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Inter-Industrial Factor Allocation in the Short and Long Runs in Chamberlinian Monopolistic Competition

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Abstract

This study reviews the allocation of factors among industries distinguishing between the short run and long run in a monopolistically competitive world. Without any conditions on scale economy, non-homotheticity, and factor intensity, it is shown that the long-run equilibrium characterized by factor-price equalization between industries is always locally stable. In addition, the direction of non-homothetic bias does not have qualitative effects on the allocation of factors, while it works as key elements to determine the direction of the change in relative industry size. The direction of the change in factor allocation mainly depends on the gross substitutability of factors which is magnified by non-homotheticity and scale economies.

JEL classification: F11, L13, L16

Keywords: Non-Homotheticity, Scale Economy, Factor Intensity, Elasticity of Substitution

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1. Introduction

In the past three decades, the Dixit and Stiglitz (1977) type of monopolistic competition model has been used widely in economic analysis. This enables us to investigate easily the issue of firm-level scale economies in a general equilibrium framework. In particular, the rich implications for industrial structure between countries or regions have been studied in detail in the new trade theory and new economic geography.¹ However, the standard models for this have tended to assume that production factors are instantaneously mobile between industries, and to ignore the fact that they may be specific to particular use, at least in the short run.

There are a number of studies that have found substantial inter-industrial wage gaps remain even after controlling for individual and job characteristics, human-capital variables, and geographic location (e.g., Dickens and Katz, 1987; Krueger and Summers, 1988; Edin and Zetterberg, 1992). These findings suggest that in the short run, factors may be specific to particular use and the adjustment is only gradual, even if factors move between industries to cancel out differences in factor reward in the long run.² This distinction between the short and long runs has not been the main focus in the standard models used in the new trade theory and new economic geography.

On the other hand, as early as 1936, Haberler (1936) discussed international trade distinguishing between the short and long runs. Thereafter, in the 1970s, traditional trade theory studied how such short-run factor specificity changes the effects of varying good prices and factor endowments on factor rewards and how these are adjusted to the long-run equilibrium (e.g., Mayer, 1974; Mussa, 1974; Amano, 1977; Neary, 1978a, 1978b).³ However, beginning

¹See, for example, Helpman and Krugman (1985), Fujita et al. (1999), and Combes et al. (2008).

²There are a few empirical studies that directly test the adjustment speed of inter-industrial factor allocation. Morrison and Berndt (1981) show quasi-fixity of inputs using annual US manufacturing data from 1952 to 1971. Helliwell and Chung (1986) estimate the speed of adjustment of the capital-energy ratio and show that the factor ratio changes by only 18 percent per year. In addition, Sneessens and Drèze (1986) find the capital-labor ratio adjusts by only 27 percent per year.

³These studies assume perfect competition and constant returns technology for characterizing industries. However, there are some considerable doubts about the appropriateness of perfect competition and constant returns technology for industrial characteristics, because a not-negligible number of studies show that

with Krugman (1979), during the 1980s, the short-run factor specificity and adjustment process of factor allocation among industries began to receive less attention in the new trade theory, which widely uses the Dixit–Stiglitz type of monopolistic competition.⁴ In the 1990s, the new economic geography, pioneered by Krugman (1991), became an analytical framework to investigate how factor allocation between regions is determined and adjusted, while it pays little attention to factor allocation between industries.

This study aims to review the allocation of factors among industries distinguishing between the short and long runs in monopolistically competitive world. In particular, we address the adjustment process leading to a stable allocation of factors between industries and the response of factor allocation to factor endowment changes. To this end, we develop a general equilibrium model that consists two industries characterized by the Dixit–Stiglitz type of monopolistic competition. Each industry uses two factors (capital and labor) that move between two industries gradually under a general form of production technology that may not be homothetic.

This setting of the model is more natural compared to that of traditional trade theory. According to Hall (1988), multiple US industries have markups that differ across these industries. For example, the markup rate in the construction industry is 2.196, but is 3.300 in the finance, insurance, and real estate industries.⁵ In addition, some studies suggest that assuming homotheticity on production technology may not be appropriate. Christensen and

economies of scale exist in many industries. For example, Christensen and Greene (1976) confirm economies of scale estimating the translog cost function in the US electric power industry. In addition, Noulas et al. (1990) estimate a translog cost function in the US banking industry, and show that the industry has economies of scale. If economies of scale prevail in industries, analysis using constant returns technology is not appropriate. Moreover, the assumption of perfect competition is also not appropriate, since economies of scale make the perfect competition unsustainable. In fact, Hall (1988), using data from 1953 to 1984, finds that some US industries have marginal costs well below price, which suggests that these industries are not in perfect competition.

⁴See, for example, Krugman (1980), Helpman (1981) and Horn (1983). Krugman (1981) analyzes a model in which labor is specified for a particular industry, but does not consider the adjustment of labor allocation between industries.

⁵Hall (1988) estimates these markups using data of US industries from 1953 to 1984.

Greene (1976) reject homotheticity in production using cross-sectional data for US firms producing electric power. Kim (1992) shows that a non-homothetic production function is more consistent with US manufacturing from 1947 to 1971. Thus, in this study, we assume that all industries under monopolistic competition have different markups and non-homothetic production technologies.

Some similar approaches have been studied previously. Horn (1983) constructs a two-industry, two-factor model in which one industry is in perfect competition with constant returns-to-scale technology while in the other industry, the Dixit–Stiglitz type of monopolistic competition prevails and firms exhibit internal increasing returns to scale with a general form of technology. Horn (1983) investigates the relationship between relative factor endowments and relative industry size, and shows that the effect of change in relative factor endowments on relative industry size cannot be predicted without the following special circumstance: *non-homothetic bias* is towards the factor that is *used intensively on average*.⁶ Chao and Takayama (1987, 1990) analyze the stability of long-run equilibrium characterized by the zero-profit condition of firms using the same setting as Horn (1983). They show that a key condition is non-homothetic bias toward the factor that is used intensively on average to ensure the stability of long-run equilibrium characterized by the zero-profit condition. However, these studies pay little attention to short-run factor specificity and allocation of factors between industries.

In this study, without requiring any condition on non-homotheticity and factor intensity, we show that local stability of long-run equilibrium characterized by factor price equalization between industries is ensured. The direction of the non-homothetic bias does not have qualitative effects on the allocation of factors, while it works as key elements to determine the direction of change in the relative industry size measured by output, which is caused by change in factor endowments. On the other hand, the response of factor allocation to factor endowment changes depends on the “gross” substitutability of factors which is magnified by non-homotheticity and scale economies. It is shown that an increase in capital endowments

⁶In addition, Helpman (1981) undertakes a similar investigation, but uses the Lancaster (1980) type of preference for the differentiated goods.

may make labor-intensive industry relatively large measured by input, while it makes capital-intensive industry relatively large measured by output.

The remainder of this paper is organized as follows. Section 2 presents the model and introduces measures of the degree of non-homotheticity and economies of scale in production. We describe the adjustment process of factor allocation between industries in Section 3. The effects of change of factor endowments are investigated in Section 4. Section 5 concludes.

2. Model and Introduction of Some Key Measures

In this section, we develop the model and introduce measures of economies of scale and non-homotheticity. Consider an economy with two industries (1 and 2). Both industry 1 and 2 produce a group of differentiated goods.

2.1. Preference and Production

All consumers share the same preference represented by following utility function:

$$U = X_1^{\alpha_1} X_2^{\alpha_2}, \quad 0 < \alpha_i < 1, \quad \alpha_1 + \alpha_2 = 1, \quad (1)$$

where X_i represents the composite index of the consumption of differentiated goods produced in industry i . Let us denote the consumption of each variety ω_i in industry i as $x_i(\omega_i)$. Then, the composite index X_i takes the form of a constant elasticity of substitution function as follows:

$$X_i \equiv \left[\int_{\Omega_i} x_i(\omega_i)^{\rho_i} d\omega_i \right]^{\frac{1}{\rho_i}}, \quad 0 < \rho_i < 1, \quad i = 1, 2,$$

where Ω_i is the set of available varieties in industry i , and ρ_i represents the intensity of substitution among varieties for in industry i . In this specification, $\sigma_i \equiv 1/(1 - \rho_i)$ represents the elasticity of substitution in industry i .

Maximizing utility given by (1) subject to the budget constraint, we obtain the following market demand functions:

$$x_i(\omega_i) = x_i(p_i(\omega_i), P_i, Y) \equiv \frac{\alpha_i Y}{P_i} \left(\frac{P_i}{p_i(\omega_i)} \right)^{\sigma_i}, \quad \omega_i \in \Omega_i, \quad i = 1, 2,$$

where Y is the income of the economy, $p_i(\omega_i)$ is the price for each variety ω_i produced in industry i , and P_i is the price index of industry i defined by

$$P_i \equiv \left[\int_{\Omega_i} p_i(\omega_i)^{1-\sigma_i} d\omega_i \right]^{\frac{1}{1-\sigma_i}}, \quad i = 1, 2.$$

Turning to production, firms are monopolistically competitive and require labor and capital. We assume that each firm has an increasing returns-to-scale technology and can be free to choose which varieties to produce. Then, without loss of generality, we can assume firm ω_i produces variety ω_i .

All firms in the same industry i have the same technology represented by a twice differentiable, monotonically increasing and strictly quasi-concave production function. We denote factor prices for labor and capital in industry i by w_i and r_i . Then, solving the cost minimization problem, we can obtain the cost function of each firm in industry i :

$$C_i(\omega_i) = C_i(w_i, r_i, x_i(\omega_i)), \quad \omega_i \in \Omega_i, \quad i = 1, 2,$$

which is concave and linear homogeneous in factor prices.

Using the first-order condition for profit maximization, we obtain the well-known pricing rule as follows:

$$p_i(\omega_i) = \frac{1}{\rho_i} \frac{\partial C_i(w_i, r_i, x_i(\omega_i))}{\partial x_i(\omega_i)}, \quad \omega_i \in \Omega_i, \quad i = 1, 2. \quad (2)$$

Therefore, when output is common to every active firm in industry i , the price is also common to every variety in the industry i . Because we only focus on the symmetric situation, we drop the variety label in the remainder of this paper.

2.2. Measures of Scale Economy and Non-Homotheticity

The production technologies differ in two respects from the traditional Heckscher–Ohlin model: one is the existence of scale economy and the other is that we allow non-homotheticity. To capture the degree of the first difference, we define the measure of economies of scale as follows:

$$\phi_i = \phi_i(w_i, r_i, x_i) \equiv \frac{C_i(w_i, r_i, x_i)}{x_i} \bigg/ \frac{\partial C_i(w_i, r_i, x_i)}{\partial x_i}, \quad i = 1, 2,$$

which is the partial elasticity of cost with respect to output.⁷ ϕ must take on values larger than unity if increasing returns prevail locally, while it becomes less than unity if there are

⁷A similar measure of economies of scale is used in Helpman (1981), Horn (1983), and Chao and Takayama (1990). However, they use $1/\phi$ as the measure. We use ϕ , which is what Helpman and Krugman (1985) use, because it is more natural that large ϕ means strong scale economy. There are some other concepts that measure the economy of scale. A detailed discussion of these concepts is found in Hanoch (1975) and Ide and Takayama (1987).

decreasing returns. $\phi = 1$ means that constant returns to scale prevail.

We allow free entry and exit for firms. This implies that profits must be driven to zero for any active firm. Thus, we obtain

$$C_i(w_i, r_i, x_i) = p_i x_i, \quad i = 1, 2, \quad (3)$$

which is called the Chamberlinian tangency solution. Together with the pricing rule, (3) ensures that the measure of economies of scale ϕ_i is equal to markup $1/\rho_i > 1$. This means that the degree of scale economy is determined independently from the supply side at the Chamberlinian tangency solution. We can observe easily that a weaker intensity of substitution between varieties (i.e., smaller ρ_i) results in larger ϕ_i . Note that larger ϕ_i means that firms in industry i do not exhaust scale merit and their size become smaller than the efficient level. This is because smaller ρ_i strengthens the monopoly power of firms. When ρ_i becomes large, firm must take advantage of scale merit to decrease average costs, which results in smaller ϕ_i .

Using the second-order condition for profit maximization, (3) implies

$$\eta_i \equiv -\frac{x_i}{\phi_i} \frac{\partial \phi_i}{\partial x_i} = \varepsilon_i + 1 - \frac{1}{\phi_i} = \varepsilon_i + \frac{1}{\sigma_i} > 0, \quad i = 1, 2,$$

where ε_i is the partial elasticity of marginal cost with respect to output given as

$$\varepsilon_i \equiv \frac{x_i}{\partial C_i / \partial x_i} \frac{\partial^2 C_i}{\partial x_i^2}, \quad i = 1, 2.$$

Since $-\eta_i$ is the partial elasticity of the measure of economies of scale with respect to output, the degree of scale economies in production must be decreasing locally when the output increases at the Chamberlinian tangency solution.

Next, to capture the degree of non-homotheticity in production, we define the following variables:

$$\mu_{Li} \equiv \frac{x_i}{\partial C_i / \partial w_i} \frac{\partial^2 C_i}{\partial x_i \partial w_i}, \quad \mu_{Ki} \equiv \frac{x_i}{\partial C_i / \partial r_i} \frac{\partial^2 C_i}{\partial x_i \partial r_i}, \quad i = 1, 2.$$

μ_{Li} (μ_{Ki}) is the partial elasticity of demand for labor (capital) of industry i with respect to output. Using the linear homogeneity in the factor prices for the cost function, we obtain the following relationship (cf., Horn, 1983, p. 91):

$$\theta_{Li} \mu_{Li} + \theta_{Ki} \mu_{Ki} = 1/\phi_i, \quad i = 1, 2,$$

where θ_{Li} (θ_{Ki}) is the distributive share of the cost for labor (capital) in the production of industry i given by

$$\theta_{Li} \equiv \frac{w_i}{C_i} \frac{\partial C_i}{\partial w_i}, \quad \theta_{Ki} \equiv \frac{r_i}{C_i} \frac{\partial C_i}{\partial r_i}, \quad i = 1, 2,$$

which must satisfy $\theta_{Li} + \theta_{Ki} = 1$. If the production function is homothetic, then $\mu_{Li} = \mu_{Ki} = 1/\phi_i$. So, we measure the degree of non-homotheticity in production as follows:⁸

$$\delta_i \equiv \left| \left(\mu_{Ki} - \frac{1}{\phi_i} \right) \left(\mu_{Li} - \frac{1}{\phi_i} \right) \right| = \theta_{Ki} \theta_{Li} (\mu_{Ki} - \mu_{Li})^2 \geq 0, \quad i = 1, 2.$$

Obviously, in the homothetic case, δ_i becomes zero.

2.3. Factor Proportions of Individual Firm

Finally, let us examine the relationship between factor price and factor proportion of individual firms, which plays an essential role in this study. We denote capital–labor ratio of an individual firm in industry i as $k_i \equiv (\partial C_i / \partial r_i) / (\partial C_i / \partial w_i)$. Then, the change in factor proportions is given by

$$\hat{k}_i = \sigma_{pi} (\hat{w}_i - \hat{r}_i) + (\mu_{Ki} - \mu_{Li}) \hat{x}_i, \quad i = 1, 2, \quad (4)$$

where the circumflex denotes the rate of change in each variable (e.g., $\hat{k}_i \equiv dk_i/k_i$) and σ_{pi} is the Allen's elasticities of substitution, which is derived as output is fixed as

$$\sigma_{pi} \equiv \frac{C_i}{(\partial C_i / \partial w_i)(\partial C_i / \partial r_i)} \frac{\partial^2 C_i}{\partial r_i \partial w_i}, \quad i = 1, 2.$$

The first term of the RHS in (4) represents the substitution effect which excludes the effect of change in output. So, we call σ_{pi} the *pure* elasticity of substitution between factors.

If the production technology is homothetic, the optimal capital–labor ratio is determined independently from the output level. Thus, the change in factor proportions is captured only by the first term of the RHS in (4). However, allowing non-homotheticity develops positive or negative effects from the change in output level on the factor proportion. This clearly depends on whether μ_{Ki} is bigger or smaller than μ_{Li} . $\mu_{Ki} > \mu_{Li}$ ($\mu_{Ki} < \mu_{Li}$) means that a

⁸A similar measure of the degree of non-homotheticity is used in Horn (1983) and Chao and Takayama (1990).

marginal increase in output makes the input ratio of factors biased toward capital (labor). In this sense, we say that industry i has *marginal capital-biased (labor-biased)* technology if $\mu_{Ki} > \mu_{Li}$ ($\mu_{Ki} < \mu_{Li}$).

The change in output level can be linked to the change in factor rewards as follows:

$$\hat{x}_i = \frac{\phi_i \theta_{Ki} \theta_{Li} (\mu_{Ki} - \mu_{Li})}{\eta_i} (\hat{w}_i - \hat{r}_i), \quad i = 1, 2, \quad (5)$$

which is derived using the fact that the degree of scale economy $\phi_i(w_i, r_i, x_i)$ is fixed to $1/\rho_i$ at the Chamberlinian tangency solution. This equation (5), together with (4), reveals the relationship between factor rewards and factor proportion as

$$\frac{\hat{k}_i}{(\hat{w}_i - \hat{r}_i)} = \sigma_{pi} + \frac{\delta_i \phi_i}{\eta_i} \equiv s_i, \quad i = 1, 2. \quad (6)$$

With output adjusted to the change in factor prices, we call s_i the *gross* elasticity of substitution between factors in industry i .

(4) and (6) show that the relative change in factor proportion is always magnified by the output changes brought about by the change in factor prices (i.e., $s_i > \sigma_{pi}$), if non-homotheticity prevails in production.⁹ In the homothetic case, this magnification effect does not arise and the gross elasticity of substitution s_i is equal to the pure elasticity of substitution σ_{pi} . As we see in Subsection 3.2, differences of our results from a perfectly competitive world with constant returns technology occur mainly through this magnification effect. In addition, because difference of markups between industries develops the different magnification effects for each industry, the gross elasticity substitution may differ between industries, even if all industries have similar value of the pure elasticity substitution. As Torstensson (1996) mentioned, this implies that it is difficult to rule out factor-intensity reversals in monopolistically competitive world unless all industries have homothetic production technology.¹⁰ These facts suggest that non-homotheticity is essential to address the influence of scale economy and monopolistic competition.

⁹Horn (1983) expounds why the output changes brought about by the change in factor prices always magnify the relative change in factor proportion. See Figure 1 in Horn (1983).

¹⁰Torstensson (1996) provide a simple example in which factor-intensity reversals occur without the difference in the pure elasticity substitution.

3. Equilibrium and Stability

In this section, we first describe the short-run and long-run equilibrium. Then, stability of the equilibrium and the adjustment process are analyzed.

3.1. Equilibrium

With regard to the factor market, we assume that full employment is satisfied, but labor and capital are specified to each industries in the short run. In this case, factor prices may differ between industry. This situation is described by

$$\begin{aligned} N_1 \frac{\partial C_1(w_1, r_1, x_1)}{\partial w_1} &= L_1, & N_2 \frac{\partial C_2(w_2, r_2, x_2)}{\partial w_2} &= L - L_1, \\ N_1 \frac{\partial C_1(w_1, r_1, x_1)}{\partial r_1} &= K_1, & N_2 \frac{\partial C_2(w_2, r_2, x_2)}{\partial r_2} &= K - K_2, \end{aligned} \quad (7)$$

where N_i is the number of firms in industry i , L_i (K_i) is the amounts of labor (capital) employed in industry i , and L (K) represents endowments of labor (capital), respectively.

Short-run equilibrium is given by (2), (3), (7), and the market-clearing condition in the goods markets given by

$$\frac{N_2 x_2}{N_1 x_1} = \frac{\alpha_2}{\alpha_1} p_1, \quad (8)$$

where we normalize as $p_2 = 1$. Therefore, in the short-run equilibrium, we can express the relative prices of factors, $W \equiv w_1/w_2$ and $R \equiv r_1/r_2$, as a function of L_1 , K_1 , L , and K : $W = W(L, K; L_1, K_1)$ and $R = R(L, K; L_1, K_1)$.

On the other hand, in the long run, labor and capital move from low-priced industry to high-priced industry. Thus, as long as both industries employ labor and capital, the factor prices must be equalized between the two industries in the long run, that is, $W^* = R^* = 1$ (we use $(*)$ to denote the long-run equilibrium value of each variable). We are interested in whether the short-run equilibrium is adjusted to the long-run equilibrium and how the long-run allocation of factors between industries is determined.

Let us examine the properties of $W(\cdot)$ and $R(\cdot)$. These properties are used for the analysis of the stability and adjustment processes. Applying comparative statics technique to the short-run equilibrium given by (2), (3), (7) and (8) yields the following equations (see

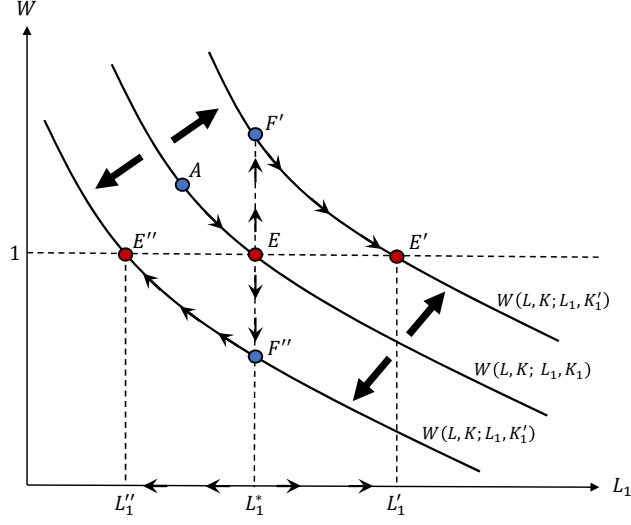


Figure 1

Appendix A):

$$\begin{aligned}
\frac{L_1}{W} \frac{\partial W}{\partial L_1} &= -\frac{1}{\lambda_{L2}} \left[1 + \lambda_{L1} \theta_{K2} \left(\frac{1}{s_2} - 1 \right) + \lambda_{L2} \theta_{K1} \left(\frac{1}{s_1} - 1 \right) \right] < 0, \\
\frac{K_1}{R} \frac{\partial R}{\partial K_1} &= -\frac{1}{\lambda_{K2}} \left[1 + \lambda_{K1} \theta_{L2} \left(\frac{1}{s_2} - 1 \right) + \lambda_{K2} \theta_{L1} \left(\frac{1}{s_1} - 1 \right) \right] < 0, \\
\frac{K_1}{W} \frac{\partial W}{\partial K_1} &= \frac{1}{\lambda_{K2}} \left[\lambda_{K1} \theta_{K2} \left(\frac{1}{s_2} - 1 \right) + \lambda_{K2} \theta_{K1} \left(\frac{1}{s_1} - 1 \right) \right], \\
\frac{L_1}{R} \frac{\partial R}{\partial L_1} &= \frac{1}{\lambda_{L2}} \left[\lambda_{L1} \theta_{L2} \left(\frac{1}{s_2} - 1 \right) + \lambda_{L2} \theta_{L1} \left(\frac{1}{s_1} - 1 \right) \right],
\end{aligned} \tag{9}$$

where $\lambda_{L_i} \equiv L_i/L$ ($\lambda_{K_i} \equiv K_i/K$) represents the share of labor (capital) of industry i . As is obvious from (9), $\partial W/\partial L_1$ and $\partial R/\partial K_1$ are always negative, but $\partial W/\partial K_1$ and $\partial R/\partial L_1$ can be either positive or negative according to whether the “gross” elasticity of substitution between factors, s_i , is smaller than one, which depends on the degree of scale economy, non-homotheticity and substitutability between factors in production.

3.2. Only Labor Can Move between Industries

We start out by constructing a case that only labor can move between industries gradually in response to gap of factor rewards. This case is described in Figure 1.

At point A in Figure 1, the wage of industry 1 is higher than that of industry 2. Thus, labor moves from industry 2 to industry 1. Because $\partial W/\partial L_1 < 0$, this movement of labor

toward industry 1 lowers W , which continues until W reaches one. Therefore, labor allocation is always adjusted to point E in Figure 1. This means that, when capital is fixed in each industry, labor allocation at point E is globally stable regardless of the degree of scale economy, non-homotheticity and substitutability between factors in production.

Suppose that labor allocation is initially given at point E in Figure 1, and then, capital allocation exogenously changes so that industry 1 becomes more capital abundant (i.e., K_1 changes to $K'_1 > K_1$). If the gross elasticities of substitution between factors, s_i , are smaller than one in both industry 1 and 2, which means that the pure elasticities of substitution, σ_{pi} , the degree of non-homotheticity, δ_i , and the degree of scale economy, ϕ_i , are sufficiently small, then $\partial W/\partial K_1$ becomes positive.¹¹ Thus, labor allocation is adjusted from point E to point E' through point F' . On the other hand, if both s_1 and s_2 are larger than one, which is caused by a sufficiently large σ_{pi} , δ_i , and ϕ_i , then we have a negative value of $\partial W/\partial K_1$. In this case, labor allocation is adjusted from point E to point E'' through point F'' . This means that labor flows out from the industry that becomes more capital abundant. In particular, if the pure elasticities of substitution in production are bigger than one in both industry 1 and 2, labor always flows out from the industry that capital flows into, even if both industry 1 and 2 have homothetic production technology.

Note that scale economy and monopolistic competition have influence only when non-homotheticity prevails in production. We show a result at (B.2) that is obtained by a calculation similar to (9) in a perfectly competitive world with a Cobb–Douglas utility function and constant returns technology (see Appendix B). Clearly, both results, (9) and (B.2), have the same form. The difference is only that (9) uses the “gross” elasticity of substitution s_i instead of the “pure” elasticity of substitution σ_{pi} . If the production technologies are homothetic in both industries, there are no magnification effects and (9) is equal to (B.2). This suggests that non-homotheticity is essential when we consider the influence of scale economy and monopolistic competition.

¹¹Christensen and Greene (1976) shows that the elasticity of substitution between labor and capital is smaller than one in the U.S. electric power industry.

3.3. Both Labor and Capital Can Move between Industries

We next turn to the case in which both labor and capital move between industries, and examine the stability of the long-run equilibrium. For the stability analysis, we postulate that adjustment towards the long-run equilibrium occurs according to

$$\begin{aligned}\dot{L}_1 &= a_L [W(L, K; L_1, K_1) - 1], \\ \dot{K}_1 &= a_K [R(L, K; L_1, K_1) - 1],\end{aligned}$$

where the dot signifies the time derivative and a_L and a_K are the positive adjustment parameter. Around the long-run equilibrium given by $W(L, K; L_1^*, K_1^*) = R(L, K; L_1^*, K_1^*) = 1$, this dynamic system is approximated by

$$\begin{bmatrix} \dot{L}_1 \\ \dot{K}_1 \end{bmatrix} = \begin{bmatrix} a_L W_{L_1}^* & a_L W_{K_1}^* \\ a_K R_{L_1}^* & a_K R_{K_1}^* \end{bmatrix} \begin{bmatrix} L_1 - L_1^* \\ K_1 - K_1^* \end{bmatrix}, \quad (10)$$

where $W_{L_1} \equiv \partial W / \partial L_1$, $W_{K_1} \equiv \partial W / \partial K_1$, $R_{L_1} \equiv \partial R / \partial L_1$, and $R_{K_1} \equiv \partial R / \partial K_1$.

The long-run equilibrium is locally stable if and only if the determinant of the coefficient matrix of (10) is positive, that is, $W_{L_1}^* R_{K_1}^* > W_{K_1}^* R_{L_1}^*$. We already know that the signs of W_{L_1} and R_{K_1} are negative. On the other hand, the signs of W_{K_1} and R_{L_1} cannot be determined in general. Even so, we can determine the sign of $W_{L_1} R_{K_1} - W_{K_1} R_{L_1}$ because, from (9), we obtain

$$\begin{aligned}\text{sign}(W_{L_1} R_{K_1} - W_{K_1} R_{L_1}) &= \text{sign}(\sigma_A), \\ \sigma_A &\equiv |\lambda| |\theta| + (\theta_{L_1} \lambda_{K_1} + \theta_{K_1} \lambda_{L_1}) s_1 + (\theta_{L_2} \lambda_{K_2} + \theta_{K_2} \lambda_{L_2}) s_2,\end{aligned} \quad (11)$$

where $|\theta|$ and $|\lambda|$ are defined by $|\theta| \equiv \theta_{K_1} \theta_{L_2} - \theta_{L_1} \theta_{K_2}$ and $|\lambda| \equiv \lambda_{K_1} \lambda_{L_2} - \lambda_{L_1} \lambda_{K_2}$, respectively. Jones (1965) calls σ_A the *aggregate elasticity of substitution*. However, to define σ_A , the pure elasticity of substitution σ_{pi} is used in Jones (1965) instead of the gross elasticity of substitution s_i . If production technologies in both industries are homothetic, there are no magnification effects and s_i is equal to σ_{pi} . Note that elasticity of substitution in consumption between X_1 and X_2 is unity. In addition, it is satisfied $|\lambda| |\theta| + (\theta_{L_1} \lambda_{K_1} + \theta_{K_1} \lambda_{L_1}) + (\theta_{L_2} \lambda_{K_2} + \theta_{K_2} \lambda_{L_2}) = 1$. Thus, it is seen that the aggregate elasticity of substitution is a weighted average of the elasticity of substitution in production and demand.

Since $|\lambda||\theta|$ is always non-negative at the long-run equilibrium, $W_{L_1}^* R_{K_1}^* - W_{K_1}^* R_{L_1}^*$ is always positive, which yields the following proposition.¹²

Proposition 1. *The long-run equilibrium is always locally stable.*

Hence, around the long-run equilibrium, inter-sectoral factor mobility adjusts the short-run equilibrium determined by (2), (3), (7), and (8) to the long-run equilibrium, regardless of the degree of scale economy and non-homotheticity in production. This is surprising result because scale economy and non-homotheticity usually serve as drivers that make equilibrium unstable. Chao and Takayama (1990) actually show that when the degrees of scale economy and non-homotheticity are strong, the free entry-exit of firms in monopolistically competitive industries may not lead to the Chamberlinian tangency solution characterized by the zero profit condition.¹³ Proposition 1 allows us to mitigate botheration which arises when we consider stability of equilibrium in a monopolistically competitive world.

The stability of the long-run equilibrium does not depend on the degrees of scale economy and non-homotheticity, but they play key roles in the uniqueness of the long-run equilibrium and adjustment process. The following proposition is about uniqueness.

Proposition 2. *Assume that at any long-run equilibrium, either condition (a) or (b) is satisfied, where:*

$$(a) : s_1 < 1 \text{ and } s_2 < 1,$$

$$(b) : s_1 > 1 \text{ and } s_2 > 1.$$

Then, the long-run equilibrium given by $W(L_1^, K_1^*; L, K) = R(L_1^*, K_1^*; L, K) = 1$ is unique if it exists.*

¹²Through the seminal work of Jones (1965), $|\lambda|$ and $|\theta|$ have become widely used in the literature of international trade theory.

¹³Chao and Takayama (1990) construct a two-sector model with a perfectly competitive sector, a monopolistically competitive sector, and non-homothetic production function and they verify the stability of the Chamberlinian tangency solution. They derive a necessary and sufficient condition for stability which strongly depends on the degrees of scale economy and non-homotheticity. For details, see proposition 2 and the appendix of Chao and Takayama (1990).

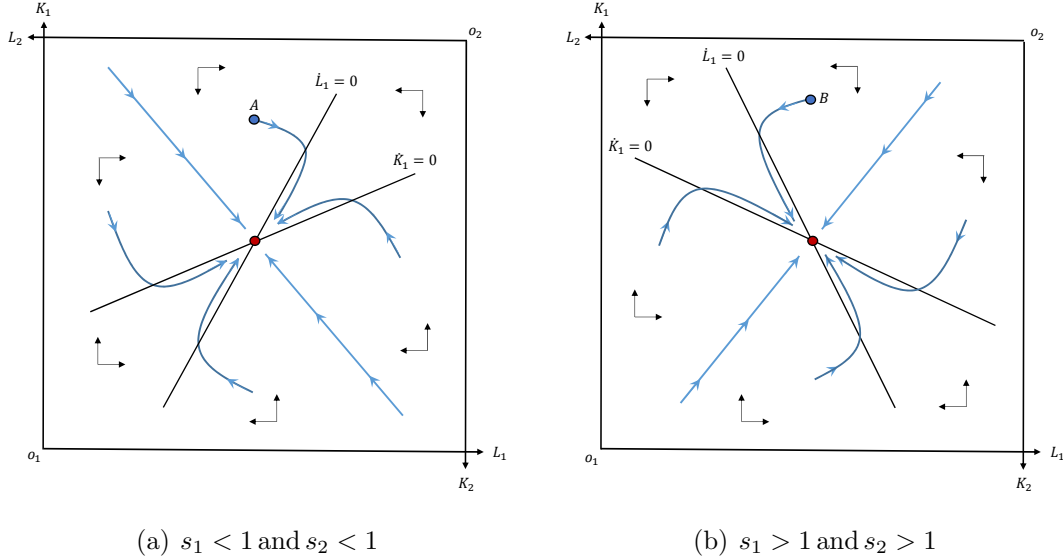


Figure 2

Proof See Appendix C.

We can determine the sign of W_{K_1} and R_{L_1} unambiguously by either condition (a) or (b) of Proposition 2. If the pure elasticities of substitution, σ_{p1} and σ_{p2} , are bigger than one, then the gross elasticities of substitution, s_1 and s_2 , are also bigger than one, and hence, condition (b) must be satisfied, regardless of the degree of scale economy, ϕ_i , and non-homotheticity, δ_i , in production. Even if the pure elasticities of substitution are smaller than one, the large magnification effects that are caused by large ϕ_i and δ_i may make s_i bigger than one. To satisfy condition (a), the pure elasticities of substitution and magnification effects must be sufficiently small. Clearly, in the non-homothetic case, the large-scale economy works to hold condition (b). At the Chamberlinian tangency solution, the degree of scale economy, ϕ_i , is equal to the markup $1/\rho_i$. Thus, satisfying condition (b) becomes easy for the smaller elasticity of substitution between varieties in consumption, which is represented by smaller ρ_i , and leads to larger the markups.

Figure 2 describes a phase diagram of the adjustment process around the long-run equilibrium corresponding to conditions (a) and (b) of Proposition 2. Consider the case $s_1 < 1$ and $s_2 < 1$, which implies $W_{K_1} > 0$ and $R_{L_1} > 0$. Point A in Figure 2(a) corresponds to a deviation from the long-run equilibrium, in which capital is excessively employed in industry

1. This case is equivalent to exogenous movement of capital from industry 2 to industry 1. At first, because movement of capital from industry 2 to industry 1 makes the wage of industry 1 relatively high compared to that of industry 2, labor flows into industry 1 (see Figure 1), while capital flows out due to $R_{K_1} < 0$. After crossing line $\dot{L}_1 = 0$, both labor and capital flow out from industry 1 until allocations of labor and capital between industries reach the long-run equilibrium. This case arises when the pure elasticities of substitution in production are smaller than 1 and the degrees of non-homotheticity and scale economy in production are sufficiently small (i.e., σ_{pi} , δ_i , and ϕ_i are sufficiently small).

On the other hand, if $s_1 > 1$ and $s_2 > 1$ hold, we have $W_{K_1} < 0$ and $R_{L_1} < 0$, which arises when σ_{pi} is bigger than 1 or δ_i and ϕ_i are sufficiently large. Point B in Figure 2(b) corresponds to a deviation from the long-run equilibrium as point A in Figure 2(a). In contrast to the case $s_1 < 1$ and $s_2 < 1$, labor flows out, at first, from industry 1, since movement of capital from industry 2 to industry 1 makes the wage of industry 1 relatively low compared to that of industry 2. After crossing line $\dot{L}_1 = 0$, labor flows into industry 1 while capital continues to flow out.

Therefore, according to which of conditions (a) or (b) of Proposition 2 is satisfied, the adjustment process changes sharply, even if the long-run equilibrium is always locally stable without any conditions. As shown earlier in this subsection, condition (b) is easy to be satisfied more in circumstances in which large degrees of non-homotheticity and scale economy prevail. In the homothetic case, only the pure elasticity of substitution is important to identify the adjustment process.

4. Capital Inflow from Outside the Economy

We consider the effects of capital inflows from outside of economy—effects of exogenous increase of capital endowment. In this case, the initial long-run equilibrium is broken, and factors are reallocated between industries toward a new long-run equilibrium. In this section, we assume that marginal cost does not decrease as output increases (i.e., $\varepsilon_i \geq 0$) and both factor is superior (i.e., $\mu_{Li} > 0$ and $\mu_{Ki} > 0$).

4.1. Definition of Marginal (Average) Factor Intensity

The long-run equilibrium is given by (2), (3), (8), and

$$\begin{aligned} N_1 \frac{\partial C_1(w, r, x_1)}{\partial w} + N_2 \frac{\partial C_2(w, r, x_2)}{\partial w} &= L, \\ N_1 \frac{\partial C_1(w, r, x_1)}{\partial r} + N_2 \frac{\partial C_2(w, r, x_2)}{\partial r} &= K. \end{aligned}$$

Taking the total differential of the long-run equilibrium condition yields

$$\begin{bmatrix} -\varsigma_L & \varsigma_L & 0 & \lambda_{L1}\mu_{L1} & \lambda_{L2}\mu_{L2} & \lambda_{L1} & \lambda_{L2} \\ \varsigma_K & -\varsigma_K & 0 & \lambda_{K1}\mu_{K1} & \lambda_{K2}\mu_{K2} & \lambda_{K1} & \lambda_{K2} \\ \tau_{L1} & \tau_{K1} & -1 & \varepsilon_1 & 0 & 0 & 0 \\ \tau_{L2} & \tau_{K2} & 0 & 0 & \varepsilon_2 & 0 & 0 \\ \theta_{L1} & \theta_{K1} & -1 & 1/\phi_1 - 1 & 0 & 0 & 0 \\ \theta_{L2} & \theta_{K2} & 0 & 0 & 1/\phi_2 - 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{w} \\ \hat{r} \\ \hat{p}_1 \\ \hat{x}_1 \\ \hat{x}_2 \\ \hat{N}_1 \\ \hat{N}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ \hat{K} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (12)$$

where ς_L , ς_K , τ_{Li} , and τ_{Ki} are defined by

$$\begin{aligned} \varsigma_L &\equiv \lambda_{L1}\theta_{K1}\sigma_{p1} + \lambda_{L2}\theta_{K2}\sigma_{p2}, & \varsigma_K &\equiv \lambda_{K1}\theta_{L1}\sigma_{p1} + \lambda_{K2}\theta_{L2}\sigma_{p2}, \\ \tau_{Li} &\equiv \frac{w_i}{\partial C_i/\partial x_i} \frac{\partial^2 C_i}{\partial w_i \partial x_i}, & \tau_{Ki} &\equiv \frac{r_i}{\partial C_i/\partial x_i} \frac{\partial^2 C_i}{\partial r_i \partial x_i}, \quad i = 1, 2. \end{aligned}$$

τ_{Li} (τ_{Ki}) represents the partial elasticities of marginal cost for the wage (rental) rate in each industry i , which is in the following relationship with μ , θ , and ϕ :

$$\tau_{ji} = \phi_i \theta_{ji} \mu_{ji}, \quad i = 1, 2, \quad j = L, K.$$

Thus, it is satisfied that $\tau_{Li} + \tau_{Ki} = 1$. In particular, τ_{ji} must be equal to θ_{ji} in the homothetic case.

Let us define $|\mu|$ by $|\mu| \equiv \mu_{K1}\mu_{L2} - \mu_{L1}\mu_{K2}$. Using $|\mu|$, we introduce the concept of *marginal factor intensity*. This concept was introduced first by Jones (1968). Consider the isoquant map measuring capital on the vertical axis. Then, industry i is said to be marginally capital intensive (labor intensive) if its expansion path is steeper (gentler) at the production point than that of industry j . This means, intuitively, that an expansion of output in industry i would require extra capital (labor) per unit of extra labor (capital)

compared to industry j . Following Chao and Takayama (1990), we capture this concept by the following definition: *Industry 1 is marginally capital-intensive compared to industry 2 if and only if $|\mu| > 0$, while industry 1 is marginally labor-intensive compared to industry 2, if and only if $|\mu| < 0$.*

If industry j has homothetic technology, this definition implies that industry i is marginally capital intensive if $\mu_{Ki} > \mu_{Li}$. This is equivalent to the condition that industry i becomes marginally capital intensive in Chao and Takayama (1990), in which industry j is characterized by perfect competition and constant returns technology. Thus, our definition of marginal factor intensity includes that of Chao and Takayama (1990).

Consider the isoquant map again. We measure capital on the vertical axis. In contrast to the concept of marginal factor intensity, following Jones (1968) and Chao and Takayama (1990), we call industry i capital intensive (labor intensive) compared to industry j in an *average sense* if industry i has a steeper (gentler) ray from the origin passing the production point than that of industry j .¹⁴ According to this concept, we use the following definition: *Industry 1 is capital intensive compared to industry 2 in an average sense, if and only if $K_1/L_1 > K_2/L_2$ while industry 1 is labor intensive compared to industry 2 in average sense, if and only if $K_1/L_1 < K_2/L_2$.*

4.2. Effects on Prices

First, we explore the effects on the price side of the economy. Solving (12), we obtain

$$\begin{aligned}\frac{\hat{w}}{\hat{K}} &= \frac{\theta_{K2}}{\sigma_A} \left(\frac{(\phi_2 - 1)\theta_{L2}(\mu_{K2} - \mu_{L2})}{\eta_2} + 1 \right) = \frac{\theta_{K2}}{\sigma_A} \frac{(\phi_2 - 1)\mu_{K2} + \varepsilon_2}{\eta_2} > 0, \\ \frac{\hat{r}}{\hat{K}} &= \frac{\theta_{L2}}{\sigma_A} \left(\frac{(\phi_2 - 1)\theta_{K2}(\mu_{K2} - \mu_{L2})}{\eta_2} - 1 \right) = -\frac{\theta_{L2}}{\sigma_A} \frac{(\phi_2 - 1)\mu_{L2} + \varepsilon_2}{\eta_2} < 0,\end{aligned}\tag{13}$$

Similarly, we can derive the following relationship:

$$\begin{aligned}\frac{\hat{w} - \hat{p}_1}{\hat{K}} &= \frac{\theta_{K1}}{\sigma_A} \frac{(\phi_1 - 1)\mu_{K1} + \varepsilon_1}{\eta_1} > 0, \\ \frac{\hat{r} - \hat{p}_1}{\hat{K}} &= -\frac{\theta_{L1}}{\sigma_A} \frac{(\phi_1 - 1)\mu_{L1} + \varepsilon_1}{\eta_1} < 0,\end{aligned}\tag{14}$$

¹⁴In traditional trade theory, this case is just called as industry i is capital intensive (labor intensive) compared to industry j .

where we use

$$\begin{aligned} \frac{\hat{p}_1}{\hat{K}} &= \frac{1}{\sigma_A} \left(\frac{\theta_{K2}}{\eta_2} \left((\phi_2 - 1)\mu_{K2} + \varepsilon_2 \right) - \frac{\theta_{K1}}{\eta_1} \left((\phi_1 - 1)\mu_{K1} + \varepsilon_1 \right) \right) \\ &= -\frac{1}{\sigma_A} \left(|\theta| + (\phi_1 - 1) \frac{\theta_{K1}\theta_{L1}(\mu_{K1} - \mu_{L1})}{\eta_1} + (\phi_2 - 1) \frac{\theta_{K2}\theta_{L2}(\mu_{L2} - \mu_{K2})}{\eta_2} \right). \end{aligned} \quad (15)$$

(13) and (14) indicate the effect of a change of capital endowment on real factor rewards. By the inflow of capital from outside the economy, real factor rewards improve in labor but not capital regardless of the degree of scale economy, non-homotheticity, and substitutability between production factors.¹⁵ In addition, this depends on neither marginal nor average factor intensity. Thus, capital inflow always works in labor's favor but operates against capital owners. Analogously, we can see that labor inflow from outside the economy works in favor of capital owners but operates against labor.

From (15), a sufficient condition to ensure that p_1/p_2 falls is given by

$$|\theta| > 0 \quad \text{and} \quad \frac{\mu_{K1}}{\mu_{L1}} \geq 1 \geq \frac{\mu_{K2}}{\mu_{L2}}.$$

The first term in this condition states that industry 1 is capital intensive in an average sense.¹⁶ On the other hand, the second term means that industry 1 has marginally capital-biased technology while industry 2 has marginally labor-biased technology, which implies that industry 1 is marginally capital intensive compared to industry 2. Similarly, we see that a sufficient condition that p_1/p_2 rises is that industry 1 is labor intensive in an average sense and industry 1 has marginally labor-biased technology but industry 2 has marginally capital-biased technology, which implies that industry 1 is marginally labor intensive compared to industry 2. Thus, to determine whether p_1/p_2 falls or rises unambiguously, the non-homothetic bias of each industry must be towards the factor which is used intensively in an average sense. This requires that average factor intensity and marginal factor intensity move in the same direction, that is, $|\theta||\mu| \geq 0$.¹⁷

¹⁵Note that we normalize p_2 to unity.

¹⁶Note that the condition that industry 1 is capital intensive (labor intensive) compared to industry 2 is equivalent to the condition that $|\theta|$ is positive (negative).

¹⁷A similar result is obtained by Horn (1983) in which there is only one differentiated goods industry.

The effect of a change of capital endowment on P_1/P_2 can be revealed by (15) and

$$\begin{aligned}\frac{\hat{N}_1}{\hat{K}} &= \frac{1}{\sigma_A} \left(\lambda_{L1}\theta_{K1}s_1 + \lambda_{L2}\theta_{K2}s_2 + |\theta|\lambda_{L2} + \frac{\theta_{K1}\theta_{L1}(\mu_{L1} - \mu_{K1})}{\eta_1} \right), \\ \frac{\hat{N}_2}{\hat{K}} &= \frac{1}{\sigma_A} \left(\lambda_{L1}\theta_{K1}s_1 + \lambda_{L2}\theta_{K2}s_2 - |\theta|\lambda_{L1} - \frac{\theta_{K2}\theta_{L2}(\mu_{K2} - \mu_{L2})}{\eta_2} \right).\end{aligned}\tag{16}$$

(15) and (16) yield the following equation:

$$\begin{aligned}\frac{\hat{P}_1 - \hat{P}_2}{\hat{K}} &= \frac{\hat{p}_1}{\hat{K}} + \frac{1}{1 - \sigma_1} \frac{\hat{N}_1}{\hat{K}} - \frac{1}{1 - \sigma_2} \frac{\hat{N}_2}{\hat{K}} \\ &= -\frac{1}{\sigma_A} \left(|\theta|(\lambda_{L1}\phi_2 + \lambda_{L2}\phi_1) + (\phi_1 - \phi_2)(\lambda_{L1}\theta_{K1}s_1 + \lambda_{L2}\theta_{K2}s_2) \right),\end{aligned}\tag{17}$$

which states that if industry i is capital intensive in an average sense and has a greater degree of scale economy than that of industry j , an inflow of capital from outside the economy lowers the price index of industry i relative to that of industry j . We have seen that the degree of scale economy ϕ_i is equal to the markup $1/\rho_i$ at the Chamberlinian tangency solution. Thus, a large markup that is caused by lower substitutability between varieties works as a device that lowers the price index relatively due to an increase in production resources.

4.3. Effects on Output

Next, let us turn to the effect of the capital inflow on output. It is easy to show that (13) and (14) give

$$\begin{aligned}\frac{\hat{x}_1}{\hat{K}} &= \frac{\phi_1}{\phi_1 - 1} \left(\theta_{L1} \frac{\hat{w} - \hat{p}_1}{\hat{K}} + \theta_{K1} \frac{\hat{r} - \hat{p}_1}{\hat{K}} \right) = \frac{\phi_1}{\sigma_A} \frac{\theta_{L1}\theta_{K1}(\mu_{K1} - \mu_{L1})}{\eta_1}, \\ \frac{\hat{x}_2}{\hat{K}} &= \frac{\phi_2}{\phi_2 - 1} \left(\theta_{L2} \frac{\hat{w}}{\hat{K}} + \theta_{K2} \frac{\hat{r}}{\hat{K}} \right) = \frac{\phi_2}{\sigma_A} \frac{\theta_{L2}\theta_{K2}(\mu_{K2} - \mu_{L2})}{\eta_2}.\end{aligned}\tag{18}$$

This simply shows that a single firm's size in an industry that has marginally capital-biased (labor-biased) technology increases (decreases) with an increase in capital endowment. Factor intensity and scale economy do not influence the sign of \hat{x}_i . The sign of \hat{x}_i mainly depends on the direction of non-homothetic bias. In particular, in the homothetic case, firm size never changes even if factor endowments change.

On the other hand, what can we say about the relative industry size? It is well known that an increase in capital endowment makes the capital-intensive industry relatively large in the usual Heckscher–Ohlin framework, which is known as the Rybczynski theorem. However,

in our model, we characterize all industries by monopolistic competition instead of perfect competition. In addition, there are scale economies and non-homotheticity in all industries. Thus, an increase in capital endowment need not to always increase capital intensive industry compared to labor intensive industry. We now check this point.

Let us measure industry i 's size, Y_i , by the sum of output, that is, $Y_i \equiv N_i x_i$. Then, (8) reveals that the change in the ratio of industry size, Y_1/Y_2 , is negatively related to the change of p_1 , as follows:

$$\frac{\hat{Y}_1 - \hat{Y}_2}{\hat{K}} = -\frac{\hat{p}_1}{\hat{K}}. \quad (19)$$

We cannot see whether an inflow of capital from outside the economy expands the capital-intensive industry rather than the labor-intensive industry, in general. To determine the sign of $\hat{Y}_1 - \hat{Y}_2$ unambiguously, we need to suppose that each industry's non-homothetic bias is toward the factor that is used intensively in an average sense (see (15)). As already shown, this condition requires that average and marginal factor intensity move in the same direction, that is, $|\theta||\mu| \geq 0$. Thus, non-homotheticity plays an essential role in the direction of the change in the ratio of industry size. If both industries have homothetic technology, the capital-intensive industry always becomes bigger relatively to the labor-intensive industry due to the inflow of capital. This case makes (19) equivalent to the equation (11) of Jones (1965).

This result is obtained measuring industry size by sum of output. However, it may be considered that we should measure industry size by the quantity index instead of the sum of output, because we regard P_i as a price index of industry i . Thus, we also check the effect of the capital inflow on the ratio of industry size measuring industry i 's size by the quantity index X_i .

The preference on the composite indexes of differentiated goods, X_1 and X_2 , is defined by the Cobb–Douglas form. Thus, the change in relative industry size is given as follows:

$$\frac{\hat{X}_1 - \hat{X}_2}{\hat{K}} = -\frac{\hat{P}_1 - \hat{P}_2}{\hat{K}}.$$

From (17), we can see that if the gap of the degree of scale economies (markup) between industries is small, the economy has a relatively large capital intensive industry as capital endowment increases. On the other hand, if the gap of factor intensity between industries

is small, then an increase in capital makes the industry that has a larger degree of scale economies (markup) relatively large. Thus, the relative size of the two industries is determined by a tug of war of two gaps (factor intensity and degree of scale economies). Unlike the case in which we measure industry size by sum of output, the gap of degree of scale economies (markup) plays an essential role rather than non-homotheticity. In fact, even if the technologies of both industries are homothetic, the sign of the change in the relative size of the industries still depends on the gap of degrees of scale economies (markup) between industries, but if there is no gap of degrees of scale economies (markup), the inflow of capital from outside the economy always increases the size of the capital-intensive industry relative to that of the labor-intensive industry.

4.4. Effects on Factor Allocation

Finally, we turn to the effect of the capital inflow from outside the economy on factor allocation. From here, without loss of generality, we assume that industry 1 is labor intensive in an average sense compared to industry 2, that is, $K_1/L_1 < K_2/L_2$. This implies that both $|\theta|$ and $|\lambda|$ are negative.

We first check the change of labor allocation. Because $L_1 = N_1(\partial C_1/\partial w)$ and labor endowments do not change, using (13), (16) and (18) yields the following relationship:

$$\begin{aligned} \frac{\hat{L}_1}{\hat{K}} &= -\frac{\lambda_{L2}}{\lambda_{L1}} \frac{\hat{L}_2}{\hat{K}} = \frac{\hat{N}_1}{\hat{K}} + \mu_{L1} \frac{\hat{x}_1}{\hat{K}} + \theta_{K1} \sigma_{p1} \frac{\hat{r} - \hat{w}}{\hat{K}} \\ &= \frac{\lambda_{L2}}{\sigma_A} \left(|\theta| (1 - s_1) + \theta_{K2} (s_2 - s_1) \right) \\ &= \frac{\lambda_{L2}}{\sigma_A} \left(|\theta| + \theta_{K1} s_2 \left(\frac{\theta_{K2}}{\theta_{K1}} - \frac{s_1}{s_2} \right) \right). \end{aligned}$$

Therefore, we can determine the signs of \hat{L}_1/\hat{K} and \hat{L}_2/\hat{K} unambiguously if either condition (a)' or (b)' given by

$$(a)' : s_2 \leq s_1 < 1,$$

$$(b)' : s_2 \geq s_1 > 1,$$

is satisfied. When condition (a)' is satisfied, then labor moves from industry 1, which is labor intensive, to industry 2, which is capital intensive, due to the increase of capital endowment, while labor moves from industry 2 to industry 1 when condition (b)' is satisfied. The latter

result may be surprising because an increase in capital endowments usually expands capital-intensive industry (i.e., industry 2).

Similarly, we can determine the signs \hat{L}_1/\hat{K} and \hat{L}_2/\hat{K} if

$$(c) : |\theta| \left(\frac{\theta_{K2}}{\theta_{K1}} - \frac{s_1}{s_2} \right) > 0$$

is satisfied. If this condition is satisfied, an increase in capital endowments causes labor movement from the labor-intensive industry to the capital-intensive industry. Because we assume that industry 2 is capital intensive in an average sense, $|\theta|$ is negative. This means that condition (c) requires $\theta_{K2}/\theta_{K1} < s_1/s_2$. Thus, s_1 must be larger than s_2 , because θ_{K2}/θ_{K1} is bigger than one. Therefore, for an increase in capital endowments to cause labor movement to the capital-intensive industry, the labor-intensive industry must have a sufficiently large gross elasticity of substitution between factors compared to the capital-intensive industry. This is because the increase in capital endowments makes the capital price relatively lower than the labor price, which causes a substitute from labor to capital in each industry. If the gross elasticity of substitution is too large in the capital intensive industry, then labor moves from the capital-intensive industry to the labor-intensive industry.

We turn to capital allocation. When industry 2 is capital intensive in an average sense, we can state that an increase in capital endowments always increases the capital employment of industry 2, because we have

$$\frac{\hat{K}_2}{\hat{K}} = \frac{s_2}{\sigma_A} + \frac{\hat{L}_2}{\hat{K}} = \frac{\lambda_{L2}}{\sigma_A} s_2 + \frac{\lambda_{L1}}{\sigma_A} (\theta_{K1} s_1 + \theta_{L2} s_2 - |\theta|),$$

which means that the capital-labor ratio in industry 2, k_2 , always rises. On the other hand, we cannot determine whether the capital employment of industry 1 increases or decreases, while the capital-labor ratio in industry 1 must rise, which is shown by

$$\frac{\hat{K}_1 - \hat{L}_1}{\hat{K}} = \frac{s_1}{\sigma_A}.$$

Figure 3 depicts the shift and adjustment process from the initial long-run equilibrium to the new one in the Edgeworth box, which is accompanied by the change in capital endowments under conditions (a)' and (b)'. The origin of industry 2 shifts from O_2 to O_2' in response to the increase in capital endowments. If capital-labor ratios in both industries do

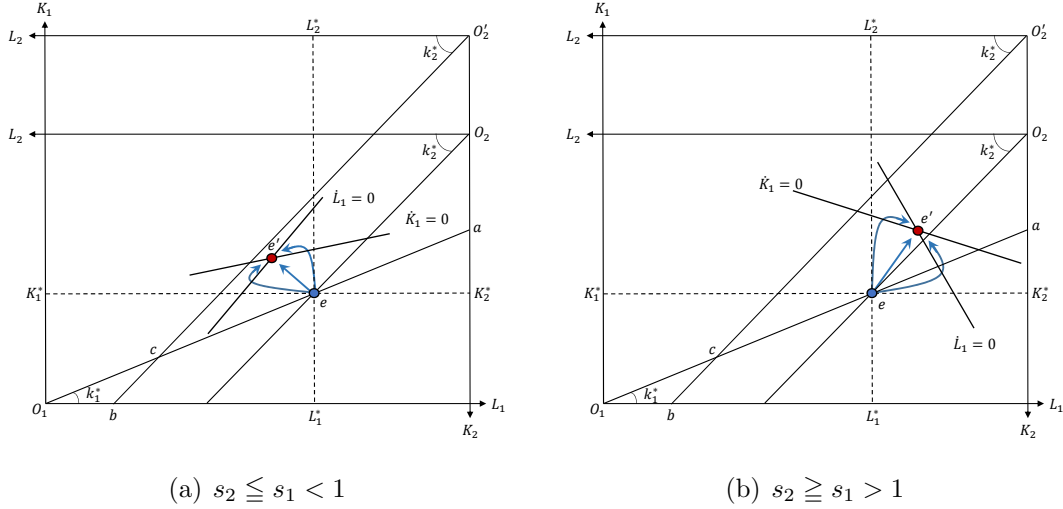


Figure 3

not change, then the long-run equilibrium shifts from the initial one, which is given by point e , to point c . This shift of the production point corresponds to the Rybczynski theorem which is derived by holding prices constant. However, in our model, prices must adjust to the change in factor endowments. In this case, an increase in capital endowments raises the capital–labor ratio k_i in both industries regardless of the degree of scale economies and non-homotheticity. Thus, the new equilibrium, which is given by point e' , must be in the area between the lines O_1a and $O_2'b$. Figure 3(a) corresponds to condition (a)', in which labor moves to the capital-intensive industry 2 from the labor-intensive industry 1.¹⁸ Note that condition (a)' implies condition (a). Thus, the adjustment path to the new long-run equilibrium becomes that which corresponds to Figure 2(a). On the other hand, under condition (b)', labor moves from the capital-intensive industry 2 to the labor-intensive industry 1, which is described in Figure 3(b). Because, condition (b)' implies the condition (b), the adjustment path to the new long-run equilibrium becomes the same as Figure 2(b).

Obviously, the effects of the change in factor endowments on factor allocation do not depend on technology bias and factor intensity in a marginal sense. The effects on factor allocation depend mainly on the “gross” elasticity of substitution between factors and factor

¹⁸Note that the employment of capital in industry 1 may decrease under condition (a)'.

intensity in an average sense. In particular, it should be noted that if the “gross” elasticities between factors are large in both industries and these differences between industries is small, which is a condition to rule out factor-intensity reversals, the shares of capital and labor, λ_{K_i} and λ_{L_i} , may increase in the labor-intensive industry due to increasing in capital endowments. In fact, if we set $s_1 = s_2 > 1$, then we obtain

$$\frac{\hat{L}_1}{\hat{K}_1} = \frac{\lambda_{L2}}{\sigma_A} \left(|\theta| (1 - s_1) + \theta_{K2} (s_2 - s_1) \right) > 0,$$

$$\frac{\hat{\lambda}_{K1}}{\hat{K}} = \frac{\lambda_{K2}}{\sigma_A} \left(|\theta| (1 - s_1) + \theta_{L2} (s_1 - s_2) \right) > 0,$$

where industry 1 is a labor-intensive industry in an average sense. In this case, an increase in capital endowments may make the labor-intensive industry relatively large measured by the input of factors, while it makes the capital-intensive industry relatively large measured by output.

5. Conclusion

In this study, we constructed a two-industry, two-factor model to analyze factor allocation among industries. In particular, by characterizing both industries as the Dixit–Stiglitz type of monopolistic competition and allowing a general form of the production function, we examined the effects of scale economies and non-homotheticity of production technology on factor allocation.

The results showed that the long-run equilibrium characterized by factor-price equalization between industries is always locally stable without any condition on scale economy, non-homotheticity, and factor intensity. This result stands in sharp contrast to the fact that the non-homothetic bias toward the factor that is used intensively on average is a key condition to ensure the stability of the long-run equilibrium characterized by the zero-profit condition, which is in line with Chao and Takayama (1987, 1990). In addition, the direction of non-homothetic bias is not important for determining the direction of the change in factor allocation. The direction of the change in factor allocation caused by change in factor endowments mainly depends on the gross substitutability of factors, which is magnified by non-homotheticity and scale economies. For example, if the gross elasticity of substitution

between factors is less than one in all industries and becomes large in the labor-intensive industry compared to the capital-intensive industry, then an increase in capital endowments causes labor to move to the capital-intensive industry from the labor-intensive industry. It should be noted that if the gross elasticity between factors is bigger than one in both industries and has only small a difference between industries, which is a condition for absence of factor-intensity reversals, then the increase in capital endowments may make the labor-intensive industry relatively large measured by input of factors. These results suggest that if factors can move between countries or regions as in the framework of the new economic geography, industrial structure and agglomeration pattern may strongly depend on the condition of the gross substitutability of factors.

The analysis of this study is limited as there is only one region. To address the allocation of factors between regions, we must establish multiple regions. Expanding the model into multiple regions will give us more interesting views like the agglomeration pattern. However, if there are multiple regions, we need to consider trade, which makes the analysis difficult because the economy is not closed in one region. We leave it for future research to address the issue of multiple regions.

Appendix A.

Taking the total differential of the short-run equilibrium condition given by (2), (3), (7), and (8), we obtain

$$J \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \\ \hat{r}_1 \\ \hat{r}_2 \\ \hat{p}_1 \\ \hat{x}_1 \\ \hat{x}_2 \\ \hat{N}_1 \\ \hat{N}_2 \end{bmatrix} = \begin{bmatrix} \hat{L}_1 \\ \frac{1}{\lambda_{L2}}(\hat{L} - \lambda_{L1}\hat{L}_1) \\ \hat{K}_1 \\ \frac{1}{\lambda_{K2}}(\hat{K} - \lambda_{K1}\hat{K}_1) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (\text{A.1})$$

where the circumflex denotes the rate of change in each variables (e.g., $\hat{w}_i \equiv dw_i/w_i$), and J is the Jacobian matrix given by

$$J = \begin{bmatrix} -\theta_{K1}\sigma_{p1} & 0 & \theta_{K1}\sigma_{p1} & 0 & 0 & \mu_{L1} & 0 & 1 & 0 \\ 0 & -\theta_{K2}\sigma_{p2} & 0 & \theta_{K2}\sigma_{p2} & 0 & 0 & \mu_{L2} & 0 & 1 \\ \theta_{L1}\sigma_{p1} & 0 & -\theta_{L1}\sigma_{p1} & 0 & 0 & \mu_{K1} & 0 & 1 & 0 \\ 0 & \theta_{L2}\sigma_{p2} & 0 & -\theta_{L2}\sigma_{p2} & 0 & 0 & \mu_{K2} & 0 & 1 \\ \tau_{L1} & 0 & \tau_{K1} & 0 & -1 & \varepsilon_1 & 0 & 0 & 0 \\ 0 & \tau_{L2} & 0 & \tau_{K2} & 0 & 0 & \varepsilon_2 & 0 & 0 \\ \theta_{L1} & 0 & \theta_{K1} & 0 & -1 & 1/\phi_1 - 1 & 0 & 0 & 0 \\ 0 & \theta_{L2} & 0 & \theta_{K2} & 0 & 0 & 1/\phi_2 - 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{bmatrix},$$

where τ_{Li} and τ_{Ki} are given by

$$\tau_{Li} \equiv \frac{w_i}{\partial C_i / \partial x_i} \frac{\partial^2 C_i}{\partial w_i \partial x_i}, \quad \tau_{Ki} \equiv \frac{r_i}{\partial C_i / \partial x_i} \frac{\partial^2 C_i}{\partial r_i \partial x_i}, \quad i = 1, 2.$$

θ , μ , τ , and ϕ have the following relationship:

$$\tau_{ji} = \phi_i \theta_{ji} \mu_{ji}, \quad i = 1, 2, \quad j = L, K.$$

Thus, $\tau_{Li} + \tau_{Ki} = 1$ ($i = 1, 2$). Solving (A.1), then, we obtain (9).

Appendix B.

If the economy is in a perfectly competitive world, the utility function is Cobb–Douglas and constant returns to scale prevail in both industries, then (A.1) is rewritten as follows:

$$\begin{bmatrix} -\theta_{K1}\sigma_{p1} & 0 & \theta_{K1}\sigma_{p1} & 0 & 0 & 1 & 0 \\ 0 & -\theta_{K2}\sigma_{p2} & 0 & \theta_{K2}\sigma_{p2} & 0 & 0 & 1 \\ \theta_{L1}\sigma_{p1} & 0 & -\theta_{L1}\sigma_{p1} & 0 & 0 & 1 & 0 \\ 0 & \theta_{L2}\sigma_{p2} & 0 & -\theta_{L2}\sigma_{p2} & 0 & 0 & 1 \\ \theta_{L1} & 0 & \theta_{K1} & 0 & -1 & 0 & 0 \\ 0 & \theta_{L2} & 0 & \theta_{K2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} \hat{w}_1 \\ \hat{w}_2 \\ \hat{r}_1 \\ \hat{r}_2 \\ \hat{p}_1 \\ \hat{X}_1 \\ \hat{X}_2 \end{bmatrix} = \begin{bmatrix} \hat{L}_1 \\ \frac{1}{\lambda_{L2}}(\hat{L} - \lambda_{L1}\hat{L}_1) \\ \hat{K}_1 \\ \frac{1}{\lambda_{K2}}(\hat{K} - \lambda_{K1}\hat{K}_1) \\ 0 \\ 0 \\ 0 \end{bmatrix}. \quad (\text{B.1})$$

Solving (B.1), then, we obtain

$$\begin{aligned}
\frac{L_1}{W} \frac{\partial W}{\partial L_1} &= -\frac{1}{\lambda_{L_2}} \left[1 + \lambda_{L_1} \theta_{K_2} \left(\frac{1}{\sigma_{p_2}} - 1 \right) + \lambda_{L_2} \theta_{K_1} \left(\frac{1}{\sigma_{p_1}} - 1 \right) \right] < 0, \\
\frac{K_1}{R} \frac{\partial R}{\partial K_1} &= -\frac{1}{\lambda_{K_2}} \left[1 + \lambda_{K_1} \theta_{L_2} \left(\frac{1}{\sigma_{p_2}} - 1 \right) + \lambda_{K_2} \theta_{L_1} \left(\frac{1}{\sigma_{p_1}} - 1 \right) \right] < 0, \\
\frac{K_1}{W} \frac{\partial W}{\partial K_1} &= \frac{1}{\lambda_{K_2}} \left[\lambda_{K_1} \theta_{K_2} \left(\frac{1}{\sigma_{p_2}} - 1 \right) + \lambda_{K_2} \theta_{K_1} \left(\frac{1}{\sigma_{p_1}} - 1 \right) \right], \\
\frac{L_1}{R} \frac{\partial R}{\partial L_1} &= \frac{1}{\lambda_{L_2}} \left[\lambda_{L_1} \theta_{L_2} \left(\frac{1}{\sigma_{p_2}} - 1 \right) + \lambda_{L_2} \theta_{L_1} \left(\frac{1}{\sigma_{p_1}} - 1 \right) \right].
\end{aligned} \tag{B.2}$$

Appendix C.

$\dot{L}_1 = 0$ line, $K_1 = f(L, K; L_1)$, and $\dot{K}_1 = 0$ line, $K_1 = g(L, K; L_1)$, are defined by $W(L, K; L_1, f(L, K; L_1)) \equiv 1$ and $R(L, K; L_1, g(L, K; L_1)) \equiv 1$, respectively. Then, we obtain

$$\frac{\partial f(L, K; L_1)}{\partial L_1} = -\frac{W_{L_1}}{W_{K_1}}, \quad \frac{\partial g(L, K; L_1)}{\partial L_1} = -\frac{R_{L_1}}{R_{K_1}}.$$

Suppose that condition (a) in Proposition 2 is satisfied. Then, both W_{K_1} and R_{L_1} are positive. Since we already know both W_{L_1} and R_{K_1} are always negative (see (9)), this makes both $\partial f/\partial L_1$ and $\partial g/\partial L_1$ positive. Moreover, in this case, we obtain

$$\text{sign} \left(\frac{\partial f(L, K; L_1^*)}{\partial L_1} - \frac{\partial g(L, K; L_1^*)}{\partial L_1} \right) = \text{sign}(\sigma_A) > 0.$$

Thus, we can conclude that both the $\dot{L}_1 = 0$ and $\dot{K}_1 = 0$ lines become upward-sloping curves that intersect once at most.

On the other hand, if condition (b) in Proposition 2 is satisfied, then both W_{K_1} and R_{L_1} become negative, and thus, both the $\dot{L}_1 = 0$ and $\dot{K}_1 = 0$ lines are downward-sloping curves. Furthermore, since we obtain

$$\text{sign} \left(\frac{\partial f(L, K; L_1^*)}{\partial L_1} - \frac{\partial g(L, K; L_1^*)}{\partial L_1} \right) = -\text{sign}(\sigma_A) < 0,$$

In this case too, the $\dot{L}_1 = 0$ and $\dot{K}_1 = 0$ lines intersect once at most.

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