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Rates of Time Preference and the Current Account in a Dynamic Model of Perpetual Youth -Should "Global Imbalances" always be Balanced?-

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Rates of time preference and the current account in a dynamic model of perpetual youth

-Should "global imbalances" always be balanced?- ♦

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Abstract

A two-country version of the Blanchard model enables us to investigate the cross country effects of different rates of time preference in a well behaved manner. A patient country runs the current account surplus and becomes a creditor; a less patient country runs the current account deficit and becomes a debtor. Even a small difference in the rate of time preference produces a sustainable current account deficit/surplus. For example, the difference in the rate of time preference by 0.25 percent enables the impatient country to run the current account deficit of 4.8 percent of GNP. Our analysis and calibration results challenge the common sense view that global imbalances should be always balanced.

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1. Introduction

"To what extent should surplus countries expand; to what extent should deficit countries contract?" asked Mundell (1967). These questions remain as important now as they did in 1967. In the 1960s and 1970s, many studied capital accounts in macro dynamic models (See for example, Hamada 1966, 1969; Bardhan 1967, Onitsuka 1974, Ruffin 1979, and others).

Now the "inter-temporal approach to the current account" (see Obstfeld and Rogoff, 1995) provides a standard theoretical foundation for policy analyses of external balances, international debt, and equilibrium real exchange rates. This approach is based on a sound micro-foundation along the tradition of Irving Fisher, viewing the current account balance as the result of agents' forward-looking inter-temporal decisions on savings and investment.

Key predictions of this approach are, however, at odds with the reality in the world economy. This approach suggests that the US, with its current state of heavy international indebtedness, has to run substantial current account surpluses into the future to restore its external sustainability. As a matter of fact, the US owes a huge net foreign debt, and keeps running the current account deficits persistently, yet any adjustment process through a large depreciation of the US dollar has not taken place.

We argue in this paper that applications of this intertemporal approach are generally limited by the assumption that two nations have identical time discount rates and accordingly the identical savings ratios. In reality, savings rates differ substantially across countries. For example, the same individual would behave over time as if she or he is located in a country with reverse mortgage or in a country without it.

Many proposals have been posited to explain the difference in savings rates, including the effects of demographics, varying levels and growth rates of GDP, disparate social security systems, and housing price differentials. These variables, however, appear to explain only part of variations in savings rates across countries. Cultural differences may explain the difference in savings rates. For example, Carroll et al (1994) succeed in explaining the savings rate differential of cultural differences by comparing savings patterns of immigrants to Canada from different cultural backgrounds. Guido, Spienza, and Zingales (2006) find that countries in which people value thrift, tend to have higher savings rates. Keith Chen (2014) shows that the difference in language with respect to the grammatical association between the present and the future explains the cross-country difference in the country's savings rates.

Researchers should then be motivated to build a model of open economies consisting of countries with different rates of time preference. As Obstfeld (1990) pointed out, however, any infinitesimal difference in the rate of time preference in the infinite horizon model will end up with an extreme wealth distribution where the world wealth concentrates in one country with the lowest time preference. This is probably why, in his Ely lecture (Obstfeld, 2012) does not consider the difference in time preference a reason for global imbalances. His lecture carefully sorts out the other possible causes of imbalances, but falls short of explaining them.

In order to relax this extreme bang-bang property, we introduce an overlapping generation structure with disconnected cohort budget constraints. Buiter (1981), in his pioneering work, demonstrates that models of finite overlapping lifetimes can produce a unique, non-degenerate, long-run distribution of wealth in the presence of flows of capital across countries. To reconcile the standard infinite horizon model with the Buiter model in a more realistic setting, remembering that humans are mortal, we rely on the Blanchard (1985) model with agents of perpetual youth for our analysis of capital flows.

We construct a two country version of the Blanchard model that supports well-behaved features on the international allocation of wealth between creditor and debtor countries. We establish in this paper the existence and local stability of the long run equilibrium path of capital accumulation and international assets/debt. A more patient country runs current account surpluses, accumulating substantial but limited amounts of foreign assets, and a less patient country runs current account deficits, owing foreign debt. A less patient country need not make up for its foreign debt fully with current account surpluses even in the long run.

Calibration results raise a question on the currently popular argument that the "global imbalance" should be balanced. Even a small difference in the rate of time preference explains a significant size of the current account deficit/surplus to be sustainable. The difference in the rate of time preference by 0.25 percent enables the impatient country to run the current account deficit of 4.8 percent of GNP. We find far higher sustainable net foreign liabilities than the current values of liabilities of the United States.

Literature reviews

Extensive arguments have been made from several dimensions to explain the sustainability of the US current account deficit and foreign debt. Obstfeld and Rogoff (2005) consider an adjustment process through the global reallocation of demand for traded versus non-traded domestic and foreign goods. In their analysis, the sustainability requires the reversal of the current account deficit, followed by a large real depreciation of the dollar. Blanchard et al (2005) take a portfolio balance approach, focusing on the dual role of the exchange rate in allocating portfolios between imperfectly substitutable domestic and foreign assets as well as the role of affecting relative demands through the terms of trade. Their model predicts the substantial depreciation of the US dollar since the exchange rate is the only variable to force the rebalancing of the current account. The US owes a huge net foreign debt, and runs persistent current account deficits, but until now, we have not yet witnessed any large depreciation of the US dollar.

Hamada and Iwata (1989) calculate the future external positions among several countries in a Solow-type growth model with different savings rates. Their simulation shows that the foreign debt of the US in terms of capital stock will rise to 30-40 percent over the long run if its low savings rate continues.

Engel and Rogers (2006) attempt to explain the sustainability by the difference in the TFP growth between the US and the rest of the world. In their two-country endowment economy, the debtor country can sustain the deficit for some period when its future share of world output is higher than the current share. Unfortunately, this condition does not seem to be congruent with the fact that the US's share of world output has declined persistently since 2000.

Gourinchas and Rey (2007) focused on the evaluation of US foreign assets that arose from the persistent dollar depreciation, and stressed that the surplus necessary to reduce the imbalance is overestimated.

Momota and Futagami (2005) develop a two-country version of the Blanchard model with a different demographic structure, showing that the difference in population dynamics affects the international asset positions. A country with high birth and death rates becomes a debtor country given the population growth rates being equal.

Sakuragawa and Hamada (2001) develop a model in which the difference in the country's financial development affects international asset positions. Caballero, Farhi and Gourinchas (2008) provide a model that explains the sustained rise in the US current account deficit in an environment where there is heterogeneity in the country's ability to produce sound financial assets.

This paper is organized as follows. Section 2 sets up the closed economy version of the model. Section 3 studies the two country version of the model. Section 4 conducts simulations.

2. Model

The world economy consists of two countries, 1, 2. Both countries are identical except for the rate of time preference. Agents in country 1 are patient and has low time preference θ_1, while those in country 2 are impatient and has high time preference θ_2, with $\theta_1 < \theta_2$. There is the final good that is consumed or invested in capital. The production of the final good follows the CRTS production technology using two factors of production, capital and labor, and is described as $\tilde{F}(K_j(t), N_j(t))$, $(j = 1,2)$, where $K_j(t)$ is the stock of capital, and $N_j(t)$ is the labor force measured in efficiency unit, the size of which grows at g, given the population size that is equal to unity as stated below. Letting δ be the depreciation rate of capital, we define the net output as $F(K_j, N_j) \equiv \tilde{F}(K_j, N_j) - \delta K_j$.

At any instant of time, a large cohort, whose size is normalized to be p, is born. Each agent throughout his life faces a constant probability of death p. The assumption that cohorts are large implies that, although each agent is uncertain about the time of death, the size of a cohort declines non-stochastically through time. A cohort born at time zero has a size, as of time t, of pe^{-pt}, and the size of the population at any time t is stationary to satisfy $\int_{-\infty}^{t} pe^{-p(t-s)} ds = 1$.

In the absence of insurance, uncertainty about death implies that agents may leave unanticipated bequests although they have no bequest motive. They may also be constrained to maintain a positive wealth position if they are prohibited from leaving debt heirs. Private markets may, however, provide insurance risklessly, and it is reasonable and convenient to assume that they do so. There exist life insurance companies. Agents may contract to make (or receive) a payment contingent on their death.

Because of the large number of identical agents, such contracts may be offered risklessly by life insurance companies. Given free entry and a zero profit condition, and given a probability of death p, agents will pay (receive) a rate p to receive (pay) one good contingent on their death. In the absence of a bequest motive, and if negative bequests are prohibited, agents will contract to have all of their wealth (positive or negative) return to the life insurance company contingent on their death. Thus, if their wealth is w, they will receive pw if they do not die and pay all the wealth if they die.

Variables are measured in efficiency unit term, to satisfy $x_j(s,t) = \hat{x}_j(s,t)/e^{g(t-s)}$. Denote by $c_j(s,t), y_j(s,t), w_j(s,t),$ and $h_j(s,t)$, consumption, non-interest income, nonhuman wealth, and human wealth, measured in efficiency unit term, of an agent born at time s, in country $j(=1,2)$, as of time t. Under the assumption that instantaneous utility is logarithmic, the agent maximizes $E_t\left[\int_t^\infty \log \hat{c}_j(s,v)e^{\theta_j(t-v)}dv\right]$, where E_t is the expectation operator. The agent facing the constant probability of death p turns out to maximize $\int_t^\infty \log c_j(s,v)e^{(\theta_j+p-g)(t-v)}dv$, with the effective discount rate $(\theta_j + p - g)$. Even if either θ_j or g is equal to zero, agents will discount the future if p is positive. If an agent has wealth $\hat{w}_j(s,t)$ at time t, he receives $r(t)\hat{w}_j(s,t)$ in interest and $p\hat{w}_j(s,t)$ from the insurance company. Let $r(t)$ be the interest rate at time t. Thus its dynamic budget constraint in efficiency term is $\dfrac{dw_j(s,t)}{dt} = [r(t) + p - g]w_j(s,t) + y_j(s,t) - c_j(s,t)$. An additional transversality condition is needed to prevent agents from going infinitely into debt and protecting themselves by buying life insurance. We impose a condition that is the extension of that used in the deterministic case; $\lim_{v\to\infty} e^{-\int_t^v [r(\mu)+p-g1d\mu} w_j(s,v) = 0$. With this condition, the budget constraint can be integrated to give $\int_t^\infty c(s,v)e^{-\int_t^v [r(\mu)+p-g]d\mu}dv = w(s,t) + h(s,t)$, where $h(s,t)=\int_t^{\infty} y(s,v)e^{-\int_t^v [r(\mu)+p-g1d\mu}dv$. Under the log utility, individual consumption depends on total individual wealth, with propensity $(\theta+p-g)$. The path of consumption satisfies $c(s,t)=(p+\theta-g)[w(s,t)+h(s,t)]$.

Denote aggregate variables by upper letters. The relation between any aggregate variable $X_j(t)$ and an individual counterpart $x_j(s,t)$ is $X_j(t)=\int_{-\infty}^{t} x_j(s,t)pe^{-p(t-s)}ds$. Let $C_j(t), Y_j(t), W_j(t)$, and $H_j(t)$ denote aggregate consumption, non-interest income, nonhuman wealth, and human wealth, measured in efficiency unit, in country j at time t, respectively. Then aggregate consumption is a linear function of aggregate human and nonhuman wealth, given by $C_j(t)=(p+\theta-g)[H_j(t)+W_j(t)]$.

The next step is to characterize the dynamics of both components of aggregate wealth. Human wealth is given by

$$H_j(t)=\int_{-\infty}^{t} h_j(s,t)pe^{p(s-t)}ds=\int_{-\infty}^{t} \left\{\int_t^{\infty} y_j(s,v)e^{-\int_t^v\{r(\mu)+p-g\}d\mu}dv\right\}pe^{p(s-t)}ds.$$

Changing the order of integration gives

$$H_j(t)=\int_t^{\infty} \left[\int_{-\infty}^{t} y_j(s,v)pe^{p(s-v)}ds\right]e^{-\int_t^v\{r(\mu)+p-g\}d\mu}dv.$$

This has a simple interpretation. The term in parentheses is labor income accruing at time v to agents already alive at time t. Human wealth is thus the present value of future labor income accruing to those currently alive. To characterize the dynamic behavior of $H(t)$, we need to specify the distribution of labor income across agents. Technological progress spills over equally to living agents so that they have the same productivity irrespective of age; $y(s,v)=Y(v)$ for all s. Thus all agents have the same human wealth and $H(t)$ is given by $H_j(t)=\int_t^{\infty} Y_j(v)e^{-\int_t^v 1r(\mu)+p-g1d\mu}dv$, or in differential equation form, $\dfrac{dH_j(t)}{dt}=[r(t)+p-g]H_j(t)-Y_j(t)$, with

$\lim_{v\to\infty} H_j(v)e^{-\int_t^v 1r(\mu)+p-g1d\mu}=0.$ Nonhuman wealth is given by

$W_j(t)=\int_{-\infty}^{t} w_j(s,t)pe^{p(s-t)}ds.$ Differentiating with respect to time gives

$$\frac{dW_j(t)}{dt} = w_j(t,t) - pW_j(t) + \int_{-\infty}^{t} \frac{dw_j(s,t)}{dt} pe^{p(s-t)} ds.$$

The first term on the right is the financial wealth of newly born agents, which is equal to zero. The second term is the wealth of those who die. The third is the change in the wealth of those alive. We rewrite

$$\frac{dW_j(t)}{dt} = \{r(t) - g\}W_j(t) + Y_j(t) - C_j(t).$$

Whereas individual wealth accumulates, for those alive, at rate $r + p - g,$ aggregate wealth accumulates at rate $r. - g$ This is because the amount pW is a transfer, through life insurance companies, from those who die to those who remain alive; it is not therefore an addition to aggregate wealth.

Denoting $dX(t)/dt \equiv \dot{X}$, collecting equations, gives a first characterization of aggregate consumption: $C_j = (p + \theta - g)(H_j + W_j)$, $\dot{H}_j = (r + p - g)H_j - Y_j$, and $\dot{W}_j = (r - g)W_j + Y_j - C_j.$ The following two equations replaces the three-equation system by:

(1) $\dot{C}_j = (r - \theta_j)C_j - p(p + \theta_j - g)W_j$, and

(2) $\dot{W}_j = (r - g)W_j + Y_j - C_j$.

We first investigate the closed economy version of the model. Since then $W_j = K_j$, we rewrite

(3) $\dot{C}_j = \{r(K_j) - \theta_j\}C_j - p(p + \theta_j - g)K_j$

(4) $\dot{K}_j = F(K_j) - C_j - gK_j$,

where $r(K) \equiv F'^{-1}(K)$. If agents have infinite horizons, that is, $p = 0$, equation (3) reduces to the standard equation. If $p > 0$, the rate of change in the aggregate consumption depends also on nonhuman wealth. At the steady state we should have $\theta_j < r(K^*) < \theta_j + p$ [see Blanchard 1985, p232] $(g = 0)$ so that the interest rate is greater than the rate of time preference. Note that even if p is positive, individual consumption follows $\dot{c} = (r - \theta)c.$ Thus if $r = \theta$, individual consumption should be constant but the aggregate consumption will decline, a contradiction. The positive nonhuman wealth requires the interest rate to being greater than the rate of time preference. This property arises from the fact that the interest rate faced by individual agents $r+p$, while the one in the society is r.

We next turn to the open-economy version of the model, with the difference in the rate of time preference. For simplifying analysis, let ω_j denote the noninterest income that is exogenous and let W_j denote the holding of net foreign assets. In the steady state we have $C_j = \dfrac{p(p+\theta_j)}{r-\theta_j} W_j \ (g=0)$. If $r > \theta_j$, living agents are accumulating wealth, and as a result, the level of foreign assets is positive, while if $r < \theta_j$, agents are decumulating wealth and the level of foreign assets is negative. We expect the possible case of $\theta_1 < r^* < \theta_2$ in this developed version.

3. Analysis of Two-Country Model

In the two-country version of the model, nonhuman wealth of each country W_j consists of domestic capital K_j and foreign assets F_j, with $F_j(<0)$ being foreign liabilities. Let ω_j denote the wage income. Consumption and non-human wealth evolves, in each country, as

(5) $\dot{C}_1 = (r-\theta_1)C_1 - p(p+\theta_1 - g)W_1$

(6) $\dot{W}_1 = (r-g)W_1 + \omega_1 - C_1$

(7) $\dot{C}_2 = (r-\theta_2)C_2 - p(p+\theta_2 - g)W_2$

(8) $\dot{W}_2 = (r-g)W_2 + \omega_2 - C_2$.

When both countries open their capital markets, the sum of foreign assets should be zero, namely, $F_1 + F_2 = 0$, which is replaced by

(9) $W_1 + W_2 = K_1 + K_2$.

Finally, the rate of return to capital should be equal to the common world interest rate;

(10) $F'(K_1) = r$,

(11) $F'(K_2) = r$.

Severn equations (1), (2), (3), (4), (5), (10), and (11) determine a sequence of seven variables $\{C_1, C_2, K_1, K_2, W_1, W_2, r\}_{t=0}^{\infty}$, given the initial conditions $W_j(0), (j = 1,2)$, and the transversality conditions.

We first solve the steady state. Capital should be equal with each other so that (9) reduces to

(12) $$W_1 + W_2 = 2K,$$

where $K \equiv K_1 = K_2$ and $\omega_1 = \omega_2 \equiv \omega(K)$.

We impose the following restriction in order to exclude explosive solutions.

Assumption A; $p + r > g$

We rigorously solve the steady-state equilibrium. From (5), (6), (7), and (8), we have

(13) $$W_1 = \frac{(r-\theta_1)\omega(K)}{(p+r-g)\{\theta_1 + p - r\}}$$

(14) $$W_2 = \frac{(r-\theta_2)\omega(K)}{(p+r-g)\{\theta_2 + p - r\}}.$$

From (12), (13), (14), we have

(15) $$\frac{2K(r)}{\omega(K(r))} = \frac{r-\theta_1}{(p+r-g)(\theta_1 + p - r)} + \frac{r-\theta_2}{(p+r-g)(\theta_2 + p - r)} \equiv \phi(r),$$

where $K(r)$ is the inverse image of $F'(K) = r$. We impose the following assumption.

Assumption B; $\dfrac{K}{\omega(K)}$ is not decreasing.

The function $F(K) = K^{\alpha} (0 < \alpha < 1)$ satisfies this assumption. Under Assumption B, the LHS of (15) is decreasing, while the RHS is increasing for each of distinct intervals so that there will exist well-defined solutions.

Two cases are to be distinguished; (i) $\theta_2 - \theta_1 < p$ and (ii) $\theta_2 - \theta_1 > p$. We first study the case of $\theta_2 - \theta_1 < p$ in which the difference $\theta_2 - \theta_1$ is small relative to p. As Figure A illustrates, the LHS of (15) is decreasing, while the RHS is increasing for over $(\theta_1, \theta_1 + p)$ so that there exists an intersection over this interval.

Furthermore, we distinguish between two cases, according to if $\frac{2K(\theta_2)}{w(K(\theta_2))} \geq \phi(\theta_2)$ or not. If the inequality holds, as Figure A illustrates, we have $\theta_2 \leq r^* < \theta_1 + p$ so that the real interest rate is higher than the rate of time preference of either country. Otherwise, as Figure B illustrates, we have the converse.

We turn to the case of $\theta_2 - \theta_1 > p$, in which the difference of the rate of time preference is large relative to p. Figure C illustrates this case. We obtain the following.

Proposition 1:

(i) Suppose $\theta_2 - \theta_1 < p$. There exists a unique steady state equilibrium with $\theta_1 < r^* < \theta_2$ if

$$(*) \qquad \phi(\theta_2) \equiv \frac{\theta_2 - \theta_1}{(p + \theta_2 - g)(\theta_1 + p - \theta_2)} > \frac{2K(\theta_2)}{w(K(\theta_2))},$$

while otherwise, there exists a unique steady state equilibrium with $\theta_1 < \theta_2 \leq r^* < \theta_1 + p$.

(ii) Suppose $\theta_2 - \theta_1 > p$. There exists a unique steady state equilibrium with $\theta_1 < r^* < \theta_2$.

Proof. (i) As Figures A and B illustrate, we have an intersection E over $(\theta_1, \theta_1 + p)$. We have another intersection F over the region $(\theta_1 + p, \infty)$. Individual consumption follows $\dot{c} = (r^* - \theta_1)c$, and if r^* is greater than $\theta_1 + p$, individual consumption grows at a rate greater than p, the rate of death so that aggregate consumption should grow forever, a contradiction to the existence of the well-defined solution. The point E is the unique steady state.

(ii) As Figure C illustrates, we have an intersection E over $(\theta_1, \theta_1 + p)$. Since $\theta_2 - \theta_1 > p$ by assumption, $\theta_1 < r^* < \theta_2$ directly follows. Eliminating another intersection F from the equilibrium comes from the same reason as (ii). Q.E.D.

Having solved r and thus K, we turn to the world distribution of assets. Equations (14) and (8R) are rewritten as

$$(16) \qquad W_j = \frac{\{r(K) - \theta_j\}\omega(K)}{\{p + r(K) - g\}\{\theta_j + p - r(K)\}}$$

Since W_j is decreasing in θ_j, we have $W_1 < K < W_2$. A more patient country holds more non-human wealth than physical capital and becomes a creditor country, while a less patient country holds less and becomes a debtor country. Capital flows from the less patient to the more patient country. In addition, we have $C_2 < C_1$. Formally,

$$(17) \qquad C_j = \frac{p(p + \theta_j)\omega(K)}{\{p + r(K) - g\}\{\theta_j + p - r(K)\}}.$$

People of the patient country enjoy higher consumption than those of the impatient one.

Suppose $F(K) = K^{\alpha}$. We have $\dfrac{2K(r)}{w(K(r))} = \dfrac{2\alpha}{(1-\alpha)r}$ so that the condition (*) is replaced by a simpler condition:

$$(**) \qquad \frac{\theta_2 - \theta_1}{(p + \theta_2 - g)(\theta_1 + p - \theta_2)} > \frac{2\alpha}{(1-\alpha)\theta_2}.$$

The condition (**) is more likely to hold if the labor share (1-α) is large or θ_1 is large relative to θ_2. When agents can borrow by collateralizing the future more labor income or when the difference of time preference is large, the real interest rate tends to lie between the two time preferences. At the steady state living agents of the patient country are accumulating wealth over their life, while those of the impatient country are accumulating liabilities over their life.

As follows from Proposition 1, the less patient country may face the smaller real interest rate than the rate of time preference; $r < \theta_2$. We then have $W_2 < 0 < W_1$.

The less patient country has the negative asset position, but is solvent because citizens of this country have the flow of labor income. The intertemporal budget constraint of the less patient country, combined with $\lim_{v \to \infty} W_2(v) e^{-r(v-t)} = 0$, is written as

$$\int_t^{\infty} C_2(v) e^{-r(v-t)} dv = \int_t^{\infty} \omega(v) e^{-r(v-t)} dv + W_2(t).$$

Even if $W_2(t)$ is negative, if the sequence of future labor income is sufficiently positive, the sequence $\{C_2(t)\}_t^\infty$ will be positive. This never occurs in Buiter (1981) because agents cannot borrow by putting up the future labor income as collateral in his overlapping generation model with two-period-lived agents.

In Buiter, a country with small rate of time preference becomes a creditor, and the one with large rate of time preference a debtor, but the net asset position of the debtor country should be positive, that is, $W_2 = K + F_2 > 0$. We have a stronger result than Buiter; the net asset position of the debtor country should be positive if the interest rate is smaller than the rate of timer preference of that country.

We turn to the current account balance. We derive the aggregate savings function as

(18) $$S_j = \frac{r-\theta_j}{p+r-g}\omega(K) + (r+g-p-\theta_j)W_j .$$

The aggregate savings may be an increasing or decreasing function of the growth rate, depending on whether the interest rate is greater than θ_j or not. If $r > \theta_j$, this country has a positive non-human wealth so that the aggregate savings are increasing in the growth rate, while otherwise, it is decreasing. On the other hand, the aggregate investment function is $I_j = gK_j$. Therefore, we have the current account as

(19) $$CA_j = \frac{r-\theta_j}{p+r-g}\omega(K) + (r-p-\theta_j)W_j + gF_j = gF_j .$$

Note that the sum of first two terms in the RHS is zero at the steady state. Accordingly, we drive the trade account as $TA_j = (g-r)F_j$. If the foreign asset F_j is not zero at the steady state, neither the current account nor the trade account need be balanced in the growing economy. A creditor country with low time preference runs the current account surplus, while a debtor one with high one the current account deficit.

This finding suggests that impatient countries should not hurry to repay their liabilities by targeting the zero current account. The current account sustainability may be consistent with the reversal around above or below zero. In G7 countries, past thirty years, the mean of the current account is systematically positive for Japan and France, while it is systematically negative for Canada, U.K., and U.S. Clarida et al (2005) estimates the mean of the current account for G7 countries, finding it is significantly different from zero for all.

We turn to the dynamics.

Proposition 2: There exists a unique saddle stable path that converges to the steady state, given $W_j(0)(j=1,2)$ in a small neighborhood of the steady state

Proof: See the Appendix.

4. Simulations

Table 1 shows some calibration results. We fix three parameters at $\alpha = 0.3, \theta_1 = 0.02,$ and $\delta = 0.1$, and change other three parameters, $\theta_2, p,$ and g. Note that we evaluate output as the gross output including depreciation of capital. The table is divided into five groups. In each of them, θ_2 varies from 0.02 to 0.03, given p and g. Upper three groups show savings, the current account, and the net asset position as the growth rate changes. As the growth rate rises, the saving rate of country 1, S_1, is higher, while that of country 2, S_2, is smaller. Accordingly, the current account surplus of country 1, CA_1, and the current account deficit of country 2, CA_2, are both greater, in terms of GDP. For example, allowing for the difference of time preference by 0.25 percent, the current account difference is 5.3 percent at $g = 0.01$, and it comes up to 9.1% at $g = 0.02$. Lower three groups show the changes in variables as the death rate changes. As the death rate p goes down, the differences in savings and the current account balance gets larger.

We try to match the model with a reality by assuming $p = g = 0.02$. The smaller difference in the rate of time preference can explain a significant size of the current account imbalance. When θ_2=0.0225, and the difference is 0.25 percent point, the impatient country runs the deficit by 4.8 percent of GDP and holds net foreign liabilities by 241 percent of GNP. When θ_2=0.025, the impatient country runs the current account deficit by 9.5 percent of GNP, and can hold net foreign liabilities by 476 percent of GDP, the figure of which is even higher than is generally argued.

A small difference in the rate of time preference enables the impatient country to have even higher level of net foreign debt position than is typically anticipated. Obstfeld and Rogoff (2005) say, “A simple calculation shows that if the U.S. nominal GDP grows at 6 % a year and the current account deficit remains at 6 % of nominal GDP. The ratio of U.S. foreign debt to GDP will asymptotically approach 100%. Few countries have ever reached anywhere near that level of indebtedness without having a crisis of some sort”. However, calculation results show that even higher ratios of foreign debt to GDP will be sustainable. U.S. could borrow more!

4. Conclusion

We have developed a framework of analyzing capital flows in a two country version of perpetual youth. The model has more affinity to reality than a rigid overlapping generation model, and does not exhibit the bang-bang capital flight property seen in the infinite horizon model. The model supports well-behaved features of the international allocation of wealth between creditor and debtor countries. We establish the existence and local stability of the long run equilibrium.

The foreign net asset predicted in the model is somewhat larger than the observed imbalance in the United States. Relaxing the logarithmic utility to the CRRA type may be a promising direction. See Horii and Kamihigashi (in progress 2016) for an attempt to narrow this gap. Another useful step is to incorporate the default risk into the model. Exploring the introduction of the stochastic elements may be too demanding, but would be worthwhile.

Appendix

Proof of Proposition 2

We characterize the dynamic system by the following four equations:

$$\dot{C}_{1t} = (F'(K_t) - \theta_1)C_{1t} - p(p + \theta_1 - g)W_{1t} \tag{A1}$$

$$\dot{C}_{2t} = (F'(K_t) - \theta_2)C_{2t} - p(p + \theta_2 - g)(2K_t - W_{1t}) \tag{A2}$$

$$\dot{W}_{1t} = (F'(K_t) - g)W_{1t} + (F(K_t) - F'(K_t)K_t) - C_{1t} \tag{A3}$$

$$\dot{K}_t = F(K_t) - gK_t - \frac{C_{1t} + C_{2t}}{2} \tag{A4}$$

We linearize (A1)～(A4) around the steady state:

$$\begin{pmatrix} \dot{C}_{1t} \\ \dot{C}_{2t} \\ \dot{W}_{1t} \\ \dot{K}_t \end{pmatrix} = M \begin{pmatrix} C_{1t} - C_1 \\ C_{2t} - C_2 \\ W_{1t} - W_1 \\ K_t - K \end{pmatrix}$$

$$M \equiv \begin{pmatrix} F' - \theta_1 & 0 & -p(p+\theta_1 - g) & F''C_1 \\ 0 & F' - \theta_2 & p(p+\theta_2 - g) & F''C_2 - 2p(p+\theta_2 - g) \\ -1 & 0 & F' - g & F''(W_1 - K) \\ -1/2 & -1/2 & 0 & F' - g \end{pmatrix}$$

Two variables (C_{1t}, C_{2t}) are jump variables, and two variables $(W_{1t},\ K_t)$ are predetermined variables. We can state that there is a unique saddle path that converges to the steady state given the predetermined variables if two eigenvalues are positive real numbers (or conjugate complex numbers with positive real parts), and two eigenvalues are negative real numbers (or conjugate complex numbers with negative real parts).

Letting λ denote the eigenvalue of the matrix M, four λ's satisfy the following equation:

$$\begin{vmatrix} F' - \theta_1 - \lambda & 0 & -p(p+\theta_1 - g) & F''C_1 \\ 0 & F' - \theta_2 - \lambda & p(p+\theta_2 - g) & F''C_2 - 2p(p+\theta_2 - g) \\ -1 & 0 & F' - \lambda - g & F''(W_1 - K) \\ -1/2 & -1/2 & 0 & F' - \lambda - g \end{vmatrix} = 0$$

Using $x \equiv F' - \lambda$, we rewrite the above equation:

$$0 = \frac{1}{2}(x + p - g)\{2(x + p - g)(x - p - \theta_1)(x - p - \theta_2)$$
$$+ F''[C_1(x - p - \theta_2) + C_2(x - p - \theta_1) - p(\theta_2 - \theta_1)(W_1 - K)]\}$$

One of solutions is

$$x = -p + g$$

and the corresponding eigenvalue is

$$\lambda = F' + p - g > 0$$

which is positive under Assumption A. We obtain the remaining three eigenvalues as solutions that satisfy the following equation:

$$E \equiv 2(x + p - g)(x - p - \theta_1)(x - p - \theta_2)$$
$$+ F''[C_1(x - p - \theta_2) + C_2(x - p - \theta_1) - p(\theta_2 - \theta_1)(W_1 - K)] = 0,$$

Using $x \equiv F' - \lambda$, we rewrite

$$[\lambda - (F' + p - g)][\lambda - (F' - p - \theta_1)][\lambda - (F' - p - \theta_2)] + \frac{F''}{2}\{C_1[\lambda - (F' - p - \theta_2)]$$
$$+ C_2[\lambda - (F' - p - \theta_1)] + p(\theta_2 - \theta_1)(W_1 - K)\} = 0$$

We rearrange this by

$$\lambda^3 + Z_1\lambda^2 + Z_2\lambda + Z_3 = 0$$

where

$$Z_1 \equiv -3F' + p + \theta_1 + \theta_2 + g$$
$$Z_2 \equiv (F' + p - g)(F' - p - \theta_1) + (F' + p - g)(F' - p - \theta_2)$$
$$+ (F' - p - \theta_1)(F' - p - \theta_2) + \frac{F''}{2}(C_1 + C_2)$$
$$Z_3 \equiv -(F' + p - g)(F' - p - \theta_1)(F' - p - \theta_2)$$
$$+ \frac{F''}{2}[-C_1(F' - p - \theta_2) - C_2(F' - p - \theta_1) + p(\theta_2 - \theta_1)(W_1 - K)]$$

We know that among the three solutions, one is a positive real number, and the remaining two are either negative real numbers or conjugate complex numbers with negative real parts either if (i) $Z_1 > 0$ and $Z_3 < 0$, or if (ii) $Z_2 < 0$ and $Z_3 < 0$.

First of all, the following is established.

Result 1 $Z_3 < 0$

Proof: We have $\theta_1 < F' < p + \theta_1$ at the steady stat. We have $W_1 - K \geq 0$ under the assumption of $\theta_1 \leq \theta_2$. We have $F' + p - g > 0$ under Assumption A. Given these, $Z_3 < 0$. Q.E.D.

We turn to the next finding.

Result 2

(a) $Z_1 > 0$ holds if $g > 3F' - p - \theta_1 - \theta_2$.

(b) $Z_2 < 0$ holds if $g \le 3F' - p - \theta_1 - \theta_2$.

Proof: Property (a) is immediate from the definition of Z_1. We prove (b) in the following procedure. We define

$$\varphi \equiv (F' + p - g)(F' - p - \theta_1) + (F' + p - g)(F' - p - \theta_2) + (F' - p - \theta_1)(F' - p - \theta_2)$$

We rewrite Z_2 as $Z_2 = \varphi + \frac{F''}{2}(C_1 + C_2)$. From $\frac{F''}{2}(C_1 + C_2) < 0$, if $\varphi \le 0$, $Z_2 < 0$ holds. When $\theta_1 < F' < p + \theta_1$, we have $F' - p - \theta_1 < 0$ and $F' - p - \theta_2 < 0$, and so φ is increasing in g. Thus, if $\varphi \le 0$ holds at $g = 3F' - p - \theta_1 - \theta_2$, (b) is supported.

We define φ as a function of F', when $g = 3F' - p - \theta_1 - \theta_2$ holds, by

$$\varphi = -3(F')^2 + 3(2p + \theta_1 + \theta_2)F' + (p + \theta_1)(p + \theta_2) - (2p + \theta_1 + \theta_2)^2$$

This function realizes the maximum at $F' = p + \frac{\theta_1 + \theta_2}{2}$ $(\ge p + \theta_1)$. If $\varphi \le 0$ at $F' = p + \theta_1$, $\varphi < 0$ holds for $\theta_1 < F' < p + \theta_1$. Indeed, at $F' = p + \theta_1$, $\varphi = -(\theta_1 - \theta_2)^2 \le 0$. This proves (b). Q.E.D.

Results 1 and 2 jointly state that among the three solutions, one is a positive real number, and the remaining two are either negative real numbers or conjugate complex numbers with negative real parts. Therefore, two eigenvalues are positive real numbers, and other two eigenvalues are negative real numbers (or conjugate complex numbers with negative real parts).

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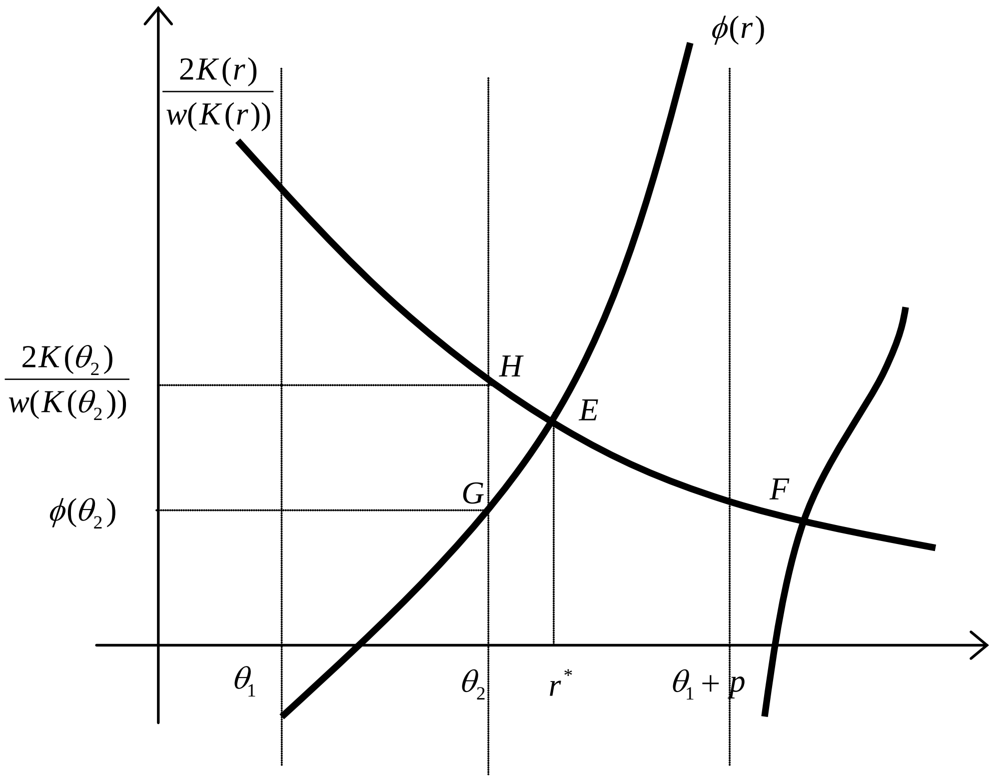
Figure A



$\phi(r)$
$\frac{2K(r)}{w(K(r))}$
$\frac{2K(\theta_2)}{w(K(\theta_2))}$
$\phi(\theta_2)$
H
E
G
F
θ_1
θ_2
r^*
$\theta_1 + p$

Figure B

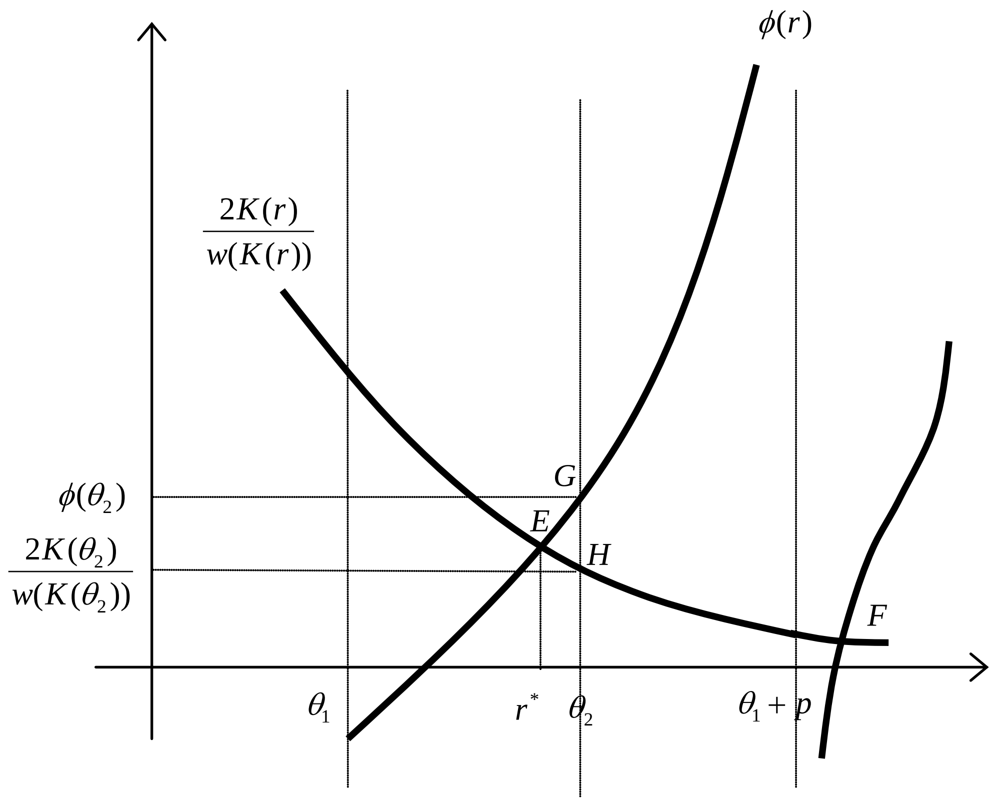

$\phi(r)$
$\frac{2K(r)}{w(K(r))}$
$\phi(\theta_2)$
$\frac{2K(\theta_2)}{w(K(\theta_2))}$
G
E
H
F
θ_1
r^*
θ_2
$\theta_1 + p$

Figure C

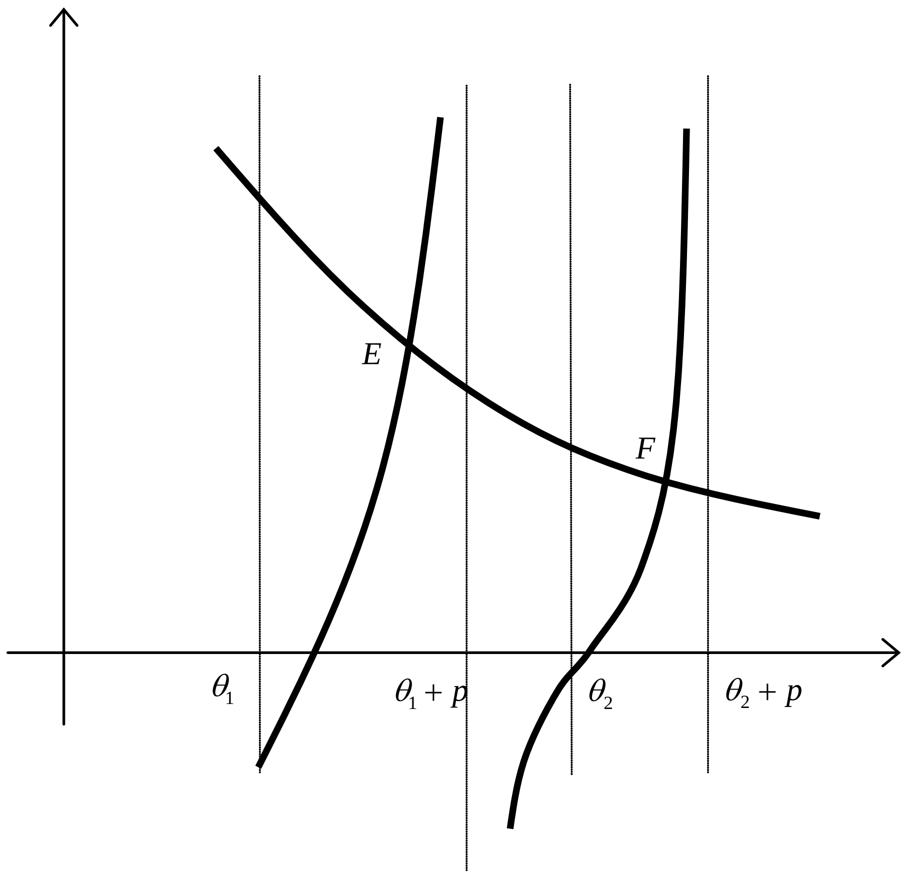


Table1 Calibration Results

θ_1	θ_2	p	g	r	S_1	S_2	CA_1	CA_2	F_1	F_2
0.02	0.02	0.02	0.01	2.24	0.263	0.263	0	0	0	0
	0.0225			2.36	0.267	0.254	0.026	-0.027	1.61	-1.75
	0.025			2.46	0.269	0.245	0.029	-0.036	2.95	-3.55
θ_1	θ_2	p	g	r	S_1	S_2	CA_1	CA_2	F_1	F_2
0.02	0.02	0.02	0.02	2.17	0.295	0.295	0	0	0	0
	0.0225			2.29	0.320	0.259	0.043	-0.048	2.15	-2.41
	0.025			2.40	0.341	0.226	0.080	-0.095	3.99	-4.76
θ_1	θ_2	p	g	r	S_1	S_2	CA_1	CA_2	F_1	F_2
0.02	0.02	0.01	0.02	2.04	0.299	0.299	0	0	0	0
	0.0225			2.17	0.400	0.175	0.155	-0.176	7.74	-8.79
	0.025			2.24	0.443	0.002	0.223	-0.432	11.16	-21.64
θ_1	θ_2	p	g	r	S_1	S_2	CA_1	CA_2	F_1	F_2
0.02	0.02	0.015	0.02	2.10	0.297	0.297	0	0	0	0
	0.0225			2.24	0.350	0.247	0.086	-0.069	4.28	-3.44
	0.025			2.31	0.373	0.175	0.123	-0.174	6.15	-8.71