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Abstract

This paper examines multinational public goods provision under multilateral income transfers and productivity differences across countries. We assume the existence of a planner who uses linear approximation for utility maximization for all countries. The main findings are: (i) A country is an income receiver if it has an advantage in producing public goods; (ii) the planner country can determine the values of transfers for all countries with an adjustment cost; (iii) all countries obtain an identical level of utility; (iv) the country with the lowest adjustment cost is the best candidate for the planner country. All results are derived based on well-known information regarding the cost of producing the public goods and on income levels.

Keywords: international public goods; multilateral income transfers; productivity difference; planner; adjustment cost; welfare

JEL classification: H41, F1

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1. Introduction

International public goods (possibly including laws & safety, policies, infrastructure, organizations and sometimes even personnel, etc.) are usually systematically underprovided by private market forces, and such under-provision can have important cross-border externality effects. For instance, pollution originating in some countries may affect health status in others, terrorists may travel from one country to another, financial volatility (such as the Federal Reserve Bank's recent QE1 and QE2 (quantitative easing)) in one nation may generate follow-on fragility elsewhere, and so on. Often, national policymakers are less likely to consider the well-being of foreign citizens in setting their own policies regarding public goods. Thus, the provision of international public goods requires some form of *multilateral coordination* and even income transfers, especially when multiple countries are involved.

The present paper examines the international provision of a public good in a setup of *multiple* countries, by introducing *multilateral* income transfers and adjustment costs under productivity differences across countries. We assume the existence of a planner country who adopts linear approximation for utility maximization and an adjustment cost for income transfers. The following main results are obtained: First, country i becomes an income receiver if it has an advantage in producing public goods; second, the particular size of multilateral income transfer can be pinned down for each country; third, all countries obtain an identical level of utility; fourth, a decrease in adjustment cost leads to a Pareto improvement, and thus, the country with the lowest adjustment cost is the best candidate for the planner country (e.g., a country with efficient institutions or strong political powers). These results are obtained under well-known information on the cost of producing public goods and on the income levels for all countries.

A large body of literature has evolved since Warr's (1983) seminal work that shows the real equilibrium is unaffected by income transfers when public goods are privately provided.[1] Bergstrom et al. (1986, 1992) generalize and reinforce this result by giving proofs for existence and uniqueness of equilibrium. Using a two-country model, Ihori (1996) investigates the welfare effects of public goods provision with productivity differences across countries, and shows that an income transfer from a low productivity (or high cost) country to a high productivity country brings a Pareto improvement. As an example, Murdoch and Sandler (1986, p.84) study the provision of weapons within the North Atlantic Treaty Organization (NATO). Subsequently, a few authors have examined *bilateral income transfers* between two or more countries under productivity differences (e.g., Ihori (1999), Caplan et al. (2000), Kim and Shim (2006) and Cornes and Hartley (2007)). Cornes and Itaya (2010) examine the case of several public goods and show that voluntary contribution typically generates not only too low a level of public good provision but also the wrong mix of public goods.

However, there is little literature on *multilateral income transfers* across *multiple* countries, especially with regard to the following issues. First, how are the different productivities among the multiple countries compared, and which countries are income senders or receivers? Second, how much do income receivers receive? Third, which country manages the income transfer? To our knowledge, the present paper is the first to investigate the effects of *multilateral income transfers* across multiple countries with an explicit planner rule, particularly emphasizing the three questions raised above. In

[1] Ihori (1996) and Cornes and Hartley (2007) provide very informative surveys.

addition, the methods we adopt in the model, e.g., linear approximation for utility maximization by the planner and the adjustment cost for income transfers, are convenient but still not widely used in the literature.

The rest of the paper is organized as follows. Section 2 outlines the basic model. Section 3 builds the benchmark case of bilateral income transfers. Section 4 deals in detail with multilateral transfers among multiple countries. Section 5 illustrates our equilibrium with the case of two countries. And finally Section 6 concludes.

2. The Basic Model

Consider N countries with one public good and one private good. Country i ($i=1\ldots N$) consumes an amount x_i of the private good and contributes an amount g_i to the supply of the international public good. The total supply of the international public good, G, is just the sum of g_i provided by each country. Country i's utility is given by $U_i = U_i(x_i, G)$, where U_i is strictly increasing and quasi-concave, and x_i and G are normal goods for each country. Country i's budget constraint is given by $x_i + p_i g_i = y_i$, where $y_i > 0$ is its exogenously given national income and $p_i > 0$ is the relative price (cost of production) of public goods in terms of private consumption in country i. As in Ihori (1996), a low (high) p_i means a high (low) productivity in producing the public good. We also make the Cournot–Nash assumption that each country believes that the contributions of others are independent of its own. Then, we can rewrite $G = g_i + \sum_{j \neq i} g_j$ as the total public good, where $\sum_{j \neq i} g_j$ is the sum of g_j provided by countries j other than i.

Definition 1. A Cournot–Nash equilibrium in this model is such that for each *i (i* = l,2,...,*N),* (x_i^*, G^*) solves:

$$\begin{aligned} &\max_{x_i,G} \quad U_i = U_i(x_i, G) \\ &s.t. \quad x_i + p_i G = y_i + p_i \sum\nolimits_{j\neq i} g_j^*, \\ &x_i \geq 0, \;\; G \geq \sum\nolimits_{j\neq i} g_j^* \geq 0, \quad i = 1,2,..,N \end{aligned} \tag{1}$$

As is well known, in (1), each country implicitly chooses not only its own supply g_i^* of international public goods contribution, but also in effect the equilibrium level of the total supply G^*, and each country consumes the same level of G^*. The existence and uniqueness of this solution are proved by Cornes and Hartley (2007). The first order condition for each *i* is $p_i \partial U_i / \partial x_i = \partial U_i / \partial G$ (*i* = 1,2,..,*N*). Utilizing the implicit function theorem, the *N* equations can be solved to give x_i^* (*i* = 1,2,…,*N*).[2]

Next, we use Definition 1 and the expenditure function to analyze the effects of income transfers, following Ihori (1996). Define the expenditure function at the Cournot–Nash equilibrium as:

$$E_i \equiv \min_{x_i,G} x_i + p_i G \qquad s.t. \;\; U^i(x_i, G) = \bar{U}_i^* \tag{2}$$

where $\bar{U}_i^* \equiv U_i(x_i^*, G^*)$ is the utility level at the equilibrium for country i. Then, $E_i = E_i(\bar{U}_i^*, p_i)$, and at the equilibrium, it is equal to the income in (1):

$$E_i(\bar{U}_i^*, p_i) = y_i + p_i \sum\nolimits_{j\neq i} g_j^*, \tag{3}$$

[2] Warr (1983) sums the budget constraints of the *N* countries and then obtains G^* under $p_i = 1$ for all *i*. In contrast, $p_i \neq 1$ in our model, and the equilibrium G^* will be derived soon as in (5a) and (5b).

where the income on the right-hand side contains actual income y_i and the externalities from other countries' provision of the public good.

By Shephard's Lemma, we can derive:

$$\frac{\partial E_i(\bar{U}_i^*, p_i)}{\partial p_i} \equiv G_i(\bar{U}_i^*, p_i), \quad G_i(\bar{U}_i^*, p_i) = G^*, \tag{4}$$

where $G_i(\bar{U}_i^*, p_i)$ is the compensated demand function of country i for the international public good. From (3) and (4), the equilibrium can be summarized by:

$$\sum_{i=1}^{N} \Phi_{-i} E_i(\bar{U}_i^*, p_i) = \sum_{i=1}^{N} \Phi_{-i} y_i + \Phi(N-1)G^*, \tag{5a}$$

$$G_i(\bar{U}_i^*, p_i) = G^*, \quad (i = 1, 2, ..., N), \tag{5b}$$

where $\Phi = \prod_{i=1}^{N} p_i$, $\Phi_{-i} = \prod_{j \neq i}^{N} p_j$ and (5a) is obtained by multiplying (3) by Φ_{-i} and summing up. These N+1 equations determine the utilities $\bar{U}_i^*$ of the N countries and the amount of the international public good G^* at the equilibrium. This formulation is similar to Ihori (1999, p.48-49), extending Ihori's (1996) two-country model to N countries.

Let us now use the above framework to examine both bilateral and multilateral income transfers. Taking the total derivatives of (5a) and (5b) consisting of N+1 equations gives:

$$\sum_{i=1}^{N} \Phi_{-i} E_{iU} d\bar{U}_i^* - \Phi(N-1) dG^* = \sum_{i=1}^{N} \Phi_{-i} dy_i, \tag{6a}$$

$$G_{iu} d\bar{U}_i^* - dG^* = 0, \quad (i = 1,2,..,N), \tag{6b}$$

where $\partial E_i(\bar{U}_i^*, p_i)/\partial \bar{U}_i^* \equiv E_{iU}$ and $\partial G_i(\bar{U}_i^*, p_i)/\partial \bar{U}_i^* \equiv G_{iU}$. Substituting (6b) into (6a), we can decrease the number of equations from N+1 to N. Moreover, by using the relationship of the expenditure function (2) for country 1,

$$E_{1U} = x_{1U} + p_1 G_{1U}, \tag{7}$$

we can rewrite (6a) and (6b) as follows:

$$\Phi_{-1}(x_{1U} + p_1 G_{1U}) d\bar{U}_1^* + \sum_{i=2}^{N} \Phi_{-i} E_{iU} d\bar{U}_i^* - \Phi(N-1) G_{1u} d\bar{U}_1^* = \sum_{i=1}^{N} \Phi_{-i} dy_i, \tag{8a}$$

$$G_{iu} d\bar{U}_i^* - G_{ju} d\bar{U}_j^* = 0 \;, (i,j = 1,\ldots,N), \tag{8b}$$

which can be arranged in vector and matrix format,

$$\begin{bmatrix} \Phi_{-1}(x_{1U} + p_1 G_{1U}(2-N)), \Phi_{-2}E_{2U}, \ldots\ldots\ldots\ldots, \Phi_{-N}E_{NU} \\ \qquad -G_{1U}, \quad G_{2U}, \; 0 \;\; , 0 \ldots\ldots\ldots , 0 \\ 0 \qquad\qquad , \; -G_{2U}, G_{3U}, 0 \ldots\ldots\ldots , 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ 0 \qquad\qquad , \; 0 \;\; , \; 0 \;\; , 0 \ldots -G_{(N-1)U}, G_{NU} \end{bmatrix} \bullet \begin{bmatrix} d\bar{U}_1^* \\ d\bar{U}_2^* \\ .. \\ .. \\ .. \\ d\bar{U}_N^* \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{N} \Phi_{-i} dy_i \\ 0 \\ . \\ . \\ . \\ 0 \end{bmatrix}, \tag{9a}$$

where the determinant Δ of the matrix on the left hand side is:

$$\begin{aligned} \Delta &= \Phi_{-1}(x_{1U} + p_1 G_{1U}(2-N))\Delta_{11} + G_{1U}\Delta_{21} \\ &= \Phi_{-1}(x_{1U} + p_1 G_{1U}(2-N))(G_{2U} \cdot\cdot G_{NU}) + G_{1U} \sum_{i=2}^{N-1} \Phi_{-i} E_{iU} (\Pi_{j \neq i} G_{2U} \cdot\cdot G_{jU} \cdot\cdot G_{NU}), \end{aligned} \tag{9b}$$

and Δ_{11}, Δ_{12} are sub-determinants of Δ. Using (7), we have $\Phi_{-i} E_{iU} = \Phi_{-i} x_{iU} + \Phi_{-i} p_i G_{iU}$,

which can be inserted into (9b) to yield:

$$\begin{aligned} \Delta &= \Phi_{-1}(x_{1U} + p_1 G_{1U}(2-N))(G_{2U} \cdot\cdot G_{NU}) + (G_{1U}\, G_{2U} \cdot\cdot G_{NU}) N\Phi \\ &\quad + G_{1U} \sum_{i=2}^{N-1} \Phi_{-i} x_{iU} (\Pi_{j \neq i} G_{2U} \cdot\cdot G_{jU} \cdot\cdot G_{NU}) \\ &= \Phi_{-1}(x_{1U} + 2p_1 G_{1U})(G_{2U} \cdot\cdot G_{NU}) + G_{1U} \sum_{i=2}^{N-1} \Phi_{-i} x_{iU} (\Pi_{j \neq i} G_{2U} \cdot\cdot G_{jU} \cdot\cdot G_{NU}). \end{aligned} \tag{10}$$

The partial derivatives of the compensated demand function, x_{iU} and G_{iU}, are positive in terms of utility because they are normal goods. Then, the determinant Δ is positive.

3. Bilateral Income Transfers

Under bilateral income transfers, income is transferred from country *j* to *i* if $p_j > p_i$; that is, in (9a): $dy_i = -dy_j > 0,\ \ dy_k = 0$ for all $k \neq i, j$. Then,

$$\sum_{i=1}^{N} \Phi_{-i} dy_i = (\Phi_{-i} - \Phi_{-j}) dy_i \qquad (11)$$

The welfare effects of this transfer are derived using Cramer's rule on (9a):

$$\frac{d\overline{U}_i^*}{dy_i} = \frac{\Phi(p_j - p_i)(-1)^{2i} \Pi_{k\neq i}^{N} G_{kU}}{p_j \cdot p_i \Delta} > 0\,, \qquad (12a)$$

$$\frac{d\overline{U}_j^*}{dy_i} = \frac{\Phi(p_j - p_i)(-1)^{2j} \Pi_{k\neq j}^{N} G_{kU}}{p_j \cdot p_i \Delta} > 0\,. \qquad (12b)$$

Conditions (12a) and (12b) are the same as in Ihori (1996), Cornes and Hartley (2007), etc. That is, a transfer from country *j* with low productivity (high price p_j) of providing the public good to country *i* with high productivity (low price p_i) improves welfare in both countries.

4. Multilateral income transfers

Now, we consider multilateral income transfers among many countries. We first determine the planner, which can be anyone but only one among the *N* countries. The planner country *h* calculates the utility increase $d\overline{U}_h^*$ from the present level by taking a *first order linear approximation* around the present utility level, as in Miyakoshi et al. (2010). Intuitively, the planner uses a gradient method to transfer the income in order to achieve the highest utility. This can be done by taking a linear approximation starting at the present levels of all players, and then step by step, approaching a higher level.

Given the above, the income transfers are

$$\bar{U}_h^*(y_1+\Delta y_1, y_2+\Delta y_2, ..., y_N+\Delta y_N) \cong \bar{U}_h^*(y_1, y_2, ..., y_N) + \sum_{i=1}^{N} (\partial \bar{U}_h^* / \partial y_i) \Delta y_i \tag{13a}$$

In derivative format,

$$\begin{aligned} d\bar{U}_h^* &\equiv \bar{U}_h^*(y_1+dy_1, y_2+dy_2, ..., y_N+dy_N) - \bar{U}_h^*(y_1, y_2, ..., y_N) \\ &\cong \sum_{i=1}^{N} (\partial \bar{U}_h^* / \partial y_i) dy_i \end{aligned} \tag{13b}$$

where dy_i is income transfer to country i, and $\partial \bar{U}_h^* / \partial y_i$ is the partial derivative of utility in income (solved from (9a)).Then, the planner maximizes the utility $\bar{U}_h^*$ (or $d\bar{U}_h^*$) by transferring income to each country.

With multilateral income transfers, it is often the case that while country A is a receiver from country B, it may be also a sender to country C. Here we define a 'receiver (sender)' as a net receiver (sender) of income. The planner country collects from the net senders and redistributes to net receivers, for which we assume a transfer fee is required, that can be interpreted as administrative costs for back and forth complicated transactions among multiple countries. Observe that this type of transfer fee to our knowledge has not been proposed yet. We assume that the fee is paid with past donation or endowment, which significantly simplifies our analysis.[3]

Then, the maximization problem of the planner can be reformulated as:

$$\begin{aligned} &\underset{\{\tilde{y}_i\}}{Max}\, \tilde{U}_h = \sum_{i=1}^{N} a_{hi} \tilde{y}_i, \\ &s.t. \quad \sum_{i=1}^{N} \tilde{y}_i = 0, \quad \sum_{i=1}^{N} \tilde{y}_i^2 \le \zeta^2, \end{aligned} \tag{14a}$$

[3] As will be proven later, the planner can obtain Pareto-improvement after income transfers. Thus, this fee can be paid with endowment first and returned later with a fraction of the income gains, as long as the fee is sufficiently small.

where

$$\tilde{U}_h \equiv d\overline{U}_h^*, \quad \tilde{y}_i \equiv dy_i, \quad a_{hi} \equiv \partial \overline{U}_h^* / \partial y_i, \quad \zeta > 0 \qquad (14b)$$

The variables with a ' tilde' are deviations from an initial allocation, as contained in (14b). The first constraint in (14a) means that the income transfers are implemented with fixed total income across the N countries; the second constraint implies all income transfers are implemented with a given total cost $\zeta^2 > 0$, where the unit cost is $\tilde{y}_i^2$ in any country i (it is squared because for a positive transfer, $\tilde{y}_i > 0$ (income receiver), and for a negative transfer, $\tilde{y}_i < 0$ (income sender)). Technically, since we have adopted linear approximation for utility maximization, in order to ensure the existence of an interior solution, some form of nonlinear constraint is needed, and thus we assume the transfer fee takes a nonlinear form as in the second constraint.

Finally, a_{hi} is derived from (9a) as follows:

$$a_{hi} \equiv \frac{\partial \overline{U}_h^*}{\partial y_i} = \frac{1}{\Delta}\left[\Phi_{-i}\Pi_{k\neq h}^{N} G_{kU}\right] > 0, \quad (i = 1,2,.h,.,N). \qquad (15)$$

Then, the Lagrange equation and Kuhn-Tucker condition for (14a) are respectively:

$$L(\lambda, \tilde{y}, \phi) = \sum_{i=1}^{N} a_{hi}\tilde{y}_i + \lambda(0 - \sum_{i=1}^{N} \tilde{y}_i) + \phi(\zeta^2 - \sum_{i=1}^{N} \tilde{y}_i^2) \qquad (16a)$$

and

$$\begin{array}{l} (i).\ a_{hi} - \lambda - 2\phi\tilde{y}_i = 0, \quad (ii).\ \sum_{i=1}^{N} \tilde{y}_i = 0, \\ (iii).\ \zeta^2 \geq \sum_{i=1}^{N} \tilde{y}_i^2, \quad (iv).\ \phi(\zeta^2 - \sum_{i=1}^{N} \tilde{y}_i^2) = 0, \quad (v).\ \phi \geq 0 \end{array} \qquad (16b)$$

The multiplier ϕ is nonnegative and depends on the price parameters out of a_{hi} in (14a) and (15). In particular, ϕ is zero when p_i is the same across countries, while it is positive when p_i differs.[4] λ can be solved by inserting condition (i) into condition (ii) in (16b) if $\phi > 0$ (i.e., at least one price is different). Then, at least one a_{hi} for country i is different in (15) such that $a_{hi} - \lambda \neq 0$. Hence $\phi > 0$, and $\tilde{y}_i \neq 0$ in condition (i) in (16b).

To be more specific, when $\phi > 0$, condition (iv) in (16b) can be rewritten as $\zeta^2 = \sum_{i=1}^{N} \tilde{y}_i^2$. Using condition (i) in (16b), we get $\tilde{y}_i$.

$$\sum_{i=1}^{N} \frac{a_{hi} - \lambda}{2\phi} = 0, \quad \lambda = \frac{1}{N}\sum_{i=1}^{N} a_{hi}, \quad \tilde{y}_i = \frac{1}{2\phi}(a_{hi} - \frac{1}{N}\sum_{i=1}^{N} a_{hi}). \tag{17}$$

On the other hand, by inserting $\tilde{y}_i$ in (17) into condition (iv) in (16b), we have:

$$\phi = \frac{\sqrt{\sum_{i=1}^{N}(a_{hi} - \frac{1}{N}\sum_{i=1}^{N} a_{hi})^2}}{2\zeta}. \tag{18}$$

Also we obtain $\tilde{y}_1, \tilde{y}_2, ..., \tilde{y}_N$ by substituting (18) into (17):

$$\tilde{y}_j = \frac{(a_{hj} - \frac{1}{N}\sum_{i=1}^{N} a_{hi})\zeta}{\sqrt{\sum_{i=1}^{N}(a_{hi} - \frac{1}{N}\sum_{i=1}^{N} a_{hi})^2}}, \qquad (j = 1,2,.h,.,N). \tag{19}$$

Then, by inserting (15) into (19) and denoting $b_i \equiv 1/p_i$ (country i's productivity of producing the public good), we can get the following solution $\{\tilde{y}_j\}$ ($\forall j = 1,2,.h,.,N$):

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[4] If all prices are the same, (15) gives $a_{hi} = \lambda$, identical for all i. In fact, aggregating condition (i) and using condition (ii) in (16b), we have $\sum_{i=1}^{N} a_{hi} - N\lambda - 2\phi \sum_{i=1}^{N} \tilde{y}_i = \sum_{i=1}^{N} a_{hi} - N\lambda = 0$. Then condition (i) in (16b) means either $\phi = 0$ (in this case $\tilde{y}_i$ is not determined) or $\tilde{y}_i = 0$ for all i.

$$\tilde{y}_j = \frac{\zeta \dfrac{\Pi_{k\neq h}^{N} G_{kU}}{\Delta}\left(\Phi_{-j} - \dfrac{1}{N}\sum_{i=1}^{N}\Phi_{-i}\right)}{\sqrt{\sum_{i=1}^{N}(a_{hi} - \frac{1}{N}\sum_{i=1}^{N}a_{hi})^2}} = \frac{\zeta\left(b_j - \dfrac{1}{N}\sum_{i=1}^{N}b_i\right)}{\sqrt{\sum\limits_{i=1}^{N}\left(b_i - \dfrac{1}{N}\sum_{i=1}^{N}b_i\right)^2}} \tag{20}$$

The denominator is not zero, given different prices across countries.

Now let us interpret (20). The first finding is, when country j ‘s productivity b_j is higher than the average ($\frac{1}{N}\sum_{i=1}^{N}b_i$), then it has *an advantage in producing the public good*, and thus it becomes an income receiver ($\tilde{y}_j$>0).

Secondly, given a particular level of the adjustment cost ζ, the planner can determine the size of income transfers for all countries,[5] by maximizing the utility for each country. Specifically, for the planner country h, using information on x_{1U} for Δ and $\{G_{1U}, G_{2U}, ..., G_{NU}\}$, the optimal utility is obtained by inserting (20) into (14a):

$$d\bar{U}_h^* = \sum_{j=1}^{N}\frac{1}{\Delta}\left[\Phi_{-j}\Pi_{k\neq h}^{N}G_{kU}\right]\frac{\zeta\left(b_j - \dfrac{1}{N}\sum_{i=1}^{N}b_i\right)}{\sqrt{\sum\limits_{i=1}^{N}\left(b_i - \dfrac{1}{N}\sum_{i=1}^{N}b_i\right)^2}}\,. \tag{21}$$

And for any other country o , we can approximate its utility by inserting (20) into (13b):

$$d\bar{U}_o^* = \sum_{j=1}^{N}\frac{1}{\Delta}\left[\Phi_{-j}\Pi_{k\neq o}^{N}G_{kU}\right]\frac{\zeta\left(b_j - \dfrac{1}{N}\sum_{i=1}^{N}b_i\right)}{\sqrt{\sum\limits_{i=1}^{N}\left(b_i - \dfrac{1}{N}\sum_{i=1}^{N}b_i\right)^2}}\,. \tag{22}$$

[5] Cornes and Hartley (2007, Corollary 4.2) imply a solution for bilateral income transfers where only the country with the highest productivity provides the public good. However, their solution does not show how much public good there is, how much income is transferred, and whether this country still remains the most productive when it produces a large amount.

Note that if the planner is switched from country h to country o, the optimal utility level of country o is obtained by replacing $\Pi_{k\neq h}^{N} G_{kU}$ with $\Pi_{k\neq o}^{N} G_{kU}$ in (21), which is the same as in (22), unsurprisingly.

Thirdly, from the above, all countries obtain the same maximum utility level, $d\overline{U}_i^*$. When the transfer fee is treated as sunk as in (20), then the optimal income transfers do not depend on the identity of the planner, i.e., any country could be the planner.

Finally, however, if the unit adjustment cost ($1\cdot \tilde{y}_i^2$ as in (14a)) for income transfers is reduced, the utility levels for all countries increase, yielding a Pareto improvement. Thus, the country with the lowest adjustment cost is the appropriate candidate for the planner country. In reality the level of adjustment cost may depend on country size, institution efficiency, political powers, and so on.

Note especially that the above findings are all derived based on well-known information on the cost p_i of producing public goods, and on the income level y_i.

5. An illustration of Multilateral Income Transfers

We provide an illustration using the example of $N = 2$. Due to (14a), the maximization problem of a planner country (say country 1) is as follows:

$$\underset{\{\tilde{y}_1,\tilde{y}_2\}}{Max}\ \tilde{U}_1 = a_{11}\tilde{y}_1 + a_{12}\tilde{y}_2, \tag{23a}$$

$$s.t.\quad \tilde{y}_1 + \tilde{y}_2 = 0, \tag{23b}$$

$$\tilde{y}_1^2 + \tilde{y}_2^2 \leq \zeta^2. \tag{23c}$$

Moreover, using (15) we have:

$$a_{11} = \Phi_{-1} G_{2U} / \Delta > 0, \quad a_{12} = \Phi_{-2} G_{2U} / \Delta > 0 \quad \text{and } \Phi_{-1} = p_2, \;\; \Phi_{-2} = p_1. \qquad (24)$$

Then, the objective function in (23a) can be rewritten as:

$$\tilde{U}_1 = G_{2U} \left[p_2 \tilde{y}_1 + p_1 \tilde{y}_2 \right] / \Delta = A \left[p_2 \tilde{y}_1 + p_1 \tilde{y}_2 \right]: \quad A \equiv G_{2U} / \Delta > 0 \qquad (25)$$

Insert Figure 1 about here

In Figure 1, the term $\tilde{U}_1 / P_1 A$ identifies the utility level, constraint (23a) is the downward sloping solid line and constraint (23c) is the region within the circle. Thus, without the transaction fee, constraint (23c) disappears and there is no circle.

If $P_1 = P_2$, the objective function (25) coincides with constraint (23b). Then the optimum transfers are on constraint (23b) and within (23c), but not uniquely decided; and the optimum utility level is zero.

Next consider when $P_1 \neq P_2$ (we take the case of $P_1 > P_2$). The objective function (25) reaches the maximum utility for planner country 1 at point *C*. The optimum income transfer exists at point *C** with $\tilde{y}_1^* < 0$ and $\tilde{y}_2^* > 0$ and the utility level is $A\left[p_2 \tilde{y}_1^* + p_1 \tilde{y}_2^*\right]$. Also note that if constraint (23c) does not exist, the optimal income transfer $\tilde{y}_1^* < 0$ endlessly decreases to obtain the maximum utility, by moving upwards on the downward sloping solid line. This formulation of maximization without (23c) is used in previous studies. Meanwhile the non-planner country 2 receives income from planner country 1 and obtains utility level: $B\left[p_2 \tilde{y}_1^* + p_1 \tilde{y}_2^*\right]$ where $B \equiv G_{1U} / \Delta$.

On the other hand, if country 2 becomes the planner, its objective function is $B\left[p_2 \tilde{y}_1 + p_1 \tilde{y}_2\right]$ where $B \equiv G_{1U} / \Delta$. When $P_1 > P_2$, the optimal income transfer is given at point C^* in Figure 1 and its utility level is obtained as $B\left[p_2\tilde{y}_1^* + p_1\tilde{y}_2^*\right]$, which is the same as that of country 2 in the previous case when it is not a planner. The reason is as follows. The optimal transfers are decided only by the constraints (23b) and (23c). Moreover, the utility increase (see (23a)) is approximated by a linear function of the income transfers of all countries. The difference between the utility increases in countries 1 and 2 is only a constant coefficient A or B, which does not affect the optimal $\tilde{y}_1^*$ or $\tilde{y}_2^*$. Thus whichever country becomes the planner, it necessarily obtains the intersection of the two constraints (23b) and (23c).

Finally, when the unit adjustment cost of income transfers increases, constraint (23c) becomes higher and in turn the utility for each country decreases. The same logic holds valid in a model of N (>2) countries.

6. Concluding Remarks

When countries differ in productivity, how should income be transferred among multilateral countries so that proper public goods are provided? This paper has proposed an explicit rule that is effective: Assuming a planner among N countries, that can maximize its utility by linear approximation and transferring income to each country for a fixed adjustment cost. This adjustment cost is required to form agreements among income-sending and income-receiving countries. The planner country uses only well-known information on the cost of producing public goods and on the income levels of all countries. More importantly, all countries obtain the same level of utility and the country with the lowest adjustment cost is the best candidate for the planner country.

A very interesting extension of our model is to introduce outsourcing of public goods provision. Our mechanism is similar to the market of emission permits. To combat global warming, a country can buy emission permits from other countries (see Bohringer (2005), Rosendahl (2008) and Flachsland et al. (2009)). Our mechanism allows countries to outsource the production of the public good, which can serve as an alternative to income transfers. This remains a fruitful avenue for future research.

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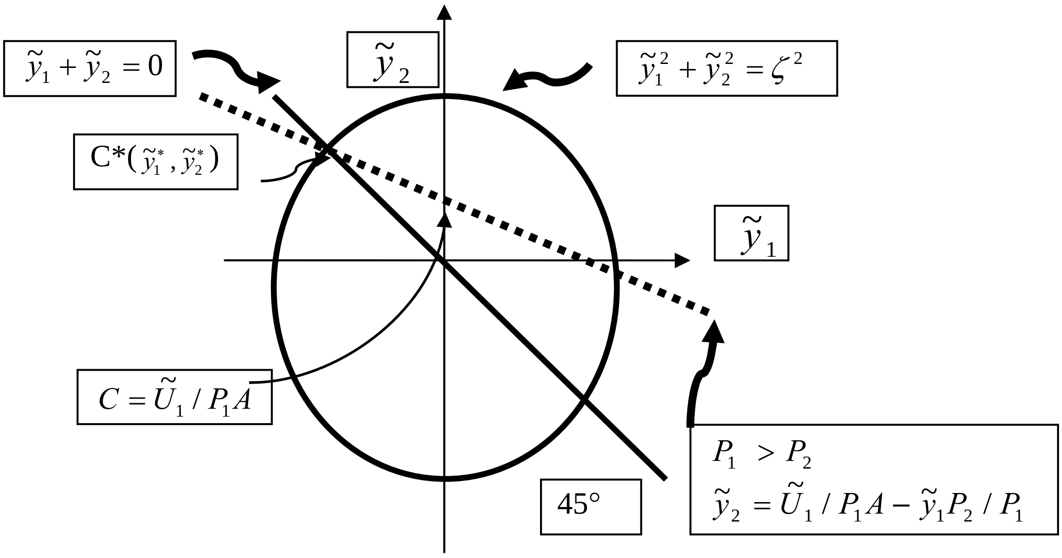

Figure 1: The case of *N* = 2