Discussion Paper Series

RIEB

Kobe University

DP2012-29

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Revised December 6, 2012

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Immigration Conflicts

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November 12, 2012

Abstract

Almost all existing literature assumes immigrants immediately assimilate in the receiving country. In contrast, the present paper considers the case of non-immediate assimilation, and analyzes various immigration conflicts in an overlapping generations dynamic system. We examine three types of conflicts that arise when immigrants come in: skill conflicts that affect the capital rental and also cause the wage gap to change between skilled and unskilled workers; intergenerational conflicts that lead to different impacts on the young and old generations; and distributional conflicts that affect each generation's life-time utility unequally. The degree of substitution between natives and immigrants in production plays a key role. We also analyze the welfare composition in detail generation by generation, and provide policy recommendation for each case.

Keyword : immigration, overlapping generations, inequality, welfare
JEL classification : F22

1 Introduction

The current wave of globalization is characterized by freer mobility of not only goods and capital across countries, but also freer mobility of human resources. With the launch of the European Union, NAFTA, APEC, etc., international labor migration has increased substantially. The world total stock of international migrants reached 214 million in 2010, representing about 3% of world population. Richer or bigger countries are the main hosts of these migrants, in the order of the U.S., Russia, Germany, Saudi Arabia, Canada, France and England, etc. Immigration is a routine issue in presidential campaigns in the US, Europe and Australia. Even in Japan, a country that has traditionally been closed to foreign migrants, labor shortage especially in agriculture and heavy manual work is forcing the government to reconsider immigration among other alternatives (Already small numbers of seasonal foreign workers and nurses are

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being introduced). Facing population aging, sooner or later more lenient immigration policies may have to be adopted.

Nevertheless, there is evidence that some unskilled workers, local communities and law enforcement officers are more negative on this issue. They are afraid that immigrants may compete away jobs and lower wage rates, free ride on public services such as schooling, parks, medicare, public universities and others, cause cultural conflicts, increase crime, etc., and eventually lower the net welfare of natives.[1] Especially during the current financial and economic crisis, some US law makers demand a tightening of immigration laws. As a consequence of all these, despite being an integral part of globalization, immigration is perceived negatively in public opinion. Often when related issues appear in the media, they are about illegal immigration, or some other negative images such as taking jobs away and depressing wages, etc.

A particularly relevant issue is assimilation. While most theoretical analysis in the literature has treated immigrants and natives identically, recent empirical studies have found that immigrants do not exert enough effort to absorb the mainstream culture, customs and language, even though the potential economic returns (increased productivity and enhanced earnings) to assimilation are high. It is also observed that during a recession, firms tend to lay off foreigner workers first; foreign guest workers are limited to a certain sector, or in occupations with a specific skill only.[2]

The present paper attempts to examine the above immigration issues in an overlapping generations (OLG) model, especially incorporating non-assimilation. To keep the model tractable, we consider an economy without trade but allows optimal immigration every period. Workers are divided into skilled and unskilled (also referred to as non-skilled or low skilled) groups, who are combined with capital to produce a single good, with skilled workers being more efficient. Households consume this good and an impure public good, the latter of which is financed through income taxes. To incorporate non-assimilation, we assume that non-skilled and skilled immigrants have different degrees of substitutability with natives, such that the possession of skill makes a worker less substitutable. We examine the different impacts of allowing skilled and unskilled immigration on the receiving economy.

[1] Hanson (2005) vividly documents that opinions on immigration are sharply divided in the U.S., even within each interest group, such as within the Democrat and Replican parties, labor groups, environmental groups, etc. He argues that there are two main sources of opposition to immigration: one concern is labor-market pressures and a second is about public finances. Hatton and Williamson (2005) and Hanson, Scheve and Slaughter (2007) find evidence that while OECD countries have lowered barriers to foreign trade and capital in recent decades, they have not commensurately reduced barriers to immigration.

[2] For instance, Both Lazear (1999) and Bauer et al. (2000) find assimilation incentives to be low if it is costly for immigrants to learn the receiving country's culture and language, or if immigrants' culture is strongly represented in the host country (such as by a large immigrant community); Djajic (2003) argues that immigrants assimilate at different rates which may also differ from those of their children; Stark and Fan (2006) and Fan and Stark (2007) explain that non-assimilation arises because immigrants want to keep close ties with families, friends and cultural roots at home. They even find that the richer the natives, the weaker the effort to assimilate, ceteris paribus; More recently, Harrie et al. (2008) argue that immigrants' utility will be lowered if they assimilite because they have to lose important parts of their own culture.

In particular, we find that immigration brings three types of conflicts, even under full employment:[3] i). "Skill conflicts" that cause the wage gap to change between skilled and unskilled workers; ii). "Intergenerational conflicts" that lead to different impacts on the young and old generations, stemming from changes in the capital stock, the interest rate and the impure public good; iii). "Distributional conflicts" that affect each generation's lifetime utility unequally, and in particular, the receiving generation seems to be hurt the most. The details of each conflict depends crucially on the degree of substitution between natives and immigrants.

Specifically, these conflicts stem from the following effects: A *productivity effect*–immigration changes the productivity of native skilled and unskilled workers, depending on the degree of skill complementarity, and a *capital allocation effect*–immigration increases the labor supply, lowering per-capita capital for production. The intercourse of these effects cause the high and low skilled wages to change, and thus the capital return, public-goods contribution and steady states will all change.

For instance, the arrival of immigrants increases the total scale of production, raising the productivity and wages of native skilled workers. On the other hand, the capital allocation effect lowers the wages for both skilled and unskilled workers. While the skilled wage may increase or decrease, the unskilled wage increases less or falls more, leading to a "skill conflict".

Intergenerational conflicts occur when different generations may face different levels of capital stock, interest rates and levels of public goods. Immigration seems to benefit the old generation more, because they earn income from savings while the young generation earns wage income, which could lead to conflicts between workers and 'capital owners'; Thus, while old cohorts may enjoy cheaper products made by immigrants, young natives tend to worry about their jobs and wages being 'competed away'.

Also we find that distributional conflicts arise among cohorts (both young and old) living in periods $t-1$, t and $t+1$. To be specific, the generations living in $t-1$ and t enjoy the same level of capital stock because the economy stays in the same steady state when immigrants arrive in period t. However, their utilities differ due to the wage difference caused by an increase in the number of workers in period t. Further, the capital stock changes after t, leading to more conflicts.

Our strategy is to divide the impacts of immigration into three cases. In Case I, both the high and low skilled wages increase and the capital stock also rises. In Case II, the exact opposite occurs such that both wages fall. In Case III, the high skilled wage rises but the low skilled wage falls. Subsequently in each case, we calculate the welfare changes for the skilled and unskilled natives respectively. Using these comparisons, we determine whether a particular immigration conflict is likely to arise or not. And finally, depending on the source and type of conflict, we provide some policy recommendations.

By conventional wisdom, natives worry that immigration may lower wages. We amplify the problem by assuming that the wage effect dominates other effects (e.g., the capital-stock effect, the interest-rate effect, the public-goods effect). Nevertheless,

[3] One might naturally think that immigration could reduce native employment in some sectors, which certainly causes conflicts. In the present paper, our concern is that even with full employment of both natives and immigrants, various conflicts still arise.

we find that immigration can benefit the receiving country in many cases. But conflicts can still arise.

The various cases and conflicts we examine are inspired by a long list of empirical studies, whose findings sharply contrast with each other. The results in the present model can help to explain these contrasting findings. For instance, Smith and Edmonston (1997) and Borjas (1999) find weak positive complementarity effects of immigration and estimate the net benefit from the present stock of immigrants to be approximately $10 billion in the U.S. Ottaviano and Peri (2005) find strong complementarities between comparably skilled immigrants and natives such that overall immigration generates a large positive effect on the average wages of US-born workers. On the other hand, Bauer and Zimmermann (1999) calculate that an annual inflow of 200,000 nationals from central and east European countries would potentially decrease the wages of the European Union workers by about 0.8 percent; Davis and Weinstein (2002) show negative terms of trade effects of immigration based on technology superiority. Borjas, Freeman, and Katz (1997) and Borjas (2003) find that immigration depresses wages for native workers who are likely to substitute for immigrant labor. Borjas, Grogger and Hanson (2010) find that a 10-percent immigrant-induced increase in the supply of a particular skill group reduced the black wage by 2.5 percent, lowered the employment rate of black men by 5.9 percentage points, and increased the incarceration rate of blacks by 1.3 percentage points.

There is voluminous theoretical literature on how immigration affects host-country welfare. With regard to models that analyze dynamics effects of immigration, Galor and Stark (1991) study the pattern of migration in an OLG model with two countries that are different in technology. Dolado, Goria, and Ichino (1994) show that immigration can increase the stock of human capital in the host country, and so facilitate growth. Lundborg and Segerstrom (2000, 2002) argue that since immigrants lower wages, firms can spend more on R&D, raising growth. In Stark and Wang (2002), migration to a richer foreign country, by raising both the level of human capital and the average level of human capital of non-migrants in the home country, can enhance welfare and nudge the economy toward the social optimum. Ben-Gad (2008) shows that an influx of high-skilled immigrants lowers the wages of skilled workers, raises the wages of unskilled workers, and substantially raises the return to native-owned capital. With regard to static models, Ethier (1985) studies the relationship between international trade and labor mobility in the host country, focusing on how immigrants help preserve import-competing industries with their cheap labor. Felebermayr and Kohler (2007) decompose the native welfare effect into a standard complementarity effect, augmented by a Stolper-Samuelson effect, and a terms-of-trade effect. They calibrate this model to a generic OECD economy and provide simulation results. However, in all these models, immigrants and natives are "undifferentiated", i.e., immigrants assimilate immediately when arriving in the receiving country. A most recent paper by Stark and Jakubek (2013) studies how migrants help each other out by forming migration networks and there exists an optimal size of such networks and hence multiple networks coexist.

The rest of the paper is organized as follows. Section 2 constructs the basic model, section 3 examines various immigration conflicts, section 4 analyzes the impacts on welfare, and section 5 provides some policy recommendations and concluding remarks. Analysis of the dynamic system and the steady state is relegated to the Appendix.

2 The Basic Model

Consider an overlapping-generations economy where people work while young and retire when old. Workers can be either high skilled or low skilled. Skills are born with and cannot be changed. The initial native population size is at $2N_t = N_t^y + N_t^o$, with N_t^y and N_t^o being the numbers of young and old at t.

Before time t, only natives live in the host country. From time t, the host country receives I_t number of immigrants (young generation only) every period, I_t^h of which being high skilled and $I_t^l = I_t - I_t^h$ being low skilled. Immigration makes the total number of workers in the host country $2N_t + I_t$ at t. We assume that one immigrant bears one child in the next period, and this child is treated as a native since it is born in the host country. Then the total population at time $t+1$ becomes $TP_{t+1} = N_{t+1}^y + N_{t+1}^o + I_{t+1}^o + I_{t+1}^y$, where N_{t+1}^y is the number of young native people, including the children of natives and immigrants, i.e., $N_{t+1}^y = N_t + I_t$; I_{t+1}^o represents immigrants arriving in t and becoming old at $t+1$; and I_{t+1}^y stands for immigrants arriving in $t+1$.

2.1 Consumers

We consider a one-good economy. Let j denote the four types of cohorts: respectively for high skilled natives and immigrants, and low skilled natives and immigrants. Each type of cohorts maximizes the per capita utility as follows:

$$\begin{aligned} \max U &= \log c_t^{y,j} + \log g_t + \beta\left(\log c_{t+1}^{o,j} + \log g_{t+1}\right), \qquad (1)\\ s.t. c_t^{y,j} &= (1-\tau)w_t^j - s_t^j, \\ c_{t+1}^{o,j} &= R_{t+1}s_t^j, \quad \text{where} \quad R_{t+1} \equiv 1 + \rho_{t+1}, \end{aligned}$$

where $c_t^{y,j}$ is cohort j's consumption when young at time t and $c_{t+1}^{o,j}$ is the counterpart when he becomes old; g_t is an *impure* public good per capita at time t, supported by tax at $t-1$. Parks, roads, bridges, hospitals, clean environment, schools and libraries are some examples of the impure public good; β is the time preference; τ is the income tax rate; s_t^j is savings; w_t^j is the wage rate, to be determined in (11) soon; ρ is the rate of return from savings; and R is the gross rate of return. All tax revenue, T, is used to produce the impure public good for the next period,[4]

$$\begin{aligned} G_t &= T_{t-1} = \tau(w_{t-1}^h N_{t-1}^h + w_{t-1}^l N_{t-1}^l), \qquad (2)\\ G_{t+1} &= T_t = \tau\left\{w_t^h(N_t^h + I_t^h) + w_t^l(N_t^l + I_t^l)\right\}, \qquad (3) \end{aligned}$$

where G_t is the aggregate public goods, and N^h and N^l are the number of high and low skilled natives respectively. The per-capita public goods decreases with immigration,

[4] To simplify the model, only labor income is taxed. Interest income is not taxed because the source of which is labor income.

e.g., hospitals, roads, schools and parks could become crowded and congested.[5] Utility maximization results in consumption for the young and old and the savings as:

$$c_t^{y,j} = \frac{1}{1+\beta}(1-\tau)w_t^j, \tag{4}$$

$$c_{t+1}^{o,j} = \frac{\beta}{1+\beta}(1-\tau)w_t^j R_{t+1}, \tag{5}$$

$$s_t^j = \frac{\beta}{1+\beta}(1-\tau)w_t^j. \tag{6}$$

2.2 Production

To model the difference between natives and immigrants, we assume that they are combined slightly differently with capital to produce the final good, by the following CES (constant elasticity of substitution) type production function,

$$Y_t = \left(K_t^h\right)^\alpha \left\{\gamma^h \left(N_t^h\right)^{\theta^h} + \gamma^h b \left(I_t^h\right)^{\theta^h}\right\}^{\frac{1-\alpha}{\theta^h}} + \left(K_t^l\right)^\alpha \left\{\gamma^l \left(N_t^l\right)^{\theta^l} + \gamma^l b \left(I_t^l\right)^{\theta^l}\right\}^{\frac{1-\alpha}{\theta^l}}, \tag{7}$$

where $K_t^x \ \ (x = h, l)$ denotes the physical capital used by high (h) and low (l) skilled workers at t; and γ^x is a parameter that regulates the shares of natives and immigrants in their respective contributions to output. Production requires both capital and labor. But for labor, either natives or immigrants or both can be used, and both of them can be either high or low skilled. We assume $1 > \left(\gamma^h\right)^{\frac{1}{\theta^h}} > \left(\gamma^l\right)^{\frac{1}{\theta^l}}$; that is, high skilled cohorts are relatively more productive than low skilled ones. The parameter b captures possible impacts of the distinct cultures and customs of immigrants that may stem from different institutions, education, region, etc., which may improve production or cause frictions that delay production. We assume $b < 1$, roughly implying that immigrants are less productive than natives for each type of workers.[6]

Finally, we describe the parameter $\theta^x \ \ (x = h, l)$, which represents the degree of substitutability between natives and immigrants, for high and low skilled agents respectively. Following a long research tradition on immigration,[7] we assume that skilled workers possess specific skills that enable them to be less substitutable. A worker becomes more substitutable if he is less skilled, and in the extreme, unskilled natives and unskilled immigrants are perfect substitutes. It is then natural to assume that $\theta^h < \theta^l$, i.e., the substitutability of high skilled workers must be lower. More generally, immigrants and natives can be either substitutes or complements in production, depending on the value of θ^x, as will be examined in detail later.

[5] Hanson (2005, p1) states that "taxpayers in high-immigration U.S. states shoulder most of immigration's fiscal costs, which they bear in the form of higher taxes that go to pay for public services used by immigrant households."

[6] If $b > 1$, then in equilibrium the number of new immigrants must be bigger than natives each period. We exclude such an abnormal case in the present model by assuming $b < 1$.

[7] For instance, in Borjas (1995) immigrants and natives with equal skills are perfect substitues. Other studies assume a certain degree of imperfect substitutability, see for example, Ethier (1985), Angrist and Kugler (2003) and Ottaviano and Peri (2005).

Let the final good be the numeraire. Then the profit of the representative firm can be written as

$$\pi_t = Y_t - \left(K_t^h + K_t^l\right) R_t - w_t^h N_t^h - w_t^{I,h} I_t^h - w_t^l N_t^l - w_t^{I,l} I_t^l.$$

Profit maximization yields the return to capital, which must be equalized working with either high or low skilled labor, and this condition then determines capital allocation between the two types of labor:

$$R_t = \alpha \left(K_t^x\right)^{\alpha-1} \left\{\gamma^x \left(N_t^x\right)^{\theta^x} + \gamma^x b \left(I_t^x\right)^{\theta^x}\right\}^{\frac{1-\alpha}{\theta^x}}. \tag{8}$$

Firms producing the final good choose the quantity of each type of labor input for a given wage type.

$$w_t^x = \left(K_t^x\right)^{\alpha} (1-\alpha) \left\{\gamma^x \left(N_t^x\right)^{\theta^x} + \gamma^x b \left(I_t^x\right)^{\theta^x}\right\}^{\frac{1-\alpha}{\theta^x}-1} \gamma^x \left(N_t^x\right)^{\theta^x-1}, \tag{9}$$
$$w_t^{I,x} = \left(K_t^x\right)^{\alpha} (1-\alpha) \left\{\gamma^x \left(N_t^x\right)^{\theta^x} + \gamma^x b \left(I_t^x\right)^{\theta^x}\right\}^{\frac{1-\alpha}{\theta^x}-1} \gamma^x b \left(I_t^x\right)^{\theta^x-1}. \tag{10}$$

We assume that the immigration policy set by the receiving-country government is, it allows only those immigrants employed by domestic firms and that immigrants receive the same wage rates as natives (e.g., H visa holders in the U.S.), given each worker's type. This can be justified on the grounds that in the absence of illegal immigration, it is unlawful to pay legal immigrants a discriminatory wage. Then firms will hire immigrants up to the point where their marginal productivity is equal to that of native workers, which is again equal to their wage, resulting in

$$w_t^{I,x} = w_t^x. \tag{11}$$

However in equilibrium the number of immigrants must be smaller than that of natives due to the assumption of $b < 1$. There is full employment in this economy. Conditions (9) and (10) give the ratio of the number of immigrants and natives as,

$$I_t^h / N_t^h = b^{\frac{1}{1-\theta^h}}, \quad I_t^l / N_t^l = b^{\frac{1}{1-\theta^l}}. \tag{12}$$

That is, more immigrants are hired as their productivity rises and as the elasticity of substitution becomes higher.

Using the above, the production function can be rewritten as

$$Y_t = \left(K_t^h\right)^{\alpha} \left(\Delta^h\right)^{1-\alpha} \left(E^h\right)^{1-\alpha} + \left(K_t^l\right)^{\alpha} \left(\Delta^l\right)^{1-\alpha} \left(E^l\right)^{1-\alpha},$$

where $E^x \equiv \{1 + b^{\frac{1}{1-\theta^x}}\}^{\frac{1}{\theta^x}} > 1$ ($x = h, l$ as before), which can be called *the immigration effects,* since they stem from immigration of respectively high and low skilled workers, and $\Delta^x \equiv \left(\gamma^x\right)^{\frac{1}{\theta^x}} \left(N_t^x\right)$.

2.3 Market Equilibrium

By Walras' law, there are two market equilibrium conditions we need to take care of. One is in the goods market, where final output is consumed by the young and old generations, re-invested as capital and collected as government tax revenue:

$$Y_t = \sum_{x}^{N+I} C_t^{y,x} + \sum_{x}^{N} C_t^{o,x} + K_{t+1} + T_t. \tag{13}$$

where C is the aggregate consumption. The other is in the capital market, where total capital in each period is combined respectively with high and low skilled workers to produce the final output:

$$K_t = K_t^h + K_t^l. \tag{14}$$

The dynamic system and the steady state are analyzed in the Appendix.

3 Basic Effects of Immigration

Since agents born at t are affected by economic conditions in periods $t-1$, t and $t+1$ only, we assume that they do not care about other periods and consider only those conflicts that arise among the three generations. For analytical tractability, we assume that half of the initial natives are high skilled and the other half low skilled, $N^h = N^l$.

Immigration increases labor supply, which affects the productivity of natives and the wages of both skills. These lead to changes in their consumption and saving choices, which further affect the tax revenue and capital stock economywide. Hence, in this section, we first consider changes in the marginal productivity and compare the wages with and without immigration. Then we analyze how immigration affects other endogenous variables. In the next section we shall examine the welfare effects under different types of immigration conflicts.

3.1 Marginal Productivity and Wages

First consider how the marginal productivities and wages of high and low-skilled native workers change with immigration.[8] The production function *before* immigration is obtained by setting $I^h = I^l = 0$ in (7); and in the steady state, we also have the wages as (for all $x = h, l$):

$$w_{t-1}^x = (1-\alpha)\left(\gamma^x\right)^{\frac{1}{\theta^x}} \left(\frac{K^*}{\Delta^h + \Delta^l}\right)^{\alpha}, \tag{15}$$

where $(\gamma^x)^{\frac{1}{\theta^x}} \equiv$ *the skill difference effect between high and low skilled workers;* $\frac{1}{\Delta^h+\Delta^l}$ measures the effectiveness of the labor force (meaning it is adjusted by the share parameter γ^x. As their names show, they denote the effect coming from the difference

[8]Note that they are undefined at $\theta^h = 0$ and θ^l.

parameter γ^x between skilled and unskilled workers when θ^x changes. Hence, $\frac{K^*}{\Delta^h+\Delta^l}$ is the steady-state effective capital-labor ratio, or the capital per unit of effective labor.

After immigration takes place, we obtain,

$$w_t^x = (1-\alpha)\left(\gamma^x\right)^{\frac{1}{\theta^x}} \left(E^x\right)^{1-\theta^x} \left(\frac{K^*}{\Delta^h E^h + \Delta^l E^l}\right)^{\alpha} \tag{16}$$

With immigration, the capital allocation per worker decreases, since the existing capital stock must be shared among more workers, as is shown by $\frac{K^*}{\Delta^h E^h+\Delta^l E^l}$ in (16). On the other hand, due to technology complementarity, each worker's productivity increases by $(E^x)^{1-\theta^x}$.[9]

Comparing the marginal productivity conditions before and after immigration yields:

Lemma 1 *Given $\theta^h < \theta^l$ and $b < 1$ by assumption, we have $E^h - E^l > 0$, i.e., the immigration effect is stronger for high skilled workers than for low skilled workers.*

Proof. Note that $\frac{1}{\theta^h} > \frac{1}{\theta^l}$. From the definition of E^h and E^l, we see that the sign of $E^h - E^l$ depends on their component difference $b^{\frac{1}{1-\theta^h}} - b^{\frac{1}{1-\theta^l}}$. Since $b < 1$, then $b^{\frac{1}{1-\theta^h}} - b^{\frac{1}{1-\theta^l}} > 0$, leading to $E^h - E^l > 0$. ■

3.2 Wage Changes under Immigration

Next we examine how wages change after immigration. Taking the ratio of the high-skilled wage between periods $t-1$ and t gives

$$\frac{w_t^h}{w_{t-1}^h} = \left(\frac{\Delta^h + \Delta^l}{\Delta^h E^h + \Delta^l E^l}\right)^{\alpha} \left(E^h\right)^{1-\theta^h}, \tag{17}$$

where $\left(\frac{\Delta^h+\Delta^l}{\Delta^h E^h+\Delta^l E^l}\right)^{\alpha}$ can be called the *capital allocation effect* and $\left(E^h\right)^{1-\theta^h}$ the *productivity effect,* both for high skilled workers. The former stems from changes in the capital allocation per worker and the latter from changes in the productivity post immigration. The capital allocation effect decreases the wage, since immigration increases the supply of workers in the host country. On the other hand, the productivity effect increases the wage. The intersection of these two opposing effects determines whether the high-skilled wage increases or not. Analogously, a similar condition can be obtained for the ratio of the low-skilled wage between periods $t-1$ and t.

As shown above, the capital allocation effect and the productivity effect cause changes in the marginal productivity and wages of native skilled workers. Recall that the capital allocation effect is negative, but the productivity effect is positive. Combining both effects, we can divide the wage impact of immigration into three cases: Case I, both native wages increase; Case II, both wages decrease; Case III, the high skilled wage increases, but the low skilled wage decreases.

[9] Without immigration, $I^h = I^l = 0$ and $E^h = 1 = E^l$, then (16) boils down to (15).

3.3 Effects on Capital Goods and Interest Income

The wage changes just considered may affect the household's behavior in consumption and savings patterns, eventually leading to changes in the capital stock, public goods, and interest rate. We examine them in detail now.

Comparing (44) and (45) in the Appendix yields

$$\frac{K_{t+1}}{K^*} = \frac{w_t^h}{w_{t-1}^h} \frac{\left(\gamma^h\right)^{\frac{1}{\theta^h}} \left(N_t^h + I_t^h\right)}{\left(\gamma^h\right)^{\frac{1}{\theta^h}} \left(N_t^h\right) + \left(\gamma^l\right)^{\frac{1}{\theta^l}} \left(N_t^l\right)} + \frac{w_t^l}{w_{t-1}^l} \frac{\left(\gamma^l\right)^{\frac{1}{\theta^l}} \left(N_t^l + I_t^l\right)}{\left(\gamma^h\right)^{\frac{1}{\theta^h}} \left(N_t^h\right) + \left(\gamma^l\right)^{\frac{1}{\theta^l}} \left(N_t^l\right)}. \tag{18}$$

K^* is from natives only, but K_{t+1} is generated by the disposable income of the four types of agents. The ratio of capital stock before and after immigration depends on not only the ratio of high and low skilled wages, but also their respective contribution to the capital stock. Each ratio is less than 1, and they sum up to 1 without immigration but more than 1 with immigration[10].

To examine in detail the effects of immigration on capital goods, we rewrite (18) as

$$\frac{K_{t+1}}{K^*} = \left(\frac{\Delta^h E^h + \Delta^l E^l}{\Delta^h + \Delta^l}\right)^{1-\alpha} \tag{19}$$

Since $\frac{\Delta^h E^h + \Delta^l E^l}{\Delta^h + \Delta^l} > 1$, the capital stock increases with immigration in all three cases.

Further, the ratio of interest rates can be derived as

$$\frac{R_{t+1}}{R_t} = \left(\frac{K_{t+1}}{K^*}\right)^{\alpha-1}. \tag{20}$$

Thus, changes in the interest rate depend on changes of the capital stock, in the opposite direction though.

Note that the impure public goods in each period is provided with the tax revenue of the previous period as shown in (2) and (3). Then the difference between pre and post immigration is,

$$\frac{G_{t+1}}{G_t} = \left(\frac{\Delta^h E^h + \Delta^l E^l}{\Delta^h + \Delta^l}\right)^{1-\alpha}. \tag{21}$$

Similar to the capital stock, the provision of public goods increases after immigrants come in.

4 Immigration Conflicts

Having investigated the various effects of immigration, now we put them together and consider the possible conflicts immigration may cause in the receiving country.

[10] The ratio of capital stock pre and post immigration depends on the population structure. Recall the assumption $N^h = N^l$. Since high skilled immigrants possess more complimentary technology than low skilled ones, the economy accepts more of the former immigrants. As time passes, the contribution ratio of high skilled immigrants will be larger, and the wage effect becomes stronger for them. However, we do not need to care too much about this issue since we focus on only periods t and $t+1$.

4.1 Wage Related Conflicts

First, let us look into the skill conflict on wages between high and low skilled workers. Table 1 combines both the capital allocation effect and the productivity effect, and summarizes the main comparative statics results on wages. In Case I, both θ^h and θ^l are less than $1-\alpha$ and both types of immigrants tend to be complementary to natives, hence both wages increase with immigration. But Case II is the exact opposite, where both θ^h and θ^l are bigger than $1-\alpha$, and both wages decrease. In these two cases, since both wages move in the same direction, immigration is less likely to cause conflicts between high and low skilled workers. In contrast, in Case III, the high skilled wage increases, but the low skilled wage decreases. Hence a conflict arises, and we name it the skill conflict.

Table 1 Skill conflicts

	Case I	Case II	Case III
w^h	$+$	$-$	$+$
w^l	$+$	$-$	$-$
conflict	no	no	yes

These results are summarized in the following proposition.

Proposition 2 *A skill conflict arises in Case III when the high- and low-skilled wages move in opposite directions since the productivity effect dominates the capital allocation effect for the high skilled, but exactly the opposite arises for the low skilled; In contrast, skill conflicts are less likely to occur in Cases I and II, where both types of immigrants tend to be either complementary or substitutable to natives simultaneously, leading both wages to move in the same direction.*

The next question is, how does the skill-premium (i.e., the wage gap between high and low skilled workers) change after immigration takes place? Straightforward calculations yield $\frac{w_t^h}{w_t^l} - \frac{w_{t-1}^h}{w_{t-1}^l} = \frac{\left(\gamma^h\right)^{\frac{1}{\theta^h}}}{\left(\gamma^l\right)^{\frac{1}{\theta^l}}}[\frac{\left(E^h\right)^{1-\theta^h}}{\left(E^l\right)^{1-\theta^l}} - 1] > 0$, which arises since $\left(E^h\right)^{1-\theta^h} > \left(E^l\right)^{1-\theta^l}$ and by using Lemma 1. That is, the wage gap widens with immigration. Specifically, in Cases I and II, while both high and low skilled wages move in the same direction, the high-skilled wage changes more in Case I and less in Case II than the low-skilled wage. And in Case III, immigration raises (reduces) the high (low) skilled wage as shown. Therefore,

Proposition 3 *The wage gap between high and low skilled natives will be enlarged with immigration, regardless if both wages rise or fall.*

Indeed, Scheve and Slaughter (2001) find that opposition to immigration is higher among the less educated, in a U.S. survey of public opinion on immigration policy, as the above Proposition would predict.

4.2 Welfare Related Conflicts

Now we examine how immigration impacts the host country's per capita welfare of generation t. From previous sections, we know that immigration affects not only the current generation but also future generations through changes in the amount of capital goods. Moreover, it causes income redistribution which may lead to conflicts. Here we focus on the case when immigration increases capital goods; that is, conflicts may still arise even though the host country as a whole benefits from immigration.

Using (1)~(5), substitution and rearrangement of equations lead to

$$\begin{aligned} & U_t^{with,Nl} - U_t^{without,Nl} \\ = \; & (1+\beta)\log \underbrace{\frac{w_t^{with,l}}{w_t^{without,l}}}_{\text{wage effect}} + \beta \log \underbrace{\frac{R_{t+1}^{with}}{R_{t+1}^{without}}}_{\text{interest rate effect}} \\ & + \beta \log \underbrace{\frac{G_{t+1}^{with}}{G_{t+1}^{without}}}_{\text{public-goods effect}} + (1+\beta)\log \underbrace{\frac{2N}{2N+I}}_{\text{scale effect}} \end{aligned} \tag{22}$$

The immigration-induced change in the per-capita welfare can be decomposed into four effects: *the wage effect, the interest rate effect, the public-goods effect,* and *the scale effect*, which can be explained as follows. By (1) utility is obtained from consumption at young and old ages and from the impure public goods. Consumption at young age comes from wage income, and that at old age depends on savings and interest income. Moreover, the public goods at $t+1$ is produced by tax revenue at t, which also comes from wage income. Therefore, the welfare difference with and without immigration depends on the difference of wages, interest rates, public goods and the scale effect of immigration–the ratio of immigrants to total population (We assume the scale effect to be dominated by other effects; Otherwise if the scale effect dominates, there are too many immigrants and welfare always falls). The ratio of interest rates depends negatively on the ratio of capital stock from (20); and the ratio of public goods is (21), which works in the opposite direction as the interest rate effect.

Consider a small open economy. Capital-stock increases via immigration do not affect the interest rate in the host country, which is given by the world market, and extra capital is lent to other countries. In this setting, we have:

$$U_t^{with,nx} - U_t^{without,nx} = (1+\beta)\log \underbrace{\frac{w_t^{with,x}}{w_t^{without,x}}}_{wage\ effect} + (1+\beta)\log \underbrace{\frac{2N}{2N+I}}_{scale\ effect} . \tag{23}$$

That is, the welfare only depends on the wage effects, and we obtain Table 1 again. Then, we can calculate the welfare difference according to skills, as in the following subsections.

4.2.1 Low-skilled natives

First, we analyze the immigration-induced change in the welfare of low-skilled natives. Substituting (20) and (21) into (22) yields the welfare difference with and without immigration. Using (19) and ignoring the scale effect, we get

$$\begin{aligned} & U_t^{with,nl} - U_t^{without,nl} \\ = \; & \log\left(\frac{w_t^l}{w_{t-1}^l}\right)^{1+\beta}\left(\frac{\Delta^h E^h + \Delta^l E^l}{\Delta^h + \Delta^l}\right)^{(1-\alpha^2)\beta} \end{aligned} \tag{24}$$

The welfare difference stems from the wage difference, and includes not only the wage effect but also the interest and public goods effects.

In Case I, the welfare is higher with immigration than without. For Cases II and III, we rewrite the above as,

$$U_t^{with,nl} - U_t^{without,nl} = \log\left(\frac{\Delta^h + \Delta^l}{\Delta^h E^h + \Delta^l E^l}\right)^{\alpha-\beta+\beta\alpha^2+\beta\alpha}\left(E^l\right)^{(1+\beta)\left(1-\theta^l\right)}. \tag{25}$$

If $\alpha + \beta\alpha^2 + \beta\alpha < \beta$, since both the productivity and capital allocation effects are positive, welfare increases with immigration; If $\alpha+\beta\alpha^2+\beta\alpha > \beta$, the capital allocation effect becomes negative but the productivity effect is still positive, resulting in an ambiguous net effect. Thus we can summarize the above discussion in Table 2L.

Table 2L

Effect name	Case I	Case II	Case III
wage	$+$	$-$	$-$
interest	$+$	$+$	$+$
public goods	$+$	$+$	$+$
scale	$-$	$-$	$-$
total	$+$	$+$ if $\alpha + \beta\alpha^2 + \beta\alpha < \beta$,	$+$ if $\alpha + \beta\alpha^2 + \beta\alpha < \beta$,
		ambiguous if $\alpha + \beta\alpha^2 + \beta\alpha > \beta$	*ambiguous* if $\alpha + \beta\alpha^2 + \beta\alpha > \beta$

4.2.2 High-skilled natives

Analogously, the high skilled natives' welfare changes as follows

$$U_t^{with,nh} - U_t^{without,nh} = \log\left(\frac{\Delta^h + \Delta^l}{\Delta^h E^h + \Delta^l E^l}\right)^{\alpha-\beta+\beta\alpha^2+\beta\alpha}\left(E^h\right)^{(1+\beta)\left(1-\theta^h\right)} + (1+\beta)\log\frac{2N_t}{2N_t + I_t}. \tag{26}$$

Ignoring the scale effect and using the same method above, the welfare difference can be summarized as in Table 2H.

Table 2H

Effect name	Case I	Case II	Case III
wage	$+$	$-$	$+$
interest	$+$	$+$	$+$
public goods	$+$	$+$	$+$
scale	$-$	$-$	$-$
total	$+$	$+$ if $\alpha+\beta\alpha^2+\beta\alpha<\beta$	$+$
total		*ambiguous* if $\alpha+\beta\alpha^2+\beta\alpha>\beta$	

4.2.3 Summary on welfare

From Tables 2H and 2L, the welfare of both high and low skilled workers move in the same direction, both increasing in Case I but both decreasing in Case II. In these two cases, skill conflicts are less likely to occur. In other words, if high- and low-skilled immigrants bring complementary technology with high- and low-skilled natives, welfare increases for both types of natives, as in Case I. On the other hand, if the host country accepts immigrants with high θ^h and θ^l (i.e., more substitutable technology), then the welfare of natives (irrespective of skill) will fall, as in Case II. However, in Case III, since welfare for the low-skilled falls but that for the high skilled rises, skill conflicts are more likely to occur.

4.3 Intergenerational Conflicts

In this subsection, we examine how immigration affects different generations living at time t, which could possibly lead to *intergenerational conflicts*.

The t-period welfare of the old and young generations[11] are respectively

$$U_t^y=\log C_t^y+\log G_t,\quad U_t^o=\log C_t^o+\log G_t,$$

where C_t^y (C_t^o) stands for the total consumption of the young (old) cohorts. Since the public goods at t is provided by tax collected at $t-1$, both generations enjoy the same amount of G_t (since immigrants arrive at t). Then the welfare difference is determined by the difference in consumption (or income),

$$\begin{aligned}\frac{U_t^y-U_{t-1}^y}{U_t^o-U_{t-1}^o} &= \frac{\log\frac{w_t^lN_t^l+w_t^hN_t^h}{w_{t-1}^lN_{t-1}^l+w_{t-1}^hN_{t-1}^h}}{\log\frac{R_t}{R_{t-1}}}\\ &= 1+\frac{\log\frac{\Delta^l\left(E^l\right)^{1-\theta^l}+\Delta^h\left(E^h\right)^{1-\theta^h}}{\Delta^hE^h+\Delta^lE^l}}{\log\left(\frac{\Delta^h+\Delta^l}{\Delta^hE^h+\Delta^lE^l}\right)^{\alpha-1}}\end{aligned}\tag{27}$$

This expression states that the welfare difference depends on the total-income ratio and the interest-rate ratio. In the second term on the last line, since $E^h>E^l>1$, the antilogarithm in the numerator is less than 1, leading the numerator to be negative. Similarly, the antilogarithm in the denominator is also less than 1 but with a negative

[11] We consider all young and old agents irrespective of their skills.

power, so the denominator is positive. Thus, the second term is negative. Then the RHS of (27) becomes less than 1. That is, the welfare increase is smaller for the young generation than for the old generation, which gives:

Proposition 4 *The old generation enjoys more benefits from immigration than the young generation, leading to an intergenerational conflict.*

The intuition can be explained as follows. Post immigration, the wage is affected by the capital allocation effect and the productivity effect, while the interest-rate increase is caused by the fall in the amount of capital per worker and the immigration effect shown by (8). Old cohorts benefit more from immigration because their income comes from savings.

4.4 Distributional Conflicts

Finally, we look into how immigration affects the life-time utility of each generation. It may seem that immigration at t lowers the welfare of the natives born at $t-1$ irrespective of their skill levels, because the public goods they contributed must be divided by immigrants also, i.e., by the number $2N+I$ at t. In other words, immigrants can enjoy the benefits at t, although they did not pay tax for the public goods at $t-1$. However, since the steady state and all variables change, natives born at $t-1$ are not always harmed by immigration, compared with t or $t+1$ generations. Here we investigate this issue in detail.

4.4.1 Welfare of low-skilled natives

Low-skilled natives born at $t-1$ First, the welfare difference between generations $t-1$ and t is,

$$\begin{aligned} & U_t^{nl} - U_{t-1}^{nl} \\ = \ & \beta \log(\frac{K_{t+1}}{K^*})^{\alpha-1} + \beta \log(\frac{K_{t+1}}{K^*}) + (1+\beta)[\log \frac{w_t^l}{w_{t-1}^l} + \log \frac{2N_{t-1}}{2N_t + I_t}]. \end{aligned} \tag{28}$$

Note that the difference between (24) and (28) lies in the interest rates. In (24), the production functions at $t+1$ with and without immigration are different, in addition to the capital-stock difference. On the other hand, in (28), the shape of the production function is the same at $t+1$ and t; then the interest difference between the two periods stems only from the capital-stock difference.

As previously shown, the capital stock effect works in the same direction as the public-goods effect, but opposite to the interest-rate effect. Using (19) and ignoring the scale effect, we can rewrite (28) as,

$$U_t^{nl} - U_{t-1}^{nl} = \log \left(\frac{\Delta^h + \Delta^l}{\Delta^h E^h + \Delta^l E^l} \right)^{\alpha(1+\beta\alpha)} \left(E^l\right)^{1-\theta^l}. \tag{29}$$

In Case I, since the capital allocation effect is dominated by the productivity effect, the welfare difference is positive. On the other hand, in Case II, the former effect dominates the latter effect; moreover, (29) lies above the curve E^l_{II} due to the bigger power of the capital allocation effect, giving rise to a negative sign for (29). And this holds true in Case III too. Then, we obtain Table 3L1.

Table 3L1 Welfare difference between t and $t-1$

Effect name	Case I	Case II	Case III
wage effect	$+$	$-$	$-$
interest effect	$-$	$-$	$-$
public goods effect	$+$	$+$	$+$
scale effect	$-$	$-$	$-$
total	$+$	$-$	$-$

Low skilled natives born at $t+1$ Similarly, the welfare difference between generations t and $t+1$ can be obtained as,

$$\begin{aligned} & U^{nl}_{t+1} - U^{nl}_{t} \\ = \; & (1+\beta)\log \underbrace{(\frac{K_{t+1}}{K^*})}_{wage\ effect} + \beta \log \underbrace{(\frac{K_{t+2}}{K_{t+1}})^{\alpha-1}}_{interest\ rate\ \text{effect}} + \beta \log \underbrace{(\frac{K_{t+1}}{K^*})}_{public\ \text{goods}\ effect} + \log \frac{TP_t}{TP_{t+1}} \end{aligned}$$

where $\frac{TP_t}{TP_{t+1}} = \frac{\left(1+\left(E^h\right)^{\theta^h}\right)+\left(1+\left(E^l\right)^{\theta^l}\right)}{\left(E^h\right)^{\theta h}\left(1+\left(E^h\right)^{\theta^h}\right)+\left(E^l\right)^{\theta^l}\left(1+\left(E^l\right)^{\theta^l}\right)}$. At $t+1$, young immigrants who came to the host country at t become old. By (12), I_{t+1}/N_{t+1} is a constant, and thus the immigration effect at $t+1$ is the same as at t. Then, both the per-capita capital allocation ratio, $1/(\Delta^h E^h + \Delta^l E^l)$ and the productivity effect remain the same in these two periods, and the wage ratio is affected by the capital goods effect only. Thus, wages are different only because the capital stock has changed. Agents who save at t receive interest income at $t+1$. Further, the difference in interest income between $t+1$ and $t+2$ depends on $K_{t+2}/K_{t+1} > 1$.[12] Therefore, the interest rate effect lowers the welfare difference between $t+1$ and t. We thus obtain Table 3L2.

Table 3L2 Welfare difference between t and $t+1$

Effect name	Case I	Case II	Case III
wage effect	$+$	$+$	$+$
interest effect	$-$	$-$	$-$
public goods effect	$+$	$+$	$+$
scale effect	$-$	$-$	$-$
total	$+$	$+$	$+$

Using Tables 3L1 and 3L2, if the scale effect is dominated by the other three effects, we further obtain Table 3L3 below, which provides the details on the patterns of gains from immigration.

[12] See the Appendix.

Table 3L3 Welfare Pattern 1

Effect name	Case I	Case II	Case III
wage effect	$P1$	$P3$	$P3$
interest effect	$P2$	$P2$	$P2$
public goods effect	$P1$	$P1$	$P1$
scale effect	$-$	$-$	$-$
total	$P1$	$P3$	$P3$

where V_i is value at period i, and $P1 \equiv \{V_{t-1} < V_t < V_{t+1}\}$, $P2 \equiv \{V_{t+1} < V_t < V_{t-1}\}$, $P3 \equiv \{V_t < V_{t+1}, V_{t-1}\}$ and $P4 \equiv \{V_t > V_{t+1}, V_{t-}1\}$ stand for the patterns in which the wage, interest-rate and public goods effects are compared. For example, $P1$ in the wage row in Case I means $w_{t-1} < w_t < w_{t+1}.$

Table 3L3 implies that distributional conflicts may occur among different generations. While the welfare difference between t and $t-1$ comes from the capital allocation and productivity effects after immigration, that between t and $t+1$ stems from capital stock changes. Note that the per capita welfare of generation t is given by

$$U = \underbrace{(1+\beta)\log w_t^l + \beta \log R_{t+1}}_{\text{total consumption}} + \underbrace{\left(\log w_{t-1}^l + \beta \log w_t^l\right)}_{\text{total public goods}}, \tag{30}$$

which shows that total consumption depends on the wage income and the interest income, because households save a part of wages for consumption in old age.

In Case I, both the high and low skilled wages increase after immigration, so that the capital stock also increases at $t+1$. Though the interest rate is lowered by the increase in the capital stock, aggregating all effects we find that welfare is the highest in $t+1$.

In Cases II and III, welfare at t is the lowest, because the wage is lower at t than at $t-1$ due to the capital allocation effect, and it is also lower than at $t+1$ due to the capital stock effect.

4.5 Welfare of high-skilled natives

High-skilled natives born at $t-1$ Using the same procedure as in the previous subsection yields the welfare difference between high skilled workers of generations t and $t-1$ as follows,

$$\begin{aligned} & U_t^{nh} - U_{t-1}^{bh} \\ = \; & \beta \log(\frac{K_{t+1}}{K^*})^{\alpha-1} + \beta \log(\frac{K_{t+1}}{K^*}) + (1+\beta)[\log \frac{w_t^h}{w_{t-1}^h} + \log \frac{2N}{2N+I}], \end{aligned}$$

which is the same as equation (26) without the interest rate effect. We thus have

Table 3H1 Welfare difference between t and $t-1$

Effect name	Case I	Case II	Case III
wage effect	$+$	$-$	$+$
interest effect	$-$	$-$	$-$
public goods effect	$+$	$+$	$+$
scale effect	$-$	$-$	$-$
total	$+$	$-$	$+$

High skilled natives born at $t+1$ Given the scale effect is dominated, the welfare difference between high skilled workers at generations t and $t+1$ is,

$$\begin{aligned} & U_{t+1}^{nh} - U_t^{nh} \\ = \ & (1+\beta)\log(\frac{K_{t+1}}{K^*}) + \beta\log(\frac{K_{t+2}}{K_{t+1}})^{\alpha-1} + \beta\log(\frac{K_{t+1}}{K^*}). \end{aligned}$$

This difference depends on the capital stock for each period, since the immigration effect does not change between $t+1$ and t. We thus obtain Table 3L2 as Table 3H2 again. Combining Tables 3L2 and 3H1 also gives Table 3H3.

Table 3H3 Welfare Pattern 2

Effect name	Case I	Case II	Case III
wage effect	$P1$	$P3$	$P1$
interest effect	$P2$	$P2$	$P2$
public goods effect	$P1$	$P1$	$P1$
scale effect	$-$	$-$	$-$
total	$P1$	$P3$	$P1$

The welfare difference between t and $t-1$ stems from changes in the wage and capital goods effect. In contrast, the difference between t and $t+1$ comes from capital stock changes. Table 3H3 shows that in Cases I and III, the welfare at $t+1$ is the highest among the three periods. The wage at t is higher than at $t-1$, because the productivity effect dominates the capital allocation effect. Moreover, since the capital stock increases at $t+1$, the wage at $t+1$ is higher than at t. Hence, Pattern $P1$ emerges in Cases I and III. In Case II, the welfare at t is the lowest; i.e., welfare falls after immigration, following the wage decrease.

Summarizing the above yields:

Proposition 5 *Regardless of skills, those born in generation t cannot obtain the highest welfare among the three generations, $t-1$, t, and $t+1$. As such, generation t natives may oppose immigration, even though their wages increase.*

4.6 Generation $t-1$

In this subsection, we consider how the welfare of generation $t-1$ is affected by immigration, since they become old at t when immigrants come in. Because the effects are similar for both high and low skilled workers, we only consider the former ones. The welfare difference between generations $t-1$ and $t-2$ can be written as

$$U_{t-1}^{h} - U_{t-2}^{h} = \beta \left\{ \log \frac{R_t}{R_{t-1}} + \log \frac{2N}{2N+I} \right\}. \tag{31}$$

With immigration at t, the capital stock increases in $t+1$ and after. However, the interest rate at t changes due to the immigration effect. It follows that the welfare of generation $t-1$ (who becomes old at t) changes via the interest rate and scale effects. Substituting the interest rate using (8) into (31) gives

$$U_{t-1}^{h} - U_{t-2}^{h} = \beta \left\{ \log \frac{R_t}{R_{t-1}} \frac{2N}{2N+I} \right\}.$$

We find that the welfare of $t-1$ generation increases if the following condition is met,

$$\left(\frac{R_t}{R_{t-1}} - 1 \right) 2N > I,$$

which requires the scale of immigration to be small. This arises because the increase in the interest rate exceeds the decrease in public goods per capita.

5 Policy Discussion and Concluding Remarks

We now use the above results to derive some policy implications. In Case I, since immigration increases the wages of both high and low skilled workers and the natives' welfare, skill conflicts are less likely to arise and immigration can be more actively adopted. But distribution problems occur since the skill premium is widened. The government must take into account intergenerational conflicts between the young and old, and the skill premium between the high and low skilled workers. In contrast, in Case II, both native wages and welfare fall, and the welfare of those born at t is the lowest among the three generations, leading to conflicts between generations t and $t-1$ or $t+1$. It is no wonder that natives at t may oppose immigration in this case. Finally in Case III, high skilled natives will benefit from immigration, but the low skilled will be hurt, leading to a higher possibility of conflict. In fact, such skill conflicts are not difficult to find in developed countries. In this case, fine tuning, such as transfers and other redistribution measures are needed. In addition, since the root is a wage shock stemming from immigration, allowing free trade might mitigate these conflicts.

To smoothe out the impacts of these immigration conflicts, a rights-based approach proposed by Hanson (2005) seems practical. Such a policy aims to phase in slowly over time immigrant access to government benefits, by creating a graduated set of rights to draw on public benefits to which immigrants would gain access after having worked in the host country for a specific time period.

Our results have been based on non-immediate assimilation. If immigrants can assimilate soon after coming into the host country, then $b = 1$ in our model. The detailed discussion of this case is contained in the Appendix.

We have also abstracted from population growth in the formal model. In the Appendix, we discuss this issue in more detail. Since high skilled immigrants are more complimentary than low skilled ones, the host country may accept more immigrants with higher skills by Lemma 1, which will affect the accumulation of capital goods.

The Appendix

Figures

If immigration did not affect both high and low skilled wages, (17) would be equal to 1. Then we obtain the relationship between E^h and E^l as,

$$E^l = \frac{\left(\Delta^h + \Delta^l\right)\left(E^h\right)^{\frac{1-\theta^h}{\alpha}}}{\Delta^l} - \frac{\Delta^h}{\Delta^l}E^h. \tag{32}$$

In Figures 1–3 to follow, we name the curve from (32) $E_i^h(i = I, II, III)$ for case i. Similarly we can also obtain

$$E^h = \frac{\left(\Delta^h + \Delta^l\right)\left(E^l\right)^{\frac{1-\theta^l}{\alpha}}}{\Delta^h} - \frac{\Delta^l}{\Delta^h}E^l, \tag{33}$$

which is represented by the curve $E_i^l(i = I, II, III)$ for case i.The slopes of these curves on $E^h - E^l$ space can be calculated as in general,

$$\frac{\partial E^l}{\partial E^h} = \frac{\Delta^h + \Delta^l}{\Delta^l}\frac{1-\theta^h}{\alpha}\left(E^h\right)^{\frac{1-\theta^h-\alpha}{\alpha}} - \frac{\Delta^h}{\Delta^l}, \tag{34}$$

$$\frac{\partial E^l}{\partial E^h} = \frac{\Delta^h\alpha}{\left(\Delta^h + \Delta^l\right)\left(1-\theta^l\right)\left(E^l\right)^{\frac{1-\theta^l}{\alpha}-1} - \Delta^l\alpha}. \tag{35}$$

To determine their signs for each case, we make the following assumption:

ASSUMPTION 1: $\Delta^h / \Delta^l > -(1-\alpha-\theta^h)/(1-\theta^h)$ and $\Delta^h / \Delta^l > -(1-\alpha-\theta^l)/(1-\theta^l)$.

Assumption 1 simply implies that the slopes are positive at the origin $E^h = E^l = 1$.

Next, setting (34) and (35) equal to 0, we find the slopes change at $E^{x*} = (\frac{\Delta^h}{\Delta^h+\Delta^l}\frac{\alpha}{1-\theta^x})^{\frac{\alpha}{1-\theta^x-\alpha}}$, where $x = l, h$. When $\theta^x < 1-\alpha$, E^{x*} does not lie in the region $1 < E^l < E^h$, because the values in brackets in E^{x*} are less than 1; when $\theta^x > 1-\alpha$, E^{x*} lies in the region $E^l < E^h < 1$. Thus, we obtain

LEMMA 2: The slopes of (34) and (35) do not change signs in the region $1 < E^l < E^h$, and their values at $E^h = E^l = 1$ are positive when $\theta^h < 1-\alpha$ and $\theta^l < 1-\alpha$; When $\theta^h > 1-\alpha$ and $\theta^l > 1-\alpha$, the corresponding slopes change signs at $E^{h*} = (\frac{\Delta^h}{\Delta^h+\Delta^l}\frac{\alpha}{1-\theta^h})^{\frac{\alpha}{1-\theta^h-\alpha}}$ and $E^{l*} = (\frac{\Delta^l}{\Delta^h+\Delta^l}\frac{\alpha}{1-\theta^l})^{\frac{\alpha}{1-\theta^l-\alpha}}$.

Note that the threshold of the slope of (35) is not based on the horizontal axis but the vertical axis. When $E^{l*} = (\frac{\Delta^h}{\Delta^h+\Delta^l}\frac{\alpha}{1-\theta^l})^{\frac{\alpha}{1-\theta^l-\alpha}}$, the expression for E^h becomes $\frac{\Delta^h+\Delta^l}{\Delta^h}[(\frac{\Delta^h}{\Delta^h+\Delta^l}\frac{\alpha}{1-\theta^l})^{\frac{1-\theta^l}{1-\theta^l-\alpha}}] - \frac{\Delta^l}{\Delta^h}(\frac{\Delta^h}{\Delta^h+\Delta^l}\frac{\alpha}{1-\theta^l})^{\frac{\alpha}{1-\theta^l-\alpha}}$, at which point the slope of (35) changes.

Finally, the second derivatives from (34) and (35) can be found as

$$\frac{\partial\left(E^l\right)^2}{\partial^2 E^h} = \frac{\left(\Delta^h+\Delta^l\right)}{\Delta^l}\frac{1-\theta^h}{\alpha}\frac{1-\theta^h-\alpha}{\alpha}\left(E^h\right)^{\frac{1-\theta^h-\alpha}{\alpha}-1}, \tag{36}$$

$$\frac{\partial\left(E^l\right)^2}{\partial^2 E^h} = \frac{-\alpha E^l\left\{\left(1-\theta^l\right)-\alpha\right\}\left[\left(E^h+E^l\frac{\Delta^l}{\Delta^h}\right)\left(1-\theta^l\right)+\alpha\frac{\Delta^l}{\Delta^h}\left(E^h-E^l\right)\right]}{\left[E^h\left(1-\theta^l\right)+E^l\frac{\Delta^l}{\Delta^h}\left\{\left(1-\theta^l\right)-\alpha\right\}\right]^3} \tag{37}$$

Thus, (36) is positive when $\theta^h < 1-\alpha$ but negative otherwise, and similarly, (37) is negative when $\theta^l < 1-\alpha$ but positive otherwise. The denominator in (37) is always positive because the first term always dominates the second term. Since $\theta^l < 1$ and $E^h > E^l$, the sign of (37) is determined by the sign of the first term in the numerator.

Summing up the above derivations, we can determine the shapes of (32) and (33). If $\theta^h < 1-\alpha$, the locus of (32) is depicted by the convex shaped curve named E^h_I on E^h~E^l space in Figure 1. Analogously, if $\theta^l < 1-\alpha$, the locus of (33) is depicted by the concave shaped curve named E^l_{II}.

From Figure 1, (17)< 1 on the left hand side (the curve shown by (32)) and (17)> 1 on the right hand side. It means that when $E^l < E^h$, the high skilled wage increases after immigration. Similarly we can do the analysis for the low skilled wage. The shaded area in Figure 1 represents the region where both wages increase. In other words, this region indicates the combination of θ^h and θ^l that gives rise to higher wages with immigration for both types of workers. It occurs when both immigration effects are big under small θ^h and θ^l, which leads to high productivity effects that dominate the capital allocation effects. Under E^l_I, Case III arises, where E^l is too small compared with E^h, and the capital allocation effect dominates the productivity effect, resulting in a fall in the low skilled wage.

On the other hand, if $\theta^h > 1-\alpha$, the locus of (32) is depicted by the inverse-U shaped curve named E^h_{II} in Figure 2. Analogously, if $\theta^l > 1-\alpha$, the locus of (33) is depicted by the inverse C-shaped concave named E^l_{II} on E^l~E^h space. The shaded region indicates the combination of θ^h and θ^l that leads to lower wages with immigration for both types of workers; That is, Case II arises in this region, because the high (low) skilled wage decreases above the curve $E^h_{II}(E^l_{II})$.Similar to the above, under E^h_{II}, Case III arises again.

Finally to cases $\theta^h < 1-\alpha$ and $\theta^l > 1-\alpha$, where we obtain E^h_{III} and E^l_{III} . The former is convex and the latter is inverse C-shaped in Figure 3. Then, under the curve $E^h = E^l$, the high skilled wage increases but the low skilled wage decreases.

Summing up, in Case I under the conditions $\theta^h < 1-\alpha$ and $\theta^l < 1-\alpha$, Case II under $\theta^h > 1-\alpha$ and $\theta^l > 1-\alpha$, and Case III when E^h is sufficiently small, if immigrants bring skills that are complementary with natives, both the high- and low-skilled wages increase. But if immigrant skills are substitutable with natives, both wages decrease.

The Dynamic System

This subsection considers the dynamic impacts of immigration on the host country. After entering in period t, immigrants start to contribute to capital accumulation and government tax. Rewriting (13) using the budget constraints gives the capital movement equation,

$$K_{t+1} = S_t. \tag{38}$$

where S_t is the aggregate savings. Substituting (6) into (38) gives,

$$K_{t+1} = \frac{\beta}{1+\beta}(1-\tau)(w_t^{n,h} N_t^h + w_t^{n,l} N_t^l + w_t^{I,h} I_t^h + w_t^{I,l} I_t^l). \tag{39}$$

From (8) we have the capital allocation equation,

$$K_t^l = \frac{\Delta^l E^l}{\Delta^h E^h} K_t^h, \tag{40}$$

where $\Delta^x \equiv (\gamma^x)^{\frac{1}{\theta^x}} (N_t^x)$. Substituting (40) into (14) further gives,

$$K_t^x = \frac{\Delta^x E^x}{\Delta^h E^h + \Delta^l E^l} K_t. \tag{41}$$

That is, capital allocation between high-skilled and low-skilled workers depends on labor productivity. An increase in labor productivity (of either type of workers) attracts more capital to work with it, leaving less capital for the other type of workers. This mechanism works through demand linkages in both the capital and labor markets, causing reallocation of capital between workers of different skills.

And substituting (9) and (41) into (39) gives the equation of motion for capital as

$$K_{t+1} = \frac{\beta}{1+\beta}(1-\tau)(1-\alpha)\left(\Delta^h E^h + \Delta^l E^l\right)^{1-\alpha} K_t^{\alpha} \tag{42}$$

Similarly, we can derive the equation of motion for public goods as

$$G_{t+1} = \tau(1-\alpha)\left(\Delta^h E^h + \Delta^l E^l\right)^{1-\alpha} K_t^{\alpha} \tag{43}$$

Equations (42) and (43) together describe the dynamic system of this economy.

Steady state

In order to isolate the impacts of immigration, we assume that the economy is initially in a steady state.

Definition 6 *The original steady state is defined as:*

The amount of aggregate capital in period t is equal to that in period $t-1$, i.e., $K_t = K_{t-1} = K^*$.

The production function in $t-1$ period can be rewritten as

$$Y_{t-1} = \left(K_{t-1}^{h}\right)^{\alpha} \{\gamma^{h} \left(N_{t-1}^{h}\right)^{\theta^{h}}\}^{\frac{1-\alpha}{\theta^{h}}} + \left(K_{t-1}^{l}\right)^{\alpha} \{\gamma^{l} \left(N_{t-1}^{l}\right)^{\theta^{l}}\}^{\frac{1-\alpha}{\theta^{l}}}.$$

Equalization of capital earnings gives

$$K_{t-1}^{l} = \frac{\Delta^{l}}{\Delta^{h}} K_{t-1}^{h}.$$

Capital in period t is determined by savings in period $t-1$,

$$\begin{aligned} K_t &= S_{t-1} N_{t-1} \\ &= \frac{\beta}{1+\beta} (1-\tau)(1-\alpha) \left(\Delta^{h} + \Delta^{l}\right)^{1-\alpha} K_{t-1}^{\alpha}. \end{aligned}$$

The value of capital at the steady state is then

$$K^{*} = [\frac{\beta}{1+\beta} (1-\tau)(1-\alpha) \left(\Delta^{h} + \Delta^{l}\right)^{1-\alpha}]^{\frac{1}{1-\alpha}}. \tag{44}$$

Since the capital stock is determined by the disposable income, it becomes higher under a lower tax rate and with a higher ratio of skilled workers, which in turn leads to higher output. Substituting (44) into (42), we can further obtain K_{t+1},

$$K_{t+1} = \frac{\beta}{1+\beta} (1-\tau)(1-\alpha) \left(\Delta^{h} E^{h} + \Delta^{l} E^{l}\right)^{1-\alpha} K_{t}^{\alpha} \tag{45}$$

Note that (44) and (45) may be different from each other, since population grows with immigration, increasing labor input. Population grows at

$$\begin{aligned} \frac{TP_{t+n+1}}{TP_{t+n}} - 1 &= \frac{(1-b^{\frac{t+n}{1-\theta^{h}}})(E^{h})^{\theta^{h}}[(E^{h})^{\theta^{h}}+1] + (1-b^{\frac{t+n}{1-\theta^{h}}})(E^{l})^{\theta^{l}}[(E^{l})^{\theta^{l}}+1]}{(1-b^{\frac{t+n}{1-\theta^{h}}}+1-b^{\frac{t+n-1}{1-\theta^{h}}})(E^{h})^{\theta^{h}} + (1-b^{\frac{t+n-1}{1-\theta^{l}}}+1-b^{\frac{t+n}{1-\theta^{h}}})(E^{l})^{\theta^{l}}} - 1 \\ &= \tilde{b} \end{aligned}$$

See the subsection "Population structure" below and (51).

If the economy moves to a new steady state where $(\frac{K_{t+n+1}}{K_{t+n}} - 1)/(\frac{TP_{t+n+1}}{TP_{t+n}} - 1) = 1$, i.e., $K_{t+n+1}/K_{t+n} = 1 + \tilde{b}$, after some periods we must have,

$$\begin{aligned} K_{t+n+1} &= \frac{\beta}{1+\beta} (1-\tau)(1-\alpha) \left(\Delta^{h} E^{h} + \Delta^{l} E^{l}\right)^{1-\alpha} K_{t+n}^{\alpha} \\ \tilde{K} &= \{\frac{\frac{\beta}{1+\beta}(1-\tau)(1-\alpha)\left(\Delta^{h} E^{h} + \Delta^{l} E^{l}\right)^{1-\alpha}}{1+\tilde{b}}\}^{\frac{1}{1-\alpha}} \end{aligned} \tag{46}$$

where $\tilde{K}$ stands for the capital labor ratio at the new steady state.

The difference between (44) and (45) (or (46)) is one of the sources that cause various conflicts between natives and immigrants in the present model.

Capital Stock Ratio $\frac{K_{t+2}}{K_{t+1}}$

In calculating the welfare difference between t and $t+1$, we need to know K_{t+2}/K_{t+1}. K_{t+2} is from the aggregate capital accumulation at $t+1$.

$$\begin{aligned} K_{t+2} &= \frac{\beta}{1+\beta}(1-\tau)(w_{t+1}^{n,h}N_{t+1}^{h}+w_{t+1}^{n,l}N_{t+1}^{l}+w_{t+1}^{I,h}I_{t+1}^{h}+w_{t+1}^{I,l}I_{t+1}^{l}) \\ &= \frac{\beta}{1+\beta}(1-\tau)\left(1-\alpha\right)\left(\Delta_{t+1}^{h}E^{h}+\Delta_{t+1}^{l}E^{l}\right)^{1-\alpha}K_{t+1}^{\alpha}, \end{aligned}$$

where $\Delta_{t+1}^{x}=\left(\gamma^{x}\right)^{\frac{1}{\theta^{x}}}N_{t+1}^{x}$. The population structure is different at t and $t+1$,because the numbers of high and low skilled immigrants differ. Using K_{t+1} and K_{t}^{*}, and rearranging give

$$K_{t+1}=\frac{\beta}{1+\beta}(1-\tau)\left(1-\alpha\right)\left(\Delta^{h}E^{h}+\Delta^{l}E^{l}\right)^{1-\alpha}K_{t}^{\alpha}$$

$$\frac{K_{t+2}}{K_{t+1}}=\{\frac{\left(\gamma^{h}\right)^{\frac{1}{\theta^{h}}}\left(E^{h}\right)^{1+\theta^{h}}+\left(\gamma^{l}\right)^{\frac{1}{\theta^{l}}}\left(E^{l}\right)^{1+\theta^{l}}}{[\frac{\left(\gamma^{h}\right)^{\frac{1}{\theta^{h}}}+\left(\gamma^{l}\right)^{\frac{1}{\theta^{l}}}}{\left(\gamma^{h}\right)^{\frac{1}{\theta^{h}}}E^{h}+\left(\gamma^{l}\right)^{\frac{1}{\theta^{l}}}E^{l}}]^{\alpha}[\left(\gamma^{h}\right)^{\frac{1}{\theta^{h}}}E^{h}+\left(\gamma^{l}\right)^{\frac{1}{\theta^{l}}}E^{l}]}\}^{1-\alpha}>1.$$

Welfare

Using (1), (2), (3), (4) and (5) , we obtain the welfare difference with and without immigration as;

$$\begin{aligned} U_{t}^{with,nx}-U_{t}^{without,nx} &= (1+\beta)[\log(w_{t}^{with,nx})-\log(w_{t}^{without,nx})] \\ &\quad +\beta\log R_{t+1}^{with}-\beta\log R_{t+1}^{without} \\ &\quad +\beta\log\tau(w_{t}^{with,h}N^{h}+w_{t}^{with,l}N^{l}+w_{t}^{with,h}I_{t}^{h}+w_{t}^{with,l}I_{t}^{l}) \\ &\quad -\beta\log\tau(w_{t}^{without,h}N^{h}+w_{t}^{without,l}N^{l}) \\ &\quad +(1+\beta)\left(\log 2N-\log(2N+I)\right). \end{aligned}$$

Rearranging this leads to (22).

Immediate Assimilation

Under immediate assimilation of immigrants, the production function is changed to

$$Y_{t}=\left(K_{t}^{h}\right)^{\alpha}\{\gamma^{h}\left(N_{t}^{h}\right)^{\theta^{h}}+\gamma^{h}\left(I_{t}^{h}\right)^{\theta^{h}}\}^{\frac{1-\alpha}{\theta^{h}}}+\left(K_{t}^{l}\right)^{\alpha}\{\gamma^{l}\left(N_{t}^{l}\right)^{\theta^{l}}+\gamma^{l}\left(I_{t}^{l}\right)^{\theta^{l}}\}^{\frac{1-\alpha}{\theta^{l}}}.$$

In this case, wage rates are

$$w_{t}^{x} = \left(K_{t}^{x}\right)^{\alpha}\left(1-\alpha\right)\{\gamma^{x}\left(N_{t}^{x}\right)^{\theta^{x}}+\gamma^{x}\left(I_{t}^{x}\right)^{\theta^{x}}\}^{\frac{1-\alpha}{\theta^{x}}-1}\gamma^{x}\left(N_{t}^{x}\right)^{\theta^{x}-1}, \tag{47}$$

$$w_{t}^{I,x} = \left(K_{t}^{x}\right)^{\alpha}\left(1-\alpha\right)\{\gamma^{x}\left(N_{t}^{x}\right)^{\theta^{x}}+\gamma^{x}\left(I_{t}^{x}\right)^{\theta^{x}}\}^{\frac{1-\alpha}{\theta^{x}}-1}\gamma^{x}\left(I_{t}^{x}\right)^{\theta^{x}-1}, \tag{48}$$

Immigrants and natives receive the same wage for their type. Setting (47) equal to (48) we obtain the number of immigrants the host country accepts,

$$N_t^x = I_t^x.$$

Moreover, the interest rate is given by

$$R_{t+1} = \alpha \left(K_t^l\right)^{\alpha-1} \{\gamma^l \left(N_t^l\right)^{\theta^l} + \gamma^l \left(I_t^l\right)^{\theta^l}\}^{\frac{1-\alpha}{\theta^l}}.$$

And then the capital allocation between skilled and low skilled is determined by

$$\left(\frac{K_t^l}{K_t^h}\right) = \frac{\{2\gamma^l \left(N_t^l\right)^{\theta^l}\}^{\frac{1}{\theta^l}}}{\{2\gamma^h \left(N_t^h\right)^{\theta^h}\}^{\frac{1}{\theta^h}}}.$$

Given $\theta^h < \theta^l$,we have $1/\theta^h > 1/\theta^l$. Combining the above and (14) yields,

$$K_t^x = \frac{2^{\frac{1}{\theta^x}}\Delta^x}{2^{\frac{1}{\theta^h}}\Delta^h + 2^{\frac{1}{\theta^l}}\Delta^l}K^*,$$

where $\Lambda_0 = \frac{\{\gamma^l\left(N_{t-1}^l\right)^{\theta^l}\}^{\frac{1}{\theta^l}}}{\{\gamma^h\left(N_{t-1}^h\right)^{\theta^h}\}^{\frac{1}{\theta^h}}} = \frac{\Delta^l}{\Delta^h}$. Comparing the wages of the assimilation case with that of no immigration , we have

$$\frac{w_t^x}{w_{t-1}^x} = 2^{\frac{1-\theta^x}{\theta^x}}[\frac{\Delta^h + \Delta^l}{2^{\frac{1}{\theta^h}}\Delta^h + 2^{\frac{1}{\theta^l}}\Delta^l}]^{\alpha},$$

The immigration effect depends on α, θ^h and θ^l. While qualitatively it is the same as when immigrants do not immediately assimilate, the quantitative level is different. Moreover, under immediate assimilation, the host country accepts the same number of immigrants as natives, since their productivity is the same. This means the population structure will not change. In contrast, in the non-assimilation case, the population structure gradually changes because the numbers of high-skilled and low-skilled immigrants the economy accepts are different.

Population Structure

Total population

Only natives live in the host country before time t. From time t, the host country receives I_t number of immigrants every period. Then there are N_{t-1}^y workers and N_{t-1}^o retirees at $t-1$, since the number of original natives does not grow. The total population at $t-1$ is

$$TP_{t-1} = N_{t-1}^{h,y} + N_{t-1}^{l,y} + N_{t-1}^{h,o} + N_{t-1}^{l,o} = 2N_{t-1}$$

At t, with immigrants $I_t = I_t^{h,y} + I_t^{l,y}$, the total population becomes

$$TP_t = 2N_t + I_t = N_t^{h,y} + N_t^{l,y} + N_t^{h,o} + N_t^{l,o} + I_t^{h,y} + I_t^{l,y}$$

The representative firm's optimization determines the number of immigrants accepted, so that we have (12). Then, $N_t^{x,y} + I_t^{x,y} = N_t^{x,y}[1 + (b)^{\frac{1}{1-\theta^x}}] = N_t^{x,y}(E^x)^{\theta^x}$. Using these and $N_t^{h,o} + N_t^{l,o} = N_{t-1}^{h,y} + N_{t-1}^{l,y} = N_t^{h,y} + N_t^{l,y}$, we obtain another expression for total population

$$TP_t = N_t^{h,y}[1 + (E^h)^{\theta^h}] + N_t^{l,y}[1 + (E^l)^{\theta^l}] + N_t^o \tag{49}$$

Next, the total population at $t+1$ is

$$TP_{t+1} = N_{t+1}^{h,y} + N_{t+1}^{l,y} + N_{t+1}^{h,o} + N_{t+1}^{l,o} + I_{t+1}^{h,y} + I_{t+1}^{l,y} + I_{t+1}^{h,o} + I_{t+1}^{l,o}$$

Here, the number of original native olds at $t+1$ is equal to that of original native youngs at t, i.e., $N_{t+1}^{h,o} + N_{t+1}^{l,o} = N_t$. Similarly, since young immigrants coming at t become old at $t+1$, we have $I_{t+1}^{h,o} + I_{t+1}^{l,o} = I_t$.Optimization determines the number of immigrants, so that we have (12) in each period. Then, since $N_{t+1}^{x,y} + I_{t+1}^{x,y} = N_{t+1}^{x,y}\left(1 + (b)^{\frac{1}{1-\theta^x}}\right) = N_{t+1}^{x,y}(E^x)^{\theta^x}$, we obtain

$$N_{t+1}^{h,y} + N_{t+1}^{l,y} + I_{t+1}^{h,y} + I_{t+1}^{l,y} = N_t^{h,y}[(E^h)^{\theta^h}]^2 + N_t^{l,y}[(E^l)^{\theta^l}]^2.$$

Moreover, since immigrants coming in at t become old at $t+1$,

$$N_{t+1}^{h,o} + N_{t+1}^{l,o} + I_{t+1}^{h,o} + I_{t+1}^{l,o} = N_t^{h,y}(E^h)^{\theta^h} + N_t^{l,y}(E^l)^{\theta^l}.$$

In addition, we have another expression from (49),

$$TP_{t+1} = N_t^{h,y}(E^h)^{\theta^h}[1 + (E^h)^{\theta^h}] + N_t^{l,y}(E^l)^{\theta^l}[1 + (E^l)^{\theta^l}]. \tag{50}$$

From (49) and (50), the ratio of total population at t and $t+1$ is

$$\frac{TP_{t+1}}{TP_t} = \frac{(E^h)^{\theta^h}[1 + (E^h)^{\theta^h}] + (E^l)^{\theta^l}[1 + (E^l)^{\theta^l}]}{[1 + (E^h)^{\theta^h}] + [1 + (E^l)^{\theta^l}]} \tag{51}$$

Subtracting the numerator from the denominator gives

$$\begin{aligned} & (E^h)^{\theta^h}[1 + (E^h)^{\theta^h}] + (E^l)^{\theta^l}[1 + (E^l)^{\theta^l}] - \{[1 + (E^h)^{\theta^h}] + [1 + (E^l)^{\theta^l}]\} \\ = \; & [1 + (E^h)^{\theta^h}][(E^h)^{\theta^h} - 1] + [1 + (E^l)^{\theta^l}][(E^l)^{\theta^l} - 1] > 0, \end{aligned}$$

since $(E^h)^{\theta^h}$ and $(E^l)^{\theta^l}$ are larger than 1. Therefore, the host-country population grows by the number of immigrants. However, since we only focus on periods t and $t+1$, the number of immigrants are too small compared to native population and we can thus ignore the scale effect in population when we consider welfare.

Population growth at the new steady state

After period t, the host country receives both high and low skilled immigration at constant but different rates, and thus the population grows at different rates. Let us compare between periods $t+n$ and $t+n+1$. The rate is expressed by

$$\begin{aligned} & \frac{TP_{t+n+1}}{TP_{t+n}} \\ = \;& \frac{N^{h,y}_{t+n+1}+N^{l,y}_{t+n+1}+N^{h,o}_{t+n+1}+N^{l,o}_{t+n+1}+I^{h,y}_{t+n+1}+I^{l,y}_{t+n+1}+I^{h,o}_{t+n+1}+I^{l,o}_{t+n+1}}{N^{h,y}_{t+n}+I^{h,y}_{t+n}+N^{l,y}_{t+n}+I^{l,y}_{t+n}+N^{h,o}_{t+n}+N^{l,o}_{t+n}} \\ = \;& \frac{(1-b^{\frac{t+n}{1-\theta^h}})\{A(E^h)^{\theta^h}[(E^h)^{\theta^h}+1]+B(E^l)^{\theta^l}[(E^l)^{\theta^l}+1]\}}{(1-b^{\frac{t+n}{1-\theta^h}})\{(E^h)^{\theta^h}A+[(E^l)^{\theta^l}]B\}+(1-b^{\frac{t+n-1}{1-\theta^h}})\{[1+(E^h)^{\theta^h}]A+[1+(E^l)^{\theta^l}]B\}}, \end{aligned}$$

where $A \equiv 1-b^{\frac{1}{1-\theta^l}}$ and $B \equiv 1-b^{\frac{1}{1-\theta^h}}$. Generally the growth rate of the total population depends on which period we choose. However, when n is sufficiently large, the growth rate becomes constant. In particular, when n approaches infinity, expressions $b^{\frac{t+n}{1-\theta^h}}$ and $b^{\frac{t+n-1}{1-\theta^h}}$ become 0.

Acknowledgement: We wish to thank B. Chen, K Futagami, S. Ikeda, T. Kikuch, K. Mino, A. Momota, K. Tabata, Y. Wang, Q. Xia, J. Yoshida, Y. Yoshida, A. Yanase and other participants of seminars at Kobe, Kyoto and Osaka Universities and at various academic meetings for their helpful comments and suggestions on previous versions. The usual disclaimer applies.

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