversion of the aggregate (representative) individual. Approximated from a continuous-time model to a discrete-time model as $(r_{Mt} - r_{ft})/\sigma_{Mt}^2 \approx (\mathrm{E}[r_{Mt}|\psi_{M,t-1}] - r_{ft})/\mathrm{Var}[r_{Mt}|\psi_{M,t-1}]$, it corresponds to eq. (2) and (4). Therefore,

$$\gamma\big(z(\psi_{Mt};\mu_M,\kappa)\big) \approx \lambda_{M,t+1|t} \tag{23}$$

Thus, the price of variance risk, $\lambda_{M,t+1|t}$, can be roughly interpreted as the relative risk aversion of a representative individual in the market M, and considered to be time-varying and counter-cyclical.

2.3.2. Prospect theory, and Chan/Kogan model

Chan and Kogan (2002) assume that all individuals are risk averse. This assumption is reflected in the condition that $\gamma^{ind} > 1$. However, this does not apply if individual preferences change

[6] Strictly speaking, we should consider the (expected) return and dividend of asset i. In this paper, capital gains and income gains are not distinguished and are collectively referred to as r_{it}.

depending on profit or loss, as indicated by Kahneman and Tversky (1979) and Tversky and Kahneman (1992). They propose the following value function.

$$v(x) = \begin{cases} x^{\alpha} & \text{if} \quad x \geq 0 \\ -\theta(-x)^{\beta} & \text{if} \quad x < 0 \end{cases}, \qquad \alpha, \beta > 0.$$

This implies that an individual becomes risk averse when the outcome is above the reference point (in the gain domain) and risk loving or "loss averse" when the outcome is below the reference point (in the loss domain). Their study suggests that $\theta > 1$, that is, the slope is steeper in the loss region than in the gain region.

Interestingly enough, the idea of reference points in prospect theory matches that of habit levels in habit formation models. Habit formation models focus on the difference between consumption and habit level just as prospect theory focuses on the difference between outcomes and reference points. However, in the habit formation model, individuals do not change their risk attitudes even if consumption falls below habit levels. We take this into account and extend the CK model.

In the CK model, the risk attitude of a representative individual (to be exact, an aggregate individual) depends on the state of the economy $z_{Yt} = z(\psi_{Yt}; \mu_Y, \kappa)$. Eq. (20) can be rewritten as follows.

$$z_{Yt} \approx \sum_{s \in \mathcal{S}^+} \kappa(1-\kappa)^{s-1} r_{Y,[t-s,t]} + \sum_{s \in \mathcal{S}^-} \kappa(1-\kappa)^{s-1} r_{Y,[t-s,t]}$$

$$\text{where} \quad \begin{cases} r_{Y,[t-s,t]} \geq 0 & \text{if} \quad s \in \mathcal{S}^+ \\ r_{Y,[t-s,t]} < 0 & \text{if} \quad s \in \mathcal{S}^- \end{cases}.$$

Consequently, the following relations approximately hold.

$$z_{Yt} \geq 0 \quad \Leftrightarrow \quad \sum_{s \in \mathcal{S}^+} \kappa(1-\kappa)^{s-1} r_{Y,[t-s,t]} \geq \left| \sum_{s \in \mathcal{S}^-} \kappa(1-\kappa)^{s-1} r_{Y,[t-s,t]} \right|$$

$$z_{Yt} < 0 \quad \Leftrightarrow \quad \sum_{s \in \mathcal{S}^+} \kappa(1-\kappa)^{s-1} r_{Y,[t-s,t]} < \left| \sum_{s \in \mathcal{S}^-} \kappa(1-\kappa)^{s-1} r_{Y,[t-s,t]} \right|$$

The reference point of investor with s-periods rebalancing cycle corresponds to Y_{t-s} and their outcome corresponds to Y_t. The difference between them corresponds to $r_{Y,[t-s,t]}$. Investors with $s \in \mathcal{S}^+$-periods rebalancing cycle will recognize positive returns or "gains". That is, $\sum_{s \in \mathcal{S}^+} \kappa(1-\kappa)^{s-1} r_{Y,[t-s,t]}$ means the weighted average influence of investors with gains. [7] Similarly, $\left|\sum_{s \in \mathcal{S}^-} \kappa(1-\kappa)^{s-1} r_{Y,[t-s,t]}\right|$ means the weighted average influence of investors with losses. Therefore, the positive (or negative) state of the economy implies the influence of investors with gains is greater (or smaller) than that of investors with losses. Intuitively, the representative investor would be expected to be risk averse as before when the state of the economy is positive. On the other hand, when the state of the economy is negative, the representative investor may become loss averse.

To confirm this expectation, we focus on individual risk attitudes γ^{ind}, rather than representative

[7] The weight is the (positive) returns recognized by such investors.

investor's risk attitude $\gamma(z_{Yt})$. Chan and Kogan (2002) assume $\gamma^{ind} \in (1,\infty)$ since they only assume risk-averse individuals. However, we also consider loss-averse individuals with $\gamma^{ind} \in (-\infty,0)$ and the rest of (risk-averse) individuals with $\gamma^{ind} \in (0{,}1)$. The case where each of these investor groups becomes dominant in the market is expressed using the regime switching variable $G_t \in \{1{,}2{,}3\}$. Considering the correspondence with prospect theory, it would be natural to assume that the dynamics of such regime variables depend on the state of the economy; $G_t = G(z_{Yt})$. The utility function of the representative individual in our extended CK model from eq. (22) are as follows.

$$U(Y_t, X_t) = \sum_{i=1}^{3} \Pr[G_t = i]\, U^{(i)}\,, \qquad G_t = G(z_{Yt}) \in \{1{,}2{,}3\} \tag{24}$$

where

$$U^{(1)}(Y_t, X_t) = \sup_{\{c_t;\gamma^{ind}\in(1,\infty)\}} \left\{ \int_1^{\infty} f(\gamma^{ind}) \frac{1}{1-\gamma^{ind}} \left(\frac{c_t}{X_t}\right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind} \quad \mathrm{s.t.} \int_1^{\infty} c_t \mathrm{d}\gamma^{ind} \le Y_t \right\}$$

$$U^{(2)}(Y_t, X_t) = \sup_{\{c_t;\gamma^{ind}\in(0,1)\}} \left\{ \int_0^{1} f(\gamma^{ind}) \frac{1}{1-\gamma^{ind}} \left(\frac{c_t}{X_t}\right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind} \quad \mathrm{s.t.} \int_0^{1} c_t \mathrm{d}\gamma^{ind} \le Y_t \right\}$$

$$U^{(3)}(Y_t, X_t) = \sup_{\{c_t;\gamma^{ind}\in(-\infty,0)\}} \left\{ \int_{-\infty}^{0} f(\gamma^{ind}) \frac{\theta(\gamma^{ind})}{1-\gamma^{ind}} \left(\frac{c_t}{X_t}\right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind} \quad \mathrm{s.t.} \int_{-\infty}^{0} c_t \mathrm{d}\gamma^{ind} \le Y_t \right\}$$

$$c_t = c_t(Y_t, X_t; \gamma^{ind})$$

$U^{(1)}(Y_t, X_t)$ and $U^{(2)}(Y_t, X_t)$ are the utility functions of the representative individuals composed of risk averse individuals. $U^{(3)}(Y_t, X_t)$ is the utility function of the representative individual composed of loss averse (or risk loving) individuals. We assume that the slope adjustment parameter $\theta^{ind} > 0$ in the loss averse individual's utility function is a function of γ^{ind}, that is, $\theta^{ind} = \theta(\gamma^{ind})$.[8]

Then we can prove the following properties (see Appendix for detail):

$$\gamma^{(1)}(z_{Yt}) \equiv -\frac{Y_t U_{YY}^{(1)}(Y_t, X_t)}{U_Y^{(1)}(Y_t, X_t)} \in (1,\infty), \qquad \frac{\mathrm{d}\gamma^{(1)}(z_{Yt})}{\mathrm{d}z_{Yt}} < 0 \tag{25}$$

$$\gamma^{(2)}(z_{Yt}) \equiv -\frac{Y_t U_{YY}^{(2)}(Y_t, X_t)}{U_Y^{(2)}(Y_t, X_t)} \in (0{,}1), \qquad \frac{\mathrm{d}\gamma^{(2)}(z_{Yt})}{\mathrm{d}z_{Yt}} < 0 \tag{26}$$

$$\gamma^{(3)}(z_{Yt}) \equiv -\frac{Y_t U_{YY}^{(3)}(Y_t, X_t)}{U_Y^{(3)}(Y_t, X_t)} \in (-\infty,0), \qquad \frac{\mathrm{d}\gamma^{(3)}(z_{Yt})}{\mathrm{d}z_{Yt}} > 0 \tag{27}$$

Consequently, $\gamma(z_{Yt})$ will be represented by eqs. (25), (26), and (27) in each regime 1, 2 and 3. $\Pr[G(z_{Yt}) = 1]$ and $\Pr[G(z_{Yt}) = 2]$ are expected to be large when $z_{Yt} \ge 0$. At this time, $\gamma(z_{Yt})$

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[8] This assumption allows us to adopt $f^{\star}(\gamma^{ind})$ defined as follows as the distribution of individuals.

$$f^{\star}(\gamma^{ind}) \equiv f(\gamma^{ind})\theta(\gamma^{ind}) \Big/ \int_{-\infty}^{0} f(\gamma^{ind})\theta(\gamma^{ind})\mathrm{d}\gamma^{ind}$$

$$\text{where} \quad \int_{-\infty}^{0} f^{\star}(\gamma^{ind})\mathrm{d}\gamma^{ind} = \int_{-\infty}^{0} f(\gamma^{ind})\theta(\gamma^{ind})\mathrm{d}\gamma^{ind} \Big/ \int_{-\infty}^{0} f(\gamma^{ind})\theta(\gamma^{ind})\mathrm{d}\gamma^{ind} = 1.$$

Therefore,

$$U^{(3)}(Y_t, X_t) = \int_{-\infty}^{0} f(\gamma^{ind})\theta(\gamma^{ind})\mathrm{d}\gamma^{ind} \sup_{\{c_t\}} \left\{ \int_{-\infty}^{0} f^{\star}(\gamma^{ind}) \frac{1}{1-\gamma^{ind}} \left(\frac{c_t}{X_t}\right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind} \quad \mathrm{s.t.} \int_{-\infty}^{0} c_t \mathrm{d}\gamma^{ind} \le Y_t \right\}.$$

will be countercyclical from eqs. (25) and (26). Conversely, when $z_{Yt} < 0$, $\gamma(z_{Yt})$ will be procyclical from eq. (27). It is expected that $\gamma(z_{Yt})$ will be maximum when z_{Yt} is near zero.

From the above discussion, the structure of relative risk aversion of representative investor, $\gamma(z_{Yt})$, cannot be approximated by linear function. The structure of the price of variance risk, $\lambda_{M,t+1|t} \approx \gamma(z_{Mt})$ from eq. (23), is similar. We consider the following four models that approximate the structure of the price of variance risk.

$$\begin{aligned} \text{Linear model:} \quad & \lambda_{M,t+1|t} = \delta_0 + \delta_1 \hat{z}_{Mt} \\ \text{Quadratic model:} \quad & \lambda_{M,t+1|t} = \delta_0 + \delta_1 \hat{z}_{Mt} + \delta_2 \hat{z}_{Mt}^2 \\ \text{Cubic model:} \quad & \lambda_{M,t+1|t} = \delta_0 + \delta_1 \hat{z}_{Mt} + \delta_2 \hat{z}_{Mt}^2 + \delta_3 \hat{z}_{Mt}^3 \\ \text{Quartic model:} \quad & \lambda_{M,t+1|t} = \delta_0 + \delta_1 \hat{z}_{Mt} + \delta_2 \hat{z}_{Mt}^2 + \delta_3 \hat{z}_{Mt}^3 + \delta_4 \hat{z}_{Mt}^4 \end{aligned} \tag{28}$$

where $\hat{z}_{Mt} \equiv 1 + z_{Mt}$ in order to align the units of δs. We select the most appropriate model from these using criteria such as AIC (Akaike's information criterion).

2.3.3. Extension to international analysis

The argument so far holds for international analysis as it is by replacing M with the market capitalization of each country. However, some interstate assumptions need to be imposed on the parameters such as κ in eq. (23) and δs in eq. (28). It is natural that investors' rebalancing cycles can vary significantly from country to country. We assume the following:

$$z_{Mt} = z(\psi_{Mt}; \mu_M, \kappa) \equiv z(\psi_{Mt}; \mu_M, \kappa_M) \tag{29}$$

where κ_M is a parameter that defines the distribution of investor rebalancing cycles in market M.

On the other hand, we assume that δs, parameters that define such behavioral tendencies, do not depend on the country (or market) M. This means that the function of risk attitude regime switching variable $G(.)$, the individual consumption function $c_t(.)$, and the distribution function of individual risk attitudes $f(.)$ do not depend on the country. Human behavior regarding loss avoidance and consumption may be considered universal, but the distribution of investor risk attitudes may vary from country to country. Note that this is a rather restrictive assumption.

3. Data

In Section 2, we have constructed an empirical model based on many theoretical arguments. Our model requires stock market returns and market capitalization for each country, r_{Mt} and w_t, respectively. We can use the weekly stock market indices of each country (market) as a proxy variable for the former, but the market capitalization is not known from these indices. Thus, we use the Market Capitalization Index (weekly) provided by Bloomberg as a proxy variable for stock market capitalization. This index cannot be used as a proxy for market returns as the effects of ex-dividends are not adjusted. Furthermore, the moving average value for the past one year is used in order not to have an effect of ex-dividend on w_t. Treasury bill rates as risk free rates are also from Bloomberg.

The sample period is from October 17, 2003 to June 26, 2020. In fact, we set a burn-in period of about a year and a half when constructing z_{Mt}. The main analysis period is from July 1, 2005 to June 26, 2020 (783 periods, 15 years).

We analyze 25 markets, including G20 countries and comparable large markets. S&P 500 (the US) is set to reference market i in eqs. (6) - (15) to prevent complication due to considering a huge combination of various markets. The remaining 24 markets are set to market j and divided into four groups: G7 ex the US ("G7"), other large developed markets ("LDM"), large emerging markets "LEM", and the rest of the G20 emerging markets ("G20").[9] "G7" includes TSX (Canada), CAC 40 (France), DAX (Germany), FTSE MIB (Italy), TOPIX (Japan), and FTSE 100 (UK). "LDM" includes ASX (Australia), HSI (Hong Kong), AEX (Netherlands), IBEX (Spain), OMXS (Sweden), and SMI (Switzerland). "LEM" includes Bovespa (Brazil), SSE (China), SENSEX (India), RTS (Russia), KOSPI (South Korea), and TWSE (Taiwan). "G20" includes Merval (Argentina), JCI (Indonesia), Mexican Bolsa (Mexico), Tadawul (Saudi Arabia), JSE (South Africa), and XU 100 (Turkey).

Table 1 shows the summary statistics of weekly returns [%] on these large stock markets.[10] The mean of the US market return is 0.156%, almost all developed market returns are smaller than the US and almost all emerging market returns are larger than the US. Markets with large average returns tend to have large standard deviations. Most skewness values indicate that the distribution is negatively skewed, and all kurtosis values indicate that the distribution is fat-tail. Correlation with the US tends to be high in developed markets and low in emerging markets. Japan and Hong Kong are classified as developed markets but have low correlation, and Brazil and Mexico are classified as emerging markets but have high correlation. Intuitively, developed markets seem to be integrated with the US, and emerging markets are segmented from the US. Alternatively, it seems that American and European markets are integrated, and Asian markets are segmented. This intuition may certainly be correct as one aspect of describing stock market integration. However, this simple analysis does not distinguish between spurious correlations and intrinsic correlations based on market integration.

[Table 1]

4. Results

4.1. Setting and model selection

To get started, we will describe how to construct $z_{Mt} = z(\psi_{Mt}; \mu_M, \kappa_M)$. The initial value is defined as μ_M/κ_M but we cannot calculate the exact value of μ_M. The average return for all sample periods can be considered as a proxy variable for μ_M, but it is not desirable that the initial value be a

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[9] The classification of developed market and emerging market is based on the MSCI (Morgan Stanley Capital International) classification.

[10] This paper sets the US dollar as the numéraire. In other words, the return of each market includes the growth rate of each country's currency.

function of future returns. Alternatively, it is possible to use the average of the period from the initial point to several years ago as a proxy variable, but this is not desirable because it wastes the sample size. After all, the effect of the initial value almost disappears by setting a certain burn-in period when constructing z_{Mt}. The mode of investor rebalancing cycle is expressed as $1/\kappa_M$. Thus, it is advisable to set a burn-in period that greatly exceeds $1/\kappa_M$. We consider the following 9 patterns as candidates for κ_M (the mode of rebalancing cycle); $\kappa_M = 0.02$ (about 1 year), $\kappa_M = 0.025$ (about 9 months), $\kappa_M = 0.0325$ (about 7 months), $\kappa_M = 0.0375$ (about 6 months), $\kappa_M = 0.045$ (about 5 months), $\kappa_M = 0.055$ (about 4 months), $\kappa_M = 0.075$ (about 3 months), $\kappa_M = 0.115$ (about 2 months), and $\kappa_M = 0.25$ (about 1 months). Figure 1 shows these distributions. The data we have available are the 872 weekly stock market indices and the market capitalization index from October 17, 2003 to June 26, 2020. By setting a burn-in period of 89 weeks, a z_{Mt} of 783 weeks (15 years) from July 1, 2005 to June 26, 2020 will be constructed. This greatly exceeds the rebalancing cycle mode of 50 weeks in the candidate of $\kappa_M = 0.02$, and the effect of the initial value is almost negligible in all candidates. Therefore, we set zero as the initial value z_{M0} or μ_M.

Next, we analyze one-market model of the US market in order to get appropriate δs and κ_i. Table 2 shows AICs of each polynomial model expressed as eq. (28). Each row shows the value of a given κ_i. The AIC is the smallest in the cubic function model given $\kappa_i = 0.0375$. As expected, the linear function is extremely poorly fitted. On the other hand, the quadratic function is also very poorly fit, and it can be seen that the approximation of $\gamma(z_{Mt}) \approx \lambda_{M,t+1|t}(z_{Mt})$ requires an order of three or more. Focusing on the column of the cubic function, it can be seen that AIC decreases as κ_i approaches 0.0375. This implies that the most typical investors in the US have rebalancing cycles of around 6 months.

[Table 2]

Based on the above results, we will analyze the two-market ($i = \mathrm{US}$ and j), two-regime (integration and segmentation regime) model. Table 3 shows AICs of each country and each κ_j. It is assumed that $\lambda_{j,t+1|t}(z_{jt})$ is represented by a cubic function of z_{jt} and parameters that determine the distribution of rebalancing cycles for US investors, κ_i, is 0.0375 in all markets $j \in \{\mathrm{G7}, \mathrm{LDM}, \mathrm{LEM}, \mathrm{G20}\}$. The model with the smallest AIC is underlined. For example, Canada has the lowest AIC of 6,295.499 when $\kappa_j = 0.0375$. This means that the most typical investors in the Canadian market have rebalancing cycles of around 6 months, similar to the US. Unfortunately, giving any κ_j does not give any results in Japan and the UK. It can be seen that the rebalancing cycle of about half of all (12 markets) is as long as or longer than that in the US. Markets with relatively slow growth tend to have longer rebalancing cycles, but there are many exceptions.

[Table 3]

4.2. Estimated parameters

Tables 4 shows the estimated parameters of the two-regime switching model in “G7”, “LDM”, “EDM” and “G20”. All parameters related to the GARCH structure ($\alpha_{i0}, \alpha_{j0}, \alpha_{i1}, \alpha_{j1}, \beta_i$ and β_j) are statistically significant in any markets. The estimates of α_{i0}, α_{i1}, and β_i should be equal in all markets since the market i is fixed in the US market in any analysis of the market j. In almost all countries, $\alpha_{i0} \approx 0.45, \alpha_{i1} \approx 0.2$ and $\beta_i \approx 0.7$. Therefore, it can be said that the GARCH structure in the US is well estimated. On the other hand, the estimates of α_{j0}, α_{j1}, and β_j may be different for each market.

The estimates of the constant conditional correlation with the US, ρ_{ij}, are generally high in developed markets and low in emerging markets. However, Australia and Hong Kong have low ρ_{ij} even in developed markets (0.508 and 0.589, respectively)., and Brazil, Mexico, Saudi Arabia and South Africa have high ρ_{ij} even in emerging markets (0.668, 0.752, 0.709 and 0.695, respectively). The probabilities of staying the integration regime, $p_{1|1}^{(i.j)}$, are above 0.9 in all markets except Australia and South Africa. It is indicated that once the market integrates with the US, its regime is extremely sustainable. On the other hand, the probabilities of staying the segmentation regime, $p_{0|0}^{(i.j)}$, are lower than $p_{1|1}^{(i.j)}$ except Australia but these heights are polarized. France, Italy, Netherlands, Spain, South Korea, Taiwan, Argentina and Indonesia have extremely low $p_{0|0}^{(i.j)}$. This implies that the segmentation regime of these markets hardly lasts. In markets with such high $p_{1|1}^{(i.j)}$ and low $p_{0|0}^{(i.j)}$, the integrated regime will almost always apply. On the other hand, the $p_{0|0}^{(i.j)}$ of other countries is relatively high, and once it becomes a segmentation regime, it continues for a while. In markets with high $p_{1|1}^{(i.j)}$ and high $p_{0|0}^{(i.j)}$, the regime transition will be smooth. Using smoothed probabilities makes time-series of regimes easier to understand. Details will be described later in Subsection 4.5.

The estimated values of δ_0, δ_1, δ_2 and δ_3 (and δ_4) all show significant values in almost all markets except China, Russia and South Africa. It is likely that the structures of risk-pricing, $\gamma(z) \approx \lambda(z)$, have been successfully captured in these markets. It is common that δ_0 and δ_2 are positive and δ_1 and δ_3 are negative while the coefficient values vary greatly from market to market. We will analyze these parameters in more detail in the next subsection.

[Table 4]

4.3. Structure of risk-pricing.

In this subsection, we analyze whether the structures of risk-pricing, $\gamma(z) \approx \lambda(z)$, are equal in each market. For that purpose, it is not enough to compare the values of the coefficients shown in the previous subsection. What is important is not the parameters themselves estimated in market j,

$\left(\hat{\delta}_0^{(j)}, \hat{\delta}_1^{(j)}, \hat{\delta}_2^{(j)}, \hat{\delta}_3^{(j)}\right) \equiv \hat{\boldsymbol{\delta}}^{(j)}$, but what value $\lambda\left(z; \hat{\boldsymbol{\delta}}^{(j)}\right) \equiv \hat{\lambda}^{(j)}(z)$ takes when z is given.

Figure 2 illustrates the shape of $\hat{\lambda}^{(j)}(z)$ in the domain $[-0.33, 0.14]$. This is because 99% of the state of the economy in the US, z_{it}, is included in this interval. It can be seen that the $\hat{\lambda}^{(j)}(z)$s estimated in the G7 markets have very similar shape except Germany (and missing Japan and the UK). In these markets, $\hat{\lambda}^{(j)}(z)$ is a decreasing function of z less than 0.23, an increasing function of z in the interval $[-0.23, 0]$, and a decreasing function of z above 0. These characteristics also apply to almost all other markets except China and South Africa. From the results in the previous subsection, some or all of $\hat{\delta}^{(j)}$s are not statistically significant in China, Russia, and South Africa. This suggests that the distribution of investor rebalancing cycles in the stock markets of China and South Africa has not been well approximated, or that the distribution of investor risk attitudes $f(.)$ differs significantly from that of the US.

[Figure 2]

Next, we interpret the shape of $\lambda(z) \approx \gamma(z)$ based on the Chan and Kogan (2002) model (CK model). Their model suggests that $\gamma(z)$ is a decreasing function of z, and $\gamma(z) > 1$. These conditions are satisfied from the local maximum point of $z \approx 0$ to the point of $z \approx 0.1$. The positive state of the economy suggests that most recent market returns are positive since the state of the economy is a cumulative weighted average of market returns. Therefore, when z is in this interval $[0, 0.1)$, the economy is likely to be upward but not overheated, that is a Goldilocks economy. Since the CK model holds in this interval, γ is counter-cyclical and can be interpreted as the relative risk aversion. The third column in Table 5, "Goldilocks", shows the interval of z such that $\gamma(z)$ has such characteristics and the ratio of z_{jt} of each market included in the interval. The interval of Goldilocks, for example, in Canadian market is $[-0.018, 0.085)$. The ratio of z_{jt} included in this interval is 43.0%. The average interval in developed markets (G7 & LDM) is $[-0.001, 0.096)$ and the average ratio of z_{jt} in the Goldilocks interval is 32.8%. Similarly, the average interval in emerging markets (LEM & G20) is $[0.007, 0.111)$ and the average ratio of z_{jt} in the Goldilocks interval is 23.5%. Therefore, the period for which the CK model holds is less than half of the total even in developed markets.

We consider the case where z is less than about 0. The price of variance risk $\lambda(z) \approx \gamma(z)$ is an increasing function of z in the interval from the minimum point of $z \approx -0.23$ to the point of $z \approx 0$. The CK model partially fails in this interval because the condition that $\gamma(z)$ is a decreasing function of z is not satisfied. This result is as expected from the perspective of prospect theory. The negative state of economy means that investors who perceive that the current stock price is lower than the reference stock price are more influential. Consequently, the probability of a loss-averse regime, $\Pr[G_t = 3]$, increases, and $\gamma(z)$ becomes procyclical (see also 2.3.2 in this paper, for detail). The second column in Table 5, “Recession”, shows the interval of z such that $\gamma(z)$ has such characteristics. The ratio of z_{jt} included in this interval is, for example, 16.6% in Canada. The average of developed markets is about 24.2%, and that of emerging markets is about 29.2%. Interpreting γ as the mean-variance ratio λ, the reason for this is thought to be that investors are pessimistic and the excess expected returns will not be required to the level originally required, even though volatility increases with the economic downturn.

The above results are as expected, but in reality, unexpected results appeared at both ends. When z exceeds about 0.1, $\gamma(z)$ has a negative value while maintaining the countercyclical property. On the other hand, when z is below about 0.23, $\gamma(z)$ has countercyclical property again. The former is associated with the overheated economy and the latter is associated with the crisis. Figure 3 shows the state of the economy and these economic stages. Especially in the crisis interval, most markets respond to the global financial crisis of 2008 and the Coronavirus crisis of 2020. The first and forth columns in Table 5, “Crisis” and “Overheated”, show these intervals. The ratios of z_{jt} included in the crisis and overheated intervals are, for example, 4.0% and 36.4% in Canada, respectively. The averages of developed markets are about 3.3% and 39.8%, and these of emerging markets is about 6.4% and 40.9%. If $\gamma(z)$ is interpreted as the mean-variance ratio or pseudo-Sharpe ratio, $\gamma(z)$ below 0 means that the short-term expected excess return becomes negative due to overheating of the economy.[11] It is a so-called “overbought” market, and it is expected that short-term selling will be expected. On the other hand, the interpretation of the crisis interval is very difficult. Loss-averse investors will be screened out when a shock like a financial crisis occurs. As a result, $\gamma \approx \lambda$ may become countercyclical.

[Table 5]

[Figure 3]

4.4. The accuracy of estimation results

Due to the design of the model in this paper, $\gamma\left(z_{\mathrm{US}t}; \hat{\boldsymbol{\delta}}^{(j)}\right) \approx \hat{\lambda}^{(j)}(z_{\mathrm{US}t})\ \forall t$ must be the same in all markets. Note that i is the United States. Using this property, it is analyzed whether or not the structure of γ is the same in each market by comparing $\hat{\lambda}^{(j)}(z_{\mathrm{US}t})\ \forall t$ with respect to the market j. In particular, the characteristics of investors in Canada are similar to those of the US, and the economic ties between the two countries are deep. Table 5 compares the correlation and mean absolute distance

[11] In the CAPM, the negative expected excess return cannot exist under equilibrium. This is because investors are implicitly constrained to have the same rebalancing cycle. When a negative excess return is expected, the price level will be unbuyable. As a result, the price falls to the level where the expected excess return is positive. On the other hand, this does not apply if the rebalancing cycle of investors is different. Even if the expected excess return for investors with short-term rebalancing cycle is negative, if the expected excess rate of return for investors with longer-term rebalancing cycle is positive, then the transaction is completed at that price. Therefore, the “short-term” expected excess return can be negative.

between $\widehat{\lambda}^{(\mathrm{Canada})}(z_{\mathrm{US}t})$ and $\widehat{\lambda}^{(j)}(z_{\mathrm{US}t})$ for $j \in \{\mathrm{France}, \mathrm{Germany}, \dots, \mathrm{Turkey}\}$ and all t. "Group I" includes markets with correlations above 0.7, and the shape of $\widehat{\lambda}^{(j)}(z_{\mathrm{US}t})$ in such markets may be considered relatively close to that in Canadian market. "Group II" includes markets where the correlation is greater than 0.7, the mean absolute distance is less than 2.0, and all δs are significant at the 5% level. All of the markets classified as Group II have a correlation of more than 0.95 with the Canadian market. It can be considered that the $\widehat{\lambda}^{(j)}(z_{\mathrm{US}t})$ in such a market has a shape very close to the $\widehat{\lambda}^{(\mathrm{Canada})}(z_{\mathrm{US}t})$.

The most accurate group, Group II, includes 15 markets; Canada, France, Italy, Australia, Hong Kong, Netherlands, Spain, Sweden, Switzerland, Brazil, South Korea, Taiwan, Indonesia, Mexico and Saudi Arabia. Results for the remaining 9 markets, Germany, Japan, the UK, China, India, Russia, Argentina, South Africa and Turkey, need to be carefully interpreted. The distribution of investor risk attitudes in these markets may differ significantly from that in the US.

[Table 6]

4.5. International integration measure – the smoothed probability of the integration regime

We analyze the existence of international integration, which is the subject of this paper. In the regime switching model, the smoothed probability of regime $S \in \{0,1,2,\dots\}$ defined as $\Pr[\phi_t = S|\psi_T]$ for $t \in \{0,1,\dots,T\}$, where ϕ_t denotes the switching variable and ψ_T indicates the information available at the terminal time T, represents the probability of realizing the regime at time t from the posterior viewpoint at time T. See also Hamilton (1988), Hamilton (1989), Hamilton (1990), and Kim and Nelson (1999) for the definition and detail. The smoothed probability of international integration regime, $\Pr\left[\phi_{ijt} = 1|\psi_{W_{ij}T}\right]$, is defined to be "international integration measure" in this paper.

If the markets i and j are internationally integrated, a model using the return on the integrated market W_{ij} as a pricing factor is likely to hold. In this case, the rational investors hold the factor mimicking portfolio of the integrated market W_{ij} so that they can be interpreted as non-home-biased investors. Therefore, a large international integration measure means that non-home-biased investors have a greater influence on the market, or that the non-home-biased strategies such as international diversification have an advantage over home-biased strategies such as domestic concentration at that time.

Figure 4 shows the international integration measure in each country from 2003 to 2019 and the color coding based on each interval defined in Table 5. International integration measures in most markets are very high. It is suggested that these markets are highly integrated with the US. On the other hand, international integration measures in Australia, China and South Africa tend to be low. Restricting to markets classified as Group II suggests segmentation only in Australia and integration in the remaining 14 markets. In addition, the international integration measure tends to decrease just before the crisis in 2008 and increase during the crisis. This trend is noticeable in the following 12 markets; Canada, France, Germany, Italy, Hong Kong, Netherlands, Spain, Sweden, Brazil, South Korea, Saudi Arabia and Turkey. Some similar trends can be seen in Switzerland, India, Russia, Taiwan and Indonesia. This implies that the influence of home-biased investors was increasing just before the crisis. That is, international diversification during this period was not an effective strategy. However, when a crisis occurred immediately after that, the positions of both were exchanged. Considering this tendency, it can be said that international diversification is a more effective strategy than domestic concentration over the entire period. This is also the case when considering only Group II.

[Figure 4]

4.6. Are developed markets more integrated than emerging markets?

The first column of Table 7 shows the time series average of international integration measure in each market. Similar to 4.5, it turns out to be a very high level except for Australia, China and South Africa. Remaining 5 columns, "Diff" (difference), "Welch" (Welch test), "BM" (Brunner-Munzel test), "Ex Prob" (excess probability) and "Corr" (correlation) are calculated with Canadian market as the reference market. Six (two) developed markets showed significantly higher (lower) international integration measure than Canada, and Hong Kong's international integration measure is not significantly different from Canada. Four (six) emerging markets showed significantly higher (lower) international integration measure than Canada, and international integration measure in Brazil and India is not significantly different from Canada. The results suggest that developed markets tend to be slightly more integrated with the US than emerging markets.

The Brunner-Munzel test (Brunner and Munzel, 2000) has a null hypothesis that the probability that the randomly sampled international integration measure of the market will be greater than the randomly sampled international integration measure of Canadian market is 0.5. "Ex Prob" shows such excess probability. From this perspective, the markets that are more integrated with the US than Canada are Sweden, Argentina, Indonesia and Mexico. Randomly sampled international integration measures in Germany, Brazil, South Korea and Saudi Arabia do not differ significantly from randomly sampled international integration measures in Canada. The average excess probability for developed markets is 0.366, and that for emerging markets is 0.415. Therefore, it is suggested that emerging markets are more integrated with the US than developed markets. Little correlation with Canadian international integration measure is observed (highest correlation is 0.558 in Saudi Arabia). Consequently, the dynamics of international integration measures in each market are largely independent of each other, except for the financial crisis of 2008.

Table 8 compares international integration measures across developed markets and these across emerging markets. When compared in all markets, the Welch test shows that the developed market is more integrated with the US than the emerging market, and the Brunner-Munzel test shows the opposite result. This is almost the same as the result of Table 7. The results, however, will be different, limiting to markets where the results are reliable as discussed in 4.4. In both Group I and Group II, the Welch and Brunner-Munzel tests suggested that emerging markets are more integrated with the US than developed markets. It is expected that the distribution of investor risk attitudes in these markets will be similar to the US. Therefore, we conclude that the traditional notion that developed markets are integrated and emerging markets are segmented is false, at least in markets where the distribution of investor risk attitudes is similar to the US.

The US investors are encouraged to make international diversified portfolio in Group II markets with high international integration measure if the results of this paper continue to hold. Such markets are following 14 markets; Canada, France, Italy, Hong Kong, Netherlands, Spain, Sweden, Switzerland, Brazil, South Korea, Taiwan, Indonesia, Mexico and Saudi Arabia. The average of most recent international integration measures in these 14 markets is 0.948, which makes international diversification extremely advantageous. Furthermore, the average staying probability of the integration regime is 0.968, and it can be said that this integration tendency is likely to continue in the future. On the other hand, the average staying probability of the segmentation regime is 0.389, and even if domestic concentration becomes advantageous, that situation will not last long.

[Table 8]

5. Conclusions

In this paper, we analyzed the degree of integration between large stock markets by using a bivariate GARCH-in-Mean model with two regimes in which the integration regime and the segmentation regime of the market are switched. The smoothed probability of the integration regime was interpreted as an "international integration measure". A high international integration measure means that non-home-biased strategies such as international diversification have advantages over home-biased strategies such as domestic concentration. In addition, the model was estimated by paying attention to the characteristic of the price of variance risk (or, relative risk aversion) suggested by the Chan and Kogan (2002) model and prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992).

The main results of the paper are reported below. First, we found that the structures of representative investor's risk attitude, or that of the price of variance risk, in each of the following 15 markets are almost the same; Canada, France, Italy, Australia, Hong Kong, Netherlands, Spain, Sweden, Switzerland, Brazil, South Korea, Taiwan, Indonesia, Mexico and Saudi Arabia. Second, the international integration measure in these markets was on average high (except Australia), declining before the 2008 global financial crisis (except Australia and Mexico), but rising again after the crisis (except Australia). This means that non-home-biased strategies have advantages over home-biased strategies except just before the crisis. Third, the difference between the international integration measures of developed and emerging markets was extremely small, both intuitively and statistically. In other words, being an emerging market does not mean that the market is segmented. Summing up the results, international diversification is strongly recommended in these 15 markets (except Australia) regardless of market or period.

Note that the effect of exchange rates is not fully considered in this paper. This is because this paper uses the US dollar as the numéraire and the US treasury-bill as the risk-free rate. Considering that the home bias is affected by the volatility of the real exchange rate as the Fidora, Flatzscher and Thimann (2007) pointed out, the analysis in this paper can be improved. One way to deal with this problem is to use a conditional two-factor model with market and exchange factors as proposed by Bekaert and Harvey (1995). Alternatively, a new three regime switching model may solve this problem. In this model, three regimes are switched: a segmentation regime, a first-country dominant integration regime, and a second-country dominant integration regime. In the segmentation regime, the excess return of each country's market is denominated in that country's currency, calculated at that country's risk-free rate, and formulated based on its volatility. In the dominant integration regime, the excess return of the country's market is denominated in that country's currency, calculated at that country's risk-free rate, and formulated based on the covariance between the market return of the country and the market return of the other country. The ratio of the probabilities of the first-country and second-country dominant integration regime represents the ratio of stock purchasing power of both countries. Therefore, this is expected to be close to the ratio of the market capitalization of them. If these are different, we can measure the under and over foreign investment between the two countries.

Appendix

We extend the Chan and Kogan (2002) model (CK model) and consider the case where the individual's risk attitude γ^{ind} is included in the real-valued set excluding 0 and 1. That is, we prove eqs. (26) and (27). Eq. (25) does not need to be proved because it is the same setting as the original CK model.

To get started, we consider the following utility function of representative investor for any $a, b \in \mathbb{R}$ such that $\gamma^{ind} \in (a, b)$ satisfies $\gamma^{ind} \in \mathbb{R} \setminus \{0,1\}$.

$$U(Y_t, X_t; a, b) = \sup_{\{c_t\}} \left\{ \int_a^b f(\gamma^{ind}) \frac{1}{1-\gamma^{ind}} \left(\frac{c_t}{X_t} \right)^{1-\gamma^{ind}} \mathrm{d}\gamma^{ind} \quad \text{s. t.} \quad \int_a^b c_t \mathrm{d}\gamma^{ind} \leq Y_t \right\}$$

$$\text{where} \quad c_t = c_t(Y_t, X_t; \gamma^{ind}).$$

Chan and Kogan show that the optimal consumption ratio, $\alpha_t(Y_t, X_t; \gamma) = c_t(Y_t, X_t; \gamma)/Y_t$, is represented by:

$$\alpha_t(z_t; \gamma) = f(\gamma)^{1/\gamma} \exp\left\{ -\frac{1}{\gamma} h(z_t) - z_t \right\} \quad \text{where} \quad \int_a^b f(\gamma)^{1/\gamma} \exp\left\{ -\frac{1}{\gamma} h(z_t) - z_t \right\} \mathrm{d}\gamma = 1$$

The following equation is obtained by differentiating both sides of above equation by z_t and rearranging.

$$h'(z_t; a, b) = -\left(\int_a^b \frac{1}{\gamma} f(\gamma)^{1/\gamma} \exp\left\{ -\frac{1}{\gamma} h(z_t) - z_t \right\} \mathrm{d}\gamma \right)^{-1} \equiv -A(z_t; a, b)^{-1} \qquad \text{(A1)}$$

They also prove the following equation.

$$-h'(z_t; a, b) = -\frac{Y_t U_{YY}(Y_t, X_t; a, b)}{U_Y(Y_t, X_t; a, b)} \equiv \gamma(z_t; a, b) \qquad \text{(A2)}$$

Let $B(z_t; a, b) \equiv \int_a^b \frac{1}{\gamma^2} f(\gamma)^{1/\gamma} \exp\left\{ -\frac{1}{\gamma} h(z_t) - z_t \right\} \mathrm{d}\gamma$. The following inequality holds from the Cauchy-Schwartz inequality.

$$B(z_t; a, b) \geq A(z_t; a, b)^2 \qquad \text{(A3)}$$

From eq. (A1),

$$h'(z_t; a, b) = -A(z_t; a, b)^{-1} \;\Leftrightarrow\; A(z_t; a, b) h'(z_t; a, b) = -1$$

The following equation is obtained by differentiating both sides of above equation by z_t and rearranging.

$$A(z_t; a, b) h''(z_t; a, b) = A(z_t; a, b) h'(z_t; a, b) + B(z_t; a, b) \{h'(z_t; a, b)\}^2 \qquad \text{(A4)}$$

Equations and inequation (A1) - (A4) is not affected by the integral interval of γ^{ind}, that is, (a, b).

Next, we consider the case of $\gamma^{ind} \in (0,1)$. In this case, from eq. (A1),

$$A(z_t; 0,1) = \int_0^1 \frac{1}{\gamma} f(\gamma)^{1/\gamma} \exp\left\{ -\frac{1}{\gamma} h(z_t) - z_t \right\} \mathrm{d}\gamma > \int_0^1 f(\gamma)^{1/\gamma} \exp\left\{ -\frac{1}{\gamma} h(z_t) - z_t \right\} \mathrm{d}\gamma = 1$$

$$\Leftrightarrow \quad 0 < A(z_t; 0,1)^{-1} < 1 \quad \Leftrightarrow \quad 0 < -(-A(z_t; 0,1)^{-1}) < 1$$

$$\therefore \quad 0 < -h'(z_t; 0,1) < 1$$

From eq. (A2),

$$0 < \gamma(z_t; 0{,}1) < 1 \tag{A5}$$

From inequation (A3),

$$
\begin{aligned}
& B(z_t; 0{,}1) \geq A(z_t; 0{,}1)^2 \\
\Leftrightarrow\ & -B(z_t; 0{,}1) \leq -A(z_t; 0{,}1)^2 \\
\Leftrightarrow\ & -B(z_t; 0{,}1)\{-A(z_t; 0{,}1)\}^{-1} \geq A(z_t; 0{,}1) \qquad \because\ -A(z_t; 0{,}1) < 0 \\
\Leftrightarrow\ & -B(z_t; 0{,}1)h'(z_t; 0{,}1) \geq A(z_t; 0{,}1) \\
\Leftrightarrow\ & A(z_t; 0{,}1) + B(z_t; 0{,}1)h'(z_t; 0{,}1) \leq 0 \\
\Leftrightarrow\ & A(z_t; 0{,}1)h'(z_t; 0{,}1) + B(z_t; 0{,}1)\{h'(z_t; 0{,}1)\}^2 \geq 0 \quad \because \quad h'(z_t; 0{,}1) < 0 \\
\Leftrightarrow\ & A(z_t; 0{,}1)h''(z_t; 0{,}1) \geq 0 \quad \because \quad \text{eq. (A4)} \\
\Leftrightarrow\ & h''(z_t; 0{,}1) \geq 0 \quad \because \quad A(z_t; 0{,}1) > 1
\end{aligned}
$$

$$\therefore\ \ h''(z_t; 0{,}1) \geq 0 \ \ (h \text{ is convex.}) \tag{A6}$$

Finally, we consider the case of $\gamma^{ind} \in (-\infty, 0)$. In this case, from eq. (A1),

$$A(z_t; -\infty, 0) = \int_{-\infty}^{0} \frac{1}{\gamma} f(\gamma)^{1/\gamma} \exp\left\{-\frac{1}{\gamma}h(z_t) - z_t\right\} \mathrm{d}\gamma < 0$$

$$\Leftrightarrow \quad A(z_t; -\infty, 0)^{-1} < 0 \quad \Leftrightarrow \ -(-A(z_t; -\infty, 0)^{-1}) < 0$$

$$\therefore \quad -h'(z_t; -\infty, 0) < 0$$

From eq. (A2),

$$\therefore \quad \gamma(z_t; -\infty, 0) < 0 \tag{A7}$$

From inequation (A3),

$$
\begin{aligned}
& B(z_t; -\infty, 0) \geq A(z_t; -\infty, 0)^2 \\
\Leftrightarrow\ & -B(z_t; -\infty, 0) \leq -A(z_t; -\infty, 0)^2 \\
\Leftrightarrow\ & -B(z_t; -\infty, 0)\{-A(z_t; -\infty, 0)\}^{-1} \leq A(z_t; -\infty, 0) \qquad \because\ -A(z_t; -\infty, 0) > 0 \\
\Leftrightarrow\ & -B(z_t; -\infty, 0)h'(z_t; -\infty, 0) \leq A(z_t; -\infty, 0) \\
\Leftrightarrow\ & A(z_t; -\infty, 0) + B(z_t; -\infty, 0)h'(z_t; -\infty, 0) \geq 0 \\
\Leftrightarrow\ & A(z_t; -\infty, 0)h'(z_t; -\infty, 0) + B(z_t; -\infty, 0)\{h'(z_t; -\infty, 0)\}^2 \geq 0 \quad \because \quad h'(z_t; -\infty, 0) > 0 \\
\Leftrightarrow\ & A(z_t; -\infty, 0)h''(z_t; -\infty, 0) \geq 0 \quad \because \quad \text{eq. (A4)} \\
\Leftrightarrow\ & h''(z_t; -\infty, 0) \leq 0 \quad \because \quad A(z_t; -\infty, 0) < 0
\end{aligned}
$$

$$\therefore\ \ h''(z_t; -\infty, 0) \leq 0 \ \ (h \text{ is concave.}) \tag{A8}$$

From inequations (A5) and (A6),

$$\gamma^{(2)}(z_{Yt}) \equiv -\frac{Y_t U_{YY}^{(2)}(Y_t, X_t)}{U_Y^{(2)}(Y_t, X_t)} \in (0{,}1), \qquad \frac{\mathrm{d}\gamma^{(2)}(z_{Yt})}{\mathrm{d}z_{Yt}} < 0 \tag{26}$$

From inequations (A7) and (A8),

$$\gamma^{(3)}(z_{Yt}) \equiv -\frac{Y_t U_{YY}^{(3)}(Y_t, X_t)}{U_Y^{(3)}(Y_t, X_t)} \in (-\infty, 0), \qquad \frac{\mathrm{d}\gamma^{(3)}(z_{Yt})}{\mathrm{d}z_{Yt}} > 0 \tag{27}$$

Q.E.D.

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Table 1
Summary statistics

		Mean	SD	Skew	Kurt	Min	Med	Max	Corr
Ref	US	0.156	2.418	-0.754	11.529	-18.195	0.238	12.102	1.000
G7	Canada	0.109	2.307	-1.158	13.359	-16.089	0.327	13.675	0.816
	France	0.091	2.892	-1.047	10.823	-22.159	0.282	13.238	0.809
	Germany	0.194	3.003	-0.812	10.537	-21.610	0.423	16.116	0.809
	Italy	0.023	3.231	-1.142	9.490	-23.296	0.304	11.040	0.727
	Japan	0.090	2.810	-0.755	7.895	-19.763	0.347	13.737	0.608
	UK	0.074	2.400	-1.153	15.506	-21.047	0.210	13.411	0.823
LDM	Australia	0.096	2.210	-1.091	9.182	-15.649	0.300	9.542	0.682
	Hong Kong	0.131	2.885	-0.134	5.744	-16.319	0.313	12.433	0.588
	Netherlands	0.107	2.813	-1.307	13.715	-24.990	0.263	13.287	0.798
	Spain	0.056	3.117	-0.910	8.736	-21.201	0.362	11.740	0.723
	Sweden	0.159	2.761	-0.821	8.980	-20.171	0.324	13.060	0.766
	Switzerland	0.106	2.394	-1.395	17.413	-22.277	0.314	14.071	0.734
LEM	Brazil	0.269	3.600	-0.227	7.103	-20.010	0.465	18.345	0.678
	China	0.145	3.344	-0.057	5.389	-13.841	0.118	14.964	0.174
	India	0.282	2.956	-0.273	6.440	-15.954	0.424	14.078	0.540
	Russia	0.206	4.762	0.454	14.787	-21.167	0.367	46.040	0.529
	South Korea	0.167	2.779	-0.670	10.417	-20.490	0.356	18.568	0.613
	Taiwan	0.115	2.514	-0.672	5.359	-10.650	0.262	9.867	0.540
G20	Argentina	0.569	4.855	-0.611	7.495	-31.447	0.702	19.904	0.525
	Indonesia	0.283	2.891	-0.936	9.091	-20.782	0.418	12.285	0.437
	Mexico	0.217	2.721	0.098	10.080	-16.413	0.273	20.416	0.723
	Saudi Arabia	0.126	3.550	-1.071	8.996	-22.041	0.454	16.808	0.356
	South Africa	0.242	2.750	0.037	8.697	-15.577	0.315	19.627	0.648
	Turkey	0.310	3.659	-0.261	5.014	-17.530	0.535	17.067	0.482

Note: Results are based on market index returns [%, weekly] from 17 Oct 2003 to 26 Jun 2020 (872 weeks). The reference market (Ref) is the US stock market (S&P 500). Analyzed markets are the G7 markets, large developed markets (LDM), large emerging markets (LEM), and the rest of the G20 markets. The statistics of "Corr" means the correlation with the US stock returns.

Table 2

AIC of each polynomial model (US / domestic)

	Linear	Quadratic	Cubic	Quartic
κ = 0.0200 (about 1 year)	3,326.589	3,328.525	3,323.229	3,325.165
κ = 0.0250 (about 9 months)	3,326.528	3,328.409	3,322.157	3,324.531
κ = 0.0325 (about 7 months)	3,326.481	3,328.230		3,323.733
κ = 0.0375 (about 6 months)		3,328.111	**3,321.535**	
κ = 0.0450 (about 5 months)		3,327.950	3,321.932	3,323.981
κ = 0.0550 (about 4 months)	3,327.703	3,327.806	3,322.537	3,324.502
κ = 0.0750 (about 3 months)	3,326.692	3,327.792		
κ = 0.1150 (about 2 months)	3,326.957	3,328.313		3,326.366
κ = 0.2500 (about 1 month)	3,327.166	3,329.159	3,328.546	3,330.176

Table 3

Model selection (AIC)

		κ = 0.0200	κ = 0.0250	κ = 0.0325	κ = 0.0375	κ = 0.0450	κ = 0.0550	κ = 0.0750	κ = 0.1150	κ = 0.2500
G7	Canada	6,296.305	6,296.211	6,295.755	<u>6,295.499</u>	6,295.688	6,295.755	6,296.293	6,296.792	6,296.851
	France	6,684.174		<u>6,683.271</u>			6,691.188			
	Germany	6,816.012	6,815.635	6,815.113	6,814.815	6,814.054	6,813.807	6,812.545	<u>6,809.704</u>	
	Italy	<u>6,960.292</u>	6,966.166	6,965.369						6,960.374
	Japan									
	UK									
LDM	Australia	6,644.013	6,643.390	6,643.047	6,642.663	6,642.198	6,642.001	<u>6,641.615</u>	6,642.148	6,642.111
	Hong Kong	6,860.696	6,860.686	6,862.529	6,862.749	6,862.882				<u>6,860.331</u>
	Netherlands						<u>6,711.177</u>			
	Spain				<u>6,873.462</u>					
	Sweden	6,745.011	6,745.517	6,745.927	6,745.932	6,745.815	6,745.599	6,745.387	6,744.936	<u>6,744.272</u>
	Switzerland	6,348.646	<u>6,346.408</u>	6,346.628	6,346.998	6,347.465	6,348.277	6,349.208	6,350.028	6,350.405
LEM	Brazil	7,613.441	7,614.482	7,612.065	<u>7,611.750</u>	7,613.419	7,614.543	7,617.384	7,619.706	7,621.737
	China	7,303.463	7,303.616	7,299.233	7,302.304	7,296.479	7,299.396	7,297.703	7,297.520	<u>7,293.411</u>
	India		<u>7,098.556</u>	7,107.781	7,107.391	7,107.320	7,107.496		7,108.042	7,106.833
	Russia	7,486.310	7,487.413	7,488.641	7,490.148	7,489.368	7,488.832	7,487.868	7,488.820	<u>7,483.609</u>
	South Korea	7,044.303	<u>7,044.143</u>	7,047.053			7,051.933			
	Taiwan		<u>6,825.465</u>	6,826.714					6,827.905	6,828.500
G20	Argentina		7,869.453	<u>7,844.306</u>	7,870.200	7,856.265	7,859.302	7,865.195		
	Indonesia		<u>7,204.589</u>	7,215.750		7,241.561	7,242.162		7,242.632	7,236.398
	Mexico	6,927.712	6,927.886	6,927.854	6,927.722		6,927.629	<u>6,927.464</u>		
	Saudi Arabia	7,146.632	7,145.421	<u>7,143.814</u>	7,143.938	7,144.551	7,144.711	7,145.311	7,146.060	7,146.724
	South Africa	7,121.067	7,120.667	7,120.277	7,118.987	7,118.167	7,116.192	7,114.409	7,113.539	<u>7,112.838</u>
	Turkey	7,801.027	7,801.286	7,801.276	7,801.252		7,798.961	7,790.890	<u>7,788.905</u>	7,789.149

Note: The model with the smallest AIC is underlined.

Table 4

Estimated parameters (G7)

	Canada		France		Germany		Italy		Japan	UK
α_{i0}	0.490	***	0.418	***	0.440	***	0.396	***		
	(0.110)		(0.093)		(0.095)		(0.091)			
α_{j0}	0.406	***	0.422	**	0.592	**	0.369	*		
	(0.098)		(0.138)		(0.190)		(0.146)			
α_{i1}	0.182	***	0.186	***	0.195	***	0.189	***		
	(0.028)		(0.030)		(0.031)		(0.030)			
α_{j1}	0.163	***	0.100	***	0.088	***	0.096	***		
	(0.024)		(0.019)		(0.019)		(0.019)			
β_i	0.714	***	0.726	***	0.715	***	0.729	***		
	(0.041)		(0.039)		(0.039)		(0.039)			
β_j	0.789	***	0.855	***	0.853	***	0.872	***		
	(0.028)		(0.029)		(0.032)		(0.026)			
ρ_{ij}	0.810	***	0.712	***	0.706	***	0.645	***		
	(0.016)		(0.020)		(0.021)		(0.024)			
$p_{1\|1}$	0.983	***	0.972	***	0.990	***	0.974	***		
	(0.013)		(0.022)		(0.007)		(0.022)			
$p_{0\|0}$	0.840	***	0.001	-	0.871	***	0.002	-		
	(0.131)		(0.019)		(0.080)		(0.087)			
δ_0	961	**	1,550	***	1,093	**	1,095	***		
	(322)		(341)		(352)		(305)			
δ_1	-3,430	**	-5,374	***	-3,686	**	-3,863	***		
	(1,123)		(1,173)		(1,176)		(1,063)			
δ_2	4,025	**	6,124	***	4,078	**	4,475	***		
	(1,283)		(1,326)		(1,289)		(1,215)			
δ_3	-1,547	**	-2,291	***	-1,479	**	-1,699	***		
	(481)		(495)		(466)		(457)			

Note: Values in parentheses denote standard errors. ***,** and * denote statistical significance at 0.1, 1 and 5% levels, respectively.

Table 4

Estimated parameters (LDM)

	Australia	Hong Kong	Netherlands	Spain	Sweden	Switzerland
α_{i0}	0.369 ***	0.481 ***	0.420 ***	0.393 ***	0.487 ***	0.459 ***
	(0.092)	(0.107)	(0.094)	(0.088)	(0.103)	(0.100)
α_{j0}	0.191 *	0.325 **	0.539 **	0.422 *	0.584 **	0.549 ***
	(0.084)	(0.126)	(0.181)	(0.167)	(0.188)	(0.167)
α_{i1}	0.244 ***	0.219 ***	0.195 ***	0.196 ***	0.171 ***	0.207 ***
	(0.037)	(0.034)	(0.032)	(0.032)	(0.027)	(0.031)
α_{j1}	0.059 ***	0.074 ***	0.129 ***	0.106 ***	0.100 ***	0.141 ***
	(0.015)	(0.020)	(0.028)	(0.023)	(0.020)	(0.031)
β_i	0.695 ***	0.684 ***	0.718 ***	0.725 ***	0.720 ***	0.716 ***
	(0.042)	(0.042)	(0.040)	(0.038)	(0.038)	(0.038)
β_j	0.897 ***	0.884 ***	0.810 ***	0.854 ***	0.847 ***	0.780 ***
	(0.028)	(0.031)	(0.043)	(0.034)	(0.033)	(0.048)
ρ_{ij}	0.508 ***	0.589 ***	0.688 ***	0.679 ***	0.760 ***	0.800 ***
	(0.117)	(0.046)	(0.025)	(0.029)	(0.016)	(0.020)
$p_{1\|1}$	0.716 ***	0.953 ***	0.950 ***	0.923 ***	0.992 ***	0.930 ***
	(0.150)	(0.055)	(0.037)	(0.050)	(0.006)	(0.026)
$p_{0\|0}$	0.735 ***	0.481 -	0.002 -	0.000 -	0.823 ***	0.569 ***
	(0.108)	(0.343)	(0.026)	(0.000)	(0.136)	(0.129)
δ_0	1,051 ***	1,850 ***	1,076 **	1,194 *	1,297 *	1,037 ***
	(261)	(344)	(391)	(526)	(550)	(306)
δ_1	-3,649 ***	-6,462 ***	-3,782 **	-4,142 *	-4,591 *	-3,636 ***
	(901)	(1,182)	(1,363)	(1,819)	(1,905)	(1,053)
δ_2	4,162 ***	7,431 ***	4,368 **	4,719 *	5,339 *	4,184 ***
	(1,022)	(1,340)	(1,559)	(2,062)	(2,162)	(1,190)
δ_3	-1,557 ***	-2,809 ***	-1,653 **	-1,764 *	-2,036 *	-1,578 ***
	(383)	(503)	(587)	(768)	(806)	(442)

Note: Values in parentheses denote standard errors. ***,** and * denote statistical significance at 0.1, 1 and 5% levels, respectively.

Table 4

Estimated parameters (LEM)

	Brazil		China		India		Russia		South Korea		Taiwan	
α_{i0}	0.493	***	0.426	***	0.425	***	0.414	***	0.427	***	0.448	***
	(0.112)		(0.100)		(0.096)		(0.098)		(0.100)		(0.105)	
α_{j0}	1.735	**	0.292	*	0.417	**	0.930	***	1.046	***	0.227	**
	(0.559)		(0.116)		(0.137)		(0.258)		(0.273)		(0.079)	
α_{i1}	0.188	***	0.264	***	0.227	***	0.219	***	0.186	***	0.213	***
	(0.030)		(0.040)		(0.035)		(0.034)		(0.030)		(0.034)	
α_{j1}	0.099	***	0.105	***	0.116	***	0.134	***	0.149	***	0.078	***
	(0.018)		(0.025)		(0.022)		(0.022)		(0.027)		(0.017)	
β_i	0.709	***	0.669	***	0.694	***	0.705	***	0.723	***	0.697	***
	(0.042)		(0.043)		(0.040)		(0.041)		(0.040)		(0.043)	
β_j	0.824	***	0.868	***	0.841	***	0.816	***	0.744	***	0.891	***
	(0.035)		(0.029)		(0.030)		(0.028)		(0.045)		(0.023)	
ρ_{ij}	0.668	***	0.390	***	0.562	***	0.621	***	0.569	***	0.516	***
	(0.025)		(0.048)		(0.036)		(0.036)		(0.025)		(0.032)	
$p_{1\|1}$	0.981	***	0.995	***	0.965	***	0.940	***	0.979	***	0.952	***
	(0.008)		(0.004)		(0.018)		(0.031)		(0.010)		(0.038)	
$p_{0\|0}$	0.841	***	0.994	***	0.613	**	0.702	***	0.011	-	0.000	-
	(0.075)		(0.005)		(0.194)		(0.142)		(0.077)		(0.009)	
δ_0	813	**	539	*	721	***	643	-	1,245	***	1,270	***
	(264)		(273)		(198)		(333)		(254)		(284)	
δ_1	-2,933	**	-1,773	-	-2,536	***	-2,280	-	-4,435	***	-4,497	***
	(931)		(931)		(668)		(1,193)		(878)		(982)	
δ_2	3,477	**	1,904	-	2,926	***	2,658	-	5,184	***	5,222	***
	(1,070)		(1,036)		(737)		(1,398)		(993)		(1,115)	
δ_3	-1,348	***	-666	-	-1,103	***	-1,014	-	-1,984	***	-1,985	***
	(403)		(379)		(268)		(537)		(369)		(416)	

Note: Values in parentheses denote standard errors. ***,** and * denote statistical significance at 0.1, 1 and 5% levels, respectively.

Table 4

Estimated parameters (G20)

	Argentina	Indonesia	Mexico	Saudi Arabia	South Africa	Turkey
α_{i0}	0.427 ***	0.461 ***	0.472 ***	0.479 ***	0.365 ***	0.441 ***
	(0.102)	(0.108)	(0.102)	(0.106)	(0.088)	(0.100)
α_{j0}	0.703 **	0.779 **	1.237 ***	1.073 **	0.311 ***	3.346 *
	(0.225)	(0.267)	(0.323)	(0.341)	(0.079)	(1.698)
α_{i1}	0.179 ***	0.230 ***	0.166 ***	0.198 ***	0.226 ***	0.231 ***
	(0.029)	(0.036)	(0.026)	(0.030)	(0.036)	(0.037)
α_{j1}	0.124 ***	0.186 ***	0.144 ***	0.110 ***	0.203 ***	0.100 ***
	(0.021)	(0.036)	(0.024)	(0.019)	(0.026)	(0.029)
β_i	0.731 ***	0.682 ***	0.729 ***	0.700 ***	0.712 ***	0.691 ***
	(0.039)	(0.044)	(0.037)	(0.040)	(0.040)	(0.041)
β_j	0.859 ***	0.746 ***	0.758 ***	0.814 ***	0.785 ***	0.753 ***
	(0.022)	(0.051)	(0.040)	(0.036)	(0.025)	(0.095)
ρ_{ij}	0.549 ***	0.412 ***	0.752 ***	0.709 ***	0.695 ***	0.565 ***
	(0.028)	(0.032)	(0.017)	(0.026)	(0.046)	(0.038)
$p_{1\|1}$	0.988 ***	0.982 ***	0.995 ***	0.987 ***	0.345 *	0.983 ***
	(0.005)	(0.008)	(0.004)	(0.013)	(0.137)	(0.013)
$p_{0\|0}$	0.007 -	0.000 -	0.968 ***	0.912 ***	0.391 **	0.919 ***
	(0.050)	(0.000)	(0.030)	(0.082)	(0.145)	(0.059)
δ_0	1,818 ***	945 ***	1,110 **	1,008 **	693 -	971 *
	(332)	(209)	(350)	(326)	(488)	(425)
δ_1	-6,513 ***	-3,368 ***	-3,857 **	-3,541 **	-2,328 -	-3,378 *
	(1,150)	(702)	(1,212)	(1,133)	(1,698)	(1,478)
δ_2	7,656 ***	3,937 ***	4,405 **	4,092 **	2,564 -	3,848 *
	(1,302)	(771)	(1,377)	(1,290)	(1,936)	(1,684)
δ_3	-2,950 ***	-1,505 ***	-1,650 **	-1,550 **	-925 -	-1,434 *
	(485)	(278)	(515)	(482)	(725)	(629)

Note: Values in parentheses denote standard errors. ***,** and * denote statistical significance at 0.1, 1 and 5% levels, respectively.

Table 5

Inflection points and four economic stages (G7 & LDM)

		Crisis (countercyclical)	**Recession (procyclical)**	**Goldilocks (counter & $\gamma > 1$)**	**Overheated (counter & $\gamma \leq 1$)**
G7	Canada (κ = 0.0375)	[-0.569, -0.248) 4.0%	[-0.248, -0.018) 16.6%	[-0.018, 0.085) 43.0%	[0.085, 0.317] 36.4%
	France (κ = 0.0325)	[-0.342, -0.219) 4.2%	[-0.219, 0.000) 20.7%	[0.000, 0.090) 34.6%	[0.090, 0.394] 40.5%
	Germany (κ = 0.1150)	[-0.292, -0.200) 1.8%	[-0.200, 0.040) 23.5%	[0.040, 0.140) 34.1%	[0.140, 0.475] 40.6%
	Italy (κ = 0.0200)	[-0.596, -0.237) 7.0%	[-0.237, -0.008) 31.3%	[-0.008, 0.093) 29.9%	[0.093, 0.396] 31.8%
	Japan				
	UK				
LDM	Australia (κ = 0.0750)	[-0.252, -0.221) 0.3%	[-0.221, 0.003) 33.7%	[0.003, 0.095) 24.5%	[0.095, 0.277] 41.5%
	Hong Kong (κ = 0.2500)	[-0.466, -0.221) 2.8%	[-0.221, -0.013) 17.8%	[-0.013, 0.080) 31.8%	[0.080, 0.479] 47.6%
	Netherlands (κ = 0.0550)	[-0.463, -0.235) 4.3%	[-0.235, -0.005) 15.5%	[-0.005, 0.098) 41.3%	[0.098, 0.424] 39.0%
	Spain (κ = 0.0375)	[-0.327, -0.221) 2.7%	[-0.221, 0.003) 36.9%	[0.003, 0.095) 28.6%	[0.095, 0.355] 31.8%
	Sweden (κ = 0.2500)	[-0.677, -0.237) 3.4%	[-0.237, -0.015) 24.4%	[-0.015, 0.085) 25.3%	[0.085, 0.337] 46.9%
	Switzerland (κ = 0.0250)	[-0.422, -0.232) 2.2%	[-0.232, 0.000) 21.3%	[0.000, 0.098) 35.0%	[0.098, 0.236] 41.5%
Average G7 & LDM (κ = 0.0898)		[-0.441, -0.227) 3.3%	[-0.227, -0.001) 24.2%	[-0.001, 0.096) 32.8%	[0.096, 0.369) 39.8%

Table 5

Inflection points and four economic stages (LEM & G20)

		Crisis (countercyclical)	**Recession (procyclical)**	**Goldilocks (counter & $\gamma > 1$)**	**Overheated (counter & $\gamma \leq 1$)**
LEM	Brazil	[-0.672, -0.258)	[-0.258, -0.023)	[-0.023, 0.090)	[0.090, 0.604]
	(κ = 0.0375)	10.0%	24.4%	10.1%	55.6%
	China	[-0.413, -0.190)	[-0.190, 0.095)	[0.095, 0.209)	[0.209, 0.906]
	(κ = 0.2500)	3.2%	64.4%	12.4%	20.1%
	India	[-0.601, -0.242)	[-0.242, 0.011)	[0.011, 0.130)	[0.130, 0.585]
	(κ = 0.0250)	3.6%	23.2%	30.3%	42.9%
	Russia	[-1.158, -0.245)	[-0.245, -0.008)	[-0.008, 0.103)	[0.103, 0.672]
	(κ = 0.2500)	10.2%	26.9%	22.2%	40.6%
	South Korea	[-0.810, -0.245)	[-0.245, -0.013)	[-0.013, 0.087)	[0.087, 0.445]
	(κ = 0.0250)	3.4%	23.1%	27.6%	45.8%
	Taiwan	[-0.561, -0.240)	[-0.240, -0.008)	[-0.008, 0.093)	[0.093, 0.291]
	(κ = 0.0250)	3.4%	16.7%	40.0%	39.8%
G20	Argentina	[-0.734, -0.245)	[-0.245, -0.023)	[-0.023, 0.066)	[0.066, 0.670]
	(κ = 0.0325)	10.0%	16.5%	9.6%	64.0%
	Indonesia	[-0.750, -0.248)	[-0.248, -0.008)	[-0.008, 0.101)	[0.101, 0.508]
	(κ = 0.0250)	5.0%	15.7%	30.5%	48.8%
	Mexico	[-0.614, -0.224)	[-0.224, 0.003)	[0.003, 0.101)	[0.101, 0.463]
	(κ = 0.0750)	4.9%	32.7%	20.8%	41.6%
	Saudi Arabia	[-0.737, -0.232)	[-0.232, -0.008)	[-0.008, 0.095)	[0.095, 0.432]
	(κ = 0.0325)	5.2%	25.2%	30.1%	39.5%
	South Africa	[-0.597, -0.200)	[-0.200, 0.048)	[0.048, 0.148)	[0.148, 0.675]
	(κ = 0.2500)	7.8%	46.1%	30.7%	15.5%
	Turkey	[-0.751, -0.227)	[-0.227, 0.016)	[0.016, 0.109)	[0.109, 0.605]
	(κ = 0.1150)	10.7%	35.0%	17.6%	36.7%
Average LEM & G20		[-0.700, -0.233)	[-0.233, 0.007)	[0.007, 0.111)	[0.111, 0.571)
(κ = 0.0952)		6.4%	29.2%	23.5%	40.9%

Table 6

Accuracy of estimated price of risk

		Correlation	Abs. Distance	Significance	Group I	Group II
G7	Canada	1.000	0.000	100%	x	x
	France	0.961	1.111	100%	x	x
	Germany	0.489	4.035	100%		
	Italy	0.991	1.002	100%	x	x
	Japan					
	UK					
LDM	Australia	0.951	1.311	100%	x	x
	Hong Kong	0.985	1.944	100%	x	x
	Netherlands	0.986	1.362	100%	x	x
	Spain	0.950	1.235	100%	x	x
	Sweden	0.998	0.754	100%	x	x
	Switzerland	0.973	1.244	100%	x	x
LEM	Brazil	0.998	0.850	100%	x	x
	China	-0.277	4.021	25%		
	India	0.890	3.222	100%	x	
	Russia	0.990	1.519	0%	x	
	South Korea	0.998	1.161	100%	x	x
	Taiwan	0.990	1.180	100%	x	x
G20	Argentina	0.998	3.839	100%	x	
	Indonesia	0.987	1.892	100%	x	x
	Mexico	0.959	1.457	100%	x	x
	Saudi Arabia	0.990	0.943	100%	x	x
	South Africa	0.336	3.229	0%		
	Turkey	0.847	2.079	100%	x	

Note: Correlation and absolute distance are calculated with Canada as the reference country. "Significance" indicates the ratio of significant δs at the 5% level. Group I includes markets with correlations above 0.7. Group II includes markets where the correlation is greater than 0.7, the absolute distance is less than 2 and the coefficients δs are all significant at 5% level.

Table 7
International integration measure

		Mean	Diff	Welch		BM		Ex Prob	Corr
G7	Canada	0.907	0.000	0.000	-	0.000	-	0.500	1.000
	France	0.973	0.066	9.203	***	-4.524	***	0.428	-0.037
	Germany	0.932	0.025	2.750	**	1.048	-	0.515	-0.088
	Italy	0.975	0.068	9.609	***	-3.760	***	0.440	-0.013
	Japan								
	UK								
LDM	Australia	0.482	-0.425	-48.165	***	-54.259	***	0.070	-0.145
	Hong Kong	0.917	0.010	1.359	-	-17.405	***	0.257	0.275
	Netherlands	0.952	0.045	6.224	***	-11.447	***	0.326	-0.046
	Spain	0.928	0.021	2.799	**	-16.073	***	0.269	-0.010
	Sweden	0.958	0.051	6.157	***	8.602	***	0.621	0.204
	Switzerland	0.861	-0.046	-4.784	***	-20.743	***	0.230	0.174
LEM	Brazil	0.892	-0.015	-1.482	-	-1.092	-	0.484	0.076
	China	0.500	-0.407	-23.047	***	-24.556	***	0.218	0.278
	India	0.916	0.009	1.118	-	-9.783	***	0.357	0.093
	Russia	0.834	-0.073	-8.015	***	-24.060	***	0.201	0.347
	South Korea	0.979	0.072	9.998	***	0.883	-	0.514	0.055
	Taiwan	0.954	0.047	6.604	***	-11.449	***	0.325	0.064
G20	Argentina	0.986	0.079	10.676	***	16.428	***	0.711	0.068
	Indonesia	0.980	0.073	9.825	***	5.193	***	0.578	0.077
	Mexico	0.877	-0.030	-2.473	*	14.309	***	0.702	0.037
	Saudi Arabia	0.878	-0.029	-2.752	**	0.040	-	0.501	0.558
	South Africa	0.483	-0.424	-43.310	***	-53.190	***	0.075	0.095
	Turkey	0.835	-0.072	-6.726	***	-14.072	***	0.310	-0.038
Average G7 & LDM		0.889	-0.018					0.366	0.131
Average LEM & G20		0.843	-0.064					0.415	0.142
Total average		0.864	-0.043					0.392	0.137

Note: Diff (difference), Welch (Welch test), BM (Brunner-Munzel test), Ex Prob (excess probability) and Corr (correlation) are calculated with Canada as the reference country.

Table 8

International integration measure between DM and EM

	G7 & LDM	LEM & G20	Diff	Welch		BM		Ex Prob
All	0.889	0.843	0.046	13.131	***	-5.911	***	0.474
Group I	0.884	0.913	-0.029	-9.641	***	-24.174	***	0.389
Group II	0.884	0.927	-0.043	-12.341	***	-34.061	***	0.325

Note: Diff (difference), Welch (Welch test), BM (Brunner-Munzel test), Ex Prob (excess probability) are calculated between developed markets (G7 & LDM) and emerging markets (LEM & G20)

Figure 1
Distribution of investors' rebalancing cycles

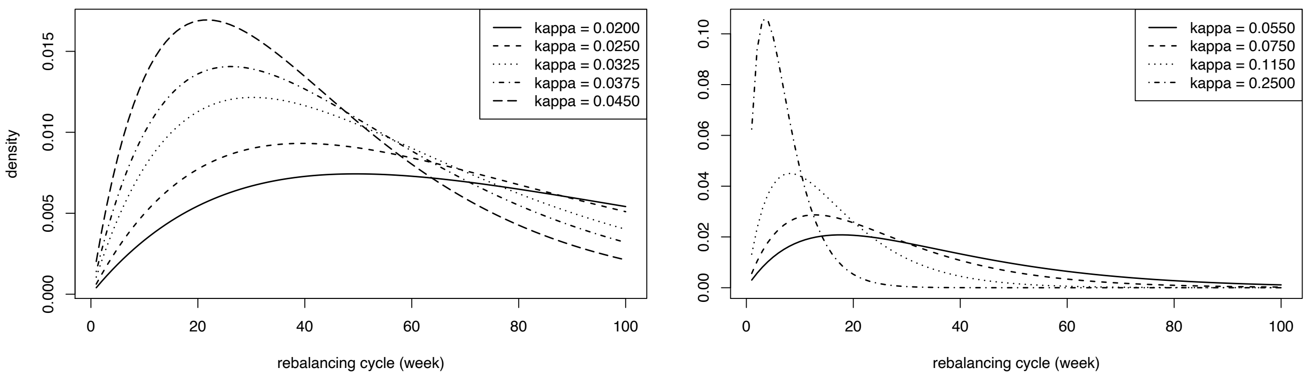

Figure 2
Structure of risk-pricing

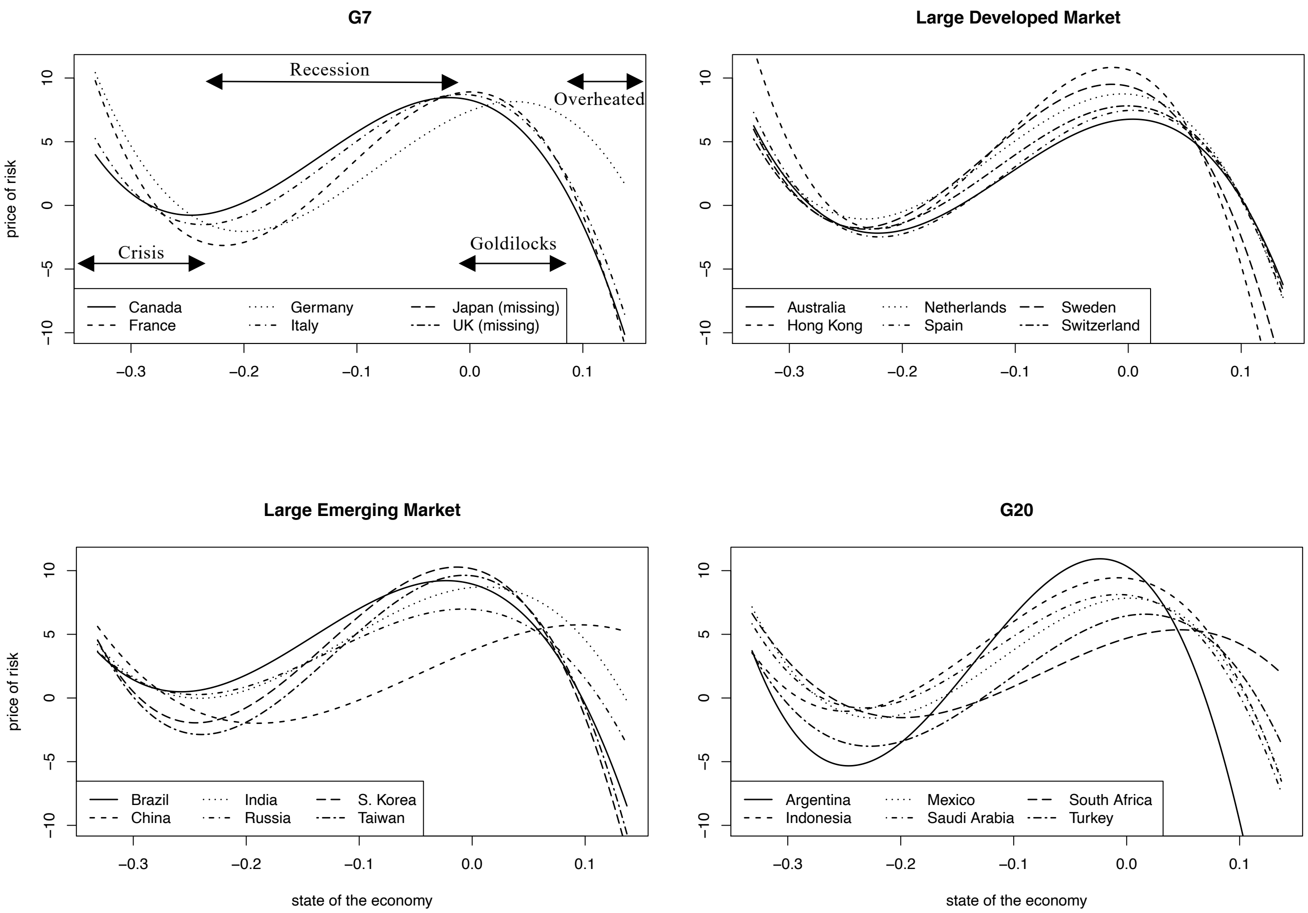

Figure 3

State of the economy and economic stages (1/3)

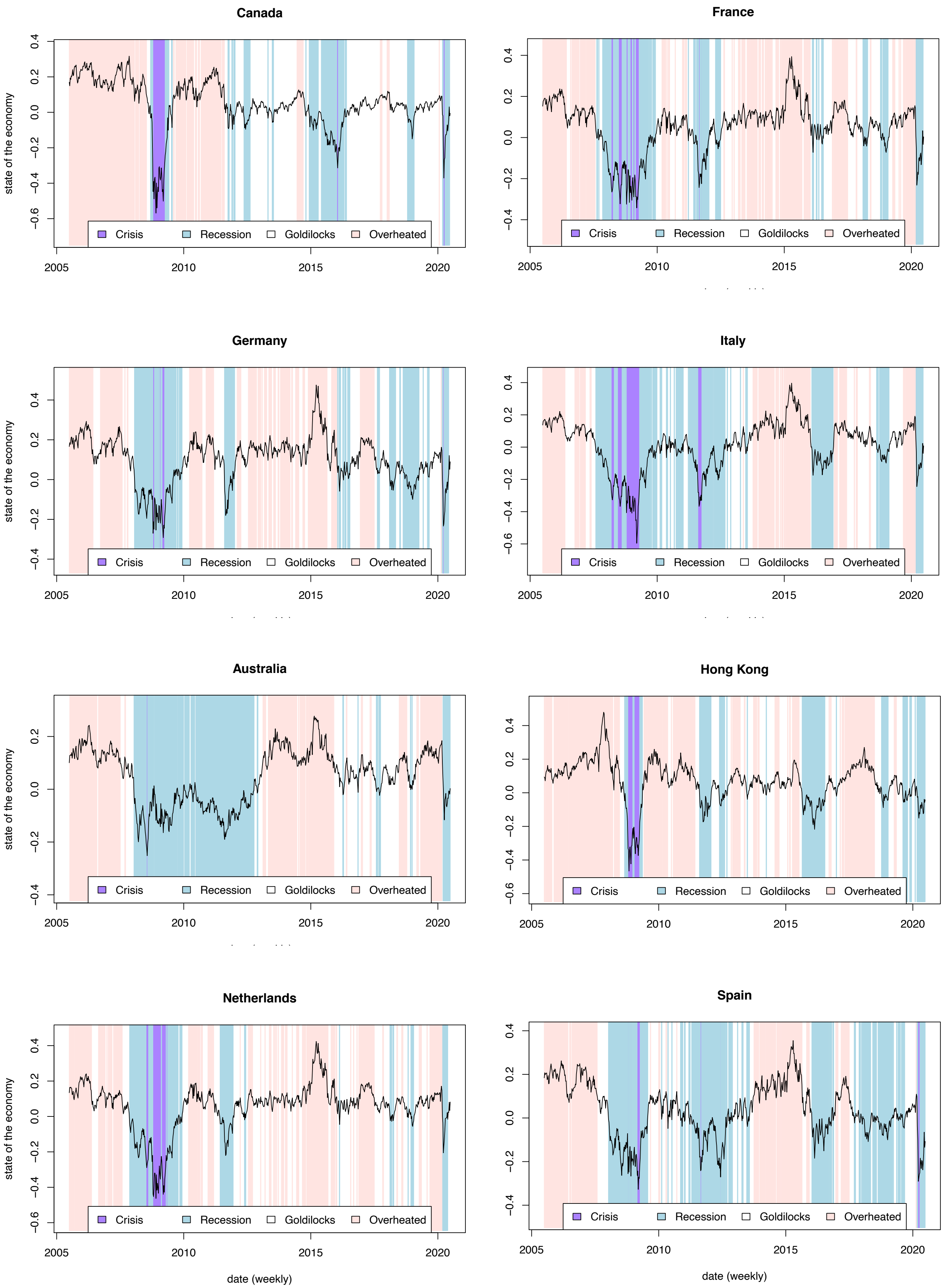

Figure 3
State of the economy and economic stages (2/3)

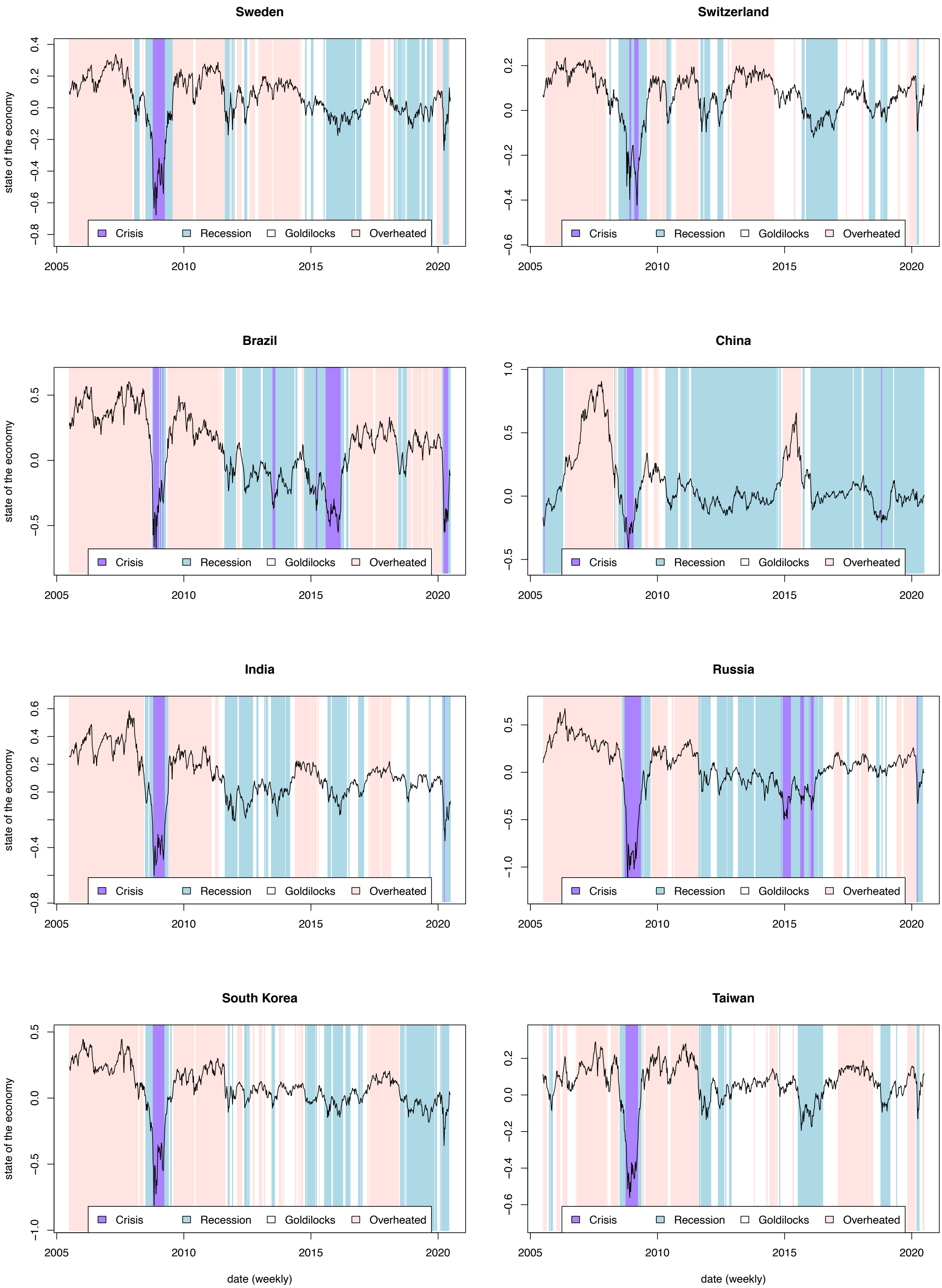

Figure 3

State of the economy and economic stages (3/3)

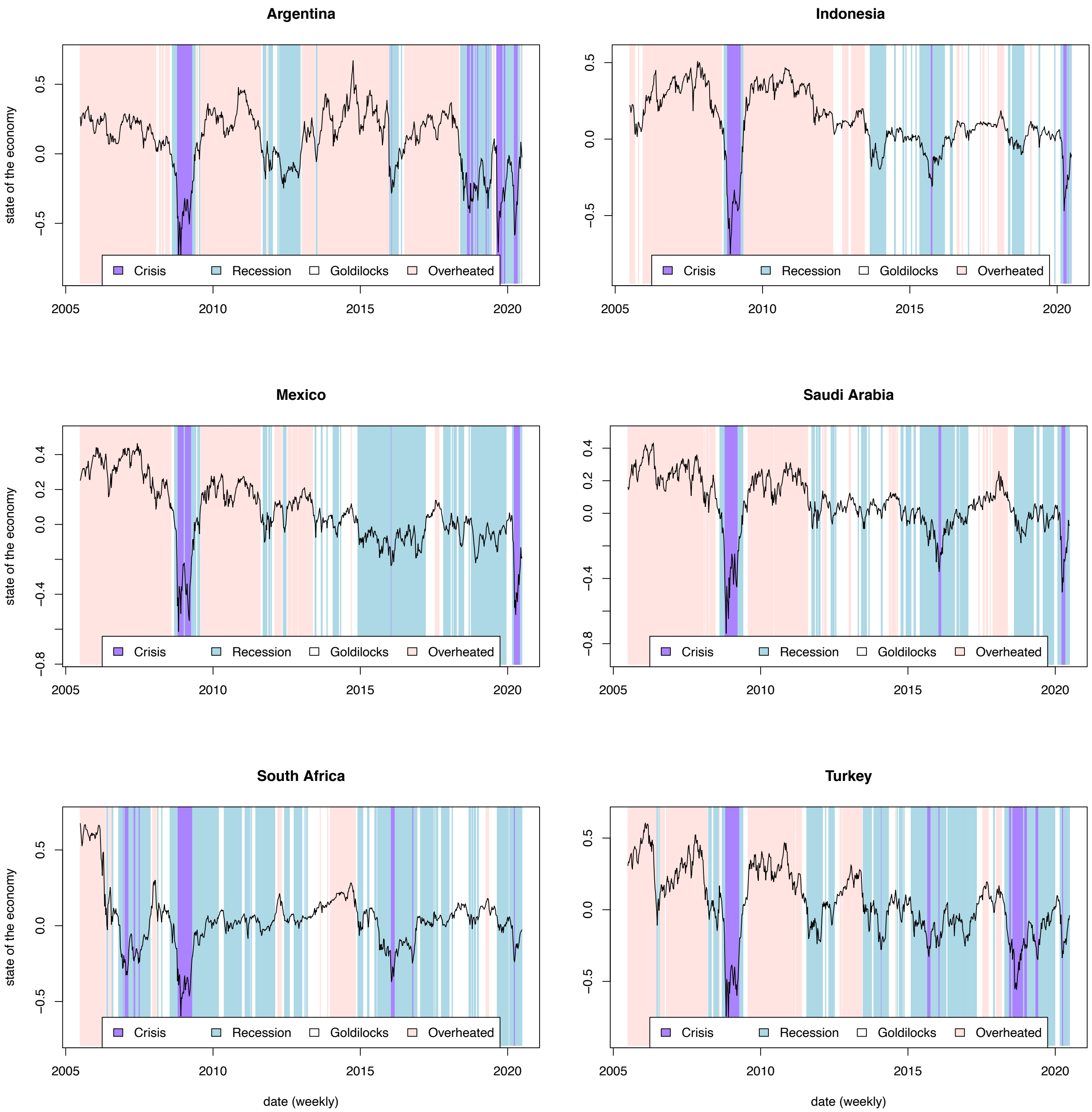

Figure 4
International integration measure (1/3)

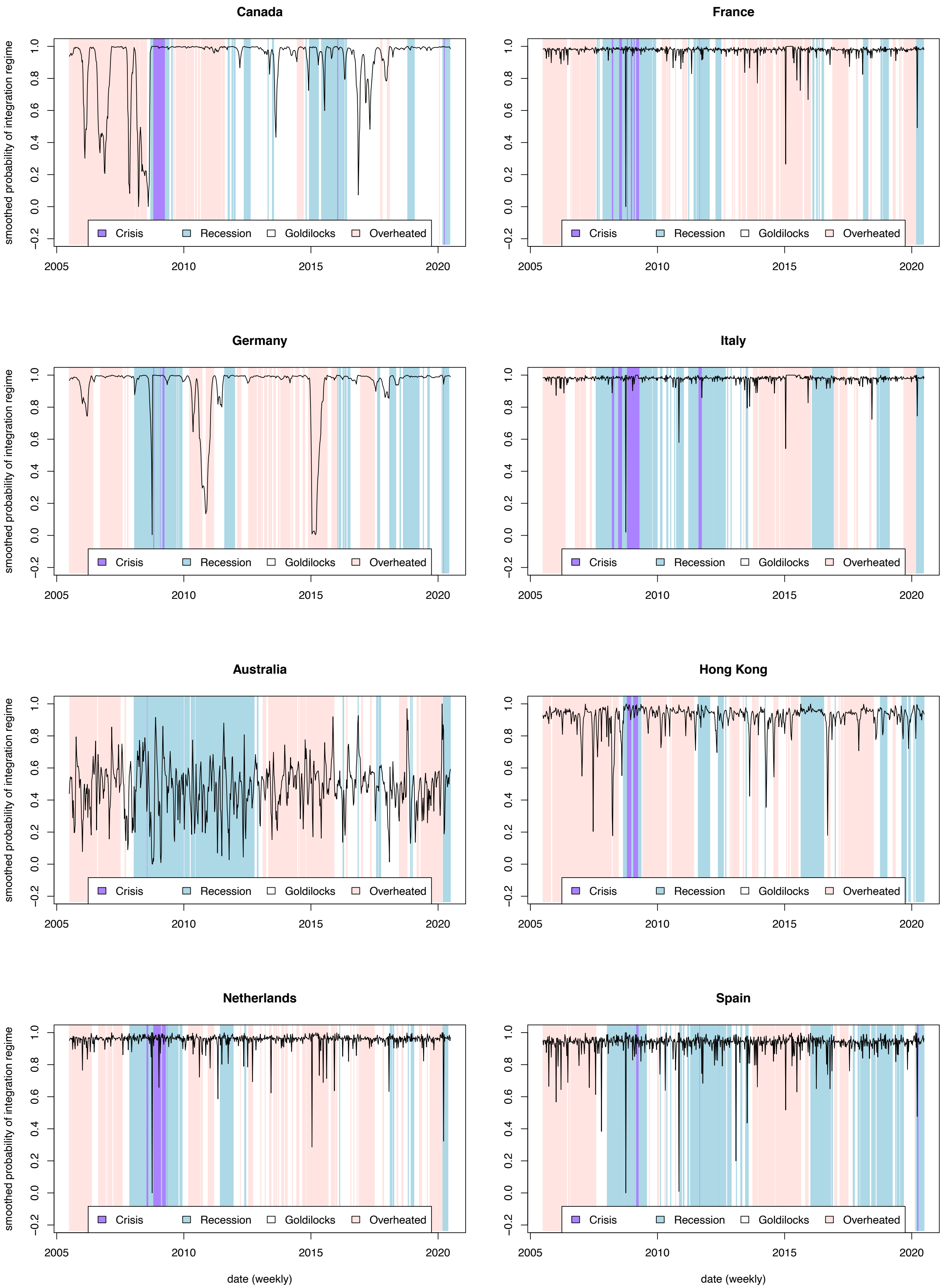

Figure 4

International integration measure (2/3)

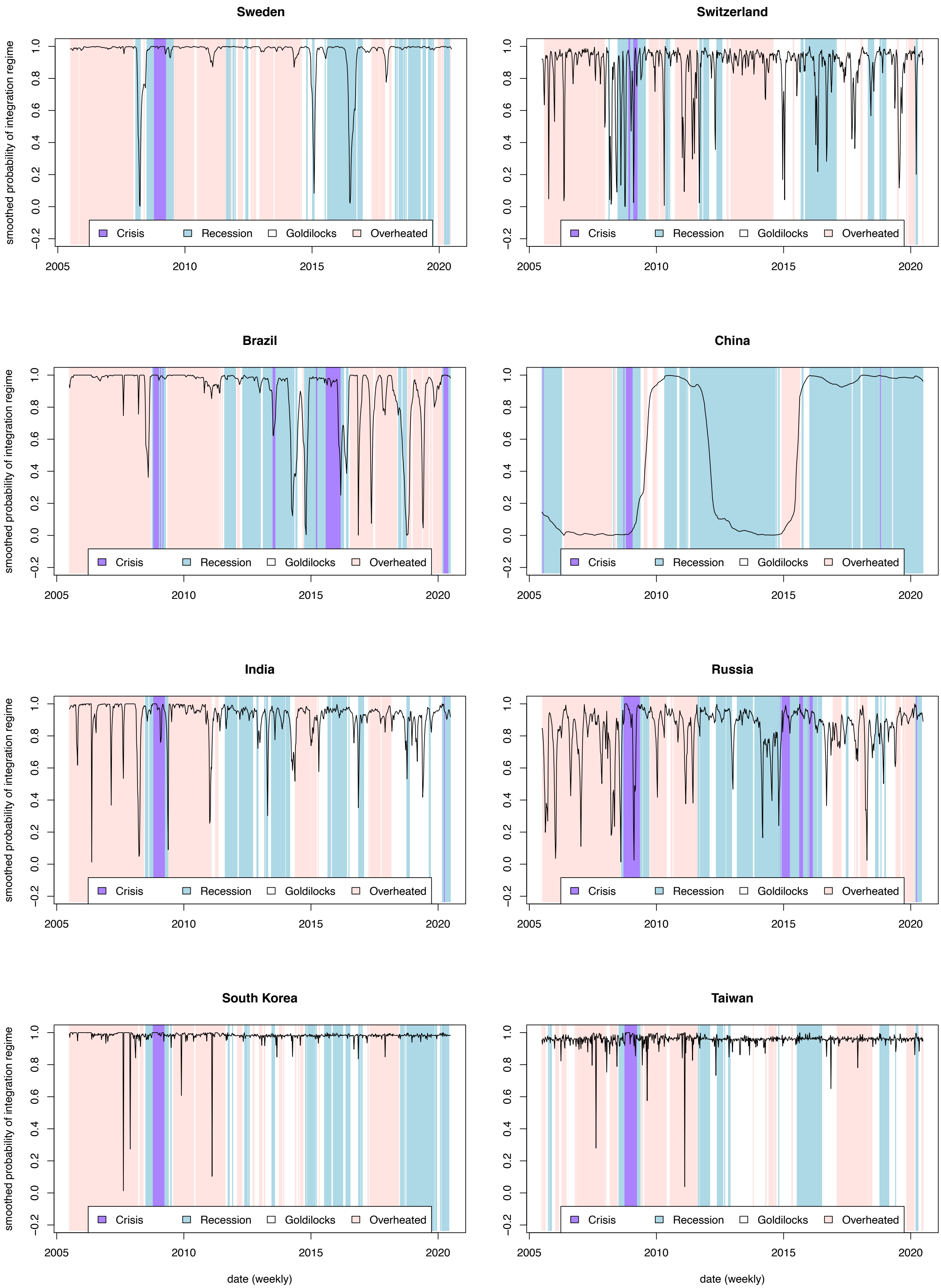

Figure 4

International integration measure (3/3)