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Walrasian Dynamics with Endowment Changes: The Gale Example in a Laboratory Market Experiment[*]

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We study the stability of price dynamics when endowments change by a laboratory market experiment of double auction. We consider a two-commodity exchange economy based on the Gale example, in which there are consumers with non-smooth preferences and exists a unique interior equilibrium. Keeping preferences and total amounts of the commodities fixed, we change individual endowments without informing subjects of when to switch so that stability of the equilibrium in Walrasian tatonnement dynamics is reversed in the middle of the experiment. We observe that, the tendency of divergence from the equilibrium is switched to convergence to the equilibrium when endowments change from an unstable setting to a stable setting, and vice versa. Theoretical predictions on movements of transaction prices work better when the interior equilibrium is unstable than when it is stable. Moreover, efficiency is higher when the interior equilibrium is unstable than when it is stable. Our observations reinforce experimental results of the Walrasian dynamics by examining a simple economy in which implausible outcomes are predicted.

JEL codes: C92, D51

Keywords: Walrasian Stability, Experiments, Double Auction

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1. Introduction

We experimentally investigate the predictability of classical economic theories of competitive market dynamics to predict paths of prices converging, diverging and switching. Smith (1965) has firstly discovered that price convergence in market experiments shows a close connection between theory of perfect competition originated by Walras (1874) and data drawn from a completely different environment. The abstract theory of the price change process is predictive even in environments with many noises that are totally different from the abstract model of perfect competition. The classical analysis of market behaviors does not address either the details of market institutions or individual decision makings (for details, see Negishi (1962), and Arrow and Hahn (1971)). In spite of that, Smith (1965) has shown by the continuous double auction that outcomes of the experimental market converge to predictions by the partial equilibrium models. Many studies of the experimental market have surprisingly shown that the market tends to balance with a pattern of prices that approximates the equilibrium of the fictional Walrasian auctioneers. It means that the standard market exchange follows the economic factors as if the Walrasian auctioneer manipulated prices. The experiment of continuous double auction, originated by Smith (1965), has been providing support for convergence to the competitive equilibrium and results of comparative statics with respect to shifts of excess demand function in the classical supply-demand model at which the equilibrium price is unique and dynamically stable in Walras' sense.

It is well known that Scarf (1960) presents an example of three-commodity three-consumer exchange economy which generates limit cycles of the Walrasian price dynamics. His general equilibrium model cannot be reduced to a partial equilibrium model. Hirota (1981, 1985) and Mukherji (2007, 2012) have shown by using the exchange economy model of Scarf (1960) that, even given the total amount of commodities, the Walrasian price dynamics become either monotone convergence or limit cycles depending on distributions of individual endowments. Following Smith's pioneering experiment, Anderson et al. (2004), Goeree and Lindsay (2016) and Gillen et al.(2020) used the Scarf example to prove that the theory of unstable equilibrium is supported by data from experiments of continuous double auction.

Gale (1963) constructs a model of exchange economy with two kinds of commodities and two types of consumers having Leontief utility functions (L-shaped indifference curves). The Gale example can be reduced to a partial equilibrium model by normalizing the price of one good to be unity. This simplification enables us to consider the Walrasian price dynamics in terms of the market excess demand function for one commodity, the functional form of which depends on the initial allocation of the two commodities to the consumers. In the Gale example, a positive equilibrium price uniquely exists, the price falls to zero once the excess demand for the good becomes negative, and the price keeps goes to infinity once the excess demand becomes positive. Crockett, Oprea and Plott (2011) call the unique equilibrium the "interior equilibrium," and zero and infinity as the "corner equilibria," and they gave strong support to the Walrasian price dynamics in their double auction experiment on the Gale example. They kept the excess demand curve so that stability properties of equilibria remain unchanged throughout the experiment: the interior equilibrium is unstable and conversely the corner equilibria are stable in the Walrasian price dynamics.

In this paper, we investigate the price dynamics in the Gale example by changing distributions of individual endowments such that the stability and instability of the three equilibria are reversed in the middle of the experiment. In our experiment, both of the supply and demand functions shift so that stability of the equilibria is reversed. We never inform previously subjects of when to change. More specifically, our experiment consists of two patterns: Treatments SU (Stable-Unstable) and US (Unstable-Stable). Treatment SU corresponds to the experimental setting of initial allocation with which the interior equilibrium is stable in Walrasian tatonnement dynamics in the first half of periods and unstable in the second half of periods. Treatment US means the experimental setting with which the interior equilibrium is unstable in the first half of periods and stable in the second half of periods. All subjects are never previously informed when their initial holdings of the commodities are to change although they know that their holdings may change from round to round of the experiment. Through this experiment, we test not only whether the Walrasian tatonnement system works for both of the stable and unstable equilibria but also whether the market mechanism recognizes changes in economic environment and how quickly it makes price adjustments.

The main results obtained from our continuous double auction experiment are as follows. We observe that when the interior equilibrium is unstable in Walrasian tatonnement dynamics, transaction prices show a tendency diverging from the equilibrium. When the interior equilibrium is stable, prices show a tendency converging to the equilibrium. However, theoretical predictions on transaction prices movement work out better when the interior equilibrium is unstable than when the interior equilibrium is stable. In addition, efficiency is higher when the interior equilibrium is unstable than when the interior equilibrium is stable. Moreover, no matter when the interior equilibrium is unstable or stable, the mean payoffs of subjects who have more endowment in the commodity are higher than those who have fewer, that is, commodity suppliers receive larger payoffs than commodity demanders do because of high transaction prices.

Our study is closely related to the following papers. Plott and George (1992), Plott and Smith (1999), and Plott (2000) examined economies with multiple competitive equilibria such that the Walrasian model and the Marshallian model of price dynamics provide opposite predictions on the stability property of each equilibrium. They investigated which model is appropriate by conducting the continuous double auction experiments in which the stability of equilibria was reversed by changing demand or supply functions in the middle of their experiments. After several periods in each session of the experiment, Plott and George (1992) changed demand functions while keeping the same downward-sloping supply curve attributed to "forward-falling" individual supplies due to external economies of scale. Plott and Smith (1999) modified supply curves in the middle of the experiment, while keeping the same demand function with a positive slope because of the existence of a consumption externality. Both observed that stability is supported by the Marshallian model of dynamics. On the other hand, Plott (2000) reported that in the case of the downward-sloping supply curve derived from "backward-bending" individual supplies due to negative income effects, stability is captured by the Walrasian model. These works are concerned with partial equilibrium models of economies with externalities. We find support for the Walrasian dynamics, in particular, in the case where the interior equilibrium is Walrasian unstable in our experiment. As Plott (2000) pointed out, which theory of Walrasian or Marshallian dynamics is appropriate for a double auction market depends on the underlying reasons for supply and demand shapes.

Crockett, Oprea and Plott (2011) conducted the first experiment of the Gale example and gave support to the Walrasian model of price dynamics in double auction experiment. They write: "In this paper we provide a particularly strong test of the Walrasian hypothesis by experimentally studying a simple economy in which Walrasian dynamics predict highly implausible outcomes" (Crockett, Oprea and Plott, 2011, P.3197, ll.18-20). Their point was to carry out a "stress-test" on the predictions of Walrasian dynamics by using the original Gale example, in which the interior equilibrium is Walrasian unstable and the two corner equilibria are Walrasian stable. We offer in this paper a "harder stress-test" than theirs by conducting intrinsically two experiments subject to different initial conditions in one sequence without telling subjects previously when the conditions swich. Our observations thus reinforce results on the predictability of the Walrasian dynamics in a continuous double auction experiment with multiple units of virtual commodities.

The paper is organized as follows. In Section 2, we present the model of an exchange economy with three competitive equilibria which we used to conduct our experiment. In Section 3, we explain the design and procedures of our experiment. Namely, we describe how we transformed the theoretical model into the experiments. In Section 4, we analyze the results of the experiment to find tendencies of the data and effects of our scientific controls. Section 5 is for concluding remarks. The appendices contain a theoretical analysis on the stability properties of equilibria in our model as well as our experimental instructions.

2. The Gale Example of an Exchange Economy

2.1. The Basic Model

Following Crockett, Oprea and Plott (2011), we consider the variation of Gale's (1963) exchange economy model with two kinds of goods called *X* (commodity) and *Y* (money as numeraire) and two types of consumers named 1 and 2. The utility functions of consumers 1 and 2 are of Leontief types in the following forms:

$$U_i(x_i, y_i) = \min\{a_i x_i, y_i + b_i\} \qquad (i = 1, 2)$$

Given consumer 1's endowment, (w_1^x, w_1^y), and consumer 2's, (w_2^x, w_2^y), a unique interior competitive equilibrium price for good *X* relative to good *Y* is given by $p^* = -(a_1 m_2 + a_2 m_1)/(m_1 + m_2)$, where $m_i = w_i^y + b_i - a_i w_i^x$, if it exists.

Throughout the experiment, we fix the preference parameters of consumers as $a_1 = 25,\ a_2 = 720,\ b_1 = -2000,$ and $b_2 = 4000$ and the total available amount of each good as $w_1^x + w_2^x = 20$ and $w_1^y + w_2^y = 6000$.[1] However, we prepare two different endowment distributions as described below. Under one endowment distribution, the interior equilibrium is Walrasian stable, whereas it is Walrasian unstable under the other.

2.2. The Stable Case

In the first case, we set the endowment of each type as $(w_1^x, w_1^y) = (2, 3500)$ and $(w_2^x, w_2^y) = (18, 2500)$. This endowment distribution and the preference parameters specified in the previous section generate an interior competitive equilibrium (ICE)

[1] The total available amount of each good is the same as that in the experiment by Crockett, Oprea, and Plott (2011), but the endowment distribution and the preference parameters are different.

price, $p^* = \dfrac{88250}{501} \approx$ 176.14 and the ICE allocation is given by

$$(x_1, y_1; x_2, y_2) = \left(16 - \frac{944}{139}, 2230 + \frac{30}{139}; 4 + \frac{944}{139}, 1970 - \frac{30}{139}\right).$$

Figure 1 illustrates the expansion paths of consumers and the ICE in an Edgeworth box.

Figure 2 plots net supply and demand functions of good X with respect to the endowment in this economy. At any price above (below) the ICE price, type 2's supply for good X is larger (smaller) than type 1's demand, that is, there is an excess supply (demand), so that the price decreases (increases) according to Walrasian price adjustment process. Hence, the ICE is stable.[2]

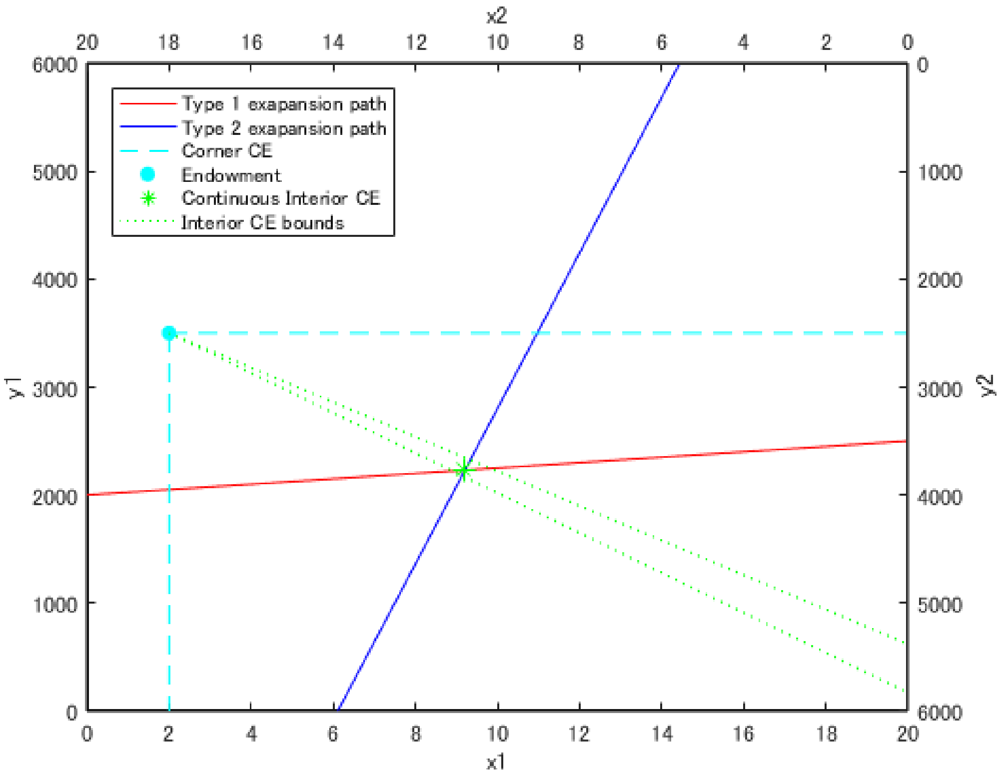

Figure 1. Exchange Economy with a Stable Interior Equilibrium

[2] See the Appendix 1 for the formal definitions of Walrasian price adjustment process and stability as well as the derivations of market demand functions.

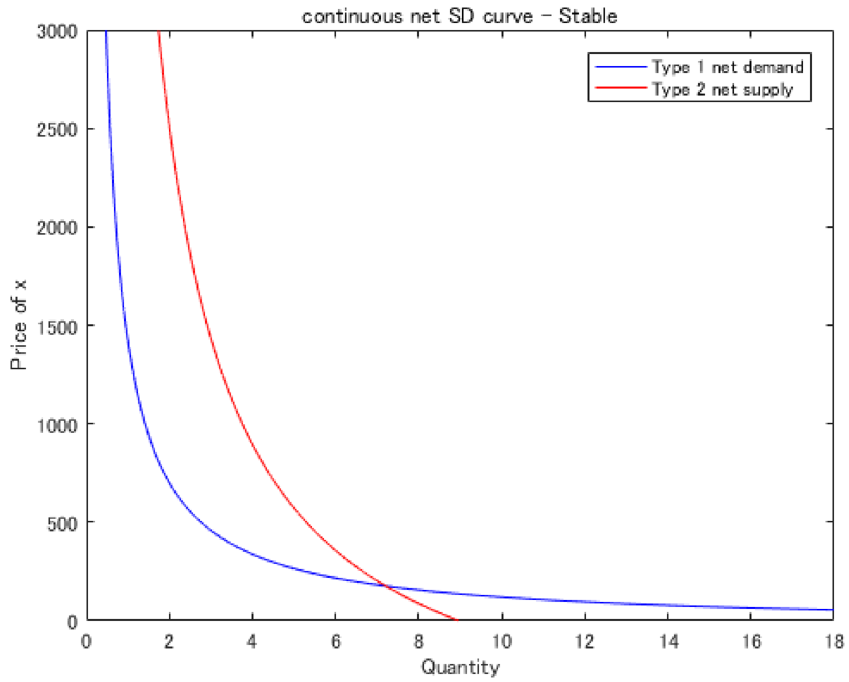

Figure 2. Supply and demand Curves for Good *X* with a Stable Interior Equilibrium

In our experiment, subjects chose integers as trading units, not real numbers as in usual theory. Thus, it is significant to examine a discrete version of the exchange economy corresponding to the experimental setting to make a rigorous theoretical prediction. Figure 3 depicts the offer curves of two consumers and the ICE allocations in the Edgeworth box for the discretized exchange economy. The set of ICE allocations is the intersection of the two offer curves. It is the set of feasible integer allocations satisfying $x_1^* = 9$ and $y_1^* \in [2205, 2380]$ in terms of consumer 1's consumption bundles. Figure 4 demonstrates net supply and demand curves for this discretized exchange economy. The set of ICE prices is drawn as an interval, $p^* \in [160,185]$, with green dotted lines in Figure 4 (a cone with green dotted lines in Figure 1).

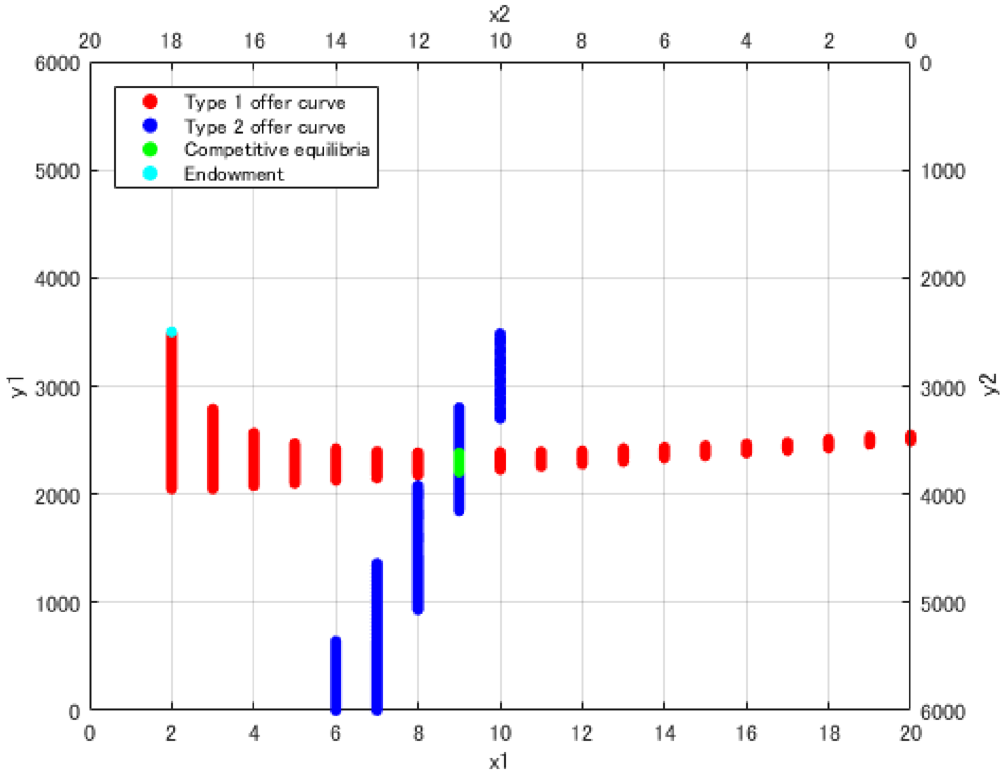

Figure 3. Discrete Version of the Exchange Economy with Stable Interior Equilibria

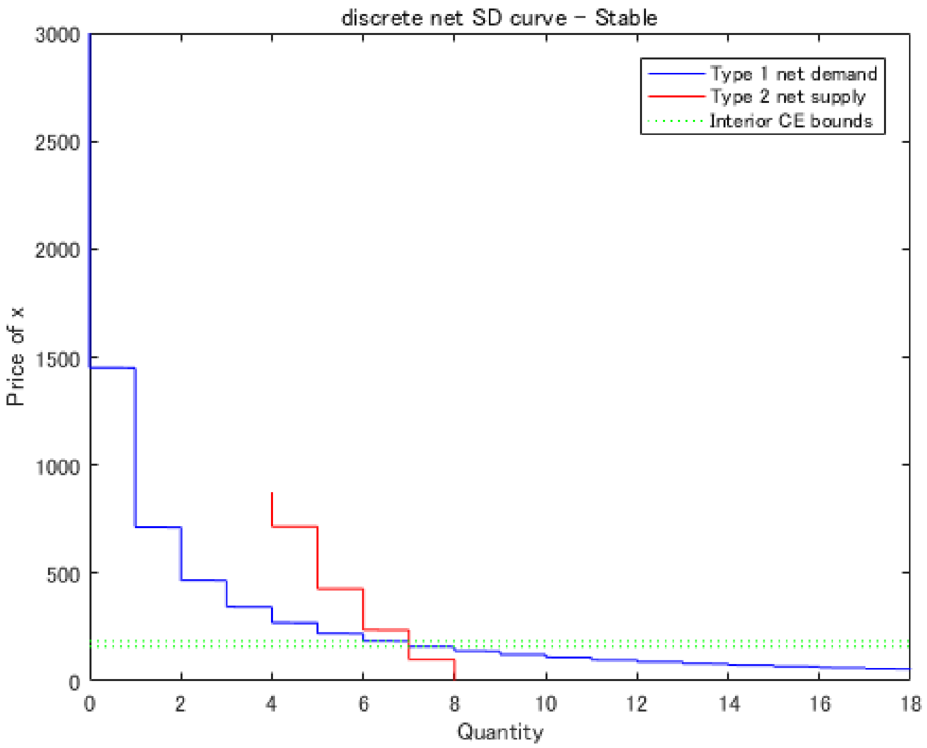

Figure 4. Discretized Supply and demand Curves with Stable Interior Equilibria

2.3. The Unstable Case

In the second case, we set the endowment distribution as $(w_1^x, w_1^y) = (16,1100)$ and $(w_2^x, w_2^y) = (4,4900)$. With this endowment distribution and the preference parameters described in Section I.1, a unique ICE price is given by $p^* = \dfrac{39275}{236} \approx$ 166.42 and the ICE allocation is given by

$$(x_1, y_1; x_2, y_2) = \left(16 - \frac{944}{139}, 2230 + \frac{30}{139}; 4 + \frac{944}{139}, 1970 - \frac{30}{139}\right),$$

which is identical with the ICE allocation in the stable case. Figure 5 shows the expansion paths of consumers and the ICE in an Edgeworth box.

Figure 6 illustrates net supply and demand functions of good X with respect to the endowment. At any price above (below) the ICE price, type 1's supply for good X is smaller (larger) than type 2's demand, that is, there is an excess demand (supply), so that the price increases (decreases) according to Walrasian price adjustment process. Thus, the ICE is unstable.

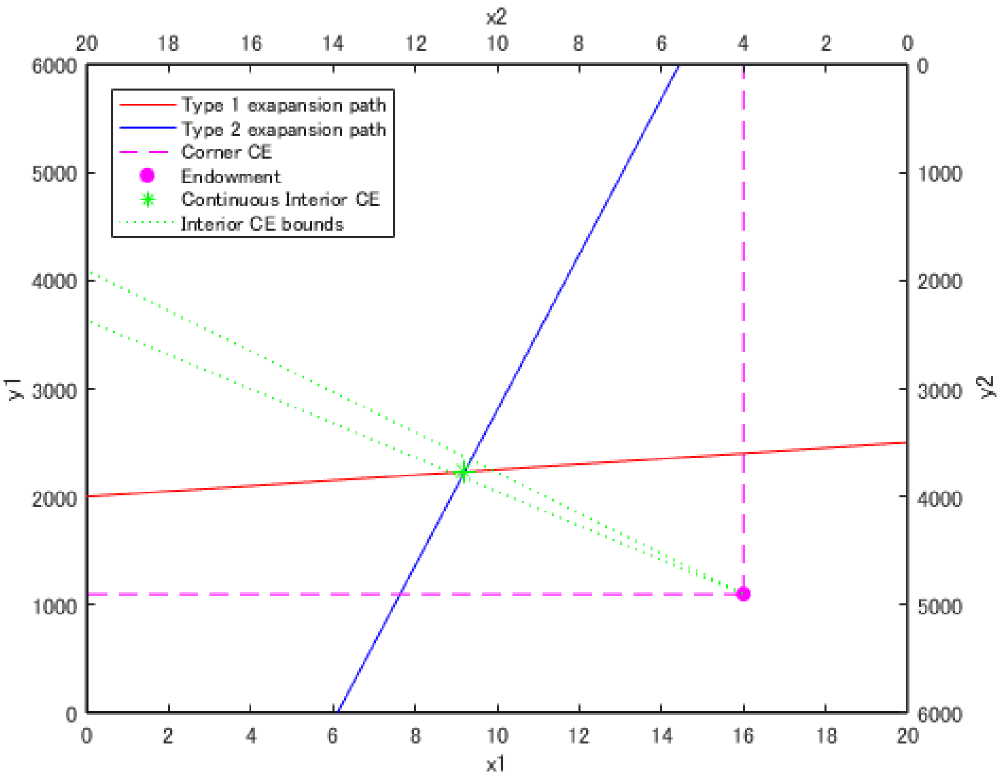

Figure 5. Exchange Economy with an Unstable Interior Equilibrium

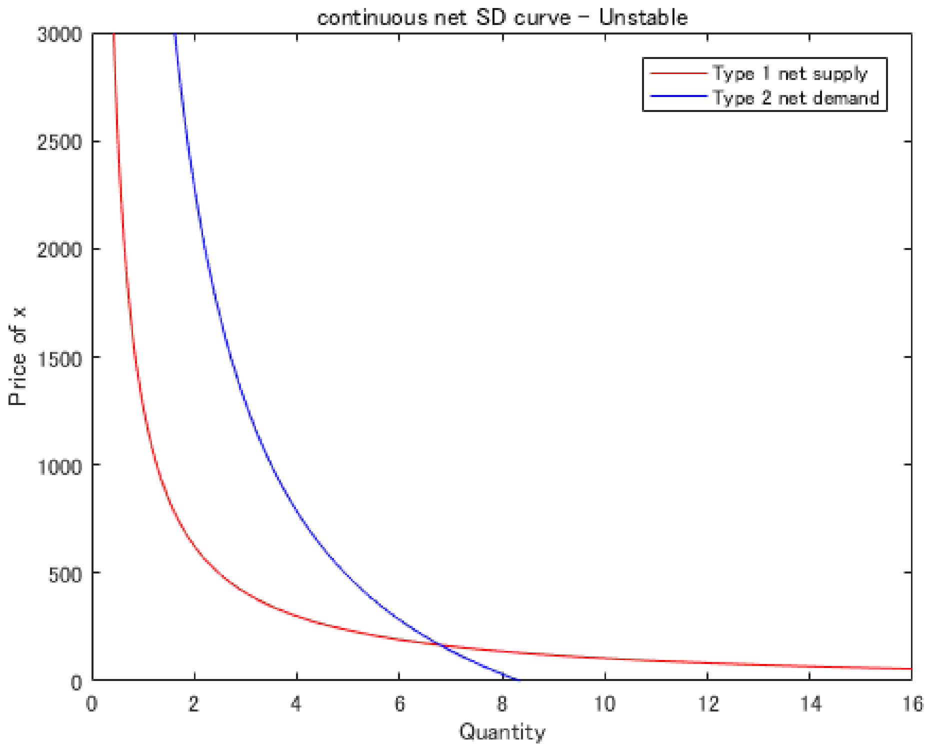

Figure 6. Supply and demand Curves for Good X with an Unstable Interior Equilibrium

Figure 7 demonstrates the offer curves of two consumers and the ICE allocations in the Edgeworth box for the discretized exchange economy. The set of ICE allocations, which is the intersection of the two offer curves, is the set of feasible integer allocations satisfying $x_1^* = 9$ and $y_1^* \in [2206, 2409]$ in terms of consumer 1's consumption bundles. There are also corner equilibrium allocations satisfying $x_1^* = 15$ and $y_1^* \in [3351, 6000]$. Figure 8 shows net supply and demand curves for the discretized exchange economy. The set of ICE prices is drawn as an interval, $p^* \in [158, 187]$, with green dotted lines in Figure 8 (a cone with green dotted lines in Figure 5).

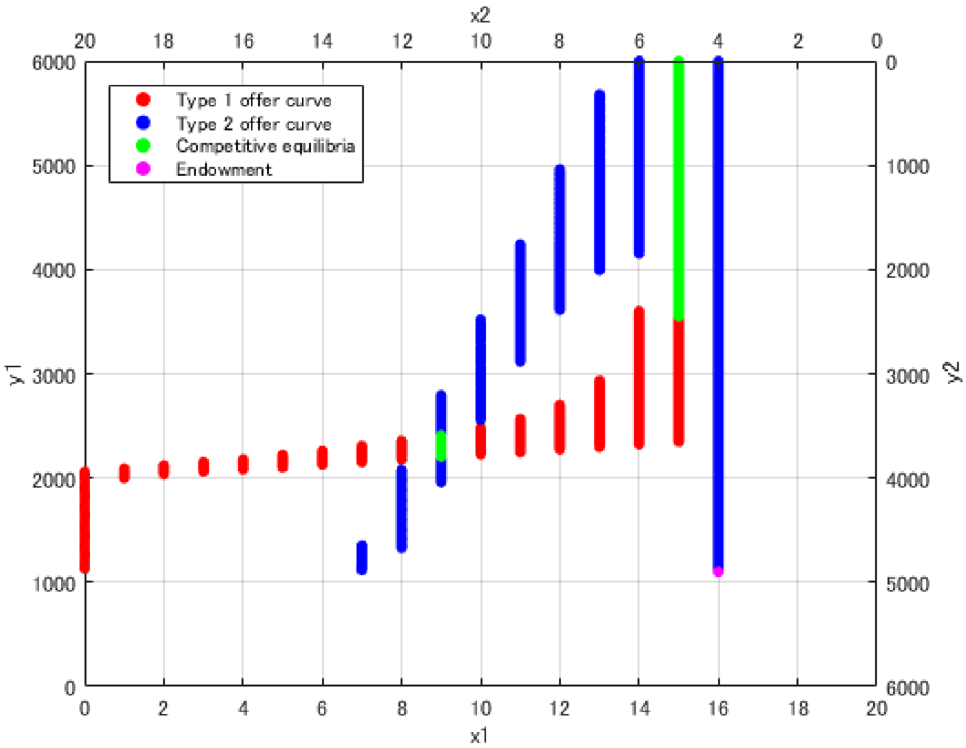

Figure 7. Discrete Version of the Exchange Economy with Unstable Interior Equilibria

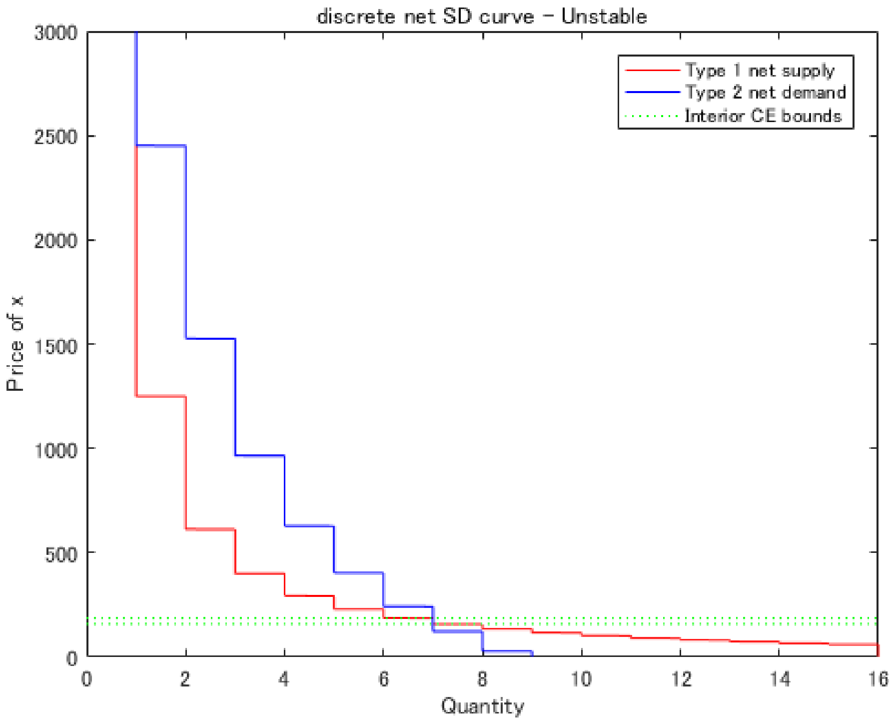

Figure 8. Discretized Supply and demand Curves with Unstable Interior Equilibria

Table 1 summarizes the equilibrium predictions. Notice that the set of discrete equilibrium prices and the set of discrete equilibrium allocations are almost the same between in the stable case and in the unstable case. In particular, there is no difference in the equilibrium distribution of good *X*. Moreover, the Euclidean distance between the endowment point and an interior equilibrium allocation point in the Edgeworth box is almost the same. Nevertheless, the stability property of equilibrium is opposite in the two cases.

Table 1. Theoretical Predictions about Discrete Interior Equilibria

	Endowment		Price	Allocation	
Walrasian Stability	Type 1	Type 2	Px/Py	Type 1	Type 2
stable	$w_1^x = 2$ $w_1^y = 3500$	$w_1^x = 18$ $w_1^y = 2500$	[160, 185]	$x_1 = 9$ $y_1 \in [2205, 2380]$	$x_2 = 11$ $y_2 \in [3620, 3795]$
unstable	$w_1^x = 16$ $w_1^y = 1100$	$w_1^x = 4$ $w_1^y = 4900$	[158, 187]	$x_1 = 9$ $y_1 \in [2206, 2409]$	$x_2 = 11$ $y_2 \in [3591, 3794]$

3. Experimental Design and Procedures

Our experiment studies the effects of endowment changes on price adjustment dynamics. It consists of two treatments: Treatments SU and US. Treatment SU corresponds to the setting in which the interior equilibrium is stable in Walrasian tatonnement dynamics with the endowments in each of the first half of periods, whereas it is unstable with the endowments in each of the second half of periods. Treatment US corresponds to the setting in which the interior equilibrium is unstable with the endowments in the first half of periods, whereas it is stable with the endowments in the second half of periods.

We conducted two sessions in each of the two treatments at Tokyo Institute of Technology during December of 2016 and July of 2017. Twenty subjects participated in each session (80 separate subjects in total). We recruited the student subjects by campus-wide advertisement. These students were told that there would be an opportunity to earn money in a research experiment. None of them had prior experience in a market experiment. No subject attended in more than one session. Each session took approximately three hours to complete. The mean payoff per subject was $43.22 ($1=100 yen) in Treatment SU and it was $42.59 in Treatment US.

In each of four sessions, the twenty subjects were seated at computer stations that were separated with visual partitions in the Experimental Economics Laboratory at Tokyo Institute of Technology. In each session, half of subjects played the role of type 1 consumer and the other half did the role of type 2 consumer, forming a replica of the economy described above. Their roles were fixed throughout the experiment. Subjects are possible to buy and sell units of x using the numeraire y as the medium of exchange. Trade was conducted via computerized continuous double auction using the z-Tree program (Urs Fischbacher, 2007).

At the beginning of a session, each subject received one experimental instruction, one record sheet, and one payoff table indicating how his/her payoff depends on the amounts of x and y. Subjects were possible to track their potential earnings using their payoff tables that allowed them to calculate the payoff consequences of prospective trades. We explicitly noticed to each of them that he/she was not allowed to reveal any information regarding his/her payoff table or endowment to any other subject. We also told that his/her initial holdings of the commodities may change from round to round of the experiment although he/she was not informed when such a change is to happen.

Each session was divided into a sequence of 13 trading periods, each lasting 6 minutes. We conducted a stationary repetition procedure, following the assumption of tatonnement price adjustment dynamics with a fixed endowment. Holdings of commodities were reset at the end of each period: allocations and induced payoffs were returned to endowment levels for the next period of trade.[3] In Treatment SU, we changed subjects' endowments at the beginning of period 7. The interior equilibrium is stable with the endowments for periods 1-6, while it is unstable with the endowments for periods 7-13. On the other hand, in Treatment US, we altered subjects' endowments at the beginning of period 8. The interior equilibrium is unstable with the endowments for periods 1-7, while it is stable with the endowments for periods 8-13.

4. Results

4.1. Transaction Price

Figures 9 displays the plotted raw transaction prices from each session of the experiment. The vertical green lines refer to periods, the vertical purple lines indicate that endowments were changed after this period, and the horizontal dotted and solid red lines represent the bounds of the equilibria.

As shown in the figure, prices begin similarly near the equilibrium in each session. Then, they show a tendency away from the bounds of the equilibria in the two sessions

[3] Subjects earned cash payments based on payoffs of their allocations at the end of one period that was randomly chosen from 13 periods at the end of the session.

of Treatment US (especially in US1) and a tendency close to the bounds in the two sessions of Treatment SU (especially in SU2). After changing endowments, prices primarily away from the equilibria bounds in the sessions of US1 and US2 start to fall and reach near to the equilibria bounds at the end. In contrast, those primarily near to the equilibria bounds in the sessions of SU1 and SU2 begin to rise and never fall in their bounds of the equilibria at the end. These provide us with our first observation.

OBSERVATION 1: *When the interior equilibrium is unstable in Walrasian tatonnement dynamics, transaction prices show a tendency moving away from the equilibrium. When the interior equilibrium is stable, prices show a tendency moving toward the equilibrium.*

Figure 10 shows the weighted average prices per period in each session of the experiment. The weighted average prices per period are the mean of subjects' weighted prices per period. A subject's weighted price per period is calculated by dividing the sum of traded y by the sum of traded x. As indicated in the figure, the weighted average prices obviously re-exhibit OBSERVATION 1.

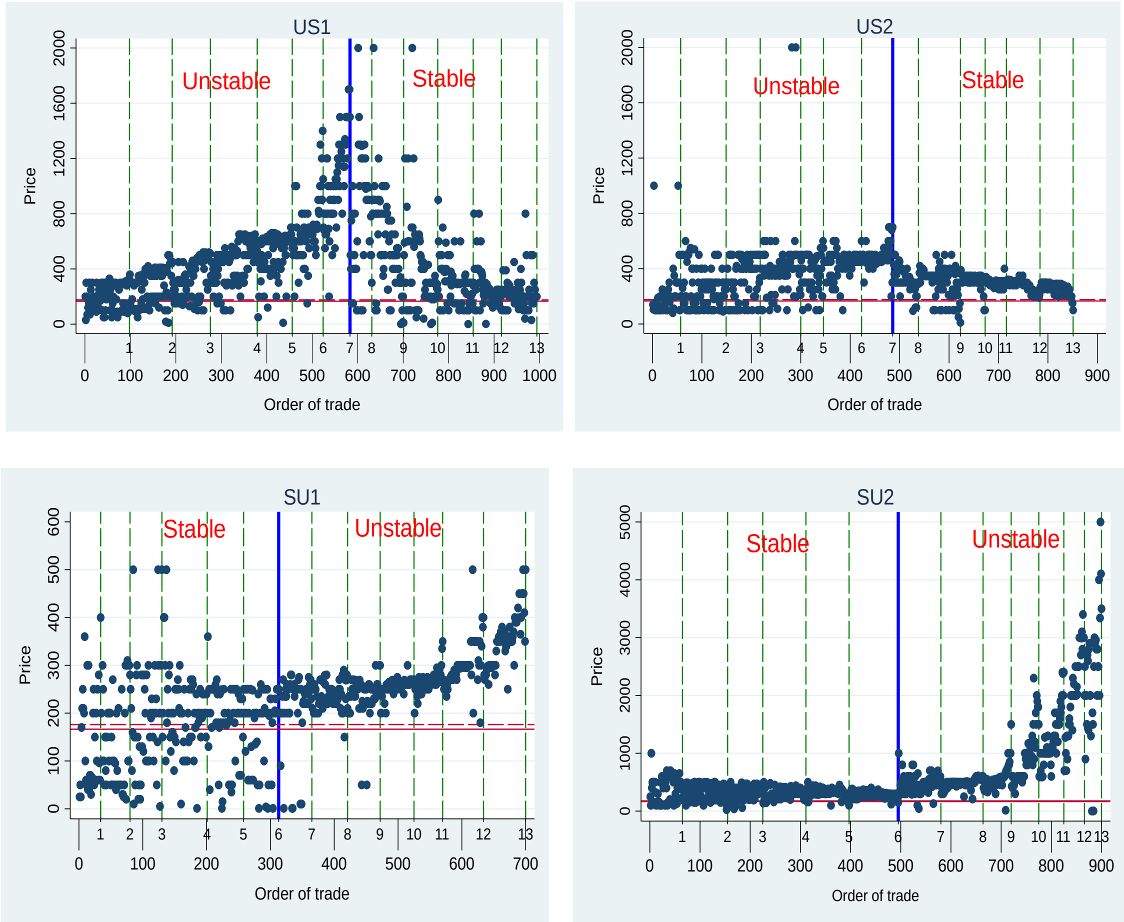

Figure 9. Prices Plotted in Four Sessions

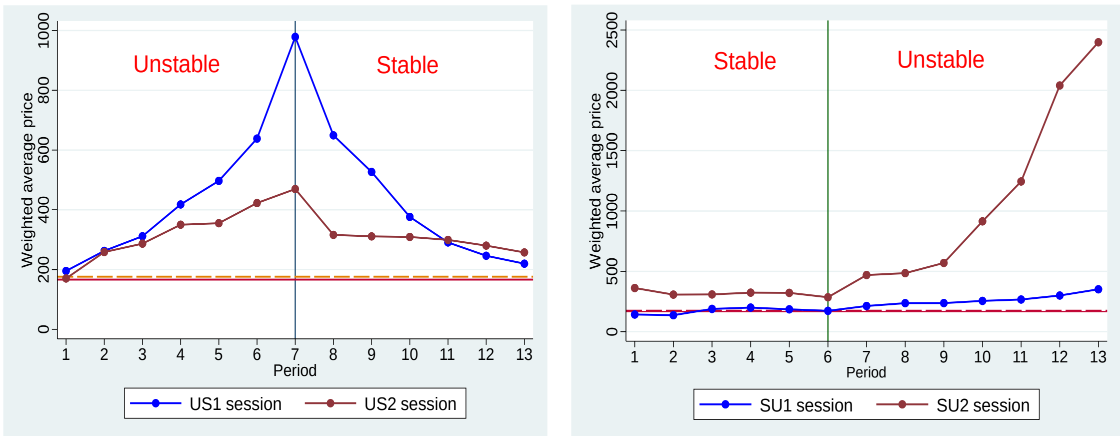

Figure 10. Weighted Average Prices Per Period in Four Sessions

To test OBSERVATION 1, we estimate the following linear equation by fixed effects model for panel data.

$$ln(PRICE_{isp}) = \alpha_{is} + \beta_s \times Period + \gamma_s \times Period \times Unstable + \lambda_s \times Unstable + \varepsilon_{isp} \qquad (4.1)$$

where $ln(PRICE_{isp})$ is a log specification of the weighted price of individual i in period p of session s. *Unstable* is a dummy for the periods with an unstable equilibrium.

Regression results are presented in Table 2. In the US1 session (resp. US2 session), the estimates of β_s is – 0.218 (resp. –0.042), which indicates that there is a significant 19.6% (resp. 4.1%) decrease in transaction price each period when the interior equilibrium is changed from the unstable to the stable. The estimates of γ_s are significantly positive at 0.500 (resp. 0.195) in the US1 session (resp. US2 session), which demonstrates a clear difference between price movements in the periods with unstable and stable interior equilibria. Summing β_s and γ_s and exponentiating it, we find that transaction prices rise by 32.6% (resp. 16.5%) per period in the US1 session (resp. US2 session) when the interior equilibrium is unstable.

Table 2. Regressions of Transaction Price

	US1 session	US2 session	SU1 session	SU2 session
Constant	8.152	6.125	4.786	5.698
	(0.176)	(0.128)	(0.062)	(0.056)
Period	–0.218	–0.042	0.090	0.001
	(0.019)	(0.012)	(0.006)	(0.015)
Period×Unstable	0.500	0.195	–0.026	0.294
	(0.021)	(0.134)	(0.020)	(0.018)
Unstable	–3.331	–1.030	0.116	–1.906
	(0.186)	(0.134)	(0.143)	(0.130)
R^2 (overall)	0.676	0.487	0.393	0.838
Observations	258	258	259	260

Notes: Standard errors are in the parentheses.

The signs of λ_s are estimated significantly negative, which means that the initial prices in the periods with unstable equilibrium are significantly lower than those with stable equilibrium. In the SU1 and SU2 sessions, the significance of our estimates is mixed. Prices in the SU1 session are implied a significantly positive increase in the periods with a stable equilibrium. Although Figure 9 displays an increase tendency in the latter periods with an unstable equilibrium, this tendency is not significant. In contrast, prices in the SU2 session are indicated a significantly positive increase in the periods with an unstable equilibrium, but the estimates also suggest that there is no significant tendency in the periods with a stable equilibrium.

To confirm the above regression results, we also conduct the Mann-Kendall trend test on *period weighted average price*, *pooled individual raw price*, and *weighted price of each subject*. These results are presented in Table 3. In periods with unstable equilibrium, both *period weighted average price* and *pooled individual raw price* indicate highly significant increasing trends. In periods with stable equilibrium, both *period weighted average price* and *pooled individual raw price* indicate significant decreasing trends in the two sessions of Treatment US, while in the two sessions of Treatment SU, the results are mixed with significantly positive trend, significantly negative trend, and no significant trend. With respect to *weighted price of each subject*, in periods with unstable equilibrium, 95%, 70%, 90%, and 95% subjects show significant (at the 0.05 level or below) increasing trends in the US1, US2, SU1, and SU2 sessions, respectively. In periods with stable equilibrium, 75% and 40% subjects show significant (at the 0.05 level or below) decreasing trends in the US1 and US2 sessions, respectively, while most of the subjects do not show significant trends in the SU1 and SU2 sessions. These statistical evidences obtained from the regression analysis and Mann-Kendall trend test generate our first result.

RESULT 1: *Theoretical predictions on transaction prices movement work better when the interior equilibrium is unstable in Walrasian tatonnement dynamics than when the interior equilibrium is stable.*

Table 3. Results of Mann-Kendall Trend Test

Period weighted average price	US1 session	US2 session	SU1 session	SU2 session
Periods with stable equilibrium	– (p=0.009)	– (p=0.003)	+ (p=0.26)	– (p=0.452)
Periods with unstable equilibrium	+ (p=0.003)	+ (p=0.003)	+ (p=0.003)	+ (p=0.003)
Pooled individual raw price	**US1 session**	**US2 session**	**SU1 session**	**SU2 session**
Periods with stable equilibrium	– (p<0.0001)	– (p<0.0001)	+ (p=0.001)	– (p=0.004)
Periods with unstable equilibrium	+ (p<0.0001)	+ (p<0.0001)	+ (p<0.0001)	+ (p<0.0001)
Weighted price of each subject	**US1 session**	**US2 session**	**SU1 session**	**SU2 session**
Periods with stable equilibrium*	75% –	40% –	10% +; 5% –	5% +; 5% –
Periods with unstable equilibrium*	95% +	70% +	90% +	95% +

Notes: + and – stand for the positive and negative trends, respectively. The p values of testing whether these trends are significant or not are provided in the parentheses.
*Results in this row denote the percentages of the significant positive or negative trends at the 0.05 level or below.

4.2. Payoffs

We plot mean payoffs of types 1 and 2 subjects by stable and unstable conditions in Figure 11. The payoff for the interior equilibrium is marked red diamond in the figure (i.e., 3075 for type 1 subjects and 7200 for type 2 subjects). It is shown that in each session the mean payoffs of type 1 subjects are higher when the interior equilibrium is unstable than when the interior equilibrium is stable. This is because the prices moved upward from the interior equilibrium when it is unstable, which made the type 1 subjects who had more endowments in x better off. In contrast, the prices did not fall toward to the interior equilibrium when it is stable, which made the type 1 subjects who had fewer endowments in x worse off. However, it is oppositely observed for type 2 subjects. Their mean payoffs in each session are higher when the interior equilibrium is stable than when the interior equilibrium is unstable. The above-mentioned price movements can also explain this phenomenon. These provide us with our second observation.

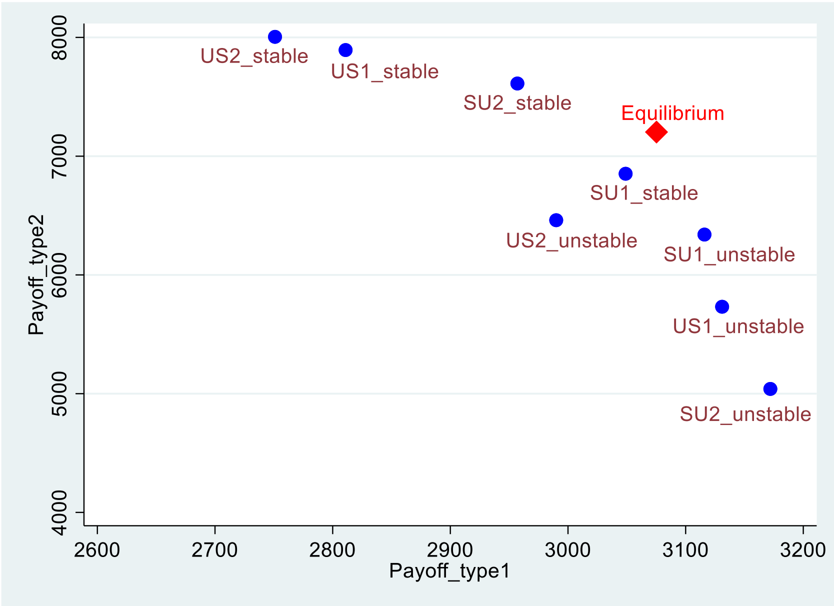

Figure 11. Mean Payoffs of Types 1 and 2 Subjects by Stable and Unstable Conditions

OBSERVATION 2: *No matter when the interior equilibrium is unstable or stable in Walrasian tatonnement dynamics, the mean payoffs of subjects who have more endowment in x are higher than those who have fewer.*

In addition, we also plot mean payoffs of types 1 and 2 subjects by periods and sessions in Figure 12. The horizontal axis refers to the mean payoff of 10 type 1 subjects in one session, and the vertical axis refers to the mean payoff of 10 type 2 subjects in the same session. The numbers from 1 to 13 correspond to period 1 to period 13. The payoff for the interior equilibrium is marked red, and the utility possibility frontier is exhibited by the blue line in the figure. As can be seen from the figure, most of the data are plotted around the utility possibility frontier, which indicates that subjects seem to have performed efficiently in the experiment. In the next subsection, we discuss the issue of efficiency more formally and test statistically whether efficiency is the same between when the interior equilibrium is stable and when the interior equilibrium is unstable.

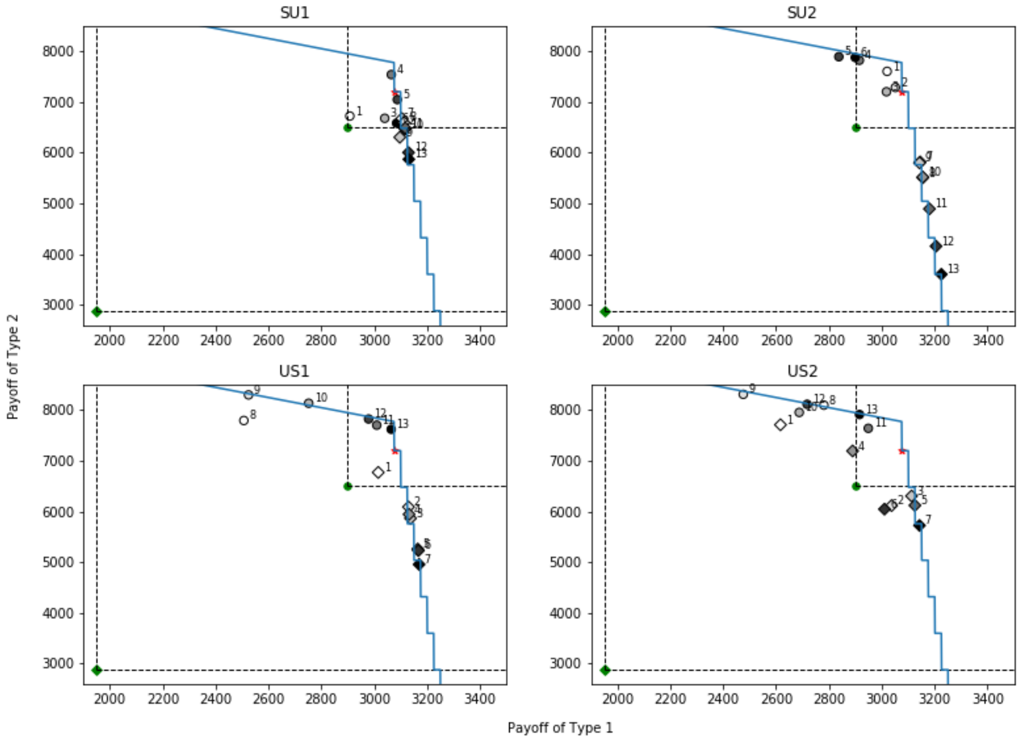

Figure 12. Mean Payoffs of Types 1 and 2 Subjects by Periods and Sessions

4.3. Efficiency

We create an index named frequency of inefficient pairs to investigate the issue of efficiency. To calculate this index, at the end of each period, for each pair of 20 subjects, we examine whether their allocations could be Pareto-improved by trading. Then this index is defined as the number of the improved pairs divided by the number of all pairs (190). Figure 13 displays the frequency of inefficient pairs in each session, and provides us with our third observation.

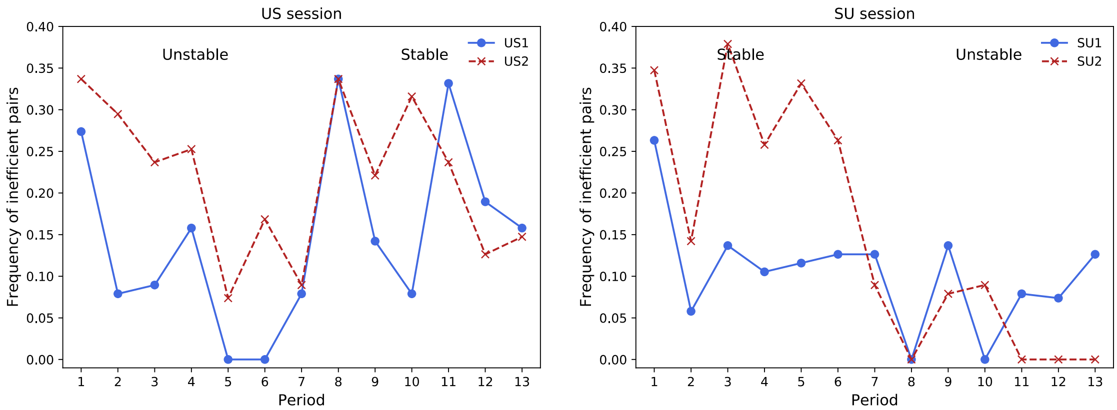

Figure 13. Frequency of Inefficient Pairs in Four Sessions

OBSERVATION 3: *Efficiency is higher when the interior equilibrium is unstable than when the interior equilibrium is stable, because the frequency of inefficient pairs is lower when the interior equilibrium is unstable than when the interior equilibrium is stable, especially in US1 and SU2 sessions.*

To test OBSERVATION 3, we use the random effect panel probit model in the regression. The dependent variable is a dummy variable of *Inefficiency* that equals to 1 if a subject can improve his/her efficiency, 0 otherwise. *Unstable* is a dummy for the periods with an unstable equilibrium. Table 4 reports the regression results. In US1, US2, and SU2 sessions, *Unstable* is estimated with significantly negative signs, indicating that the probability of a subject can improve his/her efficiency (i.e., a subject behaves inefficiently) is lower when the interior equilibrium is unstable than when the interior equilibrium is stable. In addition, the marginal effect of *Period* is also significantly negative in these sessions, which suggests that the probability of *Inefficiency* decreases with the passing the trading periods. Furthermore, both *Unstable* and *Period* are not significant in SU1 session. These statistical evidences generate our second result.

RESULT 2: *Efficiency is higher when the interior equilibrium is unstable in Walrasian tatonnement dynamics than when the interior equilibrium is stable.*

Table 4. Probit Regressions of Inefficiency (Marginal Effects)

	US1 session	US2 session	SU1 session	SU2 session
Period	–0.015	–0.022	–0.004	–0.010
	(0.004)	(0.004)	(0.003)	(0.004)
Unstable	–0.188	–0.122	–0.030	–0.181
	(0.029)	(0.032)	(0.024)	(0.027)
log-likelihood	–1002.665	–1268.640	–810.442	–882.511
Observations	2470	2470	2470	2470

Notes: Standard errors are in the parentheses.

5. Concluding Remarks

We have studied price dynamics by changing endowments so that the stability of equilibria is reversed in the middle of our double auction experiment. We observed that when the interior equilibrium was unstable (resp. stable) in Walrasian tatonnement dynamics, transaction prices displayed a tendency moving away from (resp. toward to) the equilibrium. However, theoretical predictions on transaction prices movement worked better when the interior equilibrium was unstable than when the interior equilibrium was stable. In addition, efficiency was higher when the interior equilibrium was unstable than when the interior equilibrium was stable.

There are several open questions to be examined. In this paper we have focused on a double auction as one type of market organization. However, there are much recent experimental work pointing out that market organization is potentially important. Plott and George (1992) confirmed robustness of their results by studying sealed bid/offer auctions and the tatonnment process in addition to double auctions. In order to examine the stability of competitive equilibrium of Scarf's (1960) economy, Anderson et al. (2004) conducted a double auction market experiment and Goeree and Lindsay (2016) investigated a laboratory schedule market. Plott and Pogorelskiy (2017) showed that in their call market experiments, the Newton-Jaws model based on the Newton method provides a better description of how the markets operate than the Walrasian model. Shen et al. (2016) and Qin et al. (2018) analyzed a trading pit market experiment regarding a model of exchange economy with three equilibria. It remains to check whether our results in the double auction market hold in these market organizations.

In our experiment, we allowed prices to initiate freely and there was no price control over all periods. On the other hand, Crockett, Oprea and Plott (2011) controlled the sign of initial excess demand using price controls. Following several periods with no price control, they tried to reverse observed price dynamics by switching the sign of the market's excess demand using price controls. They found that dynamics once established were typically sticky and difficult to reverse with price controls. It is interesting to check whether dynamics are sticky once seeded by introducing price controls, such as imposing a price floor above the interior equilibrium or imposing a price ceiling below the interior equilibrium, in our experiment.

Appendix A. Global Stability and Instability of Equilibrium in the Gale Example

A.1. Basic Model

In the exchange economy model under investigation, there are two consumers of types 1 and 2, who trade goods X and Y. Denote by x_i and y_i consumption level by $i \in \{1,2\}$ of X and Y, respectively. The consumer of type $i \in \{1,2\}$ has a Leontief utility function

$$U_i(x_i, y_i) = \min\{a_i x_i, y_i + b_i\},$$

where a_i is a positive real number and b_i is a real number. For consumer i, the ray $y_i = a_i x_i - b_i$ is the expansion path, which is the locus of kinked points of L-shaped indifference curves. Let $\bar{x}_i$, and $\bar{y}_i$ represent consumer i's endowment of X and Y, respectively. Then, a Leontief exchange economy is characterized by the list of parameters (a_1, b_1, $\bar{x}_1$, $\bar{y}_1$; a_2, b_2, $\bar{x}_2$, $\bar{y}_2$). We choose Y as numeraire so as to normalize its price to be one. Denote by p the price of X, which is a positive real number. Then the budget constraint of consumer i is:

$$px_i + y_i \le p\bar{x}_i + \bar{y}_i .$$

Consumer i solves the following utility maximization problem under the budget constraint:

$$\text{Maximize } \min\{a_i x_i, y_i + b_i\}$$
$$\text{subject to } px_i + y_i \le p\bar{x}_i + \bar{y}_i .$$

Then the individual demand functions x_i* and y_i* for goods X and Y of consumer i are derived as follows:

Case 1: $b_i - a_i\bar{x}_i \le 0$

$$x_i*(p) = \begin{cases} \bar{x}_i + \dfrac{\bar{y}_i + b_i - a_i\bar{x}_i}{p + a_i} & if\ p > -\dfrac{b_i + \bar{y}_i}{\bar{x}_i} \\ 0 & otherwise \end{cases}$$

$$y_i*(p) = \begin{cases} \bar{y}_i - \dfrac{p(\bar{y}_i + b_i - a_i\bar{x}_i)}{p + a_i} & if\ p > -\dfrac{b_i + \bar{y}_i}{\bar{x}_i} \\ p\bar{x}_i + \bar{y}_i & otherwise \end{cases}$$

Case 2: $b_i - a_i\bar{x}_i > 0$

$$x_i*(p) = \begin{cases} \bar{x}_i + \dfrac{\bar{y}_i + b_i - a_i\bar{x}_i}{p + a_i} & if\ p < \dfrac{a_i\bar{y}_i}{b_i - a_i\bar{x}_i} \\ \bar{x}_i + \dfrac{\bar{y}_i}{p} & otherwise \end{cases}$$

$$y_i*(p)=\begin{cases}\bar{y}_i-\dfrac{p(\bar{y}_i+b_i-a_i\bar{x}_i)}{p+a_i} & if\ \ p<\dfrac{a_i\bar{y}_i}{b_i-a_i\bar{x}_i}\\ 0 & otherwise\end{cases}.$$

The market excess demand function E for good X is defined by

$$E(p)=x_1*(p)-\bar{x}_1+x_2*(p)-\bar{x}_2.$$

By Walras' law, $y_1*(p)+y_2*(p)=\bar{y}_1+\bar{y}_2$ if $x_1*(p)+x_2*(p)=\bar{x}_1+\bar{x}_2$. Namely, the market clearing of Y is achieved by that of X. Thus, we only need to focus on the market of X in order to investigate the price adjustment process out of equilibrium. We define the key concepts in the theory of competitive market with two commodities.

Definition. The positive price $p*$ is an ***ICE price*** (interior competitive equilibrium price) if $E(p*)=0$. ***The adjustment process is Walrasian*** if there is a positive real number λ such that $\dot{p}=\lambda E(p)$, where $\dot{p}$ is the derivative of p with respect to time. The ICE price $p*$ is ***globally stable*** if the adjustment process is Walrasian and p converges to $p*$ given any $\lambda>0$ and any initial value of p. The ICE price $p*$ is ***globally unstable*** if the adjustment process is Walrasian and p never converges to $p*$ given any initial value of $p\neq p*$.

By definition, $p*$ is globally stable if and only if $E(p)>0$ for all $p<p*$ and $E(p)<0$ for all $p>p*$, and $p*$ is globally unstable if and only if $E(p)<0$ for all $p<p*$ and $E(p)>0$ for all $p>p*$.

A.2. The Stable Case

The stable case is a Leontief exchange economy with the parameters

$a_1=25$, $b_1=-2000$, $\bar{x}_1=2$, $\bar{y}_1=3500$;

$a_2=720$, $b_2=4000$, $\bar{x}_2=18$, $\bar{y}_2=2500$

Then,

$b_1-a_1\bar{x}_1=-2000-25\cdot 2<0,$

$b_2-a_2\bar{x}_2=4000-720\cdot 18<0.$

The utility functions of types 1 and 2 are respectively:

$U_1(x_1,y_1)\ =\ \min\{25x_1,y_1-2000\}$, and

$U_2(x_2,y_2)\ =\ \min\{720x_2,y_2+4000\}$.

The expansion paths of types 1 and 2 are respectively:

$25x_1=y_1-2000$

$720x_2=y_2+4000$

The intersection of the two paths is:

$$(x_1,\ y_1;x_2,\ y_2) = \left(\frac{1280}{139},\frac{310000}{139};\frac{1500}{139},\frac{524000}{139}\right)$$
$$= \left(16-\frac{944}{139},2230+\frac{30}{139};4+\frac{944}{139},1970-\frac{30}{139}\right)$$

The individual demand functions for X are:

$$x_1*(p) = \begin{cases} 2+\dfrac{3500-2000-25\cdot 2}{p+25} = 2+\dfrac{1450}{p+25} & if\ p > -\dfrac{-2000+3500}{2} = -750 \\ 0 & otherwise \end{cases}$$

$$x_2*(p) = \begin{cases} 18+\dfrac{2500+4000-720\cdot 18}{p+720} = 18-\dfrac{6460}{p+720} & if\ p > -\dfrac{4000+2500}{18} = -\dfrac{3250}{9} \\ 0 & otherwise \end{cases}$$

As long as we consider positive prices only, we have

$$x_1*(p)-\bar{x}_1 = \frac{1450}{p+25}$$

$$x_2*(p)-\bar{x}_2 = -\frac{6460}{p+720}$$

Then, the market excess demand function for X is:

$$E(p) = \frac{5010}{(p+25)(p+720)}\left(\frac{88250}{501}-p\right)$$

The unique ICE price is $p^* = \frac{88250}{501}$ =176.14… The associated consumptions x_1* and x_2* of X are:

$$x_1*(p^*) = \bar{x}_1+\frac{1450}{p^*+25} = 2+\frac{1450}{(88250/501)+25} = 16-\frac{944}{139}\text{,and}$$

$$x_2*(p) = \bar{x}_2-\frac{6460}{p^*+720} = 18-\frac{6460}{(88250/501)+720} = 4+\frac{944}{139},$$

which are consistent with the intersection of the expansion paths. The associated utilities of consumers 1 and 2 are 32000/139 and 1080000/139, respectively. In addition, $E(p) > 0$ if $p < p^{**}$, and $E(p) < 0$ if $p > p^{**}$. Hence, the ICE price $p^{**} = \frac{88250}{501}$ is globally stable.

Remark. In the stable case, there are two "extreme competitive prices": one is 0, which gives the horizontal budget line, and the other is ∞, which gives the vertical budget line. These two budget lines support the upper contour sets of both types of consumers. Recall that $E(p) > 0$ if $p < p^{**}$, then price never converges to 0. In addition, $E(p) < 0$ if

$p > p^{**}$ implies that price never diverges to ∞.

Remark. An ICE price is not always globally unstable even if it is not globally stable.

A.3. The Unstable Case

The unstable case is a Leontief exchange economy with the parameters

$a_1 = 25$, $b_1 = -2000$, $\bar{x}_1 = 16$, $\bar{y}_1 = 1100$;

$a_2 = 720$, $b_2 = 4000$, $\bar{x}_2 = 4$, $\bar{y}_2 = 4900$

Note that the values of $(a_1, b_1; a_2, b_2)$ are the same as in the stable case. In addition,

$b_1 - a_1\bar{x}_1 = -2000 - 25 \cdot 16 < 0$,

$b_2 - a_2\bar{x}_2 = 4000 - 720 \cdot 4 > 0$.

The utility functions of types 1 and 2 are respectively:

$U_1(x_1, y_1) = \min\{25x_1, y_1 - 2000\}$, and

$U_2(x_2, y_2) = \min\{720x_2, y_2 + 4000\}$.

The expansion paths of types 1 and 2 are respectively the same as in the stable case, so that the intersection of the two paths remains unchanged:

$$(x_1,\ y_1; x_2,\ y_2) = \left(\frac{1280}{139}, \frac{310000}{139}; \frac{1500}{139}, \frac{524000}{139}\right)$$
$$= \left(16 - \frac{944}{139}, 2230 + \frac{30}{139}; 4 + \frac{944}{139}, 1970 - \frac{30}{139}\right)$$

The individual demand functions for X are:

$$x_1^*(p) = \begin{cases} 16 + \dfrac{1100 - 2000 - 25 \cdot 16}{p + 25} = 16 - \dfrac{1300}{p + 25} & if\ p > -\dfrac{-2000 + 1100}{16} = \dfrac{225}{4} = 56.25 \\ 0 & otherwise \end{cases}$$

$$x_2^*(p) = \begin{cases} 4 + \dfrac{4900 + 4000 - 720 \cdot 4}{p + 720} = 4 + \dfrac{6020}{p + 720} & if\ p < \dfrac{720 \cdot 4900}{4000 - 720 \cdot 4} = 3150 \\ 4 + \dfrac{4900}{p} & otherwise \end{cases}$$

Thus,

$$x_1^*(p) - \bar{x}_1 = \begin{cases} -\dfrac{1300}{p + 25} & if\ p > 56.25 \\ -16 & otherwise \end{cases}$$

$$x_2^*(p) - \bar{x}_2 = \begin{cases} \dfrac{6020}{p + 720} & if\ p < 3150 \\ \dfrac{4900}{p} & otherwise \end{cases}$$

Then, the market demand function for X is:

$$E(p)=\begin{cases} \dfrac{6020}{p+720}-16=\dfrac{-16p-5500}{p+720}<0 & if\ p<56.25 \\ \dfrac{4720}{(p+720)(p+25)}\left(p-\dfrac{39275}{236}\right) & if\ 56.25\le p\le 3150 \\ \dfrac{3600p+122500}{p(p+25)}>0 & if\ p>3150 \end{cases}$$

The ICE price is $p^*=\dfrac{39275}{236}$ =166.42… Notice that 56.25 $<p^*<$ 3150. The associated consumptions x_1* and x_2* of X are:

$x_1*(p^*)=\ \bar{x}_1-\dfrac{1300}{p^*+25}=16-\dfrac{1300}{(39275/236)+25}=\ 16-\dfrac{944}{139}$, and

$x_2*(p)=\bar{x}_2+\dfrac{6020}{p^*+720}=\ 4+\dfrac{6020}{(39275/236)+720}=4+\dfrac{944}{139}$,

which are consistent with the intersection of the expansion paths. The associated utilities of consumers 1 and 2 are 32000/139 and 1080000/139, respectively. In addition, $E(p)<0$ if $p<p^*$, and $E(p)>0$ if $p>p^*$, so that $p^*=\dfrac{39275}{236}$ is globally unstable.

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