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December 21, 2015

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International Transmission of Bubble Crashes in a Two-Country Overlapping Generations Model*

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Abstract

We study the international transmission of bubble crashes by analyzing stationary sunspot equilibria in a two-country overlapping generations exchange economy with stochastic bubbles. We consider two cases of sunspot shocks. In the first case, we assume that only the foreign country receives a sunspot shock, while in the second, we assume that both countries independently receive sunspot shocks. In the first case, a bubble crash in the foreign country is always accompanied by a bubble crash in the home country. In the second case, a bubble crash in the foreign country can have a positive or negative effect on the home bubble. We also show that there exists a unique locally isolated stationary sunspot equilibrium, and that it is locally unstable.

JEL Classifications: D91, E44, F30.
Keywords: International transmission; stochastic bubbles; stationary sunspot equilibria; financial integration.

*This research was initiated while the first author was visiting RIEB, Kobe Univeristy in 2013. Earlier versions of this paper were presented at the Conference on Financial and Real Interdependencies, Lisbon, 2015, the 2015 ADRES Conference, Paris, and the 2014 EDGE Jamboree, Copenhagen. We would like to thank Thomas Seegmuller, Alain Venditti, Yiannis Vailakis, Jean-Pierre Drugeon, and Bertrand Wigniolle for helpful comments and suggestions. We are grateful to two anonymous referees for their comments, which have helped up significantly improve the quality of the paper. Financial support from JSPS KAKENHI 15H05729 is gratefully acknowledged.

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1 Introduction

The history of financial markets overflows with episodes of asset bubbles (e.g., Kindleberger and Aliber, 2005). Due to global financial integration over the past few decades, financial markets are now highly interdependent across countries (e.g., Tsutsui and Hirayama, 2010; Ehrmann et al., 2011; Madaleno and Pinho, 2012). As a consequence, the bursting of an asset bubble in one country can have significant impacts on financial markets in other countries. This is what may have happened during the global financial crisis of 2007-2008.

Since this event, the macroeconomic literature on asset bubbles has been growing rapidly. Much of the recent literature considers models of bubbles based on financial frictions and examines the real effects of bubbles; see, e.g., Farhi and Tirole (2012), Martin and Ventura (2012), and Miao and Wang (2012a, 2012b).[1] On the other hand, somewhat surprisingly, very little theoretical work has been done on the international transmission of bubble crashes in highly integrated financial markets.

There have of course been some closely related studies. For example, Ventura (2012) showed that bubbles may comove across countries in an overlapping generations model consisting of multiple countries with different levels of productivity. However, in his model, financial markets in different countries are completely segregated. Tandon and Wang (2003) studied currency substitution in a small open overlapping generations model by analyzing the dynamics of a stochastic bubble, but their analysis was restricted to the deterministic dynamics of the bubble prior to its burst. A recent paper by Martin and Ventura (2015) considers the international transmission of credit bubbles, but in their model the bubbles are affected by a common state variable and assumed to comove.

The purpose of this paper is to analyze the international transmission of bubble crashes in fully integrated financial markets. In other words, we wish to understand the effect of the bursting of a bubble in one country on a bubble in another. For example, if a bubble in one country bursts, then what happens to a bubble in another country when the relevant financial markets are fully integrated? This type of question cannot be answered if the bubbles are assumed to comove at the outset.

[1] See Miao (2014) for a recent survey. See Kamihigashi (2001, 2008, 2015) and the references therein for discussion on the earlier literature on bubbles.

For this purpose we construct a two-country version of the overlapping-generations exchange economy developed by Weil (1987).[2] The countries, called "home" and "foreign," are perfectly symmetric in terms of fundamentals. There is a unique consumption good worldwide, and each country has an intrinsically useless asset, or a "bubble." The good and asset markets are fully integrated internationally; agents in either country have full access to the good and asset markets in both countries.

In this setting, we consider two cases of sunspot shocks. In the first case, we assume that only the foreign country receives a sunspot shock, which has no direct influence on the fundamentals of the economy. A sunspot shock occurs only once over the infinite horizon, with a constant probability in each period. We assume that the bubble in the foreign country bursts if a sunspot shock occurs, and remains at a constant level otherwise. How does the bubble in the home country react to the bursting of the foreign bubble when the home bubble is not required to react at all? We show that the home bubble inevitably bursts simultaneously in response to the bursting of the foreign bubble.

In the second case, we assume that both countries receive sunspot shocks independently. In each country, a sunspot shock occurs only once over the infinite horizon with a constant probability in each period. But this probability is assumed to differ across countries. As in the previous case, we assume that the foreign bubble bursts if a sunspot shock occurs in the foreign country. Likewise, the home bubble bursts if a sunspot shock occurs in the home country. We show that if the foreign bubble bursts, then the home bubble either bursts simultaneously or jumps to a higher level. Hence, unlike in the previous case, a bubble crash in one country can have a positive or negative effect on the bubble in the other country.

The stationary sunspot equilibrium in which a bubble crash in one country has a positive effect on the bubble in the other country is locally isolated. Any other stationary sunspot equilibrium shown in this paper belongs to a continuum of stationary sunspot equilibria. Analyzing the local dynamics around the locally isolated stationary sunspot equilibrium, we show that this equilibrium is locally unstable. Thus to achieve this stationary sunspot equilibrium, the economy must initially jump to this equilibrium.

As discussed above, this paper builds upon the work of Weil (1987) and

[2]See Wigniolle (2014) for a recent extension of Weil's model based on a rank-dependent utility function.

is most closely related to the literature on asset bubbles. Another strand of literature related to this paper is that on sunspot equilibria in two-country overlapping models initiated by Spear (1989) and Manuelli and Peck (1990). In particular, our model is similar to that of Manuelli and Peck, and shares some properties including a common portfolio across countries. However, this literature mostly focuses on exchange rate fluctuations without explicitly considering bubble crashes; see, e.g., Barnett (1992), Betts and Smith (1997), and Russell (2003). To our knowledge, very little is known as to how a bubble crash in one country affects a bubble in the other country. Focusing on this particular issue, this paper seems to be distinct.

A large body of literature in international finance emphasizes the roles of fundamentals (e.g., Kaminsky and Reinhart, 2000), imperfect or asymmetric information (e.g, Allen and Gale, 2000), and financial constraints (e.g., Devereux and Yetman, 2009) as potential sources of international transmission of financial crises and shocks. This paper complements this literature by showing that the international transmission of bubble crashes is an inevitable consequence of financial integration within a simple framework without introducing any friction or fundamental uncertainty.

The rest of the paper is organized as follows. In Section 2 we review the case of a closed economy and show some preliminary results. In Section 3 we introduce the two-country economy, define equilibria, and show some basic results. In Section 4 we assume that only the foreign country receives a sunspot shock. In Section 5 we assume that both countries receive sunspot shocks. In Section 6 we study the local stability of the unique locally isolated stationary sunspot equilibrium. In Section 7 we provide some concluding remarks and discuss possible extensions. All omitted proofs appear in appendices unless otherwise noted.

2 The Closed Economy

In this section, we consider a closed economy that is essentially the same as the exchange economy of Weil (1987). The results in this section are presented for later reference; we do not claim originality here.

2.1 General Structure

In each period $t \in \mathbb{Z}_+$, a new generation of homogeneous two-period-lived agents are born. They are called young in the first period of their life, and old in the second period. There is no population growth, and the population of each generation is normalized to one. There is a single consumption good, and each agent is endowed with $e_1 > 0$ units of the good when young, and $e_2 > 0$ units when old. There is also an intrinsically useless asset that agents can buy when young, and sell when old. We regard this asset as a bubble whenever its market price is strictly positive.

Each agent born in period $t \in \mathbb{Z}_+$ solves the following maximization problem:

$$\max_{c_t, x_t, d_{t+1} \geq 0} \quad u(c_t) + \mathbb{E}_t v(d_{t+1}) \tag{2.1}$$

$$\text{s.t.} \quad c_t + b_t x_t = e_1, \tag{2.2}$$

$$d_{t+1} = e_2 + b_{t+1} x_t, \tag{2.3}$$

where c_t is consumption when young, d_{t+1} is consumption when old, $u, v : \mathbb{R}_+ \to [-\infty, \infty)$ are the utility functions for the first and second periods, respectively, b_t is the price of the asset, x_t is asset holdings at the end of period t as well as at the beginning of period $t+1$, and $\mathbb{E}_t$ is the expectation conditional on the information set in period t (to be specified below).[3] An old agent in period 0 simply consumes all his wealth:

$$d_0 = e_2 + b_0 x_{-1}. \tag{2.4}$$

The market-clearing conditions for the consumption good and the asset are as follows:

$$c_t + d_t = e_1 + e_2, \qquad \forall t \in \mathbb{Z}_+, \tag{2.5}$$

$$x_t = 1, \qquad \forall t \in \mathbb{Z}_+. \tag{2.6}$$

Throughout the paper we assume the following.

[3]Formally, let $(\Omega, \mathscr{F}, P)$ be a probability space, and let $\{\mathscr{F}_t\}_{t \in \mathbb{Z}_+}$ be a filtration. The conditional expectation $\mathbb{E}_t$ at time t is defined as the expectation conditional on $\mathscr{F}_t$. All stochastic processes indexed by $t \in \mathbb{Z}_+$, including the sunspot processes defined below, are assumed to be adapted to this filtration.

Assumption 2.1. $u, v : \mathbb{R}_+ \to [-\infty, \infty)$ are continuous, C^1 on $(0, \infty)$, and strictly increasing. Furthermore

$$\lim_{c \downarrow 0} u'(c) = \infty. \tag{2.7}$$

Given $x_{-1} = 1$, an *equilibrium* of this economy is defined as a set of nonnegative stochastic processes $\{c_t, d_t, x_t, b_t\}_{t \in \mathbb{Z}_+}$ such that (i) the pair (d_0, x_{-1}) satisfies (2.4), (ii) for each $t \in \mathbb{Z}_+$, the triple (c_t, x_t, d_{t+1}) solves the maximization problem (2.1)–(2.3), and (iii) the market-clearing conditions (2.5) and (2.6) hold.

The following result shows that the equilibria of this economy are characterized by the Euler equation for the maximization problem (2.1)–(2.3) along with the budget constraints and the market-clearing conditions.

Lemma 2.1. *A set of nonnegative stochastic processes* $\{c_t, d_t, x_t, b_t\}_{t \in \mathbb{Z}_+}$ *is an equilibrium if and only if it satisfies (2.2)–(2.6) and*

$$e_1 - b_t > 0, \qquad \forall t \in \mathbb{Z}_+. \tag{2.8}$$

$$u'(e_1 - b_t) b_t = \mathbb{E}_t v'(e_2 + b_{t+1}) b_{t+1}, \qquad \forall t \in \mathbb{Z}_+. \tag{2.9}$$

We call a nonnegative stochastic process $\{b_t\}_{t \in \mathbb{Z}_+}$ an *equilibrium bubble process* if there exist nonnegative stochastic processes $\{c_t, x_t, d_t\}_{t \in \mathbb{Z}_+}$ such that $\{c_t, x_t, d_t, b_t\}$ is an equilibrium. The following result characterizes equilibrium bubble processes.

Lemma 2.2. *A nonnegative stochastic process* $\{b_t\}_{t \in \mathbb{Z}_+}$ *is an equilibrium bubble process if and only if it satisfies (2.8) and (2.9).*

Proof. The "only if" part follows from Lemma 2.1. To see the "if" part, define $\{x_t\}$ by (2.6). Define $\{c_t\}$ and $\{d_t\}$ using (2.2)–(2.4). Then (2.5) holds. Since we already have (2.8) and (2.9), it follows by Lemma 2.1 that $\{c_t, d_t, x_t, b_t\}$ is an equilibrium. □

2.2 Stationary Sunspot Equilibria

To define stationary sunspot equilibria of the type studied by Weil (1987), we assume that there is a two-state sunspot process $\{s_t\}_{t \in \mathbb{Z}_+}$ obeying the following:

$$s_t = 0 \quad \Rightarrow \quad s_{t+1} = \begin{cases} 0 & \text{with probability } q, \\ 1 & \text{with probability } 1 - q, \end{cases} \tag{2.10}$$

$$s_t = 1 \quad \Rightarrow \quad s_{t+1} = 1, \tag{2.11}$$

where $q \in (0, 1)$ is a constant. Following Weil (1987), we can interpret state 0 as meaning "no sunspot" and 1 as "sunspots," though this interpretation is not necessary for our results.

Given the sunspot process defined by (2.10) and (2.11), consider an equilibrium $\{c_t, d_t, x_t, b_t\}_{t \in \mathbb{Z}_+}$ such that for some constant $b > 0$, we have

$$b_t = \begin{cases} b & \text{if } s_t = 0, \\ 0 & \text{if } s_t = 1. \end{cases} \tag{2.12}$$

An equilibrium of this type is often called a *stationary sunspot equilibrium.* Equation (2.12) means that the bubble b_t is constant until a sunspot shock occurs (i.e., s_t switches from 0 to 1), when it collapses to zero and never reappears.

Note from (2.2), (2.3), and (2.6) that under (2.12) we have

$$s_t = 0 \quad \Rightarrow \quad \begin{cases} c_t = e_1 - b, \\ d_t = e_2 + b, \end{cases} \tag{2.13}$$

$$s_t = 1 \quad \Rightarrow \quad \begin{cases} c_t = e_1, \\ d_t = e_2. \end{cases} \tag{2.14}$$

It follows from (2.9) and (2.12) that

$$u'(e_1 - b) = qv'(e_2 + b). \tag{2.15}$$

It is easy to see that this equation has a solution $b^* > 0$ if and only if

$$q > u'(e_1)/v'(e_2). \tag{2.16}$$

The solution is unique by strict concavity of u and v.

The preceeding discussion together with Lemma 2.2 establishes the following result.

Proposition 2.1. *There exists an equilibrium bubble process $\{b_t\}$ satisfying (2.12) if and only if (2.16) holds. Under (2.16), any equilibrium bubble process $\{b_t\}$ satisfying (2.12) satisfies $b = b^*$, where b^* is the unique solution to (2.15).*

Essentially the same result is shown by Weil (1987, Proposition 1). We refer to Weil (1987) for further discussion of asset bubbles in this closed economy.

3 The Two-Country Model

Consider a world economy consisting of two countries indexed by $i \in \{H, F\}$, where H and F stand for "home" and "foreign," respectively. There is a single consumption good worldwide, while there is an intrinsically useless asset in each country. The markets for the consumption good and the assets are fully integrated internationally, and the countries are entirely symmetric in terms of fundamentals.

Each agent born in period $t \in \mathbb{Z}_+$ in country $i \in \{H, F\}$ solves the following maximization problem:

$$\max_{c_t^i, x_t^{i,i}, x_t^{i,j}, d_{t+1}^i \geq 0} \quad \mathbb{E}_t[u(c_t^i) + v(d_{t+1}^i)] \tag{3.1}$$

$$\text{s.t.} \quad c_t^i + b_t^i x_t^{i,i} + b_t^j x_t^{i,j} = e_1, \tag{3.2}$$

$$d_{t+1}^i = e_2 + b_{t+1}^i x_t^{i,i} + b_{t+1}^j x_t^{i,j}, \tag{3.3}$$

where c_t^i is consumption when young, d_{t+1}^i is consumption when old, b_t^k with $k \in \{H, F\}$ is the price of the asset in country k, $x_t^{i,k}$ with $k \in \{H, F\}$ is holdings of the asset in country k, and j is given by $j \in \{H, F\}$ with $j \neq i$; unless otherwise specified, we maintain this definition of j whenever $i \in \{H, F\}$ is given. An old agent in period 0 in country $i \in \{H, F\}$ simply consumes all his wealth:

$$d_0^i = e_2 + b_0^i x_{-1}^{i,i} + b_0^j x_{-1}^{i,j}. \tag{3.4}$$

The market-clearing condition for the consumption good is

$$c_t^H + d_t^H + c_t^F + d_t^F = 2e_1 + 2e_2, \qquad \forall t \in \mathbb{Z}_+. \tag{3.5}$$

Since each asset is intrinsically useless, its supply can be set to any value without significantly affecting the analysis. For convenience we normalize the supply of each asset to 2. Thus the market-clearing condition for the asset in country $k \in \{H, F\}$ is

$$x_t^{H,k} + x_t^{F,k} = 2, \qquad \forall t \in \mathbb{Z}_+. \tag{3.6}$$

Given $x_{-1}^{i,k} \geq 0$ for $i, k \in \{H, F\}$ such that $x_{-1}^{H,H} + x_{-1}^{F,H} = x_{-1}^{H,F} + x_{-1}^{F,F} = 2$, an *equilibrium* of this two-country economy is defined as a set of nonnegative stochastic processes $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}_{t \in \mathbb{Z}_+, i \in \{H,F\}}$ such that (i) for each

$i \in \{H, F\}$, the triple $(d_0^i, x_{-1}^{i,i}, x_{-1}^{i,j})$ satisfies (3.4), (ii) for each $t \in \mathbb{Z}_t$ and $i \in \{H, F\}$, the quadruple $(c_t^i, x_t^{i,i}, x_t^{i,j}, d_{t+1}^i)$ solves the maximization problem (3.1)–(3.3), and (iii) the market-clearing conditions (3.5) and (3.6) hold.

Before we turn to stationary sunspot equilibria, it is useful to exploit the implications of the symmetry of the countries:[4]

Lemma 3.1. *A set of nonnegative stochastic processes* $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}$ *satisfying (3.2)–(3.4) and (3.6) is an equilibrium if and only if for all* $t \in \mathbb{Z}_+$ *we have*

$$c_t^H = c_t^F = e_1 - (b_t^H + b_t^F) > 0, \tag{3.7}$$

$$d_{t+1}^H = d_{t+1}^F = e_2 + (b_{t+1}^H + b_{t+1}^F), \tag{3.8}$$

$$u'(c_t^i) b_t^k = \mathbb{E}_t v'(d_{t+1}^i) b_{t+1}^k, \qquad \forall i, k \in \{H, F\}. \tag{3.9}$$

For an equilibrium $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}$ such that the bubble processes $\{b_t^H\}$ and $\{b_t^F\}$ are not perfectly correlated for any $t \in \mathbb{Z}_+$, it is possible to show that $x_t^{H,k} = x_t^{F,k} = 1$ for each $k \in \{H, F\}$. However, since the bubble processes can even be identical, the equilibrium values of $x_t^{i,k}$ for $i, k \in \{H, F\}$ are in general indeterminate. For example, there is an equilibrium with $b_t^H = b_t^F = 0$ for all $t \in \mathbb{Z}_+$, in which case the values of $x_t^{i,k}$ are essentially irrelevant. Another case in point is an equilibrium in which both $\{b_t^H\}$ and $\{b_t^F\}$ follow an identical stochastic process such as (2.12); in this case the distinction between $x_t^{i,H}$ and $x_t^{i,F}$ is irrelevant for agents. Since we are primarily interested in equilibrium bubble processes, we do not seek to obtain a full characterization of asset holdings $x_t^{i,k}$.

We say that a pair of nonnegative stochastic processes $\{b_t^H, b_t^F\}_{t \in \mathbb{Z}_+}$ is a *bivariate equilibrium bubble process* if there exist nonnegative stochastic processes $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F},\}$ such that $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}$ is an equilibrium. We close this section by showing a result that characterizes bivariate equilibrium bubble processes. The following result is a two-country version of Lemma 2.1.

Proposition 3.1. *A bivariate nonnegative stochastic processe* $\{b_t^H, b_t^F\}$ *is a bivariate equilibrium bubble process if and only if for all* $t \in \mathbb{Z}_+$ *we have*

$$b_t^H + b_t^F < e_1, \tag{3.10}$$

$$u'(e_1 - (b_t^H + b_t^F)) b_t^k = \mathbb{E}_t v'(e_2 + b_{t+1}^H + b_{t+1}^F) b_{t+1}^k, \quad \forall k \in \{H, F\}. \tag{3.11}$$

[4]The same type of symmetric structure is exploited by Manuelli and Peck (1990).

For the rest of the paper, we say that a bivariate nonnegative process $\{b_t^H, b_t^F\}$ satisfies (3.11) if it in fact satisfies both (3.10) and (3.11). This convention applies to various versions of (3.11).

4 Sunspots Only in the Foreign Country

In this section we assume that there is a sunspot process obeying (2.10) and (2.11) only in the foreign country; there is no other source of uncertainty. We consider a stationary sunspot equilibrium such that

$$b_t^F = \begin{cases} b^F & \text{if } s_t = 0, \\ 0 & \text{if } s_t = 1, \end{cases} \tag{4.1}$$

$$b_t^H = \begin{cases} b^H & \text{if } s_t = 0, \\ \tilde{b}^H & \text{if } s_t = 1. \end{cases} \tag{4.2}$$

This is the simplest type of equilibrium in which the bursting of the foreign bubble affects the home bubble. Equation (4.1) means that the foreign bubble bursts when a sunspot shock occurs. The question is: How does this event affect the home bubble? In (4.2), the home bubble is not forced to react to this event since it is possible that $b^H = \tilde{b}^H$, in which case the home bubble is not affected by the bursting of the foreign bubble at all. If $0 < \tilde{b}^H < b^H$, then the home bubble partially collapses in response to the bursting of the foreign bubble. It is even possible that $b^H < \tilde{b}^H$, in which case the home bubble becomes larger when the foreign bubble collapses to zero.

To avoid trivial cases, in this section we assume the following unless otherwise indicated:

$$b^H > 0, \quad b^F > 0. \tag{4.3}$$

A stationary sunspot equilibrium here consists of three nonnegative reals b^H, b^F, and $\tilde{b}^H$. We say that a triple $(b^H, b^F, \tilde{b}^H) \in \mathbb{R}^2_{++} \times \mathbb{R}_+$ is a *stationary sunspot equilibrium of the form (4.1)–(4.3)* if the bivariate nonnegative process $\{b_t^H, b_t^F\}$ given by (4.1)–(4.2) is a bivariate equilibrium bubble process. By Proposition 3.1, a stationary sunspot equilibrium $(b^H, b^F, \tilde{b}^H)$ of the form (4.1)–(4.3) satisfies

$$u'(e_1 - b^W)b^F = qv'(e_2 + b^W)b^F, \tag{4.4}$$

$$u'(e_1 - b^W)b^H = qv'(e_2 + b^W)b^H + (1-q)v'(e_2 + \tilde{b}^H)\tilde{b}^H, \tag{4.5}$$

where b^W is the world bubble:

$$b^W = b^H + b^F. \tag{4.6}$$

The following result shows that the home bubble inevitably bursts when the foreign bubble bursts, as long as both bubbles are strictly positive until a sunspot shock occurs.

Proposition 4.1. *A triple* $(b^H, b^F, \tilde{b}^H) \in \mathbb{R}^2_{++} \times \mathbb{R}_+$ *is a stationary sunspot equilibrium of the form (4.1)–(4.3) if and only if it satisfies (4.4) and*

$$\tilde{b}^H = 0. \tag{4.7}$$

While Proposition 4.1 characterizes stationary sunspot equilibria of the form (4.1)–(4.3), the following result provides a necessary and sufficient condition for existence of a stationary sunspot equilibrium of the same form.

Proposition 4.2. *There exists a stationary sunspot equilibrium of the form (4.1)–(4.3) if and only if (2.16) holds. Under (2.16), there exist a continuum of stationary sunspot equilibria of the form (4.1)–(4.3). Specifically, under (2.16), for any* $b^H, b^F > 0$ *satisfying*

$$b^H + b^F = b^*, \tag{4.8}$$

there exists a unique stationary sunspot equilibrium $(b^H, b^F, \tilde{b}^H)$ *of the form (4.1)–(4.3) (with* $\tilde{b}^H = 0$*).*

Note that (4.7) means that the home bubble bursts when the foreign bubble bursts. In other words, both bubbles burst simultaneously. Hence even though there are two bubbles in this economy, there is effectively only one bubble under (4.7). Not surprisingly, the sum of the two bubbles must be equal to b^*, which indicates that the economy here is effectively equivalent to the closed economy studied in Subsection 2.2.

Although it is rather straightforward to show Proposition 4.1 based on (4.4) and (4.5) (as in the proof in Appendix A.4), one can easily derive (4.7) using a simple arbitrage argument as follows.[5] Note from (4.1) and (4.2) that the stochastic return on the foreign bubble is 0 if a sunspot shock occurs, and is 1 otherwise. The return on the home bubble is also 1 if a sunspot

[5] We thank an anonymous referee for suggesting the following argument.

shock does not occur. If the return on the home bubble were strictly positive in the other event, then the home bubble would be a dominating asset. But since both bubbles are held here, the return on the home bubble must also be equal to 0 if a sunspot shock occurs.

Figure 1 shows three examples of bivariate equilibrium bubble processes generated by stationary sunspot equilibria of the form (4.1)–(4.3).[6] In all three cases, the bursting of the foreign bubble is accompanied by that of the home bubble. In particular, in panel (c), the foreign bubble is considerably smaller than the home bubble before they burst. As discussed above, no-arbitrage requires only the rates of returns on the two bubbles to be equated, but has no bearing on their levels.

On the other hand, if the foreign bubble is not valued at all, then the home bubble is not forced to burst even when a sunspot shock occurs in the foreign country.

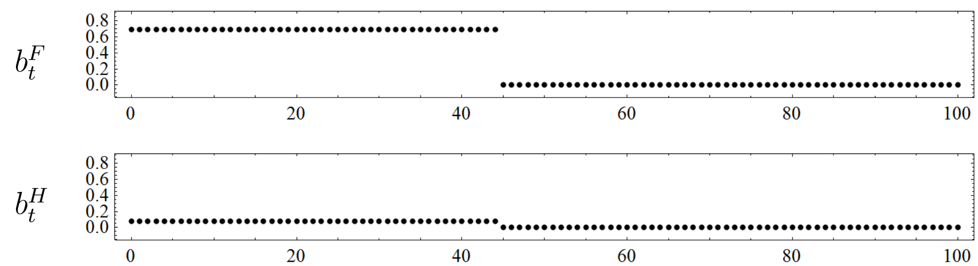
Proposition 4.3. *Under (2.16), there exists a bivariate equilibrium bubble process* $\{b_t^H, b_t^F\}$ *such that* $b_t^F = 0$ *and* $b_t^H = b > b^*$*for all* $t \in \mathbb{Z}_+$ *for some constant* $b > 0$*.*

Proof. Since $1 > q > u'(e_1)/v'(e_2)$ by (2.16), there exists a unique solution $b \in (0, e_1)$ to $u'(e_1 - b) = v'(e_2 + b)$. Since $q < 1$, it is easy to see that $b > b^*$ (recall that b^* solves (2.15)). With $\{b_t^H, b_t^F\}$ defined as above, this bivariate process satisfies (3.11). Thus it is a bivariate equilibrium bubble process by Proposition 3.1. □

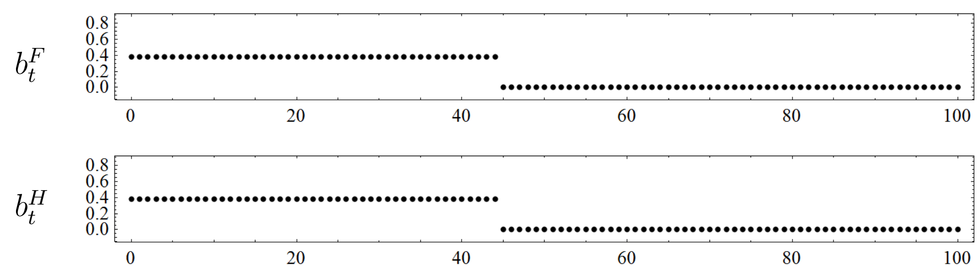
Figure 2 illustrates the bivariate equilibrium bubble process given by Proposition 4.3. The process here is deterministic, and the home bubble never bursts even if a sunspot shock occurs in the foreign country. In this sense, a sunspot shock in the foreign country does not force the home bubble to burst.

It should be mentioned that the bivariate equilibrium bubble process in Figure 2 cannot be obtained as the limit of bivariate equilibrium bubble processes of the type depicted in Figure 1 by letting b^F go to zero. Indeed, if we let b^F go to zero in Figure 1, the limiting process is still stochastic. Figure 3 depicts the limiting bivariate equilibrium bubble process obtained this way, indicating that the home bubble can still burst in response to a

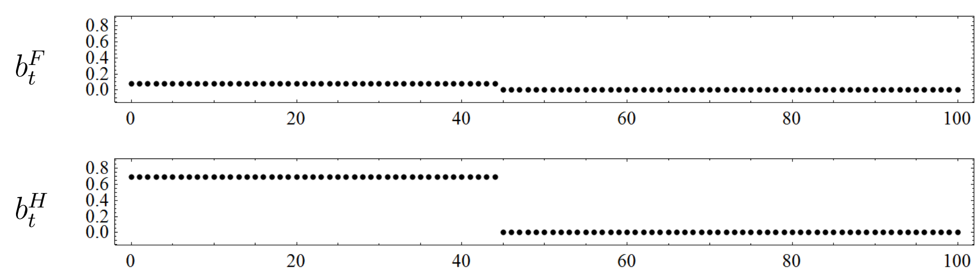
[6] All numerical examples of bivariate equilibrium bubble processes in this paper are computed by assuming that $u(c) = \ln c, v(c) = 0.8 \ln d$, and $q = 0.99$.



(a) $b^F = 0.9b^*$, $b^H = 0.1b^*$



(b) $b^F = 0.5b^*$, $b^H = 0.5b^*$



(c) $b^F = 0.1b^*$, $b^H = 0.9b^*$

Figure 1: Bivariate equilibrium bubble processes satisfying (4.1)–(4.8)

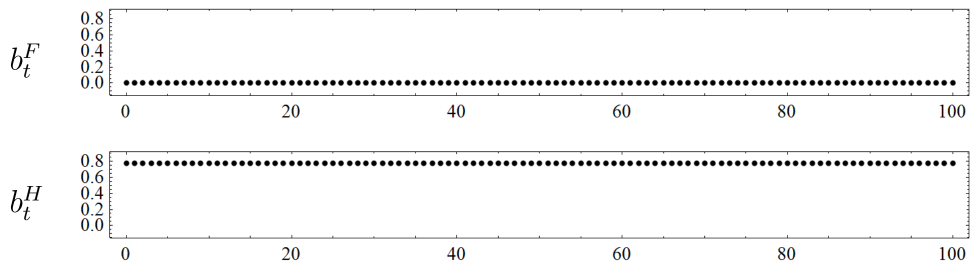


Figure 2: The deterministic bivariate equilibrium bubble process given by Proposition 4.3

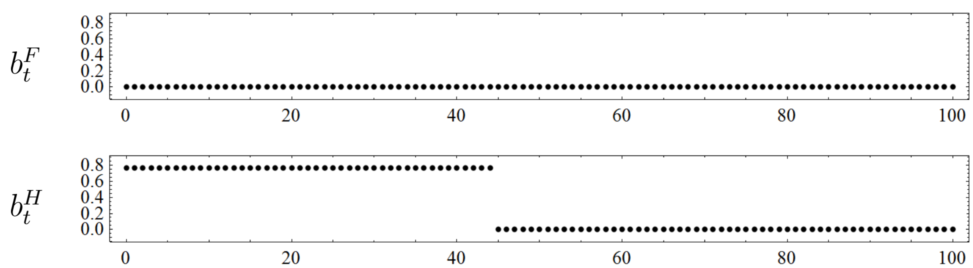


Figure 3: The limiting bivariate equilibrium bubble process obtained by letting b^F go to zero in Figure 1 ($b^F = 0$ and $b^H = b^*$)

sunspot shock in the foreign country even if the foreign bubble is not valued at all.

5 Sunspots in Both Countries

In this section we assume that both countries receive sunspot shocks. Let $\{s_t^H\}$ and $\{s_t^F\}$ be sunspot processes in the home and foreign countries, respectively. For each $i \in \{H, F\}$, we assume that $\{s_t^i\}$ follows the following process:

$$s_t^i = 0 \quad \Rightarrow \quad s_{t+1}^i = \begin{cases} 0 & \text{with probability } q^i, \\ 1 & \text{with probability } 1 - q^i, \end{cases} \tag{5.1}$$

$$s_t^i = 1 \quad \Rightarrow \quad s_{t+1}^i = 1, \tag{5.2}$$

where $q^i \in (0, 1)$ is a constant. We also assume that the two sunspot processes are independent. If this were not the case, bubble crashes in the two coun-

b_t^H, b_t^F

s_t^H \ s_t^F	0	1
0	b^H, b^F	$\tilde{b}^H, 0$
1	$0, \tilde{b}^F$	0, 0

Table 1: Dependence of (b_t^H, b_t^F) on (s_t^H, s_t^F)

tries would naturally be correlated, and the transmission of bubble crashes could be a consequence of this correlation. We rule out such an exogenous correlation by assuming independent sunspot processes.

We consider stationary sunspot equilibria of the following form:

$$b_t^F = \begin{cases} b^F & \text{if } s_t^H = s_t^F = 0, \\ \tilde{b}^F & \text{if } s_t^H = 1 \text{ and } s_t^F = 0, \\ 0 & \text{if } s_t^F = 1, \end{cases} \tag{5.3}$$

$$b_t^H = \begin{cases} b^H & \text{if } s_t^H = s_t^F = 0, \\ \tilde{b}^H & \text{if } s_t^F = 1 \text{ and } s_t^H = 0, \\ 0 & \text{if } s_t^H = 1. \end{cases} \tag{5.4}$$

Table 1 summarizes how the pair (b_t^H, b_t^F) depends on (s_t^H, s_t^F). As in the previous case, we do not require the home bubble to react to the bursting of the foreign bubble since it is possible that $b^H = \tilde{b}^H$. Likewise, the foreign bubble is not required to react to the bursting of the home bubble.

To focus on nontrivial cases, we assume the following inequalities for the rest of the paper:

$$b^H > 0, \quad b^F > 0. \tag{5.5}$$

A stationary sunspot equilibrium here consists of four nonnegative reals $b^H, b^F, \tilde{b}^H$, and $\tilde{b}^F$. We say that a quadruple $(b^H, b^F, \tilde{b}^H, \tilde{b}^F) \in \mathbb{R}_{++}^2 \times \mathbb{R}_+^2$ is a *stationary sunspot equilibrium of the form (5.3)–(5.5)* if a bivariate stochastic process $\{b_t^H, b_t^F\}$ satisfying (5.3)–(5.5) is a bivariate equilibrium bubble process.

By Proposition 3.1, a quadruple $(b^H, b^F, \tilde{b}^H, \tilde{b}^F) \in \mathbb{R}^2_{++} \times \mathbb{R}^2_+$ is a stationary sunspot equilibrium of the form (5.3)–(5.5) if and only if

$$u'(c_{0,0})b^H = q^H q^F v'(d_{0,0})b^H + q^H(1 - q^F)v'(d_{0,1})\tilde{b}^H, \tag{5.6}$$

$$u'(c_{0,0})b^F = q^F q^H v'(d_{0,0})b^F + q^F(1 - q^H)v'(d_{1,0})\tilde{b}^F, \tag{5.7}$$

$$u'(c_{0,1})\tilde{b}^H = q^H v'(d_{0,1})\tilde{b}^H, \tag{5.8}$$

$$u'(c_{1,0})\tilde{b}^F = q^F v'(d_{1,0})\tilde{b}^F, \tag{5.9}$$

where $c_{i,j}$ is the consumption of a young agent and $d_{i,j}$ is the consumption of an old agent in state $(i, j) = (s^H_t, s^F_t) \in \{(0, 0), (0, 1), (1, 0)\}$:

$$c_{0,0} = e_1 - b^W, \qquad d_{0,0} = e_2 + b^W, \tag{5.10}$$

$$c_{0,1} = e_1 - \tilde{b}^H, \qquad d_{0,1} = e_2 + \tilde{b}^H, \tag{5.11}$$

$$c_{1,0} = e_1 - \tilde{b}^F, \qquad d_{1,0} = e_2 + \tilde{b}^F. \tag{5.12}$$

The following result shows that there are exactly two types of stationary sunspot equilibria of the form (5.3)–(5.5).

Proposition 5.1. *Let $(b^H, b^F, \tilde{b}^H, \tilde{b}^F) \in \mathbb{R}^2_{++} \times \mathbb{R}^2_+$ be given.*

(a) *If $\tilde{b}^H = 0$, then $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (5.3)–(5.5) if and only if $\tilde{b}^F = 0$ and*

$$u'(e_1 - b^W) = q^H q^F v'(e_2 + b^W), \tag{5.13}$$

where

$$b^W = b^H + b^F. \tag{5.14}$$

(b) *Suppose that $\tilde{b}^H > 0$. Then $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (5.3)–(5.5) if and only if $\tilde{b}^F > 0$ and the following equations hold:*

$$u'(e_1 - \tilde{b}^k) = q^k v'(e_2 + \tilde{b}^k), \quad \forall k \in \{H, F\}, \tag{5.15}$$

$$u'(e_1 - b^W)b^W = q^H q^F v'(e_2 + b^W)b^W$$
$$+ q^H(1 - q^F)v'(e_2 + \tilde{b}^H)\tilde{b}^H + q^F(1 - q^H)v'(e_2 + \tilde{b}^F)\tilde{b}^F, \tag{5.16}$$

$$\frac{b^H}{b^F} = \frac{(1 - q^F)u'(e_1 - \tilde{b}^H)\tilde{b}^H}{(1 - q^H)u'(e_1 - \tilde{b}^F)\tilde{b}^F}. \tag{5.17}$$

Furthermore, if $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ *is a stationary sunspot equilibrium of the form (5.3)–(5.5), then*

$$b^H < \tilde{b}^H, \quad b^F < \tilde{b}^F, \tag{5.18}$$

$$q^H \lesseqgtr q^F \quad \Rightarrow \quad b^H \lesseqgtr b^F, \quad \tilde{b}^H \lesseqgtr \tilde{b}^F. \tag{5.19}$$

It is easy to see that there exists a unique b^W satisfying (5.13); thus (5.13) uniquely determines the world bubble b^W in case (a). It is also easy to see that there exists a unique $\tilde{b}^k$ solving (5.15) for each $k \in \{H, K\}$. Given $\tilde{b}^H$ and $\tilde{b}^F$ solving (5.15), it can be shown that there exists a unique b^W satisfying (5.16). Then b^H and b^F can be determined by (5.17) and (5.14). Thus (5.15)–(5.17) uniquely determine a stationary sunspot equilibrium $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ in case (b).

This stationary sunspot equilibrium has the additional properties expressed in (5.18) and (5.19). For example, a country with a lower probability of a sunspot shock has a larger bubble; see (5.19). We comment on the other aspects of Proposition 5.1 after presenting the next result, which provides necessary and sufficient conditions for existence of the two types of stationary sunspot equilibria of the form (5.3)–(5.5) classified above.

Proposition 5.2. *There exists a stationary sunspot equilibrium* $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ *of the form (5.3)–(5.5) if and only if*

$$\min\{q^H, q^F\} > u'(e_1)/v'(e_2). \tag{5.20}$$

More specifically, we have the following resutls:

(a) *There exists a stationary sunspot equilibrium* $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ *of the form (5.3)–(5.5) with* $\tilde{b}^H = 0$ *if and only if*

$$q^H q^F > u'(e_1)/v'(e_2). \tag{5.21}$$

Under (5.21), given b^W *satisfying (5.13), there exist a continuum of stationary sunspot equilibria* $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ *of the form (5.3)–(5.5) with* $\tilde{b}^H = 0$*. In particular, for any* $b^H, b^F > 0$ *satisfying (5.14), the quadruple* $(b^H, b^F, 0, 0)$ *is a stationary sunspot equilibrium of the form (5.3)–(5.5) with* $\tilde{b}^H = 0$*.*

(b) *There exists a stationary sunspot equilibrium* $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ *with* $\tilde{b}^H > 0$ *if and only if (5.20) holds. Under (5.20), there exists a unique stationary sunspot equilibrium* $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ *of the form (5.3)–(5.5) with* $\tilde{b}^H > 0$*.*

See Figure 4 for examples of bivariate equilibrium bubble processes generated by the above two types of stationary sunspot equilibria. The bivariate equilibrium bubble process in panel (a) of Figure 4 is similar to that in panel (b) of Figure 1. There are in fact a whole range of equilibrium bubble processes here as in Figure 1.

Part (a) of Proposition 5.1 is similar to Proposition 4.1. To see this more clearly, define

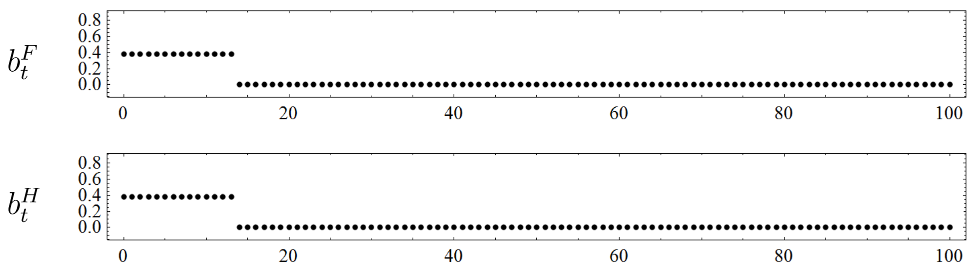
$$s_t = \begin{cases} 0 & \text{if } s_t^F = s_t^H = 0, \\ 1 & \text{otherwise.} \end{cases} \tag{5.22}$$

Then with $\tilde{b}^H = \tilde{b}^F = 0$, a bivariate equilibrium bubble process given by part (a) of Proposition 5.1 is equivalent to that given by Proposition 4.1 with $q = q^H q^F$. Indeed, (5.13) is equivalent to (2.15) with $q = q^H q^F$ and $b = b^W$. Part (a) of Proposition 5.2 is similar to Proposition 4.2. In an appropriate sense, the existence of a continuum of stationary sunspot equilibria here is inherited from the case in which there are sunspots only in the foreign country.

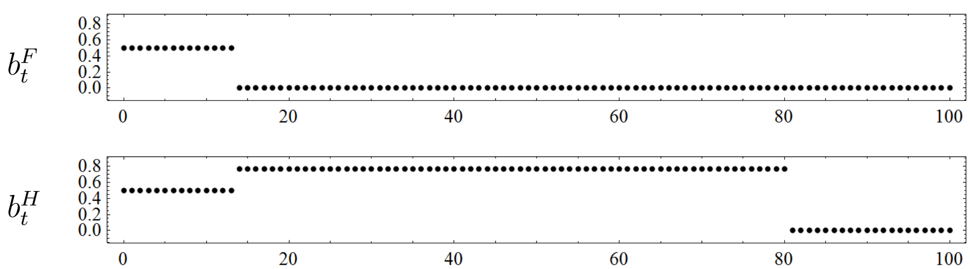
Part (b) of Proposition 5.1 shows that there is another type of stationary sunspot equilibrium, which is specific to the current setting. This equilibrium has the property that when the bubble in either country bursts, then the bubble in the other country jumps to a higher level; see (5.18). For example, the bursting of the foreign bubble has a positive effect on the home bubble; see Panel (b) of Figure 4 (which also shows that the home bubble bursts later on its own when a sunspot shock occurs in the home country). Intuitively, after the collapse of the foreign bubble, the home bubble jumps to a higher level in order to absorb the entire world's wealth.[7]

Note that condition (5.21) implies (5.20), but not vice versa. This means that for some values of q^H and q^F, only a stationary sunspot equilibrium with $\tilde{b}^H > 0$ exists. For example, this is the case if $q^H q^F = u'(e_1)/v'(e_2)$. More generally, if q^H and q^F satisfy (5.20) but each of them is sufficiently close to $u'(e_1)/v'(e_2)$, then (5.21) is violated. In other words, if sunspot shocks are sufficiently likely in both countries while maintaining (5.20), then the only possible stationary sunspot equilibrium of the form (5.3)–(5.5) follows the pattern of panel (b) in Figure 4.

[7] See Caballero (2006) for discussion of related issues.



(a) $b^H = b^H > \tilde{b}^H = \tilde{b}^F = 0$



(b) $b^H = b^F < \tilde{b}^H = \tilde{b}^F = b^*$

Figure 4: Bivariate equilibrium bubble processes satisfying (5.3) and (5.5) with $q^H = q^F$

6 Local Stability Analysis

The results in the preceding section show that there is only one locally isolated stationary sunspot equilibrium under our settings. In this section we focus on the local dynamics around this equilibrium. Any other stationary sunspot equilibrium shown to exist in the preceding section belongs to a continuum of stationary sunspot equilibria, and is not amenable to standard linearization techniques.

To analyze the local stability properties of the locally isolated stationary sunspot equilibrium $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ of the type (5.3)–(5.5), it is necessary to construct a dynamical system of which $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a steady state. One way is to allow these parameters to depend on t in (5.3) and (5.4) as follows:

$$b_t^F = \begin{cases} \beta_t^F & \text{if } s_t^H = s_t^F = 0, \\ \tilde{\beta}_t^F & \text{if } s_t^H = 1 \text{ and } s_t^F = 0, \\ 0 & \text{if } s_t^F = 1, \end{cases} \tag{6.1}$$

$$b_t^H = \begin{cases} \beta_t^H & \text{if } s_t^H = s_t^F = 0 \ , \\ \tilde{\beta}_t^H & \text{if } s_t^H = 0 \text{ and } s_t^F = 1, \\ 0 & \text{if } s_t^H = 1. \end{cases} \tag{6.2}$$

Here we use $\beta_t^F, \tilde{\beta}_t^F, \beta_t^H$, and $\tilde{\beta}_t^H$ to denote time-dependent versions of $b^F, \tilde{b}^F$, b^H, and $\tilde{b}^H$ to avoid confusion with equilibrium bubbles processes b_t^H and b_t^F.

By Proposition 3.1, a bivariate nonnegative process $\{b_t^H, b_t^F\}$ given by (6.1) and (6.2) is a bivariate equilibrium bubble process if

$$\begin{aligned} u'(e_1 - \beta_t^W)\beta_t^H &= q^H q^F v'(e_2 + \beta_{t+1}^W)\beta_{t+1}^H \\ &\quad + q^H(1-q^F)v'(e_2 + \tilde{\beta}_{t+1}^H)\tilde{\beta}_{t+1}^H, \end{aligned} \tag{6.3}$$

$$\begin{aligned} u'(e_1 - \beta_t^W)\beta_t^F &= q^F q^H v'(e_2 + \beta_{t+1}^W)\beta_{t+1}^F \\ &\quad + q^F(1-q^H)v'(e_2 + \tilde{\beta}_{t+1}^F)\tilde{\beta}_{t+1}^F, \end{aligned} \tag{6.4}$$

$$u'(e_1 - \tilde{\beta}_t^H)\tilde{\beta}_t^H = q^H v'(e_2 + \tilde{\beta}_{t+1}^H)\tilde{\beta}_{t+1}^H, \tag{6.5}$$

$$u'(e_1 - \tilde{\beta}_t^F)\tilde{\beta}_t^F = q^F v'(e_2 + \tilde{\beta}_{t+1}^F)\tilde{\beta}_{t+1}^F, \tag{6.6}$$

where

$$\beta_t^W = \beta_t^H + \beta_t^H. \tag{6.7}$$

The above system of equations (6.3)–(6.6) governs the deterministic dynamics of the quadruple $(\beta_t^H, \beta_t^F, \tilde{\beta}_t^H, \tilde{\beta}_t^F)$. In view of (5.6)–(5.9) and (6.3)–(6.6), it is easy to see that the stationary sunspot equilibrium $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a steady state of the dynamical system (6.3)–(6.6).[8]

The following result shows that the stationary sunspot equilibrium $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ as a steady state of the dynamical system is a source.

Proposition 6.1. *Assume (5.20). Suppose that*

$$\forall d \in (e_2, e_1 + e_2), \quad -dv''(d)/v'(d) \leq 1. \tag{6.8}$$

Then $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a source of the dynamical system (6.5)–(6.6), and thus is locally unstable.

Condition (6.8) means that the degree of concavity of v is relatively mild. This condition is only a sufficient condition for the above conclusion, which is obtained analytically. If v takes the CRRA form $v(d) = (d^{1-\alpha} - 1)/(1-\alpha)$, then (6.8) means that $\alpha \leq 1$.

Under this condition, Proposition 6.1 shows that the stationary sunspot equilibrium $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a source. Hence to achieve this equilibrium, the economy must initially jump to this point. This is a rather common feature in rational expectations models. For example, the stationary sunspot equilibrium analyzed by Weil (19878) has the same property.

7 Concluding Remarks

In this paper we studied the international transmission of bubble crashes by analyzing stationary sunspot equilibria in a two-country version of Weil's (1987) overlapping generations exchange economy with stochastic bubbles. We considered two types of sunspot shocks. In the first case, we assume that only the foreign country receives a sunspot shock, and that the foreign bubble bursts when a sunspot shock occurs. We showed that in this case, the home bubble inevitably bursts when the foreign bubble bursts. In the second case, we assume that both countries independently receive sunspot shocks. We showed that in this case, if the foreign bubble bursts, then the

[8] Equations (6.3)–(6.6) are sufficient, but not necessary, for the bivariate nonnegative process $\{b_t^H, b_t^F\}$ given by (6.1) and (6.2) to be a bivariate equilibrium bubble process. This is because (6.5) and (6.6) are not required to hold before a sunspot shock occurs.

home bubble either bursts simultaneously or jumps to a higher level. We also showed that there exists a unique locally isolated stationary sunspot equilibrium, and that it is locally unstable.

There are several ways to extend our analysis by relaxing some of our assumptions. First, we assumed that the countries are symmetric in terms of fundamentals. Since this assumption greatly simplified the analysis, dropping it would be a nontrivial extension. However, it is important to consider cases in which the countries are asymmetric in various aspects such as preferences, endowments, and population because in reality, no two countries are identical in terms of fundamentals.

Second, we assumed that the probability of a sunspot shock is exogenous and constant over time in each country. There are various ways to relax this assumption. For example, one may assume that the probability changes over time depending on endogenous variables, or even follows a stochastic process. Such extensions seem to be fairly easy to accommodate especially when the countries are assumed to be symmetric in all the other aspects. One may also consider more general bubble processes such as those studied in Kamihigashi (2008, 2011).

In addition to relaxing some of our assumptions, one may introduce government policies and examine their implications. Of particular interest would be a policy to minimize the effect of a bubble crash in the foreign country on the home bubble. All these extensions are left for future research.

Appendix A Proofs

A.1 Proof of Lemma 2.1

If: Let $\{c_t, d_t, x_t, b_t\}$ be a set of nonnegative stochastic processes satisfying (2.2)–(2.6), (2.8), and (2.9). By the definition of an equilibrium, it suffices to show that for each $t \in \mathbb{Z}_+$, the vector $(c_t, 1, d_{t+1})$ solves the maximization problem (2.1)–(2.3). Note that the problem can be written as

$$\max_{x \geq 0:\, b_t x \leq e_1} u(e_1 - b_t x) + \mathbb{E}_t v(e_2 + b_{t+1} x). \tag{A.1}$$

For any $x \geq 0$ with $b_t x \leq e_1$, we have

$$\begin{aligned} & u(e_1 - b_t x) + \mathbb{E}_t v(e_2 + b_{t+1} x) - [u(e_1 - b_t) + \mathbb{E}_t v(e_2 + b_{t+1})] && \text{(A.2)} \\ = {} & u(e_1 - b_t x) - u(e_1 - b_t) + \mathbb{E}_t[v(e_2 + b_{t+1} x) - v(e_2 + b_{t+1})] && \text{(A.3)} \\ \leq {} & -u'(e_1 - b_t) b_t (x - 1) + \mathbb{E}_t v'(e_2 + b_{t+1}) b_{t+1} (x - 1) && \text{(A.4)} \\ = {} & [-u'(e_1 - b_t) b_t + \mathbb{E}_t v'(e_2 + b_{t+1}) b_{t+1}](x - 1) = 0, && \text{(A.5)} \end{aligned}$$

where the inequality holds by concavity of u and v, and the last equality holds by (2.9). It follows that it is optimal to choose $x = 1$; i.e., $(c_t, 1, d_{t+1})$ solves the maximization problem (2.1)–(2.3).

Only If: Let $\{c_t, d_t, x_t, b_t\}$ be an equilibrium. Since (2.2)–(2.6) hold by definition, we only need to verify (2.8) and (2.9). Let $t \in \mathbb{Z}_+$. Consider the maximization problem (A.1). Note that (2.8) follows from (2.7). Hence (2.9) is a necessary condition for optimality provided that

$$\left. \frac{\partial \mathbb{E}_t v(e_2 + b_{t+1} x)}{\partial x} \right|_{x=1} = \mathbb{E}_t v'(e_2 + b_{t+1}) b_{t+1}. \tag{A.6}$$

To see this, note that for any $x > 1$ we have

$$0 \leq \frac{v(e_2 + b_{t+1} x) - v(e_2 + b_{t+1})}{x - 1} \leq v'(e_2 + b_{t+1}) b_{t+1} < v'(e_2) e_1. \tag{A.7}$$

For any $x \in (0, 1)$ we have

$$0 \leq \frac{v(e_2 + b_{t+1}) - v(e_2 + b_{t+1} x)}{1 - x} \leq v'(e_2 + b_{t+1} x) b_{t+1} < v'(e_2) e_1. \tag{A.8}$$

Thus by the conditional dominated convergence theorem, we have

$$\begin{aligned} & \lim_{x \to 1} \mathbb{E}_t \frac{v(e_2 + b_{t+1} x) - v(e_2 + b_{t+1})}{x - 1} && \text{(A.9)} \\ = {} & \mathbb{E}_t \lim_{x \to 1} \frac{v(e_2 + b_{t+1} x) - v(e_2 + b_{t+1})}{x - 1} && \text{(A.10)} \\ = {} & \mathbb{E}_t v'(e_2 + b_{t+1}) b_{t+1}. && \text{(A.11)} \end{aligned}$$

Now (A.6) follows.[9]

[9] See Kamihigashi (1998, pp. 112–113) for a more general argument to show (A.6).

A.2 Proof of Lemma 3.1

Only If: Let $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}$ be an equilibrium. Then (3.2)–(3.6) hold by definition. Fix $t \in \mathbb{Z}_+$. Since u and v are strictly concave, and since $(c_t^H, x_t^{H,H}, x_t^{H,F}, d_{t+1}^H)$ and $(c_t^F, x_t^{F,H}, x_t^{F,F}, d_{t+1}^F)$ solve the same problem (3.1)–(3.3), we have $c_t^H = c_t^F$ and $d_{t+1}^H = d_{t+1}^F$. Define c_t and d_{t+1} by

$$c_t = c_t^H = c_t^F, \qquad d_{t+1} = d_{t+1}^H = d_{t+1}^F. \tag{A.12}$$

Summing (3.2) and (3.3) over $i \in \{H, F\}$ and using the above equations and (3.6), we have

$$2c_t = 2e_1 - 2(b_t^H + b_t^F), \tag{A.13}$$

$$2d_{t+1} = 2e_2 + 2(b_{t+1}^H + b_{t+1}^F). \tag{A.14}$$

Dividing both equations through by 2, we obtain (3.7) and (3.8). The inequality in (3.7) follows from (2.7).

To see (3.9), let $t \in \mathbb{Z}_+$ and $k \in \{H, F\}$. By (3.6) there exists at least one $i \in \{H, F\}$ such that $x_t^{i,k} > 0$. Following the "only if" part of the proof of Lemma 2.1, we see that

$$u'(e_1 - (b_t^H + b_t^F))b_t^k = \mathbb{E}_t v'(e_2 + b_{t+1}^H + b_{t+1}^F)b_{t+1}^k. \tag{A.15}$$

Recalling (3.7) and (3.8), we obtain (3.9).

If: Let $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}$ be a set of nonnegative stochastic processes satisfying (3.2)–(3.4) and (3.6). Suppose that (3.7)–(3.9) hold for all $t \in \mathbb{Z}_+$. Then (3.5) holds. Following the "if" part of the proof of Lemma 2.1, we see that $(c_t^i, x_t^{i,i}, x_t^{i,j}, d_{t+1}^i)$ solves the maximization problem (3.1)–(3.3) for each $i \in \{H, F\}$. It follows that $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}$ is an equilibrium.

A.3 Proof of Proposition 3.1

We start by preparing the following lemma.

Lemma A.1. *Let $\{c_t^i, d_t^i, b_t^H, b_t^F\}$ be a set of nonnegative stochastic processes satisfying (3.7)–(3.9) for all $t \in \mathbb{Z}_+$. Suppose that (3.8) holds for $t = -1$ as well. Define $\{x_t^{i,k}\}_{t\in\mathbb{Z}_+, i,k\in\{H,F\}}$ by*

$$x_t^{i,k} = 1, \qquad \forall i, k \in \{H, F\}, \forall t \in \mathbb{Z}_+. \tag{A.16}$$

Then $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}$ is an equilibrium.

Proof. Let $\{c_t^i, d_t^i, b_t^H, b_t^F\}$ and $\{x_t^{i,k}\}$ be as given in the statement of the lemma. Then (3.2)–(3.6) hold. Since (3.9) holds for all $t \in \mathbb{Z}_+$ by hypothesis, it follows by Lemma 3.1 that $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}$ is an equilibrium. $\square$

To complete the proof of Proposition 3.1, note that the "only if" part of Proposition 3.1 follows from Lemma 3.1. To see the "if" part, let $\{b_t^H, b_t^F\}$ be a pair of nonnegative stochastic processes satisfying (3.10) and (3.11). Define $\{c_t^i\}$ an $\{d_t^i\}$ by (3.7) and (3.8). Then (3.9) holds by (3.11). Define $\{x_t^{i,k}\}$ by (A.16). Then by Lemma A.1, $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, b_t^H, b_t^F\}$ is an equilibrium. It follows that $\{b_t^H, b_t^F\}$ is a bivariate equilibrium bubble process.

A.4 Proof of Proposition 4.1

Only If: Let $(b^H, b^F, \tilde{b}^F) \in \mathbb{R}_{++}^2 \times \mathbb{R}_+$ be a stationary sunspot equilibrium of the form (4.1)–(4.3). Define

$$c = e_1 - (b^H + b^F), \tag{A.17}$$

$$d = e_2 + b^H + b^F, \tag{A.18}$$

$$\tilde{d} = e_2 + \tilde{b}^H. \tag{A.19}$$

Then (4.4) and (4.5) are written as

$$u'(c)b^F = qv'(d)b^F, \tag{A.20}$$

$$u'(c)b^H = qv'(d)b^H + (1-q)v'(\tilde{d})\tilde{b}^H. \tag{A.21}$$

Note from (A.20) that

$$u'(c) = qv'(d). \tag{A.22}$$

This together with (A.21) yields $(1-q)v'(\tilde{d})\tilde{b}^H = 0$. Hence (4.7) holds. Substituting (A.17) and (A.18) into (A.22) and recalling (2.15), we obtain (4.8).

If: Let $(b^H, b^F, \tilde{b}^F) \in \mathbb{R}_{++}^2 \times \mathbb{R}_+$ satisfy (4.4) and (4.7). Then (A.22) holds by (2.15) with c and d defined by (A.17) and (A.18). Note that (A.22) implies (4.4). We also have (4.5) from (A.22) and (4.7). Since (4.4) and (4.5) imply (3.11), it follows by Proposition 3.1 that $(b^H, b^F, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (4.1)–(4.3).

A.5 Proof of Proposition 4.2

Only If: Let $(b^H, b^F, \tilde{b}^F) \in \mathbb{R}^2_{++} \times \mathbb{R}_+$ be a stationary sunspot equilibrium of the form (4.1)–(4.3). Then (4.4) holds by Proposition 4.1; thus (2.16) holds.

If: Suppose that (2.16) holds. Then there exists $b^* > 0$ solving (2.16). Let $b^H, b^F > 0$ satisfy (4.8). Then we have (4.4). Let $\tilde{b}^H = 0$. Then $(b^H, b^F, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (4.1)–(4.3) by Proposition 4.1.

The other claims made in the proposition follow from the above construction.

B Proof of Propositions 5.1 and 5.2

B.1 Preliminary Lemmas

We state the following consequence of Proposition 3.1 for later reference.

Lemma B.1. *A quadruple $(b^H, b^F, \tilde{b}^H, \tilde{b}^F) \in \mathbb{R}^2_{++} \times \mathbb{R}^2_+$ is a stationary sunspot equilibrium of the form (5.3)–(5.5) if and only if it satisfies (5.6)–(5.9).*

For the rest of this subsection, we take a stationary sunspot equilibrium $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ of the form (5.3)–(5.5) as given.

Lemma B.2. *Let $k \in \{H, F\}$. If $\tilde{b}^k > 0$, then $\tilde{b}^k$ satisfies (5.15).*

Proof. This is immediate from (5.8) and (5.9). □

Lemma B.3. *We have $\tilde{b}^H = 0$ if and only if $\tilde{b}^F = 0$.*

Proof. Dividing (5.6) and (5.7) through by $b^H > 0$ and $b^F > 0$, respectively, we have

$$u'(c_{0,0}) = q^H q^F v'(d_{0,0}) + q^H (1 - q^F) v'(d_{0,1}) \tilde{b}^H / b^H, \tag{B.1}$$

$$u'(c_{0,0}) = q^F q^H v'(d_{0,0}) + q^F (1 - q^H) v'(d_{1,0}) \tilde{b}^F / b^F. \tag{B.2}$$

From these equations, we get

$$q^H (1 - q^F) v'(d_{0,1}) \tilde{b}^H / b^H = q^F (1 - q^H) v'(d_{1,0}) \tilde{b}^F / b^F. \tag{B.3}$$

This equation implies that $\tilde{b}^H = 0$ if and only if $\tilde{b}^F = 0$. □

B.2 Proof of Proposition 5.1

(a) Suppose that $\tilde{b}^H = 0$. To show the "only if" part, suppose that $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (5.3)–(5.5). Note from Lemma B.3 that $\tilde{b}^F = 0$. Thus (5.6)–(5.9) reduce to (5.13). To show the "if" part, suppose that $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ satisfies (5.13) and (5.14) with $\tilde{b}^F = 0$. Then (5.6)–(5.9) trivially hold. Thus by Lemma B.1, the quadruple $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (5.3)–(5.5).

(b) Suppose that $\tilde{b}^H > 0$. To show the "only if" part, suppose that $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (5.3)–(5.5). Then $\tilde{b}^F > 0$ by Lemma B.3. We obtain (5.15) by Lemma B.2. Adding (5.6) and (5.7), we obtain (5.16). To show (5.17), note from (B.3) that

$$\frac{b^H}{b^F} = \frac{(1-q^F)q^H v'(e_2 + \tilde{b}^H)\tilde{b}^H}{(1-q^H)q^F v'(e_2 + \tilde{b}^F)\tilde{b}^F} = \frac{(1-q^F)u'(e_1 - \tilde{b}^H)\tilde{b}^H}{(1-q^H)u'(e_1 - \tilde{b}^F)\tilde{b}^F}, \tag{B.4}$$

where the second equality uses (5.15). This completes the proof of the "only if" part of the first conclusion. The "if" part is shown in the proof of part (b) in Proposition 5.2 below.

Consider the second conclusion. To show that $\tilde{b}^H > b^H$, suppose by way of contradiction that $b^H \geq \tilde{b}^H$. Then by (5.6) we have

$$\begin{aligned} u'(e_1 - b^W) &= q^H q^F v'(e_2 + b^W) + q^H(1-q^F)v'(e_2 + \tilde{b}^H)\tilde{b}^H/b^H && \text{(B.5)} \\ &\leq q^H q^F v'(e_2 + b^W) + q^H(1-q^F)v'(e_2 + \tilde{b}^H). && \text{(B.6)} \end{aligned}$$

Since $b^W > b^H \geq \tilde{b}^H$, it follows that

$$\begin{aligned} u'(e_1 - b^W) &< q^H q^F v'(e_2 + \tilde{b}^H) + q^H v'(e_2 + \tilde{b}^H) - q^H q^F v'(e_2 + \tilde{b}^H) && \text{(B.7)} \\ &= q^H v'(e_2 + \tilde{b}^H) = u'(e_1 - \tilde{b}^H) < u'(e_1 - b^W), && \text{(B.8)} \end{aligned}$$

where the second equality in (B.8) holds by (5.15). Since (B.7) and (B.8) lead to a contradiction, we must have $\tilde{b}^H > b^H$. We can show $\tilde{b}^F > b^F$ similarly. We have verified the inequalities in (5.18).

Finally, consider (5.19). If $q^H = q^F$, then $\tilde{b}^H = \tilde{b}^F$ by (5.15), and $b^H = b^F$ by (5.17). Suppose that $q^H > q^F$.

To show that $\tilde{b}^H > \tilde{b}^F$, suppose by way of contradiction that $\tilde{b}^H \leq \tilde{b}^F$. By (5.15) we have

$$\begin{aligned} u'(e_1 - \tilde{b}^H) &= q^H v'(e_2 + \tilde{b}^H) && \text{(B.9)} \\ &> q^F v'(e_2 + \tilde{b}^H) \geq q^F v'(e_2 + \tilde{b}^F) && \text{(B.10)} \\ &= u'(e_1 - \tilde{b}^F) \geq u'(e_1 - \tilde{b}^H), && \text{(B.11)} \end{aligned}$$

where the equality in (B.11) holds by (5.15). Since (B.9)–(B.11) lead to a contradiction, we must have $\tilde{b}^H > \tilde{b}^F$.

To show that $b^H > b^F$, note that $(1-q^F)/(1-q^H) > 1$. Thus from (5.17) we have

$$\frac{b^H}{b^F} > \frac{u'(e_1 - \tilde{b}^H)\tilde{b}^H}{u'(e_1 - \tilde{b}^F)\tilde{b}^F}. \tag{B.12}$$

Since $\tilde{b}^H > \tilde{b}^F$ and $u'(e_1 - b)b$ is strictly increasing in b, the right-hand side of (B.12) is strictly greater than 1. Thus $b^H > b^F$. Since the case $b^H < q^F$ is symmetric, we have verified (5.19).

B.3 Proof of Proposition 5.2

(a) *If:* Assume (5.21). Then it is easy to see that there exists a unique $b^W > 0$ satisfying (5.13). Take any $b^H, b^F > 0$ satisfying (5.14). Let $\tilde{b}^H = \tilde{b}^F = 0$. Then $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (5.3)–(5.5) by Proposition 5.1.

Only If: Suppose that (5.21) does not hold. Then it is easy to see that there exists no $b^W > 0$ satisfying (5.13). Thus there exists no stationary sunspot equilibrium satisfying (5.3)–(5.5) by Proposition 5.1(a).

(b) *If:* Assume (5.20). Then it is easy to see that there exists a unique $\tilde{b}^k > 0$ satisfying (5.15) for each $k \in \{H, F\}$. Dividing (5.16) through by b^W, we have

$$\begin{aligned} u'(e_1 - b^W) &= q^H q^F v'(e_2 + b^W) \\ &+ [q^H(1-q^F)v'(e_2 + \tilde{b}^H)\tilde{b}^H + q^F(1-q^H)v'(e_2 + \tilde{b}^F)\tilde{b}^F]/b^W. \end{aligned} \tag{B.13}$$

The left-hand side is finite when $b^W = 0$, is strictly increasing in $b^W \in (0, e_1)$, and tends to ∞ as b^W approaches e_1. The right-hand side of (B.13) tends to ∞ as b^W approaches 0, and is strictly decreasing in $b^W > 0$. It follows that there exists a unique $b^W > 0$ satisfying (B.13), or (5.16).[10] Given b^W, there exists a unique pair $(b^H, b^F) \gg 0$ satisfying (5.17) and (5.14).

Since (5.8) and (5.9) hold by (5.15), it remains to verify (5.6) and (5.7); then we can conclude that $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (5.3)–(5.5) by Lemma B.1.

[10]The rest of the proof of the "if" part here also serves as a proof of the "if" part of part (b) in Proposition 5.1.

To this end, for $k, k' \in \{H, F\}$ with $k \neq k'$, let

$$\mu^k = q^k(1 - q^{k'})v'(e_2 + \tilde{b}^k)\tilde{b}^k. \tag{B.14}$$

Then (5.6) and the first equality in (B.4) can be written, respectively, as

$$u'(e_1 - b^W)b^H = q^H q^F v'(e_2 + b^W)b^H + \mu^H, \tag{B.15}$$

$$\frac{b^H}{b^F} = \frac{\mu^H}{\mu^F}. \tag{B.16}$$

By (5.14) and (B.16) we have

$$b^W = \left(1 + \frac{\mu^F}{\mu^H}\right) b^H = \frac{\mu^H + \mu^F}{\mu^H} b^H. \tag{B.17}$$

Substituting (B.14) and (B.17) into (B.13), we have

$$u'(e_1 - b^W) = q^H q^F v'(e_2 + b^W) + \frac{\mu^H}{b^H}, \tag{B.18}$$

which is equivalent to (B.15), or (5.6). We can show (5.7) similarly. It now follows by Lemma B.1 that $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a stationary sunspot equilibrium of the form (5.3)–(5.5).

Only If: Suppose that (5.20) does not hold. Then we have $q^H \leq u'(e_1)/v'(e_2)$ or $q^F \leq u'(e_1)/v'(e_2)$. Let $k \in \{H, F\}$ be such that $q^k \leq u'(e_1)/v'(e_2)$. Then there exists no $\tilde{b}^k > 0$ satisfying $u'(e_1 - \tilde{b}^k) = q^k v'(e_2 + \tilde{b}^k)$. This violates (5.15). Thus there exists no stationary sunspot equilibrium of the form (5.3)–(5.5) by Proposition 5.1.

C Proof of Proposition 6.1

Linearizing the system (6.3)–(6.6) with respect to β_t^H, β_t^F, $\tilde{\beta}_t^H$ and $\tilde{\beta}_t^F$ around the stationary sunspot equilibrium $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$, we find that the Jacobian matrix J takes the following form:

$$J = \begin{bmatrix} A & B \\ 0 & C \end{bmatrix}. \tag{C.1}$$

Since J is upper triangular, the eigenvalues of J are those of A and C. Thus it suffices to specify A and C. For $c, d > 0$, define

$$\varepsilon_u(c) = -cu''(c)/u'(c), \quad \varepsilon_v(d) = -dv''(d)/v'(d). \tag{C.2}$$

Then C can be written as

$$C = \begin{bmatrix} C_{11} & 0 \\ 0 & C_{22} \end{bmatrix} \tag{C.3}$$

with

$$C_{11} = \frac{1 + \varepsilon_u(c_{0,1})\tilde{b}^H/c_{0,1}}{1 - \varepsilon_v(d_{0,1})\tilde{b}^H/d_{0,1}}, \tag{C.4}$$

$$C_{22} = \frac{1 + \varepsilon_u(c_{1,0})\tilde{b}^F/c_{1,0}}{1 - \varepsilon_v(d_{1,0})\tilde{b}^F/d_{1,0}}, \tag{C.5}$$

where $c_{i,j}$ and $d_{i,j}$ are given by (5.10)–(5.12) for all $(i, j) = (0, 0), (0, 1), (1, 0), (1, 1)$. In particular, $d_{0,1} = e_2 + \tilde{b}^H$; thus $\tilde{b}^H/d_{0,1} < 1$. Likewise $\tilde{b}^F/d_{1,0} < 1$. Hence by (6.8) we have

$$\varepsilon_v(d_{0,1})\tilde{b}^H/d_{0,1} < 1, \quad \varepsilon_v(d_{1,0})\tilde{b}^F/d_{1,0} < 1. \tag{C.6}$$

These inequalities imply that $C_{11} > 1$ and $C_{22} > 1$. Since C is a diagonal matrix, its eigenvalues λ_1^C and λ_2^C are given by

$$\lambda_1^C = C_{11} > 1, \quad \lambda_2^C = C_{22} > 1. \tag{C.7}$$

We can write the matrix A as

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{C.8}$$

with

$$A_{11} = \frac{1}{\alpha}\frac{d_{0,0}c_{0,0}(1-\varepsilon_v(d_{0,0})b^F/d_{0,0}) + \varepsilon_u(c_{0,0})d_{0,0}b^H}{d_{0,0}c_{0,0}(1-\varepsilon_v(d_{0,0})b^W/d_{0,0})}, \tag{C.9}$$

$$A_{12} = \frac{1}{\alpha}\frac{b^F(d_{0,0}\varepsilon_u(c_{0,0}) + c_{0,0}\varepsilon_v(d_{0,0}))}{d_{0,0}c_{0,0}(1-\varepsilon_v(d_{0,0})b^W/d_{0,0})}, \tag{C.10}$$

$$A_{21} = \frac{1}{\alpha}\frac{b^H(d_{0,0}\varepsilon_u(c_{0,0}) + c_{0,0}\varepsilon_v(d_{0,0}))}{d_{0,0}c_{0,0}(1-\varepsilon_v(d_{0,0})b^W/d_{0,0})}, \tag{C.11}$$

$$A_{22} = \frac{1}{\alpha}\frac{d_{0,0}c_{0,0}(1-\varepsilon_v(d_{0,0})b^H/d_{0,0}) + \varepsilon_u(c_{0,0})d_{0,0}b^F}{d_{0,0}c_{0,0}(1-\varepsilon_v(d_{0,0})b^W/d_{0,0})}, \tag{C.12}$$

$$\alpha = q^H q^F v'(d_{0,0})/u'(c_{0,0}). \tag{C.13}$$

Since $d_{0,0} = e_2 + b^W$, we have $b^W/d_{0,0} < 1$. Hence by (6.8) we have

$$\varepsilon_v(d_{0,0})b^W/d_{0,0} < 1. \tag{C.14}$$

This ensures that the denominators in (C.9)–(C.12) are all strictly positive.

To determine the eigenvalues of A, note that the associated characteristic polynomial $P(\lambda)$ can be written as

$$P(\lambda) = \lambda^2 - T\lambda + D, \tag{C.15}$$

where T and D are the trace and the determinant of A, respectively:

$$D = \frac{1}{\alpha^2}\frac{1+\varepsilon_u(c_{0,0})b^W/c_{0,0}}{1-\varepsilon_v(d_{0,0})b^W/d_{0,0}}, \tag{C.16}$$

$$T = \frac{1}{\alpha}\left(1 + \frac{1+\varepsilon_u(c_{0,0})b^W/c_{0,0}}{1-\varepsilon_v(d_{0,0})b^W/d_{0,0}}\right). \tag{C.17}$$

From (C.13) and (5.6) we have $\alpha < 1$. By (C.14) we have

$$\frac{1+\varepsilon_u(c_{0,0})b^W/c_{0,0}}{1-\varepsilon_v(d_{0,0})b^W/d_{0,0}} > 1. \tag{C.18}$$

Hence $D > 1$, which implies that one eigenvalue, which we denote λ_1^A, satisfies

$$\lambda_1^A > 1. \tag{C.19}$$

By (C.16), (C.17), and (C.18), we have $\alpha D > 1$ and $T > 1$. Note that

$$P(-1) = 1 + T + D > 0. \tag{C.20}$$

Note from (C.16) and (C.17) that $T = 1/\alpha + \alpha D$. Thus

$$P(1) = 1 - T + D = \frac{(\alpha - 1)(1 - \alpha D)}{\alpha} > 0, \tag{C.21}$$

where the inequality holds since $\alpha < 1$ and $\alpha D > 1$, as shown above. Now we have $P(-1) > P(1) > P(\lambda_1^A) = 0$. This implies that the other eigenvalue is strictly greater than λ_1^A.

It follows that J has four eigenvalues outside the unit circle. Hence the steady state $(b^H, b^F, \tilde{b}^H, \tilde{b}^F)$ is a source.

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