Occupational Choice and Compensation for Losers from International Trade*

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This version: January 2005

Abstract

Trade liberalization creates winners and losers in the domestic economy. Nonetheless, many have become discontented with the current situation surrounding the explicit compensation schemes for losers. Why do we observe both a lack of compensation and the existence of overcompensation for some groups of individuals? When agents differ in their relative and absolute talents to undertake different occupations, shifts in the terms of trade will worsen the best outcome available to some agents and improve the best outcome available to others. Trade liberalization benefits some job-switchers as well as job-stayers, and harms some job-stayers as well as job-switchers. As a result, when the government cannot observe the agents' unused traits, it is impossible to design a program that ensures Pareto improvement from trade liberalization without making overcompensation to certain parts of the population. This proposition, derived rigorously in a two-good general equilibrium model with occupational choice, casts doubt on the effectiveness of existing forms of trade adjustment assistance programs. Under the conditions studied, the government faces a tradeoff between Pareto improvement and overcompensating a group of job-switching individuals.

JEL Classification: F11, F13, F16, H21, J23, J24
Keywords: occupational choice, intersectoral labor mobility, gains from international trade, adjustment assistance, compensation schemes

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*The author wishes to thank, for their comments made during discussions, the following people: Kiatipong Ariyapuchya, Lee Branstetter, Marcelle Chauvet, Avinash Dixit, Taiji Furusawa, Xavier Gabaix, Gloria González-Rivera, Jang-Ting Guo, Nobuyuki Hanaki, Gordon Hanson, Elhanan Helpman, Jota Ishikawa, Yukiko Itoh, Ronald W. Jones, Ethan Kaplan, Ryutaro Komiya, Marc Melitz, Heechul Min, Eiichi Miyagawa, Daisuke Nakajima, Douglas Nelson, Richard R. Nelson, Yoshiaki Ogura, Michihiro Ohyama, Prasanta K. Pattanaik, Mike Riordan, R. Robert Russell, Yusuke Samejima, and seminar participants at Princeton, Columbia, Hitotsubashi, Waseda, Aoyama-Gakuin Universities and University of California Riverside. The author is especially grateful to his advisors at Columbia University, for providing him with valuable guidance: Professors Ronald Findlay, Alessandra Casella and Sanjay Reddy. This research is supported by a fellowship from the SSRC Program in Applied Economics, with the funding provided by the John D. and Catherine T. MacArthur Foundation. The author also acknowledges the financial support of the government of Japan (via the International Monetary Fund) and Columbia University.

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1 Introduction

> One of the problems with free trade is we never compensate the losers. We always say that there are more winners than losers, and that's true. But there are losers, and we're not helping them. [Clyde Prestowitz, President of the Economic Strategy Institute.][1]

When academic economists teach their students about benefits from free trade, the focus of their attention tends to be on the gains in the aggregate efficiency. It is widely known that international trade, like another way of transformation than a production technology, will expand the set of feasible allocations faced by an economy as a whole. Nonetheless, international economists are quick to point out that trade liberalization almost always brings redistributive consequences among individuals within the economy.(Rodrik 1997, p.30) Thus, a change in the terms of trade favors some groups of individuals over other groups.[2] This is indeed an area where the protectionists can have the upper hand over the free traders. Of course, some economists would argue that compensation of the losers *could* take care of the problem.[3] After all, we will have a larger pie to share, and we can compensate losers in full even if we make all the beneficiaries from trade happier than they are in autarky. In the real world, however, many have grown discontented with the current situation surrouding compensation schemes.

While this section's epigraph, by Clyde Prestowitz, implies that compensation for losers is either absent or insufficient, a completely opposite opinion appears in the quotation from *The Washington Post*. It claims that the present compensation scheme, in the form of the Trade Adjustment Assistance (TAA) program, is far too magnanimous, and could put a huge strain on the federal budget. Taking it for granted that the recent expansion of the TAA program would be approved by the Senate in exchange for the president's fast-track "trade promotion authority" bill, the *Post* goes on to note that

> conservative critics are dismayed at the concessions they were forced to make, and they are hoping that budget constraints will prevent the establishment of a large new entitlement program. "Socialist governments all over the planet are trying to stop doing this kind of thing, and now we're doing it," said Sen. Phil Gramm (R-Tex.), referring to government largess for the unemployed.[4]

All of this reflects a growing sentiment among conservatives that the protectionist compensation schemes, when and if they exist, tend to shell out so much money that the society can actually end up overcompensating (some of) the losers. This paper's model seeks to explain the difficulty of ensuring Pareto gains from trade when individuals are heterogeneous and can freely move between different sectors. It concludes that no government can attain Pareto improvement unless it makes inefficiently larger transfers than are actually necessary.

[1] Los Angeles Times: Friday April 5, 2002. Part 3, Page 3, "President Pushes Lawmakers to Expand Trade Legislation: Bush seeks 'fast-track' authority. Democrats want help for U.S. workers hurt by foreign competition."

[2] For the description of a classical example, see for example Stolper and Samuelson (1941).

[3] This is known as the compensation principle. However, the compensation criterion requires only a "potential" Pareto improvement. The criterion does not ask whether the actual compensation takes place.

[4] The Washington Post: Saturday August 03, 2002. Page E01, "Trade Bill To Help Laid-Off Workers; Victims of Imports Win Added Benefits." by Paul Blustein.

The paper proposes a model of occupational choice in which we capture the realistic aspects of difficulty of identifying gainers and losers from trade by introducing agents who differ in their relative and absolute talents to undertake different occupations. The model achieves aggregate gains from trade even when individuals are allowed to switch jobs (or equivalently we observe temporally displaced workers). In order to effectively place the displaced individuals within a general equilibrium (full employment[5]) framework, I assume that each individual faces an occupational choice.[6]

Changes in terms of trade may boost the best outcome available to some agents, while worsening the best outcome for others. Some people are stuck in their industry (job-stayers), while others may switch their occupations (job-switchers) owing to a change in the economic environment. The distribution of fortune and misfortune spreads across the whole population, affecting both job-stayers and job-switchers alike. When we imagine the world of Heckscher-Ohlin or specific-factors trade models, it is not difficult to identify gainers and losers from trade.[7] In the model with occupational choice, it turns out to be very difficult to identify gainers or losers among those who switch their occupations. As a result it is hard to design a redistribution program that targets only those harmed by trade openings.

The primary reason for this difficulty is in the assymmetric information between the government and individual agents. The gains or losses from trade depend upon the relative sizes of an individual's actually-used and unused-latent talents. While we can observe actually-used talents of the individuals, we cannot observe their unused-latent talents. Even if the government can condition its taxation scheme on those variables that represent an actual use of the factors, the (infeasible) first-best compensation scheme must also base its tax rates on the latent talents of job-switchers. It is not difficult to show that there are individuals who are identical in terms of current use of their talents, and yet are either gainers or losers due to differences in size of their latent talents. Given the usual scheme of taxation and subsidy, the government has no mechanism to induce individuals to reveal their latent talents. This means that if it wishes to ensure a Pareto improvement from autarky, the government cannot avoid the overcompensation problem, and thus in some cases fails to balance its budget.

1.1 Heterogeneity of Agents in This Paper

In the model proposed in this paper, I presume the individual agents to be doubly heterogeneous, in the sense that they differ in both the absolute and relative magnitudes of their capabilities in their different occupations.

[5]Those interested in the issues of structural unemployment and gains from trade may wish to refer to the paper by Brecher and Choudhri (1994).

[6]A good justification of this full employment assumption (with occupational choice) has been provided by Daniel T. Griswold, associate director of The Center for Trade Policy Studies at The Cato Institute, a libertarian research group: "trade had little long-term impact on the overall number of jobs, because the American economy tended to create jobs in more sophisticated industries to replace those that are lost." in The New York Times: Tuesday October 29, 2002. Page 11, "TRADE WINDS; Global Trade in Elmwood Park: Familiar Saga With a Twist."

[7]For such particular results within the Heckscher-Ohlin framework, see Stolper and Samuelson (1941). For the specific-factors model, see Jones (1971) and Samuelson (1971).

Let us explain the heterogeneity used in the model via an example.

Suppose that every individual could, in principle, work either as an opera singer or as an economics professor. Naturally, every individual differs in how well he can sing opera arias. The same can be said about economic professorial skill. These differences can be called heterogeneity in absolute advantages. Of course no individual is going to be equally good at both things. Individuals' *relative* strengths are always going to vary widely. These variances can be called heterogeneity in comparative advantage. Some will be very good at singing but mediocre at economics, others the other way around, and still others good at both. A way to capture these differing absolute and comparative advantages is to assume that for every individual $j \in J$, there is a vector (θ^j, τ^j) of ability. The element θ (of the vector) measures how much "effective output of economic-professorial services" the individual can produce in a given period, while the other element τ measures how much "effective output of opera singing" the individual can produce over the same period. The size of these elements will inevitably differ across individuals, and this fact bespeaks a heterogeneity in absolute advantage. Also, the ratio of the elements of the ability vector, θ/τ, reflects the size of the comparative advantage an individual possesses in economics professorship.[8](Ruffin 1988) A relatively low θ/τ indicates a comparative advantage in opera-singing. Note that my model shares many aspects with the Roy (1951) model that had been put forth within the field of labor economics.[9]

Furthermore, every individual j faces an occupational choice in his life. Because I model this as an occupational choice, the decision is a discrete one: whether to work as an opera singer or as an economics professor. Of course the decision will depend on such economic variables as relative output price. Given a particular economic environment, an individual might choose to be an opera singer, but wish to switch to becoming an economist after a change has occurred in the terms of trade. Note as well that each element of the ability vector is indivisible and non-transferable.[10] (Dornbusch, Fischer and Samuelson 1977)

The remainder of this paper is divided into eight sections. In the next section, I present a non-technical overview of the model. In section 3, I develop a simple general equilibrium trade model having two final outputs. This model comprises a large number of heterogeneous agents who possess both generic-mobile and individual-specific occupational factors. I examine the Walrasian (trading) equilibrium of the model, and show that there are aggregate gains from trade. Section 4 is devoted to the presentation of the key result as to the existence of gainers among displaced individuals. Section 5 introduces the pertinent definitions and

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[8]The setup is somewhat similar to the model of interpersonal comparative advantage introduced in Ruffin (1988). While Ruffin's model allows for the multi-sector use of the same factors of production, I model this comparative advantage as a source of occupational choice. Also, my model allows for a continuum of varieties of individual heterogeneity, whereas Ruffin introduced a finite set of groups of individuals. Ruffin's case would violate my model's assumption of atomless agents.

[9]I was not aware of the labor literature until I had completed my analysis of a similar model as Roy. I thank Sujata Visaria for bringing my attention to the literature on labor economics.

[10]In a sense, the economy in this model has some similarity to the Ricardian economy with a large number of commodities in Dornbusch, Fischer and Samuelson (1977). Whereas the Dornbusch-Fischer-Samuelson model focuses on comparative advantage across different categories of outputs, my model emphasizes both the absolute and the comparative advantages of individuals' talents. Another difference: our model focuses primarily on the welfare change of individual agents, while also examining those compensation schemes that seek to attain the Pareto improvement.

properties of compensation schemes. Section 6 seeks to arrive at an unanticipated compensation scheme by using various taxes and subsidies based on the currently observable variables. Section 7 looks at the case in which individual agents learn about the compensation scheme and explore the disincentive problem by manipulating the mechanism. The final section offers some conclusions, and proposes a few future extensions.

2 Overview of the Model

Let me begin by providing a non-technical overview of this paper's analytical approach, postponing until the next section the formal development of the model. Parts of the model's structure bear a close resemblance to the independently discovered framework[11] first proposed in Roy (1951) and elaborated on by Rosen (1978) and Mussa (1982, pp.131-134). Consider a small open economy that faces exogenously given international output prices. Output markets are assumed to be competitive, both internationally and domestically. Putting aside the distributional concerns, it can be said that in the aggregate sense free trade is more efficient than any form of restricted trade for such an economy because there are no terms-of-trade externalities and hence no room for optimal tariffs. The economy consists of a continuum of individual agents who own two types of factor endowments: generic factors, and occupation-specific talents.

The generic-type factors are homogenous factors of production whose property rights are well defined and traded competitively via the domestic markets. Examples of these generic factors are unskilled wage labor, capital goods whose values are easily transformed into money or other types of capital goods, and all kinds of homogeneous inputs used in the production of outputs.

Occupation-specific talents (or occupational abilities) characterize the heterogeneity of individual agents in this economy. Agents vary in both their absolute and their relative strength in the different occupations. The occupation-specific talents are specific to the individual and to the industry (or chosen occupation). This can mean that human capital is sector-specific, and yet an agent still can have multiple talents in different sectors in different degrees. In addition to the specificity of talents, the other important characteristic of this specific factor is that it is intangible.[12] (Murphy 1986) Unlike the generic factors spoken of in the previous paragraph, the property rights of the specific talents (or occupational abilities) are embodied in each individual. In other words, the occupational talents are intangible and non-contractible.[13] Given that these talents belong to a

[11]It was only after I had completed my analysis that I discovered these classic works by Roy (1951), Rosen (1978) and Mussa (1982) that introduce similar setups of the model I provide here. The Roy model is used to analyze the inequality of earnings by individual workers, but never used for the analysis of international trade. Rosen applied the Roy framework for the case with 2 workers and many jobs. Mussa introduces a similar setup as a way of backing up his assumption about the convex input transformation curve. Despite our similarity of setups, however, Mussa never solves for the analysis I provide here in this paper.

[12]To put this matter differently, "human capitals are embodied in each individual" as Kevin Murphy says in his unpublished thesis.

[13]This intangible nature will explain the non-verifiability and the non-transferability of the individual's talents. The reason we assume here that the property rights are not well defined is that we seek to exclude the possible existence of both insurance and stock markets for the talents of individuals.

utility-maximizing economic agent, I postulate that the occupational abilities are indivisible.[14] I also assume that every individual agent in this economy is a residual claimant of his own specific talents that are in actual use.

Furthermore, I presume that each individual undertakes only one occupation at a time. The decision is discrete; I do not allow for the existence of individual agents who are employed in multiple sectors.[15] Usually, this type of non-convex decision-space would create difficulties for us in terms of verifying the existence of the equilibrium; here however, we are depending upon the result achieved by Hildenbrand (1974), who showed that non-convexity can be overcome by having a continuum of atomless[16] individual agents. (For the relevant cases of a large economy with non-convexities, see also Mas-Colell, Whinston and Green (1995, Section 17.I, p. 627).)

Each individual agent is a *residual claimant* who collects all the residual profits after paying the cost of production that is incurred for any generic factor of production.[17] Note that the individuals are residual claimants for the "actual use" of their talents. They may have many different kinds of "latent" (unused) talents, but to these they can lay no claim. In other words, a person chooses to produce a good by hiring as many inputs as necessary from the competitive markets, and he then earns residual profits from his activity. However good he may be at any other job, he can lay no claim to the residual profits from those activities in which he is not actually engaged. By choosing one job over another, a person forgoes his other opportunities. The opportunity cost for the person can be said to be the return from his second-best job, given the terms of trade.[18] The difference between his actual return and his second-best return will differ across individuals. Then too, the ranking of the best jobs may change when the environment changes. Nevertheless, I still can claim that a person is a residual claimant for his best talent, given the environment.

Now that we have depicted the nature of the factor endowments held by individual agents, let us now present a simplest possible general equilibrium model, namely the one with two output goods and thus with

[14]In other words, I assume that the individual will make a full effort, and thus I believe that the return for this occupational talent will appear in the form of residual profits rather than as market prices multiplied by the number of efficiency units. This is because the use of talent factors is not in the utility functions of individual agents. When the cost (disutility) of effort is zero, agents will maximize their effort-level up to the limit so that they can consume as large a set of consumption bundles as possible. For the analysis of choice of effort level when individual tastes include a disutility from making some efforts, see Spector (2001).

[15]Remember, Feenstra and Lewis (1994) allow for the supplying of one factor to multiple industries. They do not, however, allow for the existence of perfectly mobile generic factors, as we do in this paper.

[16]"Atomless" means that no point measure has a positive Lebesgue measure.

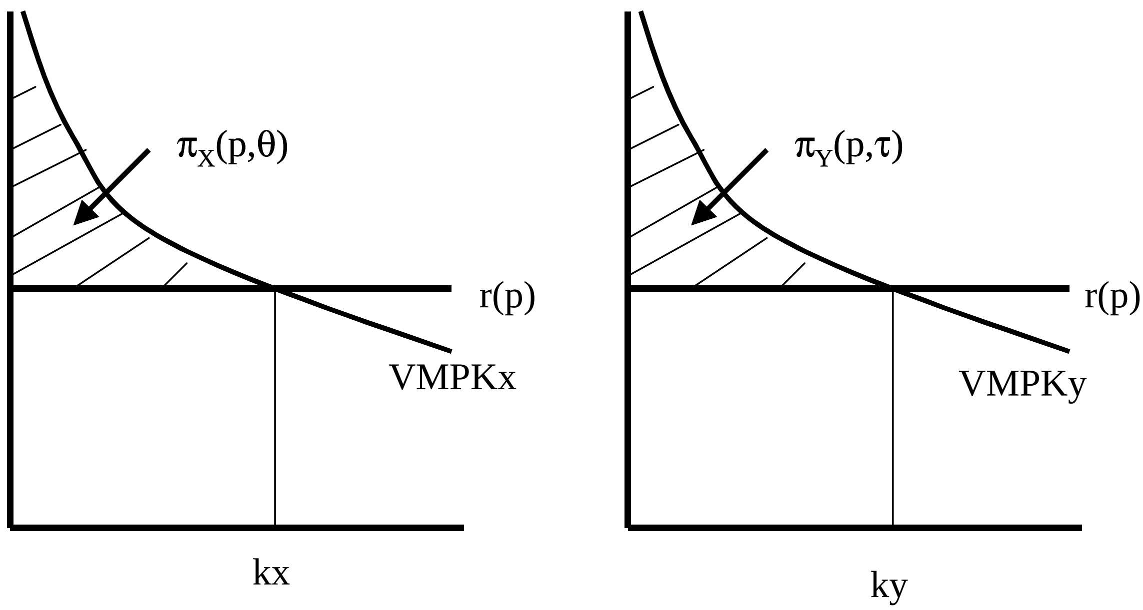
[17]This notion of residual claimant property should not be interpreted too literally. It tries to capture the specificity of a certain factor of production, and the difficulty of verifying its magnitude. Any worker in the economy possesses both the generic factors and the human-specific and industry-specific talents. One interpretation of this notion of residual claimant property is the self-employment of an agent. We are not, however, restricted to the self-employment interpretation. For even if the individual is hired by some outside firm, he still has full negotiating power to get all the residuals from production, because he still has an outside option of becoming self-employed. Thus we can assume that all the individuals in the economy are residual claimants of the talents actually used in their current production process.

[18]In this sense the size of the opportunity cost, as well as the size of the factor return, changes when there is a change in the terms of trade.

two occupations and one generic factor.[19] Let X (respectively, Y) denote the output good that is an *export* (respectively, *import*) good for home, and that is produced with the occupational ability θ (respectively, τ). Let K denote the total amount of the generic factor endowed in the economy. An individual $j \in J$ can be characterized by an occupational ability vector (θ^j, τ^j) and by an endowment of generic factor K^j. Let P_X and P_Y denote the output prices for X and Y. Let r denote the market price for the generic factor. Given each individual's endowment of ability and generic factor, he calculates the potential residual returns from every (here, two) occupational choice.

Let π_X and π_Y denote such residual returns from two sectors. Agents can freely trade generic factors at the market price r, in order to maximize their best available occupational returns. Since agents are price-takers in both the output and the generic-factor markets, they compare the expected residual returns between different occupations. Agents will choose which sector to produce as they compare: $\pi_X \gtreqless \pi_Y$. The economy takes the distribution of the ability vectors to be given by $(\theta^j, \tau^j) \sim F(\theta, \tau)$, where $F(\theta, \tau)$ represents the joint cumulative distribution function. I assume that $F(\theta, \tau)$ has a full support over a compact and convex set, and that its shape is common knowledge. Its density function $f(\theta, \tau)$ is bounded, and continuously differentiable. I also assume that the available technology (production functions for both X and Y) is common knowledge as well. The technology is characterized by constant returns to scale. Its production function is increasing in every input and is twice continuously differentiable, strictly concave, and satisfies the Inada conditions. The tastes of the consumers are assumed to be identical and homothetic. Therefore, I focus on the agents' heterogeneity with respect to their factor incomes.

Figure 1: Value of marginal product for the generic factor.



[19]Many parts of this analysis can also be applied to the basic diagrams used in the specific-factors model of trade.

The terms of trade, the relative price between X and Y (can be represented as P_X/P_Y), is the key decision variable for each individual. To see this clearly, we can utilize familiar diagrams normally used to describe specific-factor models of production. (See Fig. 1.) Given both the specification of production functions and the individual talents (θ^j, τ^j), we can draw curves representing the value of marginal product for the generic factor for both occupations. Let $VMPK_X$ and $VMPK_Y$ denote such curves. The vertical axis represents the monetary value of marginal product for the generic factor given the occupational talents of the individuals. The horizontal axis represents the quantity of generic factors being employed in the production of each output.

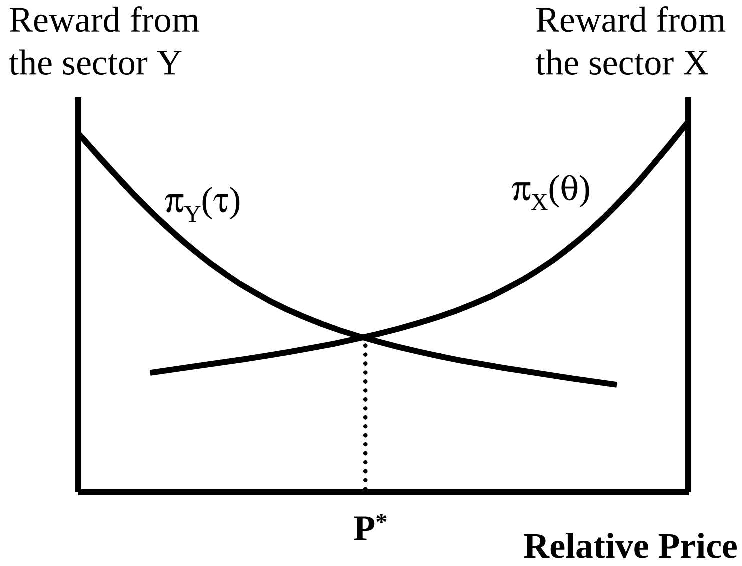
Let lower case letter k denote the *employment (use)* rather than the *endowment*, K, of generic factors. Both curves are downward-sloping in k, this showing the property of diminishing marginal product of a generic factor.

Both the elements of the ability vector (θ and τ) and the relative output price (P_X/P_Y) are the shift-parameters for the $VMPK_X$ and $VMPK_Y$ curves. The higher θ implies the higher position of $VMPK_X$. Similarly, the higher τ implies the higher position of $VMPK_Y$. The larger talent induces the corresponding value-of-marginal-product curves to shift up. An increase in the relative price of X, relative to Y, will shift the $VMPK_X$ curve up and the $VMPK_Y$ curve down. A decrease in the relative price of X induces a movement the other way around. When individuals calculate their residual profits, they take the generic factor price r as given, even though the equilibrium value of r depends on the relative price P_X/P_Y.[20] The area below the $VMPK$ curves and above the horizontal line at r represents the residual reward (or profit) π derived from the corresponding occupational talent. Given the relative price, P_X/P_Y, an individual with (θ^j, τ^j) will produce X if $\pi_X(\theta^j) > \pi_Y(\tau^j)$, will produce Y if $\pi_X(\theta^j) < \pi_Y(\tau^j)$, and will be indifferent as to producing either X or Y if $\pi_X(\theta^j) = \pi_Y(\tau^j)$. (Of course, we can deem this indifference case a measure zero event, given our atomless-agent assumptions.)

Fig. 2 shows the graph of the occupational rewards (profits), $\pi_X(\theta^j)$ and $\pi_Y(\tau^j)$, for a given individual, (θ^j, τ^j), over the possible range of relative prices $P_X/P_Y \equiv P$. The vertical axis represents the monetary value of occupational rewards, given the talent of the individual. The horizontal axis represents the relative price of output. (Note that in Fig. 2's graph the height corresponds to the area of the previous graph, Fig. 1.) Let P be a shorthand way of denoting P_X/P_Y. Let the intersection of the two occupational-reward curves occur at $P^* = (P_X/P_Y)^*$. The individual will produce Y when the level of relative output price is $P < P^*$. When $P = P^*$, he is indifferent as to producing either X or Y. He will produce X whenever $P > P^*$. Note that, for any trade liberalization, the shifts in terms of trade occur in a discrete manner. Then, for some positive discrete change $\Delta > 0$ in the relative price P, we have the ex ante price P^0 and the ex post price $P^1 = P^0 + \Delta$. When $P^0 < P^1 < P^*$, then the individual is a producer in sector Y in both periods. (One might say that he is stuck in Y production.) This sector-Y-stayer loses out owing to an increase in the relative price. When $P^* < P^0 < P^1$, then the individual is producing in the sector X in both periods. This sector-X-stayer benefits from the positive price change. In the case of this particular individual in Fig. 2, he changes his occupation when the relative price changes cross the P^* point. With respect to the case of job-switchers,

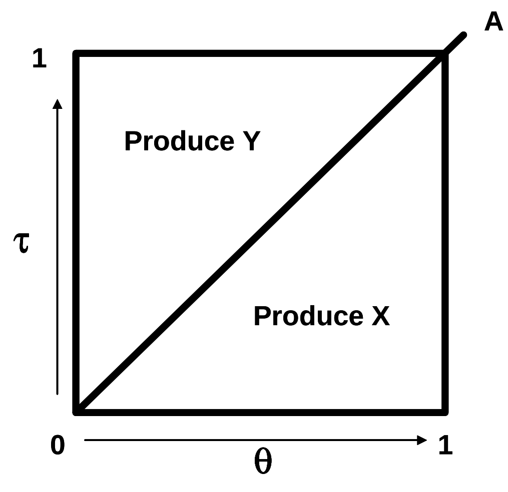
[20] And of course, the equilibrium value of r depends also on the shape of distribution $F(\theta, \tau)$ of the individuals' talents.

Figure 2: Individual occupational rewards, given output price.



$P^0 < P^* < P^1$, the welfare change is ambiguous. Note that, up to this point, our argument has not depended on the assumption about a specific distribution of talents, $F(\theta, \tau)$.

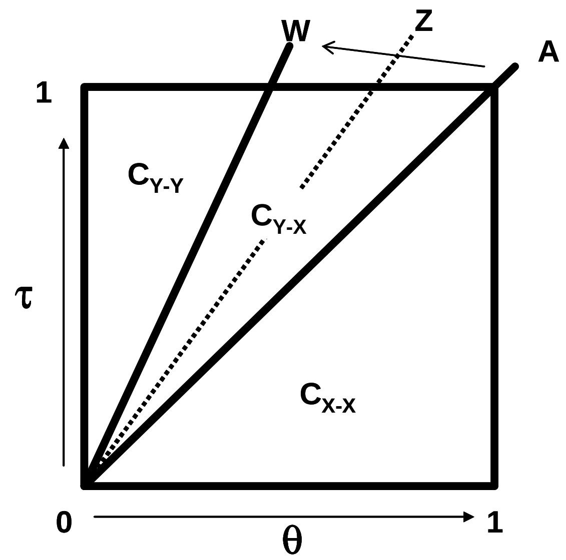
Figure 3: Ability vector space.



In order to simplify the exposition, let us assume that the ability vector (θ, τ) is distributed over the support of a unit square $[0, 1] \times [0, 1]$. (The support of unit square is not at all central to the results of this section. It is brought in here strictly for graphical convenience.) Let us also assume that the production and utility functions ensure that the autarky division of labor will occur at a 45-degree line on the unit square. This line divides the unit square in two partitions: one representing the X producers and the other the Y

producers.[21] See Fig. 3.

Figure 4: A unit square subdivided according to occupational choice.

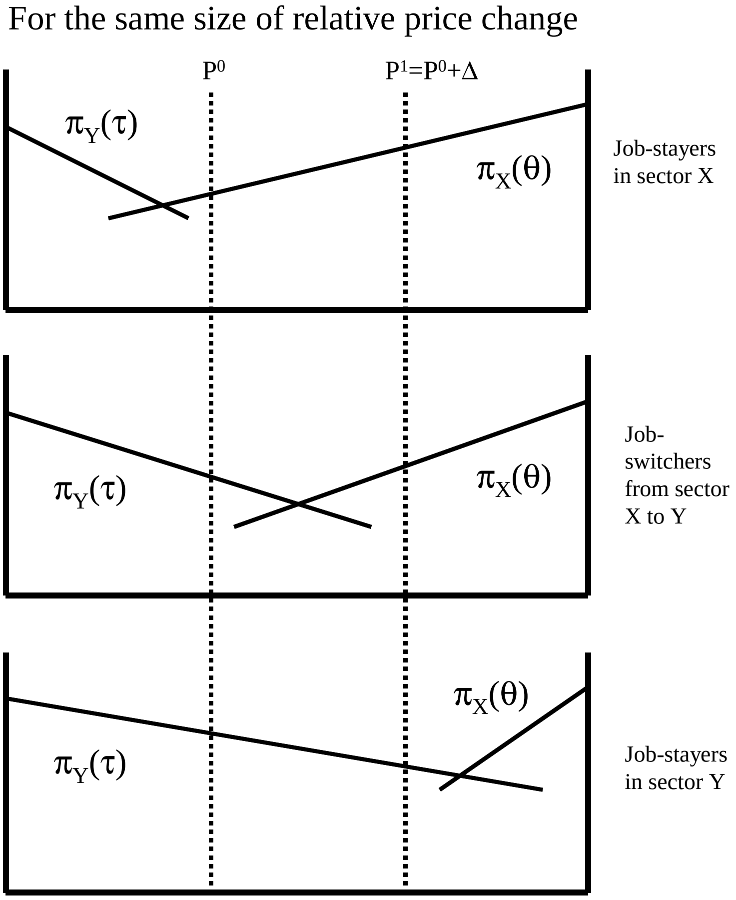


Let $P^A = (P_X/P_Y)^A$ denote the autarky relative price. In Fig. 4, the 45-degree line OA corresponds to the relative autarky price P^A. Now let us think of the case of an economy that is opening itself up to free trade. Let P^W denote the world (international) relative price. Then, because X is assumed to be a natural export good of the home country, it must hold true that $P^W > P^A$. Given the world price P^W, some individual agents may decide to switch their occupations after they have compared their present occupational rewards with those they could expect to receive in the other sector under the new price P^W. Thus, as may be seen in Fig. 4, we can draw a new ray from the origin, OW, that has a steeper slope than OA. While OA corresponds to the autarky division of occupational choice, OW represents the free-trade division of occupational choice. Next, let us partition our unit square into 3 sections. C_{X-X} denotes the partition that includes all the job-staying individuals who produced X in autarky and who keep producing X under free trade. C_{Y-Y} denotes the partition of job-stayers in the sector Y. The partition C_{Y-X} represents all the individuals who have switched occupations; for instance, someone who produced Y in autarky, and who now produces X under free trade. There are of course, given the direction of the output price change, no job-switchers in the opposite direction.

Note that there is a one-to-one correspondence between Figures 2 and 4. Each individual has a different job-switching value, P^*. The location of this trigger value depends only on the agent's comparative advantage, hence the relative size of the talents: θ/τ. Note also, in Fig. 4, that there is a one-to-one mapping between the relative size of the talents and the slope of the ray from the origin to the point that represents the individual's

[21] This assumption of a symmetric autarky division of agents, while not central to our results, does have the virtue of facilitating much easier expositions, since one need not classify one's results by the various cases.

Figure 5: A comparison of three types of individuals.

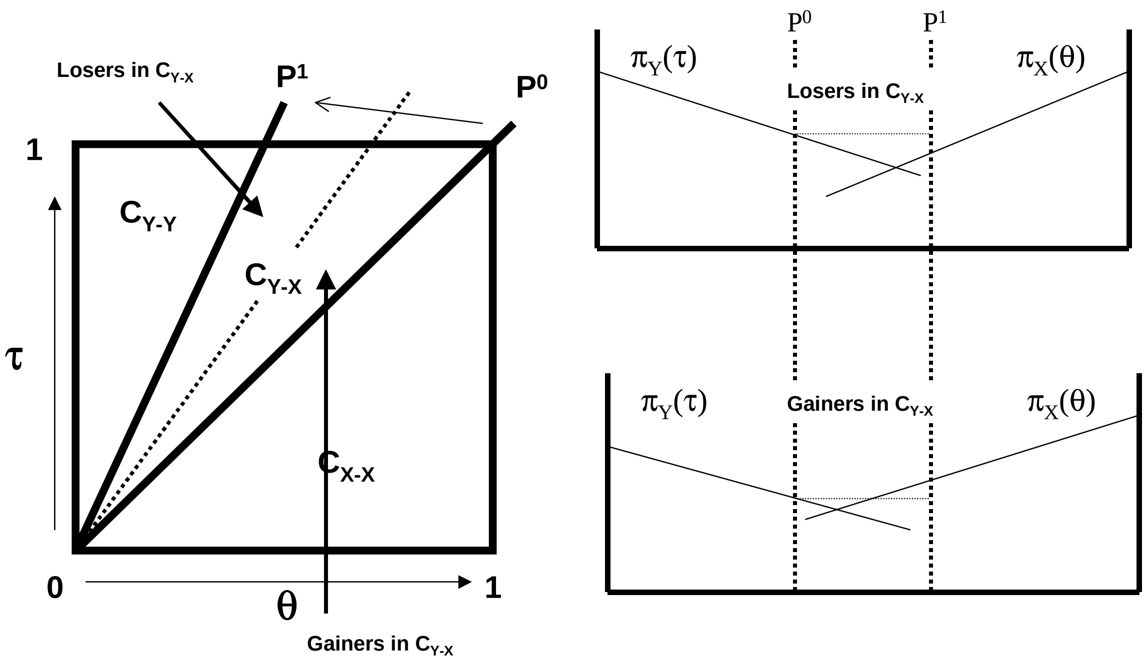


endowment. The higher the value θ/τ, the higher the comparative advantage the agent has in producing X; and therefore, the flatter becomes the slope of the ray from the origin at which the agent is located in the unit square. This can easily be seen, because the slope γ can be found to be the inverse of θ/τ by using the equation of the ray from the origin: $\tau = \gamma\theta$.

For the same relative price change, different individuals face different decisions for their occupation choices. Fig. 5 compares the residual reward values for representative agents from the three groups of individuals having different comparative advantages. Note that Fig. 5 contains the same diagrams as Fig. 2, showing three different agents with different trigger values P^*: an agent from group C_{X-X} (a job-stayer in sector X), an agent from group C_{Y-Y} (a job-stayer in sector Y), and an agent from group C_{Y-X} (a job-switcher from sector Y to sector X). The agent from group C_{X-X} has a low value of P^*, the agent from group C_{Y-X} a medium value, and the agent from group C_{Y-Y} a high value. Only the job-switchers experience that relative price change from P^0 to $P^1 = P^0 + \Delta$ which crosses over the trigger value P^*, where $\Delta > 0$. We can conclude that any rise in the relative price of X will favor the job-stayers in industry X, and disfavor the job-stayers in industry Y. The third graph, however, provides ambiguous results with respect to the job-switchers from

Y to X. In fact, we can conclude that there exist both winners and losers among those who switch their occupations.

Figure 6: Gainers and losers among job-switchers.



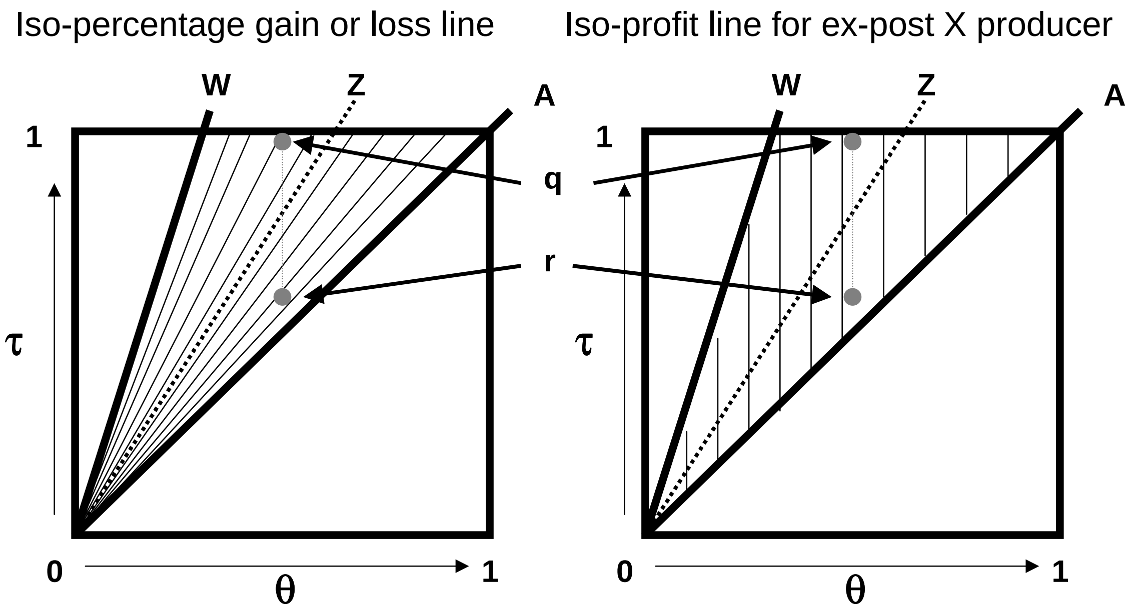
Note: There exist both gainers and losers among job switchers.

Figure 6 shows us that the factor separating the winners from the losers among job-switchers is the comparative advantage of individuals. The left-hand panel in Fig. 6 provide us with finer partitions of the group of agents from C_{Y-X} (job-switchers from the sector Y to the sector X) into gainers and losers. As for the right-hand panel in Fig. 6, these graphs represent the profit functions for the corresponding agents (gainers and losers) given a discrete price change. Among the job-switchers, each individual has a different relative size of his talents, θ/τ, and hence a different job-switching trigger-value of relative price P^*. Given the same increase in the relative price of X, it will be the agents with a higher θ/τ value who tend to be the gainers. Here in Fig. 6, I provide an example of two types of agents: a loser among job-switchers (the upper graph on the right-hand panel) and a gainer among job-switchers (the lower graph on the right-hand panel).

Given that there exists this mixture of gainers and losers among job-switching individuals, we are now able to explain the difficulty a government experiences when trying to carry out a fully Pareto improving compensation scheme while not providing overcompensation to the job-switchers. Let the government be capable of utilizing any taxation and subsidy scheme, based on the variables it can currently observe. Let us especially allow the government to use a tax-subsidy combination for both output goods and factors of production, including a residual return for the talents of individuals. Let us assume further that the tax (subsidy) base for the government can be restricted to currently observable variables. Thus, a scheme of wage insurance based on the information about individuals' previous occupations prior to their job switching is not allowed.

A Pareto-improving compensation scheme for job-staying individuals can easily be created. The direction and size of gain or loss are calculated in a manner similar to that seen in the case for specific (immobile) factors in the specific-factors model. The percentage gain or loss for job-staying individuals is the same for all of the stayers, regardless of the sizes of their talents, whether currently or previously in use.

Figure 7: Job-switching individuals.



Note: The left-hand panel depicts the iso-percentage gain-loss lines, while the right-hand panel depicts the iso-current profit lines for ex post producers of the sector X outputs.

As far as the job-switchers are concerned, the creation of a Pareto-improving compensation scheme cannot help but usher in certain complications. This is because the size and the direction of individuals' gains or losses are not necessary correlated with currently observable variables. Note in particular the percentage change in occupational residual profits will be the same for all the individuals on the same ray from origin. Nonetheless, the government cannot distinguish winners from losers within the job-switching individuals who are reaping the same amount of residual profits from the current production activities. Thus, while in Fig. 7, the iso-percentage-gain-or-loss lines are the rays from the origin, the iso-profit lines from the current production are the vertical lines showing ex-post producers from sector X. (The horizontal lines show ex-post producers from sector Y.) Thus, Fig. 7 depicts the case of job-switching individuals who move from sector Y to sector X.

The left-hand panel in Fig. 7 depicts the iso-percentage gain-loss lines, while the right-hand panel in Fig. 7 depicts the iso-current profit lines for ex post producers of the sector X outputs. The iso-percentage gain-loss lines are the rays from the origin, while the iso-current profit lines for X producers are parallel vertical lines. The closer the iso-percentage gain-loss lines are to the OA line (and hence the flatter the slope of the rays from the origin), the larger are the gains (and the smaller the losses). (Among the many rays from origin depicted in Fig. 7, it is the OZ line which represents the zero gain-loss line for job-switching individuals.) The iso-current profit lines, located toward the right of the panel, have higher values of current profits than do the ones located toward the left. While the actual sizes and directions of individuals' gains and losses depend on the knowledge of the iso-percentage gain-loss lines, the government can only observe the information based on the iso-current profit lines. For example, when we look at the two points q and r on both of the diagrams, we see that the points have the same value of θ and yet have different values for τ. The individual on the point q has a larger τ, while the individual on the point r has a smaller one. The difference of the value of τ is large enough that, when it comes to opening up to trade, the individual on the point q is a loser and the individual on the point r is a gainer. And yet they both appear to be the same from the point of view of the government, since they are making the same current profits. In other words, although the points q and r are on the same iso-current profit line, they are on different iso-percentage gain-loss lines.

The analysis of the preceding paragraph has made it clear that the government cannot both attain a Pareto-improving compensation and avoid awarding excess compensation. For the government must give the same amount of subsidy to r as q, even though the individual on the point r is actually a gainer from trade. So too, the government must provide the same amount of subsidy or tax to the individuals on the same iso-current profit line, regardless of their actual gains or losses. Indeed, if the government wants to ensure Pareto improvement, then it must see to it that the amount of subsidy is the same for all as it is for the worst individuals who are on the upper side of the square in Fig. 7. For this reason it is inevitable that the government will overcompensate the job-switching individuals, with the exception being the ones seen exactly on the line segment of the upper side of the square.

In this section, I have sought to do two things for my reader. First, provide him with an intuitive diagrammatic explanation of why there exist winners among those occupation-switchers who are facing the change in terms of trade. And second, make clear the impossibility of carrying out a compensation scheme that achieves Pareto improvement without overcompensating certain job-switchers. The formal model will be developed in the following section, in order to make the case in a more precise manner.

3 The Formal Model

Consider a continuum of agents $j \in J$, each of whom is endowed with an individual-specific occupational ability vector $(\theta^j, \tau^j) \sim F(\theta, \tau)$ and a generic factor $K^j \geq 0$.[22] Let $f(\theta, \tau) \geq 0$ denote the joint density function for $F(\theta, \tau)$, and assume that f is integrable over any partition of the ability space Θ. Agents are price takers in the output and the generic-factor markets. An economy-wide endowment of generic factors is inelastically supplied at $K = \int_J K^j$. Agents trade their generic factors freely via the competitive market. The factor price is denoted by $r > 0$. Each element of the ability vector (θ^j, τ^j) represents an occupational talent;

[22]The distribution of K^j can be quite general, since there is a competitive market for it. Therefore we will not specify its distribution function but instead simply say that the total mass is represented by K.

their magnitudes measure the innate capabilities of the agent j in the production of X and Y.

An agent decides either to produce X using θ, or Y using τ. An element of the ability vector (θ^j, τ^j) is indivisible and non-transferable. It can be considered a managerial talent of the owner, if we think of each agent as being a (self-employed) firm. An ability vector $(\theta^j, \tau^j) \in \Theta$ is unobservable to the government, but its aggregate distribution is publicly known. $\Theta \subset \mathbb{R}^2$ represents the space of individual characteristics. Θ is assumed to be a compact and convex set.

Having stipulated the individual characteristics, we are now ready to describe the technological side of the economy. Technology is a nonrival good, and every individual has access to the best available production techniques. Thus, individuals differ only in the endowment of factors. The timing of decision-making and market-clearing will be as follows.

1. The world market determines the relative output prices between P_X and P_Y. The home market takes them as given. In analyzing domestic equilibrium, we will determine relative price endogenously. But because all agents are infinitesimal, they take the equilibrium prices as given.

2. The individual agent observes one's own type vector $(\theta^j, \tau^j) \in \Theta$.

3. The agent forms a conjecture about the market factor-price r, foresees the profit-maximizing level of generic-factor employment, and calculates the occupational rewards or residual profits $\pi_X^j(P_X, r, \theta^j)$ and $\pi_Y^j(P_Y, r, \tau^j)$ to be gained from both occupational choices.

4. The agents decide (based on the expected size of rewards) in which sector to produce, and hire from the factor market the profit-maximizing level of the generic factor. They choose to produce either X or Y (not both, and not a convex combination of the two) using θ or τ. This process will determine the economy-wide size of the specific factors.

5. The generic-factor market clears. The equilibrium factor-price r should be consistent with the conjectures the agents have had.[23]

6. Given domestic production and domestic demand, the home country engages in trade with the world.

Both outputs are assumed to be produced with symmetrical production functions[24] that are twice continuously differentiable, strictly increasing, strictly concave, homogeneous of degree one. In particular, let us assume for simplicity's sake the following Cobb-Douglas specification:

$$\begin{cases} x^j(k_X^j, \theta^j) &= (k_X^j)^a (\theta^j)^{1-a} \\ y^j(k_Y^j, \tau^j) &= (k_Y^j)^a (\tau^j)^{1-a} \end{cases} \quad \text{where } a \in (0,1) \tag{1}$$

[23]This conjecture can be thought of as having emerged from the rational expectation hypothesis. Actually, however, any disequilibrium adjustment process will do the job, such as the assumption of the existence of the Walrasian auctioneer.

[24]This symmetry of the production functions is not essential to my results. It is just that by delegating all the heterogeneity to the endowment side, we are able to radically simplify the algebraic calculations.

where x^j and y^j are individual level outputs, where k_X^j and k_Y^j represent the individual-level *uses* of the generic factor, and where θ^j and τ^j represent the occupational talents. Note that the use of the generic factor is not constrained by the individual endowment K^j, because there exists a perfectly competitive market for this factor and because agents can freely buy from the market and sell portions of their endowments.[25]

Given the output prices P_X and P_Y, and the factor price r, individuals are able to compare the expected rewards (net of payments to the employed generic factors) π_X^j and π_Y^j from different occupations. Based on the regular profit-maximization program, we can depict such a comparison in the form of the following two equations.

$$\begin{cases} \pi_X^j(P_X, r, \theta^j) & = & \max\limits_{k_X} P_X \cdot x^j(k_X, \theta^j) - r \cdot k_X \\ \pi_Y^j(P_Y, r, \tau^j) & = & \max\limits_{k_X} P_Y \cdot y^j(k_Y, \tau^j) - r \cdot k_Y \end{cases} \tag{2}$$

By calculating the hypothetical employment levels of optimized generic factors, we arrive at

$$\begin{cases} k_X^j(P_X, r, \theta^j) & = & \left(\frac{aP_X}{r}\right)^{\frac{1}{1-a}} \cdot \theta^j, \text{ or} \\ k_Y^j(P_Y, r, \tau^j) & = & \left(\frac{aP_Y}{r}\right)^{\frac{1}{1-a}} \cdot \tau^j. \end{cases} \tag{3}$$

The occupational decision is based on the relative size of the post optimization level of the occupation rewards; thus, $\pi_X^j(P_X, r, \theta^j) \gtreqless \pi_Y^j(P_Y, r, \tau^j)$. And the post optimization level of the rewards can be calculated as

$$\begin{cases} \pi_X^j(P_X, r, \theta^j) & = & \left[(P_X)^{\frac{1}{1-a}} \left(\frac{1}{r}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right)\right] \cdot \theta^j, \text{ or} \\ \pi_Y^j(P_Y, r, \tau^j) & = & \left[(P_Y)^{\frac{1}{1-a}} \left(\frac{1}{r}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right)\right] \cdot \tau^j. \end{cases} \tag{4}$$

The size of the occupational rewards increases with the sizes of the agents' abilities and with their own output prices. Note now, in equation (4), the symmetry of the production functions from (1) neutralizes the effect of generic-factor price(Ruffin and Jones 1977).

Now we can partition the ability space Θ by self-selection of individual. Noting that the notation P can be utilized as a shorthand way of expressing P_X/P_Y, we can see that

$$\begin{cases} R & = & \left\{(\theta^j, \tau^j) \in \Theta : \tau^j < P^{\frac{1}{1-a}} \cdot \theta^j\right\} \\ S & = & \left\{(\theta^j, \tau^j) \in \Theta : \tau^j > P^{\frac{1}{1-a}} \cdot \theta^j\right\}, \end{cases} \tag{5}$$

where the partition R represents the individuals who produce X, and the partition S represents the Y producers. Note that the ray from origin, which can be expressed as $\tau^j = \gamma \cdot \theta^j$ where γ is a constant, is the division-line between the two partitions.[26]

Up to now, we have described how individuals choose their occupations and get involved in a production process. In (5), the partition of individual agents represents an endogenous determination of the allocation of

[25]The size of the endowment K^j matters only with respect to out calculation of the factor income for an individual. Agents can buy more than they possess, because we are implicitly assuming the existence of a perfect capital market in which people freely borrow money to pay for generic factors in excess of their possession.

[26]I am using a strict inequality for both partitions, simply because the measure of the line $\tau^j = \frac{(P_X)^{\frac{1}{1-a}}}{(P_Y)^{\frac{1}{1-a}}} \cdot \theta^j$ is zero.

specific-factors available in the economy as a whole. We now begin to examine the economy-wide allocation of specific factors.

Let A^R (respectively, A^S) denote the area-integration of the region R (respectively, S). This area-integration represents the mass of individuals in the corresponding partition. Let V_θ^R (respectively, V_τ^S) denote the volume integral with respect to the variable θ (respectively, τ) on the region R (respectively, S). This volume integral represents the economy-wide employment size of each specific factor. Our next equations bring us the mass of the individual agents in partitions R and S, respectively:

$$\begin{cases} A^R & \equiv \iint_R 1 \cdot f(\theta,\tau)d\theta d\tau \\ A^S & \equiv \iint_S 1 \cdot f(\theta,\tau)d\theta d\tau. \end{cases}$$

The economy-wide size of the specific factors can be expressed by the following equations.

$$\begin{cases} V_\theta^R & \equiv \iint_R \theta \cdot f(\theta,\tau)d\tau d\theta \\ V_\tau^S & \equiv \iint_S \tau \cdot f(\theta,\tau)d\theta d\tau. \end{cases} \tag{6}$$

When the joint density function $f(\theta,\tau)$ has a full support and is continuous, it is not difficult to show that both A^R and V_θ^R are strictly increasing in P and that A^S and V_τ^S are strictly decreasing in P.

Given the self-selection condition of individual occupational choice as depicted back in equation (5), the generic-factor market will clear, and its full-employment condition is expressed by the following equation.

$$\iint_{(\theta^j,\tau^j)\in R} k_X^j(P_X,r,\theta^j)f(\theta,\tau)d\theta d\tau + \iint_{(\theta^j,\tau^j)\in S} k_Y^j(P_Y,r,\tau^j)f(\theta,\tau)d\theta d\tau = K. \tag{7}$$

The factor-market demand, as represented by the left-hand side of equation (7), is an aggregation of all the individuals' factor-demand over each partition, R and S.

By plugging the optimized values seen in equation (3) for the employment-level generic-factors into (7), we arrive at

$$\left(\frac{aP_X}{r}\right)^{\frac{1}{1-a}} \cdot \iint_R \theta^j f(\theta,\tau)d\theta d\tau + \left(\frac{aP_Y}{r}\right)^{\frac{1}{1-a}} \cdot \iint_S \tau^j f(\theta,\tau)d\theta d\tau = K. \tag{8}$$

By utilizing the notation in (6), we can rewrite equation (8) as

$$\left(\frac{aP_X}{r}\right)^{\frac{1}{1-a}} \cdot V_\theta^R + \left(\frac{aP_Y}{r}\right)^{\frac{1}{1-a}} \cdot V_\tau^S = K. \tag{9}$$

Note that both V_θ^R and V_τ^S depend upon the relative output price P. Thus, equation (9) implicitly tell us that r, the reward for generic factor, is a function of the output prices, with K and a being parameters. We then assume that the solution of (9) for r is unique, and can be written as

$$r = r(P_X, P_Y). \tag{10}$$

In order to derive in a simple manner the properties of the reward function (10) for the generic-factor, we will postulate a specific functional form for the demand side of the economy.

3.1 Demand Side

Generally, each consumer j's problem can be depicted thus:

$$\max_{c_X, c_Y} u(c_X^j, c_Y^j) \ \text{ s.t. } P_X \cdot c_X^j + P_Y \cdot c_Y^j \leq I^j,$$

where (c_X^j, c_Y^j) represents the consumption bundle for the individual j. His income is expressed as

$$I^j = r \cdot K^j + \max_{X,Y}\{\pi_X^j(P_X, r, \theta^j), \pi_Y^j(P_Y, r, \tau^j)\}.$$

In general, the utility function shall be twice continuously differentiable, strictly quasi-concave, homothetic, and strictly increasing. For simplicity of exposition, let us assume the following Cobb-Douglas form. (Note that the constant term has been added in order to make both the Walrasian-demand and the indirect-utility functions simple.)

$$u(c_X^j, c_Y^j) = 2\sqrt{c_X^j c_Y^j}. \tag{11}$$

We now can utilize the price-normalization of $P_X = p\ P_Y = 1/p$. Note that $P = P_X/P_Y = p^2$. Given the price normalization, the indirect utility function can be normalized to the income of the individual (in terms of the parameter p).

$$v(P_X, P_Y, I^j) = \frac{I^j}{\sqrt{P_X P_Y}} = I^j(p) \tag{12}$$

Note that the last equality takes into account the dependence of income on relative output price.

By utilizing the above normalization of price parameter p, we can express the equilibrium level r as the following equation.

$$r(p) = a \cdot K^{-(1-a)} \left[p^{\frac{1}{1-a}} \cdot V_\theta^R(p) + p^{-\frac{1}{1-a}} \cdot V_\tau^S(p)\right]^{1-a} \tag{13}$$

Note that the value of the economy-wide employment of the specific factors, $V_\theta^R(p)$ and $V_\tau^S(p)$, depends on the relative-output-price parameter p.

The equilibrium-level national income can also be expressed as a function of relative output-price p.

$$I(p) = \int_{(\theta^j, \tau^j) \in \Theta} I^j(p) = r(p) \cdot K + \iint_R \pi_X^j \cdot f(\theta, \tau) d\tau d\theta + \iint_S \pi_Y^j \cdot f(\theta, \tau) d\theta d\tau \tag{14}$$

We now can state an intermediate result, concerning the relationship between national income and factor income for the generic factor.

Lemma 1 *Generic-factor income is proportional to national income with this relationship being expressed as the equation:*

$$r(p) \cdot K = a \cdot I(p). \tag{15}$$

This follows directly from equations (4), (13) and (14). This proportional relationship in (15) holds true because the production functions for the two sectors are Cobb-Douglas and symmetric. Its proof is in the Appendix.

It also is to be noted that the national factor-income is equal to the gross national product:

$$I(p) = P_X \cdot \iint_R x^j(p, r, \theta^j) \cdot f(\theta, \tau) d\tau d\theta + P_Y \cdot \iint_S y^j(p, r, \tau^j) \cdot f(\theta, \tau) d\theta d\tau. \tag{16}$$

The relationship seen in (15) can also be confirmed by using (16).

3.2 Goods Market Equilibrium

Let us now investigate the goods market equilibrium. There are two equilibria: one for autarky and the other for free trade. We will seek for the goods-market-clearing conditions for the autarky equilibrium, and examine the expression of trade volumes for the trading equilibrium.

A trading equilibrium is represented by a net import vector $m(p)$, for a given relative price p:

$$m(p) \equiv (ED_X(p), ED_Y(p)) = (C_X(p) - X(p), C_Y(p) - Y(p)),$$

where $ED_X(p)$ and $ED_Y(p)$ are the excess demand functions for sectors X and Y, respectively, and where

$$C_X(p) = \iint_{\Theta} c_X^j dF(\theta, \tau) \text{ and } C_Y(p) = \iint_{\Theta} c_Y^j dF(\theta, \tau)$$

and

$$X(p) = \iint_{R} x^j dF(\theta, \tau) \text{ and } Y(p) = \iint_{S} y^j dF(\theta, \tau).$$

Autarky is a special case where $m(p^A) = 0$. Let us now derive the conditions for the autarky equilibrium. By using the given utility function (11), we can see that the Walrasian-demand functions for goods X and Y will be written respectively as

$$\left\{ \begin{array}{lcl} c_X^j(p, I^j) & = & \frac{I^j}{2p} \\ c_Y^j(p, I^j) & = & \frac{p \cdot I^j}{2} \end{array} \right. \Longrightarrow \left\{ \begin{array}{lcl} C_X(p) & = & \frac{I(p)}{2p} \\ C_Y(p) & = & \frac{p \cdot I(p)}{2} \end{array} \right.$$

where the left panel shows the individual demand functions and the right panel shows the market demand functions. By utilizing the previous results [derived by plugging (3) into (1)], we can depict the aggregate production in terms of p.

$$\left\{ \begin{array}{lcl} x^j(k_X^j, \theta^j) & = & \left(\frac{a \cdot p}{r(p)}\right)^{\frac{a}{1-a}} \cdot \theta^j \\ y^j(k_Y^j, \tau^j) & = & \left(\frac{a}{p \cdot r(p)}\right)^{\frac{a}{1-a}} \cdot \tau^j \end{array} \right. \Longrightarrow \left\{ \begin{array}{lcl} X(p) & = & \left(\frac{a \cdot p}{r(p)}\right)^{\frac{a}{1-a}} \cdot V_\theta^R(p) \\ Y(p) & = & \left(\frac{a}{p \cdot r(p)}\right)^{\frac{a}{1-a}} \cdot V_\tau^S(p) \end{array} \right.$$

Thus, when $p = p^A$, the following equations must hold true.

$$\left\{ \begin{array}{lcl} \frac{I(p)}{2p} & = & \left(\frac{a \cdot p}{r(p)}\right)^{\frac{a}{1-a}} \cdot V_\theta^R(p) \\ \frac{p \cdot I(p)}{2} & = & \left(\frac{a}{p \cdot r(p)}\right)^{\frac{a}{1-a}} \cdot V_\tau^S(p) \end{array} \right. \tag{17}$$

By using the result seen in (15), and the condition seen in (17) can be rewritten as

$$\left\{ \begin{array}{lcl} V_\theta^R(p) & = & \frac{K}{2} \cdot \left(\frac{r(p)}{a \cdot p}\right)^{\frac{1}{1-a}} \\ V_\tau^S(p) & = & \frac{K}{2} \cdot \left(\frac{p \cdot r(p)}{a}\right)^{\frac{1}{1-a}}. \end{array} \right. \tag{18}$$

When we plug the equilibrium-level generic-factor return (13) into (18), we get the following autarky condition for the economy-wide employment of the specific occupational factors.

$$p^{\frac{1}{1-a}} \cdot V_\theta^R(p) = p^{-\frac{1}{1-a}} \cdot V_\tau^S(p) \,|_{p=p^A} \tag{19}$$

In autarky, we see that $p = p^A$ and this expression is explicitly noted in equation (19).

We know that the change with respect to each specific factor's economy-wide employment has the opposite sign; that is,

$$sign\left(\frac{dV_\theta^R}{dp}\right) = -sign\left(\frac{dV_\tau^S}{dp}\right) \text{ for some } dp.$$

Then, by taking the total derivative of the autarky condition seen in (19) with respect to p, and after reassuring ourselves that the sign will be adjusted, we arrive at

$$\frac{1}{p(1-a)} \cdot \left[p^{\frac{1}{1-a}} \cdot V_\theta^R(p) - p^{-\frac{1}{1-a}} \cdot V_\tau^S(p)\right] + \left[p^{\frac{1}{1-a}} \cdot \frac{dV_\theta^R}{dp} + p^{\frac{-1}{1-a}} \cdot \frac{dV_\tau^S}{dp}\right] = 0, \tag{20}$$

when $p = p^A$.

When we look at the case $p > p^A$, we know that the home country exports the good X. Therefore the excess demand for X is negative – i.e., $ED_X(p) < 0$ – while the excess demand for Y is positive: $ED_Y(p) > 0$. This relationship can be expressed as

$$X(p) > C_X(p) \Leftrightarrow p^{\frac{1}{1-a}} \cdot V_\theta^R(p) > p^{-\frac{1}{1-a}} \cdot V_\tau^S(p) \mid_{p>p^A} . \tag{21}$$

Similarly, we now can say that

$$p^{\frac{1}{1-a}} \cdot V_\theta^R(p) < p^{-\frac{1}{1-a}} \cdot V_\tau^S(p) \mid_{p<p^A} . \tag{22}$$

We also can derive an intermediate result, with respect to the return for the generic factor K.

Lemma 2 *Let p^A be the autarky-level price parameter. The factor price $r(p)$ can be written as a function of the relative-output-price parameter p. Its value is U-shaped around $p = p^A$; i.e., it is increasing in p when $p > p^A$, decreasing in p when $p < p^A$, and it has a slope 0 at $p = p^A$.*

Its proof is in the Appendix. When we take the above lemma along with the condition (15), we arrive at another important result about the existence of aggregate gains from trade.

Proposition 1 *Given the setup of the model, there exist aggregate gains from international trade. That is, the real-valued national income $I(p)$ is U-shaped around $p = p^A$. In other words, any deviation from the autarky price will raise the level of real valued national income.*

Proof. It is obvious from, Lemmas 1 and 2. ■

Our trade model can attain gains from trade at the level of the overall economy, even if it consists of a large number of heterogeneous individuals who have multi-talents and who are allowed to change their occupations.[27]

We have demonstrated the equilibrium property of this model characterized by heterogeneous agents who face occupational choices. We also have shown that there exist aggregate production gains from trade in this economy. Now we must shift the focus to the welfare changes of various groups within the economy.

[27]Note that such a non-convex decision space for an individual agent is usually a problem, but it turns out OK for us.

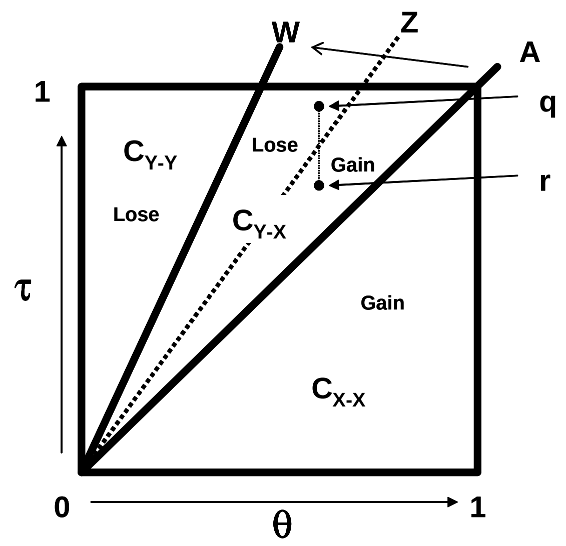
4 Welfare Changes of Individual Groups

Thus far in this paper, we have analyzed the equilibrium properties of the model, with the focus being on the comparative statics of the aggregated variables. Now we shift the focus, to the individuals within the economy. More specifically, we will be comparing the well-being of various groups (of the individuals) when there is a discrete change in output prices. The first result concerns the welfare property of the group of job-staying individuals.

Proposition 2 *Job-stayers will gain from an increase in the relative prices of their own outputs (those produced using applied talent). Job-stayers will lose from a decrease in the relative prices of their own outputs.*

These results are the same as the ones for specific-factor owners in the specific-factor model of international trade. In Fig. 8, the relative-price change from autarky to free trade – a change from p^A to p^W – is represented by a shift in the division-of-labor line from OA to OW. Partitions C_{X-X} and C_{Y-Y} each show a collection of job-staying individuals. Because the price-change is favorable to the exporting sector, the sector-X-stayers gain and the sector-Y-stayers lose. Formal proof is in the Appendix. Note that this proposition is exactly about the monotonicity of the reward values shown in Fig. 2. When the relative price p of X goes up, the reward from Y declines and the reward from X increases.

Figure 8: Individual gains and losses.



The second result concerns the well-being of job-switching individuals.

Proposition 3 *Among those who change their occupations, there exist both gainers and losers from trade without compensation. When there is a change in the relative prices, whether the job-switching individual wins or not depends on the ratio between the use of his applied and his latent talent.*

Contrary to popular belief, there are gainers among those who are "forced" to change their occupations. The sketch of the proof of this proposition goes as follows.

Proof. The proof is in three steps. First, let us show that there exist individuals who are indifferent between sector X and sector Y in autarky, i.e. – with respect to Fig. 8, this means those who are individuals right on the OA line. Under autarky, those individuals receive equal occupational returns from sector X and Y. Thus we can see, on the basis of Proposition 2, now even if they start from sector Y and switch to sector X, they will inevitably be winners from the price-change.

Second, let us show that there exist individuals who are indifferent between switching to sector X and staying in sector Y after free trade – i.e., the individuals on the OW line. Under free trade, those individuals must have equal occupational returns between sector X and Y. Therefore, regardless of whether they switched jobs or not, they are equally lost, as job-stayers in a time of trade liberalization. (This too is derived from the result in Proposition 2.)

Third, let us show that there exist individuals who are neither gainers nor losers from trade liberalization – i.e., the individuals on the OZ line. To do this, we must express the gain-loss as a function of τ/θ, the parameter of comparative advantage, and show that the function is continuous across the domain of the function. Then we can use the intermediate value theorem.

The gain-loss for a job-switcher can be expressed as $\Delta\pi(p^A, p^W, \tau^j, \theta^j) \equiv \pi^j_X(p^W, \theta^j) - \pi^j_Y(p^A, \tau^j)$, where

$$\pi^j_X(p^W, r, \theta^j) = \left[{p^W}^{\frac{1}{1-a}} \left(\frac{1}{r(p^W)}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right)\right] \cdot \theta^j = g(p^W) \cdot \theta^j$$

and

$$\pi^j_Y(p^A, r, \tau^j) = \left[{p^A}^{\frac{1}{1-a}} \left(\frac{1}{r(p^A)}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right)\right] \cdot \tau^j = g(p^A) \cdot \tau^j.$$

For the given values of K, p^A, p^W, the terms in the square brackets, which can be simplified by the functional notation $g(p)$, will be constant. Therefore, the gain-loss function can be written as $\Delta\pi = g(p^W)\cdot\theta^j - g(p^A)\cdot\tau^j$, where $g(p^A)$ and $g(p^W)$ are the corresponding constants given prices. As for the percentage-change of gain-loss, it will be

$$\%\Delta\pi\left(\frac{\theta^j}{\tau^j}\right) = \frac{\Delta\pi}{\pi^j_Y} = \frac{g(p^W)\cdot\theta^j - g(p^A)\cdot\tau^j}{g(p^A)\cdot\tau^j} = \frac{g(p^W)\cdot\theta^j}{g(p^A)\cdot\tau^j} - 1 = \frac{g(p^W)}{g(p^A)}\cdot\frac{\theta^j}{\tau^j} - 1. \tag{23}$$

Apparently, equation (23) is a continuous function of τ/θ. The value of the percentage change of gain-loss function $\%\Delta\pi\left(\theta^j/\tau^j\right)$ is positive when the value τ/θ equals the slope of the OA line, but negative when the value τ/θ equals the slope of the OW line. Since the function is continuous, we can be sure there exists a value τ/θ that will give $\%\Delta\pi = 0$. This value equals the slope OZ seen in Fig. 8. The size of gain or loss will be determined by the relative size of the actually used and the latent talents. ■

While the gains and losses for job-stayers have the same properties as those for specific-factor owners, the gains and losses for job-switchers depend on the relative size of their actually-used and unused-latent talents. Therefore we can state the following result, with respect to the limits on government policy.

Corollary 1 *When the government can observe only the current (not the past) profit, the calculation of gains* *and losses for **job-stayers** is an easy matter. The calculation of gains and losses among **job-switchers**, however, becomes formidably difficult.*

The gains or losses for the job-staying individuals can be easily calculated from Proposition 2. The difficulty of calculating the gains and losses among the job-switchers may be seen from Fig. 8. In Fig. 8, look at two points q and r. The individual q, as a producer for sector Y, has a higher ability level than does the individual r. And yet as producers for sector X, these two are equivalent. Still, the individual q belongs to the group of losers, while the individual r belongs to the group of gainers. While the government is able to observe the current profit of X, it is not able to tell the difference between q and r because their difference appears only with respect to their latent talents. Which of them will gain and which of them will lose will depend upon the relative strengths of their actually-used versus unused-latent talents. (For that matter, the iso-percentage-gain lines would be the rays from origin, while the iso-current-profit lines would be the verticals.)

What we have learned here is that the unobservability of latent talent makes it impossible for the government to distinguish gainers from losers. And it is this impossibility which will prove such a nuisance to any policymaker considering a Pareto-improving compensating redistribution scheme. We defer our discussion of such a creation of the compensation scheme, however, to section 5.

5 The Creation of Compensation Schemes

The results of the preceding analysis have shown us that there exist both gainers and losers among those who switch their occupations. Now that we have looked at the effect of a terms-of-trade change without compensation, let us turn our attention to a government redistribution policy that aims at both Pareto improvement (from opening up to trade) and a balanced budget (in other words, the avoidance of overcompensation).

Now that we are looking at the creation of a compensation scheme by the light of the informational structure of our model, we must begin by comparing the two situations: autarky (prohibitive tariffs) and free trade. The ex post situation should not necessary be the one of free trade. It can be the one of some restricted trade, but for the sake of simplicity we will focus on the autarky-versus-free-trade comparison.[28] The initial equilibrium is the one in autarky. The uncompensated free-trade equilibrium was analyzed in sections 3 and 4. When the policymaker enacts a compensation scheme, the free-trade equilibrium becomes a compensated free-trade equilibrium.

In choosing the instruments of our compensation scheme, let us follow the trend in the literature of avoiding the use of lump-sum compensation, owing to its formidable information requirement.[29] Therefore, we will examine a compensation scheme which is based on factor taxes and commodity taxes (Atkinson and Stiglitz 1980, p.20).[30] Let us now formally define the compensation scheme.

[28] For the same reason, the ex ante situation could be one of some restricted trade.

[29] See for example Feenstra and Lewis (1994, p.202).

[30] Of course, the negative taxes are the same as the subsidies. This notions of factor taxes and commodity taxes has been adopted from the standard public economics textbook of Atkinson and Stiglitz.

Definition 1 *The* ***compensation scheme*** *σ is a combination of taxes and subsidies levied on the following variables: (1) output prices, (2) generic-factor prices, and (3) occupational rewards. Tax-subsidy rates can either be linear or non-linear.*

The taxes (or subsidies) on output prices are commodity taxes, and the taxes on both generic-factor prices and occupational rewards are factor taxes. (In Dixit and Norman, "commodity taxes" embraces both commodity and factor taxes, simply because they use a general approach that does not distinguish outputs from inputs.) Following in the footsteps of Dixit and Norman (1986) and Feenstra and Lewis (1994), I too adopt a two-stage compensation procedure. Because both Dixit-Norman and Feenstra-Lewis aim to implement Pareto-improving compensation schemes, the first stage of their schemes focuses on making everyone in the economy as happy as they would be under autarky. To arrive at this end, a policymaker must utilize both commodity taxes and factor taxes – adding to these, in the case of Feenstra-Lewis, and relocation subsidies. Both Dixit-Norman and Feenstra-Lewis proved that not only will the government revenues from such first-stage schemes become non-negative, but they will be redistributed back to individuals in the economy during the second stage.

Definition 2 *The* ***compensation scheme*** *σ can be implemented in two stages: (1) In the* ***first stage****, the government tries to minimize the rents that accrue to individual agents; in other words, it seeks to capture all these rents in the form of positive revenue. Let us call this stage's result a $\sigma 1$ equilibrium – and (2) In the* ***second stage****, the government sends this positive revenue back to the individual agents by means of either a poll subsidy or a reduction of some commodity taxation. Let us call the result of this second stage a $\sigma 2$ equilibrium. This $\sigma 2$ equilibrium can also be called a σ equilibrium, since the result of the second stage is also the final result of the whole compensation scheme.*

The purpose of the first stage is to ensure Pareto gains from trade by setting as close as possible to an equilibrium in which all the individual agents in the economy are as well off as they are in autarky. The first stage may leave the government non-negative revenue (or strictly positive, if there exist strict production gains from trade). The second stage tries to distribute back to individual agents the non-negative government surplus from the first-stage equilibrium. This can be done either by poll subsidy or by lowering consumption taxes (raising factor subsidies). Since the technical requirements for the second-stage redistribution – notable among these being the Weymark conditions – are closely examined in the work by Dixit and Norman (1986), I take these results as given and will not be discussing them in this paper. Our primary focus of analysis will be on the first-stage equilibrium.

At this juncture I also would like to introduce several desirable and undesirable properties of the compensation scheme. Its single most important property is related to the concept of ex post Pareto efficiency.

Definition 3 *The compensation scheme σ is said to be* ***weakly Pareto improving*** *if every individual is at least as well off as he or she was under the autarky situation.*

Formally, the requirement for the weak Pareto improvement is written as a comparison of the welfare measure W of the individuals:

$$(W^j)^\sigma \geq (W^j)^A, \forall j \in J, \tag{24}$$

where the superscript σ means the individual's welfare "in the situation given the compensation scheme σ," and A means the individual welfare "in the autarky situation." On both sides, W, a welfare measure, indicates the real income of each individual in either situation, for in our model real income represents the value of individual's indirect utility function. [See equation (12).]

Another important property of the first-stage equilibrium is its rent neutrality. A positive rent from a particular policy or environment change is defined as an increase in the individual's welfare from such change. It is a premium or windfall profit, in the nature of Marshallian rents. As a concrete example, if an inequality

$$(W^j)^\sigma > (W^j)^A \tag{25}$$

holds true for some agent j, then we can see that this agent j has a strictly positive rent of the value $(W^j)^\sigma - (W^j)^A$, given the policy-shift from autarky to free trade under the compensation scheme σ. One of the reasons the previous literature has adopted a two-stage compensation procedure is the typical economist's love of discussing efficiency without getting into the discussion of equity issues. And indeed, we all would like to keep any economic policy rent-neutral. In other words, we certainly don't want to see an arbitrary redistribution of wealth arising out of a policy that has tried to target a different objective – in this case, the policymaker's objective of ensuring a Pareto improvement by opening his nation up to trade.

We cannot say much about the second-stage redistribution of positive government revenues in this paper. We must simply content ourselves with asserting, once again, that rent-neutrality is a desirable property of any first-stage compensation equilibrium, as evidenced by the fact that both Dixit-Norman and Feenstra-Lewis did attain rent-neutrality in their respective first-stage equilibria. Let us now codify our definition of risk-neutrality.

Definition 4 *The first-stage compensation equilibrium $\sigma 1$ is said to be* ***rent-neutral*** *if every consumer is left at exactly the same utility level as he or she was under autarky. In other words, all the positive rents shall accrue as government revenues.*

We know that the original Dixit-Norman scheme's first-stage equilibrium is rent-neutral, because all the consumers face exactly the same situation as they did in autarky in the first stage. In Dixit and Norman this scenario is arrived at by setting both the output and input prices equal to the prices of the autarky. Fixing input prices at the autarky level guarantees autarky-level incomes for the consumers. If we were to fix our output prices at the autarky level, then the consumers would be in exactly the same utility-maximizing situation as under autarky, given that income and output prices are the only parameters of the consumer's program. The same observation holds true for the Feenstra-Lewis scheme. The only difference is that, in their paper, the relocation subsidies are given to some of the consumers to compensate them exactly for that loss of income that arose out of the positive adjustment costs associated with their movement of factors from one industry to another. Under the assumptions of Feenstra and Lewis (1994), the government can pick a minimum amount of relocation subsidy such that some of the consumers are indifferent between moving and not moving to a new industry. Hence, the first-stage equilibrium in the Feenstra-Lewis scheme is also rent-neutral.

As we shall later see in greater detail, the government in this paper's model is unable to create the rent-neutral first-stage equilibrium. In order to achieve Pareto improvement from autarky, it is necessary for the government to give positive rents to some groups of individual agents. For our present purpose, we will be calling this undesirable property overcompensation.

Definition 5 *A scheme is said to **overcompensate** a group of individuals if some within that group are getting positive rents in the first-stage compensation equilibrium $\sigma 1$.*

Note that the definitions we have arrived at of overcompensation and rent-neutrality are two sides of the same coin. When the scheme is rent-neutral, it is not overcompensating any group of consumers; and by reverse token, when the scheme is overcompensating some group, it cannot be rent-neutral. We can, however, specifically identify the group for which positive rents are accruing, in accordance with our definition of overcompensation.

The other important property of the compensation scheme concerns the budget of the government.

Definition 6 *The compensation scheme σ is said to be **self-financing** if it leaves non-negative government revenue in the first-stage equilibrium $\sigma 1$:*

$$B^{\sigma 1} \geq 0, \tag{26}$$

where $B^{\sigma 1}$ is a net government balance from only the first-stage equilibrium of the scheme; i.e., the revenue from taxes minus the cost of subsidies.

This definition of a self-financing scheme has been adopted from the definition of self-financing tariffs that was introduced by Ohyama (1972, p.49). A compensation scheme containing taxes and subsidies on various economic variables is said to be self-financing if the government is able to balance the budget strictly from the net revenue earned within the scheme. The reason equation (26) does not have to contain a strict equal sign is that any positive revenue can be distributed back to the individuals in the second stage.

The procedure of implementing a compensation scheme that we will be considering here is similar to the ones considered in Dixit and Norman (1986) and Feenstra and Lewis (1994). It boils down to these two aspects. (1) A system of subsidy and taxation that leaves every consumer of the economy in the same situation as autarky, and this policy may accrue positive revenues for the government. (2) If there are some positive revenues, the government will redistribute these back to the individuals. It will do the latter via either a poll subsidy for everyone or an adjustment of the tax or subsidy. The latter is possible because, in the trading economy presumed here, the Weymark condition[31] of Dixit and Norman (1986) is automatically satisfied.

[31] The Weymark condition tells us that there exists one good for which some consumers are net buyers and none is a net seller. In the traditional trade model, in which consumers are net sellers of factors of production and net buyers of consumer output goods, the condition will automatically be satisfied.

When we discuss the compensation scheme, our focus will be on the first step of creating a system of subsidy and taxation that aims to leave all the consumers at least as well off as they were under the autarky. As for the actual implementation of the second step, this is already fully discussed in the literature.

Another important property of any compensation scheme is its feasibility. Despite the fact that much of the literature discusses the concept of "feasibility" in terms of non-negativity of governmental budgets (self-financing), this paper separates the governmental budget issues (discussed above) from the issues associated with the feasibility of a compensation scheme. In this paper, feasibility occurs when the policy instruments of the government are based on the observable variables.

Definition 7 *A scheme σ is said to be **informationally feasible** if it is based solely on the currently observable variables.*

This definition of informational feasibility is based on the observability of the variables by the government. But what are the observable variables? And which characteristics of the individuals are observable to the policymakers? I propose the following realistic, three-step assumption about observability: (1) The government keeps track of aggregate variables in record. (2) Therefore, it remembers the sizes of aggregate variables in the autarky situation. (3) The individual data can be observed at no cost only in the current situation.

This assumption makes sense, because while most aggregate data is available in various forms, it is very difficult to go back and find a past data-point that is specific to an individual. For example, the bulk of the income tax rate will be determined by the current year's income, and yet the tax rate does not usually depend upon the accumulation of multi-year income, including previous years' incomes.[32] Thus, individual data in the autarky period are presumed to be costly to verify, in the free-trade period.

Let us suppose that the government can observe the following variables:

Y1 Output prices P_X, P_Y (both at the autarky and the free-trade levels)

Y2 Generic-factor prices r (both at the autarky and the free-trade levels)

Y3 Residual return (profit) from the individual's current (free-trade) occupation

Also, we shall suppose that the government is able to observe these two characteristics of individuals:

Y4 Which industry the individual is currently working in.

Y5 Whether the individual has changed his or her occupation.

Let us further suppose that the government cannot observe the followings variables:

N1 Individual consumption vector

N2 Individual generic-factor endowment

[32] This is indeed the lack of cumulative-profit-tax system of which Columbia's late William Vickrey had been a proponent ever since the 1940s.

N3 Individual occupational-ability vector

N4 Residual return (profit) from the individual's previous (autarky) occupation

Most of the above assumptions about observability are standard in the literature. [See for example Guesnerie (1995).]

Given the assumption about observability of profit, the following result will be utilized in the ensuing analysis.

Result 1 *Given the production setup of the model, and given that the government can observe the residual profits of individuals, the profit tax will not distort their behaviors. In other words, the individuals will maximize their profits truthfully, given that the elasticity of the after-tax (subsidy) share, with respect to the profit, is larger than* -1*. Formally, they will do so whenever*

$$\varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1, \tag{27}$$

where $T(\pi) = 1 - t(\pi)$*, where* π *is the residual profit, and where* $t(\pi)$ *is an ad valorem tax rate (or if* $t(\pi)$ *is negative, a subsidy rate).*

Please see the Appendix for the proof. Note also that the linear tax has an elasticity of $\varepsilon = 0$, and thus satisfies condition (27). Also, given that the individual agents are assumed to be acting truthfully, we can conclude that their current use of their talents is revealing.

Remark 1 *Given the previous observation in Result 1 as to the truthfully maximized current levels of individuals' residual returns, the government can recalculate the size of* θ *for* X*-producers, and of* τ *for* Y*-producers. The planner can infer the size of the actual use of talent, as opposed to an agent's endowment of latent talent.*

This is straightforward. If policymakers can condition their policy on the current profit, then either

$$\left\{ \begin{array}{lcl} \pi_X^j(P_X, r, \theta^j) & = & \left[(P_X)^{\frac{1}{1-a}} \left(\frac{1}{r}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right) \right] \cdot \theta^j, \text{ or} \\ \pi_Y^j(P_Y, r, \tau^j) & = & \left[(P_Y)^{\frac{1}{1-a}} \left(\frac{1}{r}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}} \right) \right] \cdot \tau^j. \end{array} \right.$$

given the observability of such aggregate variables as the output prices P_X, P_Y and the generic-factor-return r, the inversion of profit to type is a simple calculation. One might also say that the profit is a strictly increasing function of the size of the type, in which case any tax-subsidy rate that is proportional to the observed profit could be used, almost as if the government were observing the type itself.

Now that we have defined all the necessary properties of the compensation scheme and looked at all the relevant results, we can proceed to examine the results of the possible compensation schemes. In order to do so we will investigate two distinctive cases with respect to the timing of implementation. In the first case, called *an unanticipated compensation scheme*, the trade openings are implemented prior to the announcement that the government will compensate the losers from trade. In the second case, called *an anticipated compensation scheme*, all the individual agents expect the compensation scheme to be provided later by the government, after the economy opened up its borders. In the following section we begin to look at the first such case.

6 An Unanticipated Compensation Scheme

Despite the tradition stipulating that a regular lump-sum compensation must be given prior to opening up to trade [or opening the market] (Mas-Colell et al. 1995, p.328), a more plausible and realistic policy option must include a "*post*-trade compensation scheme" (Kemp and Wan 1986, p.99) whereby the government first opens the border, then creates the compensation scheme in order to assist the losers from trade. In fact, I claim that this sort of unanticipated compensation scheme is pretty much what we saw occurring back in the 1960s. For in response to the Kennedy round of GATT multilateral tariff reductions, the United States government introduced the first TAA (trade adjustment assistance) program, in order to accommodate the high number of workers displaced by the tariff reduction.

In this section we explore a possible unanticipated post-trade compensation policy, given the informational restriction on the economy that we have posited in this paper. I will postpone to the next section both an examination of the case in which the individuals anticipate the existence of the compensation scheme, and an analysis of the way this anticipation alters individual incentives.

Whenever one goes in search of the optimal compensating redistributing scheme, the most important criterion to be kept in mind is Pareto improvement from autarky. At the same time, in pursuing the creation of such a scheme, the policymaker must always be aware of the informational feasibility constraint, given the limited observability of the unused talents of individual agents. When the scheme comprises two stages, the policymaker tries to accrue all the rents in the form of governmental revenues in the first stage. Thus the ideal first-stage equilibrium is rent-neutral. Owing to the informational feasibility constraint, however, this chapter's model does not posit any achievement of rent-neutrality in the first-stage equilibrium. That said, let us begin to explore the process of creating a compensating scheme.

For analytic convenience, we focus on the case in which the price-change occurs in one direction (the other case being completely symmetric). More specifically, this is the case in which the post-trade price is $p > p^A$, and therefore there are job-switchers from sector Y to sector X. Given the setup of our model, as described back in Section 3, we are cognizant of the following five cases (Case I. - Case V.) with respect to the gains and losses for different groups of individuals:

Case I. Generic-factor owners are all gainers, since $r(p) > r(p^A)$. More particularly, the amount of gain for those who own K^j is given by

$$\left(r(p) - r(p^A)\right) \cdot K^j = a \cdot K^{-(1-a)} \cdot \left\{\left[s(p)\right]^{1-a} - \left[s(p^A)\right]^{1-a}\right\} \cdot K^j > 0, \tag{28}$$

where

$$s(p) = p^{\frac{1}{1-a}} \cdot V_\theta^R(p) + p^{-\frac{1}{1-a}} \cdot V_\tau^S(p). \tag{29}$$

Note that this group's amount of gain from trade is proportional to the agent's endowment of generic factor K^j. The multiplier part,

$$a \cdot K^{-(1-a)} \cdot \left\{\left[s(p)\right]^{1-a} - \left[s(p^A)\right]^{1-a}\right\},$$

is invariable across all agents. Both a and K are the parameters of the model. Given the relative price change $p^A \Longrightarrow p$, the values for both $s(p^A)$ and $s(p)$ are determined in the aggregate equilibrium. Because the policymaker knows the joint distribution of the talent vector (θ, τ), he also knows the values of $V_\theta^R(p)$ and $V_\tau^S(p)$ and hence of $s(p)$ and $s(p^A)$. Thus, by imposing on the market for generic factors an ad valorem tax rate of

$$t_{r(p)} = \frac{[s(p)]^{1-a} - \left[s(p^A)\right]^{1-a}}{[s(p)]^{1-a}}, \tag{30}$$

the policymaker can make the status of all the owners of generic factors the same as it was under autarky in the first-stage equilibrium.

Case II. The job-staying individuals in sector X – those who are in the area $\tau < (p^A)^{\frac{2}{1-a}} \theta$ – are all gainers, since $\pi^j_{X1}(p) > \pi^j_{X0}(p^A)$ when $p > p^A$. More particularly, the amount of gain for those who have talent θ^j is given by

$$\pi^j_{X1}(p) - \pi^j_{X0}(p^A) = K^a(1-a) \cdot \left(p^{\frac{1}{1-a}} [s(p)]^{-a} - p^{A^{\frac{1}{1-a}}} \left[s(p^A)\right]^{-a} \right) \cdot \theta^j > 0, \tag{31}$$

where the definition of $s(p)$ is the same as it was in equation (29). Much the same as in Case I, the amount of gain from trade, for the group of job-staying individuals in sector X, is proportional to the agent's endowment of used talent θ^j. The multiplier part,

$$K^a(1-a) \cdot \left(p^{\frac{1}{1-a}} [s(p)]^{-a} - p^{A^{\frac{1}{1-a}}} \left[s(p^A)\right]^{-a} \right),$$

is invariable across all of these agents. Thus, by imposing upon the return-from-talent of job-stayers of sector X an ad valorem tax rate of

$$t_{\pi X} = \frac{p^{\frac{1}{1-a}} [s(p)]^{-a} - p^{A^{\frac{1}{1-a}}} \left[s(p^A)\right]^{-a}}{p^{\frac{1}{1-a}} [s(p)]^{-a}}, \tag{32}$$

the policymaker can make the status of these individuals the same as it was under autarky in the first-stage equilibrium.

Case III. Among the job-switching individuals, a part of them – all those who are in the area $(p^A)^{\frac{2}{1-a}} \theta^j < \tau^j < \frac{g(p^W)}{g(p^A)} \cdot \theta^j$ – are gainers, since $\pi^j_{X1}(p) > \pi^j_{Y0}(p^A)$ when $p > p^A$. More particularly, the amount of gain for those who have the talent-vector (θ^j, τ^j) is given by

$$\pi^j_{X1}(p) - \pi^j_{Y0}(p^A) = g(p^W) \cdot \theta^j - g(p^A) \cdot \tau^j > 0, \tag{33}$$

where

$$g(p^W) = p^{\frac{1}{1-a}} \left(\frac{1}{r(p)}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right)$$

and where

$$g(p^A) = p^{A^{\frac{1}{1-a}}} \left(\frac{1}{r(p^A)}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right).$$

Contrary to Cases I and II, the amount of gain for the job-switching individuals is no longer proportional to their endowments of used talent θ^j. It is true that both $g(p^W)$ and $g(p^A)$ are invariable across all these individuals, and that the policymaker can calculate the values for $g(p^W)$ and $g(p^A)$, but the amount of gain, $g(p^W) \cdot \theta^j - g(p^A) \cdot \tau^j$, depends upon both elements of the talent-vector (θ^j, τ^j), which itself is unobservable to the policymaker. Of course the policymaker could always recalculate the value of used talent θ^j based on his observations of the profits that have accrued from production of X. The value of τ^j, however, is unknown to the policymaker. To help us all see this in a more concrete manner, let us now suppose that the policymaker would like to impose an ad valorem tax rate of

$$t_{\pi X-Y} = \frac{g(p^W) \cdot \theta^j - g(p^A) \cdot \tau^j}{g(p^W) \cdot \theta^j} = 1 - \frac{g(p^A) \cdot \tau^j}{g(p^W) \cdot \theta^j}, \tag{34}$$

in order to make all of these Case III individuals as happy as they were back in the autarky. The actual tax rate that the policymaker can impose, however, should be in the form of $t_{\pi X-Y}(\pi_X(\theta))$, meaning that it should be based only on the currently observable $\pi_X(\theta)$, which will in turn depend upon the current use of talent θ.

Case IV. The other part of the job-switching individuals – who are in the area $\frac{g(p^W)}{g(p^A)} \cdot \theta^j < \tau^j < p^{\frac{2}{1-a}} \theta^j$ – are all losers since $\pi^j_{X1}(p) < \pi^j_{Y0}(p^A)$ when $p > p^A$. More particularly, the amount of loss for those who have talent (θ^j, τ^j) is given by

$$-\left(\pi^j_{X1}(p) - \pi^j_{Y0}(p^A)\right) = g(p^A) \cdot \tau^j - g(p^W) \cdot \theta^j > 0. \tag{35}$$

This case is quite similar to Case III, when it comes to both the amount of loss for each individual and the subsidy rate. The infeasible subsidy rate that the policymaker would like to impose on this group is

$$s_{\pi X-Y} = \frac{g(p^A) \cdot \tau^j - g(p^W) \cdot \theta^j}{g(p^W) \cdot \theta^j} = \frac{g(p^A) \cdot \tau^j}{g(p^W) \cdot \theta^j} - 1, \tag{36}$$

whereas the feasible subsidy rate must of course be in the form of $s_{\pi X-Y}(\pi_X(\theta))$.

Case V. The job-staying individuals in sector Y – those who are in the area $p^{\frac{2}{1-a}} \theta < \tau$ – are all losers, since $\pi^j_{Y1}(p) < \pi^j_{Y0}(p^A)$ when $p > p^A$. More particularly, the amount of loss for those who have talent τ^j is given by

$$-\left(\pi^j_{Y1}(p) - \pi^j_{Y0}(p^A)\right) = K^a(1-a) \cdot \left({p^A}^{\frac{-1}{1-a}} \left[s(p^A)\right]^{-a} - p^{\frac{-1}{1-a}} \left[s(p)\right]^{-a}\right) \cdot \tau^j > 0. \tag{37}$$

Similarly to Cases I and II, the amount of gain from trade for sector Y's job-staying individuals is proportional to their endowments of used talent τ^j. The multiplier part,

$$K^a(1-a) \cdot \left({p^A}^{\frac{-1}{1-a}} \left[s(p^A)\right]^{-a} - p^{\frac{-1}{1-a}} \left[s(p)\right]^{-a}\right),$$

is invariable across all of these agents. Thus, by imposing on the return-from-talent of the sector Y's job-stayers an ad valorem subsidy rate of

$$s_{\pi Y} = \frac{{p^A}^{\frac{-1}{1-a}} \left[s(p^A)\right]^{-a} - p^{\frac{-1}{1-a}} \left[s(p)\right]^{-a}}{p^{\frac{-1}{1-a}} \left[s(p)\right]^{-a}}, \tag{38}$$

the policymaker can make the status of all the job-staying individuals in sector Y the same as it was under autarky in the first-stage equilibrium.

It is always instructive to look at a first-best case, even if in reality it is impossible to implement such a scheme. Thus let us now posit the following first-best scheme:

Scheme 1 *As a first-stage equilibrium, tax the winning groups (Cases I,II, and III) and subsidize the losing groups (Cases IV and V) in amounts equal to their gains and losses, so that every individual is in the same situation as he or she was back in autarky. Such tax and subsidy rates have been well expressed by our equations (30), (32), (34), (36), and (38).*

If we could implement this fictitious first-best case, we would have a rent-neutral scheme. But while the taxation and subsidy schemes for Cases I,II, and V are feasible, the determination of the tax and subsidy rates for the job-switchers, Cases III and IV, must be based on a combination of observable and unobservable variables. The government cannot distinguish between the Cases III and IV groups because it cannot observe the relative size of (θ^j, τ^j) for each individual. The policymaker can observe only the profit that is accruing from current production, and thus can observe, in this case of $p > p^A$, only the profit from sector-X-production. The policymaker cannot observe (or condition his taxation scheme on) the counter-factual profit from sector Y that is proportional to the agent's unused latent talent τ. In terms of Fig. 8, for instance, this means that there is no way for the government to distinguish the points q and r, because in the equilibrium the individuals at both q and r earn the same profit and produce the same amount of product X. All of which leads us to the following result.

Proposition 4 *Given the setup of the model in this chapter, if the government is aiming to achieve a* ***Pareto improvement*** *from autarky, there is no* ***informationally feasible*** *first-stage compensated equilibrium that is* ***rent-neutral****.*

By consulting our equations (28), (31), and (37), which depict the gains and losses for the various groups of individuals, we are able to establish the taxation and subsidy rates for, and to make as happy as they were back in autarky, these three groups of individuals: (a) generic-factor-K owners at the rate (30); (b) sector-X job-stayers at the rate (32); and (c) sector-Y job-stayers at the rate (38). We can do this because these individuals' gains and losses are proportional to their factor-returns (both their residual-profits and generic-factor returns), and thus also proportional to the sizes of their actually employed talents (or factor endowments). In this case, all we need to do is simply setup a linear tax or subsidy system. (We recall, from Result 1 in section 5, that any linear tax-subsidy system is incentive compatible.)

As we shift our focus now to the job-switching individuals, we find that things are not so easy. Look at equations (33) and (35), showing that the amount of an individual's gain or loss depends on the relative size of his actually used talent θ and his unused latent talent τ. Because the policymaker does not have access to each individual's data – history of profits and losses – he can only base the taxation-subsidy scheme on the currently observable variables. In this case, the current profit from sector-X production is observable. In effect, the policymaker can observe θ, but not τ. (The policymaker observes the profits of the individual agents. If a profit is reported truthfully, the policymaker can recalculate the size of the used talent. See

Remark 1 in section 5.) Thus, the policymaker cannot make all the job-switching individuals exactly as happy as they were under autarky, with the exception of one border case that we will be looking at shortly. Given all of this, we conclude the following.

Proposition 5 *Given the setup of the model in this chapter, if the government is aiming to achieve a* ***Pareto improvement*** *from autarky, the* ***informationally feasible*** *sort of post-trade compensation policy must* ***overcompensate*** *the group of job-switching individuals in its first-stage equilibrium.*

If the policymaker's most pressing concern is to ensure a Pareto improvement over the autarky, then the informationally feasible scheme must overcompensate the job-switching individuals. The preceding points have taught us that the policymaker can tax and subsidize job-stayers in the rent-neutral manner, but cannot do so for the job-switchers simply because in their case he can observe only θ, not τ.

Let us go back for a moment to Fig. 7, in which we posit the unit-square support for the joint distribution of talents. The left-hand side of the figure contains the lines that represent a same percentage-change of gain or loss from trade. The right-hand side contains the lines indicating that those individuals are making the same amount of residual profit. The iso-percentage gain-loss lines are the rays from origin, and the iso-current profit lines for X producers are the parallel vertical lines.

While this first-best first-stage scheme requires that there be a linear taxation-subsidy system imposed along the iso-percentage gain-loss lines, the policymaker can observe only the differences among individuals along the iso-current profit lines. This is because the job-switching individuals appear to be the same when they are earning the same amount of profit, and hence show up on the same iso-current profit line.

Among those who are earning the same profit, it is the individual on the upper bound of the iso-current profit line who has gained the smallest (lost the largest) amount from trade. Since the policymaker cannot distinguish among the individuals on the same iso-profit line, he must compensate all the individuals on the same profit line at the same level as the least lucky individual who is on the upper bound of that line. And yet, apart from that least happy individual exactly on the upper bound, those who received the same amounts of compensation dispensed by the policymaker must carry positive rents, since their iso-percentage gain-loss lines are higher than that of the upper-bound individual.

Looking again at the two points q and r in Fig. 7, we see that they are on the same iso current-profit line. Thus they appear to be the same from the policymaker's view point, and yet one of them, q, is a loser while the other, r, is a gainer. Still, the amount of compensation must be the same for both points q and r. Even if the individual at r is in fact a gainer, he must be receiving the same amount of subsidy (as oppose to paying any tax) as the individual at point q. The point again being that the government which aims for a Pareto improvement will unavoidably overcompensate the job-switching individuals.

To help us to see this in a more concrete manner, let us define the iso current-profit set $I^{CP}(\theta^*)$.

Definition 8 *The iso current-profit set $I^{CP}(\theta^*)$ is the set of all those job-switching individuals who have the same size of talent θ^*:*

$$I^{CP}(\theta^*) \equiv \left\{(\theta^j, \tau^j) \in C_{Y-X} : \theta^j = \theta^*\right\},$$

where C_{Y-X} is a partition of job-switchers; i.e.,

$$C_{Y-X} \equiv \left\{ (\theta^j, \tau^j) \in \Theta : \; (p^A)^{\frac{2}{1-a}} \theta < \tau^j < p^{\frac{2}{1-a}} \theta \right\}.$$

Note that $I^{CP}(\theta^*)$ is a linear, one-dimensional subspace of $\mathbb{R}^2$. Let $\underline{\tau(\theta^*)}$ be the lower bound for the value of the element τ in a set $I^{CP}(\theta^*)$, and let $\overline{\tau(\theta^*)}$ be the upper bound for the same subspace. Note that $\underline{\tau(\theta^*)}$ is always equal to $(p^A)^{\frac{2}{1-a}}\theta^*$, whereas $\overline{\tau(\theta^*)}$ depends on the size of θ^*. In particular,

$$\overline{\tau(\theta^*)} = \sup \left\{ p^{\frac{2}{1-a}} \theta^*, \overline{\Theta^\tau(\theta^*)} \right\},$$

where $\overline{\Theta^\tau(\theta^*)}$ is a upper bound for the element τ in the whole Θ space when $\theta^j = \theta^*$. In the case of a unit-square support for the joint distribution, $\overline{\Theta^\tau(\theta^*)} = 1$.

Because all of the individuals in the set $I^{CP}(\theta^*)$ are the job-switchers from sector Y to sector X, they are currently producing output X. And since all the members of the set $I^{CP}(\theta^*)$ have the same size of talent θ^*, their profit will be the same: $\pi_X^j(p, r(p), \theta^*)$. Their individual gains or losses, however, will be different because they have different sizes of the latent talent τ. By working out of (33) and (35), we find that the amount of individual gains or losses can be expressed as $\left|g(p^W)\cdot\theta^* - g(p^A)\cdot\tau^j\right|$. Whether the individual j (who has the talent θ^*) gains or loses, and how much he gains or loses, will depend upon the size of τ^j. But among those who belong to the set $I^{CP}(\theta^*)$ there are all spectra of the individuals who have the latent talent τ in the interval $\left[\underline{\tau(\theta^*)}, \overline{\tau(\theta^*)}\right]$. The policymaker, however, cannot distinguish among them.

If the policymaker would like to ensure Pareto gains from trade, he must be sure he makes the least happy individual as happy as he was back in the autarky. Note also that this least happy individual must have had the largest talent in the previous sector Y, and hence have been the one with the largest latent talent $\overline{\tau(\theta^*)}$. Therefore, for all individuals $(\theta^*, \tau) \in I^{CP}(\theta^*)$, the amount of subsidy or tax must be $\left|g(p^W)\cdot\theta^* - g(p^A)\cdot\overline{\tau(\theta^*)}\right|$. The ad valorem rate for any individual having the profit $\pi(\theta^*)$ would then be

$$t_{\pi X-Y}(\pi(\theta^*)) = \left| \frac{g(p^W)\cdot\theta^* - g(p^A)\cdot\overline{\tau(\theta^*)}}{g(p^W)\cdot\theta^*} \right|. \tag{39}$$

If $g(p^W)\cdot\theta^* - g(p^A)\cdot\overline{\tau(\theta^*)} > 0$, equation (39) represents a tax rate. If $g(p^W)\cdot\theta^* - g(p^A)\cdot\overline{\tau(\theta^*)} < 0$, it represents a subsidy rate. With the exception of the individual at the point $(\theta^*, \overline{\tau(\theta^*)})$, which is measure zero, all of the individuals in the set $I^{CP}(\theta^*)$ are going to overcompensated, since the inequality

$$g(p^W)\cdot\theta^* - g(p^A)\cdot\overline{\tau(\theta^*)} < g(p^W)\cdot\theta^* - g(p^A)\cdot\tau \tag{40}$$

must hold for all those having the latent talent $\tau \in \left[\underline{\tau(\theta^*)}, \overline{\tau(\theta^*)}\right)$.

From (40), we can see that

$$\int_{\underline{\tau(\theta^*)}}^{\overline{\tau(\theta^*)}} \left\{ g(p^W)\cdot\theta^* - g(p^A)\cdot\overline{\tau(\theta^*)} \right\} f(\theta^*,\tau)d\tau < \int_{\underline{\tau(\theta^*)}}^{\overline{\tau(\theta^*)}} \left\{ g(p^W)\cdot\theta^* - g(p^A)\cdot\tau \right\} f(\theta^*,\tau)d\tau.$$

Then we also can integrate over all the job-switching individuals, thus,

$$\int_{C_{Y-X}} \int_{\underline{\tau(\theta^*)}}^{\overline{\tau(\theta^*)}} \left\{ g(p^W)\cdot\theta^* - g(p^A)\cdot\overline{\tau(\theta^*)} \right\} f(\theta,\tau)d\tau d\theta^* < \int_{C_{Y-X}} \int_{\underline{\tau(\theta^*)}}^{\overline{\tau(\theta^*)}} \left\{ g(p^W)\cdot\theta^* - g(p^A)\cdot\tau \right\} f(\theta,\tau)d\tau d\theta^*, \tag{41}$$

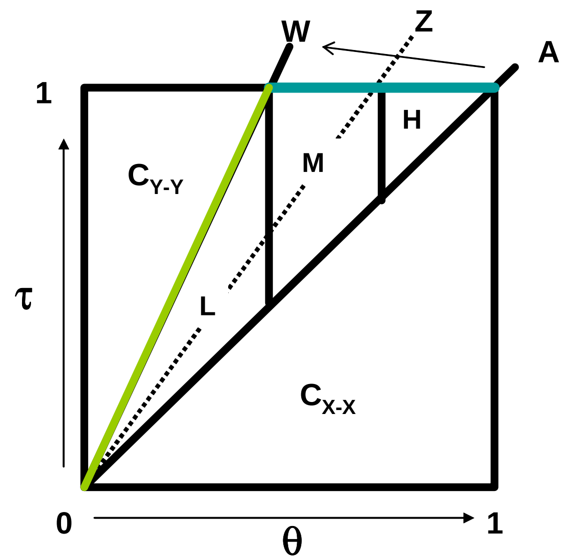
with the integration over θ^* being done for all the job-switching individuals. The difference between the right- and left-hand sides of the inequality (41) is the total amount of overcompensation for the job-switching individuals.

Given the preceding overcompensation results, we can go on to state the following proposition.

Proposition 6 *An **informationally feasible** post-trade compensation policy that achieves **weak Pareto improvement** may or may not be **self-financing**, depending upon the joint distribution of the individuals' talents.*

According to Ohyama (1972), a Pareto-improving compensation scheme will be self-financing so long as the aggregate consumption possibilities set is larger than the one under autarky, if we allow for a lump-sum transfer. In this model, however, when we impose the informational feasibility condition, a compensation scheme without a lump-sum transfer may or may not be self-financing. This is because overcompensating the job-switching individuals may absorb the positive aggregate rents the economy has seen owing to an opening up to trade. Whether the amount of overcompensation is large will depend upon the shape of the joint distribution of talents. In particular, if the total mass of job-switching individuals is large, then the total amount of overcompensation will be large as well. We can then find parameter values such that the total compensation scheme will not be self-financing.

Figure 9: The informationally feasible post-trade compensation scheme.



Let us now look at an example where the support of joint distribution is a unit square. Figure 9 illustrates the scheme for this case. For this unit-square case, we introduce a different, finer separation of the partition C_{Y-X} into two groups: a group of absolute gainers and a group combining gainers and losers – based only on the observable variables. To help us make this matter more concrete, consider the following:

(i) Generic-factor owners; same as Case I.

(ii) All of the individuals in partition C_{XX}; same as Case II.

(iii) Those individuals in partition C_{YX} who meet the condition $\theta > \frac{g(p^A)}{g(p^W)}$.

(iv) Those individuals in partition C_{YX} who meet the condition $\theta < \frac{g(p^A)}{g(p^W)}$.

(v) All of the individuals in partition C_{YY}; same as Case V.

Note that in Fig. 9, the dotted line OZ stands for the gain-zero line: $\theta = \frac{g(p^A)}{g(p^W)} \cdot \tau$. This categorization uses only the observable variables, because the distinction between the partition (iii) and the partition (iv) is based solely on θ, which can be recalculated by looking at the current profits of the individuals. Given this new categorization, let us propose a revised post-trade compensation scheme.

Scheme 2 *As a first-stage equilibrium, tax (i), (ii), (iii) and subsidize (iv) and (v). Note in particular that the tax and subsidy rates are as expressed in the equations: (30) for (i), (32) for (ii), (39) for groups (iii) and (iv), and (38) for (v).*

This scheme is all done with the observable variables. Thus, it is feasible. And yet it is only a second-best, because the groups (iii) and (iv) bring us into overcompensation. This is inevitable, given that we have no way to distinguish among the gainers and losers in this category.

In order to find the appropriate tax-subsidy rate, let us seek both the minimum subsidy rate and the maximum tax rate for each group that satisfies the weak-Pareto-improvement requirement shown in (24). Because the model in this paper uses the price normalization that assures us that the nominal income is equal to the real income, we can easily find the tax-subsidy rate for all the groups that makes everyone as well-off as they were back in the autarky. Note that the tax-subsidy base must be the observable variable or the variable that is easily recalculated. Thus the nature of the tax-subsidy for each group will be:

(i) (Linear) factor (commodity) tax on the generic factors.

(ii) (Linear) profit tax on the occupation-rewards for the job-staying producers of output X.

(iii) (Nonlinear) profit tax on the occupation-rewards for the job-switching producers of output X.

(iv) (Nonlinear) profit subsidy on the occupation-rewards for the job-switching producers of output X.

(v) (Linear) profit subsidy on the occupation-rewards for the job-staying producers of output Y.

The linear factor tax for generic-factor owners is the same as the one we saw in the first best case. Now we would like to focus on the individual heterogeneity of talents. Based on the above categorization, let us denote the partitions of the ability vector space in a finer way:

1. $C_{X-X} \equiv \left\{ (\theta^j, \tau^j) \in \Theta : \tau^j < (p^A)^{\frac{2}{1-a}} \theta^j \right\}$

2. $H = C^H_{Y-X} \equiv \left\{(\theta^j, \tau^j) \in \Theta : p^{\frac{2}{1-a}}\theta > \tau^j > (p^A)^{\frac{2}{1-a}}\theta^j \text{ and } 1 > \frac{g(p^W)}{g(p^A)} \cdot \theta^j\right\}$

3. $M = C^M_{Y-X} \equiv \left\{(\theta^j, \tau^j) \in \Theta : p^{\frac{2}{1-a}}\theta > \tau^j > (p^A)^{\frac{2}{1-a}}\theta^j \text{ and } 1/(p^{\frac{2}{1-a}}) < \theta^j < \frac{g(p^A)}{g(p^W)}\right\}$

4. $L = C^L_{Y-X} \equiv \left\{(\theta^j, \tau^j) \in \Theta : p^{\frac{2}{1-a}}\theta > \tau^j > (p^A)^{\frac{2}{1-a}}\theta^j \text{ and } 0 < \theta^j < 1/(p^{\frac{2}{1-a}})\right\}$

5. $C_{Y-Y} \equiv \left\{(\theta^j, \tau^j) \in \Theta : p^{\frac{2}{1-a}}\theta < \tau^j\right\}$

The job-stayer groups C_{X-X} and C_{Y-Y} will face the same linear tax-subsidy scheme as we saw in the first-best case. Thus, our focus here will be on the groups of job-switchers, H, M and L, all of whom are currently producing the output X. Because the government cannot distinguish among those earn the same profit from their production of X, the policymaker must take from (give to) each individual as little tax (large subsidy) as the least gainer (worst loser) among those who earn the same profit. For a given profit-level, the least gainers are those who possess the largest latent ability to make Y product. For the group H and M, the least gainers (largest losers) are the individuals with $\overline{\tau(\theta^*)} = 1$. For the group L, they are $\overline{\tau(\theta^*)} = p^{\frac{2}{1-a}}\theta^*$.

Next, we must effectively check the optimal tax rate for those who have an ability vector $(\theta^*, 1)$ where $1 \geq \theta^* > 1/(p^{\frac{2}{1-a}})$, and the optimal tax rate for those with a vector $(\theta^*, p^{\frac{2}{1-a}}\theta^*)$ where $0 < \theta^* < 1/(p^{\frac{2}{1-a}})$. Thus, the individuals in the group H who earn $\pi(\theta^*)$ will have imposed upon them a tax rate of

$$t_H(\pi(\theta^*)) = \frac{g(p^W)\cdot\theta^* - g(p^A)}{g(p^W)\cdot\theta^*} - \delta(\theta^*),$$

while the individuals in group M who earn $\pi(\theta^*)$ will be given a subsidy at the rate of

$$s_M(\pi(\theta^*)) = \frac{g(p^W) - g(p^A)\cdot\theta^*}{g(p^W)\cdot\theta^*} + \delta(\theta^*),$$

where $\delta(\theta^*) > 0$ represents an arbitrary, very small number that has a property of $\delta'(\theta^*) > 0$. The purpose of this additional small term is to avoid breaching the condition $\varepsilon = \frac{\partial T/T}{\partial\pi/\pi} > -1$, arrived at Result 1 in the previous section. Without this term $\delta(\theta^*)$, the condition must inevitably become $\varepsilon = -1$. (For the formal proof, see the Appendix.) The group-L individuals will face the linear subsidy rate:

$$s_L = \frac{g(p^A)\cdot p^{\frac{2}{1-a}}\theta^* - g(p^W)\cdot\theta^*}{g(p^W)\cdot\theta^*} = \frac{g(p^A)\cdot p^{\frac{2}{1-a}} - g(p^W)}{g(p^W)}.$$

This completes the description of the tax-subsidy scheme for the first-stage equilibrium in the unit-square case.

7 An Anticipated Compensation Scheme

In the previous section, our compensation program was enacted after trade openings. The introduction of the program is assumed to have been a surprising (unpredicted) one. It may indeed be rather close to what actually occurred in the 1960s, and yet such an analysis still may not describe at all well the more recent situations. Once a compensation scheme is in place, the individual agents start taking its very existence into account. They change their behaviors simply because the existence of the program alters their incentives.[33] In this section we analyze what we shall call an *anticipated compensation scheme*.

We begin by looking at the situation in which individual agents expect the compensation program to exist, and behave accordingly. In the previous section, we saw some agents switch their occupations before they know whether there would be a compensation scheme. In this section we posit that some of the individual agents who had changed their jobs under that scenario [without compensation] may not switch their occupations if they expect a compensating subsidy that will be given only when they stay in their declining industry. This is inevitable, since any compensation scheme must specify the tax and subsidy rates not just for job-switchers but for job-stayers as well. When job-stayers stay in their own industry, policymakers cannot tell if they are the counter-factual job-switchers. Indeed, there would be no way for us to tell which agents among the job-stayers have changed their jobs, were it not for the compensation scheme. Noting this difficulty/complication, let us turn to the creation of an anticipated compensation scheme.

We adopt the same strategy as before. In the first-stage equilibrium, the policymaker will try to make agents as happy as or happier than they were back in the autarky situation.[34] We try to generate non-negative revenues for the government, which later the policymaker can redistribute back to all agents in the second stage. Let us first announce the following tax scheme for the producers of X under autarky.

1. For those who stay in X industry, there will be a linear tax rate of

$$t_{ant} = \frac{\pi^j_{X1} - \pi^j_{X0}}{\pi^j_{X1}} = \frac{p^{\frac{1}{1-a}}\left[s(p)\right]^{-a} - {p^A}^{\frac{1}{1-a}}\left[s(p^A)\right]^{-a}}{p^{\frac{1}{1-a}}\left[s(p)\right]^{-a}}.$$

   This tax-rate can make the job-stayers in X indifferent from the autarky situation.

2. For those who switch from X to Y industry, there will be a linear tax rate of

$$t^*_{ant} > \frac{\pi^j_{X1} - \pi^j_{Y0}}{\pi^j_{X1}} = \frac{p^{\frac{1}{1-a}}\left[s(p)\right]^{-a} - {p^A}^{\frac{-1}{1-a}}\left[s(p^A)\right]^{-a}}{p^{\frac{1}{1-a}}\left[s(p)\right]^{-a}}.$$

   In reality, there will be no job-switchers in this direction, given the change in terms-of-trade.

Thus, all the members of C_{X-X} will stay in X industry, and all must pay the amount of tax that makes them indifferent from the autarky situation. No one will switch from X to Y, since paying tax at the rate t^*_{ant} makes no sense.

Now, in order to make sure that those in group C_{Y-Y} are at least as well off as they were in the autarky situation, we announce the following subsidy scheme for the producers of Y in autarky.

[33] The argument here is analogous to the Friedman-Phelps hypothesis of the natural rate of unemployment. If policymakers try to take advantage of the Phillips curve by choosing higher inflation in order to reduce unemployment, they will succeed in reducing unemployment only temporarily. Several years of a high inflation rate will shift the augmented Phillips curve upward, because people's expected level of inflation rate at the natural rate of unemployment will also rise. Thus, policymakers must wait for a long time before they can take advantage of surprise inflation. By a similar logic, the policymaker cannot take advantage of an unanticipated compensation scheme for a long time.

[34] We may have to provide some positive surplus, for informational reasons.

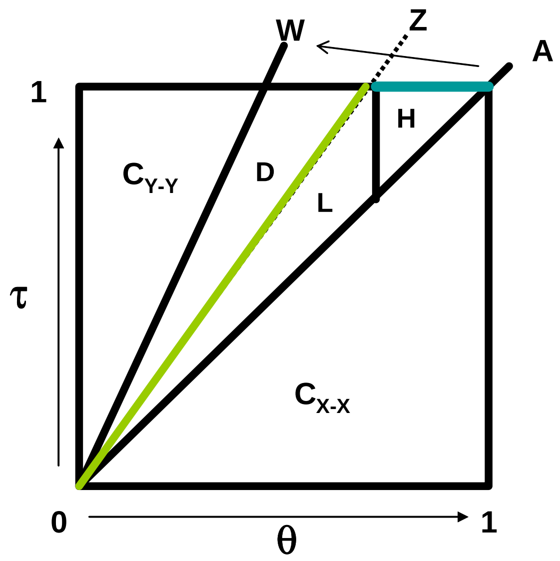
3. If any Y-producer in autarky chooses to stay in sector-Y-production after the opening up to trade, the government will provide him or her a positive subsidy – one that is proportional to his or her occupational return in Y production. The linear subsidy rate will be

$$s_{ant} = \frac{\pi^j_{Y0} - \pi^j_{Y1}}{\pi^j_{Y1}} = \frac{p^{A^{\frac{-1}{1-a}}} \left[s(p^A)\right]^{-a} - p^{\frac{-1}{1-a}} \left[s(p)\right]^{-a}}{p^{\frac{-1}{1-a}} \left[s(p)\right]^{-a}}.$$

This offer by the government will surely guarantee that no one is made worse off by the opening up to free trade, for the autarky producers of Y now have the option of staying in the same industry, with the same return as before.

The government is left to specify the tax-subsidy scheme for those who switch from sector Y to sector X – namely, the group C_{Y-X}. Now, in order to make our analysis a more concrete one, let us look at Fig. 10, which shows a case of unit-square support.

Figure 10: An ex ante compensation scheme with its partitions.



We now can divide the unit-square into five partitions. With the exception of the natural job-stayers – the groups C_{X-X} and C_{Y-Y} – there are three new groups among the counter-factual job-switchers: (1) D, comprising the individuals who were job-switchers, under free trade but who will stay in industry Y; (2) L, comprising those who were winning job-switchers under free trade but whose current profits are indistinguishable from those of the losing job-switchers; and (3) H, comprising those who were winning job-switchers under free trade and whose current profits must surely be larger than those of the losing job-switchers.

With respect to the group D, the government cannot do anything better than it did by implementing the above subsidy scheme, targeting industry-Y-stayers. As long as the latter decide to stay in sector Y, they are indistinguishable from all the other natural stayers in that sector. Therefore, let it be said that our tax scheme targets two groups above all: L and H. This entails the following:

4. Tax Exemption for group L. Those who are in this group are natural gainers from trade. Therefore, even given the subsidy for job-stayers in sector Y, the agents will find it profitable to switch their occupations, conditional on the tax-exemption in the new sector.

5. Tax the group H at the same rate as that used in the post-trade unanticipated scheme:

$$t_{ant}^{**}(\pi(\theta^*)) = \frac{\pi_{X1}^j - \pi_{Y0}^j \mid_{\tau=1}}{\pi_{X1}^j} = \frac{g(p^W)\cdot\theta^* - g(p^A)}{g(p^W)\cdot\theta^*} - \delta(\theta^*).$$

   Then, everyone except for those who have $\tau = 1$ will surely gain a positive rent. Thus, this tax rate is incentive-compatible for those who are in group H. The term $\delta(\theta^*)$ has the same property as it did in the previous section.

We can state that the scheme presented here satisfies all three conditions: informational feasibility, weak Pareto improvement, and being self-financing. It is informationally feasible, since all the tax and subsidy rates are incentive compatible. It is weakly Pareto improving, since every agent is at least as happy as he or she was under autarky. If there exist aggregate gains from trade, the tax revenues from this scheme will be larger than the costs of subsidy. It is likely that net government revenues that have been brought in by the job-staying individuals in both sectors X and Y would be positive. With respect to the job-switchers, who created a overcompensation problem in the unanticipated case, this scheme will either tax some of them or exempt some from tax; hence, the policymaker will be left with strictly positive tax revenue. Although there exist some positive rents, and hence overcompensation in the form of smaller taxes for the group H, this overcompensation will not negatively affect the government budget since it takes the form of a smaller-than-ideal tax rate.

Nevertheless, the allocation achieved in this scheme is not without its costs. The scheme attains three desirable properties – informational feasibility, weak Pareto improvement, and being self-financing – creates aggregate-level inefficiency, namely, the smaller aggregate consumption possibility set, evaluated at the world-price level. This smaller aggregate-gains arise out of the fact that there is a smaller number of job-switching individuals.

Proposition 7 *There exists an* ***anticipated (ex ante)*** *compensation program that is* ***informationally feasible, weakly Pareto improving,*** *and* ***self-financing****. The aggregate consumption possibilities set is smaller than the one under the unanticipated (ex post) case.*

Furthermore, when we look at the current TAA program, we find a striking result. Noting that our model does not have any frictional costs for occupation-switching, we propose taxing at a positive rate or at zero (tax exemption) those who switch occupations. This contradicts the results in Feenstra and Lewis (1994), which suggests a relocation subsidy for job-switchers. Our optimal scheme suggests that, to the contrary, the policymaker should give no subsidy to the job-switchers. We propose that the subsidy be given only to those job-stayers who choose to remain in the declining industry. Given the way we have set up our model to have no frictional moving (between-sectors) costs, we are not surprised to arrive at the following negative result about the current TAA, which provides a poll subsidy to occupation-switchers.

Proposition 8 *The poll subsidy for those who have changed industries has a disincentive problem. It induces an inefficient allocation of individuals.*

Given the setup of the model in this paper, the minimal subsidy for the job-switching individuals must be non-positive; i.e., it must contain a tax exemption for group L and a positive tax for group H. By giving a positive subsidy to the job-switching individuals, some of the job-stayers in sector Y (especially those individual agents who are closer to the gain-zero line OZ) may find it profitable to move to sector X. And yet, while this positive subsidy is successful in terms of inducing some counter-factual job-switchers to actually move to a more efficient sector (in the post-trade world), it also creates a huge side-effect. Because the policymaker cannot distinguish the counter-factual job-switchers from the natural (winning) job-switchers, a positive subsidy creates the overcompensation problem all over again, for the job-switchers who are on the same iso current-profit lines. It turns out that the policymaker must offer the same menu of tax-subsidy rates as that seen in the unanticipated post-trade compensation scheme, if the government is to observe the maximum number of job-switchers, and hence see the maximum aggregate production gains in the economy. With this subsidy, the same overcompensation problem, and the same ambiguity as to a violation of the scheme's self-financing, become problems.

When the policymaker's concern is in budget balancing, then some positive taxation on the job switching individuals may also be a policy option.[35] Positive tax on job switchers, if not too large, may still induce some natural job-switchers to change their occupations. Since we collect taxes from these job switchers, this policy will ease the budget problem while it may induce smaller number of individuals to switch to an efficient industry. There will be a larger number of individuals who will stay in declining industry. Thus, trade off between government budget and aggregate gains will still remain.

The preceding analysis has shown us, in the case of an anticipated compensation scheme where the government aims to attain a Pareto improvement from autarky, that there exists a tradeoff between size of aggregate production gains from trade and amount of overcompensation.

8 Conclusion

This paper has developed a model that attains aggregate production gains from trade. The model aims to depict a realistic situation which individual agents often actually find themselves in. It assumes that an individual agent must choose one job at a time, and that he is endowed with a multi-valued vector of talents in various sectors. The productivity of the agents is assumed to differ across the agents. This setup certainly creates gainers and losers from trade, but the amount of the gains and losses is based on the relative strengths of the agents' talents, between their actually-used ones and their unused-latent ones. If the government chooses to impose a realistic taxation-subsidy scheme on current factor-prices and profits, then policymakers must face up to an unavoidable tradeoff between Pareto improvement and overcompensation. In other words, if the

[35] I thank Professor Eiichi Miyagawa for pointing out the possibility of this type of policy .

policymakers do attain a Pareto improvement, the compensation scheme will necessarily be overcompensating the job-switching individuals. If, on the other hand, they rigorously avoid overcompensation because they care about a balanced-budget, their compensation program will not attain any Pareto improvement.

In addition to this tradeoff, it is the case that when a compensation scheme is anticipated by the individual agents, there emerges another tradeoff, this one being between overcompensation and size of aggregate production gains. Most policymakers are vaguely aware of all these tradeoffs, but there still haven't been many serious studies done on this issue. Thus this paper has taken as its appointed task the proposing of a theoretical framework that can explain the tradeoffs the governments face when trying to set up compensating redistribution schemes.

This paper also provides its readers with an explanation for the difficulty we all face in distinguishing winners from losers in the wake of an opening up to trade. Such identifications have been attempted successfully for such a basic trade model as that of Heckscher-Ohlin or specific-factors model. As for Feenstra and Lewis (1994), they noted their own difficulty of the identification, in their imperfectly mobile factors model, and set up as part of their investigation into heterogeneous adjustment costs. And while Feenstra and Lewis assumed positive adjustment costs for all of their imperfectly mobile factors, my model has found cases in which the adjustment costs for some agents among those who switch their occupations may become negative and hence, there are gainers. Thus, the poll subsidy for job-switching individuals (supported as a remedy by Feenstra and Lewis) may not be a good compensation policy under the setup of my model. Furthermore, any observation of current profits will not reflect the actual gains or losses from trade openings. This makes it highly difficult for any government to put in place a reliably Pareto-improving compensation scheme that bases the tax-subsidy on current variables.

This paper has provided its readers with a model of individuals' occupational-choices and welfare-changes when the economy faces a change in terms of trade, and especially, one from autarky to free trade. We have found that there exist both winners and losers among the job-switchers. And yet, although this paper's analysis can explain individuals' long-run gains and losses from moving to a new sector, the model does not take into account the short-run costs arising out of the labor adjustment process. (We have implicitly assumed that frictional unemployment costs are zero.) Therefore the paper's chief theoretical result – no positive subsidy for job-switching individuals, in a self-financing compensation scheme – should not and must not be taken too literally. Indeed, the actual government compensation provided by the United States Department of Labor through its trade adjustment assistance (TAA) program involves a relocation subsidy for those who move to a new location when job-switching owing to trade openings. Such a program may be justified, to the extent that there exist short-run frictional costs associated with job-switching.

One of the simplifying assumptions of the paper is that occupational talents are exogenously given for each individual. In reality, people may invest much of their time in expanding their skills. I have left out the possibility of such dynamic development of individual talent via a human-capital investment. Grossman and Shapiro (1982) looked at the determinants of individual talent-training, when the individual agents are identical ex ante. An interesting extension of this paper's model would bring a greater richness to a dynamic formation of specific factors, by allowing for investment in individual occupational talents. Surely this is one of the most promising areas for future research.

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A Proofs

Proof of Lemma 1. We know from (13) that

$$\left[p^{\frac{1}{1-a}} \cdot V_\theta^R(p) + p^{-\frac{1}{1-a}} \cdot V_\tau^S(p)\right] = K \cdot \left(\frac{r(p)}{a}\right)^{\frac{1}{1-a}}.$$

Then, by plugging (4) into (14) we get

$$I(p) = r(p) \cdot K + \left(\frac{1}{r(p)}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right) \left[p^{\frac{1}{1-a}} \cdot V_\theta^R(p) + p^{-\frac{1}{1-a}} \cdot V_\tau^S(p)\right].$$

Combining the above two equations yields us

$$I(p) = r(p) \cdot K \cdot \left(1 + \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right) \cdot a^{\frac{-1}{1-a}}\right).$$

By simplifying this we get

$$I(p) = \frac{r(p) \cdot K}{a},$$

which is precisely equivalent to the condition seen in (15). ■

Proof of Lemma 2. Let us first look at equation (13). Since we know that $a \cdot K^{-(1-a)} > 0$ regardless of the value of p, we would like to evaluate a derivative of

$$\left[p^{\frac{1}{1-a}} \cdot V_\theta^R(p) + p^{-\frac{1}{1-a}} \cdot V_\tau^S(p)\right]^{1-a} \tag{42}$$

with respect to p. Let $s(p) \equiv p^{\frac{1}{1-a}} \cdot V_\theta^R(p) + p^{-\frac{1}{1-a}} \cdot V_\tau^S(p)$. The derivative of equation (42) can then be expressed as

$$(1-a)\,[s(p)]^{-a}\,\frac{ds(p)}{dp}.$$

Since $(1-a)\,[s(p)]^{-a} > 0$, we need to check the signs of $s'(p) = \frac{ds(p)}{dp}$:

$$s'(p) = \frac{1}{p(1-a)} \cdot \left[p^{\frac{1}{1-a}} \cdot V_\theta^R(p) - p^{\frac{-1}{1-a}} \cdot V_\tau^S(p)\right] + \left[p^{\frac{1}{1-a}} \cdot \frac{dV_\theta^R}{dp} + p^{\frac{-1}{1-a}} \cdot \frac{dV_\tau^S}{dp}\right]. \tag{43}$$

We know from autarky condition (20) that $s'(p) = 0$ when $p = p^A$. By utilizing the conditions (21) and (22), and by noting that the second term in (43) is very small compared to the first term, we also can conclude that $s'(p) < 0$ when $p > p^A$ and that $s'(p) > 0$ when $p < p^A$. This concludes the proof. ■

Proof of Proposition 2. We express the occupational return for X producers:

$$\pi_X^j(p, r, \theta^j) = \left[p^{\frac{1}{1-a}} \left(\frac{1}{r(p)}\right)^{\frac{a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right)\right] \cdot \theta^j \tag{44}$$

By plugging equation (13) into (44), we obtain an expression of occupational reward in terms of output price:

$$\begin{aligned}\pi_X^j(p, \theta^j) &= \left[a^{\frac{-a}{1-a}} \left(a^{\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right) K^a\right] \cdot p^{\frac{1}{1-a}} \cdot \left[p^{\frac{1}{1-a}} \cdot V_\theta^R(p) + p^{-\frac{1}{1-a}} \cdot V_\tau^S(p)\right]^{-a} \cdot \theta^j \\ &= K^a(1-a) \cdot p^{\frac{1}{1-a}}\,[s(p)]^{-a} \cdot \theta^j.\end{aligned}$$

Since the constant term $K^a(1-a)$ is positive and θ^j is nonnegative by assumption, the derivative of $p^{\frac{1}{1-a}}\,[s(p)]^{-a}$ has the same sign as the derivative of $\pi_X^j(p, \theta^j)$ with respect to p. Therefore, showing that

$$\frac{d\left(p^{\frac{1}{1-a}}\,[s(p)]^{-a}\right)}{dp} > 0 \tag{45}$$

is equivalent to carrying over the truth of the above proposition to the case of the job-stayers in sector X:

$$\frac{d\left(p^{\frac{1}{1-a}}\,[s(p)]^{-a}\right)}{dp} = s^{-a} \cdot p^{\frac{a}{1-a}} \cdot a \cdot \left(\frac{1}{a(1-a)} - \frac{p \cdot s'(p)}{s(p)}\right).$$

Because we know that $0 < a < 1$ and that $p > 0$, it is clear that

$$s^{-a} \cdot p^{\frac{a}{1-a}} \cdot a > 0 \text{ and } \frac{1}{a(1-a)} > 0.$$

And because we know, from $p > p^A$, that $s'(p) < 0$, we can conclude that

$$\left(\frac{1}{a(1-a)} - \frac{p \cdot s'(p)}{s(p)}\right) > 0.$$

In this way, we have shown that (45) holds. A similar analysis could easily be carried out of the occupational rewards for Y, and hence the proof is omitted. ■

B A Profit Tax System

Let us assume that the production function is

$$x = X(k, \theta), \tag{46}$$

where x is a quantity of output, k is the amount of generic factor employed by the firm, and θ is the specific occupational factor that is indivisible and embodied in the individual agent. Let $X(k, \theta)$ be increasing in both arguments, strictly concave, infinitely continuously differentiable, and with constant returns to scale.

Let p be the output price of x. Let r be the market price for the generic factor k. The agent's profit maximization program will then be written

$$\max_k \pi(k, \theta; p, r) = p \cdot X(k, \theta) - r \cdot k. \tag{47}$$

Note that the choice variable for the agent is k only, because θ is embodied and indivisible. The regular first-order condition is written

$$\frac{\partial \pi}{\partial k} = 0 \Longleftrightarrow p \cdot \frac{\partial X}{\partial k} = r. \tag{48}$$

Strict concavity of the production function $X(\cdot, \cdot)$ guarantees that the second-order condition for the regular problem (47) holds with strict inequality:

$$\frac{\partial^2 \pi}{\partial k^2} < 0. \tag{49}$$

Now, consider the profit-tax system on the profit of the agent, given equation (47). If the ad valorem tax rate is t, then the profit-maximization program is written as

$$\max_k (1 - t) \{p \cdot X(k, \theta) - r \cdot k\}. \tag{50}$$

When t does not depend on k or θ, the profit-maximization problem faced by an individual is unchanged. Hence, the first-order condition will be the same as (48).

B.1 Tax Rate Proportional to Profit

Now let $1 - t = T(\pi)$ be the profit-tax schedule. The rate of tax depends on the observed profit of the individual. The program is now written

$$\max_k \{T(\pi) \cdot \pi\} = T(\pi) \{p \cdot X(k, \theta) - r \cdot k\}. \tag{51}$$

The first-order condition for (51) will be

$$\frac{\partial T}{\partial \pi} \cdot \frac{\partial \pi}{\partial k} \cdot \pi + T \cdot \frac{\partial \pi}{\partial k} = \frac{\partial \pi}{\partial k} \cdot \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} = 0. \tag{52}$$

Condition (52) implies that $\frac{\partial \pi}{\partial k} = 0$, except for the case where

$$\frac{\partial T}{\partial \pi} \cdot \pi + T = T \left(1 + \frac{\partial T}{\partial \pi} \cdot \frac{\pi}{T} \right) = T (1 + \varepsilon) = 0,$$

with $\varepsilon \equiv \frac{\partial T/T}{\partial \pi/\pi}$ being an elasticity of the tax rate with respect to profit. Thus we find that, unless $\varepsilon = -1$, the first-order condition (52) implies the same condition as (48).

The second-order condition for the profit-maximization will be

$$\frac{\partial^2 \pi}{\partial k^2} \cdot \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} + \frac{\partial \pi}{\partial k} \cdot \frac{\partial}{\partial k} \left\{ \frac{\partial T}{\partial \pi} \cdot \pi + T \right\} \equiv SOC < 0. \tag{53}$$

The second term of SOC will be

$$\frac{\partial \pi}{\partial k} \cdot \left\{ \frac{\partial^2 T}{\partial \pi^2} \cdot \frac{\partial \pi}{\partial k} \cdot \pi + 2 \left(\frac{\partial T}{\partial \pi} \cdot \frac{\partial \pi}{\partial k} \right) \right\}.$$

This is evaluated around the optimum point, where $\frac{\partial \pi}{\partial k} = 0$. Thus, given (49), we see that the relevant condition for the program's second-order condition will be

$$\frac{\partial T}{\partial \pi} \cdot \pi + T = T\left(1 + \varepsilon\right) > 0.$$

And since we know that $T > 0$, the condition also can be shown as

$$\varepsilon = \frac{\partial T/T}{\partial \pi/\pi} > -1. \tag{54}$$

So, unless the profit-tax rate decreases by more than 1% as the profit simultaneously increases by 1%, the agent will maximize the profit even after the tax has been imposed on the profit.

B.2 Tax Rate Proportional to Output

Now let $1 - t = T(x)$ be a new profit-tax schedule. The rate of tax depends on the observed output of the individual. The program is now written

$$\max_k \{T(x) \cdot \pi\} = T(x) \{p \cdot X(k, \theta) - r \cdot k\}. \tag{55}$$

The first-order condition is now written

$$\frac{\partial T}{\partial x} \cdot \frac{\partial X}{\partial k} \cdot \pi + T \cdot \left\{ p \cdot \frac{\partial X}{\partial k} - r \right\} = \frac{\partial X}{\partial k} \cdot \left\{ \frac{\partial T}{\partial x} \cdot \pi + pT \right\} - rT = 0. \tag{56}$$

Note that the optimal level of k is smaller than the no tax case (47), because

$$\frac{\partial T}{\partial x} \cdot \frac{\partial X}{\partial k} \cdot \{p \cdot X(k, \theta) - r \cdot k\} < 0,$$

together with $r > 0$ and $T > 0$ implies that

$$\left\{ p \cdot \frac{\partial X}{\partial k} - r \right\} > 0.$$

Thus, the profit tax system that is based on observed output will inevitably be distortionary.