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Abstract

We study how the level of trade costs and the intensity of competition can explain the existence of two-way, one-way or no trade within the same industry. As trade costs decrease from very high to very low values, the economy moves from autarky to a regime of two-way trade, through a regime of one-way trade from the larger to the smaller country. Trade is less likely when the economy gets more competitive. Finally once capital is mobile across countries, the market delivers an outcome in which capital is too much concentrated in the large country.

Keywords: trade regime; country asymmetry; capital mobility

JEL Classification: F12; H22; H87; R12

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1 Introduction

Living in a globalizing world conveys the impression that trade is the main ingredient of economic life. However trade takes several forms as it may occur within the same sector or between sectors. For instance, intra-industry trade has steadily increases ever since World War II and has served as the main motivation for the development of the new trade theories. By contrast, inter-industry trade has been the focus of standard trade theories, like the Heckscher-Ohlin and Ricardian models, that explain one-way trade by the presence of country-specific factor endowments and technologies. In response to such differences in predictions, empirical studies have aimed to measure the intensity of these two types of trade.¹ No strong conclusion emerges from the empirical studies, thus suggesting that both types of trade play a major role in international relationships. In addition, there is also evidence that many pairs of countries have no trade in many industries (Haveman and Hummels, 2004; Baldwin and Harrigan, 2007). According to Helpman *et al.* (2008), over the last few decades two-way, one-way and no trade at all respectively account for 30 – 40, 10 – 20 and 50 – 60 percent of country pairs in 158 countries. Thus, we may safely conclude that world trade is not dominated by two-way trade. The existence of asymmetric trade patterns and the absence of trade between and within sectors are other important features of the trade landscape, which deserve more attention.

To the best of our knowledge, there is no work accounting for three patterns (two-way trade, one-way trade, and no trade) within a single unified framework highlighting the impact of trade costs on the nature of trade when capital is free to move between countries.² The Japanese textile industry provides a good illustration of the various regimes of trade we want to study in this paper. The Tokugawa Shogun initiated a seclusion policy in 1639 for more than 200

¹See Hummels and Levinsohn (1995), Evenett and Keller (2002), Debaere and Demiroglu (2003), and Debaere (2005).

²Behrens (2005) studies the one-way and two-way trade cases in the core-periphery model. However, the objective of his paper is different from ours. Helpman *et al.* (2008) focus on trade when production factors are immobile.

years. Only the Dutch and Chinese were allowed to trade with Japan in a few specific ports. During this period, cotton textiles were produced and consumed under autarky. Ever since the 1853 US naval appearance in Tokyo, the pressure toward trade opening with foreign countries increased. Eventually, Japan decided to move from autarky to an open economy. To meet domestic demand, cotton textiles were imported after trade opening. This may be viewed as a period involving one-way trade in the textile sector. By way of contrast, in the 1880s and 1890s, which corresponds to the First Era of Globalization, cotton textiles started being exported and their value drastically increased to exceed that of imports from 1910 onward (Fujino *et al.*, 1979). This corresponds to a regime of two-way trade. Such shifts from autarky, one-way to two-way trade have been observed in many other manufacturing sectors such as iron-steel and chemical products in Japan. To sum up, those examples suggest that trade liberalization has been critical in the evolution of the production and trade patterns.

In this paper, we aim to study the impact of trade costs on the *nature* of trade, but also on the long run *international allocation of capital*, which is another major facet of the process of globalization. To achieve our goal, we use a linear model of monopolistic competition, which captures the following two basic features: competition gets tougher when the number of firms increases as well as when the level of trade costs falls (Ottaviano *et al.*, 2002). We then combine this model with a two-asymmetric country setting to capture the well-documented fact that market size plays a major role in trade and foreign direct investments. Depending on the level of trade costs, we show that the above-mentioned trade patterns emerge as equilibrium outcomes. More precisely, *as trade costs decrease from very high to very low values, the economy moves from autarky to a regime of two-way trade, through a regime of one-way trade from the larger to the smaller country.* In the one-way trade regime, the country accommodating the larger number of firms exports toward the country having the smaller number of firms because competition is softer therein. In other words, even though firms are homogeneous, their choice to export depends on the intensity of competition on their foreign markets. This is to be contrasted with Helpman *et al.* (2008) where the choice to export is independent from the intensity of

competition, but is determined by firms' idiosyncratic productivity and fixed costs of serving foreign countries. We thus offer an alternative explanation for the existence of no trade or one-way trade.

Our setting also allows us to parametrize the toughness of competition through a single parameter that encompasses the number and the degree of substitutability of the varieties available in all markets. Using this parameter, we show that the likelihood of regimes involving two-way or one-way trade decreases as the economy gets more competitive. This is because the penetration of foreign products becomes more difficult once the mass of domestic firms is large enough. Although the emergence of two-way trade within various sectors is often interpreted as evidence of intensified competition, our analysis thus shows that two-way trade may also be the outcome of defensive strategies adopted by firms to relax competition. By contrast, a stronger degree of competitiveness fosters the emergence of one-way trade where only the large countries export. As a result, implementing a tough competition policy before liberalizing trade deters the entry of foreign competitors in large countries.

Since the international distribution of firms is a major determinant of the nature and intensity of trade, we find it natural to ask how capital is allocated when it is free to move from one country to the other. This is an important issue because, *by changing their investment locations, capital-owners affect the intensity of competition within each country, thus making the penetration of foreign products easier or more difficult.* Put differently, we recognize that both the nature and intensity of trade vary with the mobility of capital. For example, under one-way trade, we show that the larger country is more competitive than the smaller one, which prevents the latter country from exporting to the former one. We also show that the home-market effect prevails in the three regimes of trade provided that the economy is not too competitive. In this case, the large and prosperous country attracts a disproportionate share of capital. The market size advantage stressed in economic geography thus provides an explanation to Lucas' (1992) query: Why doesn't capital flow from rich to poor countries? It is worth stressing that the above answer to Lucas' question is independent from the nature of trade between countries.

The home-market effect obtained under one-way trade that characterized many poor countries also concurs with Collier (2005) who observes that the bottom billion countries export their capital instead of attracting investments. In contrast, when the economy is very competitive the home-market effect may be reversed in the autarky and one-way trade regimes. In other words, the capital flows can run in the opposite direction when national markets are very competitive. Our analysis thus highlights *the role played by the intensity of competition, not only for the nature of trade, but also for the international allocation of capital.*

Last, we determine the efficient allocation of capital and show that the market delivers an outcome in which capital is too much concentrated in the larger country, whatever the trade regime. This result differs markedly from those derived in the standard model involving perfect competition, constant returns and zero transport costs, where capital mobility always leads to the socially optimal distribution of activities. On the contrary, once we account for imperfect competition, increasing returns and positive transport costs, *the liberalization of capital mobility favors the larger country at the expense of the smaller one.* This is to be contrasted with the case of capital immobility where the smaller country gains more than the larger one from trade liberalization because the small country's consumers get a better access to a wider range of foreign varieties.

Our paper proceeds as follows. The next section introduces the model and characterizes the demand structure under autarky, one-way and two-way trade. Section 3 explores the product market equilibrium under these various trade regimes, whereas Section 4 focuses on the equilibrium allocation of capital. Section 5 is devoted to the welfare analysis, while Section 6 concludes.

2 The model

The economy involves two tradeable goods, two production factors, and two countries $i = H, F$ with a population of size θ_H and θ_F , respectively; without loss of generality, we assume that

$\theta_H \geq \theta_F$ with $\theta_H + \theta_F = 1$. The global supply of capital is normalized to 1. Individuals are immobile and endowed with some share of capital; they are free to invest their capital wherever they want. The spatial allocation of capital is endogenous and will be determined as an equilibrium outcome. By contrast, each individual inelastically supplies a unit of labor in the country where she resides. To disentangle the various channels through which the equilibrium occurs, we will distinguish between a *short-run equilibrium* in which the capital supply is fixed in each country, and a *long-run equilibrium* when capital is freely allocated across countries.

Both countries produce a homogeneous good and a range of manufacturing differentiated varieties. Following the literature, we assume that the homogeneous good Z is produced under constant returns to scale and perfect competition by using labor as the only input and using no capital. Our purpose being to investigate how trade costs affect the spatial distribution of the firms producing the differentiated varieties, we isolate this effect by working with a setting in which workers' wage is equalized between countries, which is guaranteed by assuming that trading the homogeneous good is costless. The firms' unit input requirement being normalized to 1, the homogeneous good is used as the numéraire. As a result, profit maximization implies that the price of the homogeneous good and workers' wages are equal within each country whereas factor price equalization implies that those prices and wages are equalized across countries so that $p_i^Z = w_i = 1$, $i = H, F$. This set-up is standard in trade and economic geography, and is not restrictive for our study of the conditions under which various trade structures emerge in the manufacturing sector.³

The set of manufacturing differentiated varieties is modeled as a continuum of horizontally differentiated varieties indexed by v . Each variety is produced by a single firm under increasing returns and monopolistic competition. To operate, a firm needs f units of capital and hires a

³Introducing positive trade costs for the homogeneous good makes the analysis more involved (Picard and Zeng, 2005). Note also that factor price equalization holds provided that each country has enough labor to support the production of homogeneous goods for any international capital distribution, which we assume from now.

quantity of labor proportional to its output. So, the total mass of varieties produced in the economy is equal to $M = 1/f$. As the global supply of capital is equal to one, a larger value of M means either that more capital is allocated to the manufacturing sector or that firms display a lower degree of increasing returns. Because demands will be shown to be linear, we may normalize the marginal requirement of labor to zero. Shipping one unit of the manufactured good between the two countries requires $t > 0$ units of the numéraire.

We now turn to the consumer's preferences and demand for the two types of goods. A key-feature of the present model is that consumers may be unwilling to purchase all varieties. So, we need to distinguish the varieties that are available to consumers from those that are actually purchased by them. Recall that M is the *total* mass of varieties that are at consumer's disposal in the economy. For any given price profile of varieties $p_i(\cdot)$ faced by consumers in country i , any consumer in i chooses the quantity profile $q_i(\cdot)$ and Z_i that maximizes her utility

$$\alpha \int_0^M q_i(v)dv - \frac{\beta}{2} \int_0^M [q_i(v)]^2 dv - \frac{\gamma}{2} \left[\int_0^M q_i(v)dv \right]^2 + Z_i \quad (1)$$

subject to the budget constraint:

$$\int_0^M p_i(v)q_i(v)dv + Z_i = w_i + r + Z_0$$

where $w_i = 1$ and r the capital return defined below,⁴ and to the nonnegative consumption constraint:

$$q_i(v) \geq 0.$$

The initial endowment of the homogeneous, Z_0 , is supposed to be large enough for this good to be consumed in equilibrium. Note that, without loss of generality, the coefficients $\alpha > 0$ and $\beta > 0$ may be normalized to one. Indeed, the first coefficient may be factorized, while the unit of the differentiated good may be chosen for the second coefficient to be one. In this case, $\gamma > 0$ expresses both the substitutability between varieties (the higher γ , the closer substitutes the

⁴In this model, the distribution of capital ownership has no impact on the market outcome because of the quasi linear preferences assumed below.

varieties) and the desirability of the differentiated good with respect to the homogeneous good (the higher γ , the lower the desirability of the differentiated good).

Consumers do not necessarily purchase all varieties at their disposal at the prevailing prices. Let $N_i \leq M$ denote the number of varieties that are actually purchased and consumed in country i . As a consequence, N_i is equal to M when all varieties are consumed and is lower than M when some varieties are not consumed. As will be shown below, trade costs will inflate the price of imported varieties and could make them too expensive to be consumed in equilibrium. To determine N_i , we rank the M varieties by increasing price order: $v < v' \Leftrightarrow p_i(v) \leq p_i(v')$. The following proposition is proven in Appendix 1.

Proposition 1 *There exist a cut-off variety $\tilde{v} = N_i \leq M$ and a corresponding cut-off price $p_i(N_i)$ such that consumers purchase all the varieties that have a price below $p_i(N_i)$. The demand for each variety v is given by*

$$q_i(v) = \begin{cases} \frac{1}{1+\gamma\tilde{v}_i} - p_i(v) + \frac{\gamma}{1+\gamma N_i} \int_0^{N_i} p_i(\xi) d\xi & \text{if } v < N_i \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

while the cut-off variety solves the fixed point equation

$$p_i(N_i) (1 + \gamma N_i) = 1 + \gamma \int_0^{N_i} p_i(v) dv. \quad (3)$$

The cut-off price is smaller than one.

Quite naturally, consumers purchase only the varieties that are not too expensive. It is important to note that whereas, too expensive varieties remain at consumers' disposal, they do not affect the demand for the less expensive varieties that are actually purchased. Indeed, the cut-off variety and cut-off price are defined as the fixed point of (3), which depends only on the prices of the varieties that are actually purchased (i.e. $\xi \in [0, \tilde{v}]$). By the same token, the demand function of each purchased variety depends only upon the prices of varieties that are purchased by the consumer. Intuitively, the consumer is enticed to consume a wider range

of varieties if the cheapest varieties become less expensive, but still consumes the same range of varieties if the most expensive ones become slightly cheaper.

A variety v is consumed in country i if and only if $q_i(v) > 0$, or equivalently if,

$$p_i(v) (1 + \gamma N_i) < 1 + \gamma \int_0^{N_i} p_i(\xi) d\xi. \quad (4)$$

The consumer's demand are

$$q_i(v) = \frac{1}{1 + \gamma N_i} - p_i(v) + \frac{\gamma}{1 + \gamma N_i} P_i \quad (5)$$

where P_i is the price index

$$P_i \equiv \int_0^{N_i} p_i(\xi) d\xi. \quad (6)$$

The surplus S_i that a consumer in country i obtains from purchasing the manufactured good can be computed as

$$S_i = \frac{N_i}{2(1 + \gamma N_i)} - \frac{1}{1 + \gamma N_i} \int_0^{N_i} p_i(v) dv + \frac{1}{2} \int_0^{N_i} [p_i(v)]^2 dv - \frac{\gamma}{2(1 + \gamma N_i)} \left[\int_0^{N_i} p_i(v) dv \right]^2. \quad (7)$$

which depends on the price $p_i(\cdot)$ and number N_i of varieties that she actually purchases.

3 Short-run trade patterns

In this section, we determine the short-run equilibrium where the international allocation of capital, hence of firms, is fixed. More precisely, we show how firms set their prices and discuss how a given international allocation of capital affects the patterns of trade.

3.1 Firms' prices and export decisions

For convenience, we focus on the product market in country i . We first consider the price decision of a domestic firm in market i and then discuss the price and export decision of a foreign firm that exports from country j to country i . Let $p_{ii}(v)$ denote the consumer price of a variety v produced in country i and $p_{ji}(v)$ the consumer price of another variety v exported

from country j to country i . The demand for a variety in country i is given by (5) and the corresponding price index by (6). Because each firm is negligible, it chooses its prices and its export status taking the price indices as parameters.

A domestic firm producing variety v in country i chooses its price $p_{ii}(v)$ to maximize its domestic operating profits given by $\pi_{ii}(v) = \theta_i q_{ii}(v) p_{ii}(v)$, treating the price index P_i parametrically. When the demand for variety v is positive, it follows from (5) that all firms in country i set the same profit-maximizing price given by

$$p_{ii}(v) = p_{ii} = \frac{1}{2} \frac{1 + \gamma P_i}{1 + \gamma N_i}.$$

Because marginal costs are zero, domestic producers make positive profits on domestic sales and always supply their domestic market.

A foreign firm exporting another variety v to country i chooses its $p_{ji}(v)$ to maximize its operating profit earned abroad, $\pi_{ji}(v) = \theta_i q_{ji}(v) (p_{ji}(v) - t)$, under the constraint that it has a positive demand: $q_{ji}(v) > 0$. The firm is able to export ($q_{ji}(v) > 0$) if and only if it can set a low enough price to satisfy condition (4). Because foreign firms are symmetric, the unconstrained price and export levels are the same across all foreign firms and given by

$$p_{ji}(v) = p_{ji} = p_{ii} + \frac{t}{2}$$

and

$$q_{ji}(v) = \frac{1}{2} \frac{1 - t + \gamma P_i - t \gamma N_i}{1 + \gamma N_i}.$$

By (4), variety v is exported to country i ($q_{ji}(v) > 0$) if and only if

$$t < \frac{1 + \gamma P_i}{1 + \gamma N_i}. \quad (8)$$

When this condition does not hold, variety v is not exported ($q_{ji}(v) = 0$).

In the next section, we aggregate firms' decisions and determine the product market equilibrium.

3.2 Product market equilibrium

Let λ_i be the given capital share in country i . Because each firm produces a single variety, λ_i is also the share of varieties produced in country i . Let also $\mu_j \in [0, \lambda_j]$ be the share of varieties imported from country j . Using those definitions, the number of varieties purchased in country i is given by the sum of domestic and imported varieties: $N_i = (\lambda_i + \mu_j) M$. The corresponding price index is

$$P_i = (\lambda_i p_{ii} + \mu_j p_{ji}) M.$$

Since $p_{ji} = p_{ii} + t/2$, the price index is a function of the domestic price p_{ii} . The market equilibrium price is determined by the fixed point condition $p_{ii}(P_i(x)) = x$, which implies that the equilibrium domestic price and price index are as follows:

$$p_{ii}^* = \frac{1}{2} \frac{2 + t\mu_j\gamma M}{2 + \gamma N_i} \quad \text{and} \quad P_i^* = \frac{N_i + t\mu_j M (1 + \gamma N_i)}{2 + \gamma N_i} \quad (9)$$

while the export price is again

$$p_{ji}^* = p_{ii}^* + \frac{t}{2}.$$

Therefore, several export regimes may arise according to whether varieties are exported at the equilibrium ($\mu_j = 0$, $\mu_j = \lambda_j$ or $\mu_j \in (0, \lambda_j)$). The export regimes are determined by the value of trade costs. Let

$$\bar{t}(\lambda_i) \equiv \frac{2}{2 + \gamma\lambda_i M} \quad (10)$$

which falls with larger λ_i . We show the following lemma in Appendix 2.

Lemma. *All foreign firms, some (indeterminate number of) foreign firms or no foreign firms export in country i if $t \stackrel{\leq}{\geq} \bar{t}(\lambda_i)$. Furthermore, the equilibrium prices are continuous and weakly decreasing with respect to the level of trade costs.*

In other words, domestic consumers buy all foreign varieties if trade costs are low and/or the share of domestic competitors λ_i is sufficiently small. Conversely, foreign firms are unable to export when trade costs are high and/or the share of domestic firms is large enough. Furthermore it can be shown that, whether or not varieties are exported, the equilibrium domestic prices

converge toward marginal cost (here zero) when the manufactured good becomes homogeneous ($\gamma \rightarrow \infty$). Likewise, a smaller requirement f of capital leads to a larger number of available varieties M and therefore to lower prices. Finally, the market price prevailing in a country decreases with smaller levels of trade costs as long as some foreign firms export ($\mu_j > 0$). Stated differently, for any given allocation of capital, trade opening makes markets more competitive.

Using the foregoing Lemma, the equilibrium trade pattern may be described as follows.

Proposition 2 *Assume that the allocation of capital is fixed. Then, there is two-way trade if $t < \min\{\bar{t}(\lambda_i), \bar{t}(\lambda_j)\}$; there is one-way-trade from country i to country j if $\bar{t}(\lambda_j) \leq t < \bar{t}(\lambda_i)$; finally, there is no trade if $t \geq \max\{\bar{t}(\lambda_i), \bar{t}(\lambda_j)\}$.*

Figure 1 shows how the trade pattern changes with the level of trade costs and the international allocation of capital. It reveals several interesting and new features. First, two-way trade prevails when trade costs are very low ($t < \bar{t}(1)$). In this case, all firms export from every country to the other one regardless of the distribution of firms: geographical separation does not suffice to protect domestic firms from foreign competition. Second, there exist no trade within the manufacturing sector under very high trade costs ($t > \bar{t}(0) = 1$). In this case, no firm export and domestic firms are always protected against competition from abroad.

Insert Figure 1 about here

Third, one-way trade may arise when $\bar{t}(1) < t < \bar{t}(0)$. We distinguish between the following two configurations. On the one hand, when $\bar{t}(1) < t < \bar{t}(1/2)$, one-way trade takes place from country j to country i when few firms are located in country i (small λ_i). This is because competition is sufficiently weak in this country to allow country j -firms to export. In contrast, competition in country j is too tough to permit the entry of foreign varieties. As firms move from j to i (λ_i increases), the economy involves two-way trade. This is because competition is relaxed in country i , while trade costs are not high enough to prevent trade from i to j . By contrast, when most firms are set up in country i (high λ_i), the corresponding market is too

competitive to permit imports from j . Competition in country j is then sufficiently soft for country i -firms to export.

On the other hand, when $\bar{t}(1/2) < t < \bar{t}(0)$, one-way trade also occurs for small and large values of λ_i . However, instead of two-way trade, autarky now prevails when countries become less dissimilar in terms of capital allocation (λ_i close to $1/2$). In this case, competition becomes tougher in country j but remains intense enough in country i , implying that trade costs are sufficiently high to prevent foreign firms to export. This analysis shows that *the emergence of one-way trade may be explained by a sufficiently uneven distribution of firms that results in strong differences in market competitiveness*. Note also that country size asymmetry is necessary for one-way trade to occur. When the countries are symmetric, the economy shifts directly from autarky to two-way trade at $t = \bar{t}(1/2)$.

Before proceeding, observe that the economy moves gradually from autarky to two-way trade as γ steadily decreases. Product differentiation is, therefore, one of the main drivers of international trade, as shown by Krugman (1979) in the two-way trade case.

The above trade patterns are obtained when the allocation of capital is given. We now analyze the long-run equilibrium where capital moves to the country offering the higher return.

4 Long-run capital allocation

In the long-run equilibrium, the international allocation of capital λ_i is endogenous. By allowing capital to move from one country to the other, we allow the competitiveness of each national market to change through the number of domestic firms, which in turn affects firms' incentives to export. Hence, although firms are homogeneous, *the conditions for firms to export change with the international allocation of capital*. This in turn implies that the nature of trade may change with the relocation of firms. We first define the long run equilibrium, then study this equilibrium for each trade pattern and finally synthesize the results.

Let r_i be the return of a unit of capital invested in country i . For any allocation of capital

λ_i , each firm earns the profit $\Pi_i - fr_i$ where Π_i is the operating profit defined by

$$\Pi_i = \begin{cases} \pi_{ii} + \pi_{ij} & \text{if country-}i \text{ firms exports to country } j \\ \pi_{ii} & \text{if country-}i \text{ firms do not export.} \end{cases}$$

where $\pi_{ii} = \theta_i (p_{ii}^*)^2$ and $\pi_{ij} = \theta_j (p_{ij}^* - t)^2$. In a competitive capital market, the capital return r_i is determined by the firms' bidding process for capital, which comes to an end when no firm can earn a strictly positive profit. That is, firms' operating profits are entirely absorbed by the capital return so that $r_i = \Pi_i/f$.

In the long run or spatial equilibrium, no capital-owner can get a higher return by relocating her capital unit to another country. Formally, at an *interior* spatial equilibrium ($0 < \lambda_i^* < 1$), we have $r_H = r_F = r$, where r is such that all profits are equal to zero; we have a *corner* equilibrium at $\lambda_i^* = 1$ when r_i exceeds r_j for all values of λ_i . For conciseness, we will often use the notation:

$$\theta \equiv \theta_H/\theta_F \in [1, \infty)$$

with $\theta = 1$ if and only if the two countries have the same size. We also set

$$n \equiv \gamma M$$

which we call the *degree of competitiveness* in the economy. The larger the mass of firms and/or the lower the degree of product differentiation across varieties, the more competitive the economy.

Three cases may arise according to the nature of the trade pattern.

4.1 Two-way trade

Suppose that the manufactured good is imported from both countries. Proposition 2 then implies that the condition

$$t < \min \{\bar{t}(\lambda_H), \bar{t}(\lambda_F)\}. \quad (11)$$

holds. Since capital-owners seek the highest rate of return, the relocation incentives are given by the profit differential $(\pi_{HH} + \pi_{HF}) - (\pi_{FF} + \pi_{FH})$ in which the equilibrium prices are given

by (9) with $\mu_i = \lambda_i$, and thus $N_i = M$, for $i = H, F$. This expression has a unique zero given by

$$\lambda_H^* = \frac{1}{2} + (\theta_H - \theta_F) \frac{2-t}{nt} \geq \frac{1}{2}. \quad (12)$$

Two types of equilibria may arise.

Partial agglomeration. There is partial agglomeration in country H ($1/2 < \lambda_H^* < 1$) with two-way trade if $\lambda_H^* < 1$ and if trade costs are sufficiently low to satisfy (11). After some easy algebra, those conditions are equivalent to

$$\theta < \rho_0(t) \equiv \frac{2(2-t) + nt}{2(2-t) - nt} \quad (13)$$

and

$$\theta < \rho_1(t) \equiv \frac{2(4-3t) - tn}{t(2+n)} \quad (14)$$

where $\rho_0(t)$ increases and $\rho_1(t)$ decreases with t . The domain of trade costs and countries' asymmetry that support partial agglomeration is depicted in Figure 2.

Insert Figure 2 about here

A simple observation of the curves $\rho_0(t)$ and $\rho_1(t)$ in Figure 2, i.e. (13) and (14), leads to the following conclusions. First, two-way trade is more likely when countries are more similar (lower θ). Second, two-way trade is more likely when trade costs take intermediate values. Indeed, lower trade costs not only facilitate trade but also entice firms to move away from the smaller country so that no firms would remain to export from this country. Third, country F exports less on both the extensive and intensive margins. However, the remaining F -firms keeps exporting to country H , which means that the market integration effect dominates the competition effect generated by the larger mass of firms located in country H . Last, two-way trade becomes more likely for smaller trade costs and less likely for larger ones when the degree of competitiveness increases. Indeed, when n rises, the curve $\rho_0(t)$ is shifted upward, whereas the curve $\rho_1(t)$ is shifted downward. In sum, when the economy becomes more competitive, a

few firms seek protection in the smaller country but stop to export because the larger market remains too competitive.

Finally, under partial agglomeration, there exists a home-market effect because

$$HME \equiv \frac{\lambda_H^* - \frac{1}{2}}{\theta_H - \frac{1}{2}} = 2 \frac{2-t}{nt} > 1 \iff t < \frac{4}{2+n} = \bar{t}(1/2)$$

which holds under (14). However, the home-market effect is weaker when the economy is more competitive: firms relax competition by being more dispersed.

Full agglomeration. When $\theta \geq \rho_0(t)$, there is full agglomeration in equilibrium ($\lambda_H^* = 1$ and $\lambda_F^* = 0$) and, therefore, one-way trade from the larger to the smaller country. For trade to arise, (11) implies that

$$t < \hat{t} \equiv \frac{2}{2+n}$$

must hold. Hence, this regime emerges when the two countries are dissimilar enough and trade costs are sufficiently low to sustain trade from the larger to the smaller country (see Figure 2).

To sum up, we have shown the following result.

Proposition 3 *If $\theta < \rho_0(t)$ and $\theta < \rho_1(t)$, then there exists a unique spatial equilibrium involving partial agglomeration in the larger country with two-way trade. Furthermore, when $\theta \geq \rho_0(t)$ and $t < \hat{t}$, there is full agglomeration and one-way trade from H to F .*

4.2 No trade

Suppose now that no firms export. Proposition 2 then implies that

$$t \geq \max \{ \bar{t}(\lambda_H), \bar{t}(\lambda_F) \}. \quad (15)$$

Relocation incentives are given by $\pi_{HH} - \pi_{FF}$ in which the equilibrium prices are now given by (9) where $\mu_i = 0$, and thus $N_i = \lambda_i M$, for $i = H, F$. The unique zero of this expression is equal to

$$\lambda_H^* = \frac{1}{2} + \frac{1}{2} \frac{\sqrt{\theta_H} - \sqrt{\theta_F}}{\sqrt{\theta_H} + \sqrt{\theta_F}} \frac{4+n}{n} \geq \frac{1}{2}.$$

Therefore, the international allocation of capital remains biased toward the larger country H . This bias falls when the economy becomes more competitive (larger n). Since λ_H^* exceeds λ_F^* , market H is more competitive than market F . Therefore, it is always less profitable for firms located in country F to export than for those located in country H . This implies that (15) boils down to

$$t \geq \bar{t}(\lambda_F^*) = \frac{2}{2 + n\lambda_F^*}. \quad (16)$$

This condition becomes less stringent as the mass of firms in country F increases, thus deterring exports to F because competition therein is tougher.

Partial agglomeration. There is partial agglomeration in country H if $\lambda_H^* < 1$, or equivalently, if

$$\theta < \rho_2 \equiv \left(1 + \frac{n}{2}\right)^2$$

which is independent of t since there is no trade. Furthermore, by plugging $\lambda_F^* = 1 - \lambda_H^*$ into (16), this condition becomes

$$\theta \leq \rho_3(t) \equiv \frac{1}{4}[nt - 2(1 - 2t)]^2.$$

Since $\theta \geq 1$, it must be that $t \geq 4/(n + 4)$ for this inequality to hold. Thus, as expected, *trade costs must be sufficiently large for the no-trade case to arise*. Note also that $\rho_3(t) < \rho_2$ when $t < 1$.

In this case, there is a home-market effect if and only if

$$HME \equiv \frac{\lambda_H^* - \frac{1}{2}}{\theta_H - \frac{1}{2}} > 1 \iff \left(\sqrt{\theta_H} + \sqrt{\theta_F}\right)^2 < 1 + \frac{4}{n}.$$

Clearly, the LHS of the second inequality decreases from 2 to 1 as θ_H increases from 1/2 to 1, whereas the RHS takes values above 2 if and only if $n < 4$. Hence, if $n < 4$, there is a home-market effect regardless of countries' asymmetry. This occurs when competition is sufficiently soft, either because there are few firms in the economy or because varieties are good substitutes. By contrast, if $n > 4$, the above inequality holds only for large enough θ_H . In this case, there

is a home-market effect if and only if

$$\theta_H > \frac{1}{2} + \frac{1}{2} \sqrt{1 - \left(\frac{4}{n}\right)^2}. \quad (17)$$

Hence, country H must be big enough to overcome the competition effect triggered by a high degree of competitiveness. In contrast, when the two countries are fairly similar, there is a *reverse* home-market effect. Indeed, the domestic country is here the only firms' market. *When the degree of competitiveness n is high and countries have similar sizes, the larger country is not a sufficiently big outlet to compensate firms for the higher intensity of competition that would prevail with a proportionate share of firms.* In the same vein, (17) shows that the occurrence of a reverse home-market effect increases when the degree of competitiveness n rises. These properties, which standard CES models are silent about, stem from the pro-competitive effects that our model encompasses.

Full agglomeration. If $\theta \geq \rho_2$, there is full agglomeration in the bigger country ($\lambda_H^* = 1$). In other words, all firms set up in H when this country is sufficiently large. However, firms are not enticed to export when $t \geq 1$, as shown by (16) in which $\lambda_F^* = 0$.

To sum up, we have:

Proposition 4 *If $\theta < \rho_2$ and $\theta \leq \rho_3(t)$, then there exists a unique spatial equilibrium with partial agglomeration and no trade. Furthermore, there is full agglomeration in the larger country and no trade when $\theta \geq \rho_2$ and $t \geq 1$.*

4.3 One-way trade

Suppose, last, that firms export only from country i to country j . Proposition 2 then implies

$$\bar{t}(\lambda_i) \leq t < \bar{t}(\lambda_j). \quad (18)$$

For this condition to hold, it must be that $\lambda_i > \lambda_j$, which means that *the exporting country hosts more firms than the other.*

Under one-way trade from country i to country j , the relocation incentives are given by the operating profit differential $\Delta\pi_i = (\pi_{ii} + \pi_{ij}) - \pi_{jj}$ in which the equilibrium prices are given by (9) where country i -firms export ($\mu_i = \lambda_i$), whereas country j -firms do not ($\mu_j = 0$). Therefore, the numbers of varieties at consumers' disposal in country j and i are, respectively, $N_j = M$ and $N_i = \lambda_i M$. This yields

$$\Delta\pi_i \propto \left[\theta_i \left(\frac{1}{2 + n\lambda_i} \right)^2 - \theta_j \frac{t}{2} \left(\frac{2 + \lambda_i n t}{2 + n} - \frac{t}{2} \right) \right] \quad (19)$$

which is monotonically decreasing in λ_i ($\partial\Delta\pi_i/\partial\lambda_i < 0$). Thus, when only country- i firms export to country j , (19) has a unique zero.

In the presence of trade costs, firms always have an advantage to operate from the larger market because this one offers a better access to the larger pool of local consumers. This advantage attracts more firms but fosters competition in this market, thus reducing the foreign firms' incentives to export into this market. In Appendix 3, we prove the following: *the long-run equilibrium involving one-way trade implies trade only from the larger to the smaller country*. Given this result, we can now focus the conditions for one-way trade from H to F to arise under partial and full agglomeration.

Partial agglomeration. Under one-way trade from H to F , the unique solution to the equation $\Delta\pi_H = 0$ involves partial agglomeration if and only if operating profits in country H are lower than those made in F when all firms are located in H : $\pi_{HH} + \pi_{HF} < \pi_{FF}$ at $\lambda_H = 1$. Plugging $\lambda_H = 1$ in (19) shows that this condition is equivalent to

$$\theta < \rho_4(t) \equiv \frac{(2+n)t}{4} [2(2-t) + tn]. \quad (20)$$

Furthermore, the above solution must fulfill the conditions (18) for $i = H$ and $j = F$. It is readily verified that these two conditions are equivalent to

$$\lambda_H^* \geq 2(1-t) \frac{1}{nt} \quad \text{and} \quad \lambda_H^* > 1 - 2(1-t) \frac{1}{nt}. \quad (21)$$

The former inequality holds if and only if $\theta \geq \rho_1(t)$, while the latter holds if and only if $\theta > \rho_3(t)$. This can be shown by plugging each bound of (18) in the equilibrium condition $\Delta\pi_H = 0$.

As shown in Figure 2, the level of trade costs does not have a monotone impact on the range of countries' size asymmetries sustaining one-way trade. Consider a situation in which θ steadily decreases to $\rho_1(t)$, the economy shifts from one-way trade to two-way trade. This is because the larger country is no longer competitive enough to deter importing. In contrast, when θ steadily decreases to $\rho_3(t)$, the economy shifts to no-trade because the smaller country is more competitive and, therefore, more difficult to penetrate. Last, it can be shown that both the loci $\rho_3(t)$ and $\rho_4(t)$ are shifted upward when the degree of competitiveness n increases, while the gap between $\rho_4(t)$ and $\rho_3(t)$ widens with n .

The expression (19) may be used to uncover the impact of countries' asymmetry and trade costs. Indeed, it is readily verified that

$$\frac{\partial \Delta \pi_H}{\partial \theta} > 0 \quad \frac{\partial \Delta \pi_H}{\partial t} < 0.$$

Thus, we have:

$$\frac{d\lambda_H^*}{d\theta} = -\frac{\partial \Delta \pi_H}{\partial \theta} / \frac{\partial \Delta \pi_H}{\partial \lambda_H} > 0 \quad \frac{d\lambda_H^*}{dt} = -\frac{\partial \Delta \pi_H}{\partial t} / \frac{\partial \Delta \pi_H}{\partial \lambda_H} < 0.$$

In words, the larger country hosts a growing share of firms when its relative size rises, whereas lowering trade costs leads the manufacturing firms to get more agglomerated in the larger country. Consequently, firms that quit the less competitive and smaller country find it profitable to export their output to their country of origin. To put it differently, *even though firms are homogeneous, their locational choices determine their attitudes toward exports*. This is to be contrasted with the recent literature that explains export structure with firms' technological heterogeneity (Melitz, 2003; Melitz and Ottaviano, 2008).

Determining the domain of parameters for which the home-market effect holds in a one-way economy is a very hard task because we do not have the explicit expression for λ_H^* . Nevertheless, we can make the following inferences. First, we know that there is always a home-market effect under two-way trade. By continuity, this still holds under one-way trade when trade costs are not too high (i.e. above but close to $\rho_2(t)$ in Figure 2). Second, we know from Section 4.2 that the no-trade regime supports a reverse home-market effect when countries are similar enough,

the economy sufficiently competitive, or both. Therefore, by continuity, the same effect holds when trade costs just sustain one-way trade (i.e. above but close to $\rho_3(t)$ in Figure 2). Thus, when trade liberalization deepens, the economy continuously shifts from a reverse home-market effect to a home-market effect within the one-way trade economy since price and profits are continuous with respect to t . However, when the degree of competitiveness is low, there is always a home-market effect in the no-way trade regime. We may thus expect this effect to hold for all values of trade costs.

Figure 3 illustrates how the value of HME varies with t for $\theta = 1.3$. When the economy is not very competitive ($n = 2$), the value of HME exceeds 1 for all t , meaning that there is always a home-market effect. In contrast, when the economy is very competitive ($n = 8$), a reverse home-market effect occurs for t larger than 0.33, i.e. under autarky and a range of t -values sustaining one-way trade.

Insert Figure 3 about here

Full agglomeration. All firms agglomerate in country H ($\lambda_H^* = 1$) if $\theta \geq \rho_4(t)$. In this case, the two conditions (18) boil down to $\hat{t} \leq t < 1$.

We may summarize our results as follows.

Proposition 5 *If $\theta \geq \rho_1(t)$, $\theta > \rho_3(t)$ and $\theta < \rho_4(t)$, then there exists a unique spatial equilibrium with partial agglomeration and one-way trade from country H to country F . Furthermore, full agglomeration with one-way trade prevails when $\theta \geq \rho_4(t)$ and $\hat{t} \leq t < 1$.*

4.4 Synthesis

The above analysis describes the equilibrium conditions for the existence of a unique spatial equilibrium in the case of two-way trade, one-way trade and no trade. As illustrated by Figure 2, these conditions define several domains that form a partition of the positive quadrant in the plane (t, θ) . We may collate those conditions in the following proposition, which shows how

globalization affects the nature of trade and economic geography, where we have set

$$\hat{\theta} \equiv 1 + \frac{2n}{2+n}$$

which solves $\rho_0(\hat{t}) = \rho_1(\hat{t}) = \rho_4(\hat{t}) = \hat{\theta}$ (see Figure 2).

Proposition 6 *As trade costs steadily decrease, the economy goes through the following sequence of stages:*

(i) *For small country size asymmetries ($\theta < \hat{\theta}$), the economy involves partial agglomeration of capital and firms in the larger country with sequentially no trade, one-way trade and two-way trade; finally, it involves full agglomeration with one-way trade.*

(ii) *For intermediate values of country size asymmetries ($\hat{\theta} < \theta < \rho_2$), the economy involves partial agglomeration in the larger country with first no trade and then one-way trade; finally, it involves full agglomeration with one-way trade.*

(iii) *For large country size asymmetries ($\rho_2 < \theta$), the economy involves full agglomeration in the larger country with first no trade and then one-way trade.*

5 Efficient allocation of capital

In this section, we determine the socially optimal allocation of capital and investigate whether and how it differs from the long-run equilibrium. Specifically, we suppose that a fully informed and benevolent planner decides over the production level and over the allocation of products and capital. The planner maximizes the global welfare $W = \theta_H S_H + \theta_F S_F + \Pi$, which includes the consumer surpluses in both countries and the world profit, which here amounts to the return to capital ($rf = \Pi$). In the efficient allocation, the planner sets the product prices equal to their marginal costs, which are equal to zero for the domestic varieties and to t for the foreign ones. As a result, firms make zero profits and the socially optimal return to capital is nil. In what follows, we consider the cases of two-way trade, no trade and one-way trade.

Under two-way trade, all varieties are consumed everywhere ($N_i = M$) so that the consumer surplus (7) in country i is given by

$$S_i = \frac{M}{2(1 + \gamma M)} - \frac{1}{1 + \gamma M} \lambda_j M t + \frac{1}{2} \lambda_j M t^2 - \frac{\gamma}{2(1 + \gamma M)} (\lambda_j M t)^2 \quad (22)$$

which is a strictly concave function of λ_j . Maximizing W with respect to λ_H gives the optimal allocation of capital:

$$\lambda_H^o = \frac{1}{2} + \frac{1}{2} (\theta_H - \theta_F) \frac{2 - t}{nt}$$

where the superscript o describe efficient outcome. It is easy to check that λ_H^o is smaller than the equilibrium allocation λ_H^* under two-way trade for any $\theta_H \geq 1/2$. As a consequence, there exists excessive concentration of capital in the larger country. This result has already been noticed by Ottaviano and van Ypersele (2005) under two-way trade. Given the prevalence of zeros in the matrix of trade flows between countries, it is worth asking what this result becomes under other trade regimes (Baldwin and Harrigan, 2007; Helpman *et al.*, 2008).

Under no trade, only the domestic varieties are consumed so that the consumer surplus (7) in country i simplifies to

$$S_i = \frac{M \lambda_i}{2(1 + \gamma M \lambda_i)} \quad (23)$$

which is again a strictly concave function of λ_i . Maximizing W with respect to λ_H yields

$$\lambda_H^o = \frac{1}{2} + \frac{1}{2} \frac{\sqrt{\theta_H} - \sqrt{\theta_F}}{\sqrt{\theta_H} + \sqrt{\theta_F}} \frac{2 + n}{n}$$

which is also smaller than the equilibrium allocation under no trade.

Under one-way trade, individuals consume all varieties in the smaller country ($N_F = M$) and get the surplus (22), whereas they consume only the domestic varieties in the larger country and get the surplus (23). The first order condition is given by

$$\theta_H \left(\frac{1}{2 + 2n \lambda_H^o} \right)^2 - \theta_F \frac{t}{2} \left(\frac{2 + 2nt \lambda_H^o}{2 + 2n} - \frac{t}{2} \right) = 0.$$

The solution λ_H^o to this equation is the same as the solution to the equilibrium condition (19) when n is replaced by $2n$. Therefore, we have $\lambda_H^o(n) = \lambda_H^*(2n)$. If the equilibrium solution

$\lambda_H^*(2n)$ falls with admissible n , we get $\lambda_H^o(n) = \lambda_H^*(2n) < \lambda_H^*(n)$, so that the efficient allocation of capital is less dispersed than the equilibrium one. Although we are unable to offer a formal proof of $\lambda_H^*(2n) < 0$ under one-way trade, numerical computations show that this property holds for all admissible t and θ .

Proposition 7 *Any interior equilibrium implies excessive agglomeration of capital and firms in the larger country under two-way trade, one-way trade and no trade.*

Furthermore, if country size asymmetries increase, firms may agglomerate in equilibrium although the planner keeps them dispersed. In this case, there also is too much agglomeration from the welfare viewpoint. If country size asymmetries increase further, firms agglomerate both in the equilibrium and in the social optimum. The excessive agglomeration result vanishes only in this extreme case.

Thus, once it is recognized that product markets are not competitive, production involves increasing returns and shipping goods is costly, *in all trade regimes liberalizing capital mobility does not yield the socially optimal allocation of capital.* This runs against the standard prediction that the social optimum is always reached at the equilibrium allocation of capital. Such a prediction is commonly made in perfect competition models with constant returns and imperfect competition models with increasing returns that are based Dixit-Stiglitz iso-elastic preferences (see Baldwin *et al.* 2003, p. 257). While taxing capital is undesirable in these settings, our approach suggests that taxing capital in the larger country or subsidizing capital in the smaller one may be welfare-enhancing.

6 Concluding remarks

The main purpose of this paper was to show within a unified framework that trade liberalization may span a large range of cases, which reflects the diversity displayed by commodity flows in world trade. In particular, we have seen that the nature and intensity of trade does not depend

only upon the degree of openness of the economy, measured by the level of trade costs. How firms are distributed across countries is another critical determinant, the reason being that their locations affect the intensity of competition within each country. For example, foreign firms will find it more difficult to export to a country that accommodates a larger number of domestic firms because local prices are lower. Likewise, if the manufacturing sector supplies a poorly differentiated product, there will be little trade because each market is governed by unleashed competition. Last, when more capital is allocated to the manufacturing sector, the intensity of trade is lowered because a larger mass of firms compete in the economy. In sum, *all parameters and policies that influence the degree of competition within national economies are likely to have a strong impact on trade.*

That said, our analysis sheds light on the different roles played, on the one hand, by international market integration across countries and, on the other hand, by competition policy within each trading partner. As market integration deepens, the economy moves from autarky to two-way trade through one-way trade. By contrast, when national economies become more and more competitive, trade vanishes and countries operate under autarky. This is so when firms sell a good which is almost homogeneous, produce almost under constant returns, or both. This result is consistent with a standard result in trade theory according to which perfectly competitive agents exchange commodities as long as there are differences in relative endowments, production techniques, or preferences across locations. Otherwise, no trade will occur (Samuelson, 1939, 1962). In the same vein, lowering trade barriers fosters the concentration of capital within the bigger country. On the contrary, making the economy more competitive pushes toward a more dispersed distribution of capital because this allows firms to relax competition. Therefore, we may safely conclude that competition policy and trade policy are not good substitutes.

Last, we have seen how globalization affects the way capital is distributed across countries. Specifically, our results highlight the advantage of being big in the race for firms. For instance, for small country size asymmetries, the international distribution of capital remains the same

as long as trade costs are sufficiently high for imports to be prohibitively expensive. Once trade costs are low enough to permit the larger country to export to the smaller one, further decreases in trade costs lead to a growing concentration of capital in the larger country. The same holds when the economy operates under two-way trade. Hence, even though the trading partners have similar sizes, *market integration goes hand in hand with a more unbalanced spatial distribution of the manufacturing sector.*

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Appendix 1

Let $\lambda(v)$ be the Kuhn-Tucker multiplier of the nonnegative consumption constraint. The consumer's program may then be re-written as follows:

$$\begin{aligned} \max_{q(v)} \mathcal{L} &= \int_0^M q(v)dv - \frac{1}{2} \int_0^M [q(v)]^2 dv - \frac{\gamma}{2} \left[\int_0^M q(v)dv \right]^2 \\ &\quad - \int_0^M p(v)q(v)dv + \int_0^M \lambda(v)q(v)dv + Z_0 \end{aligned}$$

The maximization problem has a concave objective function and convex constraints. Thus, the Kuhn-Tucker conditions are necessary and sufficient condition for a maximum:

$$\begin{aligned} \frac{d\mathcal{L}}{dq(v)} &\leq 0 & q(v) &\geq 0 & \text{and} & q(v) \frac{d\mathcal{L}}{dq(v)} &= 0 \\ \frac{d\mathcal{L}}{d\lambda(v)} &\geq 0 & \lambda(v) &\geq 0 & \text{and} & \lambda(v) \frac{d\mathcal{L}}{d\lambda(v)} &= 0 \end{aligned}$$

where

$$\begin{aligned} \frac{d\mathcal{L}}{dq(v)} &= 1 - q(v) - \gamma \int_0^M q(\xi)d\xi - p(v) + \lambda(v) \\ \frac{d\mathcal{L}}{d\lambda(v)} &= q(v). \end{aligned}$$

Since both $q(v)$ and $\lambda(v)$ cannot be positive, three cases may arise. First, suppose that $q(v) > 0$. Then $d\mathcal{L}/dq(v) = 0$, which implies

$$q(v) = 1 - p(v) - \gamma \int_0^M q(\xi)d\xi.$$

This expression is strictly positive if and only if

$$p(v) < \bar{p} \equiv 1 - \gamma \int_0^M q(\xi)d\xi.$$

Second, suppose that $\lambda(v) > 0$. In this case, $q(v) = 0$ and $d\mathcal{L}/dq(v) \leq 0$, implying that

$$p(v) - \bar{p} \geq \lambda(v) > 0.$$

Last, suppose that $q(v) = \lambda(v) = 0$. Then, we have

$$d\mathcal{L}/dq(v) \leq 0 \iff p(v) - \bar{p} \geq 0.$$

Accordingly, the solution to the consumer program is given by

$$q(v) = \begin{cases} 1 - p(v) - \gamma \int_0^M q(\xi) d\xi & \text{if } p(v) < \bar{p} \\ 0 & \text{otherwise.} \end{cases}$$

We are now equipped to construct firms' demands. Since varieties are ranked by increasing price order, \tilde{v} is the first variety that is not purchased:

$$v \geq \tilde{v} \Leftrightarrow p(v) \geq p(\tilde{v}) \iff q(v) = 0$$

which implies $\int_0^M q(\xi) d\xi = \int_0^{\tilde{v}} q(\xi) d\xi$. The consumer's demands may therefore be re-written as follows:

$$q(v) = \begin{cases} 1 - p(v) - \gamma \int_0^{\tilde{v}} q(\xi) d\xi & \text{if } p(v) < \bar{p} \\ 0 & \text{otherwise.} \end{cases}$$

Integrating demands over all varieties belonging to $[0, \tilde{v}]$, we get

$$\int_0^{\tilde{v}} q(v) dv = \tilde{v} - \int_0^{\tilde{v}} p(v) dv - \gamma \tilde{v} \int_0^{\tilde{v}} q(\xi) d\xi$$

which yields

$$\int_0^{\tilde{v}} q(v) dv = \frac{\tilde{v} - \int_0^{\tilde{v}} p(v) dv}{1 + \gamma \tilde{v}}$$

so that

$$\bar{p} = p(\tilde{v}) = \frac{1 + \gamma \int_0^{\tilde{v}} p(\xi) d\xi}{1 + \gamma \tilde{v}}$$

and

$$q(v) = \begin{cases} \frac{1}{1 + \gamma \tilde{v}} - p(v) + \frac{\gamma}{1 + \gamma \tilde{v}} \int_0^{\tilde{v}} p(\xi) d\xi & \text{if } v < \tilde{v} \\ 0 & \text{otherwise.} \end{cases}$$

Since $p(v) < p(\tilde{v})$, the consumer buys the domestic varieties, which implies $p(\tilde{v}) < 1$. Q.E.D.

Appendix 2

Suppose, first, that all foreign firms export to market i ($\mu_j = \lambda_j$). Then, the number of varieties consumed in country i is $N_i = M$ so that the equilibrium price set by country- i firms is equal

to

$$p_{ii}^* = \frac{1}{2} \frac{2 + t\lambda_j n}{2 + n} \quad (24)$$

where $n \equiv \gamma M$. By (8), this regime occurs if $t < \bar{t}(\lambda_i)$ where

$$\bar{t}(\lambda_i) \equiv \frac{2}{2 + \lambda_i n}$$

is the expression in (10). In other words, *consumers buy all foreign varieties if trade costs are low, the share of domestic competitors is not too large, or both.*

Second, suppose that no foreign firm exports ($\mu_j = 0$). Then, we have $N_i = \lambda_i M$ and

$$p_{ii}^* = \frac{1}{2 + \lambda_i n} \quad (25)$$

where $p_{ii}^* < 1$. By (8), this regime occurs if $t > \bar{t}(\lambda_i)$, which means that *no foreign firms export when trade costs are high, the share of domestic firms is not too small, or both.*

Finally, an indeterminate number of foreign firms export to market i if $t = \bar{t}(\lambda_i)$. Indeed, plugging $\bar{t}(\lambda_i)$ in (24) yields

$$p_{ii}^* = \frac{1}{2 + \lambda_i n}$$

which is independent of μ_j . Therefore, the number of exported varieties μ_j can take any value in the interval $[0, \lambda_j]$.

The transition between the export regimes have the following interesting properties. On the one hand, as the value of trade cost increases, the number of varieties exported to country i suddenly switches from λ_j to 0 at $t = \bar{t}(\lambda_i)$. On the other hand, since λ_i is here exogenous, one can check

$$\lim_{t \rightarrow \bar{t}(\lambda_i)^-} p_{ii}^* = \lim_{t \rightarrow \bar{t}(\lambda_i)^+} p_{ii}^* = \frac{1}{2 + \lambda_i n}$$

which implies that p_{ii}^* is continuous at $\bar{t}(\lambda_i)$. As a result, production and profit levels are also continuous functions.

Appendix 3

We here prove that the market outcome involving one-way trade implies trade from the larger to the smaller country. Suppose indeed that there is one-way trade from the smaller country $i = F$ to the larger country $j = H$. It then follows from the trade condition (18) that $\lambda_F > 1/2$, hence $\lambda_H < 1/2$: competition in the bigger market must be soft enough for the foreign firms to supply this market, while competition in the smaller market is sufficiently fierce to prevent H -firms from exporting to the smaller market. We know that, under each trade regime, the profit differential $\Delta\pi_H$ is continuous and decreasing with respect to λ_H . Because $\Delta\pi_H$ can be written as a continuous function of all quantities, which are themselves continuous functions of λ_H , $\Delta\pi_H$ is continuous at each transition point. Therefore, the locus $\Delta\pi_H$ is continuous and decreasing over the whole interval $[0, 1]$ and has, therefore, a unique zero. As shown by Figure 1, when λ_H increases from 0, the economy moves either from one-way trade to no trade, or from one-way trade to two-way trade. Under two-way trade, it is readily verified that $\Delta\pi_H > 0$ at $\lambda_H = 1/2$, while $\Delta\pi_H > 0$ also holds $\lambda_H = 1/2$ under no trade. Therefore, for all $\lambda_H \leq 1/2$, $\Delta\pi_H$ must be positive under one-way trade. This means that there exists no equilibrium with $\lambda_H < 1/2$.

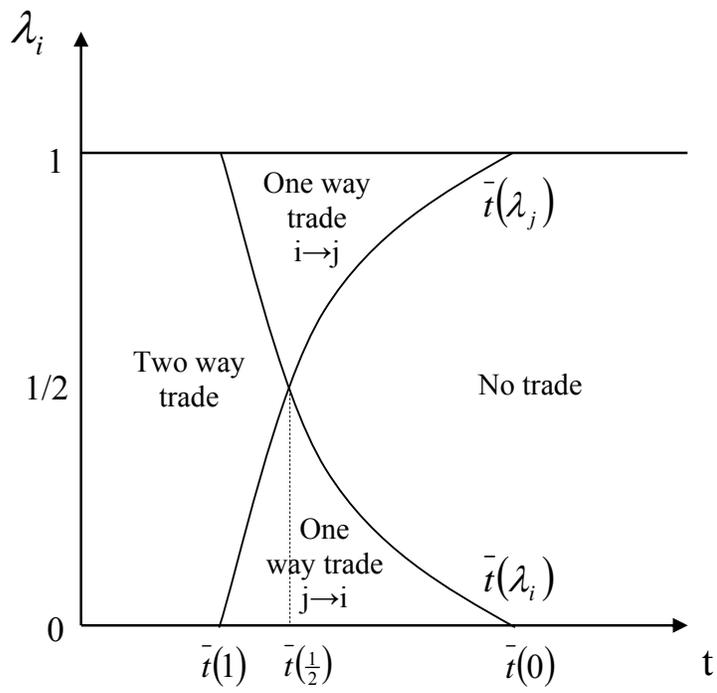


Figure 1: Structure of trade in the short run

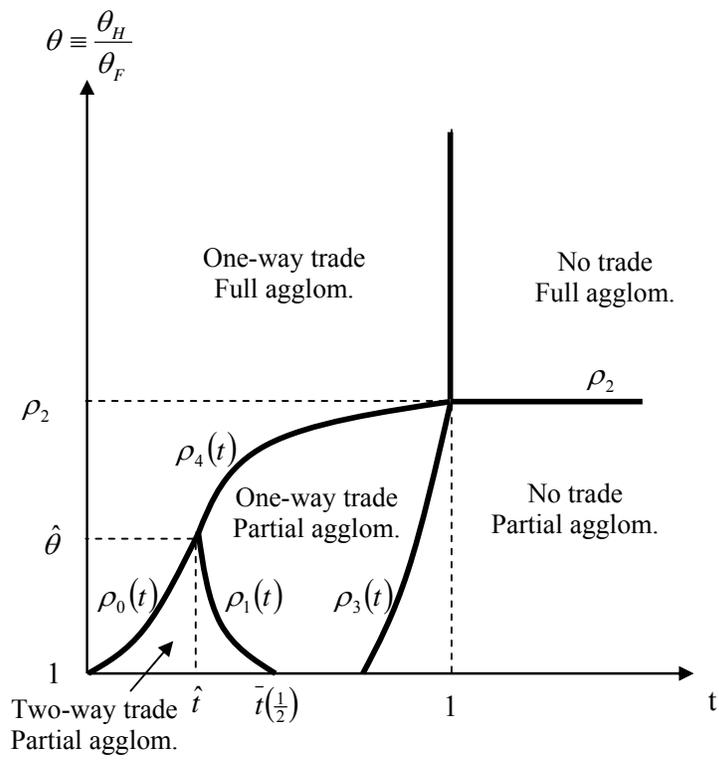


Figure 2 : Structure of trade in the long run

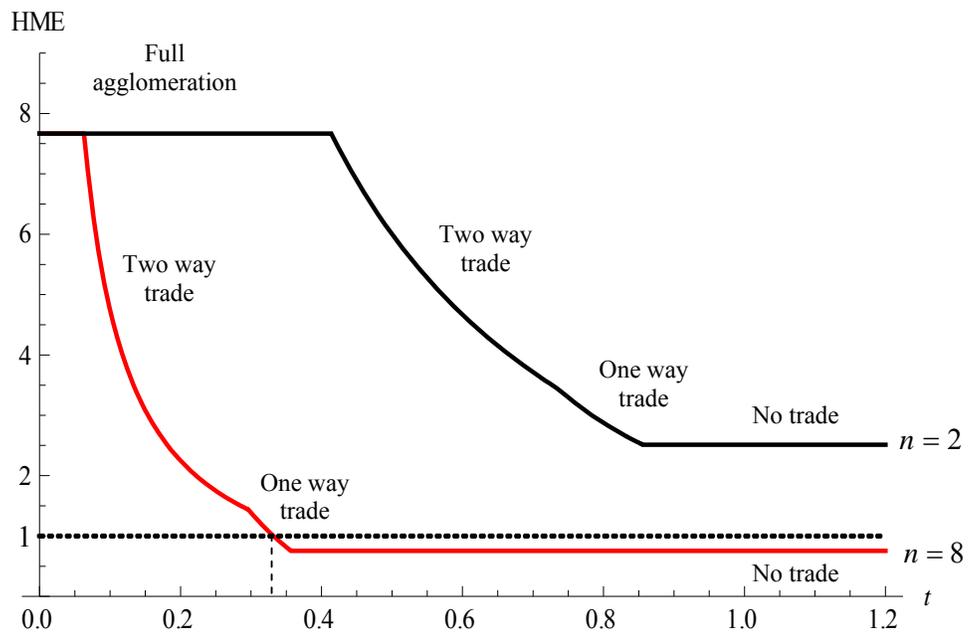


Figure 3 : Home market effect (HME)